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Essays on Labour Market Frictions and Fiscal Policy

Meri Obstbaum

Aalto University

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Abstract

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Abstract

This thesis analyzes labour market fluctuations and fiscal policy from the perspective of a small monetary union member state, namely Finland.

The first essay identifies how frictions in the labour market shape the responses of the economy to government spending shocks. The nature of offsetting fiscal measures is found to be critical for the effects of fiscal stimulus, due to the different effects of different tax instruments on the labour market. Shifting the debt-stabilizing burden towards distortionary labour taxes has detrimental effects on the labour market outcome and on general economic performance in a flexible wage regime. The results indicate that wage rigidity increases the effectiveness of fiscal policy in the short term but leads to a worse longer term development including unemployment above steady state levels. The analysis suggests that a closer look at the functioning of labour markets may help to identify fiscal policy transmission channels not captured by the standard New Keynesian model.

The second essay investigates the unemployment volatility puzzle in Finland. The ability of the New Keynesian DSGE model with labour market frictions to reproduce key volatilities found in Finnish business cycle data is explored. We offer some plausible explanations of the ″Shimer puzzle″ for the Finnish case. Specific emphasis is put on modelling the wage bargaining framework as well as the cyclical behaviour of distortionary labour taxes. Wage rigidity is found to be a promising candidate for explaining Finnish labour market volatilities in the period 1994-2010. The much higher volatilities of the period 1981-1993 are also discussed. We find that most of the specific features that characterised the Finnish economy in that period, including during the exceptionally deep recession of the early 1990′s, are such that they increase the magnitude of labour market fluctuations in the model economy.

The third essay is a first attempt to estimate a DSGE model with labour market frictions and wage rigidity for the Finnish economy. The contribution is twofold. First, estimates of nominal rigidities and wage indexation that are well in line with the existing literature are obtained, and they give an economically plausible picture of the Finnish economy. Wage rigidity is found to be empirically relevant for the business cycle. A detailed assessment of the degree of wage rigidity and indexation in different model specifications is provided. Second, the estimation approach sheds more light on the magnitude and effects of labour market shocks. Labour market shocks are found to importantly drive fluctuations of labour market variables but they also account for a non-trivial share of output fluctuations.

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Helsinki, August 2012

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Essays on Labour Market Frictions and Fiscal Policy

This thesis consists of an introduction and the following three essays:

Essay 1. The Role of Labour Markets for Fiscal Policy Transmission.
Earlier version appeared as:
Meri Obstbaum (2011): "The Role of Labour Markets for Fiscal Policy Transmission", *Bank of Finland Research Discussion Papers 16/2011*

Essay 2. The Finnish Unemployment Volatility Puzzle.
Earlier version appeared as:
Meri Obstbaum (2011): "The Finnish Unemployment Volatility Puzzle", *Ministry of Finance Discussion Paper 1/2011*

Essay 3. Labour Market Frictions and Wage Rigidity in an Estimated DSGE model of the Finnish Economy.

Introduction

This thesis analyzes labour market fluctuations and fiscal policy from the perspective of a small monetary union member state, namely Finland. A New Keynesian dynamic stochastic general equilibrium (DSGE) model is chosen as the theoretical framework. These models offer the most coherent available framework for the macroeconomic analysis of policy issues. The sources of economic fluctuations may be identified and the consequences of structural changes may be analyzed and predicted with these new model tools. The New Keynesian tradition of dynamic stochastic general equilibrium models also takes into account the many imperfections in the practical functioning of market economies and adds various frictions and rigidities in the ability of the economy to adjust to external disturbances, making it a more realistic starting point for policy analysis. The key components of the basic New Keynesian DSGE model are presented e.g. in the books by Woodford (2003) and more recently Galí (2009). The choice of the theoretical framework is, moreover, guided by specific Finnish country characteristics such as the requirements of Euro area membership, the wage negotiation tradition and rigidity in wage setting.

The first versions of New Keynesian DSGEs did not include a treatment of unemployment despite the fact that that was thought to be a major deficiency in earlier models. At the same time, however, the labour market matching model as outlined in e.g. Mortensen and Pissarides (1994) or Pissarides (2000) had become the workhorse model for analyzing unemployment. The work of Andolfatto (1995) and Merz (1995) were early contributions that enabled to merge these two approaches. In the past few years many more researchers have actively developed models that combine New Keynesian monopolistic competition and nominal rigidities with imperfections on the labour market that give rise to unemployment. The inclusion of equilibrium unemployment analysis is indeed one of the most important advances in the DSGE literature in the past few years. For example Walsh (2005) examines the consequences of embedding labour market frictions in a New Keynesian DSGE model with sticky prices but flexible wages, Trigari (2006) analyzes the impact of adding different forms of wage rigidity, and Blanchard and Galí (2010) focus on the unemployment-inflation trade-off. The focus has predominantly been on the conduct and transmission of monetary policy.

Adding more structure to the labour market block of general equilibrium models has offered new possibilities to analyze the effects of labour market policy for the overall economy, and, reciprocally, to single out what the labour market consequences of other policies or disturbances are. In addition to the incorporation of relevant sectors of the economy and new transmission mechanisms into DSGE models, important advances have been accomplished to empirically validate them. The Bayesian full information approach has become the most favoured methodology in this respect, and estimated New Keynesian DSGE's are becoming conventional tools for analyzing economic developments in many policy institutions (see e.g. Christoffel, Coenen and Warne, 2008, for the European Central Bank's model of the Euro area).

While central labour market variables can now be analyzed and predicted with improved methods, many important questions are still left open. In particular, the specific questions raised by monetary union memberhip need to be addressed. This collection of essays delivers a realistic model framework for the analysis of a small Euro area economy, and complements the literature on labour markets and the business cycle. It contributes especially to the unsettled debates on the so-called unemployment volatility puzzle and on the effects of fiscal policy. New results are obtained in three key areas: the analysis of fiscal policy in the presence of labour market frictions, the measurement and modelling of labour market fluctuations and the estimation of key mechanisms that affect the dynamic behaviour of the Finnish economy. This introductory chapter includes a review of the key themes in this branch of the literature and summarizes the three separate essays that make up this thesis.

1 Review of key themes

1.1 Fiscal policy in DSGE models

The great majority of dynamic stochastic general equilibrium models have been tailored for use at central banks, and consequently concentrate on analyzing the effects of monetary policy. As a small Euro area member state, Finland has practically no control over the common monetary policy, and therefore fiscal policy is the principal policy tool that remains in the government's own hands. Moreover, the recent economic crisis has shown that there are limitations to the use of monetary policy as an operative policy tool. The fact that nominal interest rates can not be reduced below zero implies that there is a lower bound to interest rate policy. In that situation, if governments choose to actively combat the economic recession, they have to resort to fiscal and structural policies instead. It has been shown that in these conditions fiscal policy is more effective than in normal times (see e.g. Gomes, Jacquinot, Mestre and Sousa, 2010; and Christiano, Eichenbaum and Rebelo, 2011). Thus, the crisis has brought fiscal policy issues back to the centre of policy debates. Initially the key question was whether fiscal stimulus and what kind of fiscal stimulus would be effective to counteract the recession, and more recently attention has shifted to serious indebtedness problems in several Euro area member states and in the United States, and on the various ways to repay public debt. The first essay touches upon both these issues and offers new insights into how these questions can be analyzed in the presence of labour market frictions.

Both empirical and theoretical literature on the effects of fiscal policy has so far been unable to agree on what the effects of fiscal policy really are. It is widely accepted that the effect of fiscal stimulus on output is positive, but the disagreement concerns above all the effects on private consumption. The latter, as the largest component of aggregate demand, is a key determinant of the size of government spending multipliers.

Within the empirical literature on the effects of fiscal shocks, especially of government spending shocks, on the rest of the economy two main strands can roughly be identified. Ramey and Shapiro (1998) argue that increased public expenditure just crowds out private consumption and leads to hardly any positive effects whereas Blanchard and Perotti (2002) and Fatás and Mihov (2009) are the main exponents of significant expansionary effects of fiscal stimulus on private consumption. While no definite consensus has been reached, it would seem that the empirical literature on the relationship between government spending and private consumption has more recently been converging towards a positive or, at least not significantly negative effect on private consumption. The turns and emerging consensus in the empirical literature have been summarized by Galí, López-Salido and Vallés (2007) and more recently by Ramey (2011).

Within the theoretical literature, the Old Keynesian tradition used to emphasize large multiplier effects on output. It was argued that consumption, as a function of current disposable income, rises and amplifies the effect of the expansion in government spending. In contrast, the first generation of dynamic general equilibrium models focusing on real business cycles (RBC) predicted that increased government spending would fully crowd out private consumption. The key transmission mechanism of fiscal shocks in the RBC model is the wealth effect. The increase in government spending is perceived by intertemporally optimizing agents to increase taxes in the future and therefore to reduce the present value of after-tax income.

Linnemann and Schabert (2003) were the first to analytically derive the effects fiscal policy shocks in the New Keynesian framework. The same negative wealth effect is present but is counteracted by an increase in aggregate demand brought about by sluggishly adjusting prices. The rise in aggregate demand, in turn, attenuates the fall in private consumption. Therefore fiscal stimulus yields typically higher output multipliers and less negative effects on private consumption in New Keynesian models than in RBC models. The negative wealth effect, however, always dominates and private consumption unambiguously falls. Hall (2009) analyzes in a recent paper what the effects of increased government purchases to GDP are in the New Keynesian framework.

It is useful to note that these results refer to the effects of fiscal shocks, i.e. to stochastic temporary disturbances in government spending as opposed to permanent policy changes in a deterministic framework. The approach is also unrelated to the literature on optimal fiscal policy.

The responses of employment and the real wage to fiscal shocks have received much less attention than effects on output and private consumption. Pappa (2009) studies empirically the transmission of fiscal shocks in the labour market, and finds that shocks to government consumption and investment, in all the scenarios he investigates, increase real wages and employment. The RBC model has major difficulties in accounting for this empirical relation. The wealth effect not only leads to a decrease in consumption, but also to an increase in labour supply – as long as both consumption and leisure are normal goods - leading to a fall in the real wage and contraction in employment. In contrast, in the New Keynesian framework, the increase in labour demand together with the increase in labour supply can drive up the real wage or at least make it fall by a smaller amount, also yielding a more positive employment response. This is because sticky prices raise the possibility that labour demand reacts stronger than labour supply. Therefore, it seems that the New Keynesian model would a priori be better able to approximate the empirical results of fiscal policy on the labour market than models without nominal rigidities. A recent comparison of the effects of fiscal policy in different DSGE models is provided by Coenen et al. (2010).

Compared to the extensive research on monetary policy with the help of DSGE models, their use in explaining the effects and transmission of fiscal policy has so far been limited. In particular, the progress that has been achieved in incorporating a theory of unemployment into these models has not yet benefitted fiscal policy analysis. Bridging this gap is all the more important as the objective of almost any fiscal stimulus package is decreasing unemployment. It is only after the inclusion of equilibrium unemployment into these models that this approach has been made possible. Monacelli, Perotti and Trigari (2010) are the first who investigate unemployment fiscal multipliers in a dynamic general equilibrium with labour market matching. Building on their work, the first essay analyzes the labour market consequences of fiscal stimulus in a New Keynesian model framework.

1.2 Labour market institutions in matching models

labour market institutions play an important role in the macroeconomic performance of an economy (see e.g. Layard and Nickell, 1999) but they have been relatively little studied in the context of New Keynesian business cycle models. The majority of papers which have augmented the New Keynesian business cycle model with search and matching frictions in the labour market do not, for example, incorporate distortionary taxation in their framework. However, labour costs in the EU are generally burdened by relatively high levels of taxes and social contributions which speaks for the inclusion of these labour market institutions at least in the analysis of European economies. In practise, the discussion on fiscal stimulus and the repayment of debt very much centers on whether cutting expenditures or increasing taxes is more effective, and on which tax instruments, direct or indirect, should be used to repay debt. Against this reality, surprisingly many studies of fiscal policy in the DSGE literature still assume that only lumpsum taxes are available and do not allow for debt-financing of fiscal stimulus.

The inclusion of unemployment analysis in DSGE models offers an improved framework for investigating the interaction between fiscal policy and labour markets. In the first essay, the link between fiscal policy and the labour market is introduced with the help of distortionary labour taxes which directly influence the behaviour of firms and workers on the matching market. This connection is missing in the standard approach where taxes are typically lumpsum. The detailed description of the labour market helps to better understand the transmission mechanisms of fiscal policy to private consumption, employment and the real wage.

Economic research has not reached definite conclusions on the effects of taxation and economic performance. While the example of the Nordic countries shows that high levels of taxation can be compatible with high employment, reducing taxes on labour could be a useful tool to further stimulate employment. The reasoning is that by reducing taxation of this factor, returns to labour income would become more attractive. The Mortensen-Pissarides matching model of the labour market shares this property. Lower proportional tax rates, as well as lower unemployment benefits, are associated with higher employment and output in equilibrium (see Pissarides, 2000, chapter 9). The core mechanism in the standard matching model is wage bargaining which is influenced by labour market institutions. A key concept is the relative value of non-work to work activities: taxes and benefits influence the relative attractiveness of work. This transmission channel turns out to be of central importance for the analysis of fiscal policy.

While there is a relatively large literature on the effects of labour market institutions on *equilibrium* labour market outcomes, much less research can be found on their impact on business cycle fluctuations. Den Haan et al. (2001) show that in a job matching model high initial replacement rates or tax rates lead to a larger rise in unemployment as a response to a negative technology shock than with "employment -friendly" institutions, in line with Blanchard and Wolfers' (2000) empirical evidence. In addition to the importance of the *level* of labour taxation for the dynamics of the economy, the changes in taxes over time affect the dynamics of the economy. Moreover, it has been suggested that the cyclical behaviour of taxes may have a role in explaining labour market fluctuations. For example, Burda and Weder (2010) find empirical evidence that payroll taxation tends to fall in recoveries and rise in recessions. If tax rates are cut in good times and raised in bad times this would amplify the movement of employment and unemployment in business cycles.

Including a set of distortionary taxes into the model framework, the first essay derives the equilibrium properties of the model economy, and investigates both the impact of the level of taxation on the dynamic adjustment to a fiscal shock and the effect of taxes that change over time in the form of fiscal feedback rules. The second essay, in turn, investigates the role of distortionary taxes for labour market fluctuations.

1.3 On different fiscal policy instruments

As Baxter and King (1993) first pointed out, the decision on how public spending is financed is a crucial determinant for the sign and magnitude of the response to a government spending shock. The recent financial crisis and countries' fiscal policy responses to it has spurred renewed interest in the role of different fiscal policy instruments.

Bilbiie and Straub (2004) study the effects and transmission of fiscal policy in a dynamic general equilibrium sticky-price model including distortionary taxation. As in the current study, they emphasize the role of the labour market in determining the effects of fiscal stimulus but their labour market is Walrasian. They specify a set of fiscal rules that determine how debt is paid back. This is a conventional assumption in the analysis of fiscal policy in the context of New Keynesian DSGE's (see also e.g. Coenen and Straub, 2005).

In the standard New Keynesian model, the Ricardian equivalence proposition (see Barro, 1974) holds. That is, if government expenditures are financed by non-distortive lump-sum taxes, the timing of taxes has no effect on private consumption as any rescheduling of tax liabilities keeps households' lifetime disposable income unaltered. Recent literature suggests that to account for important transmission channels of fiscal policy, the economy should be modelled as “non-Ricardian”. In particular, it has been argued that the representative-agent paradigm is not well-suited to deal with medium and long term fiscal policy and debt issues. There are various ways to introduce non-Ricardian features into DSGE models. One move away from the representative agent framework that has been found to be important for the effects of fiscal policy is the inclusion of so-called rule-of-thumb consumers (see e.g. Galí, Lopez-Salido and Valles, 2007). This modelling device is known to be able to account for a positive response of private consumption to fiscal stimulus because a share of consumers have no means to smooth their consumption.

For example, the International Monetary Fund has built a model that includes non-Ricardian features for fiscal policy analysis departing from their more standard New Keynesian open economy model, the Global Economy Model. See Botman, Karam, Laxton and Rose (2007) for a discussion on the various ways to introduce non-Ricardian features.

One way to break the Ricardian equivalence is the introduction of distortionary taxes. When there are distortionary taxes on labour, the optimality condition for the choice of consumption versus hours worked includes the proportional tax rate. The path of taxation does matter in that case, since the tax rate affects the *intratemporal* optimality condition governing the leisure-consumption trade-off. Therefore, how tax rates evolve determines consumption and labour decisions. Ricardian agents postpone work for periods when taxed less and enjoy more leisure in periods of high taxation.

Bilbiie and Straub (2004) show that the effects of fiscal stimulus depend on the intertemporal path of taxation also in the case of lump sum taxes when there are rules that determine the strength and speed of tax increases to finance the stimulus. This effect, however, purely relies on the underlying fiscal rule in the sense that a high extent of deficit financing and a strong response to debt imply a smaller wealth effect. A more aggressive debt-repayment rule means that the debt process will be less persistent, and so the implied tax process will also be less persistent

The role of distortionary taxes is analyzed throughout this collection of essays both from the point of view of labour market fluctuations and as a determinant of the effects of fiscal stimulus. Using distortionary taxes makes a considerable difference and creates significantly lower multipliers of government spending stimulus especially in the long run. Similar results have more recently been found by e.g. Drautzburg and Uhlig (2011) who extend the benchmark Smets-Wouters (2003) New Keynesian DSGE to analyze the effects of the American Recovery and Reinvestment Act (ARRA).

In the aftermath of the financial crisis, in the debate on how to repay the large debt burdens that had started to accumulate, many Western governments, including in Finland, voiced the need to increase the consumption tax but to keep labour taxes constant or lower them. The consumption tax is regarded as being less distortive and therefore a more effective instrument for tax collection. The theoretical contributions to this subject most often consider a revenue-neutral shift from direct to indirect taxation. In the first essay, the relative desirability of these two tax instruments is compared by assessing what are the consequences of using each of these tax instruments to repay debt for the overall effects of stimulus.

A point made in the tax-shifting literature is that if the consumer's take-home pay increases by a certain percentage, but the price of all goods goes up by the same percentage, there is no compelling reason why he should modify his behaviour and work longer hours. Consequently, a shift from labour taxes to consumption taxes would not have any effects on labour supply and economic performance. In practise, however, for example features of the wage bargaining system or characteristics of the unemployment benefit system affect the specific outcome of reforming the tax system.

In business cycle models with labour market matching, it is commonly assumed that the representative agent is in fact a large household that includes both working members and unemployed members. Therefore, distortionary labour taxes and consumption taxes have different effects on labour supply.

More specifically, the consumption tax is less detrimental to labour supply because, unlike the labour tax, it is also paid by the unemployed. A VAT affects all types of household income because consumption is proportional to labour income plus unemployment benefit transfers, while labour taxes only affect labour income. This relates to a more general result in the tax shifting literature that most employment gains seem to be linked to the possibility of shifting the incidence of the tax burden away from labour.

1.4 Fiscal stimulus in open economies and in a monetary union

The literature on DSGE models has mainly concentrated on closed economy versions of the model. Obstfeld and Rogoff (2000), in turn, is the standard reference for New Open Economy Macroeconomics and work by, for example, Chari, Kehoe and Mc Grattan (2002) as well as Clarida, Galí and Gertler (1999) have importantly contributed to the literature on monetary policy in open economies. Research on fiscal policy in open economies has been surveyed by e.g. Ganelli and Lane (2002) and Coutinho (2003). The focus in much of this literature is on the spillover effects of fiscal policy to other countries or on the potential benefits of fiscal policy coordination reflecting the formation of the Euro area and the emerging interest in the transatlantic transmission of policies. Accordingly, the theoretical framework is often used to model the links between two large economies. Beetsma and Jensen (2005), instead, consider a two-country monetary union model to analyze the optimality of fiscal stabilization policies and the interactions of these policies with monetary policy, while Ferrero (2009) adds distortionary taxation and government debt into the two-country monetary union model to investigate optimal policies. Stähler and Thomas (2011) describe a two-country monetary union DSGE model for fiscal policy analysis that has been developed in cooperation between the Spanish and German central banks. They calibrate the model for the Spanish economy and simulate the effects of various permanent fiscal policy measures.

The focus of this thesis is, however, on building a realistic macroeconomic model for a small Euro area member state, Finland. As a member of the Euro area, Finland misses two channels that help a standard open economy to adjust to shocks. It has practically no control over the common monetary policy, and the nominal exchange rate is fixed. Galí and Monacelli (2008) have developed a framework for analyzing the specific features of small monetary union member states. Instead of the typical two-country setup of open economy models, they model a currency union being made up of a continuum of small open economies that are subject to imperfectly correlated productivity shocks. Almeida et al. (2010) investigate the effects of fiscal stimulus in a similar small euro area economy model but introduce non-Ricardian behaviour with the help of finite lifetime households.

The lack of control over monetary policy naturally implies that fiscal policy is the principal policy tool available but it also has a profound effect on how the economy adjusts to shocks. In the preparation phase for EMU the most central question that was raised in Finland was the ability of other adjustment channels than monetary policy to ensure the stability of the Finnish economy in the face of asymmetric shocks (see Valtioneuvoston kanslia, 1997). There was wide agreement that, in this environment, in addition to the mobility of factors of production and the central role of fiscal policy, the functioning of labour markets, including the wage bargaining system will become more important. Based on these considerations, the model used throughout this collection of essays combines the New Keynesian model with labour market frictions and wage rigidity with the Galí-Monacelli perspective of an infinitely small currency union member state.

The behaviour of the real interest rate is a crucial transmission channel of fiscal policy in DSGE models. The response of the real interest rate determines the consumption path, and therefore, in New Keynesian models, the strength of the demand effect. Linnemann and Schabert (2003) show how the real interest rate effect depends on the chosen monetary policy regime. In the vast majority of DSGE models, monetary policy is modelled as a Taylor-type interest rule. According to the Taylor principle, in order to guarantee the uniqueness of the equilibrium of the model, the central bank has to adjust the nominal interest rate more than one-for-one with changes in inflation (see e.g. Woodford 2003). The application of the Taylor rule implies that the rise in prices caused by a fiscal policy shock is always accompanied with an (more than one-for-one) increase in the nominal interest rate, and thus a rise in the real interest rate. The rise in the real interest rate has a contracting effect on private consumption.

In contrast, when a small country belongs to a monetary union, the nominal interest rate controlled by the currency union central bank does not react to the speeding up of inflation in the small member state, and therefore the real interest rate always falls and attenuates the fall in private consumption. Fiscal stimulus is, thus, more effective in this kind of framework than in more conventional closed-economy New Keynesian models or in open economy models with independent monetary policy. This result is similar to the property discussed earlier that fiscal policy is more effective when there is a lower bound on monetary policy. In both cases, the nominal interest rate does not adjust to changes in prices and the resulting lower real interest rate is responsible for a less negative response of private consumption and consequently larger output multipliers.

The comparison of fiscal stimulus effects in different structural models by Coenen et al. (2010) reaches the same conclusion that the response of central banks to fiscal shocks is crucial in assessing the size of fiscal multipliers. Ilzetzki, Mendoza and Végh's (2011) estimates of fiscal multipliers seem to confirm this connection. Using a new data set of 20 high-income and 24 developing countries, they conclude that in economies with predetermined exchange rate regimes fiscal multipliers are larger than in countries with flexible exchange rate arrangements, and this difference can be attributed to the different degrees of monetary accommodation to fiscal shocks.

1.5 The unemployment volatility puzzle

The unemployment volatility puzzle has been a key challenge for modern macroeconomic modelling. Shimer (2005) argued that the conventional model of unemployment dynamics as outlined in Mortensen and Pissarides (1994), and Pissarides (2000) incorporated in a dynamic general equilibrium framework has difficulties in explaining the observed volatility of employment together with the rigidity in wages in the face of technology shocks. Shimer further asserted that the problem arises because, in the standard model, the wage is renegotiated in every period by Nash bargaining and is thereby let to adjust very easily to changes in the economic environment. In the growing body of literature that has attempted to explain the problem the focus has accordingly been on ways to amplify the response of vacancies and unemployment to shocks.

The studies that seek to answer the Shimer critique can be roughly divided into two parts: those that aim at making the match surplus small and those that add rigidity in the adjustment of wages. Hall (2005b) and Pissarides (2009) include surveys of the range of alternative models proposed to solve the unemployment volatility puzzle.

A number of contributions (see e.g. Trigari, 2006; Hagedorn and Manovskii, 2008; and Costain and Reiter, 2008), have identified the magnitude of the match surplus as a key factor contributing to explaining the unemployment volatility puzzle. The intuition is that a smaller surplus reacts more to shocks of equal size and translates into increased volatility of labour market variables. Hagedorn and Manovskii's (2008) approach has, however, been criticized for having obtained this result by calibrating the worker's negotiation parameter to abnormally low and unemployment benefit to abnormally high values.

Another direct way to address Shimer's critique is to include in the Mortensen-Pissarides framework additional rigidity in the adjustment of wages. Within this wage rigidity approach to the Shimer puzzle one strand of the literature believes that it is necessary to depart from standard Nash bargaining to solve the unemployment volatility puzzle (see e.g. Hall and Milgrom, 2008). The idea is to find an alternative wage equation to the Nash wage equation that would deliver more wage stickiness. The rest of the studies in this research area keep the assumption of Nash bargaining and introduce explicit rigidity in wage formation in order to amplify labour market fluctuations.

Hall (2005a) introduced real wage rigidity in the form of a backward looking social wage norm. The wage norm limits the adjustment capability of wages and therefore directly increases the adjustments on the labour quantity side. The wage norm has, however, been criticized for its ad hoc nature. Gertler and Trigari (2009) modify the conventional Mortensen-Pissarides model to allow for staggered multiperiod wage contracting, and are able to replicate the key labour market volatilities found in the data. In addition to the volatility measures, and unlike many other explanations of the unemployment volatility puzzle, their model is also able to match other important comovements in the data. A further advantage of the latter approach compared to a simple ad hoc wage norm is that the key primitive parameter is the average frequency of wage adjustment, as opposed to an arbitrary partial adjustment coefficient in the wage equation. This framework thus delivers the essential feature of the sticky-wage model, the sensitivity of the employer's surplus from a new hire to current economic conditions.

The most important empirical criticism of the rigid wage approach rests on the claim that the wages of new matches are not found to be rigid. Pissarides (2009) finds on the basis of microeconometric evidence that wages in new matches are volatile and consistent with the key predictions of the standard model. Haefke et al. (2009) argue likewise that wages are flexible at the start of new jobs. On the other hand, Gertler and Trigari (2009) point out that evidence is not sharp enough to support this claim for two reasons. Data in almost all cases does not match workers with firms and existing studies do not control for cyclical changes in job quality. They argue that after controlling for these compositional effects the wages of new hires are not more cyclical than existing workers' wages. In addition, Galuscak et al. (2010) find that 80 percent of the firms surveyed by the Eurosystem's Wage Dynamics Network report that internal factors such as a collective wage agreement and an internal pay scale are more important in determining the wages of new hires than external labour market conditions.

The recent book by Shimer (2010) confirms that adding wage rigidity restores the ability of the Mortensen Pissarides model to replicate data and produce realistic fluctuations in labour market variables. The model constructed in this thesis belongs to these rigid-wage explanations of the unemployment volatility puzzle. Gertler and Trigari's (2009) bargaining framework is chosen as the core mechanism generating wage rigidity but the approach differs most importantly in that the studies in this collection of essays analyze labour market fluctuations from the perspective of a small open economy and add a more detailed fiscal policy block to the underlying model framework. The wage bargaining problem is also complicated by the introduction of distortionary taxation.

The result of e.g. Den Haan et al. (2001) that in a job matching model higher tax rates lead to a larger rise in unemployment as a response to a negative technology shock (also Andrés et al., 2006, and Vanhala, 2007, get the same result) can be rationalized in the context of the unemployment volatility puzzle debate. Distortionary labour taxes are namely one way to make the surplus from working smaller, and thus make it react more strongly to current economic conditions. Distortionary labour taxes thus tend to amplify labour market fluctuations. It is a standard result from the labour market matching literature that proportional tax rates influence the division of the total surplus from a job in equilibrium, irrespective of the bargaining horizon (Pissarides, 2000). More specifically, the distortionary labour taxes on both the worker and the firm have the impact of lowering the effective bargaining weight of the worker. The introduction of distortionary taxes therefore essentially works in the same direction as Hagedorn and Manovskii's (2008) solution to the unemployment volatility puzzle, based on calibrating the worker's negotiation parameter to low and unemployment benefit to high values.

The possibility of changing taxes over time also interacts with the specific bargaining framework. In particular, when the bargaining horizon extends over multiple periods, agents also need to take into account in their bargaining behaviour the future path of taxation. The prospect of future increases in labour taxes further decreases the worker's relative effective bargaining weigth and thus amplifies the small surplus effect. This interaction complicates the bargaining model, but it is hardly an unrealistic assumption that labour taxes are one key factor that wage negotiators would consider in the real world. However, while the inclusion of proportional labour taxes takes one step towards alleviating the unemployment volatility puzzle, this effect does not turn out to be quantitatively important enough. Therefore, it does not preclude the need for additional rigidity in wage formation.

1.6 Assessing Finnish labour market volatilities

The debate on labour market volatilities and the ability of the search and matching model to replicate the stylized facts started from the United States, and analysis has accordingly until now been mostly carried out with U.S. data. There are, however, many reasons to believe that observed labour market volatilities as well as the underlying mechanisms that generate this volatility are very country-specific, and accordingly any model destined to analyze and predict the economic developments of a given country has to be tailored to that country's specific needs.

The second essay investigates labour market volatilities in Finland. As any empirical study on the Finnish economy, this analysis also faces the challenge of huge structural shifts in the data. The Finnish depression of the early 1990's was so severe that it has earned a place on Reinhart and Rogoff's (2008) list of the "Big Five" postwar rich country large-scale financial crises. A specific feature of that depression was the dramatic increase in unemployment to fivefold levels within a few years time. This exceptional period dominates the time series behaviour of any labour market variable. Due to its specific features, that time period should not primarily be analyzed with the help of a business cycle model. Moreover, any attempt to describe these labour market developments as the economy's response to just a technology shock does not make sense.

Although the Finnish experience is an extreme example of the challenges that may come up in matching a specific model to time series data, this question is surprisingly largely ignored in the unemployment volatility puzzle literature. The only feature that has been explored in more detail is the cyclicality of inflows and outflows into unemployment (see Shimer, 2005, Yashiv, 2007, Pissarides, 2009 and Elsby, Michaels and Solon, 2009). The inconclusiveness in the literature about the cyclical behaviour of inflows and outflows into unemployment suggests that this cyclical pattern may have changed over time. There is reason to believe that in addition to the cyclicality of job creation and job destruction many other economic processes are different nowadays from the 1950's when the Shimer data starts.

In Finland, there have also been significant changes during the past few decades in the openness of the economy, including a shift in the monetary regime to membership in the Euro area. The degree of openness of the economy and the extent of monetary accommodation are key factors determining the economy's adjustment to shocks. In the literature on macroeconomic adjustment within a monetary union (see e.g. Amisano, Giammarioli and Stracca, 2009, Angeloni and Ehrmann, 2007, and European Commission, 2008 for recent studies) it is well-known that an asymmetric shock that lowers the domestic price level but does not affect unionwide inflation, for example a positive technology shock, has two counteracting effects. First, in the short run, the rise in the real interest rate implied by lower domestic prices has a contracting effect on the economy that may further depress domestic prices. On the other hand, lower domestic prices translate into a real exchange rate depreciation, a gain in competitiveness, and an increase in exports that stimulates the economy and ultimately imposes a real exchange rate appreciation through speeding up of domestic inflation. These underlying mechanisms have to be taken into account when comparing Finnish labour market fluctuations to those in other countries.

The labour market response to technology shocks has been the subject of considerable debate in the literature. Galí (1999) showed that, unlike the real business cycle model, the New Keynesian model is able to replicate the empirical finding of negative comovement of employment and productivity after a technology shock. The reason is that price rigidity attenuates the aggregate demand effect of technology shocks. When productivity improves but only a fraction of firms lower prices, the rise in aggregate demand may be less than that of productivity, and employment falls. Basu, Fernald and Kimball (2006) provide recent empirical evidence that positive technology shocks are followed by a short run decline in employment. Collard and Dellas (2007) show that in an open economy a technology shock can cause a decline in employment if the elasticity of substitution between domestic and foreign goods is low. This is because the resulting deterioration in the terms of trade would discourage domestic output expansion. Thus the open economy dimension could enhance the sticky-price model's ability to account for this empirical finding (see also Tervala, 2007).

While the model built in this thesis behaves in line with these standard results on the responses to a technology shock in a small monetary union member state and in a New Keynesian economy, it extends the analysis of the labour market responses to technology shocks by including matching frictions. As the focus is on the unemployment volatility puzzle, the key variables of interest are unemployment, vacancies and labour market tightness. Prior to this thesis the literature on the unemployment volatility puzzle has concentrated on matching closed economy models to data. Of the two counteracting effects of technology shocks in a monetary union member state, the real interest rate effect has the tendency to dampen employment fluctuations, because a rising real interest rate discourages vacancy posting. On the other hand, the real exchange rate effect should amplify vacancy creation because it enforces the aggregate demand effect. The exact influence of the monetary union member state dimension on labour market fluctuations is thus ultimately an empirical question.

1.7 Labour market frictions and nominal rigidities in estimated DSGE models

The usual approach in the unemployment volatility puzzle literature is to try to quantitatively match the model moments and correlations under a single (technology) shock to unconditional data moments. While the role of technology shocks in driving business cycles is overall being increasingly questioned (see e.g. Galí and Rabanal, 2004) these doubts may be even more relevant when considering economies with a high degree of openness (see e.g. Khan and Tsoukalas, 2005). It is clear that in reality several sources of stochasticity drive aggregate economic activity and are responsible for the fluctuation of labour market variables.

Recent advances in Bayesian estimation techniques offer the possibility to empirically analyze whole macroeconomic frameworks, and permit to account for a complete range of shocks that hit the economy. While much work remains to be done to empirically validate large dynamic general equilibrium models, Bayesian full information methodology offers important economic and statistical advantages compared with other available strategies. These advantages are especially obvious in the case of medium and large scale models. An and Schorfheide (2007) review Bayesian estimation techniques that have been developed for the empirical analysis of DSGE models and the books by Canova (2007) and Dave and DeJong (2007) provide further material. While there is already a wide range of estimated DSGE models in the literature and in use in policy institutions, especially in central banks, the work on model versions that incorporate labour market frictions has just started during the past few years. Many of the estimated models take as their starting point the Smets and Wouters (2003) model, including the extension of Gertler, Sala and Trigari (2008) that allows for labour market frictions and staggered nominal wage contracting.

Gertler, Sala and Trigari (2008) find that the model with wage rigidity provides a better description of the data than a flexible wage version. Christoffel, Kuester and Linzert (2009) estimate a similar model with labour market frictions and multiperiod wage contracting but concentrate on the role of labour markets for Euro area monetary policy. Faccini, Millard and Zanetti (2010) estimate a New Keynesian model with matching frictions and nominal wage rigidities on UK data including a simplified version of Gertler, Sala and Trigari's (2008) staggered bargaining framework. The third essay employs a similar labour market structure than Gertler, Sala and Trigari (2008), and estimates the New Keynesian DSGE model built in the previous essays with Finnish data using Bayesian Maximum Likelihood methods. The framework is again modified to take into account the monetary union membership and the presence of distortionary taxes that may have a role in determining the reaction of the Finnish economy to different kinds of disturbances.

Of the estimated models that do not include labour market frictions, the following are in other respects close to the approach adopted in this thesis. Adolfson, Laséen, Lindé and Villani (2007) evaluate a small open-economy model estimated on Swedish data. Sweden and Finland share some important economic features including many labour market structures and wage negotiation traditions but these are not at the centre of the analysis by Adolfson et al.. Their model features instead a monopolistic labour market typical in New Keynesian models. Almeida (2009) estimates a New Keynesian DSGE model for the Portuguese economy which, like Finland, is a small Euro area member state. The Bank of Finland's AINO model (see Kilponen and Ripatti, 2006) was among the first DSGE models that was operative in a policy institution for both simulations and forecasting. More recently Freystätter (2010) has estimated a New Keynesian DSGE model with financial frictions with Finnish data. The third essay is, however, the first estimation of a model with labour market frictions and wage rigidity on the Finnish economy.

While sticky prices and wages are key contributions of the New Keynesian model, there still seems to be considerable uncertainty concerning the degree of rigidities and the right combination of rigity and indexation. A common feature of many of the above studies, including the ones with labour market frictions, is that they provide posterior distributions for the nominal rigidity parameters, but several report difficulties in identifying them properly. Gertler, Sala and Trigari (2008) find that it is very difficult to separately identify labour market parameters that are key determinants of effective wage rigidity, suggesting that additional labour market information could be useful. On the other hand, Christoffel, Kuester and Linzert (2009) point out the possible measurement problems related to labour market data, including the time series that are at the core of the search and matching literature. Also the choice of the right wage measure is important. While one advantage of adding labour market frictions and staggered wage bargaining is that the wage rigidity parameter gets the explicit interpretation of contract length, data often includes also other wage changes than those on base pay.

Canova and Sala (2009) argue that to address identification issues in the Bayesian estimation of DSGE models every model should at least be tested for the sensitivity with respect to different priors. Del Negro and Schorfheide (2008) have specifically pointed out the significance of the choice of priors for assessing nominal rigidities. Moreover, the assessment of nominal rigidities may importantly depend on other parameters, especially those describing exogenous processes. On the basis of the first attempts to estimate models with labour market frictions, it appears that there might be interaction between wage rigidity and labour market parameters, as well as labour market shocks that could also lead to identification issues. More specifically, part of the estimated wage rigidity inherits the properties of the bargaining power shock in the same way as estimated price rigidity inherits properties of price markup shocks (see Rabanal and Rubio-Ramirez, 2008). This implies that while, on one hand, it would seem important to include bargaining power shocks in the model to account for exogenous variations present in the negotiation process, but on the other hand, these variations are hard to quantify and prior beliefs on them heavily influences the estimation results on wage rigidity.

Despite the uncertainties related to exactly quantifying the degree of wage rigidity, existing studies seem unanimous on the fact that (some degree of) wage rigidity is an important feature in explaining macroeconomic adjustment to shocks. There are also tentative results, from the models that include labour market frictions, on the important role of shocks originating from the labour market. In particular, labour market shocks seem to importantly drive fluctuations of key labour market variables (unemployment and vacancies) but they also account for a non-trivial share of output fluctuations. This would imply in practice that the functioning of the wage formation system affects labour market outcomes, i.e. that the labour market organizations that negotiate wage contracts are also importantly responsible for labour market performance.

Next, the main features and principal results of the three separate essays are summarized.

2 Overview of the essays

2.1 The role of labour markets for fiscal policy transmission

The first essay studies the transmission of fiscal policy in the presence of labour market frictions. In order to address the question the standard open-economy New Keynesian business cycle model is extended in two dimensions: a detailed formulation of fiscal policy, and labour market matching frictions along the lines of Mortensen and Pissarides. Specifically, the transmission mechanisms of a small monetary union member state (see Galí and Monacelli, 2008) are considered.

The focus is on the effects of government spending shocks on private consumption, employment and the real wage that are still subject to considerable uncertainty in the literature. In particular, it is identified how frictions in the labour market shape these responses in the New Keynesian framework. The approach is closest to Monacelli, Perotti and Trigari (2010) who investigate output and unemployment fiscal multipliers in a real business cycle model with labour market matching. New Keynesian features are combined with debt-financing, distortionary taxes, and wage rigidity which, in the present framework, turn out to be important determinants of the effects of fiscal shocks.

