

Seasonal Customer Demand and Hedging in the Nordic Electricity Markets

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Preface

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Helsinki, September 7 2010

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Abstract
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PURPOSE OF THE STUDY

This thesis studies hedging seasonal customer demand in electricity retail business and the main objective is to estimate the cost of updating the long-term hedges with shorter-term hedges for different seasonal demand. This thesis also tests optimal time to update hedges by comparing the costs of updating the hedges at the beginning, end and in the middle of the time period when the contracts are available.

DATA AND METHODOLOGY

The data consist of Nord Pool daily closing price data from 2006 to 2009. Customer demand data is obtained from Fortum. The seasonality is modelled by a method developed by Borovkova and Geman (2006), which looks at the relationships between electricity forward prices. Seasonality is determined as the difference between monthly price and reference yearly average price. In this way the reference price does not contain seasonality. We further used the method Borovkova and Geman (2006b) developed for electricity forwards, and we use it to model the deviations of the price curve from flat de-seasoned curve, by using principal components. We then simulated possible future states of forward prices using them.

We made some adjustments to the Borovkova and Geman's methods, as we used Nord Pool data, which has different forwards available than other markets.

We also constructed an electricity forward curve based on de-seasoned prices calculated with Borovkova and Geman's (2006) method. The price curve and simulated cost of update can be combined to give a monthly electricity price for customer that includes both the latest market price information and the expected cost of updating the hedges later.

RESULTS

This thesis uses methods developed by Borovkova and Geman (2006 and 2006b) and finds support for using their methods in practice and also that it is possible to adjust them to work with Nord Pool data. The simulations gave reasonable results as, when comparing the costs of updating the long-term hedges, the demand with higher seasonality gets higher costs.

For different update timing the average costs are on same level for early, middle and late update, however, the standard deviation of the update cost is higher the later the update is done, which is consistent with Samuelson's effect of volatilities decreasing when time to maturity increases.

KEYWORDS

Commodity hedging, principal components analysis, seasonality, Borovkova and Geman

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List of Symbols

A	matrix
v	eigenvector
F	forward price
\bar{F}	average of forward price
P	de-seasoned forward price
S	spot price
T	maturity date
X	independent variable
Y	dependent variable
d	cash payout ratio
i	holding period
r	interest rate
s	seasonal component in forward curve
t	time
u	error term
v	storage cost
α	regression parameter
β	regression parameter
γ	convenience yield
δ	speed of mean reversion
λ	eigenvalue
μ	mean
σ	standard deviation

List of Acronyms

AS	Limited company (Aksjeselskap in Norwegian)
ASA	Public company (Allmennaksjeselskap in Norwegian)
CfD	Contract for difference
CME	Chicago Mercantile Exchange
CVaR	Conditional value-at-risk
EUR	Euro (currency)
GARCH	Generalized autoregressive conditionally heteroscedastic
HDD	Heating degree day
NOK	Norwegian krone
OLS	Ordinary least squares
PC	Principal component
PCA	Principal components analysis
VaR	Value-at-risk

1. Introduction

Cold winter days like the ones we experienced in the Nordic countries during winter 2009 to 2010, combined with low capacity of supply can cause extreme spikes in electricity prices. One price spike can destroy the whole year's profits or even cause bankruptcy if the electricity retailer leaves her position unhedged. Therefore it is important to understand the risk exposure and hedge against it. The subject is challenging and different kinds of problems in hedging have indeed caused bankruptcies for example in the recent energy crisis in US.

Electricity price behavior is very difficult to predict and the volatilities in prices are extremely high. The price is determined as a function of supply and demand, which are both very inelastic. Smoothing consumption would be beneficial to the whole system, as less peak capacity would be required and also peak prices would be lower when current production capacity would better meet the demand. When the risk exposure for the electricity retailer, caused by the different demand of different customer types, is quantified and measured, it can be used for basis in pricing and thus smooth the consumption by incentives to drive consumption away from peak demand.

1.1. Objective of the thesis

The energy retailer uses forwards to hedge its sales. There are always two parties involved in each forward contract. The buyer of the forward has a long position and the seller has a short position. This means the buyer benefits if the underlying increases in value and the seller benefits when the underlying decreases in value. To help us understanding the context in the case of electricity forwards, we could think that the two parties of the contract make now a deal for a future time period, in which they fix the price of electricity exchanged in some future time period. In the future when we enter the delivery period, the buyer of the forward is then consuming the electricity and the seller of the forward is delivering it, however, the actual contracts do not involve any delivery of the physical asset, electricity. The actual contract works so that in the settlement of the contract it pays the difference between the agreed price and the realized market price. In this way the effect for the two parties of the contract is quite the same

had they actually also agreed to deliver the electricity as well. As electricity cannot be stored the underlying, electricity, is delivered with constant flow, which is typically measured on hourly level. If electricity price would increase, then the party being long would benefit. When signing a contract for delivering electricity to its customer, the electricity retailer enters automatically to a short position as she offers a fixed price to the customer and buys the electricity herself from the market. This position can be neutralized by buying electricity forward contracts and thus fix also the price electricity is bought with.

In this study we do not look at speculative use of derivatives, instead the hedges are made on a so called energy neutral principle, meaning the total amount of energy is fully hedged. This means that hedging is done by buying a yearly forward for the average demand. We do this because the forwards in the Nordic electricity marketplace, Nord Pool, are made for constant demand. This leaves us with basis risk i.e. a risk that cannot be hedged away. For simplicity let us think again in the framework that the buyer of the forward consumes electricity and the seller produces it, although in reality no physical delivery of electricity takes place. Now, if we have for example a quarterly forward for quarter one for 10 MW, it means that we have agreed a fixed price in advance for a delivery of 10 MW for each hour in that quarter. The buyer of the contract cannot consume different amounts of electricity at different times. It is not possible to use for example 15 MWh / hour in January and 5 MWh / hour in February and 10 MWh / hour in March. The buyer has to consume 10 MWh / hour in every hour and the seller produce the same amount constantly. The challenge is that it is seldom the case that we actually have a constant demand; normally the demand varies especially between summer and winter.

The real challenge in long-term hedging is that in Nord Pool the long-term forwards are available only for calendar years, and forwards for delivery periods in calendar months and quarters become available only closer to the delivery period. As the demand is seasonal, buying the yearly forward leaves part of the exposure unhedged and part of the exposure overhedged. The hedge needs to be balanced closer to delivery of electricity by buying and selling quarterly and monthly forwards when they become available. For storable commodities this would not be a problem, as one could choose the time point when to consume, but for electricity the supply and demand are constantly in balance and the time of consumption plays an important role in hedging.

This thesis studies hedging seasonal customer demand in electricity retail business and the main objective is to answer the questions, what is the cost of updating the long-term hedges to shorter-term hedges for different seasonal demand. The thesis also tests what is the optimal time to update hedges by comparing the costs of updating the hedges early, late and in the middle of the time period when the contracts are available. Figure 1 illustrates the point of the hedging updates needed. In the example the original hedge is a yearly forward for 2012 assuming constant demand. As yearly forwards are available for 5 years and quarterly forwards only for 2 to 3 years we can only use the yearly forwards when making long-term hedging. Later when the quarterly forwards become available, we can then update the difference compared to original hedge using the quarterly forwards. Finally we will also update the hedges from quarterly level to monthly level as monthly forwards become available. This is the end point of our interest, as in this study we are only looking at updating the hedges to monthly level; we are not focusing on the settlement phase of the contracts which is done using spot price.

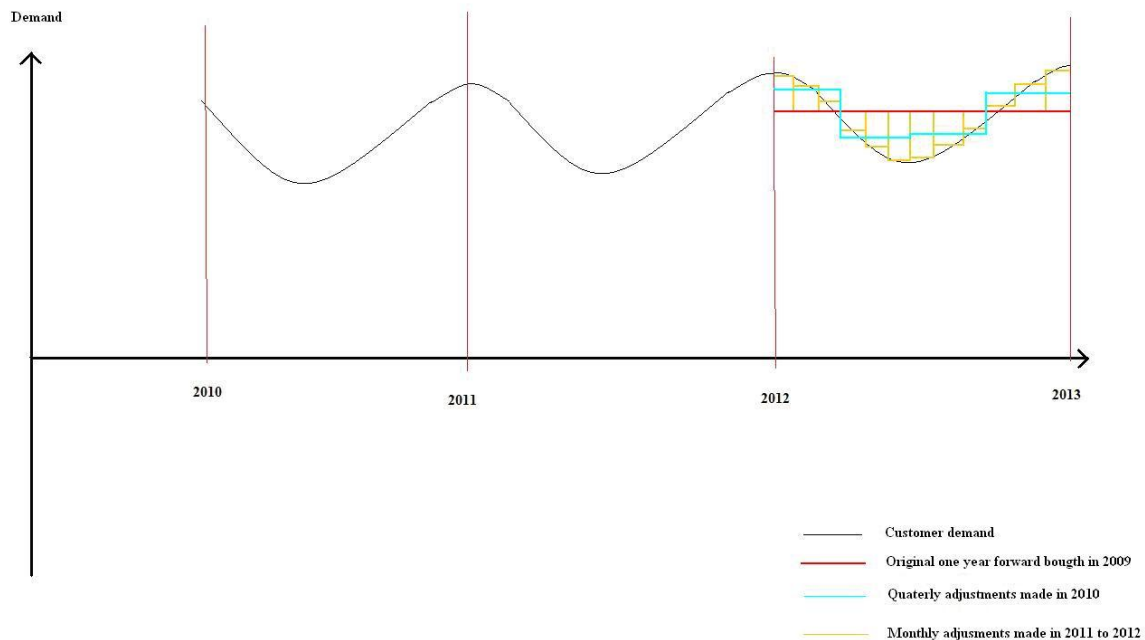


Figure 1 Idea behind the updates of hedges. The original hedge is made in year 2009 for 2012 and it is later updated with quarterly and monthly forwards as they become available.

1.2. Contribution to existing literature

There is a lot of research in electricity markets on short-term hedging using futures, but less can be found on long-term hedging using forwards. In addition there are a number of studies about valuation of derivatives in electricity markets, but less on how they should be used in practice. Usually futures and forward prices of electricity are modelled based on expected spot price¹. This approach is suitable for short term hedging, but the relationship between spot and forwards vanishes after a few weeks (Malo 2009) and for example in Nord Pool the correlation between spot and nearby futures was found to be in the range of 0.65 and -0.15 (Borovkova and Geman 2006b). Thus using a different approach, that looks at the forward prices separately from spot prices, is used in this study to model the behaviour of forwards with longer maturity.

This thesis uses an approach developed by Borovkova and Geman (2006) to model the commodity forward prices on a seasonal forward curve based on deterministic seasonal forward premium. Their approach is modified to our needs and used to tackle the issue of estimating the risk in seasonal demand for long-term contracts, given the agreed delivery to customers. Different hedging timing is also compared with simulation, to see which fits best the risk management need of long term contracts. Simulation is also used to get risk estimates for the update costs, in this way we look at historical forward prices as only one snapshot, or in other words, one possibility of the outcome of forward prices. We assume that the prices will behave similarly as they did in the past, but with simulation we get a family of possible outcomes for the prices. In the simulation we repeat the price behaviour process many times. So instead of basing our calculations on only one history, we now get as many (simulated) “histories” as we wish. The fundamentals for simulations come still from the history, as all parameters in the models are calibrated based on historical values.

The original Borovkova and Geman’s (2006) method was made for commodity markets, which have monthly forwards available for relative long time period. In Nord Pool we have less than needed monthly forwards available for the model to work in the way planned originally. Thus this thesis contributes in commodity derivatives literature not only by testing Borovkova and

¹ Look for example Bessembinder and Lemmon (2002), Lucia and Swchwartz (2002) and Malo (2009)

Geman's (2006) model in practice, but also by looking what modifications are needed for it to work with Nord Pool data. In addition this thesis gives answer to the question when should the electricity retailer update her hedging portfolio on long term contracts as shorter maturity contracts become available, a question still remaining unanswered.

1.3. Limitations of the study

This thesis looks at using of electricity derivatives for purely hedging purposes, thus the point is not to look for optimal speculative use of derivatives. In addition, we focus only on electricity markets and as electricity markets differ a lot from other commodities markets, caution should be used when generalizing the methods or results to other markets.

This thesis does not look at volumetric risk which arises from the mismatch of both prices and loads from their expected values. Those interested in volumetric risk in Nord Pool are advised to look at a previously study by Laitasalo (2004). In this study the scope is on long-term and we assume that the monthly demand is known and that the intra-monthly demand is constant for all customer types. Because of this assumption, we have no need to look at spot price behaviour, as we are already fully hedged, when we have bought the monthly forward. This thesis does not look either at the balancing of the derivatives contracts that takes place close to delivery and thus spot price behaviour is left out of the scope of this study.

This work is not an econometrics study and it does not aim to develop a new econometric model. The goal is to model the electricity prices with enough accuracy but still keep the model understandable. The idea behind choosing the model is that it is better to be approximately correct than accurately wrong. Thus simplifying assumptions will be made.

1.4. Structure of the study

This thesis is divided to seven Chapters. Next Chapter describes the research problem in more details. As electricity markets differ a lot compared to other markets it is important to understand the properties of it, thus the third Chapter looks at the properties of the Nordic Electricity market, Nord Pool. The following chapter focuses on the theoretical background of the study discussing hedging in electricity markets.

In the fifth Chapter we describe the data received from Nord Pool and Fortum and also describe the methods used in the study. Next Chapter looks at the results while the last Chapter summarizes and concludes.

2. Research Problem

This Chapter presents the research problem. There are two research questions in this study which relates to long-term hedging of electricity retailers contract portfolio. The questions consider the risk in different customer demand and when to update hedges.

2.1. How do different customer demands affect the cost of hedging updates?

The first research question relates to the risk in seasonal customer demand and the differences in the demand of different customer types. The first question is

How do different customer demands affect the cost of hedging updates?

In this study we estimate the expected cost of updating the hedges as well as quantify the risk in the costs for customers having different demands. When one customer or customer group, like for example direct electricity heater, uses a lot of electricity in the peak load, some other customer, like for example a Google's server farm, can have quite the opposite demand profile and actually reduce the risk. Customers using direct electricity heating need electricity the most, when temperature is lowest. Low temperature drives the total demand up, which usually also means the prices are on high level. Thus direct electricity heaters are expected to have a high risk contribution. On the other hand Google's servers need electricity for cooling and their demand is highest on summer time when the outside temperature is highest making their risk contribution low.

By answering the first question we can estimate the costs of updating the hedges from long-term to shorter-term, for different customer types, based on the seasonality in their demand. This cost is then used for pricing long-term contracts for different customers.

2.2. When should the hedges be updated?

Second question relates to the point in time when the yearly forward is updated with quarterly forwards and the quarterly forwards is updated with monthly ones. Samuelson's effect (Samuelson 1965) claims that volatility in prices increases when the exercise comes closer. This study tries to find out if this phenomenon also affects Nordic Power markets and if it has any influence on the optimal time point when to update the hedges.

The optimal time point for updating the long-term forwards with shorter maturity forwards is studied by comparing early, middle and late update. Early update would mean the yearly hedge is updated soon after the shorter maturity products are available. Late update means the update is delayed to close to delivery period. Middle update is in between them. The second question is

What is the optimal time point to make the adjustments in long-term hedges?

By answering to the second question, we get a guideline for timing the updates of hedges.

3. Nordic electricity markets overview

This Chapter discusses the nature of electricity. Lack of storability, price spikes, seasonality and mean-reversion are explained. We also look at the properties of the Nordic power market Nord Pool and the Chapter ends by discussing the efficiency of the Nordic power markets.

3.1. Nature of electricity

Electricity is a special kind of commodity, because unlike other commodities it is practically impossible to store electricity. In fact, it has to be produced and consumed at the same time. Thus it is not possible to transport electricity in traditional way, but instead it is transferred in real time using power systems. This limits arbitrage opportunities in electricity financial markets.

3.1.1. Characteristics of electricity prices

Electricity markets have three well know properties, which makes them different from other markets. Electricity price face extreme spikes, they have seasonal behaviour and are mean-reverted. (Malo 2009)

Price spikes are caused by the fact that storing electricity is not possible, at least in economical sense. Unlike with other commodities, inventories of electricity can not be used to smooth the gaps between supply and demand, which could lead in extremely high prices as can be seen in Figure 2 for Nord Pool spot price. For example in the end of 2002 we experienced a huge price spike upwards when supply could not meet the demand.

