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Tails, volatility risk premium, and equity index returns

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Master's thesis

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Abstract

This thesis examines the explanatory power of equity index options and volatility indexes on short-term equity index returns. The work relates to literature that employs options to show what risks are compensated in financial assets and how to assess the riskiness of investments in general. The contribution of this thesis is to provide a comprehensive assessment of the main sources of predictive power by employing a wide range of option-implicit predictors. Moreover, global evidence is provided. The predictive information is based on risk-neutral nonparametric probability distributions of equity index returns, volatility indexes, and realized returns. The main focus is in using information available before the return periods to explain subsequent returns.

The considered predictive variables comprise of option-implied volatility, option-implied skewness, volatility risk premium, skew risk premium, and two tail risk measures. This thesis answers whether the variables are consistently behaving and statistically significant predictors across indexes. The involved indexes are S&P 500 (the U.S.), FTSE 100 (the U.K.), and DAX 30 (Germany). In addition, information on S&P 500 is used to explain returns of ten equity indexes globally, the remaining seven being Euro STOXX 50 (Europe), Nasdaq OMX Helsinki (Finland), Hang Seng (Hong Kong), Nikkei 225 (Japan), MXIPC35 (Mexico), MERVAL (Argentina), and S&P/ASX 200 (Australia). The sample ranges from February 2006 to December 2014, and the option data includes options with expiries from May 2006 to December 2014. Predictive regressions are run for one-week, one-month, two-month, and three-month returns.

The results show that tail risk and volatility risk premium are the main sources of predictive power, and that the variables are robust to the inclusion of other option-implicit variables and market valuation, interest rate, and dividend based alternative explanatory variables. The results hold for FTSE 100 and DAX 30 only if the information in S&P 500 options is used to explain returns. This implies that risks and premiums are global, and that the information is best reflected in S&P 500 options. Applying the information in S&P 500 options on ten indexes globally further confirms this finding. A one standard deviation increase in the ex ante volatility risk premium of S&P 500 on average leads to a 2.2% to 4.1% increase in three-month logarithmic returns globally. A one standard deviation increase in the tail risk measure leads on average to a 3.2% to 7.8% increase. Skew risk premium seems to contain the same information as the volatility risk premium. The tail risk measure and the volatility risk premium also likely contain similar predictive information, as the inclusion of the tail risk measure at most leads to a 1% increase in explained variation for the ten equity indexes. The predictive power in the variables comes mainly from the time-varying risk of large price movements and risk aversion.

Keywords equity index returns, tail risk, volatility risk, skew risk, ex ante moments, moment premiums, equity index options, volatility indexes

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Tiivistelmä

Tämä tutkielma tarkastelee osakeindeksioptioiden ja volatiliteetti-indeksien selittävää voimaa lyhyen aikavälin osakeindeksituottoihin. Työ liittyy kirjallisuuteen, joka hyödyntää optioita tutkiessaan mitä riskejä rahoitusinstrumenttien tuotot kompensoivat ja miten sijoitustoiminnan riskisyyttä tulee arvioida. Tämän tutkielman kontribuutio on antaa kattava arviointi olennaisista tekijöistä tuottojen taustalla soveltamalla optioissa olevaa informaatiota laaja-alaisesti. Tutkielman empiirinen osio kattaa globaalisti eri indeksejä. Selittävä informaatio perustuu ei-parametrisesti estimoituihin riskineutraaleihin osakeindeksien tuottojakaumiin, volatiliteetti-indekseihin ja toteutuneisiin tuottoihin. Informaation käyttäminen ennustamaan tulevia indeksituottoja on tutkielman keskiössä.

Selittävät muuttujat tutkielmassa ovat tuottojen implisiittinen volatiliteetti, implisiittinen vinous, volatiliteettiriskipreemio, vinousriskipreemio ja kaksi eri häntäriskin mittaria. Tutkimuskysymykset kysyvät, ovatko selittävät muuttujat tilastollisesti merkittäviä ja johdonmukaisesti käyttäytyviä osakeindeksituottojen selittäjiä. Osakeindeksit S&P 500 (Yhdysvallat), FTSE 100 (Yhdistynyt kuningaskunta) ja DAX 30 (Saksa) ovat empiirisen osion pääindeksit, ja lisäksi S&P 500-indeksin optioita ja volatiliteetti-indeksiä käytetään selittämään kymmenen eri osakeindeksin tuottoja maailmanlaajuisesti. Muut seitsemän indeksiä ovat Euro STOXX 50 (Eurooppa), Nasdaq OMX Helsinki (Suomi), Hang Seng (Hong Kong), Nikkei 225 (Japani), MXIPC35 (Meksiko), MERVAL (Argentiina) ja S&P/ASX 200 (Australia). Tutkielman regressioanalyysissä selitetään viikon ja yhden, kahden ja kolmen kuukauden logaritmisia osakeindeksien tuottoja.

Tulokset näyttävät, että häntäriski ja volatiliteettiriskipreemio ovat merkittävimmät tekijät selitettyjen tuottojen taustalla. Muuttujien selitysvoima säilyy sisällyttäessä muut optioihin liittyvät muuttujat ja arvostustaso-, korko- ja osinkoperusteiset muuttujat estimoituihin malleihin. Tulokset pitävät FTSE 100 ja DAX 30 indekseille jos informaatiota S&P 500-optioissa käytetään selittämään tuottoja. Tämän perusteella riskit ja preemiot ovat globaaleja, ja S&P 500-indeksin optiot parhaiten heijastavat tätä informaatiota. Kymmenen eri osakeindeksin tuottojen selittäminen S&P 500-indeksin informaatiolla tukee tätä havaintoa. Yhden keskihajonnan kasvu ennakoidussa volatiliteettiriskipreemiossa kasvattaa keskimäärin 2.2-4.1% osakeindeksin kolmen kuukauden logaritmista tuottoa. Yhden keskihajonnan kasvu häntäriskin mittarissa vastaavasti lisää tuottoa 3.2-7.8%. Vinousriskipreemio vaikuttaa sisältävän saman informaation kuin volatiliteettiriskipreemio. Sama pätee osittain sovelletulle häntäriskimittarille ja volatiliteettiriskipreemiolle, ja häntäriskimittarin sisällyttäminen globaaleja indeksituottoja selittävään malliin lisää enimmillään selitettyä vaihtelua 1%. Selittävä voima muuttujissa tulee enimmäkseen ajassa muuttuvasta suurten hinnanmuutosten riskistä ja sijoittajien riskinkaihtamisesta.

Avainsanat osakeindeksituotot, häntäriski, volatiliteettiriski, vinousriski, ennakoidut tuottojakauman momentit, momenttipreemiot, osakeindeksioptiot, volatiliteetti-indeksit

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1. Introduction

Forward-looking information in option prices powerfully explains aggregate stock market returns. This information also enables a comprehensive view on what risks are compensated in financial asset returns, and how to assess the riskiness of investments in general. In explaining market returns, the information in option prices seems superior to the explanatory power of market valuation, dividend, or interest rate based measures particularly for under one-year horizons. Especially the variance risk premium is shown to explain subsequent aggregate returns in the U.S. (see, e.g., Bollerslev, Tauchen, and Zhou, 2009; Drechsler and Yaron, 2011) and globally (see Bollerslev et al., 2014). The variance, or volatility, risk premium is the difference between risk-neutral and true expectations of return variance, and is usually measured as the difference between option-implied and realized variance. In addition, recently applied measures of tail risk show that the time-varying risk of large price movements is compensated with a premium. Tail risk is the risk of a sudden and deep decline in market valuations. Du and Kapadia (2012) provide an option-based jump and tail index, and Kelly and Jiang (2014) provide a measure based on historical returns. Bollerslev and Todorov (2011) and Santa-Clara and Yan (2010) show that price jumps have a major contribution to the equity premium using equity index options.

Related to the mentioned findings, studies that focus on the cross-section of stocks in the U.S. stock market show that risk measures based solely on return covariance with the market and measures that do not incorporate compensation for possible jumps in prices, as the CAPM beta, fail to account fully for priced risks in individual stocks. Cremers, Halling, and Weinbaum (2015) separate aggregate jump and volatility risk by using investable option trading strategies, and stocks that hedge against increases in volatility or jump risk earn lower returns. Conrad, Dittmar, and Ghysels (2013) and Chang, Christoffersen, and Jacobs (2013) find that ex ante moments, i.e. volatility, skewness, and kurtosis, of the expected risk-neutral return distribution[1] (RND) are related to future returns. If investors have preferences over these moments on the aggregate level, they should also provide explanatory power on future returns. Higher expected variance of returns could be, for instance, associated with a higher required rate of return as in the mean-variance framework. Moreover, the probability distribution of returns can show features that are not

[1] The cross-section of equity option prices at any given point of time reveals a risk-neutral probability distribution of the asset's price at the cross-section's maturity.

captured fully by the moments, but which explain future returns. One potential feature could be the tail of the probability distribution of expected returns.

The purpose of this study is to provide new empirical evidence on the explanatory power of option-based information on aggregate stock market returns by applying the recent findings in a global setting. The contribution of this study is twofold. First, a comprehensive range of option-implicit predictors is used simultaneously to explain subsequent equity index returns. The top-down approach aims to reveal the main drivers behind future returns and assess the robustness of key variables in earlier studies. Particularly interesting is the robustness of the volatility risk premium and the jump and tail index to simultaneous inclusion of RND moments. Second, evidence is provided for a number of indexes, as opposed to the common focus on the U.S. stock market. For comparison, ex post measures of the premiums are included, but the focus is on ex ante measures which are available before the return periods. The considered variables comprise of volatility risk premium, skew risk premium, option-implied volatility, option-implied skewness, and two tail measures[2]. The tail measures are the RNDs' tail probability and the jump and tail index applied by Du and Kapadia (2012).

Specifically, three questions are answered. First, are volatility and skew risk premiums consistently behaving predictors of returns across indexes? Second, are option-implied volatility and skewness consistently behaving predictors of index returns across indexes? Third, is the tail density or jump and tail index a consistently behaving predictor of returns across indexes? Consistent behavior refers to a similar impact of variables on subsequent returns, and not to the consistency of the statistical estimators. The theoretical basis of the variables makes the empirical results interesting from an asset pricing and risk management perspective. The tail measures are explicitly measures of crash risk in a risk-neutral world. Option-implied volatility and skewness also unveil only the risk-neutral expectation of the respective moments. The volatility and skew risk premiums are in this sense different. They enable inferences of investor risk aversion and preferences that drive the differences between true and risk-adjusted expectations. The moments are labeled model-free in this study as they are based on the RNDs that do not depend on any option-pricing model, and are driven by the observed prices of traded equity index options. To clarify, the moments use the entire cross-section of options with different strikes and do not relate

[2] Table II on page 28 provides variable definitions and Table B.1 in Appendix B provides relevant terminology.

to any particular strike price as, for instance, the at-the-money (ATM) Black-Scholes (BS) option-implied volatilities do.

The approach of this study relies on information in equity index options, equity index returns, and volatility indexes. Specifically, the option-related information is considered in the form of probability distributions, RNDs. The sample period ranges from February 2006 to December 2014, and the involved options have expiry dates from May 2006 to December 2014. One-week, one-month, two-month, and three-month logarithmic returns of S&P 500 (the U.S.), FTSE 100 (the U.K.), and DAX 30 (Germany) are explained with option-based information specific to the return period. After identifying the sources of predictive power the explanatory information of S&P 500 is used to explain returns for a number of equity indexes globally. The additional indexes comprise of Euro STOXX 50 (Europe), Nasdaq OMX Helsinki (Finland), Hang Seng (Hong Kong), Nikkei 225 (Japan), MXIPC35 (Mexico), MERVAL (Argentina), and S&P/ASX 200 (Australia).

The empirical results show that the ex post volatility risk premium is positively related to subsequent returns for horizons from one to three months. The ex ante volatility risk premium is also positively related to subsequent returns. The results on the ex ante premium are statistically significant for FTSE 100 and DAX 30 only at the three-month horizon and if information in S&P 500 options is used instead of options from the British or German markets. This is in line with the finding of Bollerslev et al. (2014) that a global variance risk premium is a more powerful predictor. A one standard deviation increase in the ex ante volatility premium on average leads to a 1.9% (3.7%) increase in two-month (three-month) logarithmic returns for the S&P 500. A one standard deviation increase in S&P 500's premium leads to a 2.0% (2.8%) increase in the three-month logarithmic returns of FTSE 100 (DAX 30). Skew risk premium is also tested, which to my best knowledge has not been done before, and the results indicate that the information is similar as in the volatility risk premiums, yet not as powerful. This suggests that the premiums are driven by the same factor.

Option-implied volatility and skewness fail to provide any conclusive evidence without the inclusion of the ex post volatility premium in the model, which means that their predictive power is negligible. S&P 500's jump and tail index, defined as the difference between the model-free implied volatility measure and volatility index value, proves to be positively related to the one-, two-, and three-month returns of S&P 500, FTSE 100, and DAX 30. The results are significant at conventional levels for all two- and three-month regressions. The robustness of S&P 500's ex ante volatility premium and jump and tail index is tested by including other option-related variables and a range of alternative predictors in the estimated models. The jump and tail index and the ex ante volatility premium are the only consistently behaving and statistically significant explanatory variables, and robust to the inclusion of other variables. They decrease in significance with the inclusion of each other, indicating that they include similar explanatory information.

This motivates the last empirical test based solely on information in S&P 500 option contracts and the S&P 500 volatility index, VIX. The two measures along with option-implied volatility and skewness are used to explain three-month returns of the ten mentioned equity indexes. The jump and tail index is not included in the first regressions. For nine of the ten indexes the ex ante volatility risk premium is statistically significant and positively related to subsequent returns, and for the last index, MERVAL, the coefficient is in line with the other indexes and has a *t*-value of 1.47. A one standard deviation increase in the premium on average leads to a 2.2% to 4.1% increase in three-month logarithmic returns of an equity index. Similar results follow for the jump and tail index. Six of the ten indexes provide statistically significant coefficient estimates, and ten of the ten positive and similarly sized coefficients. Including both variables does not improve any of the models, and the variables contain similar predictive information. The inclusion of the jump and tail index at most leads to a 1% increase in explained variation of returns. To sum up the findings, this study provides evidence that the U.S. option market has explanatory power globally, and likely indicates global risk factors and premiums. The volatility risk premium and jump and tail index measure are shown to explain returns. The skew risk premium seems to contain similar information as in the volatility risk premium, but no conclusive evidence is gained.

This paper proceeds as follows. First, the empirical section is motivated with earlier research. Also the theoretical background for the explanatory power of options is provided. Second, an overview on the wide range of recent contributions is given, and literature on aggregate returns and individual stocks are both given their own subsection. Then research questions are brought up and hypotheses are formed. Then Section 5 shows the data and methodology, and Section 6 provides results. Results start with moment premiums, then move to option-implied moments, after which the two tail measures are considered. Then the robustness of results is tested, and the S&P 500 options are used in explaining equity indexes globally. Section 7 concludes.

2. Motivation and background

2.1 Motivation and theoretical background

Studying the explanatory power of options on subsequent equity index returns of S&P 500, FTSE 100, and DAX 30 is motivated by empirical asset pricing literature on equity premium and compensated risks. Table I below lists the considered variables based on the literature, and a brief motivation for their use. For reference, Table B.1 in Appendix B covers relevant terminology.

Table I
Option-implicit explanatory variables of equity index returns

Variable	**Source**	**Motivation for use**
Option-implied volatility Option-implied skewness Negative tail probability density	Risk-neutral densities (RND) – option prices are the primary source of information along with interest rate, equity index price, and dividend yield data	Equity risk is uncertainty of the future price level, and the variables characterize this uncertainty in the form of a probability distribution. Compensation for equity risk translates into a premium, which should be reflected in subsequent returns.
Jump and tail index	RND model-free volatilities and volatility index values	Difference between the measures reveals risk-neutral expectations of jumps in prices
Volatility risk premium Skew risk premium	RNDs and realized asset returns for calculation of realized volatility. Premiums equal the difference between the objective or statistical expectation and the risk-neutral expectation.	Differences between the objective expectation and risk-neutral expectation of returns translate into premiums. A measure of the second or third moment premium relate to the equity premium, if they are driven by a common factor. Serve as indicators of risk aversion or economic uncertainty.

This study specifically relies on the information content of option-implied risk-neutral densities (RND). The reasoning is that they provide market-based forward-looking expectations of future return volatility and skewness, which cannot be inferred from past returns. The RND of an asset is the expected probability density of the price or return if investors were risk-neutral. The risk-neutral measure risk-adjusts true expectations. The information in RNDs is specific to the life of the option cross-section. Therefore, the information content should optimally correspond to the future realizations of the studied equity indexes. In addition, if the density is estimated with nonparametric methods, "not only can the nonparametric method reflect the possibly complex logic used by market participants to consider the significance of extreme events, but it also implicitly brings a much larger set of information… to bear on the formulation of probability distributions" as stated by Jackwerth and Rubinstein (1996). This feature is utilized by incorporating the probability density of the distributions' negative tail to infer whether it is a signal of priced tail risk. In addition, due to the possibly complex logic market participants use to consider the future outcomes, this study strives to incorporate as much information as possible from option prices to the explanatory variables. This is covered in section 4.2. All option-implicit variables are based on the entire continuum of option strikes.

And why would the ex ante moments, volatility and skewness, be important in explaining future returns? If large negative outcomes are increasingly harmful for a risk averse investor whose utility is a monotonically increasing concave function of wealth, increased variance exposes the investor to decreased expected utility. Therefore, holding wealth in such an investment would require a higher premium, and risk is traded off for return. Thus, the CAPM predicts that the volatility of the stock market index return is a determinant of the equity premium. Moreover, keeping variance constant, if the negative skewness of the future return distribution increases, an investor is again more exposed to harmful negative outcomes. On the individual security level, regarding a parameter-preference model, Rubinstein (1973) states that "comoments are the appropriate individual measures of security risk since the comoments reflect the contribution of a marginal increase in the holdings of a security to the corresponding central moments of individual future wealth, which are the appropriate measures of portfolio risk in parameter-preference models."

If marginal increases to variance or skewness in an individual's future wealth matter, then aggregate volatility or skewness present in equity indexes also matter. The pricing of betas on market-wide volatility and skewness, i.e. asset return sensitivities to changes in option-implicit moments, has been studied by e.g. Ang et al. (2006) and Chang, Christoffersen, and Jacobs (2013). Option-implicit moments do matter in the cross-section of stock returns. The impacts are similarly inferred as a CAPM beta, but the covariance is between the assets' returns and the market volatility or skewness instead of the market return. Regarding overall explanatory power of volatility, Bollerslev, Tauchen, and Zhou (2009) in their study of expected stock returns and variance risk premium comment that "despite an extensive empirical literature devoted to the estimation of such a premium (between aggregate market returns and volatility), the search for a time-invariant expected return-volatility trade-off type relationship has largely proven elusive."

Moment premiums, i.e. volatility and skew risk premium, instead, have proven to provide significant explanatory power on subsequent aggregate market returns. As with the equity premium, variance and skew premiums are the differences between the true expectation and the risk-neutral expectation. The difference between these two is driven by investors' risk aversion and utility function. E.g. Bliss and Panigirtzoglou (2004) utilize this to estimate risk aversion, and this is possible because "knowing any two of the three functions - the risk-neutral density, the objective density, and/or the pricing kernel (equivalently the utility function) - permits us to infer the third one." This is relevant for aggregate stock market returns, because the premiums evident in volatility and skewness possibly unveil factors that also drive the equity premium. This connection between the three functions is central to the interpretation of moment premiums and aggregate returns. The objective or true expected density is not observed, requiring a substitute. In this study the ex ante and ex post realized values of volatility and skewness are used as substitutes for the objective expectation.