Different fiscal policy instruments that can be used to finance the public debt that results from increased government spending are specifically considered. The instruments include lump-sum taxes, distortionary labour taxes and consumption taxes. The analysis confirms the early finding by Baxter and King (1993) that the chosen offsetting fiscal measure is critical for the effects of fiscal stimulus. In particular, it is found that adding distortions to the labour market increases the cost of fiscal stimulus in line with earlier results by Baxter and King (1993) and e.g. Bilbiie and Straub (2004).

The role of labour market structures for fiscal policy is inspected especially in the case of wage rigidity, introduced with the help of the staggered bargaining framework of Gertler and Trigari (2009). This is because, in addition to being an intuitively important element in the modelling of a small euro area member country, wage rigidity has been found to be a central explanation for the volatile behaviour of unemployment in business cycles driven by technology shocks (see Shimer, 2010). Wage rigidity has also been found to significantly affect the transmission of monetary policy shocks to the economy (see Christoffel, Kuester and Linzert, 2009).

The main findings can be summarized as follows. First, the effects of fiscal shocks in the baseline model with flexible wages are similar to the standard New Keynesian model without labour market frictions. Output increases, the response of private consumption is negative but small, and employment and the real wage rise. While the mere presence of labour market frictions does not significantly affect the sign or size of fiscal multipliers, a clear advantage compared to the standard model is that the modelling of labour market frictions helps to identify the transmission channels of fiscal policy to the labour market. Following fiscal stimulus, firms see their future profit opportunities rise and open new vacancies, increasing labour demand. At the same time, the negative wealth effect both increases the supply of hours by each employed worker and increases the relative value from employment for all workers. labour supply increases along both the intensive and extensive margin. Real wages rise.

Second, the chosen offsetting fiscal measure is critical for the effects of fiscal stimulus due to the different effects of different tax instruments on the labour market. Most importantly, shifting the debt-stabilizing burden towards distortionary labour taxes has detrimental effects on the labour market outcome and on general economic performance in the flexible wage regime. The mechanism works as follows. As the debt-stabilizing tax rule becomes operative, the higher proportional tax rate decreases the relative value from employment, and feeds through to a higher wage. Specifically, the wage bargaining model implies that the negotiated contract wage rises to compensate workers for the otherwise falling net income. The higher wage directly implies higher labour costs to firms which reduce the number of open vacancies and unemployment starts rising. Due to this subsequent fall in employment, the contraction in private consumption is larger than when public debt is adjusted through lump-sum taxes.

Interestingly, we find that the consumption tax has less negative consequences on the labour market than labour taxes because it has a smaller negative effect on the relative value from employment. This is because, unlike the labour tax, it is also paid by the unemployed. It affects all types of household income because consumption is proportional to labour income plus unemployment benefit transfers, while labour taxes only affect labour income. This relates to a more general result in the tax shifting literature that most employment gains seem to be linked to the possibility of shifting the incidence of the tax burden away from labour. Thus, adding structure to the labour market helps us to better track down the effects of different debt-stabilizing tax instruments on labour market outcomes. In all the scenarios, the accumulation of debt implies that tax rates are increased long after the fiscal stimulus phase has finished resulting in a prolonged decline in output. Accordingly, long run multipliers are negative.

Third, wage rigidity increases the magnitude of the short-term responses of labour market variables to fiscal stimulus. Vacancies react more strongly to stimulus in the short term, since firms' profits are larger when all workers cannot internalize the expected rise in taxes, and consequently firms' labour costs do not simultaneously rise. This is in line with the literature on labour markets and business cycles (see Shimer, 2010), but different from Monacelli, Perotti and Trigari (2010). The main distinguishing features of these two approaches are the assumption on price rigidity and the behaviour of the real interest rate. In the New Keynesian framework, as opposed to a real business cycle model, rigid prices give rise to a labour demand effect as witnessed by increased vacancy creation. Rigid wages amplify this labour demand effect of fiscal stimulus on employment since firms' expected profits now rise more than with flexible wages. The importance of the extent of price rigidity for the effects of fiscal policy has been identified earlier also by e.g. Galí, López-Salido and Vallés (2007).

Finally, the open economy dimension plays a role. In the present model, the real interest rate always falls in response to a government spending shock because the rise in prices in the small currency union member state caused by increased aggregate demand is not counteracted by tightening monetary policy by the currency union's central bank. This effect is large enough to overturn the upward pressure on the real interest rate caused by the rise in the shadow value of wealth. The falling real interest rate makes fiscal policy more effective compared to a standard closed economy setup where accommodative monetary policy would counteract the rise in prices and the real interest rate would rise.

Furthermore, our results indicate that while wage rigidity would seem to make fiscal policy more effective in the short term, in the longer term, the gradual increase in the wage causes a prolonged increase in unemployment to above the steady state level. Public debt stays higher and the negative effect on private consumption is larger than when wages are flexible. Wage rigidity also affects the relative preferability of different debt-stabilizing tax instruments. Finally, expansionary tax shocks do not at first glance seem to be as effective as spending shocks, but turn out, in the longer run, to lower unemployment more than fiscal stimulus via government spending, and have a positive effect on private consumption.

2.2 The Finnish unemployment volatility puzzle

The second essay investigates the unemployment volatility puzzle in Finland. The ability of a New Keynesian dynamic stochastic general equilibrium (DSGE) model with labour market frictions to reproduce key volatilities found in Finnish business cycle data is explored. Different versions of the model are tested to find out which specification gets closest to fitting the labour market dynamics in Finland. Specific emphasis is put on modelling the wage bargaining framework as well as distortionary labour taxes that affect labour market participants' decisions. Rigidity in the adjustment of wages is added in the form of staggered bargaining initially developed by Gertler and Trigari (2009). One advantage of this approach is that wage rigidity gets the explicit interpretation of longer wage contracts.

There is no previous literature on the unemployment volatility puzzle in Finland. The debate on labour market volatilities and the ability of the search and matching model to replicate the stylized facts started from the United States (Shimer, 2005), and analysis has accordingly been mostly carried out with U.S. data. There is, however, reason to believe that the volatility of European labour market variables could differ from those in the U.S.. Gartner, Merkl and Rothe (2010) investigate labour market volatilities in Germany, and show that vacancies, labour market tightness and the job-finding rate are roughly twice as volatile compared to the volatility of labour productivity as in the United States.

We use Finnish business cycle data for the period from 1981Q1 to 2010Q4, and find that labour market volatility is also in Finland almost twice as high as in the U.S., meaning that solving the unemployment volatility puzzle is a priori even more challenging. However, the deep recession of the early 1990's clearly dominates the behaviour of Finnish labour market series suggesting that a separate explanation might be needed for this exceptional time period. Moreover, in that period many of the Finnish economy's structures significantly differed from those in place today making it possibly misleading to use the same model framework to explain fluctuations in the pre-depression years as opposed to the post-depression period.

In our main analysis we therefore concentrate on the period 1994-2010 which is found to exhibit very different labour market volatilities compared with those found in the earlier data period and under the depression years. These volatilities which are just a half of those found in the U.S. can easily be explained with the help of our New Keynesian DSGE model framework.

In addition to our calibration of the surplus from employment to match the Finnish data, including distortionary taxes on labour, wage rigidity is a particularly promising candidate for explaining Finnish labour market volatilities. Countercyclical taxes are also able to bring the model-implied volatilities close to the data, but they worsen the model in some other dimensions. Similarly, countercyclical job separations increase the volatility of unemployment but are not able to explain other key features of the data. The advantage of the model with wage rigidity is precisely that it generates, in addition to volatilities that match the data, realistic comovements between different variables.

We also discuss the much larger labour market volatilities of the period 1981Q1-1993Q4 in Finland. We find that most of the specific features that characterized the Finnish economy in that period are such that they increase the magnitude of the responses of labour market variables to shocks in our model economy. In that context we consider e.g. other than productivity shocks, the different monetary regime and more rigid wages. Some observed features of the Finnish economy that did not seem to characterize the labour market volatilities of the later data period, such as countercyclical separations, could have had a role in magnifying the shocks in the earlier period.

2.3 Labour market frictions and wage rigidity in an estimated DSGE model of the Finnish economy

The third essay empirically analyzes the role of labour market frictions and wage rigidity in shaping Finnish business cycle fluctuations, as well as the role of different shocks, including labour market shocks, in driving the Finnish economy. For this purpose, the New Keynesian DSGE model built in the earlier essays is estimated using Bayesian Maximum Likelihood methods. The objective is to find a model that best describes Finnish business cycles, in particular the behaviour of labour markets. The focus is on the mechanisms that affect the dynamic behaviour of the economy which are at the core of New Keynesian macroeconomics. We specifically assess the role of wage rigidities.

The relevance of frictions on the labour market and of wage rigidity has received a lot of attention in the recent macroeconomic literature. It has been shown that models with these features are better equipped to reproduce empirically observed business cycles (see Shimer, 2010, and Gertler and Trigari, 2009). The typical approach in this branch of the literature has been to match DSGE model-implied moments to data moments assuming that technology shocks drive economic activity. Following this approach it was found, in Obstbaum (2011), that a calibrated model for the Finnish economy with labour market matching and moderate rigidity in wage determination provided model moments that best corresponded to data moments for the time period 1994-2010.

While this moment comparison approach has become the typical one in the literature, recent advances in Bayesian estimation techniques offer a better possibility to analyze the complete macroeconomic framework, and empirically investigate the significance of different mechanisms such as wage rigidity for model performance. This full information procedure also permits to account for a complete range of shocks that hit the economy.

There are only a few previous estimation studies that feature a similar labour market structure and rigidity in wage setting than that used in this thesis. Gertler, Sala and Trigari (2008), which is the closest reference for the present approach, develop and estimate a medium-scale DSGE model that allows for labour market frictions and staggered nominal wage contracting. They find that the model with wage rigidity provides a better description of the data than a flexible wage version. In the third essay, a similar labour market structure is employed but the model is modified to account for monetary union membership and for distortionary taxes that may importantly determine the reaction of the Finnish economy to different kinds of disturbances.

As in Christoffel, Kuester and Linzert (2009) labour market data is included into the estimation procedure. They argue that despite possible measurement problems related to labour market data these series are at the core of the search and matching literature and should therefore be included in the set of observable time series in order to help the identification of parameters. As they do, in the present work labour market shocks are also added and their significance for shaping the cyclical behaviour of selected key variables is analyzed. Faccini, Millard and Zanetti (2010) estimate a New Keynesian model with matching frictions and nominal wage rigidities on UK data but they do not include labour market shocks or labour market data. Their staggered bargaining framework is a simplified version of Gertler, Sala and Trigari (2008). They establish that while wage rigidity enables the model to fit the data more closely, the model is unable to precisely identify the frequency of wage adjustment.

The third essay is a first attempt at estimating a model with labour market frictions and wage rigidity on the Finnish economy. The contribution is twofold. First, we obtain estimates of nominal rigidities and wage indexation that are in line with the existing literature and give an economically plausible picture of the Finnish economy. We establish that wage rigidity is empirically relevant for the Finnish business cycle, and provide a detailed assessment of the significance of prior beliefs on the degree of wage rigidity. We find that the data seems to support a relatively wide range of different degrees of wage rigidity conditional on the assumption of exogenous disturbances to wage negotiations. In particular, it appears that the nature of shocks to the bargaining power of workers, similarly to wage markup shocks in the conventional New Keynesian model with monopolistic labour suppliers, importantly affect the conclusions that can be made on wage rigidity.

While the exact degree of rigidity of nominal wages is left unclear, the indexation of wages to past inflation seems to be an important feature of the Finnish economy. As this can be translated into relatively high effective real wage rigidity, it corresponds well to previous evidence on the high degree of real wage rigidity in Finland. Price rigidity is estimated to be lower than in many other similar models but is again well in line with Finnish evidence.

Second, our estimation approach sheds more light on the magnitude and effects of different shocks in the Finnish economy. Of the conventional shocks, the shock to export demand is especially important as expected for a small open economy. We also add labour market shocks to account for exogenous disturbances originating in the labour market that our model is not able to fully capture although the labour market extension we apply is much more detailed than that of the standard New Keynesian model. We find that labour market shocks importantly drive fluctuations of labour market variables (unemployment and vacancies) but they also account for a non-trivial share of output fluctuations.

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I

The Role of Labour Markets for Fiscal Policy Transmission

Abstract

This paper identifies how frictions in the labour market shape the responses of the economy to government spending shocks. The open economy New Keynesian DSGE model is extended by labour market frictions of the Mortensen-Pissarides type and a detailed description of fiscal policy. The nature of offsetting fiscal measures is found to be critical for the effects of fiscal stimulus, due to the different effects of different tax instruments on the labour market. Specifically, shifting the debt-stabilizing burden towards distortionary labour taxes has detrimental effects on the labour market outcome and on general economic performance in a flexible wage regime. The results indicate that wage rigidity increases the effectiveness of fiscal policy in the short term but leads to a worse longer term development including unemployment above steady state levels. The analysis suggests that a closer look at the functioning of labour markets may help to identify fiscal policy transmission channels not captured by the standard New Keynesian model.

1 Introduction

This paper studies the transmission of fiscal policy in the presence of labour market frictions. In order to address the question we extend the standard open-economy New Keynesian (NK) business cycle model in two dimensions: a detailed formulation of fiscal policy, and labour market matching frictions along the lines of Mortensen and Pissarides (MP). We consider a small monetary union member state following Galí and Monacelli (2008).[1]

Fiscal policy is back at the centre of the policy debate. After the implementation of huge fiscal stimulus packages to counter the effects of the global financial crisis, the focus has shifted on the alternative ways to pay back the resulting large increases in government debt. At the same time, there is continuing uncertainty, both in the empirical and theoretical literature, on what the effects of fiscal policy really are. The positive effect of increased government spending on output is widely acknowledged. But the magnitude of the output multiplier as well as effects on especially private consumption and the real wage are still debated.

[1] This paper was related to a larger modelling project where the objective is to build a framework for the macroeconomic analysis of the Finnish economy. The choice of the theoretical framework is, therefore, guided by specific country characteristics such as the requirements of Euro area membership, the wage negotiation tradition and rigidity in wage setting. As the focus is on fiscal policy, the relevant tax instruments are included in the analysis.

The New Keynesian model in its standard form predicts positive output multipliers and a negative response of private consumption to government spending shocks. An increase in government spending is interpreted, by intertemporally optimizing consumers, as a future rise in taxes, and consequently as a fall in their lifetime resources. Therefore, households reduce their demand for consumption and leisure, if both are normal goods. The negative effect on private consumption is, however, typically smaller than in real business cycle (RBC) models because, when prices are rigid, firms increase labour demand as they respond to increased aggregate demand (see e.g. Linnemann and Schabert, 2003).

The responses of employment and the real wage to fiscal shocks have received much less attention than effects on output and private consumption. In the New Keynesian framework, the increase in labour demand together with the increase in labour supply can drive up the real wage, or at least make it fall by a smaller amount, and employment may increase. Hall (2009) assesses in a recent paper the effects of increased government purchases to GDP in the New Keynesian framework.

We focus on the effects of government spending shocks on private consumption, employment and the real wage, and identify how frictions in the labour market shape these responses in the New Keynesian framework. The approach is closest to Monacelli, Perotti and Trigari (2010) who investigate output and unemployment fiscal multipliers in an RBC model with labour market matching. They consider New Keynesian features as one extension to their baseline model but do not combine these features with debt-financing and distortionary taxes or with wage rigidity which, in the present framework, turn out to be important determinants of the effects of fiscal shocks. Recent research on monetary policy in the presence of labour market frictions (see e.g. Christoffel, Kuester and Linzert, 2009) is also a close reference, and indicates that these frictions may have an important role in shaping the economy's response to shocks.

We consider specifically different fiscal policy instruments that can be used to finance the public debt that results from increased government spending. This approach is motivated by the early finding by Baxter and King (1993) that the chosen financing scheme is a crucial assumption for the effects of fiscal policy. Since that finding, this question has received surprisingly little attention in the otherwise abundant literature on the effects of government spending. More recently, however, e.g. Bilbiie and Straub (2004) have recognised that the way fiscal shocks are financed, shapes the response to a government spending shock in a New Keynesian model as well. Galí, López-Salido and Vallés (2007) find, considering only lump-sum taxes, that the intertemporal path of taxation, i.e. how strongly and quickly taxes react to debt and deficit, shapes the response of the economy to government spending shocks. Corsetti, Meier and Müller (2009), in turn, have analyzed a policy where part of the stimulus is financed by reductions in spending over time in a small open-economy NK model, and find that these spending reversals significantly alter the impact of increased public spending.

The role of labour market structures for fiscal policy is inspected especially in the case of wage rigidity, introduced with the help of the staggered bargaining framework of Gertler and Trigari (2009). This is because, in addition to being an intuitively important element in the modelling of a small euro area member country, wage rigidity has been found to be a central explanation for the volatile behaviour of unemployment in business cycles driven by technology shocks (see Shimer, 2010). Wage rigidity has also been found to significantly affect the transmission of monetary policy shocks to the economy (see Christoffel, Kuester and Linzert, 2009).

Our main findings can be summarized as follows. First, the effects of fiscal shocks in our baseline model with flexible wages are similar to the standard New Keynesian model without labour market frictions. Output increases, the response of private consumption is negative but small, and employment and the real wage rise. While the mere presence of labour market frictions does not significantly affect the sign or size of fiscal multipliers in the present framework, a clear advantage compared to the standard model is that the modelling of labour market frictions helps to identify the transmission channels of fiscal policy to the labour market. Following fiscal stimulus, firms see their future profit opportunities rise and open new vacancies, increasing labour demand. At the same time, the negative wealth effect both increases the supply of hours by each employed worker and increases the relative value from employment for all workers. labour supply increases along both the intensive and extensive margin. Real wages rise.

Second, the chosen offsetting fiscal measure is found to be critical for the effects of fiscal stimulus, due to the different effects of different tax instruments on the labour market. Most importantly, shifting the debt-stabilizing burden towards distortionary labour taxes has detrimental effects on the labour market outcome and on general economic performance in the flexible wage regime. As the debt-stabilizing tax rule becomes operative, the higher proportional tax rate decreases the relative value from employment, and feeds through to a higher wage. Specifically, the wage bargaining model implies that the negotiated contract wage rises to compensate workers for the otherwise falling net income. The higher wage directly implies higher labour costs to firms which reduce the number of open vacancies and unemployment starts rising. Due to this subsequent fall in employment, the contraction in private consumption is larger than when public debt is adjusted through lump-sum taxes. Interestingly, we find that the consumption tax has less negative consequences on the labour market than labour taxes because it has a smaller negative effect on the relative value from employment. Adding structure to the labour market thus helps us also to better track down the effects of different debt-stabilizing tax instruments on labour market outcomes.

Third, wage rigidity increases the magnitude of the short-term responses of labour market variables to fiscal stimulus. Vacancies react more strongly to stimulus in the short term, since firms' profits are larger when workers cannot internalize all the expected rise in taxes, and consequently firms' labour costs do not simultaneously rise. This is in line with the literature on labour markets and business cycles (see Shimer, 2010), but in contrast to Monacelli, Perotti and Trigari (2010). The main differences in these two approaches are the assumption on price rigidity and the behaviour of the real interest rate. In the New Keynesian framework, as opposed to a RBC model, rigid prices give rise to a labour demand effect as witnessed by increased vacancy creation. Combined with rigid prices, rigid wages amplify the labour demand effect of fiscal stimulus on employment since firms' expected profits rise more than with flexible wages. The importance, for the effects of fiscal policy, of the extent of price rigidity has been identified earlier by e.g. Galí, López-Salido and Vallés (2007).

In addition, in the present model, the real interest rate always falls in response to a government spending shock because the rise in prices in the small currency union member state caused by increased aggregate demand is not counteracted by tightening monetary policy by the currency union's central bank. This effect is large enough to overturn the upward pressure on the real interest rate caused by the rise in the shadow value of wealth. The falling real interest rate makes fiscal policy more effective compared to a standard closed economy setup where accommodative monetary policy would counteract the rise in prices and the real interest rate would rise. This is in line with Hall (2009) who notes that fiscal multipliers are larger if monetary policy does not lean against the expansionary effect on increased government spending and Christiano, Eichenbaum and Rebelo (2011).discuss this issue in the natural scenario when the zero bound on nominal interest rates is binding.

Furthermore, our results indicate that while wage rigidity would also seem to make fiscal policy more effective in the short term, in the longer term, the gradual increase in the wage causes a prolonged increase in unemployment to above the steady state level. Public debt stays higher and the negative effect of private consumption is larger than when wages are flexible. Wage rigidity also affects the relative preferability of different debt-stabilizing tax instruments. Expansionary tax shocks do not at first glance seem to be as effective as spending shocks, but turn out, in the longer run, to lower more unemployment than fiscal stimulus via government spending, and have a positive effect on private consumption.

The remainder of the paper is organised as follows: Section 2 describes the model, Section 3 evaluates the steady state properties of the model and summarizes the transmission channels of fiscal policy, Section 4 presents the parameterization of the model and the results from dynamic simulations. Section 5 concludes.

2 The model

2.1 General features

The model considers a small monetary union member state and builds in this respect on Galí and Monacelli (2008). As in Corsetti, Meier and Müller (2009), however, we close the model by assuming a debt-elastic interest rate instead of complete asset markets. The home country is modelled along standard New-Keynesian practise comprising households, firms and a public sector. For simplicity, capital is not included as a factor of production. The framework is augmented by a Mortensen and Pissarides (MP) search and matching labour market model (Mortensen and Pissarides, 1994; Pissarides 2000).

The structure of the standard labour market matching model has been amended with some key features that have, in more recent literature, been found useful in capturing the data and explaining the so-called unemployment volatility puzzle.[2]. There is an emerging consensus that labour market frictions, wage rigidities and staggered price setting together are needed to explain fluctuations in unemployment, and the effects of monetary policy shocks (see e.g. Blanchard and Galí, 2010). These features are taken to be important also for analyzing fiscal policy.

The present model adds rigidity in the adjustment of wages in the form of staggered bargaining initially developed by Gertler and Trigari (2009), and applied in Gertler, Sala and Trigari (2008) and Christoffel, Kuester and Linzert (2009). One advantage of this approach is that wage rigidity gets the explicit interpretation of longer wage contracts. Lengthening the duration of wage contracts makes wages in each period less responsive to economic conditions, and shifts adjustment to the labour quantity side.

In our framework, there is only one worker per firm, and the wage and price setting decisions are separated from each other. labour market frictions arise in the intermediate good sector. The wholesale firms buy intermediate goods and re-sell them to the final goods sector. Wholesale firms operate under monopolistic competition and set prices subject to Calvo rigidities. Final goods are produced from domestic and imported intermediate inputs under perfect competition.

The other extension of the model concerns the public sector. The government's fiscal policy instruments include a lump-sum tax, a proportional wage tax paid by the employees, wage taxes paid by employers in the form of social security contributions, as well as a consumption tax. The tax instruments react to changes in the debt-to-output ratio according to simple fiscal feedback rules. Government spending is subject to shocks.

2.2 Preferences

As in similar models, we adopt the representative or large household interpretation. This implies perfect consumption insurance, a key assumption needed to embed the Mortensen-Pissarides model in a general equilibrium framework. Household members perfectly insure each other against variations in labour income due to their labour market status. This tackles the problem whereby households are identical but not all of their members are employed. As a result, the employment and unemployment rates are identical at the household level and across the population at large (see e.g. Merz, 1995).

[2]Shimer (2005) argues that, subjet to technology shocks, the MP model in its standard form does not sufficiently reproduce the relatively smooth behavior of wages and relatively volatile behavior of labor market variables observed in the data. Shimer further argues that the problem arises because, in the standard model, the wage is renegotiated in every period by Nash bargaining and therefore adjusts very easily to changes in the economic environment. The volatility of wages absorbs a large part of the fluctuation that is actually observed in employment variables. In the growing body of literature that has attempted to explain the problem, also known as the unemployment volatility puzzle, the focus has accordingly been on ways to amplify the response of vacancies and unemployment to shocks. The range of alternative models proposed to solve the unemployment volatility puzzle include both flexible and rigid wage variants and have been summarized in e.g. Hall (2005).

The representative household maximizes the expected lifetime utility of its individual members

$$\int_0^1 E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_{i,t} - \varkappa C_{t-1})^{1-\varrho}}{1-\varrho} - \delta n_t \frac{(h_{i,t})^{1+\phi}}{1+\phi} \right] \right\} di \tag{1}$$

where $C_{i,t}$ is final good consumption by household member i in period t, $\varkappa \in (0,1)$ indicates an external habit motive, C_{t-1} stands for aggregate consumption in the previous period, $h_{i,t}$ are hours worked by household member i in period t, and δ is a scaling parameter for the disutility of work. Disutility of work is experienced by those members of the household who are employed, n_t. The inverses of ϱ and ϕ are the elasticities of intertemporal substitution and of labour supply respectively. The household's (real) budget constraint is

$$\begin{aligned} & (1+\tau_t^c) C_t + \frac{B_t}{P_t} + \frac{B_t^*}{P_t} \\ = \; & n_t \frac{w_t}{P_t} h_t (1-\tau_t) + (1-n_t) b \\ & + \frac{TR_t}{P_t} + R_{t-1} \frac{B_{t-1}}{P_t} + R_{t-1}^* p\left(b_{t-1}^*\right) \frac{B_{t-1}^*}{P_t} + \frac{D_t}{P_t} \end{aligned} \tag{2}$$

The left-hand side of the equation describes the expenditures of the household. Consumption C_t is subject to a proportional tax τ_t^c. The household can buy two kinds of nominal one-period bonds, domestic B_t and foreign B_t^* which form the portfolio of its financial assets and are both denominated in the common monetary union currency. Domestic bonds are issued by the domestic government for which they represent debt. The right hand side describes the household's income sources which consist of after-tax real wage $n_t \frac{w_t}{P_t} h_t (1-\tau_t)$, unemployment benefits $(1-n_t) b$, lump-sum transfers $\frac{TR_t}{P_t}$, and of profit from firm ownership $\frac{D_t}{P_t}$. Income is also received in the form of repayment of last period's domestic or foreign bond purchases. $R_t = (1+r_t^n)$ stands for the gross nominal return on domestic bonds. The interest rate paid or earned on foreign bonds by domestic households $R_{t-1}^* p\left(b_{t-1}^*\right)$ consists, in turn, of the common currency union gross interest rate R_{t-1}^* which, for the small member state is taken to be exogenous, and a country-specific risk premium $p\left(b_{t-1}^*\right)$. The risk premium is assumed to be increasing in the aggregate level of foreign real debt as a share of domestic output $(-b_t^* = -\frac{B_t^*}{P_t Y_t})$.[3]

We leave aside for a moment the labour supply decision, which will be dealt with in the section describing the labour market. Optimal allocations are characterized by the following conditions

$$\Lambda_t = \frac{\lambda_t}{(1+\tau_t^c)} \tag{3}$$

$$\Lambda_t = \beta E_t \left[\Lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right] \tag{4}$$

[3]This is the debt-elastic interest rate assumption which is one of the mechanisms suggested by Schmitt-Grohé and Uribe (2003) to close a small open economy model. Note that with the current notation a negative (positive) deviation of the stock of foreign bonds from the steady state zero level implies that the home country as a whole becomes a net borrower (lender), and faces a positive (negative) risk premium.

$$\Lambda_t = \beta E_t \left[\Lambda_{t+1} \frac{R_t^* p\left(b_t^*\right)}{\pi_{t+1}} \right] \tag{5}$$

where $\lambda_t = (C_t - \varkappa C_{t-1})^{-\varrho}$ is the marginal utility of consumption and $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ is CPI inflation. The discount factor is the same for all optimizing agents in the economy and is hereafter defined throughout the paper as $\beta_{t,t+s} = \beta^s \frac{\Lambda_{t+s}}{\Lambda_t}$.

Combining the Euler conditions for domestic and foreign assets yields a modified uncovered interest rate parity relation where no risk is associated with exchange rate movements, as both domestic and foreign bonds are denominated in the same currency.

$$R_t = R_t^* p\left(b_t^*\right) \tag{6}$$

This arbitrage relation says that, as domestic and foreign bonds perfectly substitute each other, their nominal returns to the consumers have to be equal in equilibrium.

The risk premium on foreign bond holdings $p\left(b_t^*\right)$ follows the function

$$p\left(b_t^*\right) = \exp\left[-\gamma_{b^*}\left(b_t^* - \bar{b}\right)\right], \text{ with } \gamma_{b^*} > 0 \tag{7}$$

This should ensure the stability and determinacy of equilibrium in a small member state of the monetary union model [4]. In the steady state, the risk premium is assumed to be equal to one, and the domestic and foreign interest rates are the same. After loglinearization the arbitrage relation gets the form[5]

$$\widehat{R}_t = \widehat{R}_t^* - \gamma_{b^*} \widehat{b}_t^*$$

2.3 The labour market

The labour market brings together workers and intermediate good firms.

2.3.1 Unemployment, vacancies and matching

The measure of successful matches m_t is given by the matching function

$$m_t(u_t, v_t) = \sigma_m u_t^{\sigma} v_t^{1-\sigma} \tag{8}$$

where m_t is the flow of matches during a period t, and u_t and v_t are the stocks of unemployed workers and vacancies at the beginning of the period. The matching function is, as usual, increasing in both vacancies and unemployment, concave, and homogeneous of degree one (see Petrongolo and Pissarides, 2001). The Cobb-Douglas form implies that σ is the elasticity of matching with respect

[4] As Galí and Monacelli (2008) point out, along with accession to the monetary union the small member state no longer meets the Taylor principle since variations in its inflation that result from idiosyncratic shocks will have an infinitesimal effect on union-wide inflation, and will thus induce little or no response from the union's central bank. According to the Taylor principle, in order to guarantee the uniqueness of the equilibrium, the central bank would have to adjust the nominal interest rates more than one-for-one with changes in inflation (see e.g. Woodford, 2003)

[5] Hereafter, all variables marked with a hat denote the log deviation of that variable from its steady state level.

to the stock of unemployed people, and σ_m represents the efficiency of the matching process. The probabilities that a vacancy will be filled and that the unemployed person finds a job are respectively

$$q_t^F = q_t^F(\theta_t) = \frac{m_t}{v_t} = \sigma_m \theta_t^{-\sigma} \tag{9}$$

$$q_t^W = \theta_t q_t^F(\theta_t) = \frac{m_t}{u_t} = \sigma_m \theta_t^{1-\sigma} \tag{10}$$

and the inverse of these probabilities is the mean duration of vacancies and unemployment.

$\theta_t = \frac{v_t}{u_t}$ is labour market tightness. The tighter the labour market is, or the less there are unemployed people relative to the number of open vacancies (i.e. larger θ_t), the smaller the probability that the firm succeeds in filling the vacancy and the larger the probability that the unemployed person finds a job. Similarly, a decrease in the number of vacancies relative to unemployment (smaller θ_t) implies that the unemployed person has a smaller probability to find a job.

In the beginning of each period, a fraction of matches will be terminated with an exogenous probability $\rho \in (0, 1)$.

labour market participation is characterised as follows. The size of the labour force is normalised to one. The number of employed workers at the beginning of each period is

$$n_t = (1-\rho)\, n_{t-1} + m_{t-1} \tag{11}$$

where the first term on the right hand side represents those workers who were employed already in the previous period and whose jobs have survived beginning-of-period job destruction, and the second term covers those workers who got matched in the previous period and become productive in the current period. After the exogenous separation shock, the separated workers return to the pool of unemployed workers and start immediately searching for a job. The number of unemployed is $u_t = 1 - n_t$.

In the steady state an equal amount of jobs are created and destructed

$$JC = JD \Longleftrightarrow m = \rho n \tag{12}$$

2.3.2 Wage bargaining

Job creation takes place when a worker and a firm meet and agree to form a match at a negotiated wage. The wage that the firm and the worker choose must be high enough that the worker wants to work in the job, and low enough that the employer wants to hire the worker. These requirements define a range of wages that are acceptable to both the firm and the worker. The unique equilibrium wage is, however, the outcome of a bargain between the worker and the firm. We will call this wage the contract wage.

The structure of the staggered multiperiod contracting model applied here follows Gertler and Trigari (2009) and Gertler, Sala and Trigari (2008) but includes also the intensive margin of adjustment of the labour input (hours worked per worker) as well as distortionary taxes. For comparison, the period-by-period bargaining outcome is presented in Appendix A.3.

The idea of staggered wage bargaining is analogous to Calvo price setting. Rigidity is created by assuming that a fraction γ of firms are not allowed to renegotiate their wage in a given period. As a result, all workers in those firms receive the nominal wage paid the previous period w_{t-1}. The constant probability that firms are allowed to renegotiate the wage is labeled $(1-\gamma)$. Accordingly, $\frac{1}{(1-\gamma)}$ is the average duration of a wage contract. Thus, the combination of wage bargaining and Calvo price setting allows to give an intuitive interpretation to the source of wage rigidity instead of more or less ad hoc formulations. Period-by-period bargaining corresponds to the special case of $\gamma = 0$.

As in the standard Mortensen-Pissarides model, it is assumed that match surplus, the sum of the worker and firm surpluses, is shared according to efficient Nash bargaining. In the baseline model, wages and hours are negotiated simultaneously in each period. The firm and the worker choose the nominal wage and the hours of work to maximize the weighted product of their net return from the match. When wages are rigid, it is assumed that as they become productive, new matches enter the same Calvo scheme for wage-setting than existing matches. This is an important assumption for wage rigidity to have an effect on job creation. Gertler and Trigari (2009) argue that after controlling for compositional effects there are no empirical differences in the flexibility of new and existing worker's wages.[6]

The contract wage w_t^* is chosen to solve

$$\max \left[H_t\left(r\right)\right]^{\eta} \left[J_t\left(r\right)\right]^{1-\eta} \tag{13}$$

subject to the random renegotiation probability. $H_t(r)$ and $J_t(r)$ are the matching surpluses of renegotiating workers and firms respectively, and $0 \leq \eta \leq 1$ is the relative measure of workers' bargaining strength. The value equations describing the worker's and the firm's matching surpluses are the key determinants of the outcome of the wage bargain.

Workers The value to the renegotiating worker of being employed consists of after-tax labour income, the disutility from supplying hours of work and the expected present value of his situation in the next period[7]. In the case of non-renegotiation at time $t+1$, the worker gets the existing contract wage w_t^*

[6] E.g. Pissarides (2009) and Haefke, Sonntag and Van Rens (2009) argue the opposite: that wages of newly hired workers are volatile unlike wages for ongoing job relationships. This would mean that there is wage rigidity, but not of the kind that affects job creation and leads to more volatility in unemployment fluctuations. Before this debate is settled, we follow Gertler and Trigari (2009).

[7] In the presence of perfect consumption insurance against different labour market outcomes, it is necessary to define the worker surplus as the change in the *household's utility* of having one additional member employed. Accordingly, the worker surplus equation is obtained by differentiating the household's optimal value function with respect to the number of workers. In addition, the resulting equation is expressed in terms of current consumption, i.e. it is divided by the shadow price of consumption. For more details, see Trigari (2006).

$$
\begin{aligned}
W_t(r) &= \frac{w_t^*}{P_t} h_t (1-\tau_t) - \frac{g(h_t)}{\Lambda_t} \\
&+ E_t \beta_{t,t+1} (1-\rho) \left[\gamma W_{t+1}(w_t^*) + (1-\gamma) W_{t+1}(w_{t+1}^*) \right] \\
&+ E_t \beta_{t,t+1} \rho U_{t+1} \qquad (14)
\end{aligned}
$$

The value to the worker of being unemployed is

$$
U_t = b + E_t \beta_{t,t+1} \left[q_t^W W_{x,t+1} + \left(1 - q_t^W\right) U_{t+1} \right] \qquad (15)
$$

where the first term on the RHS is the value of the outside option to the worker, i.e. the unemployment benefit b, and the second term gives the expected present value of either finding a job or remaining unemployed in the following period. Unemployed workers do not need to take into account the probability of job destruction even if they get matched because of the timing assumption. A match that has not yet become productive cannot be destroyed. Note that the value for the worker, who is currently unemployed, to move from unemployment to employment next period is $W_{x,t+1}$, the expected *average* value of being employed. New matches are subject to the same bargaining scheme as existing matches, and therefore the new worker does not have a priori knowledge of whether the firm he will start working for will be allowed to renegotiate its wage[8].

Combining these value equations gives the expression for the surplus of those workers who renegotiate their wage in the current period

$$
\begin{aligned}
H_t(r) &= W_t(r) - U_t \\
&= \frac{w_t^*}{P_t} h_t (1-\tau_t) - \frac{g(h_t)}{\Lambda_t} - b \\
&+ E_t \beta_{t,t+1} (1-\rho) \left[\gamma H_{t+1}(w_t^*) + (1-\gamma) H_{t+1}(w_{t+1}^*) \right] \\
&- q_t^W E_t \beta_{t,t+1} H_{x,t+1} \qquad (16)
\end{aligned}
$$

For later use, it is useful to note that the value for the worker of not working consists both of the lost utility of leisure and of a fixed unemployment benefit, the second and third terms of the surplus equation respectively. Noting that the marginal rate of substitution of consumers/workers is $mrs_t = \frac{g'(h_t)}{u'(c_t)}$ allows us to rewrite the second term as $\frac{mrs_t h_t (1+\tau_t^c)}{(1+\phi)}$. This form will be used in later sections.

Intermediate firms For the firm that renegotiates the wage in the current period, the value of the occupied job is equal to the profit of the firm in the current period net of payroll taxes s_t, and the expected future value of the job

[8] Accordingly, the average surplus from working is $H_{x,t+1} = \gamma H_{t+1}(w_t) + (1-\gamma) E_t H_{t+1}(w_{t+1}^*)$. If the worker starts working in a firm that is not allowed to renegotiate, he will get last period's *average* wage. This is because in the one firm - one worker setup of this paper also firms in new matches are new, they cannot have negotiated a contract wage in the previous period.

$$\begin{aligned} J_t(r) &= x_t^H f\left(h_t\right) - \frac{w_t^*}{P_t} h_t \left(1+s_t\right) \\ &+ E_t \beta_{t,t+1} \left(1-\rho\right) \left[\gamma J_{t+1}(w_t^*) + \left(1-\gamma\right) J_{t+1}(w_{t+1}^*)\right] \end{aligned} \qquad (17)$$

where $x_t^H = \frac{P_{H,t}}{P_t} x_t$ is the relative price of the intermediate sector's good in terms of the CPI[9], and $f\left(h_t\right) = z{h_t}^{\alpha}$ is match output. The marginal product of labour is accordingly $mpl_t = \alpha z {h_t}^{\alpha-1} = \alpha \frac{f(h_t)}{h_t}$. labour-augmenting productivity z is identical for all matches and is not subject to shocks in the present inspection.

The value to the firm of an open vacancy is

$$V_t = -\kappa + E_t \beta_{t,t+1} q_t^F \left[\gamma J_{t+1}(w_t) + \left(1-\gamma\right) J_{t+1}(w_{t+1}^*)\right] + E_t \beta_{t,t+1} \left(1-q_t^F\right) V_{t+1} \qquad (18)$$

The value of a vacancy consists of a fixed hiring cost κ, and of the expected value from future matches. In equilibrium, all profit opportunities from new jobs are exploited so that the equilibrium condition for the supply of vacant jobs is $V_t = 0$. With each firm having only one job, profit maximization is equivalent to this zero-profit condition for firm entry. Setting the equation for V_t as zero in every period gives

$$\frac{\kappa}{q_t^F} = E_t \beta_{t,t+1} \left[\gamma J_{t+1}(w_t) + \left(1-\gamma\right) J_{t+1}(w_{t+1}^*)\right] \qquad (19)$$

This vacancy posting condition equates the marginal cost of adding a worker (the real cost times the mean duration of a vacancy) to the discounted marginal benefit from a new worker. After taking into account the free entry condition, the firm surplus reduces to J_t.