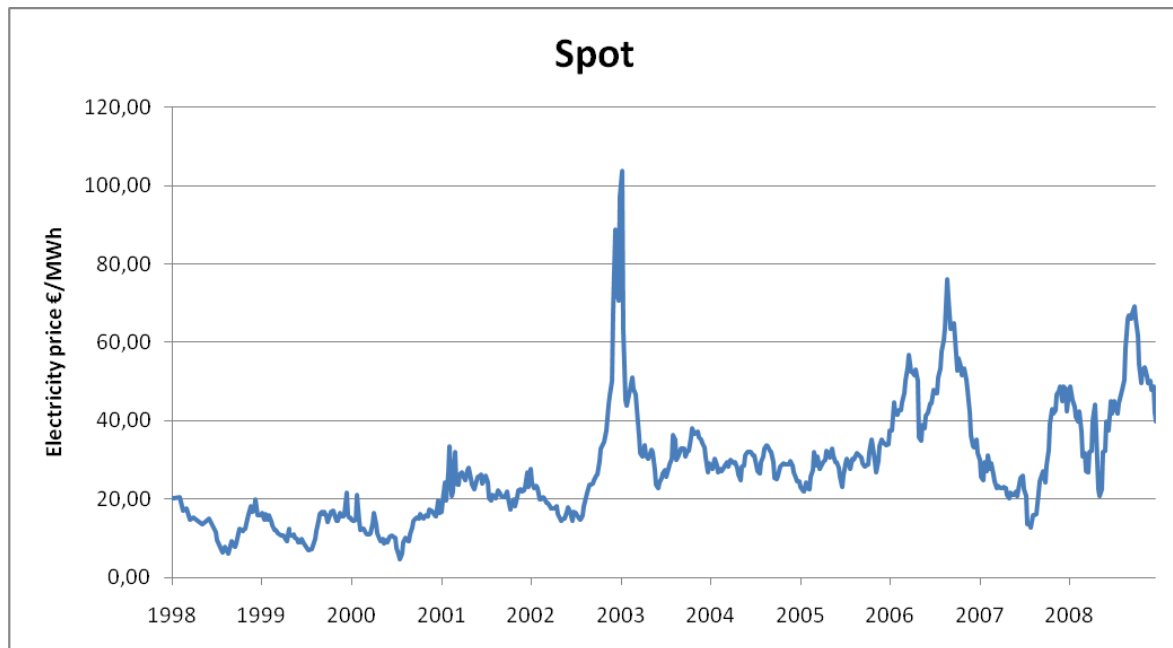


Figure 2 Electricity spot price in Nord Pool. Source for data: Nord pool.

The prices have also seasonal behaviour, meaning they follow the calendar. This is true especially in countries which have varying temperatures around the year. In Nord Pool temperature affects the demand and water reservoirs the supply. They both have a yearly cycle which drives the prices. There is also intra-day “seasonality” in the spot price as the demand is highest on the working hours and lowest at nights, however, this is out of the scope of this study as we focus on long-term effects and do not look closer to intra-day or intra-month prices.

It is also commonly known fact² that electricity prices are mean-reverted meaning the prices can have high volatilities but they have a tendency to drift towards their long-term mean value. On short term the spikes are usually caused by some disturbance in supply and after the disturbance is over, or an alternative solution is found, the prices revert. If prices would stay for a high level for a long time it would attract new investments which would eventually drive prices back down when the supply would increase. Thus high price spikes can occur and last for a time but the changes are not permanent. On the other hand if prices would be on low level for long time investments opportunities would be weak and as demand has a rising trend the prices would eventually revert towards its long-term mean value.

² Look for example Borovkova and Geman (2006) and Malo (2009)

These properties can be found both in spot and forward prices, however, as noted among others by Borovkova and Geman (2006b) the forward prices do not experience extreme price spikes. They experience smaller spikes and have the other two properties as well.

3.2. Nord Pool

Nord Pool is a voluntary marketplace for the wholesale electricity used in the Nordic countries (Finland, Sweden, Norway and Denmark). The physical markets in Nord Pool account for 70 % of the value of power used in these countries. It was established in 1993, two years after the Norwegian parliament deregulated power markets in Norway. Sweden joined in 1996, Finland in 1998 and Denmark in 1999-2000, making Nord Pool nowadays the largest and most liquid marketplace for physical and financial power contracts in Europe (Nord Pool 2010).

Nord Pool has divided power trading to two marketplaces: Nord Pool ASA and Nord Pool AS. Nord Pool ASA is the largest marketplace in the world for financial power contracts while Nord Pool AS handles contracts for physical delivery (Nord Pool 2010). The electricity price in Nordic countries is set in Nord Pool AS day ahead for the following day for each hour of delivery. It is the equilibrium price when supply curve equals demand. Supply curve gives the combined production of the producers for a given price of electricity while demand is the combined demand of the users of electricity.

There is one market or system (spot) price for all electricity traded, regardless of how it is produced. Electricity is a homogenous commodity and it can not be said ex post how it was produced, because the electricity gets mixed in the transmission grid. The spot price in the wholesale market is determined by the marginal cost of the most expensive production form. The Figure 3 below illustrates the different marginal cost for production types which represents the supply curve. A small change in production or demand could cause huge changes in prices especially if we are close to capacity limits. The Figure also illustrates the effect of increase in demand. The supply curve is illustrated in blue colour and the demand in red colour. In this example a small increase in demand results in huge increase in price.

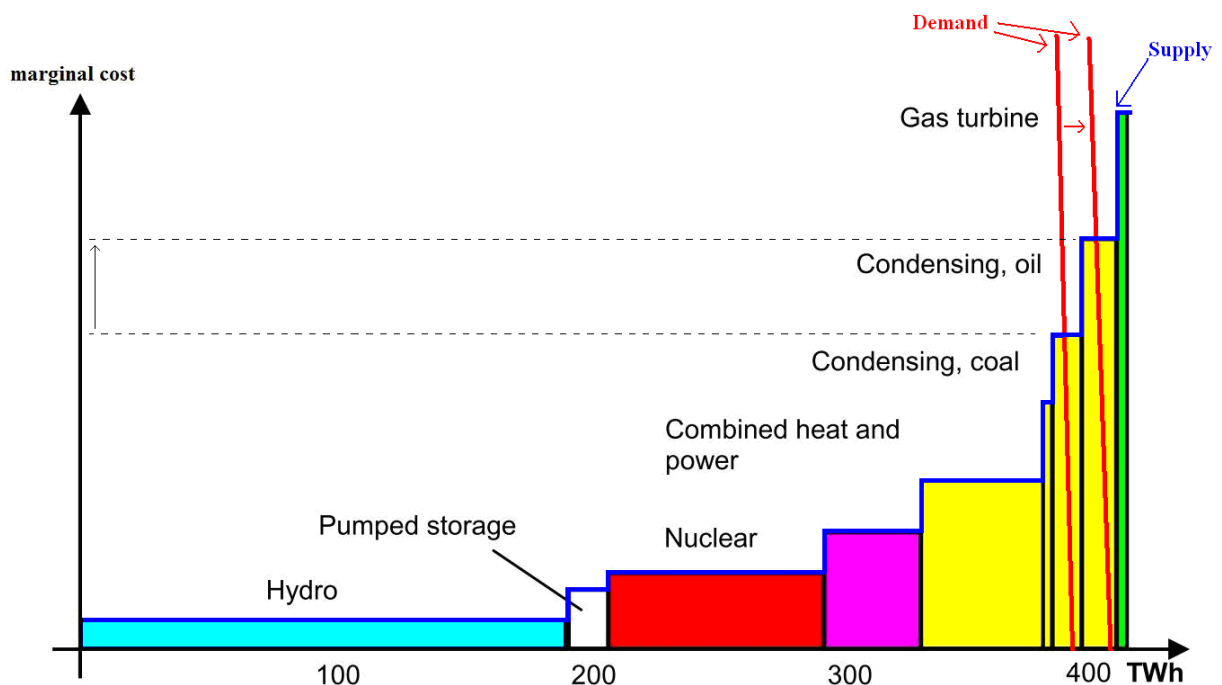


Figure 3 Schematic of marginal costs for different power plant types (Liski 2006)

3.2.1. Forwards and futures contracts in Nord Pool

As mentioned previously, electricity retailer can use forwards and futures to hedge her position. This study looks at the products traded on Nord Pool ASA, where the members can trade derivatives contracts. Nord Pool ASA is the counter party of all the contracts thus effectively eliminating the counterparty risk. The offered contracts include: (Nord Pool 2010)

- daily futures
- weekly futures
- monthly forwards
- quarterly forwards
- yearly forwards
- contracts for difference

There is some overlapping in the contracts as for example quarterly contracts are offered for the next 8 to 11 quarters (until the end of the third year from current point in time), thus one can make a yearly hedge for the next year using either quarterly or yearly forwards, as shown in Figure 4. However, this overlapping ends with higher maturities and one using long-term

hedging has to use yearly contracts as shorter maturity contracts comes available only closer to maturity.

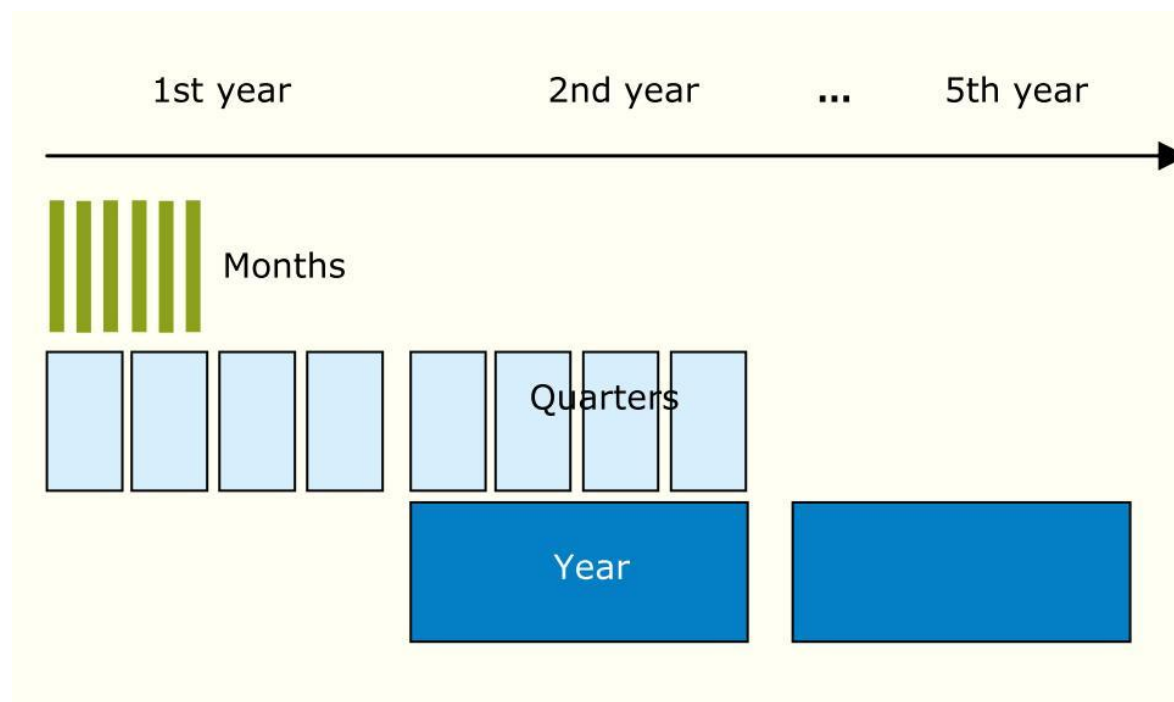


Figure 4 Forward contracts offered in Nord Pool. The shorter maturity forwards are available for shorter period of time. (Nord Pool 2008)

In Nord Pool the daily and weekly products are called futures and longer products are called forwards. The difference between futures and forwards traded at Nord Pool is in the settlement of the contracts. While futures are settled daily and cash is exchanged, forwards are settled only at the maturity of the contracts. Figure 5 and Figure 6 illustrate the difference. The difference is before the delivery starts, which is shown on the left part of the figures inside the red circles. As can be seen from the Figure 5 of futures settlement, futures use mark-to-market settlement, which means the changes in the futures prices are settled daily and at the beginning of the day the value of the futures contract for both parts is zero. Thus the value can change from zero during the day but it is balanced at the day end. For forwards (Figure 6) there is no daily settlement, but instead there is pending settlement and the changes in prices are cumulated and paid at the final settlement.

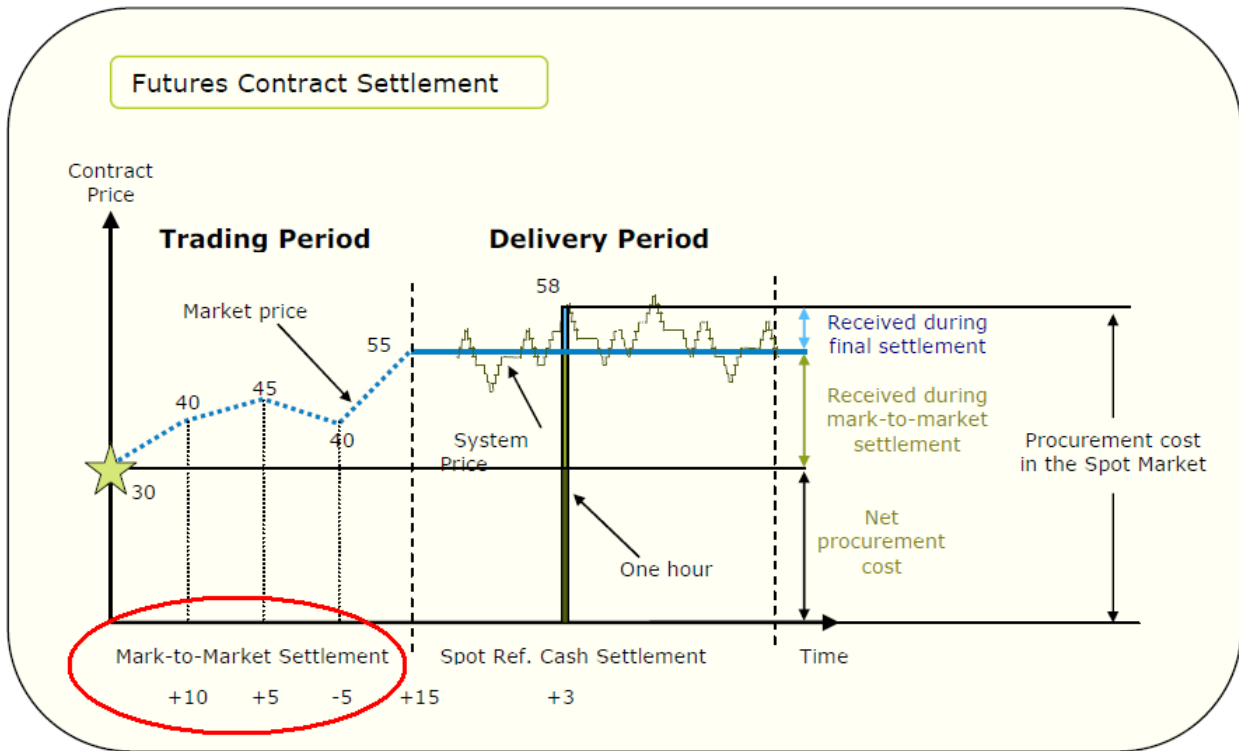


Figure 5 Mark-to-market settlement of futures contract in Nord Pool. (Nord Pool 2008)