The common perception of considering a risk-neutral measure of future return variation, e.g. the Chicago Board Options Exchange (CBOE) volatility index (VIX), as an indicator of overall risk aversion can therefore be misleading. VIX can be high due to a high objective expectation, high risk aversion or both. Bollerslev and Todorov (2011) develop an Investor Fears index that "cleanly" isolates the fear component. They also decompose the equity and variance risk premiums into diffusive risk and jump risk components. In their model jump risk of the price process directly links the equity and variance risk premiums, and has a major contribution to both. A price process of a security models how the price evolves over time, and usually option pricing models specify a process for the price and derive option prices as a function of the price process parameters. Santa-Clara and Yan (2010) form a time-varying equity premium with index options which explains following returns. Thus, it is reasonable to expect that moment premiums provide explanatory power over moments. Equity risk premium is of major importance in this study. A high required equity premium translates into a high expected return.

Volatility indexes have an alternative, yet important, use in this study. Du and Kapadia (2012) show that the difference between Bakshi, Kapadia, and Madan's (2003) model-free implied volatility (MFIV) and volatility index prices are proportional to the jump intensity in the assumed price process. According to the authors, MFIV is a measure of the variance of holding period log-returns, and VIX is constructed to measure the quadratic variation of a strictly continuous stock return process. The difference reveals the jump and tail index, which suits the purpose of this study as Du and Kapadia (2012) note that "when there are jumps, the tail of the stock return distribution is completely determined by (large) jumps. Therefore, the time variation in the difference between the quadratic variation and the VIX measure of integrated variance measures the time variation in the tail of the stock return distribution."

The jump and tail index of Du and Kapadia (2012) is replicated in this study with the model-free option-implied volatility of RNDs, since it exactly measures the option-implicit volatility of holding period log-returns. Since there are three equity indexes employed in this study, the different volatility indexes should be calculated similarly. This is important for the meaningfulness of the empirical tests, and requires attention due to changes in the methodologies in volatility index calculations. The CBOE introduced VIX in 1993, which originally measured the expected 30-day volatility implied by at-the-money (ATM) options on the S&P 100 index. In 2003 CBOE with Goldman Sachs updated the VIX, and nowadays it is calculated from a wide range of out-of-the-money (OTM) calls and puts on the S&P 500. The original index is abbreviated VXO. Similarly calculated corresponding indexes for DAX 30 and FTSE 100 are VDAX-NEW and VFTSE. Deutsche Börse and Goldman Sachs jointly created the VDAX-NEW, and the new index, similarly to VIX, applies OTM options. The new index has been calculated on a continuous basis since 18th of April 2005. The old index, which uses ATM implied volatility, is called VDAX. The VFTSE was introduced in 2008 by NYSE Euronext, and differs in methodology from the new FTSE 100 Implied Volatility Index (FTSE 100 IVI). The FTSE 100 IVI was launched on 18th of February 2013. Methodology for the included indexes can be found from the CBOE's White Paper (2003) or Deutsche Börse's Guide to the Volatility Indexes of Deutsche Börse (2007).

The concepts of RNDs, moments, moment premiums, model-free, and risk-neutral measure are central to the methodology and results. Table I gives exhaustive explanations, which are now elaborated. RNDs are basically a more intuitive way to report option prices as a probability distribution. The equity index ends to different values with different probabilities, and the probabilities sum up to one. European option prices depend on the outcome, and with many options on the same equity index one finds the probability distribution of different outcomes indirectly from option prices. This is because the option prices can be thought of as discounted expectations of future payoffs, and options have different exercise prices. Moments and the tail density simply measure certain characteristics of the probability distributions of the future asset value, the RNDs.

The option-implied moments, volatility and skewness, are labeled model-free since they totally depend on the RNDs that are not determined by any option-pricing model. Black-Scholes (BS) implied volatility, in turn, is a model-dependent way to report an option's price. The idea is that with other inputs held constant the option-implied volatility is the volatility input that matches the option pricing model's option price to an observed real option price. BS implied volatility is used as a conversion tool in the methodology, and different options usually imply different volatilities for the underlying asset since the pricing model is incorrect (see Figure 3 in Appendix C). The risk-neutral measure provides us with a probability distribution which has incorporated the pricing of risk. This is what one observes from option prices, and it gives the risk-neutral moments and tail densities. Now if one observes or assumes a true expected probability distribution, then the difference between these distributions shows risk premiums. These are the moment premiums considered in this study, and the basis for the literature on moment swaps.

2.2 Contribution and limitations of the study

The theoretical basis from which the explanatory power is drawn is broad. In turn, the empirical approach is strictly driven by the main idea behind this study: the information in the form of probability distributions reveals the assessment of the future as well as its pricing. The RNDs should incorporate all the relevant information if pricing is correct and markets efficient. By applying nonparametric methodology in estimating the risk-neutral densities it is possible to make use of all the information content and account for the possibly complex logic of market participants to price uncertainty. The contribution of this study is to explain subsequent returns with a wide range of information available in option-implicit RNDs, supplemented with the information in volatility indexes. Even though the variance premium and tail measures have proven to be important predictors, including volatility, skewness, and skew premium allows for comparison between the measures and testing their mutual explanatory nature. This comprehensive top-down approach enables inferences of the sources of explanatory power based on earlier theoretical work on the subject.

Two recent studies are the main motivators of this work, but are distinctively focused on tail measures and do not consider market returns outside the U.S. stock market. Du and Kapadia (2012) study the explanatory power of the variance risk premium and the jump and tail index along with a common set of alternative predictors also controlled for in this study. They provide results for one-month, two-month, six-month, one-year, and two-year horizons. Kelly and Jiang (2014) also focus on the tail risk measure they construct, and for comparison include variables applied in this study such as the variance risk premium, risk-neutral skewness, and risk-neutral kurtosis along with a range of non-option-based variables. The market return predictability results are reported on a univariate or bivariate level, and the horizons include one-month, one-year, three-year, and five-year horizons. Bollerslev et al. (2014) broaden the geographical perspective by including eight countries in the study, but limit the investigation entirely on the variance risk premium and its predictive power. Therefore, empirical research on the predictive abilities of option-implicit information in general and across indexes has room for new evidence.

In addition, this paper has two distinctive features. First, the empirical investigation considers a wider set of variables simultaneously. In practice this is implemented by first studying the explanatory variables and subsequent returns in isolation, and then assessing the robustness of the findings to the inclusion of the full range of predictors. Second, an attempt is made to preserve the maximal possible amount of information in options with loose filtering of input option data, and this information is used primarily in the form of probability distributions. The option prices are the channel of predictive power applied in this study, and strict filters and pre-imposed distributional structures would work against the main idea of this study. Section 5 covers the methodology in detail.

The limitations of the study result directly from the choice of approach. Using solely RNDs constrains the study to this perspective. A number of studies have exploited returns of traded option strategies and estimations of the price process to find compensated risks and relationships between option-implicit information and subsequent returns. To simplify, this study uses information on where the price can end up, but not how it gets there. The only variable containing information on the price process is the jump and tail index. Therefore, there is no direct correspondence to these strands of literature, and also there is no direct contribution to option pricing literature despite options being in the center of the methodology. Moreover, option series expiries are infrequent, which constrains the amount of observations to construct the RNDs. This could be circumvented by forming constant maturity RNDs, or directly moments, as is done by Neumann and Skiadopoulos (2013) in their study of predictable dynamics of higher-order risk-neutral moments, but little would be gained in the case of explaining returns due to a major overlap in observations. Also, the choice of methodology affects the study. The information on model-free risk-neutral moments implicit in RNDs can be extracted directly from option prices (see Bakshi, Kapadia, and Madan, 2003; and, e.g., Trolle and Schwartz, 2014; Conrad, Dittmar, and Ghysels, 2013; Buss and Vilkov, 2012, for application) or after estimating the RNDs, and the latter is done in this case due to including the probability density of the negative tail in the study.

Last but not least, this study does not take into account the true expectation of future return tail density. Volatility and skew risk premiums incorporate this information regarding the second and third moments, but no objective measure of the tail probability is applied. In general, forming objective expectations of jumps or tails is challenging because these events occur rarely. This is a limitation, because the RND reveals the effect of true probability attributable to the tail after correcting for investor preferences. Separation of these factors is not possible without statistical expectations (see Bollerslev and Todorov, 2011, for separation of diffusive and jump risk into risk-neutral and statistical expectations and their empirical application, and Kelly and Jiang, 2014, for another proposed measure of tail risk). The pricing of tails is indirectly taken into account through the variance and skew premiums, and directly with the tail risk measures.

3. Prior literature

This section covers literature on compensated risks in financial assets. Literature is covered to understand the compensated risks in individual stocks and the stock market on aggregate, and two questions get an answer. First, what risks are lately considered relevant for equity prices? Second, is there something in the option market that would indirectly imply the size of the equity premium? These considerations provide the motivation for the empirical part, and provide explanations why option-implicit information has explanatory power on equity index returns. Table B.1 in Appendix B lists relevant terminology.

3.1 Risks and premiums

3.1.1 Equity risk, volatility risk, skew risk, and moment premiums

Exposure to risk is associated with an expected compensation for being exposed. Equity risk compensates for the uncertainty of the future price level. In the traditional mean-variance framework holding wealth in an equity index is expected to have a higher return than holding wealth in an asset which is considered risk-free, i.e. variance-free, since the exposure to price changes is higher in that case. This results from risk aversion, and large negative outcomes appear to be more harmful for investors than large positive outcomes are beneficial. The difference between the expected return on the two securities can be interpreted as an equity premium, i.e. compensation for the risk. In this study the RND is applied for finding out the perceived characteristics of the future returns beforehand, and the difference between the objective return density's mean and the risk-free counterpart is the equity premium. However, this premium is not observable. Therefore, a model or indicator of the premium is needed to have explanatory power on future returns.

If investors dislike being exposed to changes in the volatility of market returns, they require being compensated for it. Volatility risk refers to the risk that volatility changes, or to the fact that returns are volatile. Option-implied volatilities usually exceed the subsequent realized volatilities, and the difference uncovers a premium (see, e.g., Carr and Wu, 2009). The difference is analogous to the difference between a long-term fixed interest rate and a floating rate. One usually pays less interest if one takes the risk that interest rates rise. The difference between the RND volatility and the objective expectation translates into a volatility risk premium. Alternatively, if volatility risk is not perceived as a risk factor itself, the observed differences between option-implied volatilities and realized volatilities mean that there are volatility arbitrage opportunities. Volatility risk premium varies in time, and has been documented as a powerful predictor of equity index returns. Therefore, there seems to be a direct link to the equity premium. Market volatility is nowadays widely traded, and by entering an over-the-counter variance swap an investor can receive a floating payoff based on the realized market volatility by paying a fixed leg. Interestingly, the expected payoff for such a contract seems to have a lot to do with market returns, most likely because the variance risk premium is correlated with the equity premium.

The average return on being short in the fixed and receiving the floating leg of the swap has been widely documented negative, and therefore hedging against market volatility costs the volatility risk premium. Carr and Wu (2009) studied five stock indexes and 35 individual stocks over a seven-year period from January 1996 to February 2003, and compared a synthetic variance swap rate, constructed from options written on the underlying, on the ex post realized return variance. Defining this difference as the variance risk premium, they find that for the S&P 500 and 100 indexes and for the Dow Jones Industrial Average the risk premiums are strongly negative. In addition, their analysis suggests that there is a systematic variance risk factor requiring a highly negative risk premium.

The negative risk premium in this context means that the realized variance, i.e. the floating leg of the variance swap, is on average lower than the fixed leg. Similarly as the difference between the RND and the objective density translate into premiums, the difference between the fixed leg and the floating leg unveils a premium. When an investor invests in a security, at least two sources of uncertainty are faced. Uncertainty about the return is captured by the return variance, and the remaining risk is variance of the return variance itself. Therefore, the security exposes the investor to both equity and variance risk. According to Carr and Wu, “it is important to know how investors deal with the uncertainty in return variance… to understand the behavior of financial asset prices in general.” The variance risk premium is tested against a range of classic risk factors, and for instance the CAPM relationship of returns of the swap and market excess returns is not sufficient in explaining variance swap returns.

Delta-hedged option strategies that are exposed to changes in volatility similarly uncover the premium associated with volatility. Bakshi and Kapadia (2003) use S&P 500 index options, and find that a delta-hedged long straddle option strategy underperforms zero. As the position holder gains from increases in market volatility, the position serves as a hedge against increases. Similar empirical investigations are numerous, and more recently Cremers, Halling, and Weinbaum (2015) use straddles in a way that separates jump and volatility risk with vega or gamma positive but delta neutral option positions. The variance premium is affected by differences in the jump tails of the risk-neutral and objective distributions, and Bollerslev and Todorov (2011) show that on average the difference explains over half of the observed variance premium. Bakshi and Kapadia (2003) tried to explain the negative variance risk premium with risk-neutral skewness, but concluded that the jump-fear explanation “cannot be the sole economic justification.” However, instead of solely looking at the risk-neutral measure, comparing it with the objective measure, as in Bollerslev and Todorov (2011), is beneficial.

Kozhan, Neuberger, and Schneider (2013) study skew and variance swaps, and “provide evidence that skew and variance premiums are manifestations of the same underlying risk factor”. The implication for explaining equity index returns is that including both premiums should not add explanatory power to a model. Relevant to this study, the variance and skew premiums embed possible expected jumps. Bollerslev and Todorov (2011) claim that jump risk drives both equity and variance premiums, and as a result higher variance premium would mean that the equity premium is higher. Drechsler and Yaron (2011) theoretically show that the variance premium, defined as the square of VIX less the expectation of realized variance, is linked to uncertainty about economic fundamentals. They show that time variation in economic uncertainty and preference for early resolution are required to generate a variance premium that is time-varying and predicts excess stock market returns. The variance premium is shown to effectively reveal the level of jump intensity, and an increase in jump intensity causes an increase in the variance and market risk premium. This idea is strongly supported by the findings of this study.

The variance premium captures time variation in the risk premium and serves as an effective predictor of market returns. In practice increased jump intensity means that price jumps become more frequent and probable. From the probability distribution perspective, increased downward jump intensity in the price process makes the negative tail fatter or return expectation more negatively skewed.

Asymmetries in the return distribution can either make the distribution negatively or positively skewed. Skew risk is the risk that skewness of this expected return distribution changes, or the fact that expected returns are skewed. For a negatively skewed distribution large negative outcomes are more probable than large positive outcomes. The RND has for long been on average negatively skewed for equity indexes (see Figure 2 and Figure 3 in Appendix C), despite historical returns show a far more symmetric distribution of returns. Consequently, either the market constantly expects a large negative movement in the index value that occurs far too rarely, or the improbable negative event is accompanied with a high required compensation. As the RND is driven by investors' objective assessment of the density and their preferences, either one of these explanations or both lead to an observed skew risk premium. Negative skewness is by construction related to tail risk, as a high probability of crashes leads to a skewed RND. This shows in the correlations between variables, and e.g. the one-month jump and tail index is correlated by -0.52, -0.73, and -0.67 with option-implied skewness for S&P 500, FTSE 100, and DAX 30 respectively (see Table B.5 in Appendix B for correlation matrices of variables).

Asymmetries in the objective distribution of returns and skew risk premium both drive the option-implicit skew, and as mentioned earlier this is quantified lately by Kozhan, Neuberger, and Schneider (2013) with a skew swap. The underlying idea is similar to the other option strategy-based studies, but as the risk exposures of strategies vary over time, they rebalance the positions to avoid spurious evidence of skew or variance premiums. The rebalancing and hedging is done in a model-free way. Again, S&P 500 index options are used to provide evidence that there exists a skew and variance risk premium, and trading strategies receiving the floating leg, i.e. realized moment, of the respective moment swap earn negative returns. Half of the option-implied skew is due to the risk premium, whereas the rest reflects the negative correlation between returns and volatility. In other words, when equity prices decline volatility simultaneously increases. Therefore, half of the skew is explained by the objective assessment of the density. As variance comes short in explaining the shapes of RNDs, skewness helps in explaining the shape.

3.1.2 Tail and jump risk

Tail or jump risk is not as unambiguous of a concept as the moments. Tail risk is usually considered as the possibility of an extreme return, and in the RND consideration a "fat" tail on the negative side of the mean would mean that tail risk is high. As an alternative, a sudden jump in an otherwise continuous price process is interpreted as the realization of tail risk. An increased tail risk in this setting would mean that the modeled jump intensity has increased, leading as well to a higher probability of an extreme return.

To list suggested measures, Du and Kapadia (2012) calculate their jump and tail index from the difference between the holding period return variance of Bakshi, Kapadia, and Madan (2003) and integrated variance as calculated in the VIX index. The difference between the two measures and the bias of VIX, in case of jumps in the price process, reveals the jump risk. Kelly and Jiang (2014) use historical stock returns, and assume that the tail risk of all assets is governed by a single process. The tail risk is then determined from the commonalities of tail risks of individual securities, tail probability being calculated as the probability that the return falls below a certain negative threshold. Bollerslev and Todorov (2011) use statistical extreme value theory approximations with high-frequency intraday data to estimate jump tails under the objective probability measure, and short maturity OTM options to estimate the risk-neutral counterpart. For comparison, in this study tail risk is simply defined as the risk-neutral probability that the future return falls below a certain threshold. The jump and tail index is used as an alternative.

3.1.3 Risks implicit in modeled asset price processes

Characteristics of the RNDs do not necessarily reveal the underlying price process. Reversely, without further assumptions an observed RND can result from any possible price process that satisfies it. By using non-path-dependent options that only derive their price from their terminal value, this study explicitly focuses on the moments and the tail of the period-end density, and any parametric restrictions on the density are avoided. However, the implications of various applied models are also relevant to cover the literature discussed.

To start with, the BS model's price process has only one source of risk, which is uncertainty about the underlying asset's price represented by the volatility input. Hedging against this one source of risk leads to a riskless portfolio, and a delta neutral portfolio should earn the risk-free rate. Therefore, the model incorporates diffusive risk but excludes jump risk. In other words, there is equity risk resulting from volatility, but no volatility risk. Also the form of the price process implies that skewness, jumps, and tails are negligible. Volatility is assumed to be time-invariant, and the RND for the asset following this difference-stationary process is lognormal. The log-return is consequently normally distributed.

The crash of 1987 and the following smirk effect (see Figure 3 in Appendix C for volatility smiles and smirks) in implied volatilities revealed that many of the assumptions behind the model are to be questioned. Heston (1993) introduced stochastic volatility, allowing the volatility of the price process itself vary in time, and Bates (2000) introduced Poisson-distributed jumps in proposing a stochastic volatility/jump-diffusion model. Later on, time-varying jump risk has been modeled by e.g. enabling the jump intensity to vary, and applied in solving for its contribution to the equity premium (see Santa-Clara and Yan, 2010; Bollerslev and Todorov, 2011). Jump discontinuities in the stock price process increase the tail or skew, and volatility of a return distribution.

3.2 Stock returns

The different sources of risk mentioned in the earlier section have been successfully used to explain the cross-section of stock returns. The results are generally robust to the inclusion of other explanatory factors, such as liquidity risk and momentum, and the choice of model. The following studies have revealed important priced risk-factors, but have also helped in assessing the magnitude of the time varying equity risk premium.

Ang et al. (2006) study the pricing of aggregate volatility risk in the cross-section of stock returns. Assets with high sensitivities to market volatility risk hedge against market downside risk, which results as a negative price of volatility risk of approximately -1% per annum. The proxy for innovations in volatility is the change of the VIX index[3]. They bring up alternative explanations from economic theory on the pricing of volatility risk. Investors may want to hedge against increases in market volatility because it makes investment opportunities worse, or alternatively the low-return stocks hedge against market downside risk because usually downward movements in stock prices are accompanied with high volatility of returns. Similar results are provided by Cremers, Halling, and Weinbaum (2015) who separate the pricing of time-varying aggregate jump and volatility risk in the cross-section of stock returns, finding that these are separately priced. The results of Ang et al. (2006) did not include this kind of separation. Cremers, Halling, and Weinbaum (2015) also find that the jump risk and volatility risk betas are uncorrelated, and therefore a stock

[3] Ang et al. (2006) note that the VIX combines both stochastic volatility and the stochastic volatility premium, and only if the volatility risk premium was zero would the VIX correctly proxy for innovations in aggregate volatility. This difference caused by the premium is analogous to the difference between the risk-neutral and statistical expectation, and is revealed by moment swaps (see e.g. Kozhan, Neuberger and Schneider, 2013; Carr and Wu, 2009).

that provides protection on one of these risk factors does not necessarily hedge for the other risk. Jump and volatility risks are mimicked by constructing straddle option positions that are either vega or gamma positive but market neutral. If a stock's returns are positively related to returns on the vega positive option position, the results suggest that keeping other factors constant the expected rate of return becomes smaller. The same holds for gamma positive option positions.