Multiperiod bargaining set up Unlike with period-to-period bargaining (see Appendix A.3), in the presence of staggered contracting, firms and workers have to take into account the impact of the contract wage on the expected future path of firm and worker surplus. Accordingly, the first order condition for wage-setting is given by

$$\eta \Delta_t J_t\left(r\right) = \left(1-\eta\right) \Sigma_t H_t\left(r\right) \qquad (20)$$

where the partial derivatives of the surplus equations w.r.t. the wage $\Delta_t = P_t \frac{\partial H_t(r)}{\partial w_t}$ and $\Sigma_t = -P_t \frac{\partial J_t(r)}{\partial w_t}$ denote the effect of a rise in the *real* wage on the worker surplus and (minus) the effect of a rise in the real wage on the firm's surplus respectively (see Appendix A.4 for details)

[9]The firm surplus is here expressed in terms of consumer prices, as opposed to producer prices, to avoid confusion in computing the wage bargaining solution. As a result, the intermediate firms get the nominal price $P_{H,t} x_t$ for their product while they value this revenue in terms of the CPI. This creates a channel for CPI-PPI differences to enter the wage bargaining problem as explained in section 2.3. See also Faia et al. (2010) for discussion on this subject.

$$\Delta_t = h_t (1 - \tau_t) + E_t \beta_{t,t+1} (1 - \rho) \gamma \Delta_{t+1} \tag{21}$$

$$\Sigma_t = h_t (1 + s_t) + E_t \beta_{t,t+1} (1 - \rho) \gamma \Sigma_{t+1} \tag{22}$$

These expressions can be interpreted as the discounting factors for the worker and the firm for evaluating the value of the future stream of wage payments. As wage contracts extend over multiple periods, agents have to take into account also the *future* probabilities of not being allowed to renegotiate the wage, or of not surviving exogenous destruction. In the limiting case of period-by-period bargaining, $\gamma = 0$, the partial derivatives of the surpluses w.r.t. the wage reduce to $\Delta_t = h_t (1 - \tau_t)$, and $\Sigma_t = h_t (1 + s_t)$, and the first order condition accordingly reduces to its period-by-period counterpart $\eta (1 - \tau_t) J_t = (1 - \eta)(1 + s_t) H_t$. The worker and the firm have, accordingly, the following effective bargaining weights: $\frac{\eta}{(1+s_t)}$ and $\frac{(1-\eta)}{(1-\tau_t)}$.

In the one firm - one worker setup, used in this paper, the discounting factors under the staggered bargaining regime would be equal across agents unless the possible changes in distortionary taxes over time were breaking this symmetry[10]. If taxes were held constant, the discounting factors would be effectively the same, just weighted with the relevant constant labour tax rate [11]. As a result, the first order condition for wage-setting would have the same form as with period-by-period bargaining $\eta (1 - \tau) J_t = (1 - \eta)(1 + s) H_t$, and the effective bargaining weights would again be accordingly $\frac{\eta}{(1+s)}$ for the worker and $\frac{(1-\eta)}{(1-\tau)}$ for the firm. So, proportional tax rates influence the *division* of the total surplus from a job in equilibrium, irrespective of the bargaining horizon, a standard result from the labour market matching literature (see Pissarides, 2000, Chapter 9). More specifically, both the worker's and the firm's marginal tax rate effectively reduce the worker's relative bargaining power, and consequently his share of the surplus.

However, when staggered bargaining is combined with the possibility of changing labour tax rates over time, workers and firms have to take into account the future path of taxation in their negotiating behaviour, and labour taxes also enter the discounting factor equations of agents. The corresponding effective bargaining weights of agents[12] now depend, in addition to the negotiation power parameter, on both labour taxes and on their effect on the agents' discounting factors. As is apparent from the loglinearized forms of the discounting factors, presented in Appendix A.2, the increase in the worker's labour tax decreases the discounting factor of the worker and the increase in the employer's labour tax increases its discounting factor. As a result, following the tax increases, firms place relatively more weight on the future than workers. The implication for the effective bargaining weights is, in addition to

[10] In Gertler and Trigari (2009), this is not the case. Differences in the worker's and the firm's optimization perspectives, a "horizon effect", arises because large firms take into account possible changes in future hiring rates.

[11] $\Delta_t = (1 - \tau) E_t \sum_{s=0}^{\infty} \beta_{t,t+s} (1 - \rho)^s \gamma^s h_{t+s}$ for the worker, and $\Sigma_t = (1 + s) E_t \sum_{s=0}^{\infty} \beta_{t,t+s} (1 - \rho)^s \gamma^s h_{t+s}$ for the firm.

[12] $\frac{\eta}{\eta(1+s_t)+(1-\eta)\frac{\Sigma_t}{\Delta_t}(1-\tau_t)}$ for the worker and $\frac{(1-\eta)}{\eta\frac{\Delta_t}{\Sigma_t}(1+s_t)+(1-\eta)(1-\tau_t)}$ for the firm.

shifting bargaining power from workers to firms contemporaneously, that the expectation of future tax increases further increases the discounting factor of firms relative to that of the workers further increasing the effective bargaining power of firms relative to that of the workers. The effect of distortionary and time-dependent taxes on the division of match surplus is thus amplified by staggered bargaining.

Given that the probability of wage adjustment is i.i.d., and all matches at renegotiating firms end up with the same wage w_t^*, the evolution of the nominal *average hourly* wage in the economy can be expressed as a convex combination of the contract wage and the average wage across the matches that do not renegotiate.

$$w_{t+1} = (1-\gamma)\, w_{t+1}^* + \gamma \int_0^{n_t} \frac{w_{it}}{n_t} di \tag{23}$$

Wage dynamics The staggered bargaining framework has implications on the behaviour of workers and firms. To describe wage dynamics in the presence of staggered contracting, we develop loglinear expressions for the relevant wage equations. The approach is in the spirit of Gertler, Sala and Trigari (2008), and is presented in detail in Appendix A.4. The contract wage is solved by first linearizing the first order condition

$$\widehat{J}_t(r) + \widehat{\Delta}_t = \widehat{H}_t(r) + \widehat{\Sigma}_t \tag{24}$$

and then plugging into the FOC the value equations and discounting factors for the worker and the firm respectively in their loglinearized form. The resulting contract wage is

$$\widehat{w}_t^* = [1-\iota]\, \widehat{w}_t^0\,(r) + \iota E_t \widehat{w}_{t+1}^* \tag{25}$$

where $\iota = \overline{\beta}\,(1-\rho)\,\gamma$. This is the optimal wage set at time t *by all matches that are allowed to renegotiate their wage*. As is usual with Calvo contracting, it depends on a wage target $w_t^0(r)$ and next period's optimal contract wage. The weight put on each of these components depends on the steady state discounting factor and on the probabilities of job survival $(1-\rho)$ and non-renegotiation γ. As the probability of not being able to renegotiate the wage approaches zero, $\gamma \longrightarrow 0$, i.e. we approach the period-by-period bargaining case, ι approaches zero, $\iota \longrightarrow 0$, and the contract wage, w_t^*, approaches the period-by-period Nash wage.

Unlike in the more conventional set up of New Keynesian models, where Calvo wage contracting is combined with a monopolistic supplier of labour, the target wage here also includes a spillover effect that brings about additional rigidity on top of that implied by the Calvo scheme alone. Gertler and Trigari (2009) show how spillover effects result from wage bargaining. The target wage can be decomposed into two parts

$$\widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H \Gamma E_t \left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right] \tag{26}$$

where $\varphi_H \Gamma = \frac{(1-\eta)\beta q^w}{(1-\iota)}$ is the spillover effect. The spillover coefficient is positive, indicating that when the expected average market wage $E_t\widehat{w}_{t+1}$ is higher than the expected contract wage $E_t\widehat{w}^*_{t+1}$, (indicating unusually good labour market conditions) this raises the target wage in the negotiations. Thus, wage rigidity and the resulting employment dynamics are not only a product of staggered wage setting, but also of the spillover effects from the Nash bargaining process.

The spillover-free component of the target wage is of exactly the same form than the period-by-period negotiated wage (presented in Appendix A.3), only adjusted for the multiperiod discounting factors.

$$\begin{aligned} \widehat{w}^0_t &= \varphi_x \left(\widehat{x}^H_t + \widehat{mpl}_t \right) + \varphi_m \widehat{mrs}_t + \varphi_H E_t \left(\widehat{q}^W_{t+1} + \widehat{H}_{t+1} \left(w^*_{t+1} \right) + \widehat{\beta}_{t,t+1} \right) \\ &\quad -\varphi_h \widehat{h}_t - \varphi_s \widehat{s}_t + \varphi_\tau \widehat{\tau}_t + \varphi_{\tau^c} \widehat{\tau}^c_t + \varphi_D E_t \left[\widehat{\Sigma}_{t+1} - \widehat{\Delta}_{t+1} \right] + \widehat{P}_t \end{aligned} \tag{27}$$

As its period-by-period counterpart, the spillover-free target wage depends on what the worker contributes to the match (the first term on the RHS) and on his opportunity cost (the second and third terms). Note that, due to the formulation of the relative price $\widehat{x^H}_t$, whenever domestic production prices rise more than the CPI, this creates an upward pressure on the target wage. Increases in both the labour tax on the employee and in the consumption tax increase the target wage whereas an increase in the labour tax on the employer lowers the target wage. The target wage depends positively on the difference between the firm's and the worker's discount factor because while an increase in the discounting factor of the firm's relative discounting factor decreases the target wage through an increase in the relative effective bargaining power of firms, this change in the relative discounting factors also has the effect - by the Nash first order condition - of decreasing the expected surplus of the worker, thereby increasing his wage demand in the current period.

Finally, combining all the relevant elements of the wage bargaining outcome: the contract wage, the average wage and the target wage, yields a second-order difference equation for the evolution of the average wage (see Appendix A.4)

$$\widehat{w}_t = \lambda_b \widehat{w}_{t-1} + \lambda_0 \widehat{w}^0_t + \lambda_f E_t \widehat{w}_{t+1} \tag{28}$$

Due to staggered contracting, the average wage in the economy $\widehat{w}_t$ depends on the lagged wage $\widehat{w}_{t-1}$, the spillover-free target wage $\widehat{w}^0_t$, and the expected future wage $E_t\widehat{w}_{t+1}$. The longer is the average duration of wage contracts, i.e. the larger is the non-renegotiation parameter γ, the more weight gets the lagged wage component in wage determination.

2.3.3 Determining hours of work

While matches are restrained to renegotiate the wage with a given exogenous probability, hours per worker can be *renegotiated at each point in time.* With efficient Nash bargaining, optimal hours of work can be found from the following first order condition obtained by differentiating the Nash maximand w.r.t hours

$$(1-\tau_t)\, x^H_t f_{h,t} = (1+s_t)\, \frac{g'(h_t)}{\Lambda_t}$$

where $f_{h,t}$ is, as before, the marginal product of the labour input i.e. hours, and which, using the expressions for the production and utility functions, can be written as

$$\left(1-\tau_t\right)x_t^H mpl_t = \left(1+s_t\right)mrs_t\left(1+\tau_t^c\right) \tag{29}$$

This optimality condition equates the value of marginal product to the marginal rate of substitution between work and leisure, and resembles, thus, to the corresponding condition in a competitive labour market. However, with labour market frictions, while the hourly wage is such that the marginal cost to the worker from working is equal to the marginal gain to the firm, neither of these measures needs to be equal to the wage. It is important to observe that the optimality condition for hours determines the optimal hours per worker, i.e. the intensive margin of labour adjustment. This individual labour input of a worker is determined *irrespective of the wage.* But the model also allows for labour adjustment in the number of workers, as defined by the vacancy posting condition and the matching function.

2.4 Final good firms

There are two types of final goods firms. One produces private consumption goods and the other type of final goods firm produces public consumption goods[13].

2.4.1 Private consumption good

The private consumption good is a composite of intermediate goods distributed by a continuum of monopolistically competitive wholesale firms at home and abroad. Wholesale firms, their products and prices are indexed by $i \in [0,1]$. Final good firms operate under perfect competition and purchase both domestically produced intermediate goods $y_{H,t}(i)$ and imported intermediate goods $y_{F,t}(i)$. They minimize expenditure subject to the following aggregation technology

$$C_t = \left[\left(1-W\right)^{\frac{1}{\varpi}}\left(\left[\int_0^1 y_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}}\right)^{\frac{\varpi-1}{\varpi}} + W^{\frac{1}{\varpi}}\left(\left[\int_0^1 y_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}}\right)^{\frac{\varpi-1}{\varpi}}\right]^{\frac{\varpi}{\varpi-1}} \tag{30}$$

where ϖ measures the trade price elasticity, or elasticity of substitution between domestically produced intermediate goods and imported intermediate goods in the production of final goods for given relative prices, and W is the weight of imports in the production of final consumption goods. The parameter $\varepsilon > 1$ is the elasticity of substitution across the differentiated intermediate goods produced and distributed within a country.

[13] This is a standard assumption in New Open Economy Macro Models that assess fiscal policy. E.g. in Obstfeld and Rogoff's (1996) extension of the Redux model, government spending is introduced as a basket of public consumption goods aggregated in the same way as for private consumption.

The optimization problem determining the allocation of expenditure between the individual varieties of domestic and foreign intermediate goods yields the following demand curves facing each wholesale firm

$$y_{H,t}(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_{H,t} \tag{31}$$

$$y_{F,t}(i) = \left(\frac{p_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} Y_{F,t} \tag{32}$$

where $P_{H,t}$ and $P_{F,t}$ are the aggregate price indexes for the domestic and foreign intermediate goods respectively

$$P_{H,t} = \left[\int_0^1 p_{H,t}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \tag{33}$$

$$P_{F,t} = \left[\int_0^1 p_{F,t}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \tag{34}$$

To determine the optimal allocation between the domestic and imported intermediate goods, the final good firm minimizes costs $P_{H,t}Y_{H,t} + P_{F,t}Y_{F,t}$ subject to its production function or aggregation constraint. This yields the demands for the domestic and foreign intermediate good *bundles* by domestic final good producers

$$Y_{H,t} = (1 - W)\left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t \tag{35}$$

$$Y_{F,t} = W\left(\frac{P_{F,t}}{P_t}\right)^{-\varpi} C_t \tag{36}$$

where P_t is the home country's aggregate price index, or consumption price index

$$P_t = \left((1 - W) P_{H,t}^{1-\varpi} + W P_{F,t}^{1-\varpi}\right)^{\frac{1}{1-\varpi}} \tag{37}$$

At the level of individual intermediate goods the law of one price holds[14]. That, together with the assumption that the weight of the home country good in the foreign consumer price index is infinitesimally small, implies that $P_{F,t}$ is equal to the foreign CPI P_t^* (see Galí and Monacelli, 2008).

2.4.2 Public consumption good

The public consumption good is composed of only domestic intermediate goods $g_t(i)$. This assumption implies full home bias in government spending. This simplifying assumption can be supported by the observation from input-output tables that the use of foreign intermediate goods in government spending is significantly lower than in private consumption.

[14]Note, however, that due to home bias in consumption the basket of consumed goods may differ in the two areas, and therefore purchasing power parity does not hold.

$$G_t = \left[\int_0^1 g_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{38}$$

Each wholesale firm i selling intermediate goods to the public consumption good producer faces the following demand schedule

$$g_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} G_t \tag{39}$$

2.5 Wholesale firms and price setting

The wholesale firms buy the homogeneous intermediate goods at nominal price $p_{H,t}x_t$ per unit and transform them one-to-one into the differentiated product. As in most models that incorporate labour market matching into the NK framework, the price setting decision is separated from the wage setting decision to maintain the tractability of the model[15]. Price rigidities arise at the wholesale level while search frictions and wage rigidity only affect directly the intermediate goods sector.

There is Calvo-type stickiness in price-setting and the relative price of intermediate goods x_t coincides with the real marginal cost faced by wholesale firms. In each period, the wholesale firm can adjust its price with a constant probability $1-\xi$ which implies that prices are fixed on average for $\frac{1}{1-\xi}$ periods. The wholesale firm's optimization problem is to maximize expected future discounted profits by choosing the sales price $p_{H,t}(i)$, taking into account the pricing frictions and the demand curve they face. It is assumed that the wholesale firm sells the home-country intermediate goods for the same price for domestic and foreign final goods producers, and for the domestic government.

The expected future discounted profit of a wholesale firm that reoptimizes at t is

$$E_t \sum_{s=0}^{\infty} \xi^s \beta_{t,t+s} \left[\left(\frac{p_{H,t}(i)}{P_{H,t+s}} \right) y_{t+s}(i) - x_{t+s} y_{t+s}(i) \right] = 0 \tag{40}$$

where $y_t(i)$ is the demand of firm i's product by domestic private consumption good firms, foreign private consumption good firms and the domestic government as outlined in the previous section

$$y_t(i) = y_{H,t}(i) + y^*_{H,t}(i) + g_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_t^D$$

where Y_t^D stands for total demand for domestic intermediate goods. All wholesale firms are identical except that they may have set their current price at different dates in the past. However, in period t, if they are allowed to reoptimize their price, they all face the same decision problem and choose

[15] Some extensions merge the intermediate and retail sectors so that there are interactions between wage and price setting at the level of the individual firm. E.g. Christoffel et al. (2009) assess the implications of that specification for inflation dynamics.

the same optimal price $p^*_{H,t}$. Using the definition of the discount factor and rearranging, the FOC can be written as

$$E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[(1-\varepsilon) \left(\frac{p^*_{H,t}}{P_{H,t+s}} \right) + \varepsilon x_{t+s} \right] \left(\frac{1}{p^*_{H,t}} \right) \left(\frac{p^*_{H,t}}{P_{H,t+s}} \right)^{-\varepsilon} Y^D_{t+s} = 0 \tag{41}$$

which can be solved for $\frac{p^*_{H,t}}{P_{H,t}}$ to yield the following pricing equation

$$\frac{p^*_{H,t}}{P_{H,t}} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} x_{t+s} \left(\frac{P_{H,t+s}}{P_{H,t}} \right)^{\varepsilon} Y^D_{t+s}}{E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left(\frac{P_{H,t+s}}{P_{H,t}} \right)^{\varepsilon - 1} Y^D_{t+s}} \tag{42}$$

where $\frac{\varepsilon}{\varepsilon-1} = \mu$ is the flexible-price markup. This is the standard Calvo result. In the absence of price rigidity, the optimal price would reduce to a constant markup over marginal costs. Log-linearizing the FOC around the steady state yields the New Keynesian Phillips Curve where domestic inflation depends on marginal costs and expected future inflation

$$\widehat{\pi}_{H,t} = \nu \widehat{x}_t + \beta E_t \widehat{\pi}_{H,t+1} \tag{43}$$

where $\nu = \frac{(1-\xi)(1-\xi\beta)}{\xi}$.

2.6 Fiscal policies

The public sector's role in this economy is to collect taxes and use them to finance unemployment benefits and lump-sum transfers as well as government spending G_t. If expenditure in any period is larger than income it can finance the deficit by issuing bonds which it repays in the next period. The various tax instruments in use are the labour tax on workers τ_t, payroll taxes on firms s_t, and a consumption tax τ^c_t. Lump-sum transfers TR_t may also be altered in response to changes in spending. The government budget constraint is

$$n_t w_t h_t (\tau_t + s_t) + \tau^c_t P_t C_t + B_t = P_{H,t} G_t + P_t b u_t + TR_t + R_{t-1} B_{t-1} \tag{44}$$

Accordingly, the government real debt $b_t = \frac{B_t}{P_t}$, evolves as

$$b_t = R_{t-1} \frac{b_{t-1}}{\pi_t} + \frac{P_{H,t}}{P_t} G_t + b u_t + \frac{TR_t}{P_t} - n_t \frac{w_t}{P_t} h_t (\tau_t + s_t) - \tau^c_t C_t \tag{45}$$

Real debt thus depends positively on repayment expenditure of previous debt,

on government spending and on unemployment benefit and other transfer payments. On the other hand tax revenue from labour taxes or consumption taxes decrease the need to issue new debt.

Fiscal policy is assumed to obey a rule whereby the chosen fiscal variable is adjusted to changes in debt as a fraction of steady state output. On the revenue side, we consider three alternative fiscal policy instruments: the consumption tax and the labour taxes on the employer and the employee.

$$TAX_t = \overline{TAX} + \Omega_d \left(\frac{b_{t-1}}{Y_{t-1}} - \frac{\bar{b}}{\overline{Y}} \right) \tag{46}$$

where $TAX_t = \tau_t^c, \tau_t, s_t$ and Ω_d is the sensitivity of the tax instrument with respect to the change in the government debt-to-output ratio. Increases in the debt ratio lead to tax increases. Similarly, on the expenditure side, lump-sum transfers TR_t can be cut to repay the debt

$$TR_t = \overline{TR} - \Omega_d \left(\frac{b_{t-1}}{Y_{t-1}} - \frac{\bar{b}}{\overline{Y}} \right) \tag{47}$$

Government spending is characterised by the following autoregressive process

$$\log(G_t) = (1 - \rho_G) \log(\overline{G}) + \rho_G \log(G_{t-1}) + \epsilon_t^G$$

where $\rho_G \in (0, 1)$, and $\epsilon_t^G \overset{iid}{\sim} N(0, \sigma_G^2)$ is the government spending shock.

2.7 Equilibrium

For each intermediate good, supply must equal total demand. The demand for good i is, as shown previously, $y_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_t^D$, where Y_t^D is total demand for domestic intermediate goods by domestic and foreign final goods firms and the domestic government. Using the expressions for the demands for domestic intermediate good *bundles* derived previously, this can be written as

$$y_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \left\{ (1 - W) \left(\frac{P_{H,t}}{P_t} \right)^{-\varpi} C_t + W \left(\frac{P_{H,t}}{P_t^*} \right)^{-\varpi} C_t^* + G_t \right\} \tag{48}$$

Following Galí and Monacelli (2008) defining an index for aggregate domestic demand $Y_t^D = \left[\int_0^1 y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$ allows us to rewrite this as

$$Y_t^D = (1 - W) \left(\frac{P_{H,t}}{P_t} \right)^{-\varpi} C_t + W \left(\frac{P_{H,t}}{P_t^*} \right)^{-\varpi} C_t^* + G_t$$

Aggregate demand for domestic intermediate goods has to equal their aggregate supply minus the resources lost to vacancy posting, leading to the home economy's aggregate resource constraint

$$Y_t = (1 - W) \left(\frac{P_{H,t}}{P_t} \right)^{-\varpi} C_t + W \left(\frac{P_{H,t}}{P_t^*} \right)^{-\varpi} C_t^* + G_t + \kappa v_t \tag{49}$$

While the above equation states that in equilibrium domestic output has to equal its usage as consumption, exports and government spending, market-clearing in the intermediate good sector also requires

$$Y_t = n_t z h_t{}^{\alpha} \tag{50}$$

The net foreign asset position is determined by the trade balance - the difference between domestic output and domestic consumption.

$$\frac{B_t^*}{P_t} - R_{t-1}^* p\left(b_{t-1}^*\right)\frac{B_{t-1}^*}{P_t} = \frac{P_{H,t}}{P_t}Y_t - C_t - \frac{P_{H,t}}{P_t}G_t - \frac{P_{H,t}}{P_t}\kappa v_t \tag{51}$$

This relation is obtained by combining the consumers' budget constraint, the government's budget constraint and the economy's aggregate resource constraint as well as the equation for total dividends accrued to households, i.e. the sum of the real profits in the intermediate and wholesale sectors (expressed in terms of consumer prices)

$$\frac{D_t}{P_t} = \frac{P_{H,t}}{P_t}Y_t - n_t\frac{w_t^*}{P_t}h_t\left(1+s_t\right) - \frac{P_{H,t}}{P_t}\kappa v_t \tag{52}$$

3 Model evaluation

3.1 Steady state properties

The majority of papers which have augmented the New Keynesian business cycle model with search and matching frictions in the labour market do not incorporate distortionary taxation in their framework. Monacelli, Perotti and Trigari (2010) look at debt-financing and distortionary taxes as one separate extension to their RBC model. Here distortionary taxes on labour and consumption are an integral part of the analysis, including in the staggered wage bargaining framework. To understand the working of the model and as a background for the dynamic simulations, it is useful first to look at how distortionary taxes and unemployment benefits affect the steady state of the model.

Comparative statics of the tax and benefit parameters, for given values of vacancy posting costs and fixed costs of maintaining a filled vacancy, reveal that cutting wage taxes, employers' social security benefits or the unemployment benefit level all decrease significantly equilibrium unemployment and the average duration of unemployment spells ($\frac{1}{q^w}$), and increase the aggregate output of the economy, as expected in a standard MP model (see Pissarides, 2000)[16]. The mechanism for the effects of all these policy instruments is the same: tax cuts decrease the relative value of non-work to work activities (in the case of payroll taxation indirectly through an increase in the wage rate), making work relatively more attractive. Similarly, cutting the unemployment benefit level increases the relative value of work activities. The working of this

[16] Calculations are available from the author upon request.

channel depends, of course, importantly on the assumption that unemployment benefits are not taxed in the same proportion as the wage or otherwise directly indexed to the wage rate. The idea is that, as the value of workers' outside option in the wage bargain decreases, they agree to negotiate a lower wage. Lower labour costs encourage firms to post more vacancies resulting in higher employment rates. At the same time, tightness in the labour market increases, and contributes, through a higher wage, to restoring the equilibrium.

The present model also shares the equilibrium property of the standard MP model, that proportional labour taxes affect the *division* of match surplus, as outlined in the previous chapter[17]. Both the wage tax on the worker and the employer's contribution to social security reduce the worker's relative share of total match surplus, which would be just equal to his bargaining power η if these taxes were set to zero.

In the recent literature on labour markets and business cycles, summarized by Shimer (2010), the magnitude of the match surplus has been identified as an important factor contributing to explaining the unemployment volatility puzzle. The intuition is that a smaller surplus reacts more to technology shocks of equal size, and this translates into increased volatility of labour market variables. Monacelli, Perotti and Trigari (2010) find that the magnitude of the match surplus is, also in the case of government spending shocks, an important factor contributing to the size of government spending multipliers. More specifically, the higher is the relative value of non-work to work activities, the smaller is the match surplus, and the more is it affected by shocks of equal size. The steady state form of the match surplus equation[18] (the sum of the worker and firm surpluses) for the present model is

$$\begin{aligned} S &= xf(h) - (\tau + s) wh - \frac{g(h)}{\Lambda} - b \\ &\quad + \left[(1-\rho) + (1-\rho-qw) \frac{\eta(1-\tau)}{(1-\eta)(1+s)} \right] \frac{\kappa}{q^F} \end{aligned} \tag{53}$$

The value of non-work activities is described by the third and fourth terms, the disutility from supplying hours of work and the unemployment benefit. Combining the relevant terms from the surplus equation, gives the following equation for the relative value of non-work to work activities

$$\frac{\frac{g(h)}{\Lambda} + b}{xf(h) - (\tau+s) wh} = \frac{\frac{mrsh(1+\tau^c)}{(1+\phi)} + b}{\frac{xmplh}{\alpha} - (\tau+s) wh} \tag{54}$$

In this model, distortionary labour taxes decrease the value of work activities, or *increase* the *relative* value of non-work to work activities. Also the consumption tax increases the relative value of non-work to work activities because it increases the relative value of leisure compared to consumption. In addition to higher unemployment benefits or higher taxes, the relative value of non-work would also increase if labour supply along the intensive margin was more elastic (a smaller ϕ, or larger Frish elasticity).

[17] This can be seen by inspecting the steady state equations for the worker's and the firm's *share of total surplus* that are obtained by rewriting the first order condition for wage setting $H = \frac{\eta(1-\tau)}{(1-\eta)(1+s)+\eta(1-\tau)} S$ and $J = \frac{(1-\eta)(1+s)}{(1-\eta)(1+s)+\eta(1-\tau)} S$

[18] The equation for match surplus, including its dynamic form, is derived in Appendix A.5.

3.2 Transmission channels of government spending shocks

There are transmission channels of government spending in the present model that are not dependent on the labour market extension.

First, as in any model with rational, intertemporally optimizing agents, government spending shocks are transmitted to the rest of the economy through their impact on the marginal value of wealth (see e.g. Monacelli and Perotti, 2008). A temporary increase in government spending is interpreted, by intertemporally optimizing consumers, as a future rise in taxes, and consequently as a fall in their lifetime resources. This effect is captured by a rise in the marginal value of wealth, Λ_t, or equivalently a tightening of the household's budget constraint. In response to this negative wealth effect, as long as leisure and consumption are normal goods, the supply of hours worked will increase and consumption will decrease.

Another important channel of transmission is the aggregate demand effect of government spending shocks, specific to New Keynesian (NK) models. Because prices are not fully flexible, the increase in government demand is larger than the decrease in private consumption, and aggregate demand rises. The rise in aggregate demand generates a rise in labour demand. This is why, in NK models, as opposed to real business cycle (RBC) models, employment and the real wage can increase.

An additional feature differentiating the responses to a government spending shock in this model from the conventional closed-economy models, is the absence of an endogenous monetary policy response that would counteract the effect of fiscal policy. The rise in the prices of the home country would, in the presence of a central bank following the Taylor rule, be compensated more than one-for-one by an increase in the nominal interest rate, implying an increase in the real interest rate. Here the rise in government spending leads unambiguously to a terms of trade appreciation (rise in the domestic price level) and to a fall in the real interest rate, attenuating the negative response of consumption and amplifying the effects of fiscal stimulus. This is in line with e.g. Coenen et al. (2010) who, in a comparison of the effects of fiscal policy in different structural models, find that the size of the response of the economy to temporary discretionary fiscal stimulus depends importantly on the extent of monetary accommodation of the higher inflation generated by the stimulus.

There are two additional channels of transmission of government spending shocks which originate from the presence of matching frictions on the labour market.

First, the interpretation of the "wealth effect" is not straightforward in this context. Whereas the wealth effect does raise the supply of individual hours worked, as in more standard NK models, the rise in the marginal value of wealth also has implications for vacancy posting and job creation, i.e. employment adjustment along the extensive margin. More specifically, the government spending shock, a higher Λ_t, decreases the disutility from supplying hours of work, and so increases the total surplus from employment. As also the firms' share of the surplus thus increases, they increase vacancy posting. Employment increases and unemployment falls. For workers who are currently unemployed the payoff from working also increases, increasing labour supply on the extensive margin. The size of these extensive margin effects partly depends, as outlined in the previous section, on the relative value of non-work to work activities.

However, the equation for match surplus

$$S_t = x_t^H f(h_t) - (\tau_t + s_t) w_t h_t - \frac{g(h_t)}{\Lambda_t} - b + continuation\ value \tag{55}$$

reveals that the government spending shock, i.e. the increase in the marginal value of wealth, Λ_t, affects directly only one component of the relative value of non-work to work activities, namely the disutility of supplying hours of work. As will become clear in the section on the parameterization of the model, quantitatively, this disutility term is much less important than the fixed unemployment benefit term, not affected by the marginal value of wealth, implying that the transmission channel of fiscal shocks working through the marginal value of non-work activities is relatively weak in this model. In the case where the value of non-work consisted only of a fixed unemployment benefit, this transmission channel would not be present at all.

More important for the magnitude of the employment effects in this model is the New Keynesian set up of sticky prices and monopolistic competition. The increase in aggregate demand raises the future profit opportunities of firms. To exploit these opportunities, firms start to open more vacancies, contributing to job creation along the extensive margin.

Second, the presence of matching frictions makes vacancy posting a forward-looking decision, and gives rise to an additional transmission channel of fiscal policy as in Monacelli, Perotti and Trigari (2010). Namely, the rise in the shadow value of wealth generated by the initial stimulus drives up the real interest rate. This produces a fall in the discounted marginal benefit from new vacancies, discouraging vacancy posting.[19]

To sum up, the general features of this model, the New Keynesian framework and the small currency area member state set up would a priori appear to be factors favouring relatively large effects of fiscal stimulus. The labour market matching extension, in turn, adds one channel which amplifies the employment effects of fiscal shocks via the marginal value of non-work to work activities, and one channel which through an increase in the real interest rate dampens the employment effects of stimulus. To see what the net effect of these different channels of transmission is, we proceed to an analysis of the model with the help of dynamic simulations.

4 Dynamic simulations

In the following, we analyze the effects of temporary fiscal stimulus, with either flexible or rigid wages. In particular, we assess the effects of government spending shocks because they are at the centre of the debate on the effects of fiscal policy, but we also provide a comparison to fiscal stimulus in the form of a tax shock. Special emphasis is put on how the public debt resulting from a spending increase is paid back. Different debt-stabilizing fiscal policy scenarios

[19] Similarly, through the real interest rate channel, expansionary fiscal policy tends to decrease the future value of employment to workers as well as the continuation value of total surplus (which consists of expected worker and firm surpluses) but these effects are small compared to the contemporaneous increases in these surpluses through other transmission channels.

are assessed, to see whether labour market frictions help us to detect different implications of different fiscal policy instruments. Tax instruments are assessed separately in order to identify the mechanisms at work with each instrument - instead of a more realistic scenario where fiscal policy would consist of a combination of instruments.

In the following simulations, the positive government spending shock generates public debt which is gradually paid back following alternative fiscal feedback rules written on lump-sum taxes, labour taxes or consumption taxes. As a baseline scenario, we analyze the case of lump-sum tax funding and flexible wages. Then, to reveal the specific properties of the present model, two other tax instruments are considered: the labour tax on employees and the consumption tax. The effects of wage rigidity and the relative importance of some other parameters are assessed separately.

The government spending shock and the tax shock are normalized so that they correspond to a 1 percent increase in steady state output. All responses are expressed in percentage deviations from respective steady state values if not indicated otherwise. The quantitative implications of the theoretical model are compared, where meaningful, to Monacelli, Perotti and Trigari's (2010) empirical results and model predictions as they are the key reference in assessing fiscal policy in the presence of labour market frictions. Reference will also be made to Coenen et al.'s (2010) comparison of fiscal stimulus in different structural models. One important difference, however, with the present analysis and that of Coenen et al. (2010) is that the present model does not include hand-to-mouth consumers, a modelling extension which is known to result in more benign responses of private consumption to fiscal shocks and thus to larger fiscal multipliers.

It is also important to note that a variety of measures for fiscal multipliers have been used in the literature, rendering comparison between models more difficult. In the following, we will use the same measure for the output multiplier as Coenen et al. (2010), namely the percentage deviation of real GDP from baseline GDP as a result of the fiscal shock. The unemployment multiplier, in turn, is reported, similarly to Monacelli, Perotti and Trigari (2010) as the change in unemployment in percentage points at the peak.

4.1 Parameterization and steady state of the model

The parameter values are chosen mostly on the basis of existing literature, and are summarized in table 1. For preferences and the labour market part, they follow mainly Christoffel et al. (2009) who mostly use quarterly data from 1984 to 2006 for the euro area, and for the open economy, parameter values are as in Corsetti, Meier and Müller (2009). Fiscal policy parameters are taken from the data of the small euro area economy of Finland.

The quarterly discount factor is $\beta = 0.992$ which corresponds to an annual interest rate of $3,3\%$. The labour supply, or Frish elasticity $(\frac{1}{\phi})$, is set to 0.1. This is in the lower range of values implied by most microeconomic studies, which estimate this elasticity to be between 0 and 0.5. Much higher elasticities have been generally used in the business cycle literature because macro elasticities also account for the variation in the employment rate[20]. The quarterly

[20] See e.g. Fiorito, R. - Zanella, G. (2008) for a recent comparison of micro and macro

separation rate is calibrated at $\rho = 0.06$. The labour elasticity of production parameter is set to $\alpha = 0.99$ which implies nearly constant returns to scale in the intermediate goods production sector, and a labour share of 75 percent.

The unemployment benefit parameter is calibrated at $b = 0.41$, and generates a replacement rate of 65 percent, defined as the ratio of net unemployment benefits to average net (after-tax) income from work $\frac{b}{wh(1-\tau)}$. This corresponds to the average replacement ratio for the euro area used in Christoffel et al. (2009), and is only slightly lower than e.g. the OECD's "Benefits and Wages, 2007" publication suggests for Finland. There, the average net replacement rate over 60 months of unemployment for Finland is 70 percent, averaging over four different family types. The unemployment benefit is not assumed to be proportional to the wage nor to be indexed to inflation. As Christoffel, Kuester and Linzert (2009) note, in labour market matching models, there is a trade-off between obtaining a reasonable labour share and a plausible replacement rate. In the present model, the wage bill is 75 percent, clearly too high compared with the data. On the other hand, this model abstracts from the use of capital as a factor of production, so we deem it more important to get the replacement rate right.

As discussed in the previous section, it is important to note that when the worker is not employed, in addition to getting the unemployment benefit, he also enjoys the increased time for leisure. As a result, the relative value of non-work to work activities consists, not only of the fixed unemployment benefit term, but also of an additional term that varies in function of hours of work. The calibration of the value of non-work to work activities term is known, in the labour market matching literature, to be of key importance for fitting the model to the data when exploring the effects of technology shocks (see Shimer, 2010). More specifically, a sufficiently high relative value of non-work to work, helps the model to generate large variations in vacancies and unemployment in response to technology shocks and consistent with business cycle facts, as shown by Hagedorn and Manowski (2008). The latter calibrated this value to 0.95 whereas Shimer (2005) set it at 0.4 interpreting it as only unemployment benefits.

In the present model, the steady state value of non-work to work activities, as defined in equation (55), is 0.74, in the mid-range of the values found in the literature. Monacelli, Perotti and Trigari (2010) find that the value of this parameter has to be calibrated in the high range of plausible values (at 0.9) in order to roughly match the size of the unemployment fiscal multiplier. And even in that case, the output multiplier remains well below the estimated one. We will provide some sensitivity analysis with respect to this important value in the end of this chapter.

The wholesale sector is calibrated in line with the literature so that the markup is at a conventional value of $\mu = \frac{\varepsilon}{\varepsilon-1} = 1.1$. The Calvo parameter is $\xi = 0.75$ on the basis of Christoffel, Kuester and Linzert's (2009) calibration from the Eurosystem Inflation Persistence Network. The average duration of prices is accordingly 4 quarters. As to wages, they are assumed to be renegotiated every one and a half years, implying a probability of non-renegotiation of $\gamma = 0.83$.

elasticities of labour supply. They estimate an individual elasticity of about 0.1 and an aggregate elasticity of about 1.

Table 1. Parameter values

Parameter	$Value$	Explanation
Preferences		
β	.992	Time-discount factor
ϕ	10	Labour supply (Frish) elasticity of 0.1
ϱ	1.5	Risk aversion
$\varkappa$	0.6	External habit persistence
Labour market		
α	0.99	Labour elasticity of production
σ	0.6	Elasticity of matches w.r.t. unemployment
σ_m	0.64	Efficiency of matching
ρ	0.06	Exogenous quarterly job destruction rate
η	0.6	Bargaining power of workers
b	0.41	Unemployment benefits
z	1.10	Technology, targets output $Y = 1$
γ	0.83	Pr(no reneg.) avg duration of w contracts 6 qrts
Wholesale sector		
ε	11	Elasticity of substitution, markup 10 percent
ξ	0.75	Calvo stickiness of prices, avg duration 4 qrts
$\nu \left(= \frac{(1-\xi)(1-\beta\xi)}{\xi}\right)$	0.085	Coefficient of mc in NK Phillips curve
Final goods sector		
$(1 - W)$	0.75	Home bias in final goods production
ϖ	0.66	Trade price elasticity
γ_{b^*}	0.005	Debt-elasticity of interest rates

The steady state values of key model variables implied by the current parameterization can be found in Table 2. The steady state equations of the model are, in turn, provided in Appendix A.1. In the steady state, output is normalized to one, so that GDP components can be interpreted directly as percent shares of GDP. The labour force is also normalised to one, and the steady state unemployment level is 9 percent. A symmetric open economy steady state is assumed where consumption levels are initially the same at home and abroad, and both the trade balance and net foreign asset holdings are zero. As no capital is included in the model, the output components of private consumption and government consumption (and the tiny amount of resources lost to vacancy posting) are scaled so that private consumption accounts for 71 percent of steady state output and government consumption is 29 percent.