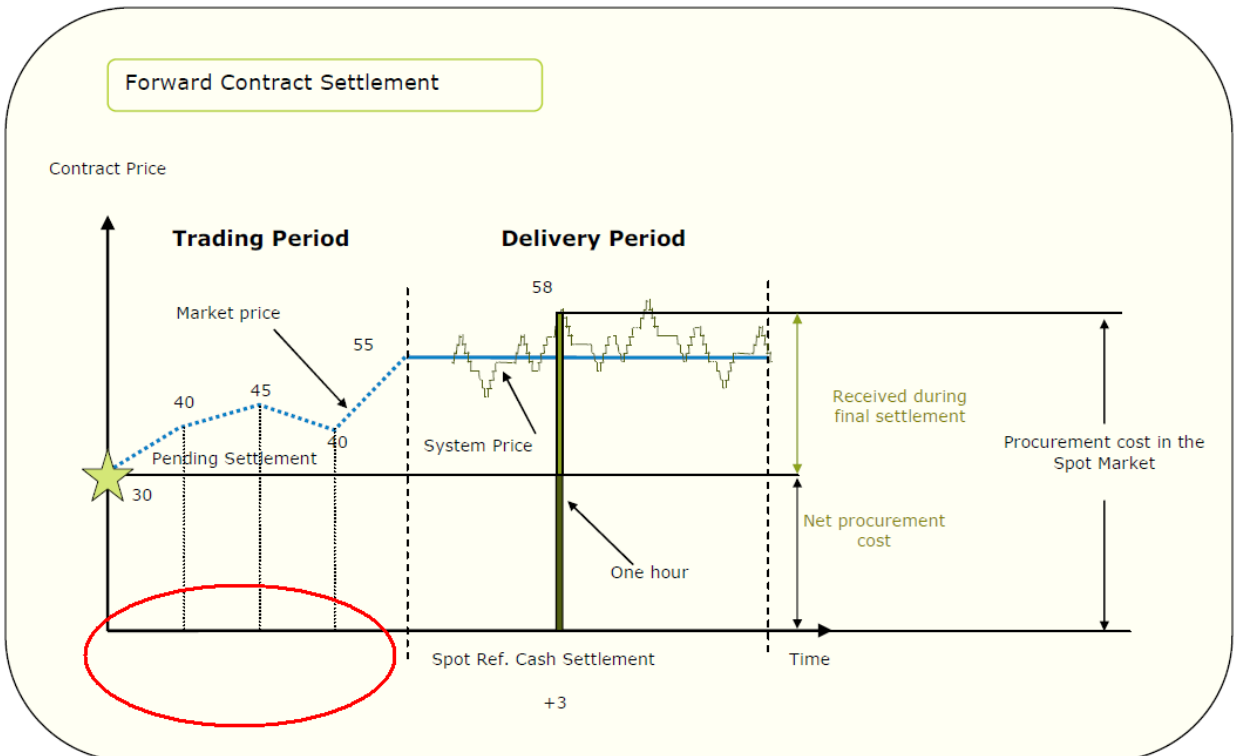


Figure 6 Pending settlement of forward contracts in Nord Pool. (Nord Pool 2008)

3.2.2. Contracts for difference

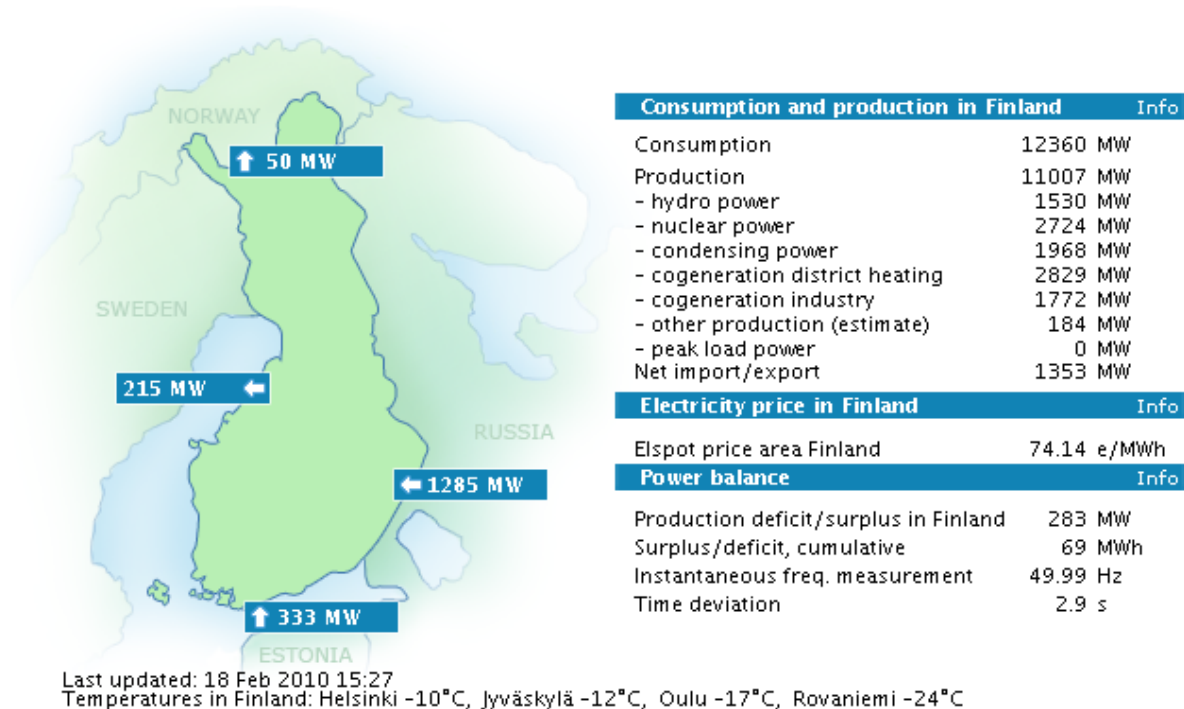
In the case when transmission capacity is not high enough to transfer electricity between distant producers and consumers the system is divided to smaller areas Finland being one of them. Other areas are Norway 1 – 4, Denmark east & west, Kontek and Sweden. If the transmission capacity is reached between two or more areas, the areas get separate prices. The contracts for difference (CfD) are derivatives that can be used to cover for the difference between area price and system price, which is based on aggregate supply and aggregate demand of the whole area.

It is relatively common not to have a same price for the whole Nord Pool area as for example in 2004 it existed only for 25.3 % of the time (Kalatie 2006). However, when considering a particular area like Finland the system price and area price are quite often the same. For example in 2001 when there existed deviations between some area and system price on half the hours in the year, the area price of Finland deviated from system price for 6 % of the time (Kalatie 2006).

The area price can deviate both up and downwards from system price depending on the transmission constraints. The Figure 7 shows the current transmission state between Finland and other countries for a particular point in time. In practice electricity is constantly imported from Russia and Estonia while the direction of power flow between Finland and Sweden and Finland and Norway may vary. When some of the transmission limits is reached then the price becomes different for these two areas. In these cases the area which exports electricity will have lower price than the area importing it.

In this thesis we do not look at the CfDs as they have different maturities and as they are a different part of risk. Also their liquidity is poorer than for system price contracts. However, one should keep in mind this additional risk component when making hedging decisions, especially if the hedging need is on an area that often reaches its transmission capacity.

State of power system



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Figure 7 The momentary transmission state in Finland. In this current state, Finland imports from Russia and Estonia and exports to Sweden and Norway. (Fingrid 2010)

3.3. Market efficiency in Nord Pool

The efficiency of power markets has recently been quite often discussed in the media. For example the current Minister of Trade and Industry of the Finnish government Mr. Mauri Pekkarinen said in spring 2010 that electricity markets lack transparency. He is also worried about large price spikes in electricity prices and suggests that the state owned companies could work together as a market participant to improve the efficiency of the market (Anon. 2010). In addition to many political statements of market efficiency, also some actual research is done on market efficiency, which mainly concludes that the wholesale market is working fairly well, while some inefficiency is found in retail markets, as observed from the studies listed below.

Liski (2006) has studied the competition in electricity markets and he says it is too early to make conclusions about market power, because of the systematic research is just beginning. He studied the market power from five perspectives: (1) the spot markets, (2) the financial markets, (3) transmission limitations, (4) hydro power and (5) retail markets. When closer looking to the financial markets he says that oligopolistic markets use financial markets both to strategic goals and risk sharing purposes. He defines competition in two forms them being price and quantity competition. With pure quantity competition he means that in equilibrium the supply does not depend on price while in price competition the relationship is price sensitive. If competition in spot market is pure price competition then financial markets can limit competition, however, if competition is focused on quantity then it increases competition. The electricity market is supply market, which does not resemble price or quantity competition. Thus Liski states further research is needed to solve in which way the competition works in Nord Pool. (Liski 2006)

Mannila and Korpinen compared the hedging methods in Nordic market and in UK. They constitute that principles are the same but the methods are different. They conclude that although the markets are very different by nature both marketplaces offer good and adequate hedging methods. Thus from hedging perspective the markets are efficient. (Mannila & Korpinen 2000)

Malo (2003) studied the efficiency in the Nordic Power markets using many different statistical methods and he finds support for efficiency in the markets as he concludes that the futures prices can be certainty equivalents of future spot prices.

Kara (2005) made a study on market efficiency in Nord Pool which has often been cited. He sees that on overall markets are functioning well, however, he criticizes the efficiency in retail markets as the consumer price follows the system price with approximately four months lag and because only a few have changed their power provider. He also points that on national level large players have very high market share and he expect the market to consolidate even further in the future. Similar results were obtained in Purasjoki's study (2006) which was based mainly on Kara's findings. Purasjoki found the derivatives market to be functioning normally but he claims that the retail market suffers from oligopolistic nature of competition in the market.

Also Hjalmarsson's (2000) study finds support for Nord Pool to be working efficiently as his study based on an extension on Bresnahan-Lau model could not reject the hypothesis of perfect market. Unlike Kara (2005), Hjalmarsson (2000) sees that there is low ownership concentration in generation in the Nordic power market and he expects that to be the reason behind the non-rejection of efficiency hypothesis.

Kristiansen (2007) found inefficiencies in pricing of newly launched monthly forwards in Nord Pool. The deviation in the average prices of synthetic and real forwards on a seasonal level was in the range of -0.18 % and 0.44 % and on a yearly level in the range of 0.01 % to 0.14 % varying between year, interest rate and settlement type. As the deviation was such that in 5 of 6 cases the synthetic contracts were more expensive than real contracts this could imply there are some, although small, cost in making a longer term synthetic contract from shorter term contracts, as the shorter term contracts, given the distribution of consumption, allows to make a more accurate hedge and thus there would be incentive to buy them at a higher price. However, Kristiansen argues that this mismatch in prices is just probably due to immaturity of the market.

When looking at the literature on electricity derivatives one should keep in mind that electricity markets are different by nature and caution needs to be used when generalizing results obtained from one market to other markets. In addition, one needs to be careful at how the metrics are defined. For example Longstaff and Wang (2004) studied the electricity markets in Pennsylvania, New Jersey and Maryland and found a positive risk premia in the forward prices. However, Botterud et al (2002) found contradicting results and evidence on negative risk premium in their study on electricity futures markets on Nord Pool. The difference is explained by the way they define risk premia, as actually they both found the forward price to be above the spot price³. This result could be seen as an inefficiency of the markets, and it could be explained by risk aversion and different hedging needs of the market participants. By reducing the risk caused by spot price behaviour it is in many cases justified to pay a risk premia. For example Longstaff and Wang (2004) conclude that prices are determined rationally by risk-averse economic agents.

³ In addition also Hanson (2007) and Torró (2008) found this in Nord Pool prices.

Malo (2009) studied the relationship between spot and futures prices in Nord Pool and he found empirical evidence that supports the efficiency of electricity futures market, because hedging with futures lead to significant risk reduction when using dynamic optimization. Thus, from electricity retailers' point of view, the electricity derivatives markets can be efficiently used for hedging.

4. Theoretical background

This Chapter starts with a discussion of the reasons for corporations to hedge. As storability in commodity hedging has a fundamental effect on hedging practices it is covered next. After that we focus on hedging in electricity markets. Then we a look at the theory of modelling electricity forwards. Finally we explain the theory behind principal components analysis, which is used in modelling the electricity forwards.

4.1. Reasons to hedge

Hedging originates from the word hedge and hedging could be seen to stand for building a fence for protection and in finance terms it is an investment, which is aimed to reduce or eliminate the risk of some other investment. One of the earliest noted uses of hedging dates back to the ancient times of Greeks, when Aristotle told a story about Thales, who had made forecast and predicted good olive harvest for next fall. He then made an agreement with olive-press owners, for a fee, to get future usage rights to the presses when harvesting period was over (Aristotle, cited in Great Books of Western World, 1990). So, already thousands of years ago it was possible to hedge exposure to changing production costs. Since then, a lot of development in organized futures exchanges has happened, but the main idea has remained the same; futures are used for neutralizing risk.

When closer looking at reasons for hedging, Smith & Stulz (1985) considers three main motives: (1) taxes, (2) cost of financial distress and (3) managerial risk aversion. Tax laws can in some cases favour the use of derivatives through lower taxes. For leveraged firms the probability of financial distress can be quite high. The transaction costs, direct and indirect, of bankruptcy reduce the firm's total value and as hedging reduces the probability of incurring these costs, it can be reasonable to use. Managerial risk aversion was the third reason Smith & Stulz found behind hedging. As managers utility is naturally a concave function of firm's value as shown in Figure 8 and as hedging reduces the variance of firm's end period value, this would lead to managers favouring high hedge ratios, unless compensation mechanism would transform the utility to a more convex shape.

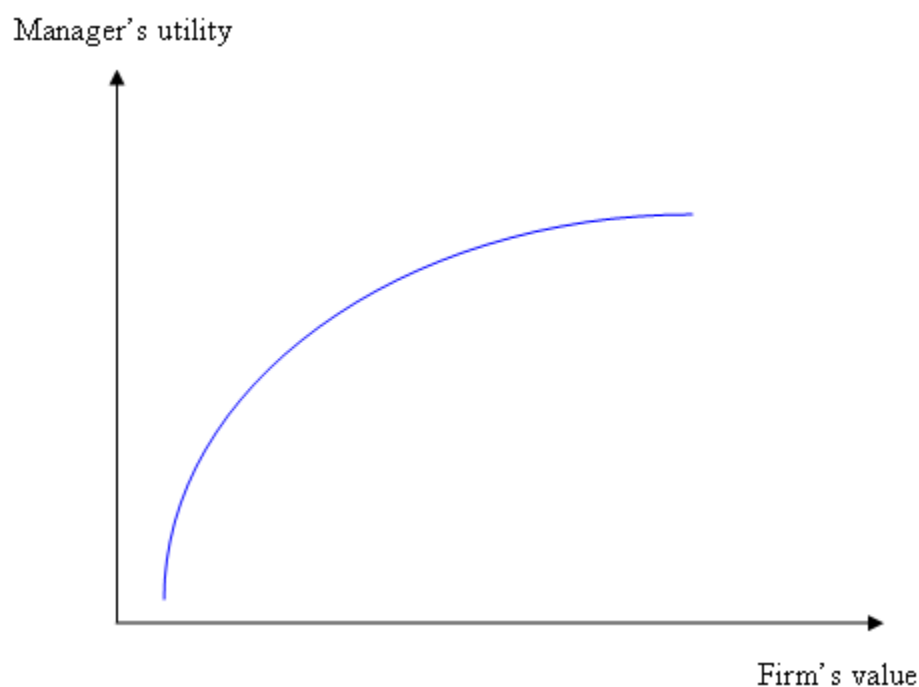


Figure 8 Manager's expected utility as a function of firm's end period value.

Graham and Rogers (2002) looked deeper into the taxing reasons and they found no evidence that firms would hedge because of tax convexity, but instead their results show that firms can hedge to increase debt capacity, or because of financial distress and firm size. They also find that the delta of CEO stock and option holdings is positively related to hedging, however, they find no significant association between option holdings and hedging. Stulz (2003) continues discussing the reasons for hedging and he states hedging being a strategic decision. He says, that first the company needs to define an objective function, which is generally to maximize shareholder wealth. Hedging decisions are then made based on that.

Hedging in commodity markets can be done also on speculative reasons, in which management changes the hedge ratio according to their view of the future commodity prices. Brown et al. (2006) studied 44 companies in gold mine industry, focusing on selective hedging and market timing and found that gold producers do in fact practice selective hedging, but they found no evidence on shareholders of those companies to get any substantial gains from that practice. Their research supports the view that managers' market views influence firms' financial decisions in a boarder view. They point out that even the managers' in the gold producing companies rarely have information that can be used in gaining from speculative positions on

commodity derivatives. Furthermore, if an electricity retailer would have information about production that other market participants do not have, she needs to disclose this inside information and temporarily suspend trading activity until the information is disclosed. However, it is still possible to undertake speculative hedging based on information that is available to all market participants.

In the case of electricity retailing the reason for hedging can be seen to be risk sharing and strategic reasons, like for example investing in own production (Liski 2006). The approach in this study is on the risk sharing needs as this thesis looks at hedging from a perspective of the retailer, without taking any stand on its own production. The retail business in the case of this study is separated from production.