In addition to using risk factors driven by changes in the VIX or changes in the value of vega or gamma exposed option positions, cross-sectional stock returns have also been studied by using the ex ante moments of the risk-neutral return distribution by Conrad, Dittmar, and Ghysels (2013). Moments are calculated from options on individual stocks following Bakshi, Kapadia, and Madan (2003). With a sample of options on individual stocks, they find that cross-sectional differences in estimates of risk-neutral volatility, skewness and kurtosis are related to subsequent returns. Volatility and skewness are negatively related to returns, whereas kurtosis is positively related. These relations are robust for controlling for firm characteristics, beta, size, book-to-market ratios, and adjustments of the Fama-French risk factors.

Rehman and Vilkov (2012) find that the ex ante option-implied skewness from individual stock options is positively related to future stock returns. In contrast to literature finding that the subsequent higher returns for negative skews are due to a risk premium, the model-free implied skewness is said to identify deviations of individual stocks from their fundamental values. A negative skew consequently shows that a stock is overvalued, and the value correction process leads to downward movements in the underlying's price. They also find that the speed of this inefficiency correction process is dependent on arbitrage risks. For the whole market's moments the opposite is found by Chang, Christoffersen, and Jacobs (2013), who report a large and negative premium for bearing market skewness risk. Also kurtosis risk is priced in the cross-section of stock returns, but results are more robust for skewness. Ang, Chen, and Xing (2006) study downside beta and the cross-section of stock returns, and show that stocks with high downside betas have high returns on average. Kelly and Jiang (2014) have confirmed the robustness of their tail risk measure to the downside beta along with liquidity, momentum, and other factors. This possibly indicates a separate role and economic interpretation of the downside beta from jump or tail risk.

3.3 Aggregate stock market returns

Option-implicit factors also have explanatory power on aggregate stock market returns. Especially options have proven to be more powerful predictors of equity index returns in the short term than other traditionally considered predictors such as dividend yield, price to earnings, default spread, or term spread. The following section sheds light on this part of literature. Santa-Clara and Yan (2010) use S&P 500 index options to "estimate the risk of the stock market as it is perceived ex ante by investors." They consider diffusion risk and jump risk, and calibrate a model to option prices which embeds time-varying stochastic volatility of diffusion shocks and jump intensity. Further on, they construct a time-varying model-implied equity premium dependent on these risks. Predictive regressions explaining realized stock returns suggest that the model-implied equity premium has significant predictive power on future stock returns for horizons up to three months, and the predictive power seems to be more related to the stochastic jump intensity. This finding is in line with the theoretical work and empirical findings of Bollerslev and Todorov (2011) and Drechsler and Yaron (2011). In their work jump risk and its pricing contribute to the size of equity and variance premiums.

In Santa-Clara's and Yan's (2010) model the average premium that compensates investors for the risks implicit in option prices is quantified at 11.8%, which is 40% higher in relative terms than the premium required to compensate the same investor for the realized volatility in stock market returns, 6.8%. Regardless of the approach taken, this highlights the important fact that realized variation, skewness, crashes, or other measures are insufficient in explaining asset returns. The higher compensation for ex ante risks compared to realized risks is said to support the Peso explanation of the equity premium puzzle, which means that the historic equity premium has been higher than what would be explained by compensation for the covariance between market returns and consumption growth with reasonable levels of risk aversion, and this is explained by rarely occurring large price movements.

Investors clearly require compensation for events that are deemed probable but occur rarely. As options are based on the perceived ex ante risks, the compensation of these events should be embedded in option prices. Santa-Clara and Yan (2010) argue that "options capture a risk that is perceived as likely by the investors even if it doesn't materialize in the realized returns." In contrast to earlier research, the model of the price process includes an independent process of jump intensity. The RNDs used in this study capture the ex ante risks in the form of a probability distribution, and the increased jump intensity in Santa-Clara's and Yan's model would increase tail probability densities and negative skewness if the price jumps were negative. The jump and tail index is directly affected by increased jump intensity.

Rarely materializing events have been widely considered as a solution for the equity premium puzzle brought up by Mehra and Prescott (1985). Rietz (1988) first considered possible, yet unlikely, market crashes as an explanation. He concludes that adding unlikely and low-probability crashes to the model proposed by Mehra and Prescott explains the high equity risk premium and low risk-free returns, thus solving the puzzle. The ex ante small probability of a stock market crash would be evident in the tail of the RND. The idea behind the rare events hypothesis, i.e. compensation for rare events explaining the equity premium puzzle, is that even though these events rarely materialize, they are still compensated in the equity premium.

Recently Julliard and Ghosh (2012), as opposed to literature supporting the rare event hypothesis, have argued that the occurred disasters in the world are too small to rationalize the puzzle, and that an unreasonably high level of relative risk aversion is needed. Ziegler (2007) also states that a very high level of relative risk aversion is needed to rationalize the stock market risk-premium, and that the rare events hypothesis is an unlikely explanation. The results also suggest that the most likely explanation is one that increases the likelihood of recessions and market crashes compared to the historical frequency. This study does not address the composition of the equity premium. The ex ante perception of the return distribution, risk premiums, and tail measures embed the expectations and pricing of rare events.

Related to the equity premium puzzle and the Peso problem, tail risk has been studied to find out whether it should be considered an additional channel of risk and risk premiums. Liu, Pan, and Wang (2005) find that the total equity premium consists of a diffusive risk premium, a jump risk premium, and a rare event premium by "solving the equilibrium asset prices in a pure-exchange economy with a representative agent who is averse not only to risk but also to model uncertainty with respect to rare events." The index option smirk, i.e. decreasing implied volatility in exercise prices, is consequently suggested to be driven by uncertainty aversion to rare events and by rare-event premiums. More recently Bollerslev and Todorov (2011) have shown that the compensation of rare events accounts for a large fraction of the average equity and variance risk premiums.

Similarly to Santa-Clara and Yan (2010), the distinct roles of continuous price variation and large jumps is studied, and their results suggest that on average close to 5% in absolute terms of the equity premium is explained by the compensation for rare events. Relevant to this study, estimates of right and left tail jump density measures, which unveil the jump risk perceived by investors, under the risk-neutral and objective measure are formed. Close-to-maturity and deeply OTM S&P 500 options are the basis for measures under the risk-neutral measure, and objective measures are based on high-frequency price data.

The important role of time-varying tail risk is also found by Kelly and Jiang (2014). Unlike the vast literature using options to infer measures of tail risk, they use firm-level price crashes of individual stocks to determine commonalities. Predictive univariate regressions on aggregate stock market returns suggest that tail risk has significant predictive power for all horizons, which are one-month, one-year, three-year, and five-year predictions. Higher tail risk leads to higher subsequent returns. A wide range of alternative predictors is used, and in relation to this study variance risk premium, risk-neutral skewness, and kurtosis are also present. Skewness has a negative, yet statistically insignificant, effect on returns.

Variance risk premium is positively and significantly related for one-month and one-year horizons, but negatively and significantly related for the five year horizon. Bivariate predictive regressions where tail risk is coupled with book-to-market, default return spread, default yield spread, dividend payout ratio, dividend price ratio, earnings price ratio, inflation, long-term return, long-term yield, net equity expansion, stock volatility, term spread, and Treasury bill rate show that the tail risk measure remains a consistently positive and significant predictor. Moreover, the explanatory power of the proposed tail measure is tested on returns of individual stocks, and the expected return is lower for stocks with low loadings on tail risk.

Related to considerations of tail risk and tail risk measures (most relevantly Bollerslev and Todorov, 2011; Kelly and Jiang, 2014), Du and Kapadia (2012) study the predictability of index returns with option data. They also propose a new measure of tail risk and explain that the VIX is not a model-free measure of stock return variability in the presence of jumps (also see Martin, 2013, for a discussion on volatility derivatives and jumps, and for reasons behind illiquid volatility derivatives markets during 2008-2009). First of all, as discontinuities, jumps, in the stock price process affect both volatility and the tail of the stock return distribution, they construct model-free volatility and tail indexes that separate these. They use the Bakshi, Kapadia, and Madan (2003) holding period return variance and VIX, and use the inaccuracy of VIX to determine time variation in tail risk, labeling it as the jump and tail index. The indexes are used to examine their predictive ability on market returns, and the study covers 1996-2009. They comment that "our analysis underscores the importance of understanding the variance risk premium from option prices, and its relation to the equity premium." The empirical estimations of the jump and tail index show significant time-variation, and therefore provide similar evidence on tail risk dynamics as Santa-Clara and Yan (2010).

Directly relevant to this study are the results of their predictive regressions. Their predictive regressions include variance risk premiums, the proposed jump and tail index, and a set of common predictor variables. Both variance risk premium and the jump tail index are positively related to returns, and provide significant predictive power for up to six-month long predictions. The results of Du and Kapadia are in line with the positive impact of variance risk premium on subsequent returns documented by e.g. Bollerslev, Tauchen, and Zhou (2009). Using the Bakshi, Kapadia, and Madan (2003) holding period variance based measure increases the predictive power of the variance risk premium. Notable is also that the variance risk premium performs best for quarterly horizons. Drechsler and Yaron (2011) also document statistically significant predictive power of the variance premium for stock returns. According to Dreschler and Yaron , "the variance premium is… interesting due to both its theoretical underpinnings and its empirical success above and beyond that of common return predictors."

Based on the earlier contributions considered above, apparently the variance, or volatility, risk premium should work in predicting market returns. The premium should unveil information about the equity premium and be positively related to subsequent returns. Ex ante moments, i.e. volatility, skewness, and kurtosis of the risk-neutral densities, have not been as successful in explaining short term returns, but from a parameter-preference perspective (e.g. CAPM trade-off between variance and mean, risk and return) should be considered. Tail risk, however, seems to be compensated for, and the question is more of how it is measured and is the effect evident in other considered variables, especially in volatility and skew risk premiums. The next section forms the research questions and brings up the hypotheses to be tested in the empirical section.

4. Research questions and hypotheses

4.1 Volatility and skew risk premium

Recent literature on the variance premium strongly suggests that the premium is positively related to returns. One possible explanation is that the explanatory power comes from investor preferences implicit in the premium of the risk-neutral value over the expected value. Rubinstein (1994) shows that in a representative agent economy, knowing any two of 1. agent's preferences, 2. the state-price density (or RND), and 3. the agent's subjective probability assessment, reveals the third. This study explicitly focuses on number 2 and the difference between 2 and 3, thus likely employing number 1 as a part of the moment premiums. Alternatively, along the lines of Santa-Clara and Yan (2010) and Bollerslev and Todorov (2011) the premiums might unveil increased jump risk. In either case, also the skew premium is affected. If the same factor drives equity, variance and skew premiums, then knowing any of them would provide explanatory power on others.

The empirical work on variance premium and its predictive power currently extensively focuses on the U.S. stock market. Drechsler and Yaron (2011) use S&P 500 and a value-weighted NYSE-Amex-Nasdaq combination. Bollerslev, Tauchen, and Zhou (2009) and Du and Kapadia (2012) focus on S&P 500, and Kelly and Jiang (2014) provide results on the CRSP value-weighted index. Bollerslev et al. (2014) study variance risk premium and returns from a geographically wider perspective[4]. The skew risk premium is not covered to the same extent, possibly because the premiums are driven by the same factor and taking either one of them into account could be sufficient. Kozhan, Neuberger, and Schneider (2013) "provide evidence that skew and variance premiums are manifestations of the same underlying risk factor". Putting these considerations together, studying variance and skew risk premiums simultaneously with other RND-related information and in isolation remains an interesting question to be answered, especially when evidence is drawn from a wider geographical perspective. The first research question is thus formed as follows.

Question 1 *Are volatility and skew premiums consistently behaving predictors of index returns?*

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[4] The considered indexes comprise of AEX (Netherlands), BEL 20 (Belgium), CAC 40 (France), Nikkei 225 (Japan), DAX 30 (Germany), SMI 20 (Switzerland), FTSE 100 (the U.K.), and S&P 500 (the U.S.).

The difference between ex ante premiums and ex post premiums is also shown. From a practical and theoretical perspective the ex ante premiums are more interesting because market declines are usually associated with higher realized volatility. In the empirical section focus is on the ex ante measure, and results on the ex post measure are provided for comparison. Due to the suggested link between the variance, skew, and equity risk premiums, the first two hypotheses are as follows.

H1 *Volatility risk premium is positively related to subsequent returns.*

H2 *Skew risk premium contains the same information as the volatility premium, and only differs in sign.*

The skew premium in Hypothesis 2 would differ in sign because negative skewness has a negative sign, and increased investor fears or risk aversion would make the skew premium even more negative, but the volatility premium more positive. Formally, all the regressions in the empirical section take the form of Equation 1, based on which the hypotheses can be tested.

$$ln\left(\frac{PI_{i,T}}{PI_{i,t}}\right) * \frac{260}{T-t} = \beta_{0,i} + \sum_{a=1}^{n} \beta_{a,i} * X_{a,i,t} + \sum_{s=1}^{m} \beta_{s,i} * X_{s,i,t} + \varepsilon_{i,t} \tag{1}$$

PI is the price index of equity index *i* observed at the option price observation date *t* and option maturity *T*. Dates are measured in days. There are $n + m$ explanatory variables. *n* refers to variables of main interest and *m* to controlled effects. In answering Question 1, the volatility and skew premiums are tested in isolation with $n = 1$ and $m = 0$. This is done both for ex ante and ex post premiums. Section 5.2 provides definitions for the dependent and independent variables, and Section 6.1 reports the results. Section 5.5 covers the econometric methodology.

4.2 Volatility and skewness of the risk-neutral density

Question 1 considers moment premiums. Moments have not been as prominent recently in explaining subsequent aggregate returns. However, from the parameter-preference perspective a highly volatile investment environment exposes the investor to increasingly unwelcome negative outcomes. This requires a premium, reflecting the CAPM trade-off between variance and return. Also, negatively skewed returns mean that negative outcomes become more probable. Volatility and skewness depict the nature of uncertainty related to future returns, and serve as possible explanatory variables on equity index returns. Therefore the second research question of this study is as follows.

Question 2 *Are option-implied volatility and skewness consistently behaving predictors of index returns?*

Section 6.2 provides answers to this question, and also briefly reviews other evidence on the explanatory nature of option-implied volatility and skewness. The tested hypothesis is based on the idea that investors dislike high volatility and negative skewness in returns.

H3 *Higher implied volatility and negative skewness require compensation and result as higher subsequent returns.*

This question is approached by including both volatility and skewness in the regression with $n = 2$ and $m = 0$, and with $m = 1$ controlling for the effect of ex ante and ex post volatility risk premiums separately.

4.3 Tail risk measures

With tail risk the consideration is not whether it is priced, but rather how it is measured. On top of the theoretical considerations, changes in tail risk provide a possible explanation for market valuation changes in the post-crisis era, as central banks have taken a more active role in managing economy-wide tail risks[5]. To recall, Santa-Clara and Yan (2010) and Bollerslev and Todorov (2011) account a large fraction of the time-varying equity risk to the jump component in the price process. Increased jump intensity increases the equity premium. When looking at the RND, increased jump intensity would increase the negative skewness and negative tail density. Kelly and Jiang (2014) suggest a historical return based tail risk measure, which performs remarkably well for horizons ranging from one month to five years. Again, increased tail risk increases subsequent returns on average. In this study, tail risk is simply the negative tail probability implicit in the RNDs. The jump and tail index is used as an alternative measure. Therefore, the last research question is the following.

[5] From a European perspective, one notable event is Mario Draghi's, the President of the European Central Bank, comment in his speech on 26th of July 2012 at the Global Investment Conference in London: "Within our mandate, the ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough."

Question 3 *Is the tail density or jump and tail index a consistently behaving predictor of subsequent returns across indexes?*

The literature on the equity premium puzzle suggests that rare events are a possible explanation. This is supported by the recent findings that jump risk is an important component in equity and variance premiums. In addition the recent empirical studies on the cross-section of stocks show that sensitivity to skewness, tails, or jumps is priced. This leads to the fourth tested hypothesis.

H4 *Tail density or the jump and tail index indicates tail or jump risk, and higher crash probability is compensated with a premium.*

First the tail density is tested in isolation with $n = 1$ and $m = 0$, and simultaneously with $n = 1$ and $m = 3$, three controls being the volatility premium, option-implied volatility, and option-implied skewness. Then the jump and tail index is studied. Results are reported in Section 6.3 for $n = 1$ and $m = 0$.

All questions are also studied by including all of the predictive variables discussed above with $n = 5$, and by controlling for other generally considered variables present e.g. in Kelly and Jiang (2014) and Du and Kapadia (2012) with $m = 5$. These m variables are dividend yield, price-to-earnings, default yield spread, term spread, and detrended risk free rate. The variables and definitions are closely covered in Section 5.2. To recall, this study focuses on returns for one-week, one-month, two-month, and three-month horizons. For these relatively short horizons, volatility risk premium is expected to play a significant role. Even if so, the behavior of other ex ante measures after controlling for volatility risk premium is highly interesting. The research questions get answers after covering the methodology to construct the explanatory variables.

5. Data and methodology

5.1 Approach to answering the research questions

The data and methodology of this study comprises of three separate parts. First (Section 5.2), the variables are defined and data sources are given for control variables. Second, the option-implicit explanatory variables are formed and option data sources are explained (Section 5.3 and 5.4). Third, the econometric methods are specified (Section 5.5) to answer the research questions brought up in Section 4.

The empirical part involves regressions explaining equity index returns. The index price levels of S&P 500, FTSE 100, and DAX 30 are used. The dependent variables are one-week, one-month, two-month, and three-month annualized logarithmic index returns (see Equation 1 on page 24). The explanatory variables can be divided into two different types. The first type includes variables implicit in option prices, volatility indexes, or index returns. Options provide information unique to the period before their maturity, and one-week, one-month, two-month, and three-month observations before an option cross-section's maturity are used to explain returns over the corresponding period. An option cross-section consists of all traded European call and put options on the underlying equity index maturing on the same day.

5.2 Variables, definitions, and control variable data sources

The main explanatory variables are option-implied volatility, option-implied skewness, volatility and skew premiums, tail density, and jump and tail index. Option-implied volatility, skewness and tail density are observed at t from options maturing at T. Ex post volatility and skew premiums are calculated as the difference between the option-implicit value at t and realized value in the period t to T. The ex ante measures are calculated over the realized value over the month preceding the observation. This means that e.g. return volatility from $t-21$ to t is subtracted from option-implied volatility observed at t. The jump and tail index is the difference between option-implied volatility and volatility index observed at t. The other type of variables consists of commonly used variables in predicting index returns, and these serve to control for their effect on index returns. These variables are the price-to-earnings ratio, dividend yield, term spread, default spread, and detrended risk-free rate, as applied in e.g. Bollerslev, Tauchen, and Zhou (2009) and Du and Kapadia (2012). Table II on the next page gives a full list of variables and definitions.

Table B.2 in Appendix B presents summary statistics for all variables from options or realized returns. Definitions for all the variables involved in the predictive regressions are given below. Summary statistics for remaining variables of interest are provided in Table B.3 in Appendix B.