The steady state tax rates for labour and consumption are computed as ten year historical averages of corresponding tax rates in Finland times the model-implied tax base for each tax category. Accordingly, labour taxes for the employee and the employer respectively amount to 30 percent and 25 percent times the wage bill and the consumption tax rate corresponds to an average of 19 percent times the size of private consumption. The government's steady state debt to GDP ratio is set at 45 percent, close to the value for the so-called EMU debt for Finland in 2010.

Table 2. Steady state values

Variable	Value	Description
Y	1	Output
C	0.70	Consumption
u	0.09	Unemployment rate
κv	0.01	Total vacancy costs
n	0.91	Employment
qw	0.6	Probability of finding a job
qf	0.7	Probability of finding a worker
$b/(wh(1-\tau))$	0.65	Net replacement rate
nwh	0.75	Wage bill
Fiscal policy		
τ^C	0.13	Consumption tax
τ	0.23	Labour tax rate on employee
s	0.19	Employers' social security contribution
TR / τ^{LS}	0.075	Lump-sum transfers
d/Y	0.45	Government debt to GDP ratio
G	0.29	Government spending
ρ_G	0.8	Autocorrelation of government spending
ϵ_t^G	0.0345	Gvmt spending shock, 1 pct of steady state output

4.2 Baseline results

The baseline response to a positive government spending (solid line in Figure 1) is similar to results obtained from standard New Keynesian models with no matching frictions (see e.g. Linnemann and Schabert, 2003). The rise in government demand has a positive effect on *real* output. The multiplier is 0.3 percent at the peak and rises to 0.8 and 1.3 percent at one and two year horizons respectively, i.e. the effect on impact is small but the multipliers at one and two year horizon get fairly close to Monacelli, Perotti and Trigari's (MPT, 2010) empirical estimates of 1.2 and 1.5 respectively. The multipliers are much larger at all horizons than those implied by the MPT model for the same relative value of non-work to work activities. The effect on private consumption is negative but small. The negative wealth effect, caused by a perceived fall in lifetime income, produces the initial drop in private consumption and an increase of hours worked per person.

The increase in aggregate demand, stemming from price rigidities, raises the expected returns of firms from a filled vacancy. Due to the timing assumption of the matching process, vacancies increase on impact but employment only starts to rise (unemployment starts to fall) from the next period on, as new matches become productive[21]. The combined increase in both labour demand and labour supply drives up the negotiated wage. Also the real wage rises contemporaneously, in line with recent findings by e.g. Pappa (2009).

[21] The timing assumption is the same as in the standard Mortensen-Pissarides model. All labour adjustment in the first period after the shock is through the intensive margin, hours worked per person, which may cause them to overreact compared to what is observed in business cycle data. Blanchard and Galí (2006) introduce contemporaneous hiring into a business cycle matching model, whereby new matches become immediately productive, shifting labour adjustment to the extensive margin, the number of workers.

The unemployment rate multiplier is about −0.2 *percentage points* at peak[22], much smaller than MPT's empirical estimate of −0.6 but larger than the unemployment multiplier implied by their model for the same relative value of non-work to work activities. The peak response of total employment is 1.3 percent, close to MPT's empirical result of 1.5 percent. After one year, total hours are about 2.5 percent higher than in the steady state, of which the contribution of the extensive margin of adjustment is 0.5 percent and hours worked per person account for the rest of the adjustment. While the magnitude of the response of total hours is close to MPT's empirical results, the relative role of the extensive and intensive margins of labour adjustment is clearly at odds with business cycle facts The strong response of hours worked may indicate the presence of an 'hours volatility puzzle' in matching models of the business cycle, as pointed out by Krause and Lubik (2010).

While the wealth effect raises the supply of individual hours worked (intensive margin) in the same way as in standard NK models, in this framework the tightening of the consumer budget constraint also affects vacancy creation. In particular, the increase in total surplus due to the increase in the marginal value of wealth encourages firms to open more vacancies. As both wages and the labour input (along both the intensive and the extensive margin) increase, the initial negative response of consumption is reversed.

The real interest rate channel works to decrease vacancy posting, but in this framework it is not significant enough to overturn the positive response of vacancies to the government spending shock stemming from the increase in aggregate demand. Also, because of the assumption of no counteracting monetary policy, the real interest rate falls as a result of faster domestic inflation.

The effect of increased government spending on the trade balance and on the terms-of-trade appear similar to what e.g. Kim and Roubini (2004) or Müller (2008) find. An increase in government spending appreciates the terms of trade and increases net exports. The terms of trade appreciation is natural in the presence of full home bias in government consumption: the export price index - which in this framework is just the domestic price index (because of producer pricing) - rises relative to the foreign price index which is not affected by fiscal stimulus in the small member state.

As to the trade balance, there are two counteracting forces. On the one hand, the trade balance improves because the value of trade increases, but on the other hand higher prices of home-produced goods have a negative effect on the trade balance through the substitution channel. Here the former effect dominates. The latter effect tends to be larger the higher the home bias in private consumption and the higher the intratemporal elasticity of substitution between the home and foreign good.

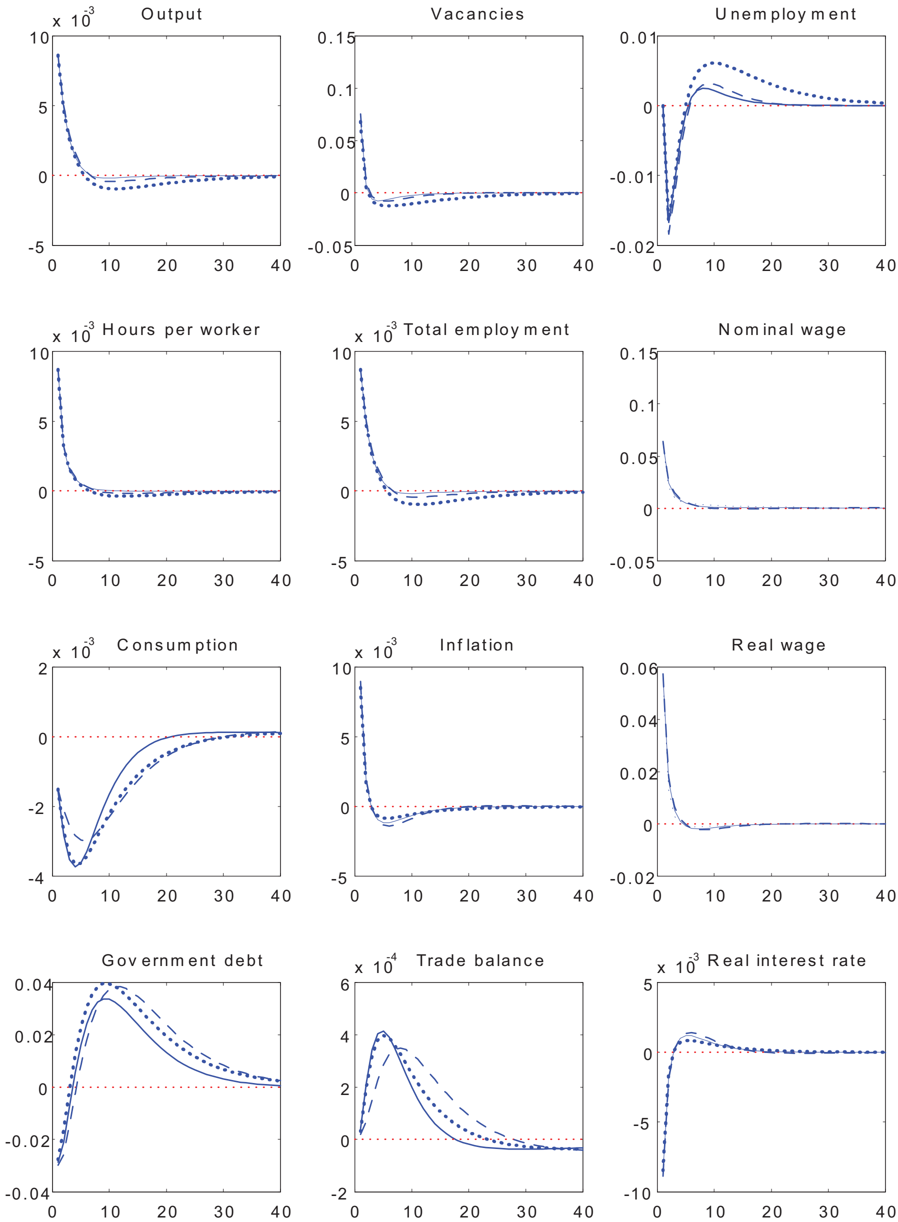
4.2.1 Alternative fiscal policy scenarios

In figure 1, two additional debt-stabilizing scenarios are presented: one where the labour tax on the employee is increased in order to finance the initial increase in government spending, and another where repayment is done through increases in the consumption tax. The coefficient for the sensitivity of the tax

[22] Thus implying a fall in the unemployment rate from the steady state 9 percent to 8.8 percent.

instrument with respect to debt, Ω_d, is set so that the initial tightening effect of the fiscal rule is equal across different tax instruments.

Figure 1. The dynamic effects of a government spending shock: baseline vs. alternative debt-stabilizing fiscal rules. Note: baseline (rigid line), labour tax rule (dotted line), and consumption tax rule (dashed line)



The results show that shifting the debt-stabilizing burden towards the distortionary labour tax (dotted line) significantly changes the response of the economy to the positive fiscal policy shock. Most importantly, after the initial identical shock, as soon as the labour tax rule becomes operative, the higher proportional tax rate lowers the total surplus from employment and discourages vacancy posting. It also feeds through to higher wages. In the current wage bargaining framework, the gradual increases in the labour tax rate imply an increase in the target wage of negotiators (as can be verified from the dynamic target wage equation). As a result, the bargained nominal wage stays above its steady state level for a prolonged period of time to compensate workers for the otherwise falling net income. The higher wage directly implies higher labour costs which further decreases the incentives of firms to open vacancies and unemployment starts rising. The corresponding fall in employment makes the private consumption response more negative than when public debt is adjusted through lump-sum taxes, despite the higher wage.

When consumption taxes, in turn, are used to stabilize debt after an increase in government spending, the negative labour market reactions are smaller. Output and vacancies rise as much as with the lump-sum tax rule, but government debt returns more quickly to its steady state level, also due to the larger tax base. Even the response of private consumption is less negative than in the case of the labour tax rule although consumption is directly taxed. The reason, interestingly, lies in the structure of the labour market. While both distortionary taxes decrease the surplus from employment, the effect of the labour tax is larger. More specifically, consumption taxes only affect one component of the relative value of non-work activities, the - quantitatively less significant - disutility from work. So while this tax does somewhat increase the relative value of non-work to work activities via the substitution effect between consumption and leisure, the effect of labour taxes is larger because they also increase the net replacement rate of unemployment benefits.

The detrimental effects of financing fiscal stimulus with distortionary labour taxes is especially apparent in long-run fiscal policy multipliers[23]. The long-run real output multiplier implied by the baseline scenario is 1.5, whereas if debt is paid back by raising consumption taxes the multiplier shrinks to 1.3. If labour taxes are raised to finance the initial stimulus the long-run output multiplier is zero, because after five quarters of positive output effects, real output remains slightly below its steady state level due to unfavourable labour market conditions. Similarly, the long-run consumption multipliers for the different scenarios are -3.0, -4.0 and -3.8 percent (for lump-sum tax, labour tax and consumption tax funding respectively), and total hours multipliers at the same horizon are 1.6, 0.2 and 1.3 percent.

The results show the importance of how the increased public spending is financed. Due to the detailed description of the labour market, we are able to track down the transmission channels of fiscal policy that shape the responses of the economy in the different debt-stabilizing scenarios. These interactions have not been captured by existing New Keynesian models.

[23]Here, long-run multipliers are calculated as the cumulative effect over ten years.

We also investigated a similar government spending shock using the labour tax on the employer as the stabilizing instrument. The results are very similar than when using the fiscal rule on the employee's labour tax. The only significant difference is that the negotiated wage does not rise in the same way as when the incidence of increased labour taxation is on the worker (indeed, the dynamic equation for the target wage shows that an increase in the employer's social security contribution has a negative direct effect on the target wage), leaving the worker's net income and the firm's labour cost approximately the same across these two scenarios. As labour costs are, however, raised directly by the tax on employers, the labour market outcome with employer contributions as the debt-stabilizing tool is similar with falling employment and rising unemployment[24].

Automatic stabilizers are at work in the present setup. The initial expansion of output and the accompanying improvement in employment after a government spending shock increase the government's labour tax revenues and decreases expenditure on unemployment benefits. However, consumption tax revenue falls as private consumption decreases and government debt increases significantly and persistently. Indeed, debt-stabilizing fiscal rules are needed to help bringing debt back to its steady state level within a reasonable time frame.

4.2.2 Wage rigidity

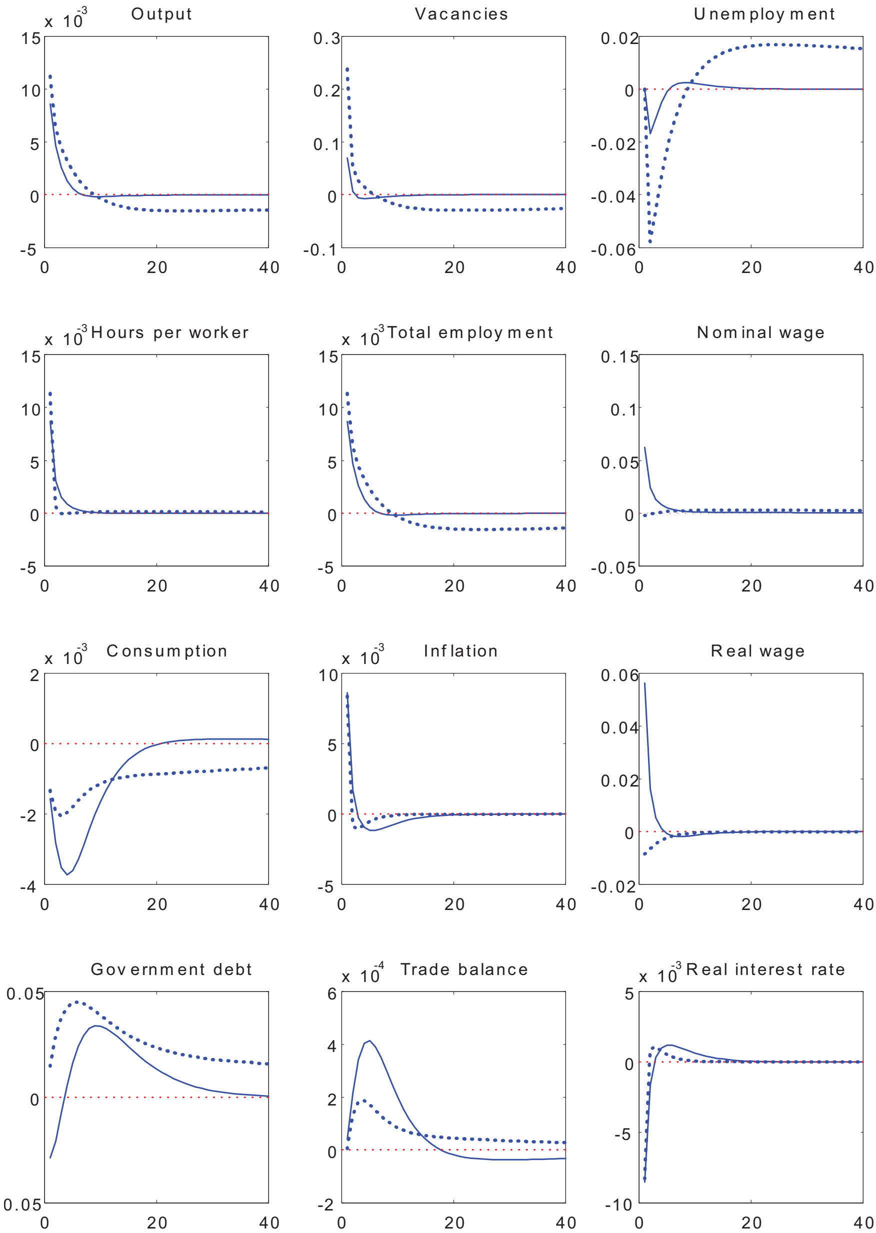
Figure 2 shows the results for the baseline model when wages are negotiated, instead of period-by-period, on average once every sixth quarter.

Making the wage more rigid increases the magnitude of the responses of labour market variables. Vacancies now react more strongly to the initial stimulus, since firms' expected profits are larger when their labour costs do not rise. Also the fall in unemployment in response to the shock is larger. While the unemployment rate multiplier at peak was about -0.2 *percentage points* in all the previous scenarios, it now increases to -0.5 *percentage points.* Total hours also increase slightly more on impact, 1.1 percent compared to 0.9 percent when wages are flexible. The difference is small, since while the number of workers increases much more, the response of hours worked per person is attenuated when the negotiated wage can not adjust upward.

The stronger response of labour market variables is in line with the literature on labour markets and business cycles, which has stressed that wage rigidity affects the cyclicality of labour market variables because it influences firms' expected gains from the match. Compared to flexible wages, when wages are rigid, firms' expected profits rise more in upturns and fall more in downturns. The more favourable labour market reaction, in the short-term, to fiscal stimulus contributes to consumption falling less than in the baseline.

[24]The simulations are available from the author on request.

Figure 2. The dynamic effects of a government spending shock: flexible wages (rigid line) vs rigid wages $\gamma = 0.83$ (dotted line).



This could lead to conclude that in an environment characterised by rigid wages, fiscal stimulus would be especially effective. However, in the longer term, the picture is very different. As the wage gradually adjusts upward, vacancies and employment start to fall and unemployment rises, as shown by the right tails of the corresponding impulse response functions. Output and private consumption remain lower than their steady state levels for a prolonged period of time. As a result, compared to the flexible wage case, the long-run output multiplier is now -1.5 percent instead of 1.5 percent, and the long-run multiplier for total hours changes from 1.6 to -1.2 percent. The unfavourable labour market outcome is especially apparent in unemployment figures. After two years, unemployment starts rising and remains over steady state levels for a prolonged period of time. Poor labour market performance, including increasing unemployment benefit payments, is also reflected in large and persistent government debt under rigid wages.

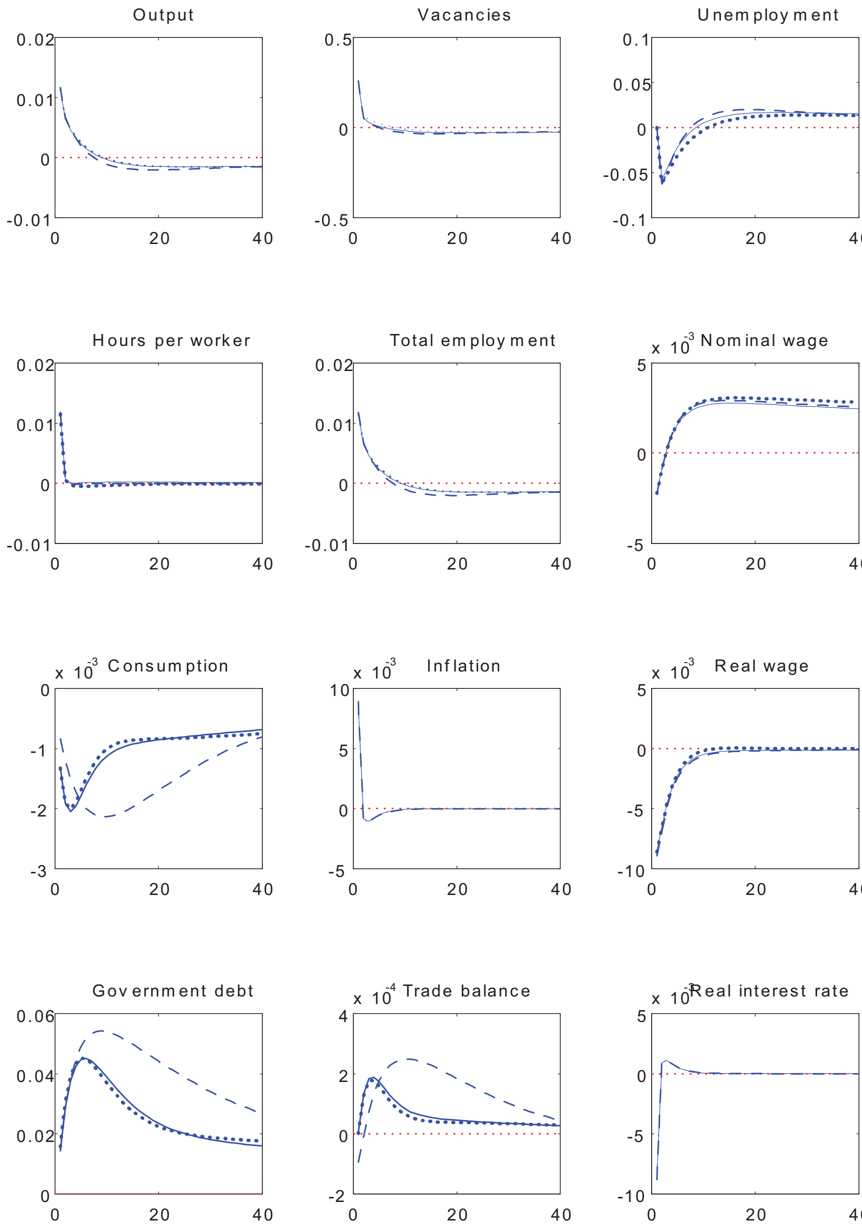
The result that wage rigidity amplifies the fiscal policy shock (in the short term) is in contrast to Monacelli, Perotti and Trigari (2010) who find that (real) wage rigidity[25] *dampens* the effect of government spending shocks on hiring. The shock increases the total surplus from the match by raising the firm's reservation wage, but also by lowering the worker's reservation wage. Of these two counteracting effects, the latter dominates in MPT's model, lowering the Nash bargained wage, and raising hiring and employment. The introduction of wage rigidity prevents the wage from falling thereby decreasing hiring incentives. In the present model, wage rigidity is combined with price rigidity. Price rigidity is needed to generate a rise in the negotiated wage in response to increased government spending (the combined effect of increased labour demand and labour supply). When wages are rigid, the profit opportunities of firms are even larger. Their share of the surplus increases and generates a positive net effect on vacancy posting. Wage rigidity thus amplifies the labour demand effect typical to the New Keynesian model.

4.2.3 Alternative pay-back rules in the presence of wage rigidity

Introducing rigidity in wage determination alters the conclusions on the relative favourability of different debt-funding instruments. Especially, it appears that if wages are rigid enough, stabilizing debt with the help of increases in the labour tax actually becomes less detrimental for the economy than raising consumption taxes. This is because the negative effect of raising labour taxes on the total surplus from employment is in direct proportion to the wage, as can be seen from equation (56). The total surplus thus diminishes if the labour tax rate is raised, but also if the relevant tax base, in this case the wage bill, gets larger. When wages are renegotiated period-by-period, the increase in labour taxes feeds through to higher wages, the wage bill increases and amplifies the negative effect of these taxes on the total surplus from employment. Therefore vacancy posting and job creation decrease. With wage rigidity, the overall effect of a similar rise in labour taxes remains smaller because the increase in the wage bill is attenuated. The negative effect on vacancy creation brought about by a smaller firm surplus from filling vacancies is now also contained by the fact that simultaneously firms' labour costs do not rise.

[25] Introduced as a simple wage adjustment rule, instead of as the result of staggered bargaining.

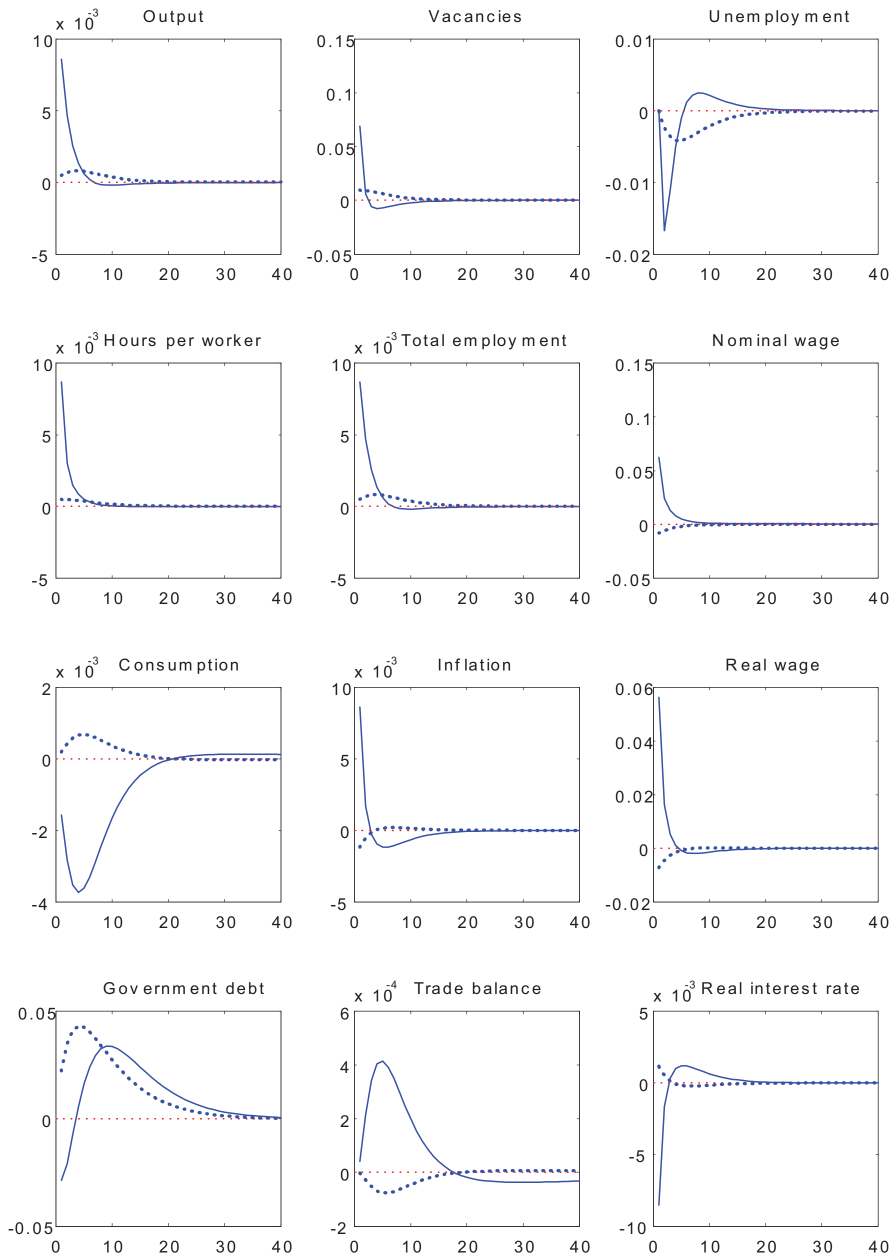
Figure 3. The dynamic effects of a government spending shock: baseline vs. alternative debt-stabilizing fiscal rules when wages are rigid, i.e. $\gamma = 0.83$. Note: baseline (rigid line), labour tax rule (dotted line), and consumption tax rule (dashed line)



4.2.4 A decrease in labour income taxes

We find, in line with Coenen et al. (2010) that multipliers from government spending stimulus are larger than the multipliers from labour income taxes (see Figure 4.). Government spending is directly translated as an increase in aggregate domestic demand whereas tax cuts operate through their effects on agents' working and spending behaviour.

Figure 4. The dynamic effects of a tax shock. Decrease in the workers' labour taxes corresponding to 1 percent of steady state output



The multiplier on real output is 0.2 percent on impact and raises to 0.5 and 0.7 percent at one and two year horizons. The unemployment multiplier is just 0.05 percentage points at the peak, but importantly, the temporary decrease in labour income taxes lowers unemployment in the long term more than any of the government spending stimulus alternatives. Also because there are now no crowding-out effects, private consumption increases (though very modestly) in response to the increase in net after-tax incomes.

4.2.5 Relative importance of other parameters

The above simulations and comparisons support the finding from earlier literature that the degree of price rigidity is a crucial parameter in shaping the economy's response to fiscal policy shocks. In figure 5, the baseline model is simulated for two different degrees of price rigidity. With more flexible prices, the effects of fiscal stimulus are significantly dampened. The *nominal* output impact multiplier shrinks from 0.9 to approximately 0.3 and unemployment is unaffected or even slightly rises. The response of vacancies is more or less flat and private consumption reacts more negatively. This illustrates the earlier point that the New Keynesian nature of the present model is an important determinant for the magnitude of fiscal multipliers.

It is known from earlier contributions (see e.g. Linnemann and Schabert, 2003) that the real interest rate is a crucial variable for the adjustment to fiscal shocks because it determines the consumption path and, consequently, the magnitude of the aggregate demand effect. As shown in the dynamic simulations of the model, the small monetary union member state framework ensures that domestic prices rise but the nominal interest rate does not react to the speeding up of inflation. As a result, the real interest rate falls and attenuates the fall in private consumption. The model is closed by the debt-elastic interest rate assumption, but the calibration of the sensitivity parameter of the interest rate to the increase in foreign indebtedness implies that this is a purely technical assumption. Assuming a sufficiently more aggressive elasticity parameter would eventually reverse the response of the real interest rate to the spending shock.

Monacelli, Perotti and Trigari (2010) find that the relative value of non-work to work activities is an important parameter in determining the size of fiscal multipliers. In the present framework, its role is not that crucial. This is because the government spending shock, i.e. the increase in the marginal value of wealth only affects one component of this value, the disutility of work. Therefore, if the relative value of non-work to work activities is increased to 0.9, the same value as in MPT, by increasing the fixed unemployment benefit, the match surplus is smaller but so is the relative role of the shock. Figure 6 shows that this change does affect the responses of labour market variables as expected. The smaller match surplus amplifies the responses of vacancies and unemployment to the shock, but the effect is rather short-lived. The difference in total hours remains tiny because hours of work, that respond as previously to the shock, account for the bulk of the employment adjustment.

If instead the relative value of non-work to work activities is increased to the same value (0.9) by increasing the elasticity of labour supply (lowering ϕ to 2.6 from 10), this does hardly change the response of employment (vacancies and unemployment) along the extensive margin of adjustment. But as firms can now more easily increase their production by increasing hours of work the output multiplier becomes larger, total hours in the economy increase and consequently the negative effect on private consumption remains smaller.

While labour taxes were found to have significant equilibrium effects, lowering them by 3 percentage points hardly affects the dynamics of the model after a government spending shock.

5 Concluding remarks

This paper contributes to the ongoing debate about the effects of fiscal policy by analyzing government spending shocks under alternative fiscal rules and rigid labour markets. For this purpose, we have introduced fiscal policy and labour market matching frictions into an open-economy New Keynesian DSGE. The link between fiscal policy and the labour market was introduced with the help of distortionary labour taxes which directly influence the behaviour of firms and workers on the matching market. The framework was adapted to the small currency union member country case, and additional rigidity in wage determination was introduced with the help of Gertler and Trigari's (2009) staggered bargaining framework.

We find that the effects of fiscal shocks, in the model with labour market frictions and lump-sum taxes, are similar to those obtained from standard New Keynesian models. Fiscal stimulus has an expansionary effect on output, and a small but negative effect on private consumption. The detailed description of the labour market, however, helps to better understand the transmission mechanisms of fiscal policy to private consumption, employment and the real wage. The negative response of private consumption is driven by the negative wealth effect but counteracted by a positive employment response, brought about by increasing real wages and increasing labour supply along both the intensive and extensive margin.

The results show that the assumption of the offsetting fiscal measure is critical for the effects of fiscal stimulus. In particular, shifting the debt-stabilizing burden towards distortionary labour taxes is detrimental for employment and general economic performance. Most importantly, the higher proportional tax rate lowers the total surplus from employment and discourages vacancy posting. It also feeds through to higher bargained wages to compensate workers for the otherwise falling net income. The higher wage directly implies higher labour costs to firms which further decrease open vacancies and unemployment starts rising. The fall in employment implies a stronger contraction in private consumption compared with the more standard case of lump-sum tax adjustment.

The negative labour market reactions are smaller when fiscal stimulus is withdrawn by means of increased consumption taxes. This is because they lower less the total surplus from employment than labour taxes by only affecting the consumption-leisure choice. labour taxes also increase the net replacement rate of unemployment benefits making non-work more attractive.

Wage rigidity was found to significantly increase the magnitude of the responses of labour market variables in the short term. Vacancies react more strongly to the initial stimulus, since firms' expected profits are larger when their labour costs do not rise. This is in line with the literature on labour markets and business cycles, which suggest that rigid wages are needed to generate larger fluctuations in labour market variables. Our results indicate also that while wage rigidity would seem to make fiscal policy more effective in the short term, in the longer term, the gradual increase in the bargained wage causes a prolonged increase of unemployment to above the steady state level. Public debt stays higher and the negative effect of private consumption is larger than when wages are flexible. Furthermore, wage rigidity alters the relative favourability of different debt-funding tax instruments. If wages are rigid enough, increases in the labour tax become less detrimental for the economy than increases of consumption taxes. This is because the overall effect of raising labour taxes has a smaller negative effect on the total surplus from employment when the tax base, i.e. the wage bill, does not increase.

While the analysis conducted highlighted important transmission channels of fiscal policy not captured by standard New Keynesian models, the more precise quantitative effects of fiscal policy on the labour market remain to be explored. The comparison of the theoretical predictions of this framework with estimates of fiscal policy effects in the benchmark small currency union member state is still work in progress. A number of potentially relevant other transmission channels have not been included in this analysis. Recent literature suggests for example that the economy should be modelled as "non-Ricardian" to account for important transmission channels of fiscal policy. A move in that direction could be the inclusion of rule-of-thumb consumers that has been found to be important for the effects of fiscal policy (see e.g. Galí, Lopez-Salido and Valles, 2007). This is left for future work.

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A Appendix

A.1 Steady state of the model economy

Euler equation

$$\beta = \frac{1}{R}$$

Marginal utility of consumption

$$\lambda = (C - \varkappa C)^{-\varrho}$$

Marginal utilility of wealth

$$\Lambda = \frac{\lambda}{(1 + \tau^c)}$$

Interest rate on foreign bonds

$$R^* = R$$

FOC of retail firm

$$x = \frac{1}{\mu} = \frac{\varepsilon - 1}{\varepsilon}$$

Matches

$$m = \sigma_m u^\sigma v^{1-\sigma}$$

Employment

$$\rho n = m$$

Unemployment

$$u = 1 - n$$

Probability of finding a worker

$$q^F = \frac{m}{v}$$

Probability of finding a job

$$q^W = \frac{m}{u}$$

Labour market tightness

$$\theta = \frac{v}{u}$$

FOC for hours

$$(1-\tau)\,x^{H} mpl = (1+s)\,mrs(1+\tau^{c})$$

where

$$mpl = \alpha z h^{\alpha-1} \text{ and } mrs = \frac{\delta h^{\phi}}{\lambda} \text{ and } x^{H} = x$$

Economy-wide resource constraint

$$\begin{aligned} Y &= (1-W)\,C + WC^{*} + G + \kappa v \\ &= C + G + \kappa v, \text{ in the symmetric steady state} \end{aligned}$$

Government budget constraint

$$(1-R)\,B = G + bu + TR - nwh(\tau+s) - \tau^{c}C$$

Market clearing / aggregate output

$$Y = nzh^{\alpha}$$

Wage

$$w = \frac{\eta}{(1+s)}\left[\frac{xmpl}{\alpha} + \frac{\kappa\theta}{h}\right] + \frac{(1-\eta)}{(1-\tau)}\left[\frac{mrs(1+\tau^{c})}{(1+\phi)} + \frac{b}{h}\right]$$

Job creation condition

$$\kappa = q^{F}\beta J$$

where the firm surplus

$$J = \frac{1}{1-\beta\,(1-\rho)}\left[xzh^{\alpha} - wh\,(1+s)\right]$$

Worker surplus

$$H = \frac{1}{1-\beta\,(1-\rho-q^{W})}\left[wh\,(1-\tau) - \frac{mrsh(1+\tau^{c})}{(1+\phi)} - b\right]$$

Worker discount factor

$$\overline{\Delta} = \frac{\overline{h}\,(1-\overline{\tau})}{1-\overline{\beta}\,(1-\rho)\,\gamma}$$

Firm discount factor

$$\overline{\Sigma} = \frac{\overline{h}\,(1+s)}{1-\overline{\beta}\,(1-\rho)\,\gamma}$$

A.2 Model dynamics

The dynamics of the model are obtained by taking a log-linear approximation around a deterministic steady state.