Whatever the original reason for hedging in the company, the hedging strategy is set by company management and in the case of this thesis, the main reason for hedging is managerial risk aversion and the hedge is made to match the short position of electricity contracts, which has emerged due to offering a fixed price for customers. The hedging strategy for the case company (electricity retailer) in this thesis is to make energy neutral hedges meaning the total amount of energy is always fully hedged. Speculation is not allowed.

4.2. The effect of storability in hedging of commodities

Inventories and storage play an important role in commodity derivatives. For example Yang and Awokuse (2003) have studied how the asset storability affects the hedging performance in commodity futures markets. Based on error correction model and using a bivariate GARCH framework, they found empirical evidence that hedging effectiveness is strong for storable but weak for non-storable agricultural commodities. They point out that although the hedging performance is poor, the economic merit of the market can be justified by the price discovery function of the commodity derivatives markets for non-storable commodities, which is already pointed out by Black (1976) as being the big benefit of the futures markets to the society. This is also an important aspect of electricity markets, as the derivatives markets help to estimate the future price of electricity and thus to plan the production in advance. This price discovery was in fact one of the reasons to create the electricity markets in the first place.

For storable commodities like gold and oil the forward price can be thought of as a function of convenience yield, storage cost and spot price. Thus the forward price can be calculated using the basic equation (Stulz 2003):

$$F = S_t e^{(r+v-\gamma-d)i} \quad (4.1)$$

where, F is the forward price, S is the spot price at time t , r is the interest rate, v is the yearly storage cost, γ is the convenience yield, i is the holding period in years and d is the continuously compounded cash payout ratio, which can be seen as dividend.

As already mentioned in previous Chapter, electricity does not have convenience yield and thus a different approach is needed. Therefore, Geman (2005) suggests that we should not use the model set used in other commodities and add cost of carry to expected spot price, but instead we should think in terms of

$$\text{Forward price} = \text{Expected Spot price} + \text{Risk premium} \quad (4.2)$$

However, later Borovkova and Geman (2006 and 2006b) introduced a new model being able to better capture the cost of carry relationship, by taking into account the seasonality in forward curves. They also give a method of to look at how the forward prices deviate from their expected prices after removing the seasonal effects in the prices, which is also used. They model forward curve based on average forward price and convenience yield according to equation

$$F(t, T) = \bar{F}_t e^{s(t) - \gamma(t, T-t)(T-t)} \quad (4.3)$$

Where \bar{F}_t is the average forward price, s is seasonal component, γ is convenience yield, t is time and T is the delivery period of the forward contract. The average forward price is the geometric average and it is calculated by equation

$$\ln \bar{F}_t = \frac{1}{N} \sum_{T=1}^N \ln F(t, T) \quad (4.4)$$

where N is the number of contracts used in the calculation. For monthly contracts it should be a multiple of 12 to make a full year or years. The usage of the model is described in more detail in Chapter 5 and it is used in this study to capture the seasonality in the prices.

As noted among others by Geman (2005) the lack of storage makes dynamic hedging, based on the Black-Scholes assumption of continuous trading in the underlying, impossible to conduct. On the other hand, water plants can control their output with some limits, by controlling the water flow and practically they have a limited possibility to store electricity in the dams. Thus there is a limited possibility to conduct dynamic hedging using hydro power. However, as arbitrage based models for dynamic hedging for stocks or other commodities are based on assumptions that the arbitrageur can hold the underlying asset until the expiration of the contract, it is clear that these models can not be applied as such. Thus a new approach is needed.

4.3. Hedging in electricity markets

Hedging in electricity markets differ from hedging in other commodity markets mainly because electricity can not be stored⁴. There are also other aspects in hedging in electricity markets which are covered next, some of which can be found on other markets as well. Seasonality makes the prices dependent on calendar month, while the poor link between spot and forward prices makes modelling forward prices challenging. In addition, electricity markets have volumetric risk, which can cause huge losses if hedging is done poorly.

4.3.1. Seasonality due to weather conditions

As noted already by Black (1976) there exists seasonality in agricultural spot prices while some other commodities like gold behave totally different. Seasonality can also be found in electricity prices, especially in markets driven by changing weather conditions. Lucia and Schwartz (2002) studied the seasonal effects in the Nordic power markets and found that seasonality is very

⁴ For more about lack of storage please look at Chapter 3.

important in explaining the shape of futures prices in Nord Pool. If seasonality is ignored, it is likely to lead to residual autocorrelation of the order of the seasonality and thus it should be taken into account in modeling financial data (Brooks 2008). Borovkova and Geman (2006) expands the study of seasonality to forward curves and with a different approach of using the actual forward prices as the reference, instead of spot prices, gives a model for forward price curve, which is applied also in this thesis.

4.3.2. The link between spot and forward prices is weak

Hedging in forward markets based on spot price distributions has been studied for example by Bessembinder and Lemmon (2002). They discuss hedging positions based on equilibrium model, which is usable close to delivery of the electricity. They look at hedging against the movements in spot prices, while this thesis looks at hedging the movements against forwards prices. As discussed in Chapter 3 it is commonly known that the spot price, futures price and forward prices of electricity behave differently. Even huge price spikes are relatively common in spot prices, while not experienced in futures or forwards.

It has been noted that the link between spot and futures holds for a few weeks but not for long time periods (Malo 2009)⁵. Thus, as the long-term forwards are not related to spot prices, a different approach is needed. Therefore, as this thesis looks at long-term hedging with a time scale of months to years, the Borovkova's and Geman's approach is justified, as the model reveals the seasonal effect and is not dependent on spot price movements, which is also the case in our hedging needs. The aim in the long-term hedge under consideration in this thesis is to protect against movements in forward prices and not against spot prices. The protection against spot prices is a different part of risk management.

4.3.3. Short term dynamic hedging in electricity markets

Byström (2003) studied the short term dynamic hedging in Nord Pool and he concludes that when transaction and clearing cost are taken to account the unconditional "buy and hold" OLS hedge is preferred over time varying moving average and GARCH hedge ratios, although some

⁵ Also Borovkova and Geman (2006 b) noted that the link between forwards and spot is weak. For example in their study with Nord Pool data the correlation on spot and nearby futures was in the range of 0.65 and -0.15.

gains could be achieved prior to taking these costs into account. Besides, the constant OLS hedge ratio resulted in lower portfolio variance. He also made hedges with longer term maturity futures against spot prices and he noticed⁶ that hedging performance with futures of higher than a few weeks maturity deteriorates compared to shorter maturity futures.

4.3.4. Volumetric risk - deviations of price and demand from their expected values

As mentioned previously, electricity can not be stored and it is a flow commodity where the time and amount of consumption are important (under transmission restrictions also the location of consumption matters). As both the amount of consumption and price are uncertain, there is risk related to them. A combination of price and quantity risk called the volumetric risk plays an important role in electricity markets. Volumetric risk is defined by Laitasalo (2004) as the product of the deviation of volume and price from their expected values. The situation in the Nordic countries is worst in cold winter days with demand and prices being on high level. If both are above expected, the losses for too low hedge can be dramatic.

Laitasalo (2004) studied volumetric risk in Nord Pool and found out that the loss distribution is highly skewed, cold days having significant risks while warmer days having low. Support to Laitasalo's findings is reported by Kettunen et al (2009) who say that spot price and loads are correlated and the correlation is strongest on high loads. Based on simulations, Laitasalo (2004) suggested the use of heating degree day⁷ (HDD) swaps and options to hedge the exposure. The approach is also supported by Geman's (2005) findings that heating degree days closely track the amount of heat used by consumers. During the conduction of Laitasalo's study weather derivatives were just launched in Europe and only most standardized derivatives having some liquidity. Now the liquidity is quite good in US and on the Chicago Mercantile Exchange (CME) one can trade HDD options for months and seasons referenced by temperature in different locations. However, the liquidity is still a problem for using them in the Nordic countries.

⁶ The results of the long term futures hedge is not presented in Byström (2003), he just presents the conclusion.

⁷ Heating degree day (HDD) is used in weather derivatives. It is calculated by subtracting the mean daily temperature from the reference temperature. If the number is negative it is set to zero. For example, if the reference temperature is 15°C and the average temperature for that day is 5°C, then the HDD for that day is $15 - 5 = 10$.

Kettunen et al (2009) are also concerned about the volumetric risk and they approach the optimization problem with a multistage stochastic optimization, in which they integrate the correlation between spot and forwards and look at forward premiums and risk preferences of the electricity retailer. They build a scenario tree and use simulation to optimize the portfolio of retail contracts. Their focus is on 6 week horizon with weekly and monthly level contracts while this study has long-term focus from months to many years. Their approach could be used also with longer-term contracts and thus be implemented in this study. However, they use spot price as the underlying and their approach is quite complex including many different variables. This thesis has a more practical approach and the method should be usable in everyday pricing of electricity contracts. Moreover in this thesis we are not looking at volumetric risk but instead we assume the demand is deterministic and thus we focus on modelling electricity prices. Therefore, Borovkova and Geman's (2006) method is favoured over Kettunen et al's.

4.4. Modelling electricity prices

Modelling electricity price is done to forecast future behaviour of them. With the help of models one can simulate the future price movements and make action decisions based on that. Simulation is widely used practice in calculating risk measures and suits well the needs of this study as it helps to quantify the risks.

4.4.1. Modelling spot prices

Majority of the research in electricity price modelling is focused on spot prices, which have high volatilities. Capturing the properties of spot prices is an extremely difficult task. There are many different approaches and for example Geman (2005) used mean-reversion component combined by jump component, which gives the possibilities for high price movements. However, the mean reversion in their model is very powerful in high spikes and will most likely make this spikes short lived, which is not optimal in modelling risk exposure to heavy frost weathers or plant outages, which could easily last for more than a few days. This issue can be tackled for example with Villaplana's (2003) two-factor jump diffusion model, which allows the probability of jump that occur to be non-constant. This approach could be used if the hedging would be made against

spot price movements, but as mentioned above the focus of this thesis is on long-term hedging involving forwards and the hedging against spot prices is left out of the study.

4.4.2. Modelling forward prices and their volatility

There are some previous research done to model the forward prices based on underlying spot prices⁸, however, as the link between spot and long term forward prices is weak and as we are not interested in the spot prices, we are looking of modelling the forward prices based on the information we have in forward prices. We also want to model how the forward prices move together.

Generalized autoregressive conditional heteroskedasticity (GARCH) model is often used in modelling volatilities. For example Longstaff and Wang (2004) have used GARCH(1,1) model in electricity markets to estimate volatilities. They are looking at day-ahead forward markets while our interest is on year-ahead level. Our goal was to get a model designed for long-term electricity forwards and modify it to fit our perhaps different data. Using models developed on short term electricity forwards would need a lot more modification and thus increase the possibility of making errors. Therefore we did not look at models developed on short-term electricity forwards any further.

Fleten and Lemming (2003) developed a model for forward curves that is based both on market data and bottom-up models. They look at bid-ask spreads and combine that information with forecasts generated by bottom up models. The bottom up model tries to forecast future spot price, while we are interested in future forward prices. A more suitable approach for our needs was found from Borovkova and Geman's (2006) model, which is appealing for the needs of this thesis as it looks at the relationships between electricity forward prices and the effect of the calendar expiry month. They model seasonality directly from historical forward prices and they replace the spot price of the model by a more robust quantity of average forward price. This suits well for our need as we are interested on the forward price behaviour on long term. In addition Borovkova and Geman (2006b) developed a method for electricity forwards to model the

⁸ Look for example Lucia and Schwartz (2002)

deviations of the price curve from average seasonal pattern by using principal components⁹, which was of particular interest in our study as we are interested in the risk in seasonal demand.

Koekebakker and Olmar (2005) constructs continuous smoothed forward curve for the Nord Pool electricity markets from daily futures and forward contracts. Their model was made in similar setting as Heat-Jarrow-Morton bond pricing model (Heat et al. 1992). Both Koekebakker and Olmar (2005) and Borovkova and Geman (2006b) apply principal component analysis (PCA) to reveal volatility structure, however, the difference is in that Koekebakker and Olmar use it on futures returns while Borovkova and Geman use it on actual futures prices. Also Clewlow and Strickland (2000, p. 143-149) have used PCA on futures returns. They model seasonality in forwards based on volatility in spot prices, which does not suit our needs as the relationship between long term electricity forwards and spot prices is not holding well¹⁰. Furthermore, Borovkova and Geman's model is explained in a clear way, while Koekebakker and Olmar leave practically all the variables in their equations unexplained. In addition in Borovkova and Geman's (2006b) model the first three principal components of de-seasoned forward curves can be visually interpreted as the level, slope and curvature which help to interpret the results and check if some fundamental errors have occurred when applying the model.

Audet et al (2004) model electricity forward dynamics based on market price data. They model the whole price curve including also the forward and spot price relationship by using a parameterized model. When using their model, they noted that forwards' correlation with the spot price decreases with time to maturity. Thus, for our needs this gives support for looking at the long-term forwards separately from spot prices. Spot and forward price relationship is needed for example when hedging the production of electricity. However, looking at the whole curve makes the model quite complicated. A model looking only at the end part of the forward curve is more suitable for our needs as we are not looking at the short-term forwards or spot prices at all.

⁹ If you are unfamiliar with principal components please check Chapter 4.5. for how they work

¹⁰ See for example (Borovkova and Geman 2006b) and (Malo 2009).

4.4.3. Simulating future prices

Simulation is widely used method for calculating the risk exposure using different risk measures. It can be used to simulate future states of the underlying. Simulation also helps to cover for the Samuelson effect found in electricity forward prices. Samuelson (1965) found that volatility drops when maturity increases. This can be covered in the Borovkova and Geman's model as the future price volatilities are based on convenience yield volatilities which depends on time to maturity. In this thesis we simulate mean reverting Ornstein-Uhlenbeck process to get a family of future forward prices and we use a simulation tool that is modified for our needs from a simulation tool, which was originally written by Smith (2010) and developed to work with Borovkova and Geman's model.

Instead of using simulation we could just look at the historical values of the prices and base our calculation on them. However, historical values are only one possibility for what could have happened. Using them as such would be similar as to driving a car by looking at the rear-view mirror. Simulation is also based on historical values, but these values are used to calibrate the simulation parameters. Simulation gives possible scenarios where the prices could be in the future if they behave in similar way they did in the past. Thus we could say that when we use simulation we expect the future to behave in similar way than past, while when using historical values we expect the future to be identical to the past

4.5. *Principal components analysis (PCA)*

In this section we go through how principal components analysis (PCA) works. It can be used to focus on the most relevant information in the data. Large number of data series can be compressed into smaller amount of series of principal components. It is especially useful when we have high correlation in the data, as then we can model the dependencies in the data with fewer factors. We do it by replacing the original data with principal components. If we would use all the principal components, we could always shift between the data and principal components without losing any information. Basically, doing principal components analysis can be understood as an axes transformation of data, as the axes are transformed along eigenvectors. The key in using principal components is, however, that the principal components explain the variation in the data in decreasing order; the first component explaining most of the variation.

Thus we can take only a few first principal components to explain majority of the data. It becomes a lot easier to model a few principal components than modelling all the data series and their correlations. To get an understanding about what happens in PCA, first we need to cover some statistical and matrix calculation concepts.

4.5.1. Statistical analysis behind PCA

The PCA uses the basic statistics tools from mean to covariance.

4.5.1.1. Mean

Mean is the simplest statistical measure and it is the arithmetic average of the data. It can be calculated with the equation below:

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n} \quad (4.5)$$

Where \bar{X} is the mean, X_i is data point, and n is the number of data points.

4.5.1.2. Standard deviation

Standard deviation measures how far away the data points are from its mean value. It can be calculated with the equation below:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}} \quad (4.6)$$

note, that the denominator is “n-1” and not “n” because we have a sample instead of whole population.

4.5.1.3. Variance

Variance is the standard deviation squared and can be calculated with equation below.

$$\text{var} = \sigma^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1} \quad (4.7)$$

4.5.1.4. Covariance

The most relevant statistical tool for the PCA is covariance which measures how much two variables change together. Covariance can be calculated with equation below:

$$\text{cov}(X, Y) = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{n-1} \quad (4.8)$$

Note, that covariance between X and X is same as variance of X. As covariance is calculated between two series of data, we can construct a matrix containing all the possible covariances.