Table II
Explanatory variables and definitions

Variable	**Abbreviation**	**Definition**
Annualized log-return	*RET*	Annualized logarithmic return from t to T
Option-implied volatility	*OI-VOL*	Model-free annualized implied volatility implicit in risk-neutral densities, observed at t from all out-of-the money options maturing at T
Option-implied skewness	*OI-SKEW*	Model-free implied skewness implicit in risk-neutral densities, observed at t from all out-of-the money options maturing at T
Volatility risk premium	*VOLPRE*	*OI-VOL* less the realized volatility from t to T
Skew risk premium	*SKEWPRE*	*OI-SKEW* less the realized skewness from t to T
Tail density	*TAILZ2*	Probability density below two standard deviations from the risk-neutral density's mean observed at t from options maturing at T
Ex ante volatility premium	*EA-VOLPRE*	*OI-VOL* less the realized volatility from *t-21 trading days* to t
Ex ante skew premium	*EA-SKEWPRE*	*OI-SKEW* less the realized skewness from *t-21 trading days* to t
Jump and tail index	*JTIX*	*OI-VOL* less the volatility index value observed at t
Price-to-earnings	*PE*	Aggregate equity index market capitalization over aggregate earnings observed at t; directly from Datastream
Dividend yield	*DY*	Aggregate equity index dividends over aggregate market capitalization observed at t; directly from Datastream
Term spread	*TERM*	The yield difference between a ten-year and three-month government liability observed at t
Default spread	*DEF*	Difference between Moody's Baa and Aaa bond yield observed at t; one series used for all three indexes
De-trended risk-free rate	*RREL*	One-month government liability yield less its trailing 12-month average observed at t

Option-implied volatility, *OI-VOL*, is the annualized standard deviation of logarithmic returns implicit in the RNDs. Skewness, *OI-SKEW*, is the third moment around the mean standardised with squared volatility, i.e. variance, to the power of 3/2. Volatility and skew premiums, *VOLPRE* and *SKEWPRE*, are calculated by subtracting the ex post observed realized values from their ex ante observed option-implied counterparts, and the sample is the forecast period of the options. For example, if we use options maturing in 21 trading days from the observation, the realized value is calculated from the 20 trading days between the observation and maturity. To recall, this study involves 5, 21, 42, and 63 trading day return horizons. Every variable previously mentioned therefore only contains information specific to the forecast period. Volatility risk premiums over the preceding month's realized values are labeled *EA-VOLPRE* and *EA-SKEWPRE*. Section 5.4 provides the calculation methodology for the variables.

Price-to-earnings, *PE*, and dividend yield, *DY*, are downloaded for the indexes from Datastream. Term spread, *TERM*, is the yield difference between a ten-year and three-month government liability. For S&P 500, these are calculated from ten-year Treasury notes and three-month Treasury bills. For FTSE 100 the basis rates are ten-year and three-month UK government liability spot rates downloaded from Datastream. For DAX 30 the benchmark ten-year Bund yield and the three-month BD EU-Mark Deposit (ECWGM3M) are used. Datatype is redemption yield (RY) except for the BD EU-Mark Deposit, for which it is interest rate (IR). *DEF* is the default spread, i.e. yield difference between Moody's Baa and Aaa rated corporate bonds from the Federal Reserve's website[6], and one value is used for the three indexes. *RREL* is the detrended risk-free rate. Yields for one-month government liabilities are taken from Datastream, and for the S&P 500 series Treasury bill rates are applied. For FTSE 100 the UK Government Liability Nominal Spot Curve 1M is used. For DAX 30, the BD EU-Mark Deposit (ECWGM1M) is used. Table B.3 in Appendix B provides summary statistics.

5.3 Estimating the risk-neutral probability densities

5.3.1 Overview on methodology, option data, and sample period

Option prices can be used to infer the risk-neutral probability density function[7,8], RND, for the underlying asset for different option maturities, unveiling market participants' risk-neutral

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[6] *http://www.federalreserve.gov/releases/h15/data.htm*

[7] E.g. Neumann and Skiadopoulos (2013): "The risk-neutral PDF is defined via the relationship that dictates that the market option price equals its theoretical price calculated as the integral of the option payoff with respect to the risk-neutral PDF."

[8] See Jackwerth (2004) for a comprehensive overview on estimating risk-neutral probability distributions.

expectations of future return volatility, skewness, kurtosis, or more broadly, the shape of the entire return distribution not necessarily characterized by parametric models of the price process. The advantage compared to historical information is that options are traded for different exercise prices, and since pricing of European options can be done as the discounted expectation of the payoffs at maturity, the information is unique to the option's remaining life and the entitled payoff. Alternatively, the price process of the asset can be modeled in a way that it satisfies the time-series features of prices and option prices, enabling similar inferences. A simple description of the applied methodology is that first a continuum of option prices along exercise prices is formed, and then the probability distribution, RND, which explains these prices, is calculated.

The usual aim is to estimate RNDs that are smooth and accurate close to the mean. In this particular case the aim is to avoid extensive smoothing to preserve information in option prices (see Figures 2 and 3 in Appendix C for examples of estimated RNDs). No assumptions on the data-generating process behind equity index returns are made, and only the properties of the RNDs are employed in explaining returns. In addition, the aim is to make use of all information in option prices with minimal limitations and no pre-imposed structure on the resulting RNDs. In filtering the input option data the requirements are not as strict as generally applied, and all options with traded volume are considered regardless of their moneyness. The RNDs are estimated by finding a discrete set of risk-neutral probabilities.

Regarding data requirements for the whole empirical section, options are important since they provide the necessary forward-looking information for constructing RNDs for the underlying indexes. The index returns of S&P 500, FTSE 100, and DAX 30 are in the focus of this study. Breeden and Litzenberger (1978) showed that with a continuous set of options with strikes from zero to infinity, the entire risk-neutral distribution is attainable by taking the second derivative of option prices with respect to the exercise price. Shimko (1993) first provided a way to implement this by interpolation and extrapolation of implied volatilities. The implied volatilities then imply the option pricing function which gives the probability density function. Rubinstein (1994) comments that the method "passes the test… since if the smile (volatility smile) is exactly horizontal, he will imply a lognormal risk-neutral probability distribution with the correct volatility." The methodology applied here follows this approach closely, but instead of the Breeden-Litzenberger result the resulting distribution is attained by finding a discrete set of probabilities based on the implied volatilities. This avoids over-smoothing of the input implied-volatilities. The method also implies a lognormal distribution with the correct volatility for a flat volatility smile.

The market prices for puts and calls for the option observation dates are converted into BS implied volatilities, and after applying filters on the data an OTM cross-section of implied volatilities is formed. Then the implied volatility set is interpolated inside the available exercise price range and extrapolated outside it to form a continuous set of implied volatilities for all possible end values of the underlying index. The implied volatility set is then discretized and state price densities are calculated to unveil the whole RND for different forecast periods.

European options on three different equity indexes are used to build the risk-neutral densities. Daily observations for market prices and trade volumes are downloaded from Datastream for European options on the S&P 500 spot index traded on CBOE, FTSE 100 index options traded on NYSE Euronext LIFFE, and DAX 30 index options traded on Eurex. Covered expiry date range is from May 1, 2006 to December 31, 2014. The S&P 500 set contains 31,005 individual options with 379 different exercise prices for 110 expiry dates, and for FTSE 100 there are 13,013 options, 225 exercise prices, and 104 expiry dates. The DAX 30 set has 17,050 options with 213 exercise prices, and 105 different expiry dates. DAX 30 and S&P 500 have few close expiries so that the number of expiry dates exceeds months in the sample. Volatility index time-series for VIX, VFTSE, and VDAX-NEW are also obtained from Datastream with names FTSE 100 Volatility Index, CBOE SPX Volatility VIX (New), and VDAX-NEW Volatility Index. The datatype is price index (PI).

The market prices used in this study, by Datastream definitions, are issued day end prices for FTSE 100 and DAX 30 options. The final settlement day for DAX 30 options is the third Friday of each expiration month if it is an exchange day, and otherwise it is the exchange day immediately preceding that day. The option can be exercised only on the final settlement day until 21:00 CET. For FTSE 100 options the last trading day is also the third Friday of the expiration month, and exercise is to be done by 19:30 CET. Settlement days are the exchange days following the last trading day for both. S&P 500 option market price is given as the last traded price provided that it is within the bid-ask range. Otherwise the nearest bid or ask price to the last trade price is given. The expiration date for S&P 500 contracts in the sample is the Saturday following the third Friday of the expiration month. A time-series for put and call option prices and trade volumes is formed for every available exercise price. Forecast periods of one week, one month, two months, and three months are formed by observing the prices of option contracts 5, 21, 42, and 63 business days before all expiry dates. If the observation date is invalid then the observation is done one or two business days before the initial date.

5.3.2 Option data filters and conversion to implied volatilities

The data is then filtered by requiring that every observation has volume traded on the specific observation date. After that, monotonicity for option prices across strikes is required, i.e. calls with lower strikes must be more or as expensive as all observed call options with higher exercise prices, and the same is required for put options with higher strikes. Therefore, the sample call option prices are monotonically decreasing in exercise price and the opposite is true for put options. Also, option prices below their intrinsic value are excluded, which is observed often for deep in-the-money (ITM) options.

Table III

Summary statistics for option cross-sections

This table provides summary statistics for available strikes from option cross-sections after matching option contract price information with all available strike prices traded on the underlying index. Traded Strikes sets contain all options with different exercise prices that had volume traded on the observation date 5, 21, 42, or 63 business days before the contract's expiry. The time range is increased for up to two business days in case of missing observations. The filtered OTM series contains all OTM contract prices from Traded Strikes – Calls/Puts after applying monotonicity and intrinsic value requirements for option prices. The sample contains all European equity index options on the underlying spot indexes with expiries from May 2006 to December 2014.

			Traded Strikes – Calls			Traded Strikes - Puts			Filtered OTM Series		
	Time to Expiry	Available forecast periods	Min.	Mean	Max.	Min.	Mean	Max.	Min.	Mean	Max.
S&P 500	1 week	103	26	48.3	102	31	65.4	126	35	64.6	106
	1 month	103	19	57.2	135	31	85.8	135	36	89.4	136
	2 months	100	13	38.6	63	17	54.9	146	22	66.6	139
	3 months	101	3	21.3	50	4	31.9	70	5	40.6	85
FTSE 100	1 week	104	9	22.6	54	11	27.2	48	14	32.0	59
	1 month	104	9	25.1	43	18	30.3	49	22	38.0	63
	2 months	104	8	17.4	29	13	21.5	38	20	30.2	44
	3 months	104	3	11.7	26	3	14.9	30	7	21.3	38
DAX 30	1 week	104	15	27.5	52	20	33.0	59	22	37.0	68
	1 month	104	19	32.0	52	25	37.9	65	30	45.9	74
	2 months	104	13	25.4	55	20	31.8	60	25	41.1	70
	3 months	100	5	19.0	40	2	22.4	64	8	33.1	88

OTM cross-sections are then formed by including call options with exercise prices above the index value and put options with exercise prices below the index value on the observation date. The OTM option series of call and put prices are then converted into BS implied volatilities. The BS model is in this stage only used to convert prices into implied volatilities. The applied methodology does not impose any form or restrictions on the resulting probability distributions, since variation in implied volatility is not restricted beyond the initial filters on price data and, therefore, it provides the needed flexibility. Inputs required are the price of the index, time to maturity, dividend yield, and the risk-free rate of return. Time to maturity is the difference between the observation date and option expiration date measured in days, and risk-free rates, dividend yields, and index prices are downloaded from Datastream. Following Bliss and Panigirtzoglou (2004) the applied risk-free rates are three-month EuroDollar London, Libor, and Euribor rates for the rates to be free from distortions of central bank activities and reflective of the actual borrowing costs faced by option traders. The rates are modified by taking into account the day count convention and converting into continuously compounded rates. Thereafter, the implied volatilities for all the option time-series are numerically computed.

Conrad, Dittmar, and Ghysels (2013) use OTM calls and puts, and require that they have the same number of both options available. The minimum number of required options is two. Moments of the density are directly calculated from option prices using the results of Bakshi, Kapadia, and Madan (2003). Similarly to this study they exclude options with no trading volume, but unlike in this study exclude options with prices less than $0.50. Table III shows that in this study the minimum number of OTM options used to infer the RND in the whole sample is five for S&P 500's three-month horizon. The minimum average is 21 options for FTSE 100's three-month sample, and maximum average is 89 options for S&P 500's one-month horizon. Using actively traded equity indexes is crucial for this study, since the RNDs can be reliably estimated with a wide range of available exercise prices. Bliss and Panigirtzoglou (2004) use OTM options, and exclude implied volatilities greater than 100%. They also discarded cross-sections with less than five available strikes. In this sample observing BS implied volatilities for deeply OTM put options exceeding 100% was not exceptional.

5.3.3 Interpolation and extrapolation of implied volatilities

In the next step the implied volatility cross-sections are interpolated using cubic spline interpolation for the one-month forecasts and linear interpolation for the rest of the forecast periods as there were no material differences between the methods. This is due to the frequency of traded OTM exercise prices. Outside the available strike range the set is extrapolated using the closest available implied volatility value, thus forcing lognormal tails with the extreme BS implied volatility to the RND. As the left tail of the density is applied as a variable in this study, forcing a specific structure on the tails is not optimal. The wide range of available strikes along with relatively short forecast periods, however, decreases the impact of this choice.

In calculating moments or RNDs from a continuum of option prices, the interpolation of option prices or implied volatilities is required. The linear or cubic interpolation applied above is done in the BS implied volatility-strike space with perfect fit to the inputs (see Figure 3 in Appendix C for examples). Similar interpolation inside and extrapolation outside the available strike range is used by Carr and Wu (2009) and Du and Kapadia (2012). Here the goal is to match the RND to the input option prices with minimal limitations. Bliss and Panigirtzoglou (2004) do not apply perfect fit to the data, and apply an interpolation methodology which trades fit for smoothness. Moreover, the interpolation is done in implied volatility-delta space, which focuses the attention more to the center of the distribution.

5.3.4 Forming the risk-neutral densities

The new BS implied volatility sets now enable us to form the RNDs, because we have a continuous set of implied volatilities and option prices at disposal. The density is estimated by finding a discrete set of risk-neutral probabilities q_k associated with m = 5,000 different outcomes $P_{j,k}$ that should satisfy all prices V of contracts j dependent on the same asset maturing after a single forecast period. Similarly, period-beginning state prices π_k with a payoff of 1 if state k occurs and 0 otherwise at period end unveils the distribution. R is the period-end price of 1 earning the risk-free rate of return. The idea is that there should be a unique set of probabilities that matches the different option payoffs to their market prices, and these probabilities sum up to one. The underlying index itself is also the discounted period-end expectation of its value, and without discounting matches the forward price of the index.

$$V_j = \frac{1}{R}\sum_{k=1}^{m} q_k \, P_{j,k} = \sum_{k=1}^{m} \pi_k P_{j,k} \tag{2}$$

The implied volatility sets are discretized to 5,000 equally-sized ticks of the exercise price with intervals of 2.5 for FTSE 100 and DAX 30, and 1 for S&P 500. Rubinstein (1994) uses prior guesses of risk-neutral probabilities and formulates the solution as a minimizing problem to find posterior probabilities in his work on implied binomial trees. This could be a solution in this case but is not tested for now. In this case, the risk-neutral node probabilities q_k are estimated using the BS formula's $N(d_2)$[9] probability that a call will be exercised. Input implied volatilities are from the interpolated and extrapolated series.

$$q_k = \frac{q_{s=k}}{\sum_{s=1}^{m} q_s}, q_s = N\big(d_2(\sigma_s, K_{s-1})\big) - N\big(d_2(\sigma_s, K_s)\big), \sum_{k=1}^{m} q_k = 1 \tag{3}$$

The estimated risk-neutral probabilities then provide an estimate of the entire density with 5,000 separate intervals. Figures 2 and 3 in Appendix C show examples of estimated RNDs. With a flat implied volatility across exercise prices the probability density of the asset's price at maturity is lognormal. To use the Breeden-Litzenberger (1978) result, volatilities should be converted to call prices, which would have to be made a twice-differentiable, convex, and monotonic function of the exercise price. This leads to smooth RNDs and loss of input information.

5.4 Volatility, skewness, and tails

5.4.1 Risk-neutral density based measures

Option-implied volatility and skewness of the RND for forecast periods are calculated based on the discrete sets of probabilities q_k as shown in Equation 4 and Equation 5. X_k is the logarithmic return from time t to T if index value equals strike K_k and μ is its expected value. Volatility is annualized by multiplying it with the square root of 260 trading days divided by days in the forecast period. Table B.2 in Appendix B shows the summary statistics. Estimated model-free option-implied volatilities are, as expected, closely similar to the corresponding volatility indexes (see Figure 1 on page 37 for an example of one-month forward looking option-implied volatilities), and on average exceed the realized values. The time-series of one-month forward looking implied

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[9] $q_s = N\left(\frac{1}{\sigma_s\sqrt{T-t}}\left[ln\left(\frac{F}{K_{s-1}}\right) + \frac{1}{2}\sigma_s^2(T-t)\right] - \sigma_s(T-t)\right) - N\left(\frac{1}{\sigma_s\sqrt{T-t}}\left[ln\left(\frac{F}{K_s}\right) + \frac{1}{2}\sigma_s^2(T-t)\right] - \sigma_s(T-t)\right)$, where $F = PI_t e^{(r-div)(T-t)}$, r is the continuously compounded risk-free rate, and div is the dividend yield.

volatilities from the risk-neutral densities show an upward bias compared to the volatility indexes in Figure 1. All the volatility series show a peak in the second half of 2008 when the financial crisis started, and other peaks in volatility occurred in May 2010 and August 2011.

$$OI - VOL = \sqrt{\sum_{k=1}^{m} q_k \, (X_k - \mu)^2} \tag{4}$$

The difference between option-implied volatilities and realized volatilities imply the volatility risk premium. Volatility indexes and the RND-based measures are based on options on the same underlying, but the methodology applied here means that the RND includes information from all OTM options satisfying the filters regardless of the options' moneyness.

$$OI - SKEW = \sum_{k=1}^{m} q_k \left(\frac{X_k - \mu}{OI - VOL}\right)^3 \tag{5}$$

Option-implied skewness is on average clearly negative for all indexes and maturities. This means that the left tails are fatter than implied by log-normality of the RND of asset's price at maturity. The absolute values decrease as maturity increases. Realized skewness, for comparison, is closer to zero implying that investor preferences make the risk-neutral expectations more skewed compared to statistical expectations. This leads to the skew premium. A widely used method to calculate risk-neutral moments is to calculate the model-free implied variance, skewness and kurtosis directly from option prices following the approach of Bakshi, Kapadia, and Madan (2003). They show that any payoff to a security can be formed and priced using options with different exercise prices on the security. Quadratic, cubic, and quartic returns on base securities, standard European call and put options, are used along with standard moment definitions. However, due to including the tail density measure, the entire distribution has to be formed.