Euler equation

$$\widehat{\Lambda}_t = E_t\left(\widehat{\Lambda}_{t+1} + \widehat{R}_t - \widehat{\pi}_{t+1}\right)$$

Shadow value of wealth

$$\widehat{\Lambda}_t = \widehat{\lambda}_t - \frac{\overline{\tau}^c}{(1+\overline{\tau}^c)}\widehat{\tau}^c_t$$

Marginal utility of consumption

$$\widehat{\lambda}_t = -\frac{\varrho}{(1-\varkappa)}\left(\widehat{C}_t - \varkappa\widehat{C}_{t-1}\right)$$

Interest rates

$$\widehat{R}_t = \widehat{R}^*_t - \gamma_{b^*}\widehat{b}^*_t$$

Matching function

$$\hat{m}_t = \sigma\hat{u}_t + (1-\sigma)\,\hat{v}_t$$

Employment dynamics

$$\hat{n}_t = (1-\rho)\,\hat{n}_{t-1} + \frac{\bar{m}}{\bar{n}}\hat{m}_{t-1}$$

Unemployment

$$\widehat{u}_t = -\frac{1-\overline{u}}{\overline{u}}\hat{n}_t$$

Transition probabilities

$$\hat{q}^F_t = \hat{m}_t - \hat{v}_t$$

$$\hat{q}^W_t = \hat{m}_t - \hat{u}_t$$

Labour market tightness

$$\widehat{\theta}_t = \hat{v}_t - \hat{u}_t$$

FOC for hours worked

$$\Longleftrightarrow \hat{x}_t^H = m\hat{r}s_t - m\hat{p}l_t + \frac{\overline{\tau}}{(1-\overline{\tau})}\widehat{\tau}_t + \frac{\overline{s}}{(1+\overline{s})}\widehat{s}_t + \frac{\overline{\tau}^c}{(1+\overline{\tau}^c)}\widehat{\tau}_t^c$$

where

$$\hat{x}_t^H = \widehat{x}_t + \widehat{P}_{H,t} - \widehat{P}_t$$

$$m\hat{p}l_t = \hat{z}_t - (1-\alpha)\,\hat{h}_t$$

and

$$m\hat{r}s_t = \phi\hat{h}_t - \hat{\lambda}_t$$

New Keynesian Phillips Curve

$$\widehat{\pi}_{H,t} = \nu\widehat{x}_t + \beta E_t\widehat{\pi}_{H,t+1}$$

where $\widehat{\pi}_{H,t} = \widehat{P}_{H,t} - \widehat{P}_{H,t-1}$ is domestic inflation

First order condition for wage setting

$$\widehat{J}_t(w_t^*) + \widehat{\Delta}_t = \widehat{H}_t(w_t^*) + \widehat{\Sigma}_t$$

Firm surplus

$$\begin{aligned}\widehat{J}_t(w_t^*) \quad = \quad & \frac{\overline{x}\overline{mplh}}{\alpha\overline{J}}\left(\widehat{x}_t^H + \widehat{mpl}_t + \widehat{h}_t\right) - \frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}\left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t\right) \\ & -\frac{\overline{w}\overline{h}\overline{s}}{\overline{J}}\widehat{s}_t + \overline{\beta}\,(1-\rho)\,E_t\left(\widehat{J}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) \\ & -\frac{\overline{\beta}\,(1-\rho)\,\gamma}{1-\overline{\beta}\,(1-\rho)\,\gamma}\frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}E_t\left(\widehat{w}_t^* - \widehat{w}_{t+1}^*\right)\end{aligned}$$

Worker discount factor

$$\widehat{\Delta}_t = (1-\iota)\,\widehat{h}_t - \frac{(1-\iota)\,\overline{\tau}}{(1-\overline{\tau})}\widehat{\tau}_t + \iota E_t\left(\widehat{\beta}_{t,t+1} + \widehat{\Delta}_{t+1}\right)$$

Worker surplus

$$\begin{aligned}\widehat{H}_t(w_t^*) &= \frac{\overline{wh}\,(1-\overline{\tau})}{\overline{H}}\left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t\right) - \frac{\overline{w}\overline{h}\overline{\tau}}{\overline{H}}\widehat{\tau}_t \\ &-\frac{1}{1+\phi}\frac{\overline{mrs}\overline{h}\,(1+\overline{\tau}^c)}{\overline{H}}\left(\widehat{mrs}_t + \widehat{h}_t\right) \\ &-\frac{\overline{mrs}\overline{h}\overline{\tau}^c}{(1+\phi)\,\overline{H}}\widehat{\tau}_t^c - \overline{\beta}\overline{q}^W E_t\left(\hat{q}_t^W + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1}\right) \\ &+\overline{\beta}\,(1-\rho)\,E_t\left(\widehat{H}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) \\ &+\frac{\overline{\beta}\,(1-\rho)\,\gamma}{1-\overline{\beta}\,(1-\rho)\,\gamma}\frac{\overline{wh}\,(1-\overline{\tau})}{\overline{H}}E_t\left(\widehat{w}_t^* - \widehat{w}_{t+1}^*\right)\end{aligned}$$

Firm discount factor

$$\widehat{\Sigma}_t = (1-\iota)\,\widehat{h}_t + \frac{(1-\iota)\,\overline{s}}{(1+\overline{s})}\widehat{s}_t + \iota E_t\left(\widehat{\beta}_{t,t+1} + \widehat{\Sigma}_{t+1}\right)$$

Optimal contract wage

$$\widehat{w}_t^* = [1-\iota]\,\widehat{w}_t^0\,(r) + \iota E_t \widehat{w}_{t+1}^*$$

Target wage

$$\widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H \Gamma E_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]$$

Spillover-free target wage

$$\begin{aligned}\widehat{w}_t^0 &= \varphi_x\left(\widehat{x}_t^H + \widehat{mpl}_t\right) + \varphi_m \widehat{mrs}_t - \varphi_h \widehat{h}_t \\ &+\varphi_H E_t\left(\widehat{q}_t^W + \widehat{H}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) + \widehat{P}_t \\ &-\varphi_s \widehat{s}_t + \varphi_\tau \widehat{\tau}_t + \varphi_{\tau^c}\widehat{\tau}_t^c + \varphi_D E_t\left[\widehat{\Sigma}_{t+1} - \widehat{\Delta}_{t+1}\right]\end{aligned}$$

Average wage

$$\widehat{w}_t = (1-\gamma)\,\widehat{w}_t^* + \gamma \widehat{w}_{t-1}$$

or

$$\widehat{w}_t = \lambda_b \widehat{w}_{t-1} + \lambda_0 \widehat{w}_t^0 + \lambda_f E_t \widehat{w}_{t+1}$$

Vacancy posting condition

$$-\hat{q}_t^F = E_t\left(\widehat{J}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) + \frac{\gamma}{1-\iota}\frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}E_t\left(\widehat{w}_{t+1}^* - \widehat{w}_t\right)$$

Trade balance

$$\widehat{TB}_t = \widehat{Y}_t - \overline{C}\widehat{C}_t + W\overline{C}\left(\widehat{P}_{H,t} - \widehat{P}^*_t\right) - \overline{G}\widehat{G}_t - \kappa\overline{v}\left(\widehat{v}_t\right)$$

Economy-wide resource constraint

$$\begin{aligned}\widehat{Y}_t &= (1-W)\,\overline{C}\widehat{C}_t + W\overline{C}^*\left(\widehat{C}^*_t + \varpi\widehat{P}^*_t\right) \\ &\quad - \left[(1-W)\,\overline{C}\varpi W\right]\left(\widehat{P}_{H,t} - \widehat{P}^*_t\right) - \overline{C}\varpi W\widehat{P}_{H,t} + \overline{G}\widehat{G}_t + \kappa\overline{v}\left(\widehat{v}_t\right)\end{aligned}$$

Consumer price index

$$\widehat{P}_t = (1-W)\,\widehat{P}_{H,t} + W\widehat{P}^*_t$$

Evolution of debt / Government budget constraint

$$\begin{aligned}\overline{b}\widehat{b}_t &= \overline{R}\overline{b}(\widehat{R}_{t-1} + \widehat{b}_{t-1} - \widehat{\pi}_t) + \overline{G}\left(\widehat{P}_{H,t} - \widehat{P}_t + \widehat{G}_t\right) + b\overline{u}\widehat{u}_t \\ &\quad + \overline{TR}(\widehat{TR}_t - \widehat{P}_t) - \overline{nw}\overline{h}\left(\overline{\tau} + \overline{s}\right)\left(\widehat{n}_t + \widehat{w}_t - \widehat{P}_t + \widehat{h}_t\right) \\ &\quad - \overline{nw}\overline{h}\overline{\tau}\widehat{\tau}_t - \overline{nw}\overline{h}\overline{s}\widehat{s}_t - \overline{\tau}^c\overline{C}\left(\widehat{\tau}^c_t + \widehat{C}_t\right)\end{aligned}$$

Market clearing / aggregate output

$$\widehat{Y}_t = \widehat{n}_t + \hat{z}_t + \alpha\hat{h}_t$$

A.3 Period-by-period Nash bargaining

In the standard MP model, it is assumed that total match surplus, $S_t = (W_t - U_t) + (J_t - V_t)$, the sum of the worker and firm surpluses is shared according to efficient Nash bargaining where wages and hours are negotiated simultaneously. The firm and the worker choose the wage and the hours of work to maximize the weighted product of the worker's and the firm's net return from the match.

$$\max_{w,h} (H_t)^{\eta} (J_t)^{1-\eta}$$

where $0 \leq \eta \leq 1$ is the relative measure of workers' bargaining strength.

The worker surplus gets the following form.

$$H_t = W_t - U_t = \frac{w_t^*}{P_t} h_t (1 - \tau_t) - \frac{g(h_t)}{\Lambda_t} - b + E_t \beta_{t,t+1} \left(1 - \rho - q_t^W\right) H_{t+1}$$

and the firm surplus is (after taking into account the free entry condition $V_t = 0$)

$$J_t = x_t^H f(h_t) - \frac{w_t^*}{P_t} h_t (1 + s_t) + E_t \beta_{t,t+1} (1 - \rho) J_{t+1}$$

The first-order condition for wage-setting is

$$\begin{aligned} \eta \frac{\partial H_t}{\partial w_t} J_t &= (1 - \eta) \frac{\partial J_t}{\partial w_t} H_t \iff \\ \eta (1 - \tau_t) J_t &= (1 - \eta)(1 + s_t) H_t \end{aligned}$$

which would, without taxes, correspond to the simple surplus splitting result where the total surplus from the match is shared according to the bargaining power parameter η.

The optimality condition for wage-setting can be rewritten as a wage equation that includes only contemporaneous variables by substituting the value equations into the Nash FOC, and making use of the expressions for the production and utility functions.

$$\begin{aligned} \frac{w_t}{P_t} &= \frac{\eta}{(1 + s_t)} \left[\frac{x_t^H mpl_t}{\alpha} \right] && (56) \\ &+ \frac{(1 - \eta)}{(1 - \tau_t)} \left[\frac{mrs_t (1 + \tau_t^c)}{(1 + \phi)} + \frac{b}{h_t} + \frac{q_t^W}{h_t} E_t \beta_{t,t+1} H_{t+1} \right] && (57) \\ &+ \frac{\eta}{h_t} E_t \beta_{t,t+1} \left(1 - \rho_{t+1}\right) J_{t+1} \left[\frac{1}{(1 + s_t)} - \frac{(1 - \tau_{t+1})}{(1 - \tau_t)} \frac{1}{(1 + s_{t+1})} \right] && (58) \end{aligned}$$

where w_t is the nominal hourly wage in a match.

The wage equation is a convex combination of what the worker contributes to the match (the first square brackets) and what he has to give up in terms of disutility from supplying hours of work and not getting unemployment benefits plus a term that accounts for possible changes in tax rates over time. Since workers and firms are homogeneous and all matches adjust their wages every period, they will all choose the same wage. The economy's wage bill is this wage rate times the total number of hours worked in the economy. It is clear from the wage equation that the introduction of taxes works to decrease the worker's relative effective bargaining power from η to $\frac{\eta}{(1+s_t)}$. Consequently, economic conditions get a smaller weight in wage determination.

A.4 Dynamics with wage rigidity

The derivation of the wage under staggered contracting follows Gertler, Sala and Trigari (GST) (2008). The Nash first order condition is in this case

$$\eta \Delta_t J_t \left(w_t^*\right) = \left(1-\eta\right) \Sigma_t H_t \left(w_t^*\right)$$

where the effect of a rise in the *real* wage on the worker's surplus is

$$\begin{aligned}
\Delta_t &= P_t \frac{\partial H_t \left(w_t\right)}{\partial w_t} \\
&= h_t \left(1-\tau_t\right) + E_t \beta_{t,t+1} \left(1-\rho\right) \gamma P_{t+1} \frac{\partial H_{t+1} \left(w_t\right)}{\partial w_t} \\
&= h_t \left(1-\tau_t\right) \\
&\quad + E_t \beta_{t,t+1} \left(1-\rho\right) \gamma \left[h_{t+1} \left(1-\tau_{t+1}\right) + E_t \beta_{t+1,t+2} \left(1-\rho\right) \gamma P_{t+2} \frac{\partial H_{t+2} \left(w_t\right)}{\partial w_t}\right] \ldots \\
&= E_t \sum_{s=0}^{\infty} \beta_{t,t+s} \left(1-\rho\right)^s \gamma^s h_{t+s} \left(1-\tau_{t+s}\right)
\end{aligned}$$

$$\Longleftrightarrow \Delta_t = h_t \left(1-\tau_t\right) + E_t \beta_{t,t+1} \left(1-\rho\right) \gamma \Delta_{t+1}$$

And similarly for the firm

$$\Sigma_t = -P_t \frac{\partial J_t \left(w_t\right)}{\partial w_t} = h_t \left(1+s_t\right) + E_t \beta_{t,t+1} \left(1-\rho\right) \gamma \Sigma_{t+1}$$

The dynamic contract wage equation is solved by first linearizing the FOC for wage setting, and then substituting the linearized worker and firm surplus equations as well as the above discount factors in their loglinearized form (see GST (2008) for more details).

First order condition

$$\widehat{J}_t(w_t^*) + \widehat{\Delta}_t = \widehat{H}_t(w_t^*) + \widehat{\Sigma}_t$$

where the loglinear forms of the discount factors are

$$\widehat{\Delta}_t = \left[1-\overline{\beta}\left(1-\rho\right)\gamma\right] \widehat{h}_t - \frac{\left[1-\overline{\beta}\left(1-\rho\right)\gamma\right] \overline{\tau}}{\left(1-\overline{\tau}\right)} \widehat{\tau}_t + \overline{\beta}\left(1-\rho\right)\gamma E_t \left(\widehat{\beta}_{t,t+1} + \widehat{\Delta}_{t+1}\right)$$

$$\widehat{\Sigma}_t = \left[1-\overline{\beta}\left(1-\rho\right)\gamma\right]\widehat{h}_t + \frac{\left[1-\overline{\beta}\left(1-\rho\right)\gamma\right]\overline{s}}{\left(1+\overline{s}\right)}\widehat{s}_t + \overline{\beta}\left(1-\rho\right)\gamma E_t\left(\widehat{\beta}_{t,t+1}+\widehat{\Sigma}_{t+1}\right)$$

and the expressions for $\widehat{J}_t(w_t^*)$ and $\widehat{H}_t(w_t^*)$ can be found as follows

Worker surplus The worker surplus can be written as

$$\begin{aligned} H_t(w_t^*) &= \frac{w_t^*}{P_t}h_t\left(1-\tau_t\right)-\left[\frac{g\left(h_t\right)}{\Lambda_t}+b+q_t^W E_t\beta_{t,t+1}H_{x,t+1}\right] \\ &\quad +E_t\beta_{t,t+1}\left(1-\rho\right)H_{t+1}(w_{t+1}^*) \\ &\quad +\gamma E_t\beta_{t,t+1}\left(1-\rho\right)\left[H_{t+1}(w_t^*)-H_{t+1}(w_{t+1}^*)\right] \end{aligned}$$

In the last term, evaluate the expression $E_t\left[H_{t+1}(w_t^*)-H_{t+1}(w_{t+1}^*)\right]$

$$\begin{aligned} & E_t\left[H_{t+1}(w_t^*)-H_{t+1}(w_{t+1}^*)\right] \\ = \; & E_t\left(\frac{w_t^*}{P_{t+1}}-\frac{w_{t+1}^*}{P_{t+1}}\right)h_{t+1}\left(1-\tau_{t+1}\right) \\ & +\left(1-\rho\right)\gamma E_t\beta_{t+1,t+2}\left[H_{t+2}(w_t^*)-H_{t+2}(w_{t+1}^*)\right] \end{aligned}$$

When linearized, this expression gets the following form

$$\begin{aligned} & E_t\left[\widehat{H}_{t+1}(w_t^*)-\widehat{H}_{t+1}(w_{t+1}^*)\right] \\ = \; & \frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}E_t\left(\widehat{w}_t^*-\widehat{w}_{t+1}^*\right) \\ & +\overline{\beta}\left(1-\rho\right)\gamma E_t\left[\widehat{H}_{t+2}(w_t^*)-\widehat{H}_{t+2}(w_{t+1}^*)\right] \end{aligned}$$

Iterating forward this can be further simplified to yield

$$E_t\left[\widehat{H}_{t+1}(w_t^*)-\widehat{H}_{t+1}(w_{t+1}^*)\right] = \frac{1}{1-\overline{\beta}\left(1-\rho\right)\gamma}\frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}E_t\left(\widehat{w}_t^*-\widehat{w}_{t+1}^*\right)$$

With the help of the above expression, the loglinear formulation of the worker surplus is found to be

$$\begin{aligned}
\widehat{H}_t &= \frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}\left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t\right) - \frac{\overline{wh}\overline{\tau}}{\overline{H}}\widehat{\tau}_t \\
&\quad -\frac{1}{1+\phi}\frac{\overline{mrs}\overline{h}\left(1+\overline{\tau}^c\right)}{\overline{H}}\left(\widehat{mrs}_t + \widehat{h}_t\right) \\
&\quad -\frac{\overline{mrs}\overline{h}\overline{\tau}^c}{\left(1+\phi\right)\overline{H}}\widehat{\tau}_t^c - \overline{\beta}\overline{q}^W E_t\left(\hat{q}_t^W + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1}\right) \\
&\quad +\overline{\beta}\left(1-\rho\right)E_t\left(\widehat{H}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) \\
&\quad +\frac{\overline{\beta}\left(1-\rho\right)\gamma}{1-\overline{\beta}\left(1-\rho\right)\gamma}\frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}E_t\left(\widehat{w}_t^* - \widehat{w}_{t+1}^*\right)
\end{aligned}$$

where$E_t\widehat{H}_{x,t+1}$ can be derived as follows

$$\begin{aligned}
E_tH_{x,t+1} &= E_t\left[\gamma H_{t+1}(w_t) + \left(1-\gamma\right)H_{t+1}(w_{t+1}^*)\right] \\
&= E_tH_{t+1}(w_{t+1}^*) + \gamma E_t\left[H_{t+1}(w_t) - H_{t+1}(w_{t+1}^*)\right]
\end{aligned}$$

Linearizing this expression using the same technique as above to evaluate the last term results in

$$E_t\widehat{H}_{x,t+1} = E_t\widehat{H}_{t+1}(w_{t+1}^*) + \frac{\gamma}{1-\overline{\beta}\left(1-\rho\right)\gamma}\frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}E_t\left(\widehat{w}_t - \widehat{w}_{t+1}^*\right)$$

Firm surplus The firm surplus can be written as

$$\begin{aligned}
J_t(w_t^*) &= x_t^H f\left(h_t\right) - \frac{w_t^*}{P_t}h_t\left(1+s_t\right) + E_t\beta_{t,t+1}\left(1-\rho\right)J_{t+1}(w_{t+1}^*) \\
&\quad +\gamma E_t\beta_{t,t+1}\left(1-\rho\right)\left[J_{t+1}(w_t^*) - J_{t+1}(w_{t+1}^*)\right]
\end{aligned}$$

In the last term, evaluate the expression $E_t\left[J_{t+1}(w_t^*) - J_{t+1}(w_{t+1}^*)\right]$

$$\begin{aligned}
&E_t\left[J_{t+1}(w_t^*) - J_{t+1}(w_{t+1}^*)\right] \\
&= -E_t\left(\frac{w_t^*}{P_{t+1}} - \frac{w_{t+1}^*}{P_{t+1}}\right)h_{t+1}\left(1+s_{t+1}\right) \\
&\quad +\left(1-\rho\right)\gamma E_t\beta_{t+1,t+2}\left[J_{t+2}(w_t^*) - J_{t+2}(w_{t+1}^*)\right]
\end{aligned}$$

When linearized this expression gets the following form

$$\begin{aligned}
&E_t\left[\widehat{J}_{t+1}(w_t^*) - \widehat{J}_{t+1}(w_{t+1}^*)\right] \\
&= -\frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}}E_t\left[\widehat{w}_t^* - \widehat{w}_{t+1}^*\right] + \overline{\beta}\left(1-\rho\right)\gamma E_t\left[\widehat{J}_{t+2}(w_t^*) - \widehat{J}_{t+2}(w_{t+1}^*)\right]
\end{aligned}$$

Iterating forward this can be further simplified to yield

$$E_t\left[\widehat{J}_{t+1}(w_t^*) - \widehat{J}_{t+1}(w_{t+1}^*)\right] = -\frac{1}{1-\overline{\beta}(1-\rho)\gamma}\frac{\overline{wh}(1+\overline{s})}{\overline{J}}E_t\left[\widehat{w}_t^* - \widehat{w}_{t+1}^*\right]$$

Finally, as with the worker surplus, the loglinear formulation of the renegotiating firm's surplus can be found with the help of the above expression

$$\begin{aligned}
\widehat{J}_t &= \frac{\overline{x}\overline{mplh}}{\alpha\overline{J}}\left(\widehat{x}_t^H + \widehat{mpl}_t + \widehat{h}_t\right) - \frac{\overline{wh}(1+\overline{s})}{\overline{J}}\left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t\right) \\
&\quad -\frac{\overline{wh}\overline{s}}{\overline{J}}\widehat{s}_t + \overline{\beta}(1-\rho)E_t\left(\widehat{J}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) \\
&\quad +\frac{\overline{\beta}(1-\rho)\gamma}{1-\overline{\beta}(1-\rho)\gamma}\frac{\overline{wh}(1+\overline{s})}{\overline{J}}E_t\left(\widehat{w}_{t+1}^* - \widehat{w}_t^*\right)
\end{aligned}$$

The Contract wage Inserting the expressions for the worker and firm surpluses, as well as those for the discount factors, into the linearized FOC yields (after collecting the wage terms to the left-hand side and using the Nash FOC for next period)

$$\begin{aligned}
\Longrightarrow\ & \left[\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}} + \frac{\overline{wh}(1+\overline{s})}{\overline{J}}\right]\widehat{w}_t^* \\
& +\frac{\iota}{(1-\iota)}\left[\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}} + \frac{\overline{wh}(1+\overline{s})}{\overline{J}}\right]E_t\left(\widehat{w}_t^* - \widehat{w}_{t+1}^*\right) \\
=\ & \frac{\overline{x}\overline{mplh}}{\alpha\overline{J}}\left(\widehat{x^H}_t + \widehat{mpl}_t\right) + \frac{1}{1+\phi}\frac{\overline{mrs}\overline{h}(1+\overline{\tau}^c)}{\overline{H}}(\widehat{mrs}_t) + \frac{\overline{mrs}\overline{h}\overline{\tau}^c}{(1+\phi)\overline{H}}\widehat{\tau}_t^c \\
& +\overline{\beta}(1-\rho)E_t\left[\widehat{J}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right] \\
& -\overline{\beta}(1-\rho)E_t\left[\widehat{J}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1} + \widehat{\Delta}_{t+1} - \widehat{\Sigma}_{t+1}\right] \\
& +\overline{\beta}\overline{q}^W E_t\left(\widehat{q}_t^W + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1}\right) + \left[\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}} + \frac{\overline{wh}(1+\overline{s})}{\overline{J}}\right]\widehat{P}_t \\
& -\left\{\left[\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}} + \frac{\overline{wh}(1+\overline{s})}{\overline{J}}\right] - \frac{\overline{x}\overline{mplh}}{\alpha\overline{J}} - \frac{\overline{mrs}}{1+\phi}\frac{\overline{h}(1+\overline{\tau}^c)}{\overline{H}}\right\}\widehat{h}_t \\
& -\left[\frac{\overline{whs}}{\overline{J}} + \frac{(1-\iota)\overline{s}}{(1+\overline{s})}\right]\widehat{s}_t + \left[\frac{\overline{wh\tau}}{\overline{H}} - \frac{(1-\iota)\overline{\tau}}{(1-\overline{\tau})}\right]\widehat{\tau}_t \\
& +\left[\overline{\beta}(1-\rho)\gamma\right]E_t\widehat{\Delta}_{t+1} - \left[\overline{\beta}(1-\rho)\gamma\right]E_t\widehat{\Sigma}_{t+1}
\end{aligned}$$

where $\iota = \overline{\beta}(1-\rho)\gamma$. Dividing by the term $\left[\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}} + \frac{\overline{wh}(1+\overline{s})}{\overline{J}}\right] = \frac{\overline{wh}(1+\overline{s})}{\eta\overline{J}} = \frac{\overline{wh}(1-\overline{\tau})}{(1-\eta)\overline{H}}$, and using the steady state equations for $\overline{\Delta}$ and $\overline{\Sigma}$, and for the Nash

FOC allows us to rewrite the contract wage equation in the following simpler form

$$\begin{aligned}
&\Longrightarrow && \widehat{w}_t^* + \frac{\iota}{1-\iota} E_t \left(\widehat{w}_t^* - \widehat{w}_{t+1}^* \right) \\
&= && \varphi_x \left(\widehat{x}_t^H + \widehat{mpl}_t \right) + \varphi_m \widehat{mrs}_t + \varphi_H E_t \left(\widehat{q}_t^W + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1} \right) \\
& && -\varphi_h \widehat{h}_t - \varphi_s \widehat{s}_t + \varphi_\tau \widehat{\tau}_t + \varphi_{\tau^c} \widehat{\tau}_t^c + \varphi_D E_t \left[\widehat{\Sigma}_{t+1} - \widehat{\Delta}_{t+1} \right] + \widehat{P}_t \\
&= && \widehat{w}_t^0(r)
\end{aligned}$$

where $\widehat{w}_t^0(r)$ is the target wage in the bargain, and its coefficients are combinations of deep parameters and model-implied steady state values

$$\begin{aligned}
\varphi_x &= \frac{\overline{x}\overline{mpl}\eta}{\alpha\overline{w}(1+\overline{s})}, \quad \varphi_m = \frac{\overline{mrs}(1-\eta)(1+\overline{\tau}^c)}{(1+\phi)\overline{w}(1-\overline{\tau})}, \quad \varphi_H = \frac{(1-\eta)\overline{H}}{\overline{wh}(1-\overline{\tau})}\overline{\beta}\overline{q}^W, \\
\varphi_{\tau^c} &= \frac{\overline{mrs}(1-\eta)\overline{\tau}^c}{(1+\phi)\overline{w}(1-\overline{\tau})}, \quad \varphi_h = \left\{ 1 - \frac{\overline{x}\overline{mpl}\eta}{\alpha\overline{w}(1+\overline{s})} - \frac{\overline{mrs}(1-\eta)(1+\overline{\tau}^c)}{(1+\phi)\overline{w}(1-\overline{\tau})} \right\}, \\
\varphi_s &= \frac{\eta\overline{s}}{(1+\overline{s})} \left[1 + \frac{(1-\iota)\overline{J}}{\overline{wh}(1+\overline{s})} \right], \quad \varphi_\tau = \frac{(1-\eta)\overline{\tau}}{(1-\overline{\tau})} \left[1 - \frac{(1-\iota)\overline{H}}{\overline{wh}(1-\overline{\tau})} \right], \\
\text{and } \varphi_D &= \left[\overline{\beta}(1-\rho)(1-\gamma)\frac{\eta\overline{J}}{\overline{wh}(1+\overline{s})} \right]
\end{aligned}$$

The target wage $\widehat{w}_t^0(r)$ is of the same form than the period-by-period negotiated wage, adjusted for the new bargaining weights. The equation for the contract wage can be further rewritten as

$$\frac{1}{(1-\iota)}\widehat{w}_t^* = \widehat{w}_t^0(r) + \frac{\iota}{(1-\iota)} E_t \widehat{w}_{t+1}^*$$

$$\Longleftrightarrow \widehat{w}_t^* = [1-\iota]\,\widehat{w}_t^0(r) + \iota E_t \widehat{w}_{t+1}^*$$

This is the optimal contract wage set at time t *by all matches that are allowed to renegotiate their wage.* As is usual with Calvo-type contracting, it depends on a wage target $w_t^0(r)$ and next period's optimal wage.

The spillover effect To derive the spillover effect, consider the worker surplus with *optimal (contract) wage* versus the expected *average market wage* $E_t\left[H_{t+1}(w_{t+1}) - H_{t+1}(w_{t+1}^*)\right]$ in the same way as above

$$E_t\widehat{H}_{t+1}(w_{t+1}) = E_t\widehat{H}_{t+1}(w_{t+1}^*) + \frac{1}{1-\overline{\beta}(1-\rho)\gamma}\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}} E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)$$

Substituting the loglinear equation for the average wage into the above equation yields

$$\begin{aligned}
E_t\widehat{H}_{t+1}(w_{t+1}) &= E_t\widehat{H}_{t+1}(w^*_{t+1}) \\
&\quad + \frac{1}{1-\overline{\beta}(1-\rho)\gamma}\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}}E_t\left((1-\gamma)\widehat{w}^*_{t+1}+\gamma\widehat{w}_t-\widehat{w}^*_{t+1}\right) \\
&= E_t\widehat{H}_{t+1}(w^*_{t+1}) + \frac{\gamma}{1-\overline{\beta}(1-\rho)\gamma}\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}}E_t\left(\widehat{w}_t-\widehat{w}^*_{t+1}\right)
\end{aligned}$$

which is exactly equal to the expression for $E_t\widehat{H}_{x,t+1}$, and we can substitute it in the target wage equation. To simplify, we denote $\frac{1}{1-\overline{\beta}(1-\rho)\gamma}\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}}=\Gamma$

$$\begin{aligned}
\widehat{w}^0_t(r) &= \varphi_x\left(\widehat{x}^H_t+\widehat{mpl}_t\right)+\varphi_m\widehat{mrs}_t \\
&\quad +\varphi_H E_t\left(\widehat{q}^W_t+\widehat{H}_{t+1}(w^*_{t+1})+\widehat{\beta}_{t,t+1}+\Gamma E_t\left[\widehat{w}_{t+1}-\widehat{w}^*_{t+1}\right]\right) \\
&\quad +\varphi_h\widehat{h}_t-\varphi_s\widehat{s}_t+\varphi_\tau\widehat{\tau}_t+\varphi_{\tau^c}\widehat{\tau}^c_t+\varphi_D E_t\left[\widehat{\Sigma}_{t+1}-\widehat{\Delta}_{t+1}\right]+\widehat{P}_t
\end{aligned}$$

$$\Longleftrightarrow \widehat{w}^0_t(r)=\widehat{w}^0_t+\varphi_H\Gamma E_t\left[\widehat{w}_{t+1}-\widehat{w}^*_{t+1}\right]$$

where the target wage $\widehat{w}^0_t(r)$ - the wage the firm and its worker would agree to if they are allowed to renegotiate, *and if firms and workers elsewhere remain on staggered multiperiod wage contracts* - is a sum of the wage that would arise *if all matches were negotiating wages period-by-period* $\widehat{w}^0_t$ and the spillover effect $\varphi_H\Gamma E_t\left[\widehat{w}_{t+1}-\widehat{w}^*_{t+1}\right]$.

Evolution of the average wage To derive the appropriate loglinear expression for the evolution of the average wage, first collect the necessary elements from previous calculations

1) The contract wage

$$\widehat{w}^*_t=[1-\iota]\,\widehat{w}^0_t(r)+\iota E_t\widehat{w}^*_{t+1}$$

2) The average wage

$$\widehat{w}_t=(1-\gamma)\,\widehat{w}^*_t+\gamma\widehat{w}_{t-1}$$

3) The target wage

$$\widehat{w}^0_t(r)=\widehat{w}^0_t+\varphi_H\Gamma E_t\left[\widehat{w}_{t+1}-\widehat{w}^*_{t+1}\right]$$

First, insert the target wage in the contract wage equation

$$\widehat{w}^*_t=[1-\iota]\left(\widehat{w}^0_t+\varphi_H\Gamma E_t\left[\widehat{w}_{t+1}-\widehat{w}^*_{t+1}\right]\right)+\iota E_t\widehat{w}^*_{t+1}$$

Then update the average wage equation by one period and take expectations

$$E_t\widehat{w}_{t+1} = (1-\gamma)\, E_t\widehat{w}^*_{t+1} + \gamma\widehat{w}_t$$

$$\Longleftrightarrow E_t\widehat{w}^*_{t+1} = \frac{1}{(1-\gamma)}\left(E_t\widehat{w}_{t+1} - \gamma\widehat{w}_t\right)$$

Use this expression to eliminate $E_t\widehat{w}^*_{t+1}$ from the contract wage equation

$$\begin{aligned}\widehat{w}^*_t &= [1-\iota]\left(\widehat{w}^0_t + \varphi_H\Gamma E_t\widehat{w}_{t+1} - \varphi_H\Gamma\left[\frac{1}{(1-\gamma)}\left(E_t\widehat{w}_{t+1} - \gamma\widehat{w}_t\right)\right]\right) \\ &\quad +\iota\left[\frac{1}{(1-\gamma)}\left(E_t\widehat{w}_{t+1} - \gamma\widehat{w}_t\right)\right]\end{aligned}$$

$$\begin{aligned}\widehat{w}^*_t &= (1-\iota)\,\widehat{w}^0_t + (1-\iota)\,\varphi_H\Gamma E_t\widehat{w}_{t+1} - (1-\iota)\,\varphi_H\Gamma\frac{1}{(1-\gamma)}E_t\widehat{w}_{t+1} \\ &\quad +[1-\iota]\,\varphi_H\Gamma\frac{\gamma}{(1-\gamma)}\widehat{w}_t + \frac{\iota}{(1-\gamma)}E_t\widehat{w}_{t+1} - \frac{\iota\gamma}{(1-\gamma)}\widehat{w}_t\end{aligned}$$

$$\begin{aligned}\Longleftrightarrow\quad \widehat{w}^*_t &= (1-\iota)\,\widehat{w}^0_t + \left[(1-\iota)\,\varphi_H\Gamma\frac{\gamma}{(1-\gamma)} - \iota\frac{\gamma}{(1-\gamma)}\right]\widehat{w}_t \\ &\quad + \left[(1-\iota)\,\varphi_H\Gamma - (1-\iota)\,\varphi_H\Gamma\frac{1}{(1-\gamma)} + \iota\frac{1}{(1-\gamma)}\right]E_t\widehat{w}_{t+1}\end{aligned}$$

Denote $\zeta = (1-\iota)\,\varphi_H\Gamma$, and use the above equation to eliminate $\widehat{w}^*_t$ from the average wage equation (equation 2)

$$\widehat{w}_t = (1-\gamma)(1-\iota)\,\widehat{w}^0_t + (\zeta\gamma - \iota\gamma)\,\widehat{w}_t + [(1-\gamma)\,\zeta - \zeta + \iota]\,E_t\widehat{w}_{t+1} + \gamma\widehat{w}_{t-1}$$

$$[1-\gamma(\zeta-\iota)]\,\widehat{w}_t = (1-\gamma)(1-\iota)\,\widehat{w}^0_t + [(1-\gamma)\,\zeta - \zeta + \iota]\,E_t\widehat{w}_{t+1} + \gamma\widehat{w}_{t-1}$$

Finally, after dividing by $[1-\gamma(\zeta-\iota)]$, the dynamic average wage equation can be expressed as

$$\Longleftrightarrow \widehat{w}_t = \lambda_b\widehat{w}_{t-1} + \lambda_0\widehat{w}^0_t + \lambda_f E_t\widehat{w}_{t+1}$$

$$\begin{aligned}\text{where } \lambda_b &= \frac{\gamma}{[1-\gamma(\zeta-\iota)]},\ \lambda_0 = \frac{(1-\gamma)(1-\iota)}{[1-\gamma(\zeta-\iota)]},\ \text{and } \lambda_f = \frac{\iota-\gamma\zeta}{[1-\gamma(\zeta-\iota)]}, \\ \text{with } \zeta &= (1-\iota)\,\varphi_H\Gamma,\ \iota = \overline{\beta}\,(1-\rho)\,\gamma,\ \Gamma = \frac{\overline{wh}\,(1-\overline{\tau})}{(1-\iota)\,\overline{H}}, \\ \text{and } \varphi_H &= \frac{(1-\eta)\,\overline{H\beta}\overline{q}^W}{\overline{wh}\,(1-\overline{\tau})}\ \text{as previously denoted.}\end{aligned}$$

A.5 Match surplus and reservation wages

Total match surplus is the sum of the worker surplus and the firm surplus

$$
\begin{aligned}
S_t &= H_t + J_t \\
&= \frac{w_t^*}{P_t} h_t (1-\tau_t) - \frac{g(h_t)}{\Lambda_t} - b \\
&\quad + E_t \beta_{t,t+1} (1-\rho) \left[\gamma H_{t+1}(w_t^*) + (1-\gamma) H_{t+1}(w_{t+1}^*) \right] \\
&\quad - q_t^W E_t \beta_{t,t+1} H_{x,t+1} + x_t^H f(h_t) - \frac{w_t^*}{P_t} h_t (1+s_t) \\
&\quad + E_t \beta_{t,t+1} (1-\rho) \left[\gamma J_{t+1}(w_t^*) + (1-\gamma) J_{t+1}(w_{t+1}^*) \right]
\end{aligned}
$$

$$
\begin{aligned}
\iff \quad & S_t = x_t^H z h_t{}^{\alpha} - \frac{w_t^*}{P_t} h_t (\tau_t + s_t) - \frac{mrs_t h_t (1+\tau_t^c)}{(1+\phi)} - b \\
& + E_t \beta_{t,t+1} (1-\rho) \left[\gamma S_{t+1}(w_t^*) + (1-\gamma) S_{t+1}(w_{t+1}^*) \right] - q_t^W E_t \beta_{t,t+1} H_{x,t+1}
\end{aligned}
$$

If wages are renegotiated each period, the surplus equation can be further written as follows

$$
\begin{aligned}
\iff \quad & S_t = x_t^H z h_t{}^{\alpha} - \frac{w_t^*}{P_t} h_t (\tau_t + s_t) - \frac{mrs_t h_t (1+\tau_t^c)}{(1+\phi)} - b \\
& + E_t \beta_{t,t+1} (1-\rho) S_{t+1}(w_{t+1}^*) - \frac{\eta}{(1-\eta)} \kappa \theta_t E_t \frac{(1-\tau_{t+1})}{(1+s_{t+1})}
\end{aligned}
$$

where the last term is derived using the Nash FOC equation and the vacancy posting condition

$$
\begin{aligned}
q_t^W E_t \beta_{t,t+1} H_{t+1}\left(w_{t+1}^*\right) &= q_t^W E_t \beta_{t,t+1} \frac{\eta}{(1-\eta)} \frac{(1-\tau_{t+1})}{(1+s_{t+1})} J_{t+1}\left(w_{t+1}^*\right) \\
&= \theta_t q_t^F E_t \beta_{t,t+1} \frac{\eta}{(1-\eta)} \frac{(1-\tau_{t+1})}{(1+s_{t+1})} \frac{\kappa}{q_t^F} \\
&= \frac{\eta}{(1-\eta)} \kappa \theta_t E_t \frac{(1-\tau_{t+1})}{(1+s_{t+1})}
\end{aligned}
$$

The loglinear version of this surplus equation is

$$
\begin{aligned}
\widehat{S}_t &= \frac{\overline{xzh}^{\alpha}}{\overline{S}} \left(\widehat{x}_t^H + \alpha \widehat{h}_t \right) - \frac{\overline{wh} (\overline{\tau} + \overline{s})}{\overline{S}} \left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t \right) \\
&\quad - \frac{\overline{mrs}\overline{h} (1+\overline{\tau}^c)}{(1+\phi)\overline{S}} \left(\widehat{mrs}_t + \widehat{h}_t \right) \\
&\quad - \frac{\overline{mrs}\overline{h}\overline{\tau}^c}{(1+\phi)\overline{S}} \widehat{\tau}_t^c - \frac{\overline{wh\tau}}{\overline{S}} \widehat{\tau}_t - \frac{\overline{whs}}{\overline{S}} \widehat{s}_t \\
&\quad + \overline{\beta} (1-\rho) E_t \left[\widehat{S}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1} \right] \\
&\quad + \frac{\eta (1-\overline{\tau})}{(1-\eta)(1+\overline{s})} \frac{\kappa \overline{\theta}}{\overline{S}} \left[\widehat{\theta}_t + E_t \left(\widehat{\tau}_{t+1} + \widehat{s}_{t+1} \right) \right]
\end{aligned}
$$

In the presence of staggered bargaining, the dynamic surplus equation is, in turn

$$\begin{aligned}
\widehat{S}_t &= \frac{\overline{xzh}^{\alpha}}{\overline{S}}\left(\widehat{x}_t^H + \alpha \widehat{h}_t\right) - \frac{\overline{wh}\left(\overline{\tau}+\overline{s}\right)}{\overline{S}}\left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t\right) \\
&- \frac{\overline{mrsh}\left(1+\overline{\tau}^c\right)}{\left(1+\phi\right)\overline{S}}\left(\widehat{mrs}_t + \widehat{h}_t\right) - \frac{\overline{mrsh}\overline{\tau}^c}{\left(1+\phi\right)\overline{S}}\widehat{\tau}_t^c - \frac{\overline{wh\tau}}{\overline{S}}\widehat{\tau}_t - \frac{\overline{whs}}{\overline{S}}\widehat{s}_t \\
&+ \overline{\beta}\left(1-\rho\right)E_t\left[\widehat{S}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right] - \overline{\beta}\overline{q}^W E_t\left(\widehat{q}_t^W + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1}\right) \\
&+ \frac{\overline{\beta}\left(1-\rho\right)\gamma}{1-\overline{\beta}\left(1-\rho\right)\gamma}\frac{\overline{wh}\left(\overline{\tau}+\overline{s}\right)}{\overline{S}}E_t\left(\widehat{w}_{t+1}^* - \widehat{w}_t^*\right)
\end{aligned}$$