4.5.2. Matrix calculus behind PCA

Matrix calculus needed in PCA involves eigenvectors and eigenvalues. In general when we multiply vectors with matrixes both the direction and the length can change. However, when we multiply the matrix with its eigenvector the direction of the vector does not change but the length of the vector do change.

4.5.2.1. Eigenvector and eigenvalue

So for a matrix \mathbf{A} the eigenvector \mathbf{v} is defined by the equation below

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{v} \quad (4.9)$$

where λ is a scalar (plain number). This scalar is called the eigenvalue of the eigenvector. \mathbf{v} is an eigenvector of \mathbf{A} if it satisfies the equation above for some scalar λ . An example illustrates the point. Let's have a matrix

$$\mathbf{A} = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

which has the characteristic equation

$$\begin{aligned} \det(\mathbf{A} - \lambda\mathbf{I}) &= \det \begin{vmatrix} -5 - \lambda & 2 \\ 2 & -2 - \lambda \end{vmatrix} = 0 \\ (-5 - \lambda)(-2 - \lambda) - 4 &= 0 \\ \lambda^2 + 7\lambda + 6 &= 0 \end{aligned} \quad (4.10)$$

The solution for this characteristic polynomial are:

$$\begin{aligned} \lambda_1 &= -1 \\ \lambda_2 &= -6 \end{aligned}$$

These are the eigenvalues of \mathbf{A} .

The eigenvector corresponding to λ_1 can be obtained from equation (4.9) above,

with substituting $\lambda = -1$

Then we would get

$$\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = -1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (4.11)$$

$$\begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

This has a solution

$$v_2 = 2v_1$$

Thus we have an eigenvector

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

And if we scale the length of the vector to be 1, we will get a unit eigenvector. To do this we divide the vector with its length. The length of the vector is

$$l_1 = \sqrt{1^2 + 2^2} = \sqrt{5} \quad (4.12)$$

Thus we can calculate the unit eigenvector:

$$\mathbf{v}_1 = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$$

The second eigenvector can be obtained similarly by substituting $\lambda = -6$, then we would get

$$\mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

and again similarly as for \mathbf{v}_1 we get the unit eigenvector

$$\mathbf{v}_2 = \begin{bmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \end{bmatrix}$$

After we have now explained how principal components work, we will in next Chapter describe Borovkova and Gemans' methods in more details, as well as show what modifications we made

to the model. We will also describe in more details how we use principal component analysis in this study.

5. Methodology and data

The research questions of hedging cost and timing are answered by a case study of data from Fortum and Nord Pool. The electricity price data is obtained from Nord Pool and data containing customer demands is obtained from Fortum.

The update cost of hedges is studied by simulating forward price combinations and then calculating the cost for the adjustments with different seasonal demand. The cost is calculated by comparing the amount of money needed for updating the hedges to the amount of money needed for buying the original hedge. The timing of hedging is also studied by changing the point when the hedge is updated. The comparison is done between early, middle and late update.

5.1. Data sources

The real market data from Nord Pool consists of daily end prices in yearly, quarterly and monthly forwards from years 2006 to 2009. The quoted periods thus include 2006 to 2014 as the most far away yearly forward is available for delivery in five years in the future. The start date was chosen to be 2006 as the first quarter products were quoted for quarter 1 in 2006. Customer demand data is obtained from Fortum and it contains the intra-year monthly distribution of user group demand. The actual results of cost of different Fortum's customers are not shown in this thesis for obvious reasons, but instead the analysis is done using a few artificial customers' user demand data as they illustrate the point in a similar way.

5.2. Seasonal premium in electricity forward prices

Borovkova and Geman (2006) developed a method for estimating the seasonal premium in electricity forward prices. We call the premium as seasonal components. This means that prices are driven by seasonal effects such as weather and the goal of Borovkova and Geman's (2006) method is to capture the seasonal premiums. The seasonal components are calculated compared to a geometric average of monthly forward prices. The average forward price should be calculated over a period of 12 months to cover the seasonal component. In this way we have an underlying time period that is not seasonal as a comparison point. Removing seasonality out of the data makes it easier to analyse it and we can for example find trends in the data that we could

not find if seasonality was left in the data. As there are only forwards available for next 6 months in Nord Pool the calculation needs some modification. Luckily Borovkova and Geman (2006) also developed a method for these situations and they show it in the appendix of their article (2006). This method was used also in this thesis.

First we need to estimate the difference between all the possible combinations of monthly prices. We do this for the entire data set. For those months that are missing we will use nearest quarterly price as the reference. We get a following matrix of the estimates.

$$\begin{bmatrix} s(1) - s(2) & & & & & \\ s(1) - s(3) & s(2) - s(3) & & & & \\ s(1) - s(4) & s(2) - s(4) & & & & \\ \cdot & \cdot & & & & \\ \cdot & \cdot & & & & \\ \cdot & \cdot & & & & \\ s(1) - s(12) & s(2) - s(12) & \cdot & \cdot & \cdot & s(11) - s(12) \end{bmatrix}$$

Next we will sum up the columns of the above matrix. The sums are denoted by Σ_i where the i refers to the column.

$$\Sigma_1 = [s(1) - s(2)] + [s(1) - s(3)] + \dots + [s(1) - s(12)] \quad (5.1)$$

Then we use the restriction that seasonal components sum up to zero

$$\sum_{M=1}^{12} s(M) = 0 \quad (5.2)$$

and we can calculate the monthly components.

$$\Sigma_1 = 11s(1) - \sum_{M=2}^{12} s(M) = 12s(1) \quad (5.3)$$

Thus we get

$$s(1) = \frac{\Sigma_1}{12} \quad (5.4)$$

And similarly for $s(2)$ we get¹¹

$$\Sigma_2 = 10s(2) - \sum_{M=3}^{12} s(M) = 11s(2) + s(1) \quad (5.5)$$

and thus we get

$$s(2) = \frac{\Sigma_2 - s(1)}{11} \quad (5.6)$$

This process is repeated for the rest of the seasonal components to get all the 12 monthly components.

An alternative approach that was also tested was to calculate seasonal components for shift from yearly forward to quarter and then from quarter to month. The logic behind it is that quarterly parameters are more robust than monthly as they are available for 8 to 11 consecutive quarters as the volatility is lowest for forwards with more distant maturity. With quarterly forwards we can get maturities further away in the future than with monthly forwards. The further away in the future the maturity is the better the price reflects the pure seasonal component as the short-term effects have less influence on them. On the other hand the liquidity of forwards with distant maturity is lower than for closer maturity. As the goal was to get as reliable estimations of seasonal components as possible we used different data selections to determine them and the one with lowest standard deviation was chosen. We used prices for the first and last quoted year as well as quarters that are 2 to 5, 3 to 6, 4 to 7 and 5 to 8 quarters till maturity. The closest quarter

¹¹ Please note, that If you compare these equations to the equations Borovkova and Geman (2006) there are some differences in the signs inside the equations, as they had made some typing errors. The equations are corrected to this thesis.

was left out of the study as it can be biased by spot price behaviour. We calculated average quarter components and their standard deviations for each of the choice of data. The standard deviation was highest for 2 to 5 quarters to maturity and lowest for the last traded quarters. Thus we used the most distant quarters available to determine the quarterly components. In accordance to Borovkova and Geman's (2006) model we constructed a synthetic yearly forward which price is the day geometric average price of the four quarter prices. The average price is calculated with the equation

$$\ln \bar{F}(t) = \frac{1}{N} \sum_{T=1}^N \ln F(t, T) \quad (5.7)$$

Where T is the delivery quarter.

Now we have the average price and we can calculate the seasonal quarter components for the given days data. Seasonal component is then calculated with equation

$$s(t)_Q = \ln F_Q(t) - \ln \bar{F}(t) \quad (5.8)$$

and the weight, which means the weighting factor for re-seasoning the yearly forward curve, is simply

$$w_Q = e^{s(t)_Q} \quad (5.9)$$

In this alternative approach the monthly components is calculated for the three consecutive months leaving out the first month so that it forms a quarter of a year (January to March, April to June, July to September or October to December). This is compared against available quarter data and the components can be added to the quarter component calculated previously. This procedure is repeated for the whole data to cover the seasonal premia parameters. Leaving out the first month is justified by the fact that it can be biased of the spot market data especially in times close to the end of the month i.e. times close to the beginning of the exercision period of that forward.

The seasonal components calculated with these different approaches are shown in Chapter 6.1. For this thesis we chose to use the seasonal components calculated with the method in the appendix of Borovkova and Geman (2006) because the components calculated this way gave most sensible relationship between the components.

5.3. Smoothing the data

Borovkova and Geman's (2006) model was written for commodities and especially commodities having delivery periods with constant time to maturity. As electricity forwards have maturities tied to calendar months and quarters, we smoothed the data to have constant maturities. By doing this we can apply the Borovkova and Geman's (2006) method to our data. We used the observation date for smoothing and assumed linear dependency of two closest de-seasoned forwards. An example illustrates the point. Let's assume the observation date is June 9 2006. Then we can calculate the de-seasoned monthly forwards for months 1 to 5 in the future. So the forward for time period of July 9 to August 9 would then be

$$F_{July9\ to\ August9} = \frac{31-9}{31} \cdot F_{July2006} + \frac{9}{31} \cdot F_{August2006} \quad (5.10)$$

where F is forward price. The same is done for the other months and quarters as well. As a result we have managed to get rid of spikes between changing of contracts and have a smooth de-seasoned forward curve, but on the set back we have one month and one quarter less for data analysis. This loss of data is not a major problem as forwards with high maturity has poorer liquidity and they are not used that much in practice anyway. So for the data analysis we got then data for the next five months and next seven quarters.

5.4. Constructing the forward curve

The traded electricity forwards include monthly prices for only next six months. When looking further in the future we have only quarterly or yearly prices available. By using the seasonality calculated with Borovkova and Geman's (2006) method, the information in the quarterly and yearly prices can be used to estimate the forward prices on monthly interval. In this approach, the

forward price curve is constructed using the seasonal components and is based on the actual forward prices. As a result we get a forward curve on monthly level, which is not directly available in the market.

The forward curve is constructed on monthly level using quarterly and yearly forwards. The idea is to use seasonal premiums / seasonal components to de-season the forward curve and then by assuming liner trend in de-seasoned prices we can calculate missing monthly values on de-seasonal prices. After having a complete de-seasoned curve on monthly level the seasonal components are added back to get a whole forward curve for up to 5 years. An example illustrates how it can be done.

First we will de-season all the quoted prices. We do it by dividing the quoted price with the seasonal weight of that price. For example if we have a price for Q3 in 2011 that is 42.6 € / MWh. Then we get the de-seasoned price by dividing the price with the seasonal weight

$$P_{Q3} = \frac{F_{Q3}}{e^{s(Q3)}} = \frac{42.6 \frac{\text{€}}{\text{MWh}}}{e^{-0.112}} = 47.63 \frac{\text{€}}{\text{MWh}} \quad (5.11)$$

Where P is the de-seasoned price, F is the forward price and s is the seasonal component. This process is then repeated for all the quoted monthly and quarterly prices.

After de-seasoning all the prices we set directly the de-seasoned six quoted monthly prices for the first six months. Then for the following months we assign the de-seasoned quoted quarterly forward to the middle of the forward. Thus for February we assign the de-seasoned price of Q1, for May Q2, for August Q3 and for November Q4. Then for the missing values we assume a linear trend. So for example de-seasoned price for January is

$$P_{Jan} = \frac{1}{3} \cdot P_{Nov} + \frac{2}{3} P_{Feb} \quad (5.12)$$

This is repeated for all the contracts that are quoted in quarterly level. Next we continue on yearly level.

On yearly level the first step is to estimate the price for the year after the yearly quotations end. We do it by assuming a linear trend for the last year. So the price for the year after the last traded year is assumed to be

$$P_{last+1} = \left(1 + \frac{P_{last} - P_{last-1}}{P_{last-1}}\right) P_{Last} \quad (5.13)$$

Then for yearly prices we also assume a linear trend inside the years. We assign the quoted yearly price for middle of the year, so that we assign the (de-seasoned) yearly price for June. The prices for other months are a weighted average of the two closest yearly prices. So for example the price for March 2014 is

$$P_{March2014} = \frac{3}{12} \cdot P_{2013} + \frac{9}{12} P_{2014} \quad (5.14)$$

As a result we have a de-seasoned forward curve on monthly level up to 5 years. Then we add the seasonal components back by multiplying the de-seasoned price with the seasonal weight

$$F_{Month} = P_{Month} \cdot e^{s(Month)} \quad (5.15)$$

And as a final result we get a forward curve that contains only monthly prices.

5.5. Comparing different timing of updating the hedges

In real life the needed amount to hedge changes constantly as the amount of electricity sold changes and the hedges are updated according to these changes daily. However, as the loads and contracts with customers are assumed to be constant for the whole period there is no need to for dynamic hedging with further updating the contracts after they are updated to shorter maturity forwards. In addition as the case company is not allowed to take speculative positions in the derivatives markets and as they should always have an energy neutral hedge, meaning they have fully hedged the demand on yearly level, we do not look at dynamic hedging in this study. The

point of interest is the time point when the hedge is updated and the updating is done only once from year to quarter and from quarter to month for each customer as we are after the update as fully hedged as possible.

The update timing that is compared includes early update, late update and updating in the middle. Early update means the portfolio is updated as soon as possible after the shorter maturity forwards become available. Late update means the long-term hedges are updated with shorter term hedges close to delivery period. Finally, middle is in between of early and late timing. The update time is tested with simulated data to cover for the most optimal time point.

5.6. Principal component analysis

Originally we used the convenience yield approach of Borovkova and Geman (2006) first paper to model the volatility, but the convenience yield volatility matrix did not have an inverse matrix as there was high correlation between the price series. Therefore we had to abandon that approach. As mentioned earlier, Borovkova and Geman wrote a second article that is focused solely on electricity forwards (Borovkova and Geman 2006b). They state that principal components analysis is particularly well suited for analysing the deviation of de-seasoned forward curve from its flat shape. After taking out the seasonal component principal component analysis can be applied. The idea in using them is to transform the data to a few axes which makes it easier to analyse. Instead of 12 data series¹² we can now look at fewer ones. These fewer series are able to explain majority of the data as well as the correlations between the original series.

Principal components transform the data to different axes according to the eigenvector loadings. When the eigenvector loadings are calculated with some mathematical software we can construct the original data from the principal components and the principal components from the data. An example is used to illustrate the point. Let's say we wish to calculate the first principal component, and then we get

¹² The 12 data series consists of 5 monthly and 7 quarterly data series.

$$PC_1 = PC_{1M1 \text{ loading}} * F_{M1} + PC_{1M2 \text{ loading}} * F_{M2} + \dots + PC_{1Q7 \text{ loading}} * F_{Q7} \quad (5.16)$$

where PC is principal component, PC_{loading} is the corresponding eigenvector loading connecting the forward prices to a particular principal components and F is forward price. When knowing the principal components we can get the price back by doing the previous process backwards we get

$$F_{M1} = PC_{1M1 \text{ loading}} * PC_1 + PC_{2M1 \text{ loading}} * PC_2 + \dots + PC_{12M1 \text{ loading}} * PC_{12} \quad (5.17)$$

where PC_{loading} is the corresponding eigenvector loading connecting the principal components to a particular forward price.

Borovkova and Geman (2006b) see the three first principal components as representatives of level, slope and curvature. When comparing the average forward price and the first principal component we can indeed see strong connection between them as illustrated in Figure 9. As the first principal component captures the average price, it can be used in modelling the first state variable in Borovkova and Geman's model, other principal components explaining the rest of the volatility. In this way we can model the movement of the principal components. As a rule of thumb we should use principal components up to an eigenvalue of 1. As Borovkova and Geman (2006b) used three first components we also used the first three principal components although the third one has an eigenvalue of less than 1. The rest 9 explain only little of the data and these components were summed to a level factor explaining the difference between the historical data series and the one constructed from principal components. These level components are shown in Chapter 6.