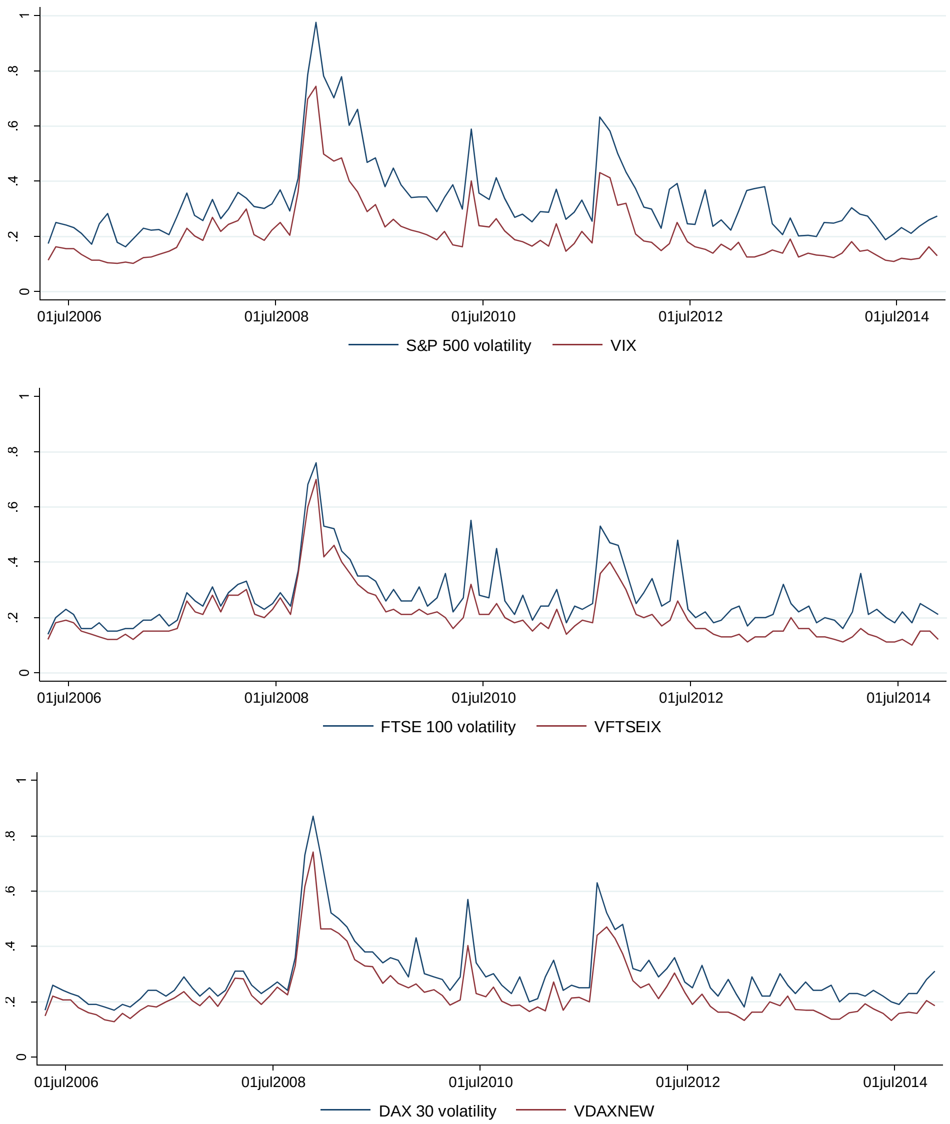
In the empirical analysis the time-series of volatility, skewness, and their premiums over the preceding one-month realizations are used. Also, the premium of the option-implied value over the realized moment over the return prediction period is used to have ex post risk premiums for comparison. The last explanatory variable inferred from risk-neutral densities is the tail density.

$$TAIL = \sum_{k=1}^{m} q_k \, , X_m < \mu - Z * OI - VOL < X_{m+1} \tag{6}$$

Figure 1

Risk-neutral density implied volatility time-series and volatility index time-series

The one-month forward-looking option-implied volatility as presented by traded volatility indexes, VIX, VFTSE, and VDAX-NEW, and the log-return volatility calculated from the formed risk-neutral densities. The volatility index and the density-based time-series have the same source of information, and for the one-month forecasts of index volatility the measures are correlated by 0.94 for S&P 500, by 0.92 for FTSE 100, and by 0.93 for DAX 30. The density-based estimates remarkably exceed the volatility index. The difference is on average 0.12 for S&P 500, 0.07 for FTSE 100, and 0.06 for DAX 30. Similarly to findings of Du and Kapadia (2012) the VIX provides a downward biased measure compared to the option-implied volatility of the holding period's log-returns (see Bakshi, Kapadia, and Madan, 2003, for a measure of model-free implied volatility extractable directly from option prices).



The tail density is formed so that by construction it is independent of the volatility level. Specifically, the tail density is the probability that the return falls below a specified number Z of standard deviations from the mean in the RND.

5.4.2 *Realized volatility, skewness, and premiums*

The realized values are calculated from daily logarithmic returns over the one-week, one-month, two-month, or three-month forecast periods, i.e. from t to T. The ex ante accessible premium is calculated from the preceding month's returns. The Datastream price index (PI) values for the underlying indexes are used as the basis. As noted in literature, skewness estimates are sensitive to outliers, and a more reliable estimate of volatility results from using high-frequency returns. Especially the one-week horizon values are subject to error. Acknowledging these shortcomings, the realized values are calculated as in Equation 7 and Equation 8, where $u = ln(PI_a/PI_{a-1})$, observations are daily, $a = t,\ldots,T$, and $n = T - t$. For the ex ante premiums, *EA-VOLPRE* and *EA-SKEWPRE*, $a = t\text{-}21,\ldots,t$ and $n = 21$. The volatilities are annualized similarly as the option-implied values.

$$VOL = \sqrt{\frac{\sum(u - \bar{u})^2}{(n-1)}} \tag{7}$$

The realized volatilities of S&P 500, FTSE 100, and DAX 30 (see Table B.2 in Appendix B) are close to each other during the sample period. For instance, the volatility sets that match the one-month option-maturity horizons show 0.18, 0.17, and 0.21 respectively. The differences in option-implied volatilities are larger, implying possibly a differing volatility premium across indexes or estimation differences. The S&P 500 sets are constructed on average with more valid option price quotes (see Table III on page 32).

$$SKEW = \frac{n}{(n-1)(n-2)} \sum \left(\frac{u - \bar{u}}{VOL}\right)^3 \tag{8}$$

Realized skewness is generally more negative than positive, but values are close to zero. This implies that actual returns are close to symmetric.

5.5 Methodology for the predictive regressions

Section 4 outlines the ordering of the analysis, and the motivation behind the empirical analysis. The time-series nature and overlapping of part of the data requires that issues with autocorrelation and conditional heteroskedasticity are considered when doing the empirical analysis. Research questions are first addressed by explaining index returns for S&P 500, FTSE 100, and DAX 30 separately in Sections 6.1, 6.2, and 6.3. In Section 6.4 the data is treated as a panel to increase the sample size and regressions are pooled. In Section 6.5 the data is treated again as a time-series.

Option maturities and sample observations have monthly intervals. Consequently, the one-week and one-month sets of returns can be reasonably thought to be serially independent. However, for the two longer horizons there is overlap, and by construction the explanatory variables and the logarithmic returns show serial correlation. For instance, at a single point in time the realized return over the last one and three months share the price innovations that occurred during the last month. As an example, the one-month logarithmic returns for S&P 500 show no clear structure in values of autocorrelation, but the two-month returns show a 0.56 correlation with its lagged value, and three-month returns are correlated by 0.76, 0.52, and 0.32 with lags from one to three months respectively.

Working with time-series data, the dependent variables should be stationary, i.e. mean and variance should be constant and unconditional on time. This is required for the estimator properties to hold. By applying the Dickey-Fuller test, a null hypothesis that the series has a unit root can be rejected for all return series except for two-month S&P 500 logarithmic returns, indicating stationarity. Therefore, no specification adjustments on the model are done due to non-stationary dependent variables. For post-estimation testing, the Breusch-Godfrey test for autocorrelation and a test for autoregressive conditional heteroskedasticity (ARCH) are applied. The errors in OLS estimations show autocorrelation, and ARCH effects are also present. To account for these, regressions are done with Newey-West standard errors with lag length of one. The samples have few missing option cross-sections, and the time-series are forced to be equally spaced. The special feature in applying the pooled regression with Newey-West standard errors for a panel in the econometric programme Stata is that observations are assumed to have zero serial correlation with the few missing observations (see Table III on page 32 for the number of available cross-sections for all indexes and maturities).

6. Results

6.1 Volatility and skew risk premium

Differences between risk-neutral and objective expectations of the future return distribution show premiums associated with different moments. Investor preferences and risk aversion determine the differences. Exposure to the first moment, mean, has a positive premium, and a risk-free bond on average has a lower return than an investment in the stock market. High mean return is good for investors. Exposure to the second moment, variance (or volatility), has a negative premium. Receiving the floating leg of a variance swap has on average negative returns. High volatility is bad for investors. Skew risk has a positive premium, and negatively skewed returns are bad for investors. Knowing any of the premiums indicates the magnitude of other premiums if they are driven by a common factor. The equity premium is interesting regarding equity index returns, and volatility and skew premiums are applied in explaining returns. This leads to the first research question.

Question 1 *Are volatility and skew premiums consistently behaving predictors of index returns across indexes?*

Table IV on page 41 provides results for the first set of predictability regressions, which consider the volatility premium. The ex post volatility premium, *VOLPRE*, is consistently positively related to subsequent returns for one-month, two-month, and three-month horizons. This holds for all three indexes. A one standard deviation increase in the ex post volatility premium means on average a 3.1%, 2.9% or 3.3% increase in the one-month logarithmic return of S&P 500, FTSE 100, and DAX 30 respectively. For the three-month horizon the impact of a one standard deviation increase is 7.4%, 6.4%, or 6.6%. Table IV shows Newey-West standard error based *t*-values in parenthesis below coefficient estimates, and the coefficients are significant at the 1% level. The impact on returns is sizable, and the positive relationship can result from the volatility premium's connection to the equity premium. An alternative and more likely explanation is that volatility tends to be lower when there is no downward trend in the stock market. Low realized volatility leads to high ex post premiums. Therefore, from a practical and theoretical perspective the ex ante volatility premium, *EA-VOLPRE*, is more interesting, because the interest is not in explaining the contemporaneous movement of return volatility and returns but rather in uncovering the predictive role.

Table IV

Volatility risk premiums and returns

OLS regressions with Newey-West standard errors. Results for univariate regressions, where logarithmic index returns over the following week (PANEL A), month (PANEL B), two months (PANEL C), and three months (PANEL D), or 5, 21, 42, and 63 trading days, are regressed on the realized volatility premium, *VOLPRE,* and the volatility premium of implied volatility over the preceding month's realized volatility, *EA-VOLPRE*. Option-implied volatility, which is the basis for the volatility premiums, is calculated from options maturing in 5, 21, 42, or 63 trading days so that the information is relevant and specific to the return prediction period. The sample period covers all option maturities with monthly intervals between May 2006 and December 2014, and observations span from February 2006 to December 2014. FTSE 100 and DAX 30 returns are also explained with S&P 500's ex ante volatility premium.

	S&P 500		FTSE 100			DAX 30		
PANEL A: 1-WEEK								
VOLPRE	-1.267		0.103			0.500		
	(0.97)		*(0.08)*			*(0.35)*		
EA-VOLPRE		-1.029		-2.242			1.253	
		(0.78)		*(0.94)*			*(0.88)*	
S&P 500 EA-VOLPRE					-0.521			-0.594
					(0.52)			*(0.41)*
Constant	0.298	0.230	-0.113	0.137	0.029	-0.052	-0.182	0.196
	(1.12)	*(0.79)*	*(0.76)*	*(0.61)*	*(0.13)*	*(0.18)*	*(0.63)*	*(0.61)*
N	103	103	104	104	103	104	104	103
Prob > F	0.33	0.44	0.94	0.35	0.60	0.73	0.38	0.69
R^2	0.03	0.01	0.00	0.01	0.01	0.00	0.01	0.00
PANEL B: 1-MONTH								
VOLPRE	**3.404**		**3.471**			**4.393**		
	(3.53)***		***(6.76)******			***(3.33)******		
EA-VOLPRE		2.071		0.767			-0.458	
		(1.52)		*(0.71)*			*(0.34)*	
S&P 500 EA-VOLPRE					1.283			1.161
					(1.21)			*(0.92)*
Constant	**-0.497**	-0.282	**-0.322**	-0.059	-0.176	**-0.357**	0.092	-0.123
	(3.46)***	*(1.22)*	***(4.70)******	*(0.47)*	*(1.00)*	***(2.75)******	*(0.59)*	*(0.54)*
N	103	103	104	104	103	104	104	103
Prob > F	0.00	0.13	0.00	0.48	0.23	0.00	0.73	0.36
R^2	0.34	0.05	0.29	0.01	0.02	0.27	0.00	0.01
PANEL C: 2-MONTH								
VOLPRE	**2.369**		**3.826**			**3.500**		
	(3.45)***		***(7.63)******			***(3.32)******		
EA-VOLPRE		**1.151**		0.181			-0.182	
		(1.73)*		*(0.18)*			*(0.33)*	
S&P 500 EA-VOLPRE					0.641			0.966
					(1.14)			*(1.52)*
Constant	**-0.254**	-0.100	**-0.269**	-0.014	-0.085	**-0.195**	0.056	-0.084
	(2.49)**	*(0.77)*	***(6.22)******	*(0.13)*	*(0.78)*	***(2.40)*****	*(0.60)*	*(0.66)*
N	103	103	104	104	103	104	104	103
Prob > F	0.00	0.09	0.00	0.86	0.26	0.00	0.74	0.13
R^2	0.44	0.06	0.52	0.00	0.02	0.37	0.00	0.03
PANEL D: 3-MONTH								
VOLPRE	**2.474**		**2.546**			**2.200**		
	(6.71)***		***(5.90)******			***(3.28)******		
EA-VOLPRE		**1.626**		0.692			0.224	
		(2.56)**		*(1.38)*			*(0.41)*	
S&P 500 EA-VOLPRE					**0.875**			**1.238**
					(2.08)**			***(2.35)*****
Constant	**-0.278**	-0.176	**-0.199**	-0.059	-0.119	**-0.114**	0.025	-0.123
	(4.72)***	*(1.55)*	***(4.62)******	*(0.88)*	*(1.48)*	***(1.79)****	*(0.34)*	*(1.22)*
N	101	101	104	104	102	100	100	98
Prob > F	0.00	0.01	0.00	0.17	0.04	0.00	0.68	0.02
R^2	0.57	0.13	0.49	0.02	0.05	0.32	0.00	0.06

* p<0.1; ** p<0.05; *** p<0.01

The ex ante volatility premium measures are inconsistent for the three indexes. The S&P 500 is the only index to show a clear positive relation between ex ante volatility premium and subsequent returns. The regression coefficients are significant for S&P 500's two-month and three-month horizons. A one standard deviation increase in the ex ante volatility premium leads on average to a 1.9% (3.7%) increase in two (three) month logarithmic returns for the S&P 500. Bollerslev et al. (2014) studied variance risk premium on eight indexes, and found that a globally weighted variance premium improves the explanatory power. This leads to ask whether the premiums are global, and could it be that the most liquid sets of options for the S&P 500 explain the other indexes better? This is studied by simply regressing the returns of FTSE 100 and DAX 30 with the ex ante volatility premium implicit in S&P 500 options and returns. This greatly improves the estimated models. The regression coefficients become consistently positive for FTSE 100 and DAX 30 for one-month, two-month, and three-month returns. The coefficients are significant at the 5% level for all indexes at the three-month horizon. A one standard deviation increase in S&P 500's ex ante volatility premium leads on average to a 3.7%, 2.0%, or 2.8% increase in three-month logarithmic returns of S&P 500, FTSE 100, and DAX 30 respectively.

These results are in line with the evidence for the U.S. stock market (see, e.g., Bollerslev, Tauchen, and Zhou, 2009; Drechsler and Yaron, 2011) and the recent extension (Bollerslev et al., 2014) to the global level. Yet, still uncovered is the skew premium's role. Skew premium in predicting equity index returns has not been addressed to my best knowledge. Research on moment swaps indicate that volatility and skew premiums are tightly linked (see Kozhan, Neuberger, and Schneider, 2013), and they should provide us with the same information. Table A.1 in Appendix A shows results for univariate regressions. Ex post skew premiums, *SKEWPRE*, are negatively related to returns for all indexes and horizons except the three-month horizon of DAX 30. S&P 500's ex ante skew premium is consistently negatively related to the returns of all indexes for one-month, two-month and three-month horizons.

The coefficients are statistically significant at the 5% level for S&P 500's one-month horizon, and for all indexes at the two-month horizon. A one standard deviation increase in the ex ante skew premium of S&P 500 leads to a 1.4%, 1.3%, or 2.1% decrease in two-month logarithmic returns for S&P 500, FTSE 100, and DAX 30 respectively. This is in line with the common factor explanation behind the premiums. Due to volatility risk premium's superior predictive performance and their similar explanatory impact, skew risk premium is not considered further. The results show that volatility and skew premiums are consistent predictors of future returns. The truly predictive ex ante variables require the use of S&P 500's option-implicit variables, indicating that the premiums are a global phenomenon or that S&P 500 options provide more relevant information.

Empirical finding 1 *Volatility and skew premiums are consistently behaving explainers of equity index returns, and the volatility risk premium is more powerful. For ex ante premiums, the results hold for FTSE 100 and DAX 30 if information implicit in S&P 500 options is used.*

The empirical findings strongly support the Hypothesis 1 that volatility premium is positively related to returns. Regarding the Hypothesis 2, information in skew premiums clearly seems similar than what is in the volatility premium, but statistically no conclusive evidence is reached. Bollerslev et al. (2014) report adjusted R^2 values for the variance risk premium and S&P 500 excess returns for their sample from 2000 to 2011, and the values are 8.9%, 8.7%, and 13.0% for one- to three-month horizons. The explanatory power with the more recent sample is similar, and Table IV reports simple R^2 values of 5%, 6%, and 13% for the same horizons. For comparison, the R^2 of the lagged equity premium of Santa-Clara and Yan (2010) for the three-month horizon is 6.6%.

6.2 Option-implied volatility and skewness

The focus in explaining short-term stock market aggregate returns has lately been in using tail measures or variance premiums. However, option-implied volatility and skewness are intuitively highly interesting, since they directly depict the nature of future uncertainty. If implied volatility is high, higher future stock return variation is likely and larger movements can be expected. Negative skewness leads to an increased probability of large negative returns. Equity premium is compensation for the uncertainty of the future price level, and therefore measures of this uncertainty are interesting. This leads to the second research question.

Question 2 *Are option-implied volatility and skewness consistently behaving predictors of index returns across indexes?*

Kelly and Jiang (2014) report the relation of stock volatility and risk-neutral skewness on subsequent returns on the CRSP value-weighted index. For one-month horizons stock volatility

Table V

Ex ante moments, volatility risk premiums, and returns

OLS regressions with Newey-West standard errors. Results for bivariate and multivariate regressions, in which logarithmic index returns over the following week (PANEL A), month (PANEL B), 2 months (PANEL C), or 3 months (PANEL D), or 5, 21, 42, and 63 trading days, are regressed on option-implied volatility, *OI-VOL*, skewness, *OI-SKEW*, and the ex post volatility premium, *VOLPRE*, or the ex ante volatility premium, *EA-VOLPRE*. The sample period covers all option maturities with monthly intervals between May 2006 and December 2014, and the observations span from February 2006 to December 2014.

	S&P 500		FTSE 100		DAX 30	
PANEL A: 1-WEEK						
OI-VOL	**-3.154**	**-4.943**	-2.296	-1.882	-1.178	-3.014
	(1.87)*	(2.14)**	(1.19)	(0.91)	(0.52)	(0.98)
OI-SKEW	-0.170	-0.004	-0.120	-0.189	-0.257	-0.099
	(1.37)	(0.02)	(0.61)	(0.75)	(1.13)	(0.35)
VOLPRE	0.471		0.729		0.745	
	(0.22)		(0.43)		(0.29)	
EA-VOLPRE		4.018		-0.352		4.555
		(1.16)		(0.09)		(0.91)
Constant	**0.742**	**1.066**	0.313	0.234	0.005	0.238
	(1.81)*	(2.12)**	(0.59)	(0.45)	(0.01)	(0.38)
N	103	103	104	104	104	104
Prob > F	0.16	0.10	0.43	0.38	0.35	0.25
R^2	0.12	0.15	0.05	0.05	0.03	0.05
PANEL B: 1-MONTH						
OI-VOL	**-2.079**	-0.608	**-1.575**	0.259	**-1.894**	0.630
	(3.89)***	(0.83)	(3.05)***	(0.35)	(2.89)***	(0.78)
OI-SKEW	**0.093**	-0.018	**0.313**	0.061	**0.396**	-0.031
	(2.50)**	(0.34)	(2.50)**	(0.42)	(2.02)**	(0.16)
VOLPRE	**5.118**		**4.939**		**6.186**	
	(7.04)***		(6.66)***		(5.98)***	
EA-VOLPRE		2.576		0.889		-1.142
		(1.52)		(0.57)		(0.66)
Constant	0.135	-0.198	**0.345**	-0.066	0.499	-0.066
	(0.86)	(0.99)	(1.74)*	(0.32)	(1.57)	(0.22)
N	103	103	104	104	104	104
Prob > F	0.00	0.31	0.00	0.78	0.00	0.88
R^2	0.48	0.07	0.39	0.01	0.37	0.01
PANEL C: 2-MONTH						
OI-VOL	**-1.678**	-0.220	**-1.150**	0.362	**-1.271**	0.481
	(4.28)***	(0.44)	(2.36)**	(0.83)	(2.29)**	(0.98)
OI-SKEW	**0.156**	0.007	**0.387**	**0.161**	**0.483**	-0.074
	(4.59)***	(0.15)	(6.09)***	(1.68)*	(4.20)***	(0.58)
VOLPRE	**4.135**		**4.709**		**5.290**	
	(11.33)***		(15.31)***		(10.07)***	
EA-VOLPRE		1.373		0.718		-0.716
		(1.31)		(0.61)		(1.00)
Constant	**0.304**	-0.050	**0.347**	0.019	**0.465**	-0.104
	(3.38)***	(0.45)	(2.87)***	(0.14)	(3.52)***	(0.64)
N	103	103	104	104	104	104
Prob > F	0.00	0.31	0.00	0.39	0.00	0.67
R^2	0.67	0.06	0.66	0.03	0.54	0.01
PANEL D: 3-MONTH						
OI-VOL	**-1.578**	-0.138	**-1.251**	0.325	**-1.315**	0.438
	(5.82)***	(0.43)	(2.98)***	(0.95)	(2.50)**	(0.84)
OI-SKEW	**0.094**	-0.006	**0.306**	0.073	**0.358**	0.153
	(2.08)**	(0.08)	(3.75)***	(0.62)	(4.32)***	(1.66)
VOLPRE	**3.461**		**3.367**		**4.165**	
	(11.86)***		(7.95)***		(9.10)***	
EA-VOLPRE		**1.656**		0.817		0.864
		(2.68)***		(1.48)		(1.24)
Constant	**0.220**	-0.144	**0.323**	-0.090	**0.387**	-0.030
	(2.76)***	(0.85)	(3.09)***	(0.68)	(3.76)***	(0.23)
N	101	101	104	104	100	100
Prob > F	0.00	0.06	0.00	0.38	0.00	0.31
R^2	0.75	0.13	0.61	0.03	0.60	0.02

* p<0.1; ** p<0.05; *** p<0.01

and risk-neutral skewness are negatively related to returns. Only stock volatility is statistically significant. With the sample at hand, bivariate regressions including option-implied volatility and skewness do not provide any consistent evidence. Applying S&P 500-based values in all regressions did not improve results, and one-week returns are only consistently related to volatility and skewness. The relationship is negative for both. Table V on page 44 reports results, in which ex ante volatility premium or ex post volatility premium is included. With the ex ante premium, volatility and skewness still fail to provide any conclusions.