The reservation wages of the worker and of the firm are, respectively, the wages at which the value of employment is exactly equal to the value of unemployment, i.e. $H_t\left(\underline{w}_t\right) = 0$, or the value of a filled vacancy is exactly equal to the value of an open vacancy, i.e. $J_t\left(\overline{w}_t\right) = 0$.

$$H_t\left(\underline{w}_t\right) = 0$$

$$\begin{aligned}
\Longleftrightarrow\quad 0 &= \frac{\underline{w}_t}{P_t}h_t\left(1-\tau_t\right) - \frac{g\left(h_t\right)}{\Lambda_t} - b \\
&+ E_t\beta_{t,t+1}\left(1-\rho\right)\left[\gamma H_{t+1}(w_t^*) + \left(1-\gamma\right)H_{t+1}(w_{t+1}^*)\right] - q_t^W E_t\beta_{t,t+1}H_{x,t+1}
\end{aligned}$$

$$\begin{aligned}
\Longleftrightarrow\quad \underline{w}_t &= \frac{P_t}{h_t\left(1-\tau_t\right)}\left[\frac{g\left(h_t\right)}{\Lambda_t} + b + q_t^W E_t\beta_{t,t+1}H_{x,t+1}\right] \\
&- \frac{P_t}{h_t\left(1-\tau_t\right)}E_t\beta_{t,t+1}\left(1-\rho\right)\left[\gamma H_{t+1}(w_t^*) + \left(1-\gamma\right)H_{t+1}(w_{t+1}^*)\right]
\end{aligned}$$

$$\begin{aligned}
J_t\left(\overline{w}_t\right) &= 0 \\
&\Longleftrightarrow\quad 0 = x_t^H f\left(h_t\right) - \frac{\overline{w}_t}{P_t}h_t\left(1+s_t\right) \\
&\qquad + E_t\beta_{t,t+1}\left(1-\rho\right)\left[\gamma J_{t+1}(w_t^*) + \left(1-\gamma\right)J_{t+1}(w_{t+1}^*)\right] \\
&\Longleftrightarrow\quad \overline{w}_t = \frac{P_t}{h_t\left(1+s_t\right)}\left[x_t^H f\left(h_t\right)\right] \\
&\qquad + \frac{P_t}{h_t\left(1+s_t\right)}E_t\beta_{t,t+1}\left(1-\rho\right)\left[\gamma J_{t+1}(w_t^*) + \left(1-\gamma\right)J_{t+1}(w_{t+1}^*)\right]
\end{aligned}$$

The steady state equations for the reservation wages are

$$\begin{aligned}
\underline{w} &= \frac{1}{h\left(1-\tau\right)}\left[\frac{g\left(h\right)}{\Lambda} + b - \beta\left(1-\rho-q^W\right)H\right] \\
&= \frac{1}{h\left(1-\tau\right)}\left[\frac{mrsh\left(1+\tau^c\right)}{\left(1+\phi\right)} + b - \beta\left(1-\rho-q^W\right)H\right]
\end{aligned}$$

$$\overline{w} = \frac{1}{h\left(1+s\right)}\left[xf\left(h\right) + \beta\left(1-\rho\right)J\right] = \frac{1}{h\left(1+s\right)}\left[\frac{xmplh}{\alpha} + \beta\left(1-\rho\right)J\right]$$

Figure 5. Impulse responses to a government spending shock with different degrees of price rigidity. Solid line: baseline, $\xi = 0.75$, dotted line $\xi = 0.25$

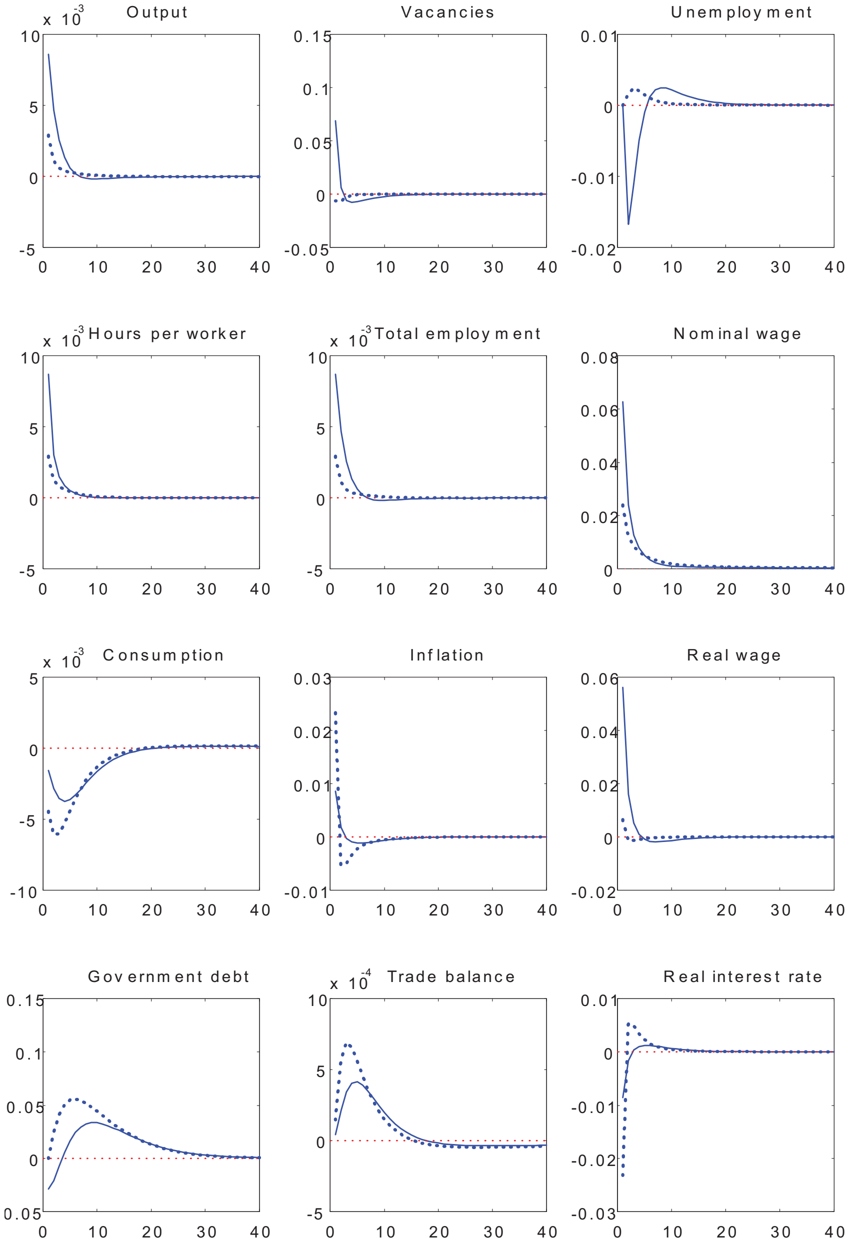


Figure 6. Impulse responses to a government spending shock with relative value of non-work to work activities 0.72 ($b = 0.41$, solid line) and 0.9 ($b = 0.53$, dotted line)

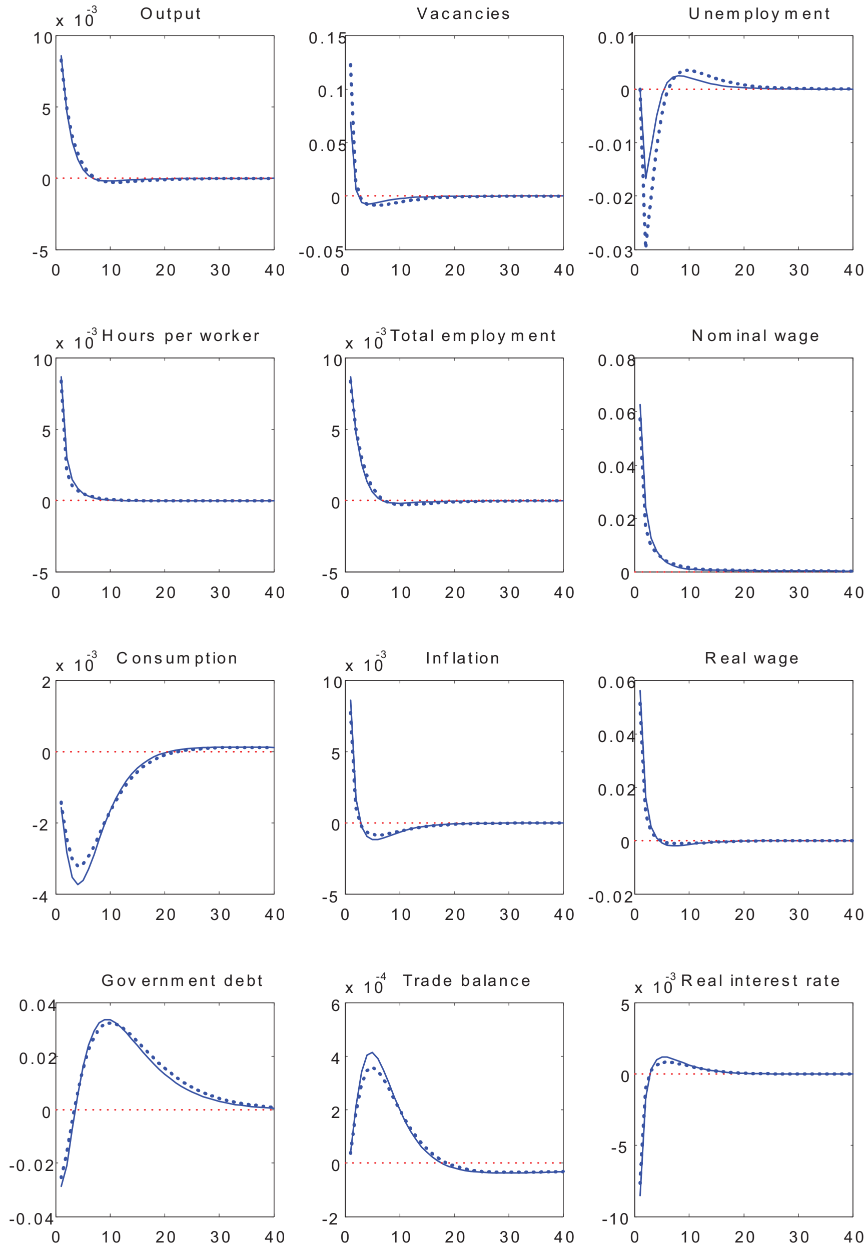
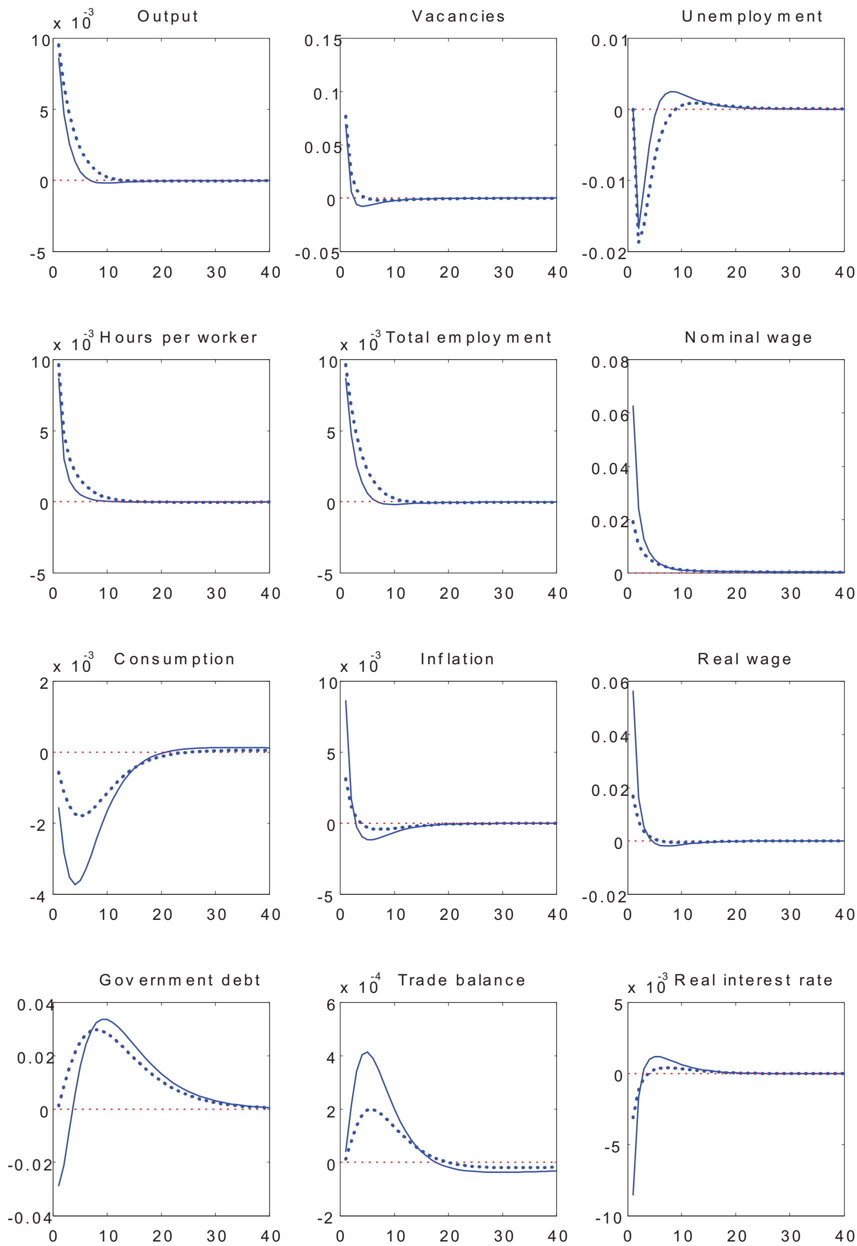


Figure 7. Impulse responses to a government spending shock with relative value of non-work to work activities 0.72 ($\phi = 10$, solid line) and 0.9 ($\phi = 2.6$, dotted line)



II

The Finnish Unemployment Volatility Puzzle

Abstract

This paper investigates the unemployment volatility puzzle in Finland. We explore the ability of a New Keynesian dynamic stochastic general equilibrium (DSGE) model with labour market frictions to reproduce key volatilities found in Finnish business cycle data. We contribute with a new data set to the Shimer debate and offer some plausible explanations for the Finnish case. Specific emphasis is put on modelling the wage bargaining framework as well as the cyclical behaviour of distortionary labour taxes. We find that wage rigidity is a promising candidate for explaining Finnish labour market volatilities in the period 1994-2010. Countercyclical taxes are not able to bring the model-implied volatilities close enough to the data if calibrated properly, and they worsen the model in some other dimensions. We also discuss the much larger labour market volatilities of the period 1981-1993. We find that most of the specific features that characterized the Finnish economy in that period, including during the exceptionally deep recession of the early 1990's, are such that they also increase the magnitude of the responses of labour market variables to shocks in our model economy.

1 Introduction

Recent literature on labour markets and business cycles has aimed at solving the unemployment volatility puzzle. Competitive models of the labour market had not been able to explain the observed relatively smooth behaviour of real wages over the business cycle together with the relatively volatile behaviour of employment. Shimer (2010) shows that adding matching frictions to the labour market does not help in explaining this inconsistency between the model and the data when fluctuations are driven by technology shocks. The root cause seems to be that in the standard Mortensen-Pissarides model, the wage is renegotiated in every period by Nash bargaining and is thereby let to adjust very easily to changes in the economic environment. The volatility of wages absorbs a large part of the fluctuation that is observed in employment variables. Indeed, when job search is time consuming but wages are flexible, search frictions aggravate the inconsistency between the model and the data.

Shimer (2010), however, also shows that wage rigidities can reconcile the search model with the data and provide a quantitatively accurate description of labour market dynamics. Rigid wages contribute to solving the unemployment volatility puzzle because they make wages in each period less responsive to economic conditions, and shift adjustment to the labour quantity side.

This paper investigates the Finnish unemployment volatility puzzle. We explore the ability of a New Keynesian dynamic stochastic general equilibrium (DSGE) model with labour market frictions to reproduce key volatilities found in Finnish business cycle data. We test different versions of the model to find out which specification gets closest to fitting the labour market dynamics in Finland. Specific emphasis is put on modelling the wage bargaining framework as well as distortionary labour taxes that affect labour market participants' decisions. We add rigidity in the adjustment of wages in the form of staggered bargaining initially developed by Gertler and Trigari (2009). One advantage of this approach is that wage rigidity gets the explicit interpretation of longer wage contracts.

There is no previous literature on Finnish labour market volatilities in the unemployment volatility puzzle context. The debate on labour market volatilities and the ability of the search and matching model to replicate the stylized facts started from the United States, and analysis has accordingly been mostly carried out with U.S. data. There is, however, reason to believe that the volatility of European labour market variables could differ from those in the U.S.. Therefore, a separate analysis has to conducted to assess whether a search and matching model is able to provide a quantitatively accurate description of these economies. Gartner, Merkl and Rothe (2010) investigate labour market volatilities in Germany, and find that vacancies, labour market tightness and the job-finding rate are roughly twice as volatile compared to the volatility of labour productivity as in the United States. They use a dynamic labour market model with heterogeneous worker productivity and explain the higher German labour market volatilities by a longer expected job duration.

We use Finnish business cycle data for the period from 1981Q1 to 2010Q4, and find that labour market volatility is also in Finland almost twice as high as in the U.S., meaning that fitting a standard search and matching model to this data is a priori likely to be even more challenging. However, the deep recession of the early 1990's associated with huge fluctuations in employment clearly dominates the behaviour of Finnish labour market series. The exceptional nature of that deep recession suggests that a separate explanation might be needed for this time period. Moreover, in the early 1990's many of the Finnish economy's structures significantly differed from those in place today making it possibly misleading to use the same model framework to explain fluctuations in the pre-depression years as opposed to the post-depression period.

In our main analysis we, therefore, concentrate on the period 1994-2010 which is found to exhibit very different labour market volatilities compared with those found in the earlier data period and under the depression years. These volatilities which are just a half of those found in the U.S. can easily be explained with the help of our New Keynesian DSGE model framework.

We find that the calibration of the surplus from employment, including distortionary taxes on labour, to match the Finnish data brings the search and matching closer to being able to replicate labour market volatilities. This is because the calibration implies a small match surplus that reacts relatively strongly to technology shocks. In addition, wage rigidity is a particularly promising candidate for explaining Finnish labour market volatilities. Imposing a wage contract length of approximately six months makes the model match our data well.

Following the finding by Burda and Weder (2010) that payroll taxation is countercyclical in many OECD countries, especially Finland, we investigate the effects of including this feature into the model economy to the empirical performance of the model. We find that countercyclical taxes are also able to bring the model-implied volatilities close to the data, but they worsen the model in some other dimensions. Similarly, countercyclical job separations, that have been identified to contribute to Finnish labour market fluctuations in bad times (see Ilmakunnas and Maliranta, 2008), increase the volatility of unemployment but are not able to explain other key features of the data. The advantage of the model with wage rigidity compared with other model specifications is precisely that it generates, in addition to volatilities that match the data, realistic comovements between different variables.

Furthermore, we discuss the much larger labour market volatilities of the period 1981Q1-1993Q4 in Finland. We find that most of the specific features that characterized the Finnish economy in that period are such that they increase the magnitude of the responses of labour market variables to shocks in our model economy. In that context we consider e.g. other than productivity shocks, the different monetary regime and more rigid wages. Some observed features of the Finnish economy that did not seem to characterize the labour market volatilities of the later data period, such as countercyclical separations, could have had a role in magnifying the shocks in the earlier period.

In section 2 we briefly summarize some key results for our approach from the growing literature on the unemployment volatility puzzle. Section 3 presents the Finnish data and establishes some key volatilities and cross-correlations to be matched. Section 4 lays out the basic ingredients of the model. In section 5 we evaluate the steady state properties of the model, the results of dynamic simulations, and the sensitivity of the model to key parameter choices. Finally, we provide some concluding remarks in section 6.

2 The unemployment volatility puzzle

Shimer (2005) argued that the Mortensen-Pissarides model in its standard form does not sufficiently reproduce the relatively smooth behaviour of wages and relatively volatile behaviour of labour market variables observed in the data when the driving force in the model is assumed to be technology shocks of plausible magnitude. Shimer further argued that the problem arises because, in the standard model, the wage is renegotiated in every period by Nash bargaining and is thereby let to adjust very easily to changes in the economic environment. In the growing body of literature that has attempted to explain the problem, known as the unemployment volatility puzzle, the focus has accordingly been on ways to amplify the response of vacancies and unemployment to shocks. The range of alternative models proposed to solve the unemployment volatility puzzle includes both flexible and rigid wage variants and has been summarized in e.g. Hall (2005b).

The majority of solutions proposed in the literature can be roughly divided into those that aim at making the match surplus small and those that add rigidity in the adjustment of wages.

A number of contributions (see e.g. Trigari, 2006; Hagedorn and Manovskii, 2008; and Costain and Reiter, 2008), have identified the magnitude of the match surplus as a key factor contributing to explaining the unemployment volatility puzzle. The intuition is that a smaller surplus reacts more to shocks of equal size and translates into increased volatility of labour market variables. Hagedorn and Manovskii (2008), however, obtained this result by calibrating the worker's negotiation parameter to abnormally low and unemployment benefit to abnormally high values. In addition, Costain and Reiter (2008) point out that the calibration can be chosen either to match unemployment volatility *or* the effect of benefits on unemployment but *not both*. More specifically, as Hornstein, Krusell and Violante (2005) also showed, the small surplus calibration solution to the unemployment volatility puzzle has the tendency to exaggerate the policy effects of the model. This is not a trivial point as policy analysis is one of the main uses of these kinds of models.

Another direct way to address Shimer's critique is to include in the Mortensen-Pissarides framework additional rigidity in the adjustment of wages. Hall (2005a) introduced real wage rigidity in the form of a backward looking social wage norm. The wage norm limits the adjustment capabilities of wages and hence increases the adjustments on the labour quantity side. A small increase in productivity immediately results in more vacancies being posted, a higher job-finding rate and lower unemployment in that setup. Krause and Lubik (2007) found that real wage rigidity in the form of a fixed wage did help to generate a negative correlation between unemployment and vacancies, and to increase the magnitude of labour market flows to more realistic values, but did not affect inflation dynamics in a significant way. Trigari (2006) investigated the different adjustment margins for labour input and showed that by modifying the characteristics of the bargaining framework or of consumers' behaviour wage rigidity does also reduce the volatility of inflation.

The wage norm has been criticized for its ad hoc nature. Gertler and Trigari (2009) modify the conventional MP model to allow for staggered multiperiod wage contracting. The gain of the latter approach over a simple ad hoc wage norm is that the key primitive parameter is the average frequency of wage adjustment, as opposed to an arbitrary partial adjustment coefficient in the wage equation. Their framework delivers the essential feature of the sticky-wage model, the sensitivity of the employer's surplus from a new hire to current economic conditions. As wages are fixed over an exogenously determined time period, they cannot respond immediately to those conditions. As a result, adjustment is shifted to the number of workers employed.

Christoffel et al. (2009) compare the success of some different labour market modeling approaches in the New Keynesian setup from the perspective of inflation dynamics and also reach the conclusion that wage rigidity is needed to explain the unemployment volatility puzzle. They confirm one point that has received relatively much attention in recent years that under efficient bargaining stickiness in wages matters for the rest of the economy only if it also affects newly-created jobs. That is, only if it affects wages at the job creation margin.

The most important empirical criticism of the rigid wage approach rests on the claim that the wages of new matches are not found to be rigid. Pissarides (2009) finds on the basis of microeconometric evidence that wages in new matches are volatile and consistent with the key predictions of the standard model. Haefke et al. (2009) argue likewise that wages are flexible at the start of new jobs. On the other hand, Gertler and Trigari (2009) state that evidence is not sharp enough to support this claim for two reasons. Data in almost all cases does not match workers with firms and existing studies do not control for cyclical changes in job quality. They argue that after controlling for these compositional effects new hire wages are not more cyclical than existing workers' wages.

While there is a relatively large literature on the effects of labour market institutions on equilibrium labour market outcomes, much less research can be found on their impact on business cycle fluctuations. An exception is the effect of unemployment benefits that has more recently gained attention in the unemployment volatility puzzle literature. The unemployment benefit is namely a major determinant of the relative value of non-work to work activities. Large unemployment benefits make the agents nearly indifferent between working and not working, decreasing the surplus from employment and increasing the magnitude of the labour market responses to shocks.

Distortionary labour taxes also represent a significant intervention in labour markets in developed economies, especially in Europe, and could have important implications for labour markets' capability to adjust to shocks. However, the majority of papers which have augmented the New Keynesian business cycle model with search and matching frictions in the labour market do not incorporate taxation in their framework. Exceptions are e.g. Den Haan et al. (2001), Andrés et al. (2006) and Vanhala (2007) who investigate the effects of different taxes or of other labour market institutions such as unemployment benefits and firing costs on the cyclical properties of the New Keynesian monetary model. They find that an increase in taxes increases the volatility of output, unemployment and employment. Den Haan et al. (2001), for instance, show that in a job matching model high initial replacement rates or tax rates lead to a larger rise in unemployment as a response to a negative technology shock than with "employment -friendly" institutions, in line with Blanchard and Wolfers' (2000) empirical evidence. This is because distortionary taxes on labour reduce the after-tax value of the match surplus, thereby decreasing the relative attractiveness of work.

While higher distortionary taxes imply a smaller match surplus and therefore contribute to explaining the unemployment volatility puzzle, e.g. Andrés et al. (2006) find that this effect is quantitatively of minor importance. Instead, Burda and Weder (2010) find strong evidence that payroll taxation is countercyclical: employer and employee contributions to social insurance tend to fall in recoveries and rise in recessions. Incorporating this feature, instead of constant tax rates, into a real business cycle model, they show that the model is able to match central labour market volatilities and interactions of different variables.

Our approach to explaining the Finnish unemployment volatility puzzle will concentrate on wage rigidity and the behaviour of distortionary labour taxes without resorting to any artificial small surplus calibration. The motivation for this approach is, first, as Hall (2005a) notes: "the primary reason that the sticky-wage case is interesting is the general impression that wages are, in fact, quite sticky". In the context of the Finnish wage negotiation tradition, it would not seem reasonable to ignore explicit wage rigidity. In international comparisons, Finland has typically been classified as a country with high union density and centralized wage bargaining. There is also recent evidence of real wage rigidity in Finland both from individual real wage changes of job stayers (Dickens et al., 2007; Böckerman et al., 2006) and industry-level data (e.g. Holden and Wulfsberg, 2007). Moreover, Gorodnichenko, Mendoza and Tesar

(2009), who study the Finnish Great Depression of the early 1990's, find that downward wage rigidity played a key role in the amplification of the downturn caused by a negative shock to external trade and positive shock to energy prices.

Second, for example the evidence from selected OECD countries presented in Burda and Weder (2010) shows that the negative correlation between payroll taxes and GDP is strongest in Finland, together with Germany, suggesting that the cyclical behaviour of taxes is a relevant alternative to explore in explaining Finnish labour market dynamics.

In addition to wage rigidity and countercyclical taxation we also test a model specification with countercyclical job separations. This is because there is evidence of increased job separations and worker outflow in bad times in Finland (see Ilmakunnas and Maliranta, 2008). If the job separation rate is higher in bad times, this would obviously cause larger movements in unemployment and could thereby contribute to the explanation of the volatility puzzle.

3 Finnish labour market developments and dynamics

This section describes labour market developments and dynamics in Finland during the period from 1981 to 2010. We discuss the time series behaviour of unemployment, vacancies, labour market tightness, the job-finding rate, wages and productivity in Finland and compare the volatility of these variables to those in the U.S. and those in Germany.

There is no previous literature on labour market fluctuations in Finland in the context of the unemployment volatility puzzle. Most of the evidence on this phenomenon is from the United States, starting with Shimer (2005). Gartner, Merkl and Rothe (2010) provide summary statistics on labour market fluctuations for Germany, and conclude that vacancies, labour market tightness and the job-finding rate are roughly twice as volatile compared to the volatility of labour productivity as in the United States. We use this available evidence to analyze and compare Finnish labour market data. Table 1 first gives an overview of the cyclical behaviour of relevant variables in the U.S. and Germany.

Table 1: Summary statistics for the U.S. 1951-2003, and for Western Germany 1977-2004

	u	v	v/u	q^W	ρ	w	z
U.S. Std deviation	0.190	0.202	0.382	0.118	0.075	–	0.020
U.S. / productivity	9.5	10.1	19.1	5.9	3.75	–	1.0
W.G. Std deviation	0.180	0.313	0.505	0.229	0.065	0.018	0.013
W.G. / productivity	13.52	23.56	37.98	17.2	4.89	1.379	1.0

Source: Shimer (2005), Gartner, Merkl and Rothe (2010)

In Finland, there are two important data sets that contain information on the labour market, the Employment Service Statistics (ESS) compiled by the Ministry of Employment and the Economy and the labour Force Survey (LFS) by Statistics Finland. The former only comprises data on the clients of so-called employment and economic development offices (TE offices), i.e. registered job seekers and vacancies. On the other hand, the labour Force Survey does not include, or does not permit to compute, all relevant variables for the labour market matching model used in this paper. Combining the available survey and register data is not either an available option, since the two data sources differ, among other things, with respect to the criteria for job-seeking activeness and labour market availability.

In order to obtain sufficiently long time series which form a coherent data set, we choose to use the Employment Service Statistics from 1981 onwards to compute the relevant empirical labour market volatilities. The fact that this data only represents some market share of the total labour market is not likely to affect our main conclusions in any significant way. To confirm this, we present a comparison between the two data sets and their most important differences in the Appendix. Wage and productivity data are taken from the quarterly national accounts data. The data for each variable and, for some variables, the calculations that have been performed to obtain the final series are described in more detail below.

To retain comparability to the U.S. and Germany, following Shimer (2005) and Gartner, Merkl and Rothe (2010), we use seasonally adjusted quarterly data and apply a Hodrick-Prescott filter with smoothing parameter $\lambda = 10^5$ to obtain the cyclical components of the variables as deviations from trend. These deviations are calculated from logged series unless stated otherwise. Before going into a more detailed description of individual variables, Table 2 shows an overview of the cyclical behaviour of relevant variables in Finland in the period from 1981Q1 to 2010Q3.

Table 2: Summary statistics and correlation table for Finland 1981Q1-2010Q3

	u	v	v/u	q^W	w	z
Std deviation	0.286	0.352	0.626	0.402	0.028	0.019
/ Productivity	14.983	18.481	32.833	21.088	1.474	1.000
Autocorrelation	0.983	0.950	0.972	0.969	0.790	0.627
			Correlation			
u Unemployment	1.000	−0.924	−0.977	−0.963	−0.710	−0.152
v Vacancies		1.000	0.985	0.965	0.595	0.306
$v/u = \theta$ Tightness			1.000	0.982	0.659	0.242
q^W Job-finding rate				1.000	0.592	0.234
w Wages					1.000	0.177
z Productivity						1.000

Finnish labour market volatility in 1981-2010 resembles more the corresponding fluctuations in Germany than in the U.S.. The standard deviations of all labour market variables are larger than in both comparison countries but the volatility of productivity is nearly as high as in the U.S.. As a result, the volatilities *relative to productivity* are for some variables smaller than in Germany. In particular, vacancies and labour market tightness seem to be less volatile relative to productivity than in Germany whereas unemployment and especially the job-finding rate are more volatile. The standard deviation of the job-finding rate relative to productivity is almost one and a half times larger than in Germany and three and a half times larger than in the U.S..

As to the cross correlations between the variables, we note a strong Beveridge relationship and a positive correlation between the job-finding rate and labour market tightness of almost one. One noteworthy difference in the Finnish 1981 to 2010 data from both U.S. and West German data is that labour market variables are less correlated with productivity. This observation suggests that other omitted driving forces of the business cycle and labour markets than technology shocks have been more important in Finland than in the two comparison countries. This seems logical in light of the exceptional events that Finland faced in the early 1990's recession and, overall, in the case of a small open economy. In particular, the low correlation of wages and productivity indicates that aggregate wage movements have been only loosely tied to current period productivity.

It is not possible to check robustness of these findings by using exactly the same sample period for Finland than has been used in the studies on the U.S. or Germany because of limited data availability. However, the time period we use is not very different from that in Gartner, Merkl and Rothe (2010), and it seems that Germany is also in other respects a closer reference for analyzing Finnish labour market volatilities.

One relevant comparison is, however, to take a shorter observation period from Finnish data, in particular, by leaving outside the exceptionally deep recession of the early 1990's. That recession has been combined with a number of exceptional events such as the collapse of the Finnish Soviet trade and the deregulation of financial markets that had taken place in the 1980's, raising doubts about the validity of analyzing the economy in this period as being driven by technology shocks. In that period, the Finnish economy experienced a profound structural change, including a shift from more traditional industrial production to the rise of the information technology sector. The comparison is important because that recession dominates the behaviour of most of the time series for labour market variables, especially that of unemployment. Table 3 shows the corresponding summary statistics for the shorter, and more recent, time period.

Table 3: Summary statistics and correlation table for Finland 1994Q1-2010Q3

	u	v	v/u	q^W	w	z
Std deviation	0.086	0.165	0.238	0.167	0.015	0.022
/ Productivity	3.894	7.459	10.801	7.583	0.681	1.000
Autocorrelation	0.956	0.965	0.972	0.965	0.946	0.955
			Correlation			
u Unemployment	1.000	−0.789	−0.906	−0.940	−0.131	−0.763
v Vacancies		1.000	0.975	0.881	0.347	0.803
$v/u = \theta$ Tightness			1.000	0.947	0.288	0.830
q^W Job-finding rate				1.000	0.164	0.766
w Wages					1.000	0.519
z Productivity						1.000

The volatilities of all labour market variables and the wage are significantly smaller for the shorter time period. While this is not a surprising result per se, it should be noted that the standard deviation of productivity, in turn, is larger than for the whole observation period, and, as a result, the volatility of labour market variables has significantly decreased also in *relative* terms. The correlation of labour market variables and of the real wage to productivity becomes significantly stronger and that of the real wage to labour market variables significantly weaker. In the subsequent data analysis we compare the volatilities of the shorter and longer time periods.

3.1 Unemployment

Quarterly unemployment is measured as mean of the seasonally adjusted monthly data. For the observation period we use to calculate labour market volatilities, the standard deviation of the cyclical component of registered unemployment is 0.286, i.e. about 15 times more volatile than productivity. Figure 1 shows the quarterly series for unemployment in levels and its trend.

There is evidence that employment agencies are a relevant market place mainly for primary educated job seekers in Finland. This bias in unemployed searching workers' composition is likely to affect positively matching performance as suggested by Lahtonen (2006), and therefore be relevant for e.g. matching function estimations. The composition of the pool of job seekers does not, however, affect the conduct of the present study because the cyclical properties of registered unemployment are very similar to the labour Force Survey's measure of unemployment which covers in a more balanced way the whole labour market. In particular, implied unemployment volatility is nearly the same. The log deviations from H-P trend are 0.32 for the ESS series and 0.33 for the LFS measure for a comparison period from 1989 to 2010.

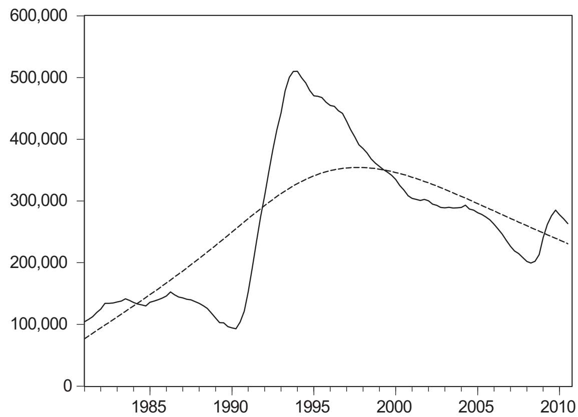


Figure 1. Quarterly unemployment and trend, 1981-2010

The exceptionally deep recession of the early 1990's dominates the behaviour of most of the time series, especially that of unemployment. Unemployment then rose to fivefold levels within a few years time and took a long time to recover. It still has not reached the levels seen before the deep recession. In contrast, during the recession that followed the global financial crisis, employment responses to the sharp fall in exports and output were very mild. As a result, the relative volatility of unemployment differs very much according to which time period is considered. In particular, shortening the observation period to the period after the deep recession decreases the relative volatility of unemployment to just 4 times that of productivity.

3.2 Vacancies

Vacancies are measured as open positions registered at employment and economic development offices at the end of each month. The quarterly series is calculated as the mean of the seasonally adjusted monthly data. The log-deviation from trend of the cyclical component of vacancies is 0.352, i.e. vacancies are about 18 times more volatile than productivity. Statistics Finland also records quarterly figures for vacancies based on a survey on working aged (15 to 74 years) Finns. While this would offer a broader coverage of the labour market this data has only been available since 2003 and cannot, therefore, be used for our purposes. Our conclusion on the volatility of vacancies does not, however, depend on which data for vacancies is used, since the log-deviation from H-P trend of registered vacancies is almost identical to that of Statistics Finland's vacancy measure which covers all vacancies, at least for the short time period the measure for all vacancies has been recorded.

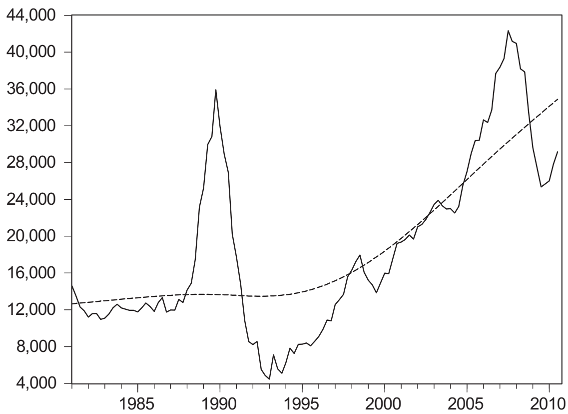


Figure 2. Quarterly vacancies and trend, 1981-2010

Vacancies peaked before the deep recession of the early 1990's and fell thereafter to record low levels. Vacancies peaked also before the recent global financial crisis but both the rise in vacancies and the subsequent fall were much more subdued. This can also be seen as decreased relative volatility of vacancies in more recent times. Taking only the later part of the time series from 1994 onwards, the relative volatility of vacancies decreases to 7.5 times that of productivity. While this is less than half of the volatility observed in whole sample, the difference between the whole data set and the shorter data set is not as large as in the case of unemployment. One further observation is that the time series for vacancies shows an upward trend from the mid 1990's onwards (see Figure 2), probably reflecting at least partly the advances in information technology at the employment offices.

A solid finding in data analysis is the negative cross correlation of vacancies and unemployment. Figure 3 shows a scatter plot of the relationship between the cyclical components of these two series, i.e. the Beveridge curve relation. The correlation of the percentage deviation of unemployment and vacancies from trend is -0.924 between 1981 and 2010, slightly stronger than in the U.S. or Western Germany. However, the Beveridge relation in Finland seems to have weakened more recently since the corresponding correlation in the shorter data sample is -0.789.