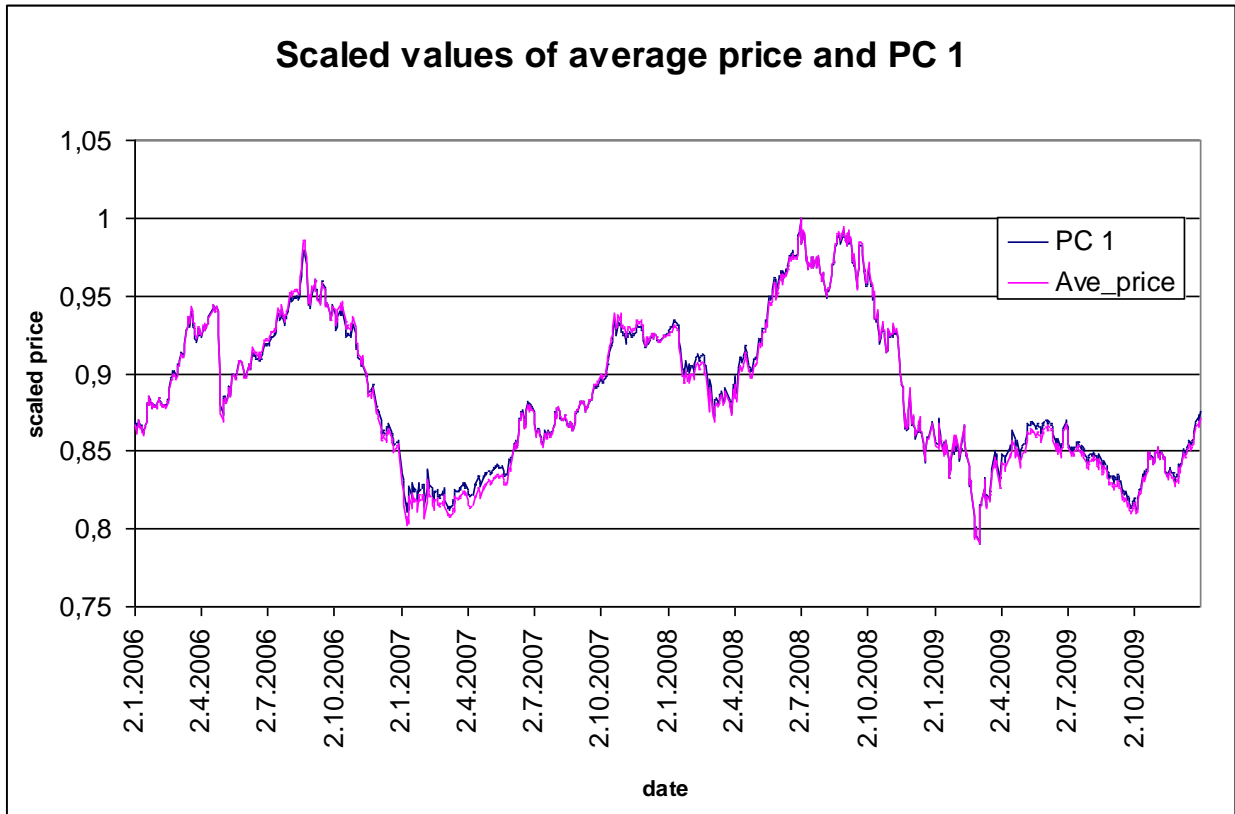


Figure 9 First principal component and average forward price scaled to have maximum value of 1.

The original Borovkova and Geman's (2006) approach modelled convenience yield as mean reverting process with a mean of zero. We developed this further using Borovkova and Geman's (2006b) second paper to more realistically capture the relationship between the prices. We applied principal components to convenience yield modelling and used two sources of uncertainty to model it, one for each principal component. The three first principal components can be seen as level, slope and curvature. In accordance to Borovkova and Geman's method we can model the principal components as mean reverting process.

5.7. Simulating future states of forward prices

To simulate future forward prices we first need to simulate the movements of principal components, which is done with Matlab. We will then later transform the simulated principal components to forward prices. We did the simulating by using a code written by Smith (2010) who is a PhD student under the supervision of Geman, who is the co-author of the two

Borovkova and Geman papers. Smith wrote his code to be applied especially to commodities markets. Smith (2010) wrote a simple code to model Ornstein Uhlenbeck process, which is a basic mean reverting process. The code is given in Appendix. It models the mean reverting process S given in equation

$$ds = \delta(\mu - S)dt + \sigma dW_t \quad (5.19)$$

where s is the variable, δ is the speed of mean reversion, μ is the long run mean, σ is the volatility and W_t is Brownian motion, so that $dW_t \sim N(0, \sqrt{dt})$. By using discrete time steps we get the equation for simulation

$$S_t = S_{t-1} + \delta(\mu - S_{t-1})\Delta t + \sigma dW_t \quad (5.20)$$

To be able to calculate the cost of updating the hedges we simulated the three first principal components using 100 repetitions. In this way we got 100 series of 4 year data of principal components. Next we transformed these series back to forward price series using the eigenvector loadings. Then we added the seasonal components back and got daily price data on 4 year time periods. As we had 100 simulations containing 4 years of data we used 400 possible states of the forward prices in the update points. As it takes time to update a whole portfolio of forwards we used five day average price around the starting date of each month to update the hedges. In this way we got the seasonality to match calendar seasonality¹³.

Next step is to calculate the money flow for updating the original hedge. An example will illustrate the point. Let's say we have a forward price for year 2015 that is 45.00 €. The customer demand on monthly level in MWh is given in Table 1 below. The total demand is thus 1105 MWh.

¹³ Please recall that we smoothed the forward curves to get rid of the change in contracts. Now we want to get back to actual contracts. When we are close to the end of month then we are also close to having the actual contract in the smoothed data. We could alternatively do the smoothing in opposite direction to get contracts matching calendar months, but then we would lose again the information of one month and one quarter.

Table 1 Example customer demand in MWh for each month.

	Demand	(Unit)
Jan	110	MWh
Feb	100	MWh
Mar	105	MWh
Apr	80	MWh
May	70	MWh
Jun	70	MWh
Jul	80	MWh
Aug	90	MWh
Sep	95	MWh
Oct	100	MWh
Nov	100	MWh
Dec	105	MWh
Total	1105	MWh

So for making an energy neutral hedge, meaning the demand is hedged on yearly level for the average demand we have an initial money flow in the hedge of

$$1105 \text{ MWh} \cdot 45.00 \frac{\text{€}}{\text{MWh}} = 49725 \text{ €} \quad (5.21)$$

Then we will use the 400 simulated forward price combinations to calculate the update cost. Again an example illustrates the point.

Let's take one simulated case. We would have the forward prices simulated shown in Table 2. We are now interested in the spreads between months inside one quarter and the spreads between the quarters. Please note that only the prices inside one quarter are taken from same simulated state of prices. By this we mean that using the simulated data the prices for Jan, Feb and Mar are taken 3 months before prices for Apr, May and Jun. The reason for this is that the actual update also incurs with same interval. We do not have the prices simultaneously available for all the months. Next we will calculate how much energy we need in each quarter and compare it the how much energy we have bought with the original hedge for each quarter. In this example it is done in Table 3. For example, quarter one we have bought with the yearly hedge 272 MWh

when we actually need 315 MWh. Thus we are short of 43 MWh in quarter 1. On the other hand we have bought too much energy for quarter 2 and we need to sell 55 MWh in quarter 2. ¹⁴

Table 2 Example of simulated forward prices.

Time period	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Price	57,53 €	56,87 €	53,84 €	40,39 €	39,40 €	39,45 €	40,40 €	42,23 €
Time period	Sep	Oct	Nov	Dec	Q1	Q2	Q3	Q4
Price	43,60 €	42,48 €	45,86 €	48,14 €	54,15 €	45,71 €	44,97 €	52,54 €

Table 3 Example of energy needed compared to what is bought in the original contract.

Quarter	Need [MWh]	Bought originally [MWh]	Difference [MWh]
Q1	315	272	43
Q2	220	275	-55
Q3	265	279	-14
Q4	305	279	26

When we next multiply the difference between bought MWh and needed MWh with the simulated electricity prices we get the money flow of the update of the hedge. In this case we get

$$\begin{aligned}
 & 43\text{MWh} \cdot 54.15 \frac{\text{€}}{\text{MWh}} - 55\text{MWh} \cdot 45.71 \frac{\text{€}}{\text{MWh}} \\
 & - 14\text{MWh} \cdot 44.97 \frac{\text{€}}{\text{MWh}} + 26\text{MWh} \cdot 52.54 \frac{\text{€}}{\text{MWh}} \\
 & = 550\text{€}
 \end{aligned} \tag{5.22}$$

Thus we would have to pay 550 € for updating the yearly contract to quarterly contracts so that we would be fully hedged. This equals to 1.11 % of the cost of original hedge. Next we would calculate the cost of updating the quarters to months in a similar way and as a result we would get a cost for the total update from year to month. In this example it would be 62 € as shown in

Table 4. The total cost would be 612 € or 1.23 % compared to original money flow for making the hedge.

¹⁴ Note that the amount of electricity bought for each quarter is not equal as the numbers of days in quarters are not equal. We have more days in quarter 4 than quarter 1 and thus we have more MWh in quarter 4 than quarter 1. Please recall that the original hedge is done using a forward with constant demand.

Table 4 example of update costs from quarter to month. Positive numbers in Money flow means you have to buy a forward for that month and negative numbers means you have to sell a forward for that month.

Month	Need [MWh]	Bought [MWh]	Difference [MWh]	Price [€]	Money flow [€]
Jan	110	109	1	57,53 €	83 €
Feb	100	98	2	56,87 €	111 €
Mar	105	108	-3	53,84 €	-183 €
Apr	80	73	7	40,39 €	302 €
May	70	75	-5	39,40 €	-195 €
Jun	70	73	-3	39,45 €	-100 €
Jul	80	89	-9	40,40 €	-375 €
Aug	90	89	1	42,23 €	30 €
Sep	95	86	9	43,60 €	374 €
Oct	100	103	-3	42,48 €	-122 €
Nov	100	99	1	45,86 €	27 €
Dec	105	103	2	48,14 €	110 €
Total	1105	1105			62 €

This same procedure is repeated 400 times to get a simulated distribution of the update costs. In this case we would get that the average update cost would be 1.01 % with a standard deviation of 0.22 %. This can then be used to price the risk for example by giving a price of cost + margin one standard deviation. This total margin would then be added to the forward price curve that is calculated previously in Section 5.4. For example it can be added evenly on all the months. As a result we have a customer specific forward price for year 2015. The same process is repeated for other years as well with a difference that for those years when we have quarterly forwards available we would use the cost of updating from quarters to months instead of years to months. In addition for the next 6 months we would use directly the quoted monthly prices as they are already available in the market. As a final result we will get a customer specific forward price that includes the cost of update plus a risk component of the update cost.

5.8. Comparing simulated cost to historical costs

To validate the simulated costs we compared to simulated update cost to update costs calculated from historical prices. We used the data set of 2006 to 2009 for the historical cost as well. We calculated the money flow in historical update and compared it to the money flow of the yearly hedge as we did with the simulated cases. We used the five customer demands shown in Table 5

for the comparison The costs are roughly on the same level for both cases, while the simulated cost is a bit lower. Results are shown in Chapter 6.7.

Table 6 Customer monthly demands (=loads) used in comparing the simulated and historical update costs. We compared 5 customers with different seasonal demands. Note that the Totally flat demand is not constant in MWh, because there are different amount of hours in each month.

Month	Demand for different customers on Monthly level that were used in this study				
	Counter cyclical demand [MWh]	Totally flat demand [MWh]	Close to flat demand [MWh]	Cyclical demand [MWh]	Cyclical demand 2 [MWh]
Jan	80	744	105	370	220
Feb	70	672	100	380	210
Mar	80	743	100	300	230
Apr	80	720	90	200	150
May	70	744	95	170	117
Jun	130	720	90	160	86
Jul	120	744	90	130	73
Aug	130	744	95	180	84
Sep	95	720	95	250	115
Oct	90	745	100	300	166
Nov	80	720	100	310	205
Dec	70	744	105	350	232

For further validating our model we also compared the spreads in historical prices and simulated prices. We calculated the average spreads for them both and in addition compared the spreads calculated from seasonal components that are behind the simulation. Results are shown in details in Chapter 6.7.

6. Analysis and results

In this Chapter we will show the results of modelling with Borovkova and Geman's methods. The seasonal factors or components represent the expected premium that particular month or quarter has compared to average price. We simulated how the different time point of updating the long-term hedges to shorter-term hedges influences the cost. We notice that the result of increasing volatility in costs with time to maturity decreasing is indeed in line with Samuelson's effect. When comparing the cost for different customer demand the simulation gives reasonable results as the costs are higher for customers who have more seasonal demand.

6.1. Seasonal components for months and quarters

The seasonal components used in Borovkova and Geman's model represent weight factors for each quarter and season. They were calculated with Nord Pool closing price data from 2006 to 2009. The monthly components calculated with the method in appendix of Borokova and Geman (2006) are shown in Table 7. Even though the standard deviation is high, the results seem to be reasonable as the changes in the components follow the changes in average temperature differences of the months. This can be seen from Figure 10 which plots the long-term average temperatures for Stockholm and inversion of seasonal components. To ease the comparison they are both scaled to have a maximum of 1 and minimum of zero. The Stockholm temperatures were chosen because Stockholm lies fairly in the middle of the Nord Pool area. We can see that the series follow each other, but there are some gaps in April and December, in which the inverted seasonal component is higher than expected by the temperature, thus the actual seasonal component in these months is lower than expected solely by the Stockholm average temperature. In August the difference is other way around as the seasonal component is higher than expected by average temperature. The point of this comparison was not to explain the seasonal components purely by the temperature in Stockholm, it was more to just illustrate that the main driver behind the seasonal behaviour in electricity prices is the temperature. There are many other things also affecting the seasonal components, like the expected rain falls in Norway.

Table 7 Monthly components (premium) in Nord Pool calculated with Borovkova and Geman's (2006) method. The components are in logarithmic form and thus also the weight factor is presented, which can be used directly to the prices.

Month	Seasonal component	Standard deviation	Median	Weight factor
Jan	0,146	0,089	0,119	1,157
Feb	0,134	0,089	0,107	1,143
Mar	0,084	0,104	0,067	1,087
Apr	-0,024	0,104	-0,008	0,976
May	-0,070	0,109	-0,070	0,932
Jun	-0,087	0,118	-0,072	0,916
Jul	-0,147	0,129	-0,115	0,863
Aug	-0,102	0,110	-0,073	0,903
Sep	-0,074	0,107	-0,051	0,929
Oct	0,005	0,115	0,024	1,005
Nov	0,054	0,075	0,049	1,056
Dec	0,082	0,067	0,071	1,085

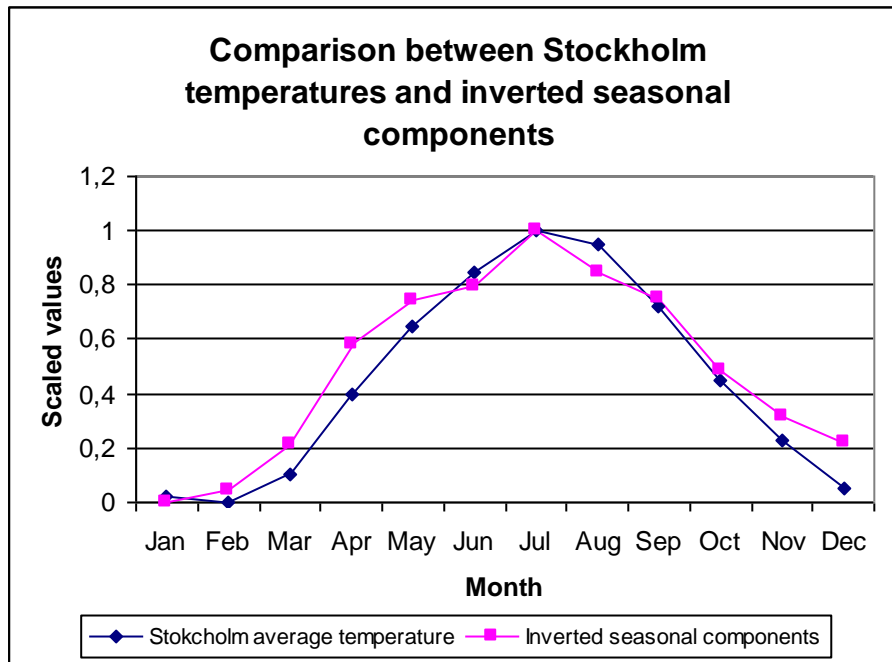


Figure 10 Scaled average temperatures in Stockholm and scaled inverted seasonal components. Temperature data is provided by Foreca (MSN Weather, 2010).