If coupled with the ex post premium, the evidence becomes consistent and highly significant for one-month, two-month, and three-month horizons. Option-implied volatility is negatively and skewness positively related to subsequent returns, and the ex post volatility premium itself remains positively related. Consistent evidence is only reached when the ex post volatility premium is included, and the Hypothesis 3 that volatility and skewness require compensation and therefore predict returns is not supported. Substituting estimated RND model-free volatilities to volatility index values does not change the results.

6.3 Tail risk measures

Du and Kapadia (2012) and Kelly and Jiang (2014) provide tail risk measures that are positively related to index returns. Bollerslev and Todorov (2011) and Santa-Clara and Yan (2010) show that jumps in the price process are important in determining the size of the equity premium. Tails or jumps are therefore clearly priced. Tail and jump risk should be present in the tails of the estimated risk-neutral densities. Alternatively, Du and Kapadia (2012) provide a jump and tail index by decomposing jump risk from different measures of implied volatility. The measure builds on the difference between model-free volatility, in this case the RND option-implied volatility, and biased volatility measures given by volatility indexes. Section 2.1 includes an explanation of the variable and Section 5.2 provides definitions. These two measures are tested as alternatives. These considerations lead to the third research question.

Question 3 *Is the tail density or jump and tail index a consistently behaving predictor of subsequent returns across indexes?*

Table A.2 in Appendix A reports results for univariate regressions, in which the estimated tail probability density is used to explain equity index returns. The univariate regressions do not provide consistent evidence. Unlike with volatility and skewness premiums, using information from options on S&P 500 does not improve the results. The coefficients are mostly negative, indicating that a fatter option-implied tail leads to drops in the index. This is not in line with the idea that an increased risk of market crashes requires a compensation. Multivariate regressions including the ex post volatility premium, option-implied volatility, and option-implied skewness, reported in Table A.2 in Appendix A, do not change the results. If something, the tail is negatively related to returns. Hypothesis 4, that the tail is compensated, is not supported. Moreover, the evidence to the opposite direction is not strong enough to end up to alternative conclusions. The next set of regressions focuses on the jump and tail index created by Du and Kapadia (2012). Table VI below reports the results. To my best knowledge the measure has not been applied earlier to explain equity index returns outside the U.S. stock market.

The jump and tail index is positively related to subsequent returns for the S&P 500. As with the volatility risk premium, explanatory power is greatly increased by using the jump and tail index inferred from S&P 500 options and VIX to explain returns on FTSE 100 and DAX 30. A one standard deviation increase in the S&P 500 jump and tail index leads on average to a 1.5%, 1.4%, or 1.9% increase in two-month logarithmic returns for S&P 500, FTSE 100, and DAX 30 respectively. For three-month logarithmic returns, a one standard deviation increase leads to a 2.7%, 2.1%, or 2.4% increase.

Table VI

S&P 500 jump and tail index and returns

OLS regressions with Newey-West standard errors. Results for univariate regressions, in which annualized logarithmic index returns for S&P 500, FTSE 100, and DAX 30 from *t* to *T* are regressed on the jump and tail index of S&P 500, *JTIX**, observed at *t*. The jump and tail index is the difference between the option-implied volatility from risk-neutral densities, *OI-VOL*, and the VIX index. *T-t* is 21 trading days (1m, one month), 42 trading days (2m, two months), or 63 trading days (3m, three months). The sample ranges from February 2006 to December 2014, with option expiries from May 2006 to December 2014. *NOTE: S&P 500's jump and tail index is applied in all regressions.

	S&P 500			FTSE 100			DAX 30		
	1m	2m	3m	1m	2m	3m	1m	2m	3m
*JTIX**	2.197	**1.181**	**1.935**	1.998	**1.063**	**1.520**	1.784	**1.414**	**1.750**
	(1.46)	***(1.84)****	***(2.09)*****	*(1.52)*	***(1.98)****	***(1.88)****	*(1.16)*	***(2.13)*****	***(1.81)****
Constant	-0.232	-0.064	-0.160	-0.224	-0.106	-0.161	-0.163	-0.096	-0.141
	(1.14)	*(0.61)*	*(1.25)*	*(1.27)*	*(1.16)*	*(1.49)*	*(0.74)*	*(0.89)*	*(1.10)*
N	103	103	101	103	103	101	103	103	97
Prob > F	0.15	0.07	0.04	0.13	0.05	0.06	0.25	0.04	0.07
R^2	0.04	0.04	0.07	0.03	0.04	0.06	0.02	0.04	0.04

* $p<0.1$; ** $p<0.05$; *** $p<0.01$

The jump and tail index coefficient is significant at the 10% level for two-month returns of S&P 500 and FTSE 100, and three-month returns of FTSE 100 and DAX 30. The coefficient is significant at the 5% level for three-month S&P 500 returns and two-month DAX 30 returns. These results encourage to apply the jump and tail index as an indicator of compensated tail risk. The Hypothesis 3 and Hypothesis 4 that tail risk is compensated and behaves consistently are supported by these findings. On the other hand, the tail density measure, *TAILZ2*, seems inconclusive and negligible at this point.

Empirical finding 2 *The jump and tail index is positively related to subsequent returns, implying compensation for tail risk. The jump and tail indexes from FTSE 100 and DAX 30 options and volatility indexes do not provide similar predictive power as the jump and tail index from S&P 500 options and VIX. The S&P 500 based jump and tail risk measure is the only statistically significant and consistently behaving return predictor on all three indexes of the tail measures.*

6.4 Robustness to alternative explanatory variables

By far, ex ante volatility premium and the jump and tail index have been consistently and significantly related to future returns. In this section the variables' robustness to the inclusion to all of the other considered option-implicit variables as well as to common alternative predictors is tested. Section 5.2 defines the variables and Table VII on page 48 reports results. The jump and tail index of S&P 500 remains positively related to future returns, and the coefficients are significant at the 5% level for the one- and three-month horizons, and at the 1% level for the two-month horizon. The jump and tail index is therefore robust to the inclusion of the wide range of alternative explanatory variables, which implies that it contains important information that is not embedded in other variables. The ex ante volatility premium is also significantly and positively related. These variables serve as the only return predictors that are stable and statistically significant across the three longer horizons.

For other variables, the results do not imply a clear and significant relationship to returns for all three longer horizons. The default spread, *DEF*, is clearly negatively related to future returns for all horizons, and the coefficient is statistically significant for one-month returns at the 5% level when the jump and tail index is involved. Du and Kapadia (2012) have a sample from January 1996 to October 2009, and they also find a negative relationship between the default spread and returns.

Table VII

Robustness to alternative explanatory variables of returns

Pooled OLS regressions with Newey-West standard errors. Results for multivariate pooled regressions, in which annualized logarithmic index returns of S&P 500, FTSE 100, and DAX 30 are regressed on a set of independent variables. The sample spans from February 2006 to November 2014. Independent variables include option-implied volatility, *OI-VOL;* skewness, *OI-SKEW*; negative tail density, *TAILZ2;* S&P 500 ex ante volatility risk premium, *EA-VOLPRE**; jump and tail index from S&P 500's *OI-VOL* and the VIX index, *JTIX**; term spread between ten-year and three-month government liability yields, *TERM*; default spread between Moody's Baa and Aaa rated U.S. corporate bonds, *DEF*; one-month government liability yield minus its 12-month moving average, *RREL*; price to earnings, *PE*; and dividend yield, *DY*. *S&P 500's variables *EA-VOLPRE* and *JTIX* applied.

	1-week		1-month		2-month		3-month	
OI-VOL	-1.741	-0.259	0.531	0.509	0.476	0.513	0.600	0.519
	(0.59)	*(0.20)*	*(0.88)*	*(0.84)*	*(1.16)*	*(1.30)*	*(1.58)*	*(1.41)*
OI-SKEW	0.016	-0.094	0.081	**0.145**	0.042	0.067	0.063	**0.091**
	(0.12)	*(0.84)*	*(1.07)*	***(1.95)****	*(0.74)*	*(1.17)*	*(1.34)*	***(1.82)****
*EA-VOLPRE**	1.896		**1.486**		**0.783**		**1.051**	
	(0.55)		***(1.68)****		***(1.78)****		***(3.20)******	
*JTIX**		-0.933		**3.178**		**1.272**		**1.906**
		(0.90)		***(2.22)*****		***(2.86)******		***(2.39)*****
TAILZ2	0.702	0.314	**-17.888**	**-19.884**	-1.071	-0.449	-4.419	-4.685
	(0.08)	*(0.04)*	***(1.85)****	***(2.14)*****	*(0.22)*	*(0.09)*	*(1.45)*	*(1.49)*
TERM	-3.160	-2.511	-0.120	-0.336	2.879	2.965	2.609	2.004
	(0.35)	*(0.27)*	*(0.03)*	*(0.08)*	*(1.02)*	*(1.08)*	*(1.05)*	*(0.71)*
DEF	-43.936	-56.923	-23.144	**-34.536**	-7.519	-12.108	-7.904	-14.693
	(0.93)	*(1.30)*	*(1.39)*	***(2.10)*****	*(0.63)*	*(0.99)*	*(0.87)*	*(1.48)*
RREL	5.646	3.618	1.083	0.890	0.910	-0.054	2.390	0.808
	(0.26)	*(0.17)*	*(0.12)*	*(0.10)*	*(0.13)*	*(0.01)*	*(0.56)*	*(0.18)*
Ln(PE)	0.772	0.713	-0.025	-0.024	0.010	0.030	-0.054	-0.060
	(1.29)	*(1.14)*	*(0.12)*	*(0.11)*	*(0.06)*	*(0.19)*	*(0.45)*	*(0.49)*
Ln(DY)	0.450	0.590	0.184	0.085	0.009	-0.020	-0.063	-0.121
	(0.95)	*(1.21)*	*(0.88)*	*(0.40)*	*(0.06)*	*(0.14)*	*(0.55)*	*(0.93)*
Constant	0.400	1.015	1.734	1.571	-0.029	-0.178	0.008	-0.084
	(0.20)	*(0.52)*	*(1.53)*	*(1.38)*	*(0.05)*	*(0.31)*	*(0.02)*	*(0.20)*
N	311	309	309	309	309	309	301	299
Prob > F	0.69	0.60	0.22	0.10	0.42	0.10	0.02	0.05
R^2	0.08	0.09	0.05	0.06	0.04	0.05	0.09	0.09

* $p<0.1$; ** $p<0.05$; *** $p<0.01$

The tail density measure is also negatively related, and significant at the 5% or 10% level for one-month returns. Option-implied volatility and skewness are positively related to future returns, and the coefficients of skewness are significant at the 10% level for one- and three-month returns when the jump and tail index is involved.

These findings are important in answering the research questions. At this point Question 1 and Question 3 have answers. S&P 500's volatility risk premium is positively related to S&P 500's, FTSE 100's, and DAX 30's returns for two- and three-month horizons. So is the jump and tail risk if it is measured with the jump and tail index from S&P 500 options and VIX. So far Hypothesis

1, Hypothesis 2, and Hypothesis 4 set up in Section 4 have received support from the empirical evidence. However, implied volatility and skewness have failed to provide predictive power on returns as expected. In general, these results provide an extension to the earlier empirical evidence. The results show that the tail measure also explains returns outside the U.S. market, and is robust to the inclusion of risk-neutral moments. This is not self-evident. Negative price jumps should affect risk-neutral skewness, because far negative returns become more probable. This makes the probability distribution more skewed. As a measure of tail risk the jump and tail index therefore seems more accurate, assuming that exposure to the tail is compensated with a premium.

6.5 Global predictive power of S&P 500 equity index options

One additional test is highly motivated by the earlier results. If the premiums and tail risk are a global phenomenon, then the variables of S&P 500 should have predictive power globally. This assumes that S&P 500 is the best source for this kind of information from the three considered indexes. This is apparent based on earlier results in Section 6.1 and 6.3. Global evidence has been reported in Bollerslev et al. (2014) for the variance risk premium and index returns. Their global variance premium, with a 0.89 correlation with the S&P 500's premium, is positively related to returns for eight indexes, and explains variation best for four- and five-month horizons. Their sample spans from January 2000 to December 2010. Adding the jump and tail index brings new evidence on two things. First, does the tail risk measure explain returns globally? Second, what is the mutual explanatory nature of the volatility risk premium and the jump and tail index? If they measure different relevant factors, they have individual predictive power even if grouped with the other. As important as the results is to consider the reason why the measures predict returns.

Predictive multivariate regressions are run for ten equity indexes. Independent variables are ex ante volatility risk premium, jump and tail index, option-implied volatility, and option-implied skewness. All variables are based on S&P 500 equity index options and the VIX. The considered indexes comprise of S&P 500 (the U.S.), FTSE 100 (the U.K.), DAX 30 (Germany), Euro STOXX 50 (Europe), Nasdaq OMX Helsinki (Finland), Hang Seng (Hong Kong), Nikkei 225 (Japan), MXIPC35 (Mexico), MERVAL (Argentina), and S&P/ASX 200 (Australia). Prices for the return calculations are in local currencies. The considered return horizon is three months and three separate regressions are run for all indexes. First excludes the jump and tail index, second excludes the volatility premium, and the third one includes all four independent variables. Table VIII on page 52 reports results.

Without the jump and tail index in the regressions, the ex ante volatility premium of S&P 500 is positively related and statistically significant for nine of the ten equity indexes, the one being MERVAL. For MERVAL the Newey-West standard error based *t*-value is 1.47 and the regression coefficient is in line with other values. The coefficients range between 0.990 and 1.820, implying that the effect of a one standard deviation increase in the premium leads on average to a 2.2% to 4.1% increase in equity indexes' three-month logarithmic returns. Without the ex ante volatility premium, the jump and tail index is positively related to subsequent returns for all of the ten indexes. The coefficient *t*-values range from 1.27 to 2.54. For six indexes the variable is significant at the 10% level, and for four indexes significant at the 5% level. Variable coefficients are between 2.347 and 5.690, and a one standard deviation increase in the jump and tail index on average leads to a 3.2% to 7.8% increase in the equity indexes' three-month logarithmic returns.

Including both the ex ante volatility premium and the tail and jump index decreases the regression coefficients' significance. Only three of the ex ante volatility premium coefficients remain significant at the 10% level, and none of the coefficients for the jump and tail index. This finding strongly suggests that they are alternative measures of the same determinant of future returns. Looking at the R^2 and F-statistics supports this. For all of the ten indexes the F-statistics imply that inclusion of the jump and tail index returns does not improve the model. Moreover, the R^2 increases at most by 1% with the inclusion of the jump and tail index. S&P 500 equity index options clearly provide explanatory power on equity index returns globally. It is apparent that the variables indicate the same return determinant, while the volatility risk premium has some additional information embedded.

Based on theory, this additional information relates to time-varying risk aversion or investor preferences. First of all, the tail and jump index measure should purely expose tail risk in a risk-neutral world. Risk aversion and investor preferences are not extractable without estimates of jumps in the true expected price process. Estimation of future price jumps is difficult, because they occur rarely. In turn, the ex ante volatility premium is based on the difference between the risk-neutral expectation and the true expectation of return volatility. The measure includes expectations of jumps and investor preferences, but these two impacts cannot be easily separated.

Both the ex ante volatility premium and the jump and tail index are significant and the models excluding the other explain roughly the similar amount of variation in returns. Therefore, tail risk is clearly important, but the additional information in the volatility premium makes it a better predictor of equity index returns globally. To sum up, this section shows that S&P 500 equity index options provide explanatory power on equity index returns globally. The ex ante volatility risk premium and the jump and tail index contain similar predictive information, and the premium is a more powerful predictor. The role of jumps or tails is important. This is in line with the results of Bollerslev and Todorov (2011) and Santa-Clara and Yan (2010). The empirical findings of this section are summed as follows.

Empirical finding 3 *S&P 500 equity index options provide explanatory power on equity index returns globally. The ex ante volatility risk premium and the jump and tail index contain similar predictive information, and the premium is a more powerful predictor. Their similar predictive nature implies that they include a common component.*

From a practical point of view we now have sufficient evidence that either the volatility risk premium or the jump and tail index serves as a good indicator of future returns. They seem superior in predicting short term equity index returns, and the choice is between the two. From a theoretical point of view separation of jump and volatility risk in the process would be interesting. The next section reviews the study, and sums up the empirical findings.

Table VIII

Global equity index returns, S&P 500 options, and the VIX

OLS regressions with Newey-West standard errors. Three-month log-returns from *t* to *T* of S&P 500 (the U.S.), FTSE 100 (the U.K.), Euro STOXX 50 (Europe), Nasdaq OMX Helsinki (Finland), Hang Seng (Hong Kong), Nikkei 225 (Japan), MXIPC35 (Mexico), MERVAL (Argentina), and S&P/ASX 200 (Australia) are regressed on option-implicit variables from S&P 500 equity index options and the VIX index. Three separate regressions consider the explanatory power of option-implied volatility, *OI-VOL*, option-implied skewness, *OI-SKEW*, ex ante volatility premium, *EA-VOLPRE*, and the jump and tail index, *JTIX*. Option-implied volatility and skewness are the model-free option-implied volatility and skewness implicit in risk-neutral densities observed at *t*. The ex ante volatility premium is option-implied volatility observed at *t* less the realized volatility from *t-21 trading days* to *t*. The jump and tail index is option-implied volatility observed at *t* less VIX observed at *t*. The sample is from February 2006 to December 2014, with option cross-section expiry dates from May 2006 to December 2014. *NOTE: Independent variables are based on equity index options on S&P 500 and the volatility index, VIX.