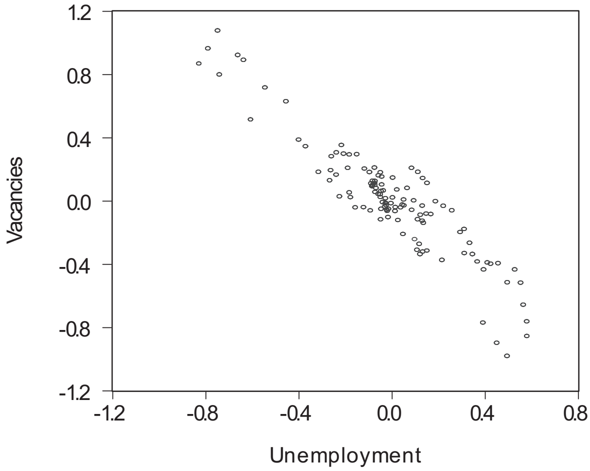


Figure 3. Quarterly Finnish Beveridge curve, 1981-2010

3.3 Labour market tightness

The Finnish vacancy-unemployment ratio, or labour market tightness, a key concept in the Mortensen-Pissarides literature, is extremely procyclical, with a standard deviation of 0.626 around its trend. This implies 32 times more volatile behaviour than that of productivity. The sharp peak in labour market tightness, preceding the deep recession of the early 1990's partly explains this unusually high figure. Indeed, the volatility of the vacancy-unemployment ratio is just 10.8 times that of productivity in the shorter data sample

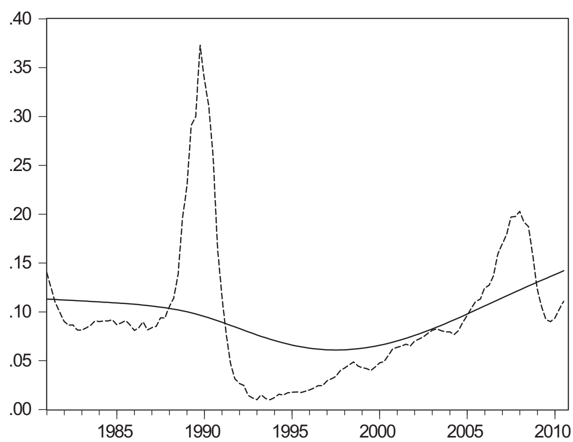


Figure 4. Quarterly labour market tightness and trend, 1981-2010

3.4 Job-finding rate

We compute the job-finding rate as filled vacancies, or matches, as a share of unemployed workers. Our time series for matches is filled vacancies each month that is available directly from the Employment Service Statistics database. This differs from Shimer's (2012) measure for the job-finding rate which is computed as the share of unemployed workers that leave unemployment within a month. As Gartner, Merkl and Rothe (2010) note these out-of-unemployment individuals may have moved out of the labour force instead of finding a job. While the advantage with Finnish data is that we have a time series for matches readily available, similar criticism applies here: the measure for matches includes all filled vacancies irrespective of the labour market status of the worker before he got matched. In other words, those workers may also have been in another job or outside the working force in the previous period.

The search and matching model implies, however, that only workers in the unemployment pool at the beginning of the period can be hired. Therefore, to obtain a model consistent measure of job finding, the measure of filled vacancies should be identified as filled vacancies *by unemployed searching workers*. As there is no straightforward way to differentiate vacancies filled by unemployed searching workers in our data set, we correct the match series with the share of unemployed job seekers of all job seekers in each quarter[1]. The share of unemployed job seekers of all job seekers tends to decrease in good times and rise in bad times. As a result, the match series we use to calculate the volatility of the job-finding rate is somewhat smoother than the series for all matches (see Figure 5). The total number of matches (solid line) increases in good times, but as a smaller share of these matches can be attributed to unemployed searching workers, the behaviour of the matches of unemployed searching workers (dotted line) are less procyclical.

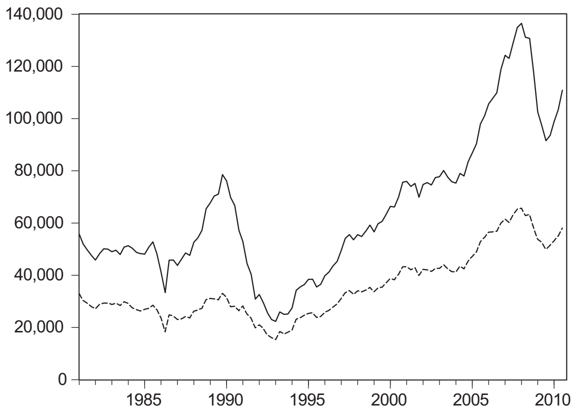


Figure 5. Quarterly job-findings / matches, 1981-2010

The job-finding rate calculated on the basis of our corrected match measure is plotted in Figure 6. The log-deviation from trend of its cyclical component

[1]The measure of all job seekers is decomposed, in the ESS database, into unemployed job seekers (including individually laid-off workers), job seekers on reduced working week, those who have work (including those in subsidized employment), job seekers not in the labour force, and unemployment pensioners.

is 0.402, i.e. the job-finding rate is 21 times more volatile than productivity. In the shorter data sample this relative volatility is 7.6.

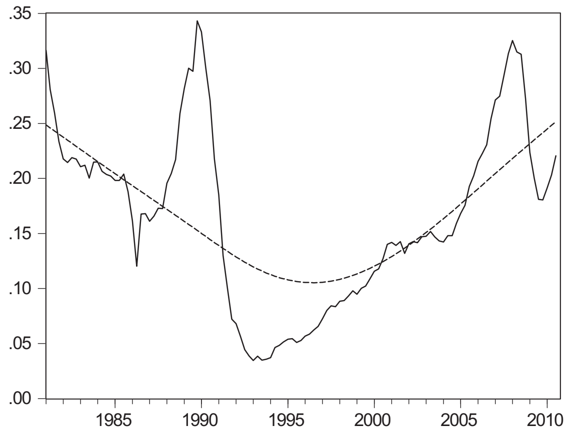


Figure 6. Quarterly job-finding rate and trend, 1981-2010

If the increasing, constant returns-to-scale matching function is a good description of the process whereby the unemployed find jobs and firms find workers, the job finding rate is an increasing function of the vacancy-unemployment ratio. In the Finnish data this seems to be the case. The correlation between the cyclical components of the job-finding rate and the labour market tightness measure is 0.98, slightly higher than in U.S. or Western German data where the corresponding correlation coefficient is 0.95. Fiqure 7 plots this ratio.

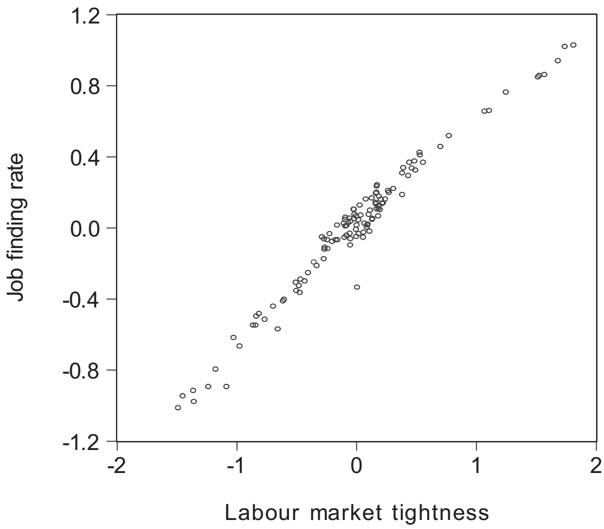


Figure 7.

3.5 Separation rate

A central question for the design of labour market matching models has been whether the inflow into unemployment varies with economic fluctuations, but the literature remains inconclusive about the driving forces. The early view on employment termination was that cyclical unemployment stems almost entirely from cyclical variation in the *inflow* rate (e.g. Darby, Haltiwanger and Plant, 1986). This view has been contradicted by e.g. Hall (2005b) who argues that the separation rate has been nearly constant over time but that the job-finding rate shows high volatility. Therefore, during a recession the unemployment rate increases not because the workers lose their jobs but because firms do not create jobs.

This "outs win" view has also generated counter-arguments more recently. Yashiv (2007) finds by re-examination of Shimer's data considerable cyclicality and volatility of both job creation and destruction. Likewise, Pissarides (2009) finds evidence that both inflows and outflows into unemployment are cyclical. He argues that the inflow rate contributes roughly one third to one half of the volatility of unemployment. Elsby, Michaels and Solon (2009) replicate Shimer's analysis and extend it. They confirm Shimer's finding that the majority of cyclical unemployment variation can be attributed to cyclicality in the outflow hazard, but they also find that increased inflows are important in most recessions, especially the most severe ones. In a recent paper, Shimer (2012) reassesses the ins and outs of unemployment and concludes that, in the U.S. since 1948, the job finding probability has accounted for three-quarters of the fluctuations in unemployment and the job separation probability for the remaining one-quarter. Moreover, Shimer finds that during the last two decades, fluctuations in the separation probability are quantitatively irrelevant.

We compute the separation rate as new unemployed workers per unemployed workers. The measure for new unemployed workers is directly available from the ESS database. The separation rate is plotted in Figure 8. It looks like there has been a regime shift from the high separations preceding the deep recession of the early 1990's to a fairly low and stable share of separations from the mid 1990's onward. The standard deviation of the cyclical component of the separation rate from H-P trend is 14 times larger than that of productivity for the observation period, but this relative volatility falls to 3.6 if the time period is shortened to the more recent period.

Our separation rate is calculated from data on the worker side. It does not, however, describe *exits from employment*, since the data makes not distinction as to what the labour market status of the new unemployed job seekers was in the previous period. They may not have been employed but out of the labour force for instance. As a consequence, our new unemployed workers is a larger group of persons than would be unemployed workers *who were employed in the previous period*. In addition to a difference in level, these two measures could have different cyclical properties if the share of new unemployed job seekers who were employed in the previous period to all new unemployed job seekers showed a cyclical pattern. To make sure that we do not make misleading conclusions on the cyclical properties of Finnish job separations, we take a closer look at some properties of our time series. We also compare our job separation measure to information from job and worker flow data where transitions from one labour market status to another can better be tracked.

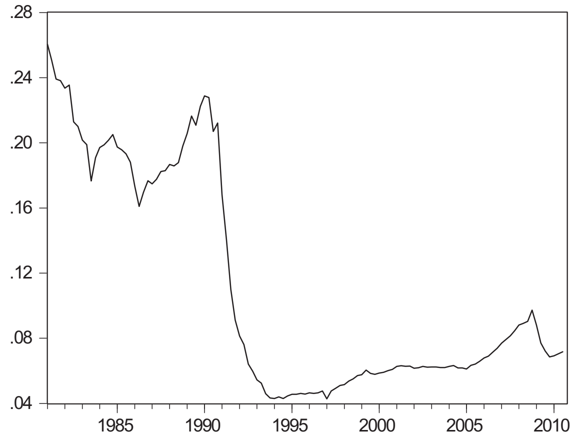


Figure 8. Quarterly job separation rate, 1981-2010

Ilmakunnas and Maliranta (2008) have recently updated the data on job and worker flows in the Finnish Business Sector. One of their main findings is that, in the period from 1991 to 2005, job creation has been procyclical and job destruction countercyclical. The job destruction rate, however, after peaking in the deep recession of the early 1990's decreased steadily until 1997 and has since then stayed fairly constant.

In the data on job and worker flows job destruction and job creation are quite distinct concepts. In the Mortensen-Pissarides model, in turn, the job destruction rate refers to *matches*, which consist in the standard model of 1 worker - 1 firm pairs. No difference is, therefore, made as to whether it is a worker that gets separated from a job or whether it is the vacancy that seizes to exist. Although our job separation rate is calculated only from the worker side in unemployment register data, and can, therefore, not be directly compared to the job-destruction rate calculated from job flow data, these two measures show a similar pattern, especially in the recent time period.

In our data set, the separation rate correlates *negatively* with unemployment and *positively* with vacancies, tightness and job-finding in 1981-2010, implying a smaller separation rate in recessions and larger in booms, in contrast to Ilmakunnas and Maliranta's (2008) findings on the countercyclical nature of job destruction. According to the ESS data, separations do rise in recessions but unemployment rises more, so that the ratio between the two decreases. No clear-cut conclusion on the cyclicality of job separations can therefore be made. Furthermore, the correlation of the cyclical component of the separation rate with that of productivity is just 0.03 in our observation period, i.e. insignificant.

With respect to the above considerations and data for Finland, we opt to model job destruction as exogenous in our analysis of Finnish labour market fluctuations. We will, however, consider cyclical job destruction as one alternative modelling strategy because of its importance in the relevant literature as well as the evidence of countercyclical separations in bad times found in Finnish labour market flow data and the uncertainty surrounding the correct way of calculating the job separation rate.

Finally, it should be noted that the assumption that recessions involve little increase in the flow of workers out of jobs does not mean that employers would adjust their labour force *only* through variations in hires. The point is that the *changes* in the separation rate that accompany employment changes at the industry or aggregate level are tiny compared to the regular flow of workers out of jobs (Hall, 2005b).

3.6 The wage

The wage is measured as gross wages in the whole economy. This measure thus includes part-time work, overtime hours, bonus payments and one-off increases in wages which tend to behave cyclically. The index of wage and salary earnings does not seem a good alternative to us since it only includes whole-time workers, and therefore, excludes significant cyclical variations in the price of labour. Gross wages is then divided by hours worked in the whole economy to avoid biases from possible changes in working time over time (e.g. changes in the share of part-time work). To obtain a measure of real wages, we deflate the hourly wage series with the series for CPI. Figure 9 plots log real hourly wages and their trend in the observation period.

The standard deviation of the wage from its trend is 0.028, approximately 1.5 times as volatile as productivity. Shimer (2005) does not report the relative volatility of wages in the U.S. or its cross correlations with other variables, but according to Gertler and Trigari's (2009) calculations using a slightly different time period, the relative volatility of wages to output in the U.S. is 0.52. The wage in Finland in the 1981-2010 period is thus much more volatile than in the U.S. and even a little more volatile than in Western Germany. However, in the period after the deep recession, the standard deviation of the wage from its trend falls to 0.015, corresponding to a volatility of 0.681 of the volatility of productivity.

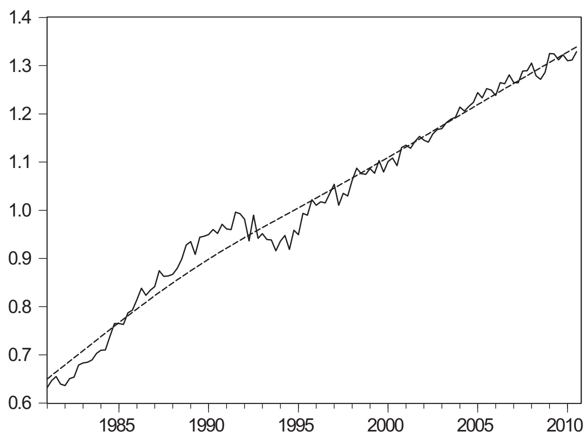


Figure 9. Quarterly log real hourly wages and their trend, 1981-2010

The Finnish wage series in 1981-2010 is clearly less correlated with labour market variables than in the comparison countries. In particular, the correlation of the real wage with productivity is only 0.177 compared to 0.56 (relative to output) in the U.S. and 0.611 in Germany. This correlation, however, does rise to 0.519 in the period after the deep recession, whereas the correlation of wages with other labour market variables weakens significantly.

3.7 Labour productivity

We compute labour productivity as output *per hour* in the total economy and not per worker. In that way, we rule out that changes in working time over the observation period affect the volatility of productivity. The standard deviation of the cyclical productivity component from its trend is 0.019. The corresponding volatility measure in the shorter time period is 0.022.

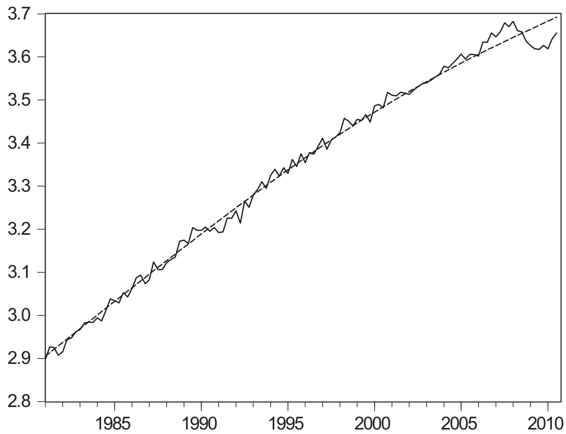


Figure 10. Quarterly labour productivity, 1981-2010

In contrast to other variables, thus, the volatility of productivity seems to have increased in recent years. This higher volatility reflects the different nature of the most recent recession from the deep recession of the 1990's. While in the latter, productivity hardly fell at all, in the recent recession the productivity fall was exceptionally large and abrupt mostly due to the sudden stop in export demand. In addition, the 1990's recession was felt in the dramatic deterioration of the labour market situation, while in the more recent recession, the huge fall in productivity did not cause large scale unemployment.

In Finland, productivity developments have been importantly driven by the information technology sector during our observation period. Since the beginning of the 1990's the rapid increase in the value added of the information technology sector is by far the most important factor that contributed to the rapid increase in productivity. In the recent boom and bust period both the peak in productivity growth preceding the financial crisis and the subsequent sharp fall in productivity would be significantly smoother without the effect of the information technology sector.

4 The model

In the light of the data analysis, it appears that building a model that would be able to reproduce Finnish business cycle dynamics of the whole period from 1981 to 2010 would be a considerable challenge, not only because of the exceptionally deep recession and profound structural change of the economy, but also because of the fundamental change in the exchange rate regime. Therefore, we opt for building a model that reflects the more recent developments in the Finnish economy, i.e. our shorter data period from 1994Q1 to 2010Q3.

Although Finland became a part of the Euro area only in the beginning of 2002, it had entered the third phase of the European Monetary Union already in January 1999. Finland also entered the second phase of the EMU in the beginning of 1995 directly upon membership in the European Union. In our modelling approach we will, consequently, abstract from monetary policy considerations reflecting our choice to focus on the more recent time period, but we will, in the subsequent analysis, specifically point out which modelling choices would help in accounting for earlier business cycle dynamics.

4.1 General features

The model considers a small monetary union member state and builds in this respect on Galí and Monacelli (2008). As suggested by Schmitt-Grohé and Uribe (2003), however, we close the model by assuming a debt-elastic interest rate instead of complete asset markets. The home country is modelled along standard New-Keynesian practise comprising households, firms and a public sector. Capital is not included as a factor of production since the focus of this paper is on the labour market.

The framework is augmented by a Mortensen and Pissarides (MP) search and matching labour market model (Mortensen and Pissarides, 1994; Pissarides 2000). The structure of the standard labour market matching model has been amended with rigidity in the adjustment of wages. Wage rigidity is introduced in the form of staggered bargaining initially developed by Gertler and Trigari (2009), and applied in Gertler, Sala and Trigari (2008) and Christoffel, Kuester and Linzert (2009). One advantage of this approach is that wage rigidity gets the explicit interpretation of longer wage contracts. Lengthening the duration of wage contracts makes wages in each period less responsive to economic conditions, and shifts adjustment to the labour quantity side.

Another extension from more standard New Keynesian models is the incorporation of distortionary labour taxes. This reflects our hypothesis that these represent a possibly significant distortion in economies like the Finnish one, and are therefore likely to affect both the equilibrium of that economy and its capacity to adjust to shocks. We explore these issues in more detail in the model evaluation part.

In our framework, there is only one worker per firm, and the wage and price setting decisions are separated from each other. labour market frictions arise in the intermediate good sector. The wholesale firms buy intermediate goods and re-sell them to the final goods sector. Wholesale firms operate under monopolistic competition and set prices subject to Calvo rigidities. Final goods are produced from domestic and imported intermediate inputs under perfect competition.

4.2 Preferences

As in similar models, we adopt the representative or large household interpretation, implying perfect consumption insurance against variations in labour income due to the labour market status of household members (see e.g. Merz, 1995).

The representative household maximizes its expected lifetime utility

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - \varkappa C_{t-1})^{1-\varrho}}{1-\varrho} - \delta n_t \frac{(h_t)^{1+\phi}}{1+\phi} \right] \right\} \tag{1}$$

where C_t is final good consumption in period t, $\varkappa \in (0,1)$ indicates an external habit motive, C_{t-1} stands for aggregate consumption in the previous period, h_t are hours worked, and δ is a scaling parameter for the disutility of work. Disutility of work is experienced by those members of the household who are employed, n_t. The inverses of ϱ and ϕ are the elasticities of intertemporal substitution and of labour supply respectively. The household's (real) budget constraint is

$$\begin{aligned} & (1+\tau_t^c)\, C_t + \frac{B_t^*}{P_t} \\ = \; & n_t \frac{w_t}{P_t} h_t (1-\tau_t) + (1-n_t)\, b + \frac{TR_t}{P_t} + R_{t-1} \frac{B_{t-1}^*}{P_t} + \frac{D_t}{P_t} \end{aligned} \tag{2}$$

The left-hand side of the equation describes the expenditures of the household. Consumption C_t is subject to a proportional tax τ_t^c. The household can buy a nominal one-period foreign bond B_t^* denominated in the common monetary union currency. The right hand side describes the household's income sources which consist of after-tax real wage $n_t \frac{w_t}{P_t} h_t (1-\tau_t)$, unemployment benefits $(1-n_t)\, b$, lump-sum transfers $\frac{TR_t}{P_t}$, and of profit from firm ownership $\frac{D_t}{P_t}$. Income is also received in the form of repayment of last period's foreign bond purchases. $R_t = (1+r_t^n)$ stands for the gross nominal return on bonds.

We leave aside for a moment the labour supply decision, which will be dealt with in the section describing the labour market, below. Optimal allocations are characterized by the following conditions

$$\Lambda_t = \frac{\lambda_t}{(1+\tau_t^c)} \tag{3}$$

$$\Lambda_t = \beta E_t \left[\Lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right] \tag{4}$$

where $\lambda_t = (C_t - \varkappa C_{t-1})^{-\varrho}$ is the marginal utility of consumption and $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ is CPI inflation. The discount factor is the same for all optimizing agents in the economy and is hereafter defined throughout the paper as $\beta_{t,t+s} = \beta^s \frac{\Lambda_{t+s}}{\Lambda_t}$.

The interest rate paid or earned on foreign bonds by domestic households R_t consists of the common currency union gross interest rate R_t^* which, for the small member state is taken to be exogenous, and a country-specific risk premium

$$R_t = R_t^* p\,(b_t^*) \tag{5}$$

The risk premium is assumed to be increasing in the aggregate level of foreign real debt as a share of domestic output $(-b_t^* = -\frac{B_t^*}{P_t Y_t})^2$. The risk premium on foreign bond holdings $p(b_t^*)$ follows the function

$$p(b_t^*) = \exp\left[-\gamma_{b^*}\left(b_t^* - \bar{b}\right)\right], \text{ with } \gamma_{b^*} > 0 \tag{6}$$

In the steady state, the risk premium is assumed to be equal to one. After loglinearization the arbitrage relation gets the form[3]

$$\widehat{R}_t = \widehat{R}_t^* - \gamma_{b^*}\widehat{b}_t^*$$

4.3 The labour market

The labour market brings together workers and intermediate good firms.

4.3.1 Unemployment, vacancies and matching

The measure of successful matches m_t is given by the matching function

$$m_t(u_t, v_t) = \sigma_m u_t^{\sigma} v_t^{1-\sigma} \tag{7}$$

where m_t is the flow of matches during a period t, and u_t and v_t are the stocks of unemployed workers and vacancies at the beginning of the period. The matching function is, as usual, increasing in both vacancies and unemployment, concave, and homogeneous of degree one (see Petrongolo and Pissarides, 2001). Lahtonen (2006) finds support for the constant returns to scale specification of the matching function in 1995Q1-2002Q8 Finnish data but not in his earlier estimation period which includes the recession years 1991-1993. The Cobb-Douglas form implies that σ is the elasticity of matching with respect to the stock of unemployed people, and σ_m represents the efficiency of the matching process. The probabilities that a vacancy will be filled and that the unemployed person finds a job are respectively

$$q_t^F = q_t^F(\theta_t) = \frac{m_t}{v_t} = \sigma_m \theta_t^{-\sigma} \tag{8}$$

$$q_t^W = \theta_t q_t^F(\theta_t) = \frac{m_t}{u_t} = \sigma_m \theta_t^{1-\sigma} \tag{9}$$

and the inverse of these probabilities is the mean duration of vacancies and unemployment.

$\theta_t = \frac{v_t}{u_t}$ is labour market tightness. The tighter the labour market is, or the less there are unemployed people relative to the number of open vacancies (i.e. larger θ_t), the smaller the probability that the firm succeeds in filling the vacancy and the larger the probability that the unemployed person finds a job. Similarly, a decrease in the number of vacancies relative to unemployment

[2] This is the debt-elastic interest rate assumption, one of the mechanisms suggested by Schmitt-Grohé and Uribe (2003) to close a small open economy model. With the current notation a negative (positive) deviation of the stock of foreign bonds from the steady state zero level implies that the home country as a whole becomes a net borrower (lender), and faces a positive (negative) risk premium.

[3] Hereafter, all variables marked with a hat denote log deviations of that variable from its steady state level.

(smaller θ_t) implies that the unemployed person has a smaller probability to find a job.

In the beginning of each period, a fraction of matches will be terminated with an exogenous probability $\rho \in (0,1)$. Later, to assess the role of countercyclical separations, we will use the following simple rule by which the separation rate depends negatively on output, i.e. there are larger separations in bad times[4].

$$\rho_t = \overline{\rho} - \Omega_\rho \left(Y_t - \overline{Y} \right) \tag{10}$$

Labour market participation is characterised as follows. The size of the labour force is normalised to one. The number of employed workers at the beginning of each period is

$$n_t = (1 - \rho)\, n_{t-1} + m_{t-1} \tag{11}$$

where the first term on the right hand side represents those workers who were employed already in the previous period and whose jobs have survived beginning-of-period job destruction, and the second term covers those workers who got matched in the previous period and become productive in the current period. After the exogenous separation shock, the separated workers return to the pool of unemployed workers and start immediately searching for a job. The number of unemployed is $u_t = 1 - n_t$.

In the steady state an equal amount of jobs are created and destructed:

$$JC = JD \Longleftrightarrow m = \rho n \tag{12}$$

4.3.2 Wage bargaining

Job creation takes place when a worker and a firm meet and agree to form a match at a negotiated wage. The wage that the firm and the worker choose must be high enough that the worker wants to work in the job, and low enough that the employer wants to hire the worker. These requirements define a range of wages that are acceptable to both the firm and the worker. The unique equilibrium wage is, however, the outcome of a bargain between the worker and the firm. We will call this wage the contract wage.

The structure of the staggered multiperiod contracting model applied here follows Gertler and Trigari (2009) and Gertler, Sala and Trigari (2008) but includes also the intensive margin of adjustment of the labour input (hours worked per worker) as well as distortionary taxes. For comparison, the period-by-period bargaining outcome is presented in Appendix A.3.

The idea of staggered wage bargaining is analogous to Calvo price setting. Rigidity is created by assuming that a fraction γ of firms are not allowed to renegotiate their wage in a given period. As a result, all workers in those firms receive the nominal wage paid in the previous period w_{t-1}. The constant probability that firms are allowed to renegotiate the wage is labeled $(1-\gamma)$. Accordingly, $\frac{1}{(1-\gamma)}$ is the average duration of a wage contract. Thus, the combination of wage bargaining and Calvo price setting allows to give an intuitive

[4] In that case, time-dependent ρ has to be included in all the relevant equations characterising labour market behaviour and wage setting. For simplicity, this is not done in the main text, but is included in the numerical simulations of chapter 5. The algebra including time-dependent ρ is available from the author.

interpretation to the source of wage rigidity instead of more or less ad hoc formulations. Period-by-period bargaining corresponds to the special case of $\gamma = 0$.

As in the standard Mortensen-Pissarides model, it is assumed that match surplus, the sum of the worker and firm surpluses, is shared according to efficient Nash bargaining. In the baseline model, wages and hours are negotiated simultaneously in each period. The firm and the worker choose the nominal wage and the hours of work to maximize the weighted product of their net return from the match. When wages are rigid, it is assumed that as they become productive, new matches enter the same Calvo scheme for wage-setting than existing matches.

The contract wage w_t^* is chosen to solve

$$\max \left[H_t\left(r\right)\right]^{\eta} \left[J_t\left(r\right)\right]^{1-\eta} \tag{13}$$

subject to the random renegotiation probability. $H_t(r)$ and $J_t(r)$ are the matching surpluses of renegotiating workers and firms respectively, and $0 \leq \eta \leq 1$ is the relative measure of workers' bargaining strength. The value equations describing the worker's and the firm's matching surpluses are the key determinants of the outcome of the wage bargain.

Workers The value to the worker of being employed consists of after-tax labour income, the disutility from supplying hours of work and the expected present value of his situation in the next period[5]. In the case of non-renegotiation, the worker gets the existing contract wage

$$\begin{aligned} W_t(r) &= \frac{w_t^*}{P_t} h_t \left(1-\tau_t\right) - \frac{g\left(h_t\right)}{\Lambda_t} \\ &+ E_t \beta_{t,t+1} \left(1-\rho\right) \left[\gamma W_{t+1}\left(w_t^*\right) + \left(1-\gamma\right) W_{t+1}(w_{t+1}^*)\right] \\ &+ E_t \beta_{t,t+1} \rho U_{t+1} \end{aligned} \tag{14}$$

The value to the worker of being unemployed is

$$U_t = b + E_t \beta_{t,t+1} \left[q_t^W W_{x,t+1} + \left(1 - q_t^W\right) U_{t+1}\right] \tag{15}$$

where the first term on the RHS is the value of the outside option to the worker, i.e. the unemployment benefit b, and the second term gives the expected present value of either finding a job or remaining unemployed in the following period. Unemployed workers do not need to take into account the probability of job destruction even if they get matched because of the timing assumption. A match that has not yet become productive cannot be destroyed. Note that the value for the worker, who is currently unemployed, to move from unemployment to employment next period is $W_{x,t+1}$, the expected *average* value of being employed. New matches are subject to the same bargaining

[5]In the presence of perfect consumption insurance against different labour market outcomes, it is necessary to define the worker surplus as the change in the *household's utility* of having one additional member employed. Accordingly, the worker surplus equation is obtained by differentiating the household's optimal value function with respect to the number of workers. In addition, the resulting equation is expressed in terms of current consumption, i.e. it is divided by the shadow price of consumption. For more details, see Trigari (2006).

scheme as existing matches, and therefore the new worker does not have a priori knowledge of whether the firm he will start working for will be allowed to renegotiate its wage[6].

Combining these value equations gives the expression for the surplus of those workers who renegotiate their wage in the current period

$$\begin{aligned} H_t(r) &= W_t(r) - U_t \\ &= \frac{w_t^*}{P_t} h_t (1-\tau_t) - \frac{g(h_t)}{\Lambda_t} - b \\ &\quad + E_t \beta_{t,t+1} (1-\rho) \left[\gamma H_{t+1}(w_t^*) + (1-\gamma) H_{t+1}(w_{t+1}^*) \right] \\ &\quad - q_t^W E_t \beta_{t,t+1} H_{x,t+1} \end{aligned} \tag{16}$$

Intermediate firms For the firm that renegotiates the wage in the current period, the value of the occupied job is equal to the sum of the profit of the firm in the current period net of payroll taxes s_t, and the expected future value of the job

$$\begin{aligned} J_t(r) &= x_t^H f(h_t) - \frac{w_t^*}{P_t} h_t (1+s_t) \\ &\quad + E_t \beta_{t,t+1} (1-\rho) \left[\gamma J_{t+1}(w_t^*) + (1-\gamma) J_{t+1}(w_{t+1}^*) \right] \end{aligned} \tag{17}$$

where $x_t^H = \frac{P_{H,t}}{P_t} x_t$ is the relative price of the intermediate sector's good in terms of the CPI[7], and $f(h_t) = z_t {h_t}^{\alpha}$ is match output. The marginal product of labour is accordingly $mpl_t = \alpha z_t {h_t}^{\alpha-1} = \alpha \frac{f(h_t)}{h_t}$. labour-augmenting productivity z_t is identical for all matches and follows

$$\log(z_t) = (1-\nu_z) \log(z) + \nu_z \log(z_{t-1}) + \epsilon_t^z$$

where $\nu_z \in (0,1)$, and $\epsilon_t^z \overset{iid}{\sim} N(0, \sigma_z^2)$. The value to the firm of an open vacancy is

$$\begin{aligned} V_t &= -\kappa + E_t \beta_{t,t+1} q_t^F \left[\gamma J_{t+1}(w_t) + (1-\gamma) J_{t+1}(w_{t+1}^*) \right] \\ &\quad + E_t \beta_{t,t+1} \left(1 - q_t^F\right) V_{t+1} \end{aligned} \tag{18}$$

[6] Accordingly, the average surplus from working is $H_{x,t+1} = \gamma H_{t+1}(w_t \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w} \right) \right]) + (1-\gamma) E_t H_{t+1}(w_{t+1}^*)$. If the worker starts working in a firm that is not allowed to renegotiate, he will get last period's *average* wage partially indexed to inflation. This is because in the one firm - one worker setup of this paper also firms in new matches are new, they cannot have negotiated a contract wage in the previous period.

[7] The firm surplus is here expressed in terms of consumer prices, as opposed to producer prices, to avoid confusion in computing the wage bargaining solution. As a result, the intermediate firms get the nominal price $P_{H,t} x_t$ for their product while they value this revenue in terms of the CPI. This creates a channel for CPI-PPI differences to enter the wage bargaining problem as explained in section 2.3. See also Faia et al. (2010) for discussion on this subject.

The value of a vacancy consists of a fixed hiring cost κ, and of the expected value from future matches. In equilibrium, all profit opportunities from new jobs are exploited so that the equilibrium condition for the supply of vacant jobs is $V_t = 0$. With each firm having only one job, profit maximization is equivalent to this zero-profit condition for firm entry. Setting the equation for V_t as zero in every period gives

$$\frac{\kappa}{q_t^F} = E_t \beta_{t,t+1} \left[\gamma J_{t+1}(w_t) + (1-\gamma) J_{t+1}(w_{t+1}^*) \right] \tag{19}$$

This vacancy posting condition equates the marginal cost of adding a worker (the real cost times the mean duration of a vacancy) to the discounted marginal benefit from a new worker. After taking into account the free entry condition, the firm surplus reduces to J_t.

Multiperiod bargaining set up Unlike with period-to-period bargaining, in the presence of staggered contracting, firms and workers have to take into account the impact of the contract wage on the expected future path of firm and worker surplus. Accordingly, the first order condition for wage-setting is given by

$$\eta \Delta_t J_t(r) = (1-\eta) \Sigma_t H_t(r) \tag{20}$$

where the partial derivatives of the surplus equations w.r.t. the wage, $\Delta_t = P_t \frac{\partial H_t(r)}{\partial w_t}$ and $\Sigma_t = -P_t \frac{\partial J_t(r)}{\partial w_t}$, denote the effect of a rise in the *real* wage on the worker surplus and (minus) the effect of a rise in the real wage on the firm's surplus respectively (see Appendix for details)

$$\Delta_t = h_t (1-\tau_t) + E_t \beta_{t,t+1} (1-\rho) \gamma \Delta_{t+1} \tag{21}$$

$$\Sigma_t = h_t (1+s_t) + E_t \beta_{t,t+1} (1-\rho) \gamma \Sigma_{t+1} \tag{22}$$

These expressions can be interpreted as the discounting factors for the worker and the firm for evaluating the value of the future stream of wage payments. As wage contracts extend over multiple periods, agents have to take into account also the *future* probabilities of not being allowed to renegotiate the wage, or of not surviving exogenous destruction. In the limiting case of efficient bargaining, $\gamma = 0$, the partial derivatives of the surpluses w.r.t. the wage reduce to $\Delta_t = h_t (1-\tau_t)$, and $\Sigma_t = h_t (1+s_t)$, and the first order condition reduces to its period-by-period counterpart $\eta (1-\tau_t) J_t = (1-\eta)(1+s_t) H_t$. The worker and the firm have, accordingly, the following effective bargaining weights: $\frac{\eta}{(1+s_t)}$ and $\frac{(1-\eta)}{(1-\tau_t)}$.

In the symmetric one firm - one worker setup, used in this paper, the discounting factors under the staggered bargaining regime would be equal across agents unless the possible changes in distortionary taxes over time were breaking this symmetry[8]. If taxes were held constant, the two discounting factors would be effectively the same, just weighted with the relevant constant labour tax rate [9]. As a result, the first order condition for wage-setting would have the same form as with period-by-period bargaining $\eta(1-\tau)J_t = (1-\eta)(1+s)H_t$, and the effective bargaining weights would again be accordingly $\frac{\eta}{(1+s)}$ for the worker and $\frac{(1-\eta)}{(1-\tau)}$ for the firm. So, proportional tax rates influence the *division* of the total surplus from a job in equilibrium, irrespective of the bargaining horizon, a standard result from the labour market matching literature (see Pissarides, 2000, Chapter 9). More specifically, both the worker's and the firm's marginal tax rate effectively reduce the worker's relative bargaining power, and consequently his share of the surplus.

However, when staggered bargaining is combined with the possibility of changing labour tax rates over time, workers and firms have to take into account the future path of taxation in their negotiating behaviour, and labour taxes also enter the discounting factor equations of agents. The corresponding effective bargaining weights of agents[10] now depend, in addition to the negotiation power parameter, on labour taxes both contemporaneously and through their effect on the agents' discounting factors.

As is apparent from the loglinearized forms of the discounting factors, presented in Appendix A.2, the increase in the worker's labour tax decreases the discounting factor of the worker and the increase in the employer's labour tax increases its discounting factor. As a result, following an increase in either tax, firms place relatively more weight on the future than workers. The implication for the effective bargaining weights is, in addition to shifting bargaining power from workers to firms contemporaneously, that the expectation of future tax increases further increases the discounting factor of firms relative to that of the workers and thus increases the effective bargaining power of firms relative to that of the workers. The effect of distortionary taxes on the division of match surplus is thus amplified by staggered bargaining.

Given that the probability of wage adjustment is i.i.d., and all matches at renegotiating firms end up with the same wage w_t^*, the evolution of the nominal *average hourly* wage in the economy can be expressed as a convex combination of the contract wage and the average wage across the matches that do not renegotiate.

$$w_{t+1} = (1-\gamma)\, w_{t+1}^* + \gamma \int_0^{n_t} \frac{w_{it}}{n_t} di \tag{23}$$

[8] In Gertler and Trigari (2009), this is not the case. Differences in the worker's and the firm's optimization perspectives, a "horizon effect", arises because large firms take into account possible changes in future hiring rates.

[9] $\Delta_t = (1-\tau)\, E_t \sum_{s=0}^{\infty} \beta_{t,t+s} (1-\rho)^s \gamma^s h_{t+s}$ for the worker, and $\Sigma_t = (1+s)\, E_t \sum_{s=0}^{\infty} \beta_{t,t+s} (1-\rho)^s \gamma^s h_{t+s}$ for the firm.

[10] $\frac{\eta}{\eta(1+s_t)+(1-\eta)\frac{\Sigma_t}{\Delta_t}(1-\tau_t)}$ for the worker and $\frac{(1-\eta)}{\eta\frac{\Delta_t}{\Sigma_t}(1+s_t)+(1-\eta)(1-\tau_t)}$ for the firm.