Table 8 shows the seasonal quarterly components and their standard deviation while Table 9 shows the monthly components and their standard deviations calculated from the alternative

approach described in Chapter 5. As can be seen from the results the standard deviation is relatively high especially for monthly components.

Table 8 Quarterly seasonal components in Nord Pool calculated with Borokova and Geman's (2006) model.

Calculated using quarters to maturity in		Calendar quarters and the corresponding components			
		Q1	Q2	Q3	Q4
Q2 to Q5	Component	0,118	-0,058	-0,094	0,034
	Standard deviation	0,061	0,04	0,067	0,051
Q3 to Q6	Component	0,109	-0,062	-0,086	0,039
	Standard deviation	0,032	0,033	0,031	0,032
Q4 to Q7	Component	0,111	-0,065	-0,091	0,046
	Standard deviation	0,025	0,019	0,029	0,022
Q5 to Q8	Component	0,111	-0,065	-0,094	0,048
	Standard deviation	0,029	0,019	0,028	0,017
Last 4 Q	Component	0,111	-0,067	-0,094	0,05
	Standard deviation	0,024	0,019	0,025	0,024

Table 9 Monthly seasonal components in Nord Pool calculated for shift from quarterly forwards to monthly forwards.

Month	Seasonal component	Standard deviation
January	0,025	0,013
February	0,033	0,009
March	-0,055	0,018
April	0,036	0,027
May	-0,018	0,014
June	-0,018	0,021
July	-0,105	0,056
August	0,017	0,010
September	0,091	0,057
October	-0,085	0,070
November	0,026	0,018
December	0,060	0,062

When the results in Tables 5 and 6 are combined one can make monthly components for shift from yearly forwards to monthly forwards via quarterly forwards. These are simply sum of monthly and quarterly components and the results are shown in Table 10. The results seem to be otherwise reasonable, but the component for September is higher than for October. This is probably due to the nature of the data which has some uncertainty as the standard deviations are

high. For this reason we decided not to use the alternative approach and instead we used quarterly components calculated with Borovkova and Geman's (2006) original method and monthly components using the method in the appendix of Borovkova and Geman (2006).

Table 10 Monthly components in Nord Pool calculated for shift from yearly to quarterly and then to monthly forwards.

Month	Seasonal component	Weight
January	0,136	1,146
February	0,145	1,155
March	0,057	1,058
April	-0,030	0,970
May	-0,084	0,919
June	-0,084	0,919
July	-0,199	0,820
August	-0,077	0,926
September	-0,003	0,997
October	-0,034	0,966
November	0,076	1,079
December	0,111	1,117

6.2. Average forward price

In Borovkova and Geman's (2006) model the average forward price is the first state variable and it is calculated over a year or multiple years of time. As in Nord Pool there are only monthly forwards available for next 6 months we used the 3 consecutive months starting from the first month of the next quarter. If next quarter starts in next month then we will take the following quarter as a starting point. For the following three quarters we used quarterly forwards and thus we have a forward price for a year constructed from 3 months and 3 quarters. The year starts from 2 to 4 months from the quoting date of the data. By this choice we can omit the closest month and quarter as they most volatile and highly influenced by current state of power system balance.

The Figure 11 illustrates the calculated average forward price. There is a huge fall in the price in April – May in 2006 when the price fell by approximately 15 € in a few days. Also in August there is a deep short lived spike upwards in the price. The average of the price for the whole period was 45.3 € and standard deviation 8.9 €. As mentioned in Chapter 5 and illustrated in

Figure 9 the principal component follows closely the average price and can be used as a representative of it.



Figure 11 Average forward price for next year. The year starts in 2 to 4 months.

6.3. Principal components of forward curve

Principal components were calculated from de-seasoned forward curves and it was done with EViews. Results are shown in Table 11 to Table 13. As we can see the first principal components explains 84 % of the volatility while three first components combined explain 99 % of it. As a rule of thumb one should use the principal components that have an eigenvalue of 1. We decided to include also the principal component 3 as it is also used in the Borovkova and Gemans (2006b) paper.

Table 11 Eigenvalues of principal components.

Number	Value	Difference	Proportion	Cumulative	Cumulative
				Value	Proportion
1	10,02	8,22	0,84	10,02	0,84
2	1,80	1,71	0,15	11,82	0,99
3	0,09	0,06	0,01	11,91	0,99
4	0,03	0,01	0,00	11,94	0,99
5	0,02	0,01	0,00	11,96	1,00
6	0,01	0,01	0,00	11,97	1,00
7	0,01	0,00	0,00	11,98	1,00
8	0,01	0,00	0,00	11,99	1,00
9	0,00	0,00	0,00	12,00	1,00
10	0,00	0,00	0,00	12,00	1,00
11	0,00	0,00	0,00	12,00	1,00
12	0,00	---	0,00	12,00	1,00

Table 12 Eigenvector loadings of principal components. PC 1 to PC 12 refers to principal components 1 to 12. M1 means forward that starts in one month and lasts for, M2 starts in 2 months etc. Similarly Q5 means a forward whose delivery period is after 5 quarters and the delivery period is one quarter.

Variable	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10	PC 11	PC 12
M1	0,26	-0,39	0,59	-0,02	0,14	-0,38	0,16	0,38	-0,17	0,24	0,06	0,11
M2	0,28	-0,34	0,31	0,14	-0,04	-0,01	0,05	-0,54	0,46	-0,42	-0,10	0,01
M3	0,29	-0,28	-0,02	0,22	-0,14	0,37	-0,50	-0,23	-0,42	0,22	0,17	0,27
M4	0,30	-0,21	-0,27	0,06	0,00	0,35	-0,09	0,65	0,44	-0,16	-0,04	0,15
M5	0,31	-0,15	-0,43	-0,18	0,25	0,06	0,59	-0,27	0,03	0,34	0,18	0,19
Q1	0,30	-0,22	-0,14	-0,04	0,02	0,08	-0,02	0,02	-0,19	0,05	-0,18	-0,87
Q2	0,31	0,00	-0,28	-0,42	-0,08	-0,39	-0,12	0,02	-0,36	-0,50	-0,21	0,23
Q3	0,30	0,21	-0,08	-0,10	-0,55	-0,39	-0,20	-0,03	0,35	0,29	0,38	-0,11
Q4	0,28	0,32	-0,03	0,56	-0,28	-0,07	0,28	0,03	-0,12	0,12	-0,54	0,13
Q5	0,28	0,35	-0,05	0,44	0,49	-0,16	-0,06	0,04	-0,09	-0,29	0,48	-0,11
Q6	0,27	0,36	0,16	-0,29	0,47	0,05	-0,38	-0,12	0,24	0,32	-0,38	0,06
Q7	0,27	0,39	0,41	-0,33	-0,23	0,51	0,30	0,05	-0,16	-0,19	0,19	-0,04

Table 13 Ordinary correlations of principal components. M1 means forward that starts in one month and lasts for, M2 starts in 2 months etc. Similarly Q5 means a forward whose delivery period is after 5 quarters and the delivery period is one quarter.

	M1	M2	M3	M4	M5	Q1	Q2	Q3	Q4	Q5	Q6	Q7
M1	1,00											
M2	0,99	1,00										
M3	0,96	0,98	1,00									
M4	0,92	0,96	0,99	1,00								
M5	0,89	0,93	0,96	0,99	1,00							
Q1	0,94	0,97	0,99	0,99	0,99	1,00						
Q2	0,81	0,86	0,91	0,95	0,97	0,95	1,00					
Q3	0,65	0,71	0,78	0,83	0,87	0,82	0,95	1,00				
Q4	0,53	0,60	0,68	0,74	0,79	0,73	0,89	0,98	1,00			
Q5	0,49	0,56	0,65	0,71	0,77	0,70	0,87	0,96	0,99	1,00		
Q6	0,49	0,56	0,63	0,70	0,75	0,69	0,86	0,96	0,98	0,99	1,00	
Q7	0,46	0,52	0,59	0,65	0,70	0,64	0,83	0,95	0,97	0,97	0,99	1,00

The level components calculated from principal components 4 to 12 to the forward prices are summarized to a level component given in Table 14.

Table 14 The level representing the last 9 principal components. M1 means forward that starts in one month and lasts for, M2 starts in 2 months etc. Similarly Q5 means a forward whose delivery period is after 5 quarters and the delivery period is one quarter.

Forward	Level
M1	-0,01
M2	0,01
M3	-0,02
M4	-0,05
M5	-0,08
Q1	0,03
Q2	0,13
Q3	0,14
Q4	-0,03
Q5	-0,11
Q6	-0,02
Q7	0,01

6.4. Descriptive statistics for series derived from principal components and real prices

We calculated descriptive statistics from the logarithmic price series covering the next 5 months prices and next 7 quarters prices. We looked for mean, variance, kurtosis and skewness and the results for series constructed from principal components and the real series are shown

Table 15. The statistics are quite close to each other and thus the principal components seem to be able to capture the properties of the data well. In addition also the series suggests that volatility decreases with time to maturity with is consistent with the Samuelson's effect.

Table 15 Descriptive statistics for forward curves constructed from principal components compared to real price data. M1 means forward that starts in one month and lasts for, M2 starts in 2 months etc. Similarly Q5 means a forward whose delivery period is after 5 quarters and the delivery period is one quarter. Prices are natural logarithms of actual prices according to Borovkova and Geman's (2006) model.

Real prices

	M1	M2	M3	M4	M5	Q1	Q2	Q3	Q4	Q5	Q6	Q7
Mean	3,68	3,71	3,74	3,76	3,78	3,77	3,79	3,80	3,80	3,80	3,79	3,79
Sample Variance	0,10	0,10	0,10	0,09	0,08	0,08	0,05	0,03	0,03	0,04	0,03	0,02
Kurtosis	-	-	-	-	-	-	-	-	-	-	-	-
Skewness	0,67	0,72	0,74	0,65	0,61	0,67	0,73	0,67	0,57	0,16	0,32	0,63
	-	-	-	-	-	-	-	-	-	-	-	-
Skewness	0,13	0,00	0,10	0,14	0,22	0,25	0,18	0,06	0,03	0,04	0,21	0,34

Reconstructed real prices from principal components

	M1	M2	M3	M4	M5	Q1	Q2	Q3	Q4	Q5	Q6	Q7
Mean	3,69	3,73	3,73	3,72	3,70	3,79	3,79	3,80	3,80	3,80	3,79	3,80
Sample Variance	0,08	0,08	0,07	0,06	0,05	0,06	0,05	0,03	0,03	0,03	0,03	0,02
Kurtosis	-	-	-	-	-	-	-	-	-	-	-	-
Skewness	0,86	0,91	0,96	1,02	1,07	1,00	1,03	0,64	0,46	0,48	0,22	0,08
	-	-	-	-	-	-	-	-	-	-	-	-
Skewness	0,15	0,18	0,16	0,12	0,07	0,16	0,15	0,24	0,19	0,16	0,26	0,35

6.5. Cost between different update timing

We calculated the update costs and standard deviations for different update timing. We compared early, middle and late update and for typical customer shown in Table 16. Based on the simulation we observe that middle update cost is on average highest, but one should keep in mind that the early prices are illiquid and buying them on large amount is not possible. When

taking into account the standard deviation we see that the average costs are close to each other as they are all inside the lowest standard deviation. So no reliable estimations can be about the cost level between the update timing. However, it is clear that the standard deviation of the update costs increases when the delivery period comes closer.

Table 16 Comparing update cost with different timing for typical customer. Cost is relative to the money flow of buying the original yearly forward.

	Cost	St_dev
Cost of early update relative to original hedge	1,59 %	0,24 %
Cost of middle update relative to original hedge	1,61 %	0,28 %
Cost of late update relative to original hedge	1,61 %	0,50 %

6.6. Relative cost of update the hedges for different demands

The pricing tool was tested with different demands and the relative cost between different demands is sensible. For example a counter cyclical demand like Google server farm would actually get discount for their electricity price relative to the real forward prices. As an example we have illustrated some demands and their update costs in Table 17. We see that the demands having high seasonality are more expensive to hedge as the updating cost is higher.

Table 17 Example of simulated update costs for different demands. Demands are same as in Table 18.

	Counter cyclical demand [MWh]	Totally flat demand [MWh]	Close to flat demand [MWh]	Cyclical demand [MWh]	Cyclical demand 2 [MWh]
Update cost [%]	-1,34 %	0,00 %	0,41 %	2,45 %	2,97 %
Standard deviation [%]	0,43 %	0,00 %	0,13 %	0,77 %	0,88 %
Total premium [%]	-0,91 %	0,00 %	0,53 %	3,22 %	3,84 %

6.7. Comparing simulated costs to historical costs

We compared the historical and simulated costs of updating the hedges to check if the simulated results are reasonable. The results are shown in Table 19. We also calculated the update costs on year to quarter and quarter to month for comparison. These are shown in Table 20 and Table 21. From Table 19 we see that the costs are close to each other but the simulated costs are a bit lower than the historical costs. When looking closer we see that the simulated cases give slightly higher

cost than historical cost on year to quarter update (Table 20), while the simulated costs of update are lower than historical costs on quarter to month level (Table 21). Standard deviations are close to the same in all cases.

Table 19 Comparing simulated and historical update costs for the hedges. The update costs are calculated for five different customer demands and they are calculated from updating the hedges from yearly level to monthly level.

Simulated	Counter cyclical	Totally flat	Close to flat	Cyclical	Cyclical 2
Update cost	-1,34 %	0,00 %	0,41 %	2,45 %	2,95 %
Standard deviation	0,43 %	0,00 %	0,13 %	0,77 %	0,93 %
Historical	Counter cyclical	Totally flat	Close to flat	Cyclical	Cyclical 2
Update cost	-1,42 %	0,00 %	0,46 %	2,76 %	3,13 %
Standard deviation	0,41 %	0,00 %	0,13 %	0,79 %	0,95 %

Table 20 Comparing simulated and historical update costs for the hedges. The update costs are calculated for five different customer demands and they are calculated from updating the hedges from yearly level to quarterly level.

Simulated	Counter cyclical	Totally flat	Close to flat	Cyclical	Cyclical 2
Update cost	-1,10 %	0,00 %	0,39 %	2,34 %	2,70 %
Standard deviation	0,37 %	0,00 %	0,13 %	0,76 %	0,93 %
Historical	Counter cyclical	Totally flat	Close to flat	Cyclical	Cyclical 2
Update cost	-1,09 %	0,00 %	0,39 %	2,33 %	2,68 %
Standard deviation	0,39 %	0,00 %	0,13 %	0,76 %	0,93 %

Table 21 Comparing simulated and historical update costs for the hedges. The update costs are calculated for five different customer demands and they are calculated from updating the hedges from quarterly level to monthly level.

Simulated	Counter cyclical	Totally flat	Close to flat	Cyclical	Cyclical 2
Update cost	-0,24 %	0,00 %	0,02 %	0,12 %	0,27 %
Standard deviation	0,14 %	0,00 %	0,03 %	0,13 %	0,17 %
Historical	Counter cyclical	Totally flat	Close to flat	Cyclical	Cyclical 2
Update cost	-0,33 %	0,00 %	0,08 %	0,43 %	0,45 %
Standard deviation	0,14 %	0,00 %	0,05 %	0,22 %	0,22 %

When deeper looking into the differences, we also compared the spreads between the monthly and quarterly prices, by comparing the prices relative to the cheapest month and quarter. We compared the monthly prices inside one quarter and quarterly prices inside one year. For example we compared the spreads between January and March and February and March. We

calculated these spreads also from the seasonal components. The results are shown in Table 22 and Table 23. We see that spreads calculated from the monthly components of Borovkova and Gemans method are lower than historical spreads. Interestingly, the alternative way¹⁵ to calculate seasonal components gives closer spreads to historical values. However, as the historical spreads as well as the alternative way of calculating the components have some illogical behaviour, we chose to go with the seasonal components calculated from Borovkova and Geman's method. By illogical behaviour we mean that the alternative approach to calculate the seasonal components gives October lower price than for September, while the historical spreads give May higher price than June. The seasonal monthly components calculated with Borovkova and Geman's approach are a bit lower but they are in reasonable relationship towards each other. When comparing the simulated costs and the spreads we can see that the lower cost comes from the seasonal components, because when we look at quarterly level both the spreads and the simulated and historical costs are all close to each other, regardless the way they are calculated. However, on the monthly level both the seasonal components and the simulated costs are lower than historical ones. The question then is should we rely on the seasonal components or historical spreads? We suggest relying on the simulated values as they are logically more plausible as explained above.