	S&P 500			**FTSE 100**			**DAX 30**			**EURO STOXX 50**			**OMXH**		
*OI-VOL**	-0.189	**-1.351**	-0.695	0.265	-0.258	0.198	0.015	-1.021	-0.461	-0.089	-0.912	-0.197	-0.278	**-1.498**	-1.034
	(0.56)	***(1.92)****	*(1.12)*	*(0.90)*	*(0.49)*	*(0.34)*	*(0.04)*	*(1.42)*	*(0.61)*	*(0.23)*	*(1.10)*	*(0.25)*	*(0.66)*	***(1.67)****	*(1.08)*
*OI-SKEW**	-0.002	0.140	0.072	0.021	0.078	0.031	0.041	0.170	0.112	0.065	0.155	0.081	0.060	0.220	0.172
	(0.03)	*(1.46)*	*(0.79)*	*(0.29)*	*(0.88)*	*(0.34)*	*(0.56)*	*(1.56)*	*(0.94)*	*(0.87)*	*(1.29)*	*(0.65)*	*(0.69)*	*(1.66)*	*(1.20)*
*EA-VOLPRE**	**1.820**		**1.330**	**0.990**		0.924	**1.597**		1.134	**1.554**		**1.450**	**1.674**		0.940
	(2.90)***		***(1.85)****	***(2.00)*****		*(1.40)*	***(2.83)******		*(1.39)*	***(2.55)*****		***(1.80)****	***(2.31)*****		*(1.03)*
*JTIX**		**4.993**	2.000		2.347	0.267		**4.439**	1.887		3.687	0.424		**5.110**	2.995
		(2.54)**	*(0.98)*		*(1.45)*	*(0.13)*		***(2.20)*****	*(0.69)*		*(1.61)*	*(0.15)*		***(1.90)****	*(0.87)*
Constant	-0.149	0.135	-0.030	-0.192	-0.061	-0.176	-0.126	0.128	-0.014	-0.128	0.078	-0.103	-0.103	0.192	0.075
	(0.87)	*(0.79)*	*(0.16)*	*(1.17)*	*(0.40)*	*(1.04)*	*(0.71)*	*(0.67)*	*(0.06)*	*(0.74)*	*(0.38)*	*(0.47)*	*(0.48)*	*(0.83)*	*(0.29)*
N	101	101	101	101	101	101	101	101	101	101	101	101	101	101	101
F	2.92	2.33	2.25	1.46	1.14	1.12	2.76	1.84	2.07	2.25	1.01	1.74	1.82	1.23	1.39
Prob > F	0.04	0.08	0.07	0.23	0.34	0.35	0.05	0.14	0.09	0.09	0.39	0.15	0.15	0.30	0.24
R^2	0.15	0.13	0.15	0.07	0.05	0.07	0.08	0.07	0.09	0.07	0.05	0.07	0.06	0.06	0.07

	HANG SENG			**NIKKEI 225**			**MXIPC35**			**MERVAL**			**S&P/ASX 200**		
*OI-VOL**	0.341	-0.494	0.292	0.020	-0.992	-0.848	0.267	-0.642	-0.122	0.284	-0.843	-0.446	-0.063	-1.066	-0.600
	(0.73)	*(0.57)*	*(0.33)*	*(0.05)*	*(1.36)*	*(0.96)*	*(0.64)*	*(0.96)*	*(0.16)*	*(0.43)*	*(0.68)*	*(0.32)*	*(0.22)*	*(1.59)*	*(0.90)*
*OI-SKEW**	**0.166**	**0.254**	0.173	-0.160	-0.017	-0.031	0.156	**0.268**	**0.214**	0.015	0.164	0.123	0.059	**0.186**	0.138
	(1.75)*	***(2.12)*****	*(1.38)*	*(1.39)*	*(0.11)*	*(0.19)*	*(1.57)*	***(2.29)*****	***(1.67)****	*(0.10)*	*(0.80)*	*(0.56)*	*(0.78)*	***(1.77)****	*(1.39)*
*EA-VOLPRE**	**1.641**		**1.593**	**1.134**		0.293	**1.431**		1.053	1.513		0.805	**1.466**		0.945
	(2.65)***		***(1.78)****	***(1.78)****		*(0.34)*	***(2.93)******		*(1.28)*	*(1.47)*		*(0.69)*	***(2.48)*****		*(1.52)*
*JTIX**		3.780	0.194		**4.092**	3.433		**3.911**	1.542		4.698	2.887		**4.253**	2.127
		(1.48)	*(0.06)*		***(1.76)****	*(1.06)*		***(2.25)*****	*(0.57)*		*(1.27)*	*(0.63)*		***(2.13)*****	*(0.99)*
Constant	-0.068	0.142	-0.057	**-0.387**	-0.147	-0.183	0.019	0.241	0.110	-0.093	0.179	0.079	-0.106	0.138	0.020
	(0.26)	*(0.59)*	*(0.23)*	***(1.84)****	*(0.68)*	*(0.71)*	*(0.09)*	*(1.13)*	*(0.48)*	*(0.27)*	*(0.50)*	*(0.19)*	*(0.63)*	*(0.79)*	*(0.11)*
N	101	101	101	101	101	101	101	101	101	101	101	101	101	101	101
F	4.57	2.15	3.41	2.26	2.21	1.73	3.50	2.58	2.71	0.91	0.77	0.68	2.15	1.83	1.65
Prob > F	0.00	0.10	0.01	0.09	0.09	0.15	0.02	0.06	0.03	0.44	0.51	0.61	0.10	0.15	0.17
R^2	0.07	0.05	0.07	0.10	0.11	0.11	0.08	0.07	0.09	0.03	0.03	0.03	0.09	0.08	0.10

* $p<0.1$; ** $p<0.05$; *** $p<0.01$

7. Conclusion

This study assesses the explanatory power of option prices on subsequent equity index returns. This is motivated by the rich forward-looking information content of options, and by the numerous recent findings of options' strong explanatory power on individual stock returns and the stock market on aggregate. These findings give a better picture on what risks are compensated in financial asset returns, and how to assess the riskiness of investments. In this study a comprehensive approach is adopted to find out the mutual explanatory nature of option-related information, and to find out the main components of explanatory power that are robust to the inclusion of other alternatives. In short, tail risk and time-varying risk aversion or investor preferences are the key components of return predictability. Equity index options, volatility indexes, and return data on S&P 500, FTSE 100, and DAX 30 are employed. One-week, one-month, two-month, and three-month returns on equity indexes are in the focus. In addition, information on the S&P 500 is used to explain future returns for the three indexes and Euro STOXX 50 (Europe), Nasdaq OMX Helsinki (Finland), Hang Seng (Hong Kong), Nikkei 225 (Japan), MXIPC35 (Mexico), MERVAL (Argentina), and S&P/ASX 200 (Australia).

The results show that the information in S&P 500 options has global explanatory power. This exceeds the explanatory power in options on FTSE 100 and DAX 30 even on the returns of their underlying indexes. Moreover, the results show that the volatility risk premium and the jump and tail index contain the relevant information in options and volatility indexes regarding short-term equity index returns. This is because they imply tail risk and risk aversion. A one standard deviation increase in the ex ante volatility premium leads on average to a 2.2% to 4.1% increase in three-month logarithmic returns for different indexes globally. A one standard deviation increase in the jump and tail index leads to a 3.2% to 7.8% increase. The two predictors contain similar explanatory information, and adding the jump and tail index leads at most to a 1% increase in explained return variation for the three-month global equity index returns. The volatility risk premium also contains information on risk aversion, whereas the jump and tail index is explicitly a measure of jumps under the risk-neutral measure. Ex ante moments, in turn, seem negligible.

The contribution of this study is twofold. First, the top-down approach reveals the option-implicit variables that are relevant and robust in explaining short term equity index returns. This is interesting from a theoretical perspective, since it enables inferences on what risks are compensated. Second, the evidence from the U.S. stock market is supplemented with new global evidence. Regarding further research, more accurate separation of risks due to large price moves and small price variation could be used to provide global evidence, as is done for the U.S. market by Bollerslev and Todorov (2011) and Santa-Clara and Yan (2010). This is encouraged by the observed small role of risk-neutral moments, and focusing the approach to studying implicit price processes seems worthwhile. A model of the price process that separates small price moves and price jumps would enable a more accurate breakdown of risks and compensation of risks. In the current approach, for instance, the model-free implied volatility includes possible price jumps in the process and jumps' contribution to the volatility risk premium. Moreover, the jump and tail index does not enable inferences of the jump risk premium, because the true expectation of large price moves is missing.

The discrepancy between information in S&P 500 options and options on FTSE 100 and DAX 30 is also interesting. Is the superior explanatory power in S&P 500 options due to more liquid options and a wider range of available strikes, or are there systematic inconsistencies in option pricing? Based on the explanatory power of S&P 500 options either one is the answer, since perceived risks and fears that are reflected in the U.S. equity index option market are clearly relevant worldwide.

To conclude, the hypothesized connection between the moment premiums is supported, and this is likely due to time-varying risk aversion. Higher skew and volatility premiums, in absolute value, lead to higher subsequent returns on average, implying a higher equity premium. Instead, moments of the RNDs do not provide information on future equity index returns. The hypothesized connection between tail risk and higher future returns is confirmed, and the question is more of how it is measured. The jump and tail index proves to be a remarkably powerful predictor of future returns.

Appendix A

Table A.1

Skew risk premium and returns

OLS regressions with Newey-West standard errors. Results for univariate regressions, in which logarithmic index returns over the following week (PANEL A), month (PANEL B), two months (PANEL C), or three months (PANEL D), are regressed on the ex post skew premium, *SKEWPRE*, and the ex ante skew premium *EA-SKEWPRE*. Option-implied skewness, which is the basis for the skew premiums, is calculated from options maturing in 5, 21, 42, or 63 trading days so that the information is specific to the return prediction period. The sample ranges from February 2006 to December 2014.

	S&P 500		FTSE 100			DAX 30		
PANEL A: 1-WEEK								
SKEWPRE	-0.057		-0.164			**-0.304**		
	(1.14)		*(1.39)*			***(2.32)*****		
EA-SKEWPRE		0.035		-0.122			-0.072	
		(0.45)		*(1.02)*			*(0.56)*	
S&P 500 EA-SKEWPRE					-0.003			-0.020
					(0.04)			*(0.20)*
Constant	-0.156	0.074	-0.314	-0.250	-0.100	-0.372	-0.050	0.010
	(0.81)	*(0.34)*	*(1.46)*	*(1.10)*	*(0.50)*	*(1.29)*	*(0.18)*	*(0.04)*
N	103	103	104	104	103	104	104	103
F-statistic	1.29	0.20	1.94	1.03	0.00	5.38	0.31	0.04
Prob > F	0.26	0.66	0.17	0.31	0.97	0.02	0.58	0.84
R^2	0.00	0.00	0.02	0.01	0.00	0.04	0.00	0.00
PANEL B: 1-MONTH								
SKEWPRE	-0.063		-0.077			-0.054		
	(1.31)		*(1.11)*			*(0.66)*		
EA-SKEWPRE		**-0.102**		-0.106			-0.096	
		(2.00)**		*(1.49)*			*(0.94)*	
S&P 500 EA-SKEWPRE					-0.065			-0.104
					(1.34)			*(1.62)*
Constant	-0.101	-0.182	-0.082	-0.109	-0.119	-0.015	-0.064	-0.167
	(0.66)	*(1.14)*	*(0.73)*	*(0.92)*	*(0.85)*	*(0.11)*	*(0.39)*	*(0.93)*
N	103	103	104	104	103	104	104	103
F-statistic	1.72	4.01	1.23	2.21	1.80	0.44	0.88	2.63
Prob > F	0.19	0.05	0.27	0.14	0.18	0.51	0.35	0.11
R^2	0.01	0.04	0.01	0.02	0.02	0.00	0.01	0.03
PANEL C: 2-MONTH								
SKEWPRE	**-0.085**		-0.022			-0.063		
	(2.77)***		*(0.30)*			*(0.66)*		
EA-SKEWPRE		**-0.080**		0.041			-0.041	
		(2.01)**		*(0.58)*			*(0.83)*	
S&P 500 EA-SKEWPRE					**-0.074**			**-0.118**
					(2.08)**			***(2.51)*****
Constant	-0.080	-0.074	-0.022	0.040	-0.118	-0.009	0.006	-0.144
	(0.86)	*(0.67)*	*(0.23)*	*(0.46)*	*(1.21)*	*(0.09)*	*(0.08)*	*(1.20)*
N	103	103	104	104	103	104	104	103
F-statistic	7.70	4.02	0.09	0.34	4.34	0.43	0.69	6.32
Prob > F	0.01	0.05	0.76	0.56	0.04	0.51	0.41	0.01
R^2	0.04	0.03	0.00	0.00	0.03	0.00	0.00	0.05
PANEL D: 3-MONTH								
SKEWPRE	**-0.093**		-0.042			0.022		
	(1.88)*		*(0.61)*			*(0.34)*		
EA-SKEWPRE		-0.069		-0.001			0.000	
		(1.63)		*(0.02)*			*(0.00)*	
S&P 500 EA-SKEWPRE					-0.045			-0.072
					(1.18)			*(1.64)*
Constant	-0.069	-0.048	-0.035	-0.004	-0.061	0.056	0.042	-0.052
	(0.76)	*(0.51)*	*(0.49)*	*(0.05)*	*(0.76)*	*(0.95)*	*(0.63)*	*(0.57)*
N	101	101	104	104	102	100	100	98
F-statistic	3.55	2.66	0.38	0.00	1.38	0.11	0.00	2.69
Prob > F	0.06	0.11	0.54	0.98	0.24	0.74	1.00	0.10
R^2	0.03	0.02	0.00	0.00	0.01	0.00	0.00	0.02

* p<0.1; ** p<0.05; *** p<0.01

Table A.2

Tail density, moments, moment premiums, and returns

OLS regressions with Newey-West standard errors. Results for univariate and multivariate regressions, in which logarithmic index returns over the following week (PANEL A), month (PANEL B), two months (PANEL C), or three months (PANEL D), or 5, 21, 42, and 63 trading days, are regressed on 1. the negative tail of the estimated risk-neutral probability density, *TAILZ2*, and 2. *TAILZ2*, option-implied volatility and skewness, *OI-VOL* and *OI-SKEW*, and volatility premium, *VOLPRE*, which is *OI-VOL* less the period's realized volatility. The sample ranges from February 2006 to December 2014.

	S&P 500		FTSE 100		DAX 30	
PANEL A: 1-WEEK						
TAILZ2	2.616	-11.440	8.666	2.778	2.175	-8.420
	(0.13)	*(0.36)*	*(0.44)*	*(0.14)*	*(0.22)*	*(0.71)*
OI-VOL		**-3.185**		-2.291		-1.166
		(1.84)*		*(1.18)*		*(0.50)*
OI-SKEW		-0.134		-0.115		-0.321
		(1.14)		*(0.59)*		*(1.57)*
VOLPRE		0.432		0.692		1.094
		(0.19)		*(0.42)*		*(0.39)*
Constant	-0.119	1.332	-0.543	0.180	-0.056	0.241
	(0.14)	*(0.75)*	*(0.56)*	*(0.16)*	*(0.11)*	*(0.30)*
N	103	103	104	104	104	104
Prob > F	0.89	0.20	0.66	0.60	0.83	0.24
R^2	0.00	0.12	0.00	0.05	0.00	0.03
PANEL B: 1-MONTH						
TAILZ2	**-12.648**	**-16.233**	-8.051	**-28.620**	-9.294	-26.456
	(1.76)*	***(2.06)*****	*(0.65)*	***(3.07)******	*(0.50)*	*(1.41)*
OI-VOL		**-2.168**		**-1.842**		**-1.963**
		(3.95)***		***(3.84)******		***(3.01)******
OI-SKEW		**0.199**		**0.416**		**0.483**
		(3.23)***		***(3.88)******		***(2.64)******
VOLPRE		**5.174**		**5.357**		**6.383**
		(7.33)***		***(7.58)******		***(6.07)******
Constant	**0.660**	**1.193**	0.477	**2.142**	0.562	**2.062**
	(2.04)**	***(2.05)*****	*(0.66)*	***(3.78)******	*(0.56)*	***(2.07)*****
N	103	103	104	104	104	104
Prob > F	0.08	0.00	0.52	0.00	0.61	0.00
R^2	0.03	0.50	0.00	0.44	0.00	0.39
PANEL C: 2-MONTH						
TAILZ2	-2.599	-0.587	3.380	-5.036	1.911	-7.782
	(0.90)	*(0.12)*	*(0.50)*	*(1.37)*	*(0.14)*	*(1.12)*
OI-VOL		**-1.675**		**-1.097**		**-1.193**
		(4.16)***		***(2.27)*****		***(2.17)*****
OI-SKEW		**0.162**		**0.394**		**0.495**
		(2.45)**		***(6.55)******		**(3.91)*****
VOLPRE		**4.142**		**4.772**		**5.365**
		(11.07)***		**(15.51)*****		**(10.10)*****
Constant	0.228	0.353	-0.235	**0.687**	-0.084	**0.966**
	(1.21)	*(0.90)*	*(0.48)*	***(2.37)*****	*(0.09)*	***(2.08)*****
N	103	103	104	104	104	104
Prob > F	0.37	0.00	0.62	0.00	0.89	0.00
R^2	0.00	0.67	0.00	0.66	0.00	0.55
PANEL D: 3-MONTH						
TAILZ2	-0.669	-0.250	**-10.305**	-5.348	-4.536	-1.800
	(0.13)	*(0.05)*	***(1.91)****	*(1.25)*	*(0.79)*	*(0.43)*
OI-VOL		**-1.577**		**-1.253**		**-1.303**
		(5.77)***		***(3.00)******		***(2.46)*****
OI-SKEW		0.096		**0.256**		**0.354**
		(1.44)		***(3.12)******		***(4.06)******
VOLPRE		**3.463**		**3.323**		**4.156**
		(11.50)***		***(7.77)******		***(9.11)******
Constant	0.083	0.237	**0.602**	**0.597**	0.277	**0.475**
	(0.27)	*(0.63)*	***(1.87)****	***(2.40)*****	*(1.00)*	***(2.21)*****
N	101	101	104	104	100	100
Prob > F	0.90	0.00	0.06	0.00	0.43	0.00
R^2	0.00	0.75	0.02	0.61	0.01	0.60

* p<0.1; ** p<0.05; *** p<0.01

Appendix B

Table B.1

Terminology of relevant literature

Risk-neutral probability density function	Defines the forward looking probability distribution of the asset price level or returns implicit in option prices. Is inferred from option prices and is specific to the period up to the option cross-section's maturity.
Objective probability density function	Unobserved true expectation of the probability distribution of the asset price level or returns. In this study characteristics of the unobserved objective distribution are taken from actual realizations of return volatility and skewness.
Equity risk premium	Equity risk is uncertainty of the future price level. Equity risk premium is the expected compensation for holding equity risk. The return difference between a risk-free asset and the equity instrument should on average equal the premium, and is generally positive.
Volatility and skew risk	Risk related to changes in volatility and skewness of returns. E.g. a delta-neutral and vega-positive option position is exposed to volatility risk but not equity risk. Can also refer to volatility and skewness of returns.
Volatility and skew risk premiums	The difference between the risk-neutral and objective or statistical expectation of volatility and skewness. Shows the compensation for holding volatility or skewness risk. Volatility risk premium is generally negative, i.e. a strategy that pays off when volatility rises has on average negative returns.
Moments	Mean, variance (or volatility), skewness, and kurtosis of returns. Moments characterise a probability density function parametrically if the distribution is known. A normal distribution is determined by its mean and variance.
Moment premiums	The difference between the risk-neutral and objective expectation of a moment. Equity, volatility, and skew risk premiums are moment premiums. E.g. if one wants to receive a fixed payoff based on future return variance and pay a floating payoff based on actual realized variance, on average the payoff is positive due to exposure to variance risk.
Moment swaps	A moment swap swaps the option-implied moment (fixed leg) to the realized moment (floating leg). Payoffs to moment swaps on average unveil the risk premium related to the specific moment. E.g. investing in the stock market and borrowing at the risk-free rate can be thought of as a moment swap on the first moment of returns.
Tail risk	Tail refers to the far end of a probability distribution. A major negative price jump would mean the realization of tail risk. A negatively skewed distribution means that far negative movements are more probable. Therefore jump risk and skewness are related to tail risk.
Diffusive risk	Risk due to small price moves. Diffusive risk can be hedged. E.g. in Bollerslev and Todorov (2011) diffusive risk is the variation in returns and volatility of returns attributable to a continuous-time stochastic volatility process.
Jump risk	Risk due to discontinuous jumps in the asset's price. In Bollerslev and Todorov (2011) compensation for jump risks drives both the variance risk premium and the equity risk premium. Modeling jumps typically involves modeling the jump size and intensity.
Delta	Position's sensitivity to changes in value of the underlying asset. Delta is model-specific, and usually refers to the Black-Scholes delta. Holding the underlying asset equals a delta of 1. Zero delta option positions can be used to show the price of volatility risk or, from an alternative perspective, do volatility arbitrage.
Vega	Position's sensitivity to changes in volatility. Vega is model-specific, and usually refers to the Black-Scholes vega. For long option positions vega is positive, and increases in volatility benefit the option holder. A delta neutral but vega positive option position is useful for assessing the volatility risk premium.
Gamma	Gamma measures delta's sensitivity to changes in value of the underlying asset. Gamma and vega positive option positions are used to assess the pricing of diffusive risk and jump risk, e.g. by Cremers, Halling, and Weinbaum (2015).