Wage dynamics The staggered bargaining framework has implications on the behaviour of workers and firms, and on wage determination. To describe wage dynamics in the presence of staggered contracting, we develop loglinear expressions for the relevant wage equations. The approach is in the spirit of Gertler, Sala and Trigari (2008), and is presented in detail in Appendix A.4. The contract wage is solved by first linearizing the Nash first order condition

$$\widehat{J}_t(r) + \widehat{\Delta}_t = \widehat{H}_t(r) + \widehat{\Sigma}_t \tag{24}$$

and then plugging into the FOC the value equations and discounting factors for the worker and the firm respectively in their loglinearized form. The resulting contract wage is

$$\widehat{w}_t^* = [1-\iota]\,\widehat{w}_t^0(r) + \iota E_t \widehat{w}_{t+1}^* \tag{25}$$

where $\iota = \overline{\beta}(1-\rho)\gamma$. This is the optimal wage set at time t *by all matches that are allowed to renegotiate their wage.* As is usual with Calvo contracting, the optimal wage depends on a wage target $w_t^0(r)$ and next period's optimal contract wage. The weight put on each of these components depends on the steady state discounting factor and on the probabilities of job survival $(1-\rho)$ and non-renegotiation γ. As the probability of not being able to renegotiate the wage approaches zero, $\gamma \longrightarrow 0$, i.e. we approach the period-by-period bargaining case, ι approaches zero, $\iota \longrightarrow 0$, and the contract wage, w_t^*, approaches the period-by-period Nash wage.

Unlike in the more conventional set up of New Keynesian models, where Calvo wage contracting is combined with a monopolistic supplier of labour, the target wage here also includes a spillover effect that brings about additional rigidity into wage dynamics on top of that implied by the Calvo scheme alone. Gertler and Trigari (2009) show how spillover effects result from wage bargaining. The target wage can be decomposed into two parts

$$\widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H \Gamma E_t \left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right] \tag{26}$$

where $\varphi_H \Gamma = \frac{(1-\eta)\beta q^w}{(1-\iota)}$ is the spillover effect. The spillover coefficient is positive, indicating that when the expected average market wage $E_t\widehat{w}_{t+1}$ is higher than the expected contract wage $E_t\widehat{w}_{t+1}^*$, (corresponding to unusually good labour market conditions) this raises the target wage in the negotiations. Thus, wage rigidity and the resulting employment dynamics are not only a product of staggered wage setting, but also of the spillover effects from the Nash bargaining process.

The spillover-free component of the target wage is of exactly the same form than the period-by-period negotiated wage (presented in Appendix A.3), only adjusted for the multiperiod discounting factors.

$$\begin{aligned}\widehat{w}_t^0 \;=\;& \varphi_x\left(\widehat{x^H}_t + \widehat{mpl}_t\right) + \varphi_m \widehat{mrs}_t + \varphi_H E_t\left(\widehat{q}_{t+1}^W + \widehat{H}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) \\ & -\varphi_h \widehat{h}_t - \varphi_s \widehat{s}_t + \varphi_\tau \widehat{\tau}_t + \varphi_{\tau^c}\widehat{\tau}_t^c + \varphi_D E_t\left[\widehat{\Sigma}_{t+1} - \widehat{\Delta}_{t+1}\right] + \widehat{P}_t\end{aligned} \tag{27}$$

As its period-by-period counterpart, the spillover-free target wage depends on what the worker contributes to the match and on his opportunity cost. Note that, due to the formulation of the relative price $\widehat{x^H}_t$, whenever domestic production prices rise more than the CPI, this creates an upward pressure on the target wage. An increase in the employee's labour tax increases the target wage whereas an increase in the employer's labour tax lowers the target wage. The target wage depends positively on the difference between the firm's and the worker's discount factor because while an increase in the firm's relative discounting factor decreases the target wage through an increase in the relative effective bargaining power of firms, the change in the relative discounting factors also has the effect - by the Nash first order condition - of decreasing the expected surplus of the worker, thereby increasing his wage demand in the current period.

Finally, combining all the relevant elements of the wage bargaining outcome: the contract wage, the average wage and the target wage, yields a second-order difference equation for the evolution of the economy's average wage (see Appendix A.4 for derivation)

$$\widehat{w}_t = \lambda_b \widehat{w}_{t-1} + \lambda_0 \widehat{w}_t^0 + \lambda_f E_t \widehat{w}_{t+1} \tag{28}$$

Due to staggered contracting, the average wage in the economy $\widehat{w}_t$ depends on the lagged wage $\widehat{w}_{t-1}$, the spillover-free target wage $\widehat{w}_t^0$, and the expected future wage $E_t \widehat{w}_{t+1}$. The longer is the average duration of wage contracts, i.e. the larger is the non-renegotiation parameter γ, the more weight gets the lagged wage component in wage determination.

4.3.3 Determining hours of work

While matches are restrained to renegotiate the wage with a given exogenous probability, hours per worker can be *renegotiated at each point in time.* With efficient Nash bargaining, optimal hours of work can be found from the following first order condition obtained by differentiating the Nash maximand w.r.t hours

$$(1-\tau_t)\, x_t^H f_{h,t} = (1+s_t)\, \frac{g'(h_t)}{\Lambda_t}$$

where $f_{h,t}$ is, as before, the marginal product of the labour input i.e. hours. Using the expressions for the production and utility functions, this can be written as

$$(1-\tau_t)\, x_t^H mpl_t = (1+s_t)\, mrs_t \left(1+\tau_t^c\right) \tag{29}$$

This optimality condition equates the value of the marginal product to the marginal rate of substitution between work and leisure, and resembles to the corresponding condition in a competitive labour market. However, with labour market frictions, while the hourly wage is such that the marginal cost to the worker from working is equal to the marginal gain to the firm, neither of these measures needs to be equal to the wage. It is important to observe that the optimality condition for hours determines the optimal hours per worker, i.e. the intensive margin of labour adjustment. This individual labour input of a worker is determined *irrespective of the wage.* But the model also allows for labour adjustment in the number of workers, as defined by the vacancy posting condition and the matching function.

4.4 Final good firms

There are two types of final goods firms. One produces private consumption goods and the other type of final goods firm produces public consumption goods[11].

4.4.1 Private consumption good

The private consumption good is a composite of intermediate goods distributed by a continuum of monopolistically competitive wholesale firms at home and abroad. Wholesale firms, their products and prices are indexed by $i \in [0,1]$. Final good firms operate under perfect competition and purchase both domestically produced intermediate goods $y_{H,t}(i)$ and imported intermediate goods $y_{F,t}(i)$. They minimize expenditure subject to the following aggregation technology

$$C_t = \left[(1-W)^{\frac{1}{\varpi}} \left(\left[\int_0^1 y_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \right)^{\frac{\varpi-1}{\varpi}} + W^{\frac{1}{\varpi}} \left(\left[\int_0^1 y_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \right)^{\frac{\varpi-1}{\varpi}} \right]^{\frac{\varpi}{\varpi-1}} \tag{30}$$

where ϖ measures the trade price elasticity, or elasticity of substitution between domestically produced intermediate goods and imported intermediate goods in the production of final goods for given relative prices, and W is the weight of imports in the production of final consumption goods. The parameter $\varepsilon > 1$ is the elasticity of substitution across the differentiated intermediate goods produced and distributed within a country.

The optimization problem determining the allocation of expenditure between the individual varieties of domestic and foreign intermediate goods yields the following demand curves facing each wholesale firm

$$y_{H,t}(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_{H,t} \tag{31}$$

$$y_{F,t}(i) = \left(\frac{p_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} Y_{F,t} \tag{32}$$

where $P_{H,t}$ and $P_{F,t}$ are the aggregate price indexes for the domestic and foreign intermediate goods respectively

$$P_{H,t} = \left[\int_0^1 p_{H,t}(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \tag{33}$$

[11] This is a standard assumption in New Open Economy Macro Models that assess fiscal policy. E.g. in Obstfeld and Rogoff's (1996) extension of the Redux model, government spending is introduced as a basket of public consumption goods aggregated in the same way as that of private consumption goods.

$$P_{F,t} = \left[\int_0^1 p_{F,t}(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \tag{34}$$

To determine the optimal allocation between the domestic and imported intermediate goods, the final good firm minimizes costs $P_{H,t}Y_{H,t} + P_{F,t}Y_{F,t}$ subject to its production function or aggregation constraint. This yields the demands for the domestic and foreign intermediate good *bundles* by domestic final good producers

$$Y_{H,t} = (1 - W) \left(\frac{P_{H,t}}{P_t} \right)^{-\varpi} C_t \tag{35}$$

$$Y_{F,t} = W \left(\frac{P_{F,t}}{P_t} \right)^{-\varpi} C_t \tag{36}$$

where P_t is the home country's aggregate price index, or consumption price index

$$P_t = \left((1 - W) P_{H,t}^{1-\varpi} + W P_{F,t}^{1-\varpi} \right)^{\frac{1}{1-\varpi}} \tag{37}$$

At the level of individual intermediate goods the law of one price holds[12]. That, together with the assumption that the weight of the home country good in the foreign consumer price index is infinitesimally small, implies that $P_{F,t}$ is equal to the foreign CPI P_t^* (see Galí and Monacelli, 2008).

4.4.2 Public consumption good

The public consumption good is composed of only domestic intermediate goods $g_t(i)$. This assumption implies full home bias in government spending. This simplifying assumption can be supported by the observation from input-output tables that the use of foreign intermediate goods in government spending is significantly lower than in private consumption.

$$G_t = \left[\int_0^1 g_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{38}$$

Each wholesale firm i selling intermediate goods to the public consumption good producer faces the following demand schedule

$$g_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} G_t \tag{39}$$

[12]Note, however, that due to home bias in consumption the basket of consumed goods may differ in the two areas, and therefore purchasing power parity does not hold.

4.5 Wholesale firms and price setting

The wholesale firms buy the homogeneous intermediate goods at nominal price $p_{H,t}x_t$ per unit and transform them one-to-one into the differentiated product. As in most models that incorporate labour market matching into the NK framework, the price setting decision is separated from the wage setting decision to maintain the tractability of the model[13]. Price rigidities arise at the wholesale level while search frictions and wage rigidity only affect directly the intermediate goods sector.

There is Calvo-type stickiness in price-setting and the relative price of intermediate goods x_t (in terms of producer prices) coincides with the real marginal cost faced by wholesale firms. In each period, the wholesale firm can adjust its price with a constant probability $1-\xi$ which implies that prices are fixed on average for $\frac{1}{1-\xi}$ periods. The wholesale firm's optimization problem is to maximize expected future discounted profits by choosing the sales price $p_{H,t}(i)$, taking into account the pricing frictions and the demand curve they face. It is assumed that the wholesale firm sells the home-country intermediate goods for the same price for domestic and foreign final goods producers, and for the domestic government.

The expected future discounted profit of a wholesale firm that reoptimizes at t is

$$E_t \sum_{s=0}^{\infty} \xi^s \beta_{t,t+s} \left[\left(\frac{p_{H,t}(i)}{P_{H,t+s}} \right) y_{t+s}(i) - x_{t+s} y_{t+s}(i) \right] = 0 \tag{40}$$

where $y_t(i)$ is the demand of firm i's product by domestic private consumption good firms, foreign private consumption good firms and the domestic government as outlined in the previous section

$$y_t(i) = y_{H,t}(i) + y^*_{H,t}(i) + g_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y^D_t$$

where Y^D_t stands for total demand for domestic intermediate goods. All wholesale firms are identical except that they may have set their current price at different dates in the past. However, in period t, if they are allowed to reoptimize their price, they all face the same decision problem and choose the same optimal price $p^*_{H,t}$. Using the definition of the discount factor and rearranging, the first order condition can be written as

$$E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[(1-\varepsilon) \left(\frac{p^*_{H,t}}{P_{H,t+s}} \right) + \varepsilon x_{t+s} \right] \left(\frac{1}{p^*_{H,t}} \right) \left(\frac{p^*_{H,t}}{P_{H,t+s}} \right)^{-\varepsilon} Y^D_{t+s} = 0 \tag{41}$$

which can be solved for $\frac{p^*_{H,t}}{P_{H,t}}$ to yield the following pricing equation

$$\frac{p^*_{H,t}}{P_{H,t}} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} x_{t+s} \left(\frac{P_{H,t+s}}{P_{H,t}} \right)^{\varepsilon} Y^D_{t+s}}{E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left(\frac{P_{H,t+s}}{P_{H,t}} \right)^{\varepsilon - 1} Y^D_{t+s}} \tag{42}$$

[13] Some contributions merge the intermediate and retail sectors so that there are interactions between wage and price setting at the level of the individual firm. E.g. Christoffel et al. (2009) assess the implications of that specification for inflation dynamics.

where $\frac{\varepsilon}{\varepsilon-1} = \mu$ is the flexible-price markup. This is the standard Calvo result. In the absence of price rigidity, the optimal price would reduce to a constant markup over marginal costs. Log-linearizing the FOC around the steady state yields the New Keynesian Phillips Curve where domestic inflation depends on marginal costs and expected future inflation

$$\widehat{\pi}_{H,t} = \nu \widehat{x}_t + \beta E_t \widehat{\pi}_{H,t+1} \tag{43}$$

where $\nu = \frac{(1-\xi)(1-\xi\beta)}{\xi}$.

4.6 The government

The public sector's role in this economy is to collect taxes and use them to finance unemployment benefits and lump-sum transfers as well as government spending G_t. The government budget is balanced in each period with the help of lumpsum transfers. In terms of consumer prices, the government's real budget constraint thus reads as

$$\frac{TR_t}{P_t} = n_t \frac{w_t}{P_t} h_t(\tau_t + s_t) + \tau_t^c C_t - \frac{P_{H,t}}{P_t} G_t - bu_t \tag{44}$$

The per period lumpsum transfers thus depend positively on tax revenue from labour taxes or consumption taxes. On the other hand, transfers have to be cut if government spending or unemployment benefit expenditure increases.

Later, to assess the role of countercyclical distortionary taxes, we use a simple rule whereby the chosen tax variable reacts to the deviation of output from its steady state level.

$$TAX_t = \overline{TAX} - \Omega_T \left(Y_t - \overline{Y} \right) \tag{45}$$

where $TAX_t = \tau_t, s_t$ and Ω_T is the output sensitivity of the tax instrument. Increases in productivity increase output and permit to cut taxes but, conversely, in bad times characterized by low productivity, tax rates are raised. Note that this modelling strategy for introducing countercyclical labour taxes differs from Burda and Weder (2010), but is consistent with their empirical finding from OECD data that the cyclical components of the payroll tax rate and of GDP are negatively correlated.

4.7 Equilibrium

For each intermediate good, supply must equal total demand. The demand for good i is, as shown previously, $y_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_t^D$, where Y_t^D is total demand for domestic intermediate goods by domestic and foreign final goods firms and the domestic government. Using the expressions for the demands for domestic intermediate good *bundles* derived previously, this can be written as

$$y_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left\{(1-W)\left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t + W\left(\frac{P_{H,t}}{P_t^*}\right)^{-\varpi} C_t^* + G_t\right\} \tag{46}$$

Following Galí and Monacelli (2008) defining an index for aggregate domestic demand $Y_t^D = \left[\int_0^1 y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ allows us to rewrite this as

$$Y_t^D = (1-W)\left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t + W\left(\frac{P_{H,t}}{P_t^*}\right)^{-\varpi} C_t^* + G_t$$

Aggregate demand for domestic intermediate goods has to equal their aggregate supply minus the resources lost to vacancy posting, leading to the home economy's aggregate resource constraint

$$Y_t = (1-W)\left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t + W\left(\frac{P_{H,t}}{P_t^*}\right)^{-\varpi} C_t^* + G_t + \kappa v_t \tag{47}$$

As a demand shock, we will later consider a shock to external demand

$$\log(C_t^*) = (1-\nu_{C^*})\log(C^*) + \nu_{C^*}\log\left(C_{t-1}^*\right) + \epsilon_t^{C^*}$$

where $\nu_{C^*} \in (0,1)$, and $\epsilon_t^{C^*} \overset{iid}{\sim} N(0,\sigma_z^2)$. While the economy's aggregate resource constraint equation states that in equilibrium domestic output has to equal its usage as consumption, exports and government spending, market-clearing in the intermediate good sector also requires

$$Y_t = n_t z_t {h_t}^{\alpha} \tag{48}$$

The net foreign asset position is determined by the trade balance - the difference between domestic output and domestic consumption.

$$\frac{B_t^*}{P_t} - R_{t-1}^* p\left(b_{t-1}^*\right)\frac{B_{t-1}^*}{P_t} = \frac{P_{H,t}}{P_t}Y_t - C_t - \frac{P_{H,t}}{P_t}G_t - \frac{P_{H,t}}{P_t}\kappa v_t \tag{49}$$

This relation is obtained by combining the consumers' budget constraint, the government's budget constraint and the economy's aggregate resource constraint as well as the equation for total dividends accrued to households, i.e. the sum of the real profits in the intermediate and wholesale sectors (expressed in terms of consumer prices)

$$\frac{D_t}{P_t} = \frac{P_{H,t}}{P_t}Y_t - n_t\frac{w_t^*}{P_t}h_t(1+s_t) - \frac{P_{H,t}}{P_t}\kappa v_t \tag{50}$$

5 Model evaluation

5.1 Parameterization and steady state of the model

The parameter values are chosen to reflect the Finnish economy in the period 1994Q1-2010Q3 and are summarized in Table 4. For those parameters where no independent evidence is found from Finnish data, we use conventional values from the business cycle and labour market matching literature.

The quarterly discount factor is $\beta = 0.992$ which corresponds to an annual interest rate of $3,3\%$. The labour supply, or Frish elasticity $(\frac{1}{\phi})$, is set to 0.1. This is in the lower range of values implied by most microeconomic studies, which estimate this elasticity to be between 0 and 0.5. The quarterly separation rate is calibrated at $\rho = 0.06$, corresponding to the average Finnish separation rate in 1994-2010. The labour elasticity of production parameter is set to $\alpha = 0.99$ which implies nearly constant returns to scale in the intermediate goods production sector, and a labour share of 70 percent. The elasticity of matches with respect to unemployment and the efficiency of matching are usual values used in the literature.

Table 4. Parameter values

Parameter	$Value$	Explanation
Preferences		
β	.992	Time-discount factor
ϕ	10	labour supply (Frish) elasticity of 0.1
ϱ	1.5	Risk aversion
$\varkappa$	0.6	External habit persistence
labour market		
α	0.99	labour elasticity of production
σ	0.6	Elasticity of matches w.r.t. unemployment
σ_m	0.6	Efficiency of matching
ρ	0.06	Exogenous quarterly job destruction rate
η	0.6	Bargaining power of workers
b	0.4	Unemployment benefits
z	1.1	Technology, targets output $Y = 1$
Wholesale sector		
ε	6	Elasticity of substitution, markup 20 percent
ξ	0.75	Calvo stickiness of prices, avg duration 4 qrts
$\nu\left(=\frac{(1-\xi)(1-\beta\xi)}{\xi}\right)$	0.085	Coefficient of mc in NK Phillips curve
Final goods sector		
$(1-W)$	0.75	Home bias in final goods production
ϖ	2.50	Trade price elasticity
γ_{b^*}	0.05	Debt-elasticity of interest rates

The unemployment benefit parameter is calibrated at $b = 0.4$, and generates a replacement rate of 66 percent, defined as the ratio of unemployment benefits to average net (after-tax) income from work $\frac{b}{wh(1-\tau)}$. The OECD's "Benefits and Wages" publication (2007) suggests an average net replacement rate over 60 months of unemployment of 70 percent for Finland, averaging over four different family types. The unemployment benefit is not assumed to be proportional to the wage nor to be indexed to inflation. As Christoffel, Kuester and Linzert (2009) note, in labour market matching models, there is a trade-off between obtaining a reasonable labour share and a plausible replacement rate. In the present model, the wage bill is 70 percent, too high compared with the data. On the other hand, this model abstracts from the use of capital as a factor of production, so we deem it more important to get the replacement rate right.

The wholesale sector is calibrated in line with the literature, so that the markup is at a conventional value of $\mu = \frac{\varepsilon}{\varepsilon - 1} = 1.2$. The Calvo parameter is $\xi = 0.75$ on the basis of Christoffel, Kuester and Linzert's (2009) calibration from the Eurosystem Inflation Persistence Network. The average duration of prices is accordingly 4 quarters.

Table 5. Steady state ratios

Variable	Value	Description
Y	1	Output
C	0.70	Consumption
u	0.1	Unemployment rate
κv	0.01	Total vacancy costs
n	0.9	Employment
qw	0.6	Probability of finding a job
qf	0.7	Probability of finding a worker
$b/(wh(1-\tau))$	0.66	Net replacement rate
nwh	0.7	Wage bill
τ^C	0.13	Consumption tax
τ	0.21	labour tax rate on employee
s	0.175	Employers' social security contribution
TR / τ^{LS}	0.03	Lump-sum transfers
G	0.29	Government spending
ρ_z	0.9	Autocorrelation of technology
ϵ_t^G	0.01	Technology shock of one percent

The steady state values of key model variables implied by the current parameterization can be found in Table 5. The steady state equations of the model are, in turn, provided in Appendix A.1. In the steady state, output is normalized to one, so that GDP components can be interpreted directly as percent shares of GDP. The labour force is also normalised to one, and the steady state unemployment level is 10 percent, corresponding to the average Finnish unemployment rate in 1994-2010. A symmetric open economy steady state is assumed where consumption levels are initially the same at home and abroad, and both the trade balance and net foreign asset holdings are zero. As no capital is included in the model, the output components of private consumption and government consumption (and the tiny amount of resources lost to vacancy posting) are scaled so that private consumption accounts for 70 percent of steady state output and government consumption is 29 percent.The steady state tax rates for labour and consumption are computed as ten year historical averages of corresponding tax rates in Finland times the model-implied tax base for each tax category. Accordingly, labour taxes for the employee and the employer respectively amount to 30 percent and 25 percent times the wage bill and the consumption tax rate corresponds to an average of 19 percent times the size of private consumption.

5.1.1 The relative value of non-work to work activities

Model calibration can be critical for the ability of the search and matching model to account for observed labour market fluctuations, i.e. for solving the unemployment volatility puzzle. In particular, the calibration of the value of the non-work to work activities term is of key importance for fitting the model to the data when exploring the effects of technology shocks (see e.g. Shimer, 2010). A sufficiently high relative value of non-work to work helps the model to generate large variations in vacancies and unemployment in response to technology shocks and consistent with business cycle facts, as shown by Hagedorn and Manowski (2008). The idea is that a smaller surplus reacts more strongly to shocks of equal magnitude. Hagedorn and Manowski calibrated this value to 0.95 whereas Shimer (2005) set it at 0.4 interpreting it as only unemployment benefits. In the present model, when the worker is not employed, in addition to getting the unemployment benefit, he also enjoys increased time for leisure. As a result, the value of non-work activities consists, not only of the fixed unemployment benefit term, but also of an additional term that varies in function of hours worked.

The steady state value of non-work to work activities[14] is 0.73, in the midrange of values found in the literature. This value is obtained by calibrating the level of benefits and the elasticity of labour supply to Finnish data and to evidence from microeconometric studies, instead of direct targeting. An important feature of the present model, that increases the value of this parameter compared to e.g. Shimer's initial calibration, is the presence of distortionary labour taxes. labour taxes namely decrease (increase) the relative value of work (non-work). So, calibrating the model to the Finnish economy, including labour taxation makes it a priori a better candidate for being able to replicate observed labour market fluctuation than the standard model with low value of non-work.

In the next section, we show that while the current parameterization brings the model one step closer to matching observed labour market volatilities, it cannot do the job alone unless wage rigidity is assumed.

5.2 Dynamic simulations

The Shimer critique stated, first, that the standard matching model cannot generate enough fluctuations in labour market variables for productivity shocks of plausible magnitude. Second, it pointed out that the standard model has difficulties to match dynamic cross-correlations between labour market variables and output and unemployment. In this section we assess the performance of our DSGE model with matching, that has been built and parameterized to take into account specific features of the Finnish economy, in replicating the Finnish data.

We initially assess three candidates that could bring the model closer to the 1994-2010 data: wage rigidity, countercyclical taxes and countercyclical separations. Then we separately point out how the model should possibly be modified in order to be able to reproduce the kind of volatilities found in the earlier data period 1981-1993.

[14] Defined as $\frac{\frac{g(h)}{\Lambda}-b}{xf(h)-(\tau+s)wh}$

5.2.1 Baseline results

The dynamic simulation of the baseline model economy shows that it cannot reproduce Finnish labour market fluctuations (see Table 6). The model with flexible wages and constant tax rates produces fairly correct cross correlations between labour market variables, e.g. a well-behaved Beveridge curve with the correlation between unemployment and vacancies of −0.781. The problem is that the responses of labour market variables are much too smooth compared to the data and the wage, in turn, adjusts too easily to the productivity shock. The standard deviation of labour market variables relative to productivity is less than a third of what the data indicates, while the volatility of the wage implied by the baseline model is more than twice as high as that found in the data. In addition, the baseline model fails to replicate the correlation of the wage with productivity. While the unemployment volatility puzzle refers specifically to the inability of the standard search and matching model to account for the large volatility of U.S. unemployment and vacancies over the business cycle when fluctuations are driven by technology shocks of plausible magnitude, the same problem arises when we try to match our fairly standard matching model to our data. This is what we call the Finnish unemployment volatility puzzle.

Table 6. Aggregate Statistics

	u	v	v/u	qw	w	z
			FI Economy, 1994-2010			
Relative std. dev.	3.894	7.459	10.801	7.583	0.681	1.000
Autocorrelation	0.956	0.965	0.972	0.965	0.946	0.955
Correlation with z	−0.763	0.803	0.830	0.766	0.519	1.000
			Model Economy, baseline			
Relative std. dev.	1.000	2.045	2.864	1.136	1.318	1.000
Autocorrelation	0.961	0.832	0.919	0.919	0.358	0.890
Correlation with z	−0.735	0.839	0.843	0.843	0.168	1.000
			Model Economy, $\gamma = 0.5$			
Relative std. dev.	2.864	5.955	8.318	3.318	0.773	1.000
Autocorrelation	0.956	0.895	0.935	0.935	0.970	0.890
Correlation with z	−0.805	0.891	0.913	0.913	0.785	1.000
			Model Economy, countercyclical taxes			
Relative std. dev.	3.045	5.909	8.681	3.455	1.681	1.000
Autocorrelation	0.977	0.950	0.970	0.970	0.741	0.890
Correlation with z	−0.791	0.976	0.940	0.940	0.534	1.000
			Model Economy, $\gamma = 0.5$ and countercyclical taxes			
Relative std. dev.	3.000	6.182	8.682	3.455	0.818	1.000
Autocorrelation	0.956	0.895	0.936	0.936	0.973	0.890
Correlation with z	−0.801	0.903	0.920	0.920	0.780	1.000
			Model Economy, countercyclical job separations			
Relative std. dev.	3.909	1.773	3.182	1.273	1.545	1.000
Autocorrelation	0.924	0.218	0.767	0.767	0.123	0.787
Correlation with z	−0.939	−0.766	0.721	0.721	−0.022	1.000

The improvement in technology allows firms to produce the same amount of output with fewer inputs. Because of matching frictions the number of workers cannot be adjusted on impact, and therefore, initially, hours per worker fall. This causes an initial fall in total hours in the economy, replicating the empirical evidence of e.g. Basu, Fernald and Kimball (2006) that positive technology shocks are followed by a short run decline in employment. The improvement in technology also implies higher expected future profits for firms which start to open more vacancies from the next period on. Unemployment starts to fall and total hours start to increase.

On impact, the negotiated wage falls because the price of the intermediate good falls (i.e. the markup of retailers rise) more than the marginal productivity of intermediate goods firms improves, lowering firms' per period profits. As a result, the firms' reservation wage falls. The reason behind this fall in the intermediate goods price is the fall in the marginal rate of substitution. As Blanchard and Galí (2010) emphasize, the behaviour of the marginal rate of substitution is central to the outcome of technology shocks also in a model with labour market frictions. They show that under specific assumptions on preferences and technology, the marginal rate of substitution and marginal productivity increase in the same proportion, leading to no fluctuations in the markup and no change in unemployment.

In the present model, the change in the marginal rate of substitution depends on the change in hours *per worker* and the change in the marginal utility of consumption[15]. With inelastic labour supply along the intensive margin ($\phi = 10$), the fall in hours worked leads to a fall in the disutility of work (rise in the utility of leisure) that is larger than the fall in the marginal utility of consumption, and therefore to a fall in the marginal rate of substitution. The price of intermediate goods is, if there are no changes in distortionary taxes, solely determined by the changes in the marginal rate of substitution and in marginal productivity[16]. With the present calibration, the fall in the marginal rate of substitution is larger than the increase in marginal productivity. Therefore, in this case, the fall in working hours produces initially a larger gain to workers in the form of increased leisure time than the gain to firms from increased productivity.

5.2.2 Wage rigidity

Adding rigidity in wage determination significantly increases the volatility of labour market variables as shown in the third panel of Table 6 and in Figure 11. In this simulation, it is assumed that wages can be renegotiated every two quarters ($\gamma = 0.5$). These results confirm that the quantitative implications of the search and matching model for fluctuations in labour market variables are sensitive to even modest amounts of wage rigidity. When productivity improves but wages are rigid, firms' expected profits rise encouraging them to open more vacancies. The nominal wage reacts much less to changed economic conditions since current conditions now only have a weight of 0.24 in wage determination.

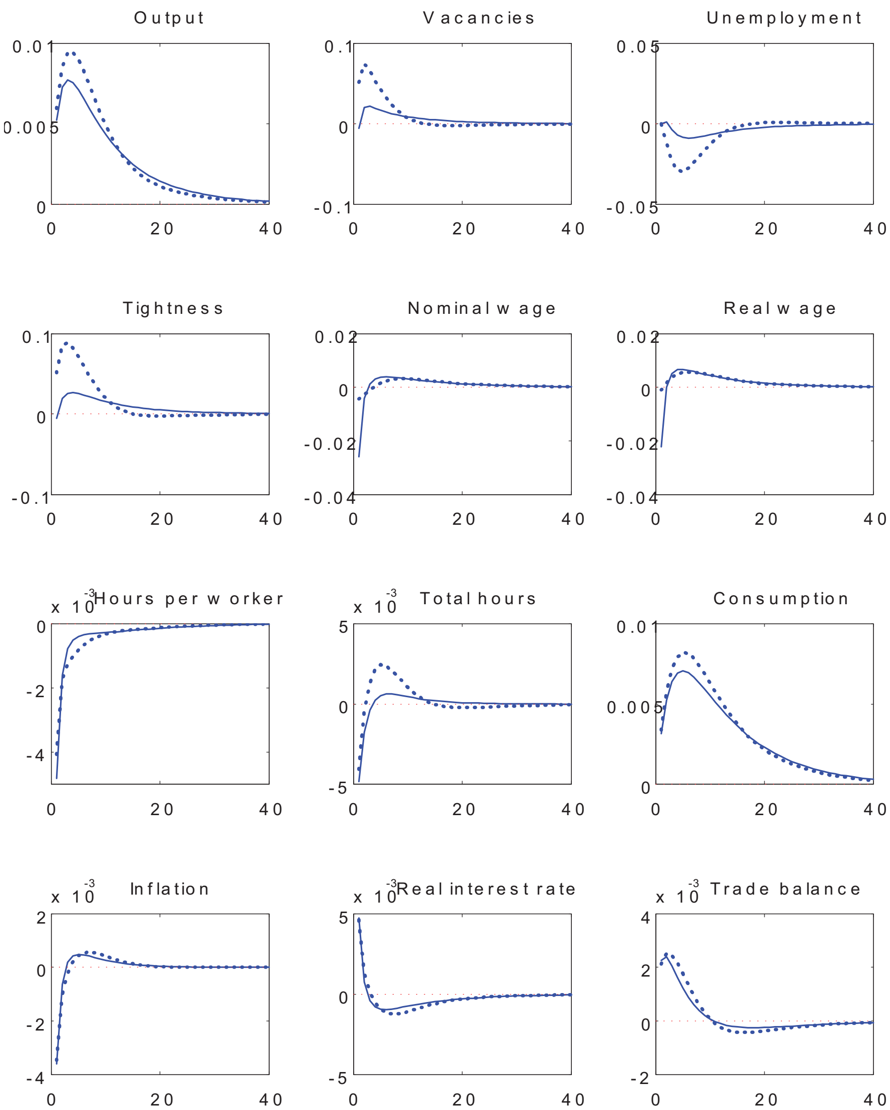
The volatilities of unemployment and vacancies are now fairly close to those observed in Finland in the later part of the observation period. Also the relative volatility of the wage to productivity is close to the value found in

[15] $m\hat{r}s_t = \phi\hat{h}_t - \hat{\lambda}_t$.

[16] $\hat{x}_t^H = m\hat{r}s_t - m\hat{p}l_t$ by the optimality condition for hours worked.

the data for that period, and the same applies for the correlation of the wage with productivity. The Beveridge relation somewhat weakens to -0.755, still staying close to the correlation found in the data.

Figure 11. Impulse responses to a one percent positive technology shock, flexible wages (solid line), rigid wages ($\gamma = 0.5$, dotted line)



It appears that a wage rigidity parameter of approximately $\gamma = 0.59$ would be most consistent with Finnish labour market volatilities in 1994-2010, if no other simultaneous modifications to the model are made to explain the unemployment volatility puzzle. This renegotiation probability of nearly twice a year is much higher than the approximate average wage agreement length in Finland of one year, observed in the few most recent wage agreement rounds.

However, the adjustment period of one year only applies to base pay in negotiated contracts. The adjustment of other wage components such as bonuses probably increases true wage flexibility significantly.

As Gertler and Trigari (2009) note, the elasticity of the wage with respect to labour productivity corresponds to the regression coefficient of log wages on log productivity. This elasticity can also be computed as the product of the correlation between wages and productivity and their relative standard deviations. Gertler and Trigari show that their model with rigid wages, unlike e.g. the small surplus solution of Hagedorn and Manovskii (2008), not only replicates the wage elasticity with respect to productivity found in the data, but also their correlation and relative standard deviations independently.

For the Finnish data from 1994 to 2010, the OLS regression coefficient of log wages on log productivity is 0.35, i.e. smaller than the finding of Gertler and Trigari (2009) from U.S. data of 0.52. The corresponding correlation and relative standard deviation of wages and productivity in the data are 0.519 and 0.681 respectively. Whereas our model with flexible wages implied an elasticity of just 0.13, being the product of a too small correlation of the wage with productivity (0.1 vs. 0.52 in the data) and a too large relative standard deviation (1.318 vs. 0.68 in the data), the model with wage rigidity implies an elasticity of 0.6, i.e. a correlation of 0.8 and relative standard deviation of 0.73. This slightly too high value compared to the OLS coefficient is thus almost entirely due to the fact that the model somewhat exaggerates the correlation of wages with productivity. This feature is natural in a framework where changes in productivity are assumed to be the only source of business cycle fluctuations. More importantly, the model is able to reproduce realistic volatility of both labour market variables and the wage.

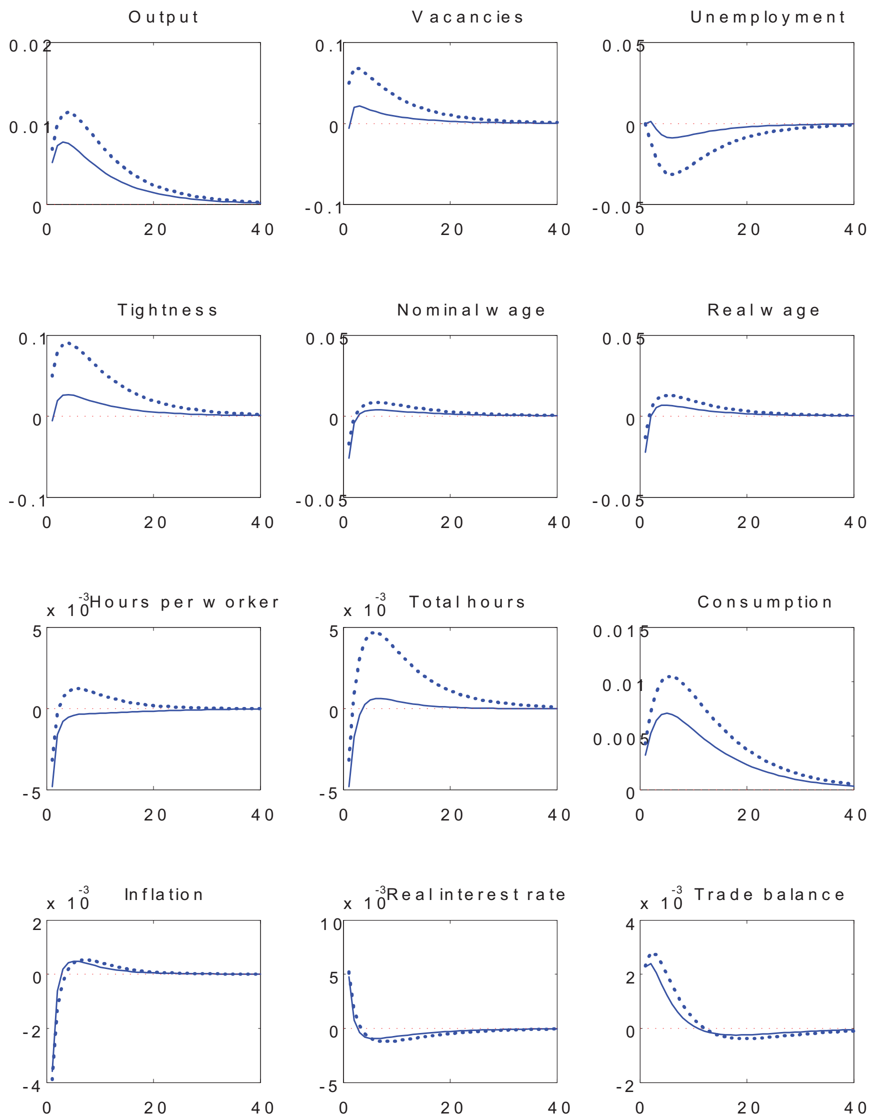
5.2.3 Countercyclical taxes

Burda and Weder (2010) show that introducing countercyclical payroll taxes into an RBC model with matching frictions solves the unemployment volatility puzzle. The assumption of countercyclical taxes generates higher labour market volatilities also in the present model which, however, differs in many ways from their model. Figure 12 shows how the economy's response to the technology shock is altered when tax rates are assumed to depend negatively on output. Tax rates are cut in good times and raised in bad times.

The volatility of labour market variables increases significantly. Firms expected profits rise following the positive shock to technology but now also their labour costs decrease due to the payroll tax cut, further improving their profits and encouraging vacancy creation. The cut in the worker's labour tax increases the relative value of work encouraging labour supply along both the intensive and extensive margins. Hours per worker rise after the initial drop and unemployment falls.

This modelling choice brings the volatilities of unemployment, vacancies and tightness fairly close to those in the Finnish data for 1994-2010 (see fourth panel of Table 6). It also increases the Beveridge relation to -0.873 which is already higher than in the data for that period. But, there are two major drawbacks with this approach.

Figure 12. Impulse responses to a one percent positive technology shock, baseline model (solid line) vs. model with countercyclical taxes (dotted line)



First, alongside the volatility of labour market variables, the volatility of the wage relative to productivity also increases from the baseline case and becomes far too large compared to the data (1.681 vs 0.681). The reason lies in the effect of taxes on match surplus and on the division of that surplus. Lower labour taxes on the worker make him accept a lower wage, and lower payroll taxes on the firm increases its willingness to pay higher wages. The bargaining set thus widens, but these effects are of roughly equal magnitude. However, as discussed previously, changes in distortionary taxes also affect the *division* of match surplus. More specifically, a decrease in the worker's labour tax and a decrease in the firm's payroll tax both have the effect of shifting bargaining power from the firm to the worker. This allows the worker to take home a larger part of the productivity improvement in the form of larger wage increases. The wage thus increases more in response to improved technology than in the baseline model.

In terms of the steady state wage equation, $w = \frac{\eta}{(1+s)}\left[\frac{xmpl}{\alpha} + \frac{\kappa\theta}{h}\right]$ $+\frac{