Table 22 Spreads between monthly prices in historical prices and simulated prices. For comparison we also show the spreads in seasonal components. The spreads are calculated inside one quarter.

Spreads [€]	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Historical	3,86	4,27	0,00	1,59	-0,27	0,00	0,00	4,21	6,97	0,00	4,25	6,89
Simulated	1,40	1,70	0,00	1,00	0,12	0,00	0,00	2,14	3,89	0,00	2,78	4,70
Seasonal Bor & Gem	3,14	2,53	0,00	2,68	0,71	0,00	0,00	1,80	2,95	0,00	2,29	3,60
Alternative seasonal	3,93	4,37	0,00	2,31	0,01	0,00	0,00	4,79	7,96	0,00	5,08	6,78

Table 23 spreads between quarterly prices in historical prices and simulated prices. For comparison we also show the spreads in seasonal components.

Spreads [€]	Q1	Q2	Q3	Q4
Historical	9,71	1,29	0,00	5,89
Simulated	9,08	1,12	0,00	6,27
Seasonal Bor & Gem	9,34	1,15	0,00	6,38

In addition, by using the seasonal components calculated with Borovkova and Geman's method, we follow the original approach of Borovkova and Geman (2006) as close as possible. If one

¹⁵ For more about the alternative way please look at chapter 5.2.

would like to replicate the historical costs, then the alternative approach to calculate the monthly seasonal components could be used. But as the idea was to model plausible future scenarios and not to mirror the past to the future, we chose to go with the seasonal components from Borovkova and Geman's (2006) method.

If we look back at Chapter 6.1. we see that the standard deviation in the monthly seasonal components is clearly higher than for quarterly components. There is thus more uncertainty on monthly level than quarterly level, which is reasonable as the monthly prices are closer to delivery and thus more influenced by spot prices. For our model this also means that the model is more reliable on quarterly level than on monthly level. Luckily, as the update cost is mainly determined by the update from year to quarter, it is more important to get the update cost on year to quarter level than on quarter to month level to be correct. And when we look at the different ways of calculating the update cost on year to quarter, we see that they all give similar results. This is relaxing and we can conclude that the model is roughly accurate, but when using it one should keep in mind that historical update costs on monthly level have been higher than what the simulated costs suggest.

7. Summary and conclusions

Electricity retailer enters automatically to a short position when she sells electricity to her customer for a fixed price. If the electricity is sold for near future, the retailer can use forward with varying maturities to hedge her short position. However, in Nord Pool, the long-term forwards are available only for calendar years and forwards for delivery periods in calendar months and quarters become available only closer to delivery period. As the demand is seasonal, buying the yearly forward leaves part of the exposure unhedged and part of the exposure overhedged. The hedge needs to be balanced closer to delivery of electricity with quarterly and monthly forwards when they become available. This thesis studied the cost of these updates as well as compared different timing of making the updates. The analysis was done by simulating possible future states of the forward prices and then calculating the cost of updating the hedges based on them.

We started our analysis by modelling seasonality in forward prices with a method developed by Borovkova and Geman (2006). The idea was to look at the relationships between electricity forward prices inside a time period that constructs a whole year. In this way the reference price does not contain seasonality. We further used the method Borovkova and Geman (2006b) published in their second paper to model the deviations of the price curve from flat de-seasoned curve using principal components analysis. By using principal components we were able to reduce the amount of data needed to model the forward prices, still being able to capture the dependencies between different prices. We modelled the movements of principal components and simulated future states of forward prices using these principal components. After simulating 100 future states we collected 400 different points of updating the hedges for each contract and then we calculated the update cost and standard deviation for the costs.

By using simulated prices we could calculate the update cost of the long-term hedges to shorter-term hedges for different customers. The costs were compared with actual Fortum customer data as well as with artificial user data and they all give logical results. The cost for customer who has highly seasonal demand is higher than the cost for user with relatively flat demand. It also gives negative update cost for user who has counter cyclical demand. As the results are reasonable, this

study gives support to using Borovkova and Geman's method for modelling seasonality and electricity price behaviour. However, when comparing the simulated costs to historical costs we observe that simulated costs are lower on monthly level than historical ones. This difference is due to the seasonal components / premia, calculated with Borovkova and Geman's (2006) method and using different method to calculate them could be used to get simulated costs close to historical ones. However, as the spreads between the monthly prices calculated with Borovkova and Geman's (2006) method, gives the most plausible relationship between the prices of different months, we suggest using them.

From the simulations we can conclude that the average cost of update is close to each other when comparing different timing of updating the hedges. However, the standard deviation of the update cost increases clearly when the updating is delayed. This is consistent with the Samuelson's effect that the volatility drops with time to maturity. Thus we suggest that the updates should be done quite early, that is when the liquidity in the market is high enough, as then we can reduce the risk of increased volatility in the update costs.

The de-seasoned curve calculated with Borovkova and Geman's (2006) method works also well for constructing a marked to market forward curve. When the prices are de-seasoned we can look at the trend inside the price curve and after adding seasonality back to the curve we get a forward curve that is based on latest market price information and contains the seasonal information of historical forward curves.

Further research could be done by expanding the analysis to contracts for difference. It could be studied if some kind of seasonality is found in them as well. In addition research in the area of improving the simulation is needed. The speed of simulation is currently not optimized and also the simulation could be improved further. One could also try to model shorter maturity forwards with similar methods. However, as the behaviour of electricity spot markets are quite different from its forward markets and as shorter maturity forwards are influenced by spot markets the model needs probably some modification. In addition also very long-term term structure which takes place inside decades and which is caused by among others shifts in the average temperature could be modelled when more data becomes available.

An interesting topic for further research would be to compare seasonality in different power markets and look if the drivers for it are the same. Temperature clearly affects the electricity price in the Nordic markets but in areas with relative stable temperature other factors could still cause prices to behave in seasonal way.

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Appendix

Matlab code that was used in simulation. The codes are based on codes written by Smith (2010) and modified to the needs for this thesis.

```
function [output] = use();

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This is the user interface for testing the model
% It is created by Matias Vitie and
% It uses functions written by Willian Smith
% Created March 23 2010 and latest modification made on May 7 2010
% It writes to excel 10 simulated series of principal components 1, 2 and 3
% The output is used for testing of the code and it is 0 until the code is
% finished (then it gives an output on 1)

%First we will read the input data directly from file
input_data_pc1 =xlsread('Filename', 'Sheet1');
input_data_pc2 =xlsread('Filename', 'Sheet2');
input_data_pc3 =xlsread('Filename', 'Sheet3');
output = 0;
%Then we will estimate the parameters from the original input data
[mu_pc1,sigma_pc1,lambda_pc1]=CalibrateOrnsteinUhlenbeckRegress(input_data_pc
1,1/251,4);
[mu_pc2,sigma_pc2,lambda_pc2]=CalibrateOrnsteinUhlenbeckRegress(input_data_pc
2,1/251,4);
[mu_pc3,sigma_pc3,lambda_pc3]=CalibrateOrnsteinUhlenbeckRegress(input_data_pc
3,1/251,4);
% next we will simulate 10 sets of data
[s1_pc1, s1_pc2, s1_pc3] =
SimulateOrnsteinUhlenbeckRough(mu_pc1,mu_pc1,sigma_pc1,lambda_pc1,mu_pc2,mu_p
c2,sigma_pc2,lambda_pc2,mu_pc3,mu_pc3,sigma_pc3,lambda_pc3,1/251,4);
[s2_pc1, s2_pc2, s2_pc3] =
SimulateOrnsteinUhlenbeckRough(mu_pc1,mu_pc1,sigma_pc1,lambda_pc1,mu_pc2,mu_p
c2,sigma_pc2,lambda_pc2,mu_pc3,mu_pc3,sigma_pc3,lambda_pc3,1/251,4);
[s3_pc1, s3_pc2, s3_pc3] =
SimulateOrnsteinUhlenbeckRough(mu_pc1,mu_pc1,sigma_pc1,lambda_pc1,mu_pc2,mu_p
c2,sigma_pc2,lambda_pc2,mu_pc3,mu_pc3,sigma_pc3,lambda_pc3,1/251,4);
[s4_pc1, s4_pc2, s4_pc3] =
SimulateOrnsteinUhlenbeckRough(mu_pc1,mu_pc1,sigma_pc1,lambda_pc1,mu_pc2,mu_p
c2,sigma_pc2,lambda_pc2,mu_pc3,mu_pc3,sigma_pc3,lambda_pc3,1/251,4);
[s5_pc1, s5_pc2, s5_pc3] =
SimulateOrnsteinUhlenbeckRough(mu_pc1,mu_pc1,sigma_pc1,lambda_pc1,mu_pc2,mu_p
c2,sigma_pc2,lambda_pc2,mu_pc3,mu_pc3,sigma_pc3,lambda_pc3,1/251,4);
[s6_pc1, s6_pc2, s6_pc3] =
SimulateOrnsteinUhlenbeckRough(mu_pc1,mu_pc1,sigma_pc1,lambda_pc1,mu_pc2,mu_p
c2,sigma_pc2,lambda_pc2,mu_pc3,mu_pc3,sigma_pc3,lambda_pc3,1/251,4);
[s7_pc1, s7_pc2, s7_pc3] =
SimulateOrnsteinUhlenbeckRough(mu_pc1,mu_pc1,sigma_pc1,lambda_pc1,mu_pc2,mu_p
c2,sigma_pc2,lambda_pc2,mu_pc3,mu_pc3,sigma_pc3,lambda_pc3,1/251,4);
```

```

[s8_pc1, s8_pc2, s8_pc3] =
SimulateOrnsteinUhlenbeckRough(mu_pc1,mu_pc1,sigma_pc1,lambda_pc1,mu_pc2,mu_p
c2,sigma_pc2,lambda_pc2,mu_pc3,mu_pc3,sigma_pc3,lambda_pc3,1/251,4);
[s9_pc1, s9_pc2, s9_pc3] =
SimulateOrnsteinUhlenbeckRough(mu_pc1,mu_pc1,sigma_pc1,lambda_pc1,mu_pc2,mu_p
c2,sigma_pc2,lambda_pc2,mu_pc3,mu_pc3,sigma_pc3,lambda_pc3,1/251,4);
[s10_pc1, s10_pc2, s10_pc3] =
SimulateOrnsteinUhlenbeckRough(mu_pc1,mu_pc1,sigma_pc1,lambda_pc1,mu_pc2,mu_p
c2,sigma_pc2,lambda_pc2,mu_pc3,mu_pc3,sigma_pc3,lambda_pc3,1/251,4);
% % Now we will write the output to an excel file
xlswrite('Filename output', s1_pc1, 'Sheet1', 'A2:A1005');
xlswrite('Filename output', s1_pc2, 'Sheet1', 'B2:B1005');
xlswrite('Filename output', s1_pc3, 'Sheet1', 'C2:C1005');
xlswrite('Filename output', s2_pc1, 'Sheet1', 'D2:D1005');
xlswrite('Filename output', s2_pc2, 'Sheet1', 'E2:E1005');
xlswrite('Filename output', s2_pc3, 'Sheet1', 'F2:F1005');
xlswrite('Filename output', s3_pc1, 'Sheet1', 'G2:G1005');
xlswrite('Filename output', s3_pc2, 'Sheet1', 'H2:H1005');
xlswrite('Filename output', s3_pc3, 'Sheet1', 'I2:I1005');
xlswrite('Filename output', s4_pc1, 'Sheet1', 'J2:J1005');
xlswrite('Filename output', s4_pc2, 'Sheet1', 'K2:K1005');
xlswrite('Filename output', s4_pc3, 'Sheet1', 'L2:L1005');
xlswrite('Filename output', s5_pc1, 'Sheet1', 'M2:M1005');
xlswrite('Filename output', s5_pc2, 'Sheet1', 'N2:N1005');
xlswrite('Filename output', s5_pc3, 'Sheet1', 'O2:O1005');
xlswrite('Filename output', s6_pc1, 'Sheet1', 'P2:P1005');
xlswrite('Filename output', s6_pc2, 'Sheet1', 'Q2:Q1005');
xlswrite('Filename output', s6_pc3, 'Sheet1', 'R2:R1005');
xlswrite('Filename output', s7_pc1, 'Sheet1', 'S2:S1005');
xlswrite('Filename output', s7_pc2, 'Sheet1', 'T2:T1005');
xlswrite('Filename output', s7_pc3, 'Sheet1', 'U2:U1005');
xlswrite('Filename output', s8_pc1, 'Sheet1', 'V2:V1005');
xlswrite('Filename output', s8_pc2, 'Sheet1', 'W2:W1005');
xlswrite('Filename output', s8_pc3, 'Sheet1', 'X2:X1005');
xlswrite('Filename output', s9_pc1, 'Sheet1', 'Y2:Y1005');
xlswrite('Filename output', s9_pc2, 'Sheet1', 'Z2:Z1005');
xlswrite('Filename output', s9_pc3, 'Sheet1', 'AA2:AA1005');
xlswrite('Filename output', s10_pc1, 'Sheet1', 'AB2:AB1005');
xlswrite('Filename output', s10_pc2, 'Sheet1', 'AC2:AC1005');
xlswrite('Filename output', s10_pc3, 'Sheet1', 'AD2:AD1005');

output = 1;
end

```



```

function [ PC1, PC2, PC3 ] = SimulateOrnsteinUhlenbeckRough( PC10, mu, sigma,
lambda, dPC20, muPC2, sigmaPC2, lambdaPC2, dPC30, muPC3, sigmaPC3, lambdaPC3,
deltat, t, PC20, PC30 )
%% Approximate Ornstein-Uhlenbeck Generator. A more accurate version is
preferred
%% and available : SimulateOrnsteinUhlenbeck.
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USE OF THIS
% SOFTWARE, EVEN IF ADVISED OF THE POSSIBILITY OF SUCH DAMAGE. This Code has
been modified by Matias Vitie on May 6 2010.

% Initializing the data series
periods = floor(t / deltat);
S = zeros(periods, 1);
PC2 = zeros(periods, 1);
PC3 = zeros(periods, 1);
dPC2 = zeros(periods, 1);
dPC3 = zeros(periods, 1);
S(1) = S0;
PC2(1) = PC20;
PC3(1) = PC30;
% Brownian motion
dWt = sqrt(deltat) * randn(periods,1);
dW2t = sqrt(deltat) * randn(periods,1);
dW3t = sqrt(deltat) * randn(periods,1);

```

```
for t=2:1:periods
    dSt = lambda*(mu-S(t-1))*deltat + sigma*dWt(t);
    S(t) = S(t-1)+dSt;
    dPC2t = lambdaPC2*(muPC2-PC2(t-1))*deltat + sigmaPC2*dW2t(t);
    PC2(t) = PC2(t-1)+dPC2t;
    dPC3t = lambdaPC3*(muPC3-PC3(t-1))*deltat + sigmaPC3*dW3t(t);
    PC3(t) = PC3(t-1)+dPC3t;
end

% OPTIM Note :
% Precalculating all dWt's rather than one-per loop makes this function
% approx 50% faster. Useful for Monte-Carlo simulations.
% OPTIM Note : I tried calculating an array of dSt's and only doing a
cumsum() at
% the end, but it doesn't speedup any more.
end
```

```

function [ mu, sigma, lambda ] = CalibrateOrnsteinUhlenbeckRegress(S, deltat,
bigt)
% Calibrate an OU process by a simple discrete time regression.
% Does not properly take the reversion into account, meaning this will
% become inaccurate for large deltat.
%
% Use CalibrateOrnsteinUhlenbeckLeastSquares if deltat is small.
%
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USE OF THIS
% SOFTWARE, EVEN IF ADVISED OF THE POSSIBILITY OF SUCH DAMAGE.
% Regressions prefer row vectors to column vectors, so rearrange if
% necessary.
if (size(S,2) > size(S,1)) S = S';
end
% Regress S(t)-S(t-1) against S(t-1).
[ k,dummy,resid ] = regress(S(2:end)-S(1:end-1),[ ones(size(S(1:end-1)))
S(1:end-1) ] );
a = k(1);
b = k(2);
lambda = -b/deltat;
mu = a/lambda/deltat;
sigma = std(resid) / sqrt(deltat);
end

```