Table B.2

Summary statistics of the explanatory variables

This table shows summary statistics of the option-implicit explanatory variables employed in return-predictive regressions. Also, the realized values of volatility, skewness, and kurtosis are presented, as well as kurtosis premium which is not used in the predictive regressions. Option-implied model-free moments, *OI-VOL*, *OI-SKEW*, and *OI-KURT* are based on the risk-neutral densities calculated from option prices (see Figure 2 and Figure 3 in Appendix C for examples). Ex post premiums, *VOLPRE*, *SKEWPRE*, and *KURTPRE*, are differences between option-implied moments and realized values during the return prediction periods. Ex ante premiums *EA-VOLPRE*, *EA-SKEWPRE*, and *EA-KURTPRE* are differences between option-implied moments and realized values occurred during 21 trading days before the prediction periods. *TAILZ2* is the probability density in the negative side tail of the risk-neutral probability distributions. *JTIX* is the difference between *OI-VOL* and volatility index value for the underlying index. Volatility indexes, VIX, VFTSE, and VDAX-NEW, are based on options maturing close to 30 days from the observation. Option-implicit volatility, skewness, and kurtosis all decrease on average in absolute value as the forecast horizon becomes longer. The same holds for the volatility, skew, and kurtosis premiums. The average size of the premiums are not dependent on whether the premium is calculated over the forecast period or from the preceding month's realized value of volatility, skewness, or kurtosis of returns. The tail density is higher for longer forecast periods.

Mean / Std. Deviation	**S&P 500**				**FTSE 100**				**DAX 30**			
	1w	**1m**	**2m**	**3m**	**1w**	**1m**	**2m**	**3m**	**1w**	**1m**	**2m**	**3m**
OI-VOL	0.41	0.33	0.31	0.31	0.28	0.27	0.25	0.26	0.38	0.30	0.28	0.28
	0.23	*0.15*	*0.15*	*0.12*	*0.13*	*0.11*	*0.09*	*0.09*	*0.16*	*0.12*	*0.11*	*0.11*
OI-SKEW	-2.5	-2.24	-1.71	-1.39	-1.21	-1.22	-1.01	-0.86	-1.35	-1.16	-0.89	-0.76
	1.69	*1.05*	*0.93*	*0.57*	*0.6*	*0.59*	*0.42*	*0.29*	*0.88*	*0.51*	*0.44*	*0.64*
OI-KURT	15.9	10.79	5.75	3.49	3.61	3.03	1.75	1.03	5.24	2.82	1.37	1.36
	17.43	*9.95*	*7.7*	*1.48*	*2.95*	*4.41*	*2.26*	*1.01*	*4.94*	*3.07*	*2.32*	*3.35*
VOL - Realized volatility	0.17	0.18	0.18	0.18	0.17	0.17	0.18	0.18	0.19	0.21	0.21	0.21
	0.15	*0.13*	*0.12*	*0.12*	*0.14*	*0.11*	*0.10*	*0.10*	*0.14*	*0.11*	*0.10*	*0.10*
SKEW - Realized skew	0.03	-0.08	-0.16	-0.22	0.08	0.06	-0.03	-0.09	0.03	0.03	-0.07	-0.11
	1.01	*0.64*	*0.60*	*0.57*	*0.99*	*0.58*	*0.47*	*0.36*	*0.89*	*0.56*	*0.46*	*0.37*
KURT - Realized kurtosis	0.24	0.75	1.02	1.25	0.3	0.41	0.6	0.72	0.25	0.62	0.76	0.89
	2.33	*1.18*	*1.23*	*1.48*	*1.94*	*1.20*	*1.24*	*0.99*	*1.92*	*1.30*	*1.14*	*1.14*
VOLPRE	0.24	0.16	0.13	0.13	0.11	0.10	0.07	0.08	0.20	0.09	0.07	0.07
	0.21	*0.11*	*0.13*	*0.12*	*0.12*	*0.10*	*0.08*	*0.1*	*0.16*	*0.09*	*0.10*	*0.12*
SKEWPRE	-2.55	-2.18	-1.57	-1.21	-1.29	-1.28	-0.99	-0.77	-1.38	-1.18	-0.82	-0.66
	2.05	*1.16*	*1.13*	*0.76*	*1.16*	*0.84*	*0.58*	*0.43*	*1.27*	*0.72*	*0.57*	*0.71*
KURTPRE	15.7	10.08	4.79	2.37	3.31	2.62	1.15	0.31	4.99	2.20	0.62	0.48
	17.47	*9.76*	*7.44*	*3.55*	*3.72*	*4.48*	*2.6*	*1.44*	*5.25*	*3.23*	*2.50*	*3.62*
EA-VOLPRE	0.23	0.15	0.13	0.13	0.11	0.10	0.07	0.08	0.18	0.09	0.07	0.07
	0.18	*0.07*	*0.10*	*0.09*	*0.07*	*0.06*	*0.06*	*0.07*	*0.10*	*0.06*	*0.07*	*0.10*
EA-SKEWPRE	-2.41	-2.15	-1.59	-1.28	-1.23	-1.17	-1.01	-0.92	-1.34	-1.17	-0.89	-0.72
	1.77	*1.30*	*1.06*	*0.86*	*0.89*	*0.79*	*0.72*	*0.71*	*1.07*	*0.76*	*0.83*	*0.91*
EA-KURTPRE	15.16	10.00	4.98	2.73	3.19	2.60	1.31	0.49	4.69	2.19	0.70	0.55
	17.23	*9.82*	*7.69*	*3.61*	*3.09*	*4.55*	*2.72*	*1.82*	*4.99*	*3.14*	*2.75*	*3.64*
TAILZ2	0.042	0.049	0.067	0.060	0.051	0.06	0.069	0.059	0.047	0.055	0.067	0.052
	0.008	*0.009*	*0.011*	*0.006*	*0.006*	*0.005*	*0.007*	*0.005*	*0.018*	*0.005*	*0.006*	*0.007*
JTIX	0.196	0.123	0.099	0.105	0.073	0.065	0.043	0.054	0.149	0.064	0.044	0.051
	0.182	*0.056*	*0.079*	*0.055*	*0.050*	*0.042*	*0.035*	*0.047*	*0.102*	*0.039*	*0.056*	*0.078*

Table B.3

Summary statistics of the alternative explanatory variables

This table shows summary statistics for term spreads, default spreads, stochastically detrended risk-free rates, price-to-earnings ratios, and dividend yields applied in Section 6.4. The means and standard deviations are calculated from monthly observations from April 2006 to November 2014, so that the information supplements the option information which covers option cross-section expiries from May 2006 to December 2014. Term spread is the difference between ten-year and one-month government liability yields. Default spread is the yield difference between Moody's Baa and Aaa rated U.S. corporate bonds, and the same values are applied for all indexes. Detrended risk-free rate is the one-month government liability yield less its 12-month trailing average. Price-to-earnings is the equity index market capitalization divided by aggregate earnings. Dividend yield equals aggregate dividends divided by market capitalization. Price-to-earning and dividend yield are directly Thomson Reuters Datastream's default datatypes. *NOTE: The applied default spread is the same for all indexes.

		Mean			**Standard deviation**		
		S&P 500	**FTSE 100**	**DAX 30**	**S&P 500**	**FTSE 100**	**DAX 30**
Term spread	*TERM*	0.0205	0.0163	0.0100	0.0116	0.0147	0.0095
Default spread*	*DEF*	0.0118	0.0118	0.0118	0.0058	0.0058	0.0058
Detrended risk-free rate	*RREL*	-0.0024	-0.0023	-0.0014	0.0070	0.0081	0.0075
Price-to-earnings	*PE*	16.53	12.26	14.64	2.25	2.24	4.48
Dividend yield	*DY*	0.0211	0.0360	0.0327	0.0031	0.0059	0.0070

Table B.4

Summary statistics of monthly logarithmic equity index returns

This table shows summary statistics for the equity index returns considered for the ten equity indexes. The statistics are based on monthly observations of monthly returns from April 2006 to December 2014. The indexes are from the U.S. (S&P 500), the U.K. (FTSE 100), Germany (DAX 30), Europe (Euro STOXX 50), Finland (OMXH), Hong Kong (Hang Seng), Japan (Nikkei 225), Mexico (MXIPC35), Argentina (MERVAL), and Australia (S&P/ASX 200). The statistics are based on returns in local currencies.

	Mean	**Standard Deviation**	**Annualized volatility**
S&P 500	0.00255	0.05253	0.18
FTSE 100	-0.00140	0.05468	0.19
DAX 30	0.00176	0.06580	0.23
EURO STOXX 50	-0.00485	0.06372	0.22
OMXH	-0.00644	0.06798	0.24
HANG SENG	0.00294	0.06693	0.23
NIKKEI 225	-0.00014	0.07197	0.25
MXIPC35	0.00793	0.05500	0.19
MERVAL	0.01422	0.09451	0.33
S&P/ASX 200	0.00002	0.05240	0.18

Table B.5

Correlation matrices of variables for one-month observations

This table shows correlation matrices for the option-implicit explanatory variables and volatility indexes employed in return-predictive regressions. The equity index option observations are done at *t*, which is 21 trading days before the option cross-section maturity *T*, and the information is used to explain returns from *t* to *T*. Option-implied model-free moments, *OI-VOL* and *OI-SKEW*, are based on the risk-neutral densities calculated from option prices (see Figure 2 and Figure 3 in Appendix C for examples). Ex post volatility premiums *VOLPRE* are differences between option-implied volatilities *OI-VOL* and realized volatilities during the return prediction periods. Ex ante premiums *EA-VOLPRE* and *EA-SKEWPRE* are differences between option-implied moments and realized values occurred on 21 trading days before *t*. *TAILZ2* is the probability density in the negative tail of the risk-neutral probability distributions. *JTIX* is the difference between *OI-VOL* and volatility index value for the underlying index. Volatility indexes VIX, VFTSE and VDAX-NEW are based on options maturing close to 30 days from the observation.

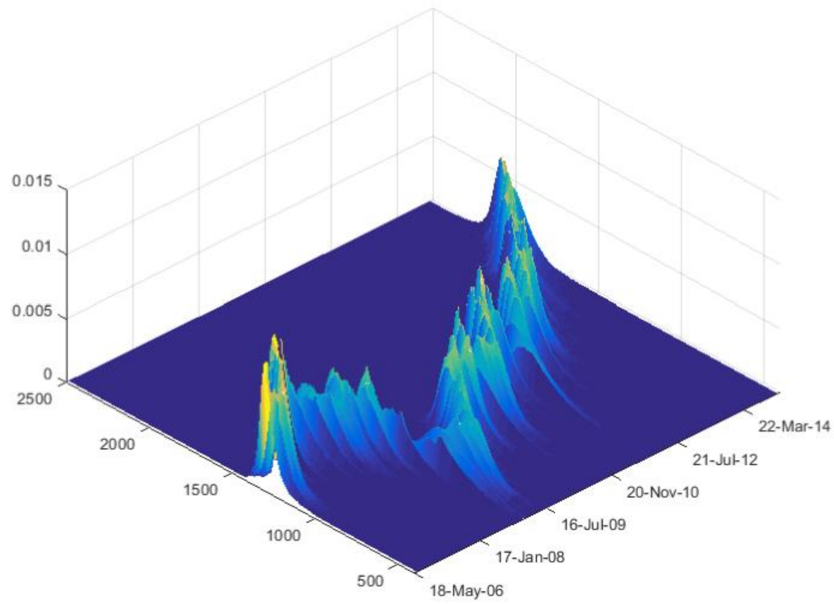
	OI-VOL	*OI-SKEW*	*VOLPRE*	*EA-VOLPRE*	*EA-SKEWPRE*	*TAILZ2*	*JTIX*	*VIX*
S&P 500								
OI-VOL	1							
OI-SKEW	0.0578	1						
VOLPRE	0.5272	-0.2982	1					
EA-VOLPRE	0.4838	-0.3486	0.5475	1				
EA-SKEWPRE	0.0509	0.8479	-0.2297	-0.2035	1			
TAILZ2	-0.0242	0.7730	-0.2411	-0.2610	0.6447	1		
JTIX	0.7184	-0.5243	0.7244	0.7596	-0.4089	-0.4388	1	
VIX	0.9380	0.3361	0.3223	0.2485	0.2697	0.1873	0.4327	1
FTSE 100								
OI-VOL	1							
OI-SKEW	0.0501	1						
VOLPRE	0.4495	-0.3502	1					
EA-VOLPRE	0.3757	-0.5118	0.4322	1				
EA-SKEWPRE	-0.0409	0.7172	-0.2466	-0.3785	1			
TAILZ2	-0.0537	0.3027	0.0416	-0.1040	0.2567	1		
JTIX	0.4548	-0.7252	0.6275	0.7067	-0.5105	-0.0889	1	
VFTSE	0.9233	0.3689	0.2325	0.1157	0.1744	-0.0218	0.0777	1
DAX 30								
OI-VOL	1							
OI-SKEW	0.0472	1						
VOLPRE	0.4633	-0.3055	1					
EA-VOLPRE	0.4162	-0.4146	0.3439	1				
EA-SKEWPRE	-0.0650	0.6329	-0.2409	-0.2490	1			
TAILZ2	0.0195	0.3178	0.0081	0.0616	0.1920	1		
JTIX	0.6072	-0.6696	0.6119	0.6243	-0.4660	-0.1062	1	
VDAX-NEW	0.9532	0.3107	0.3158	0.2553	0.1004	0.0634	0.3385	1

Appendix C

Figure 2

S&P 500 one-month forward-looking risk-neutral densities

This figure shows an example of a time-series of one-month forward-looking risk-neutral densities. The risk-neutral densities are based on all European out-of-the-money options on the S&P 500 with traded volume on the observation date. The involved options always mature in 21 trading days. Negative skewness is evident in the densities, as the densities show a long negative tail compared to the upper side of the distributions. The decrease in the underlying index during 2008 shows as a shift in the position of the density, and high volatility during the crisis results as flat distributions. An upward market is associated with highly peaked risk-neutral densities. Dimensions are time, S&P 500 price level, and probability density (Y-axis).



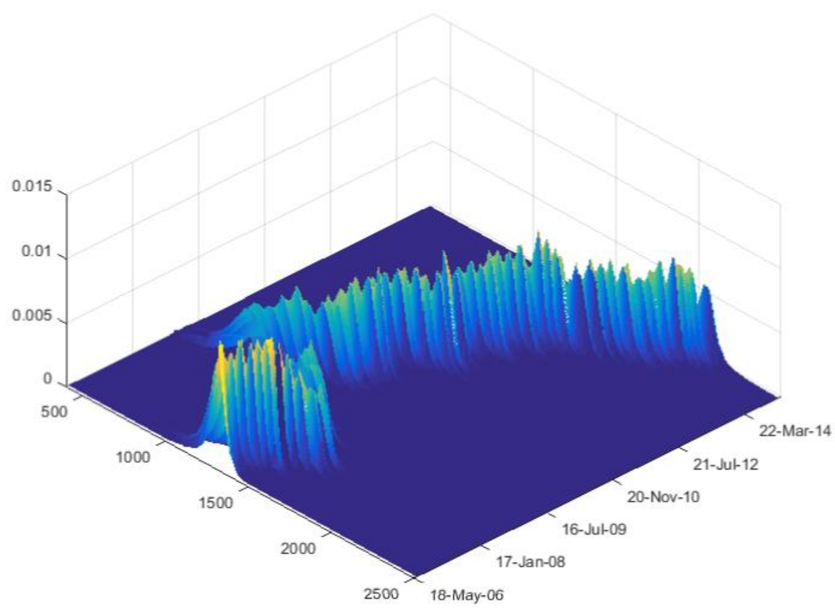
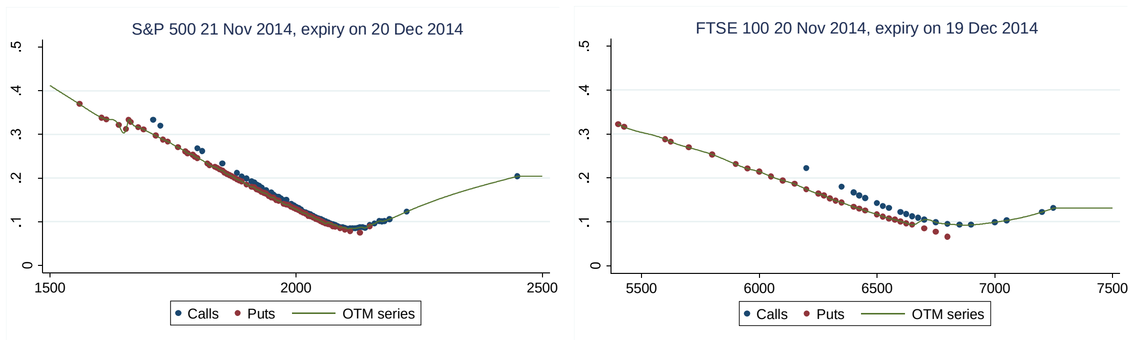


Figure 3

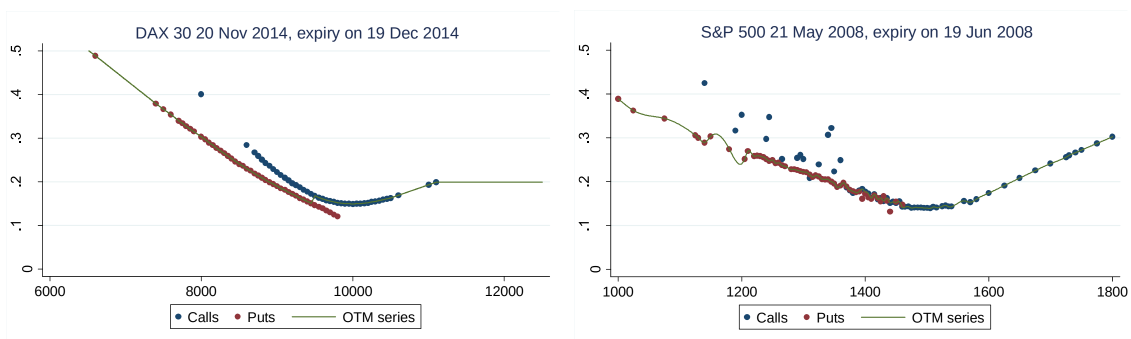
Implied-volatility cross-sections and risk-neutral densities

This figure shows four examples of Black-Scholes option-implied volatility cross-sections for put options, call options, and out-of-the-money interpolated options (line), and the corresponding risk-neutral densities of index prices at maturity. Y-axis is option-implied volatility for the scatter graphs, and probability density for the probability distributions. X-axis is the equity index level. For comparison, a lognormal probability density with at-the-money volatility is also plotted (line). The volatility skew results as negatively skewed distributions, and the exact fit of interpolated implied volatilities make the density jagged especially for S&P 500. Applying out-of-the money puts causes a jump in the density and this is clearly visible for FTSE 100 and DAX 30.



S&P 500 21 Nov 2014, expiry on 20 Dec 2014

FTSE 100 20 Nov 14, expiry on 19 Dec 2014



DAX 30 20 Nov 2014, expiry on 19 Dec 2014

S&P 500 21 May 2008, expiry on 19 Jun 2008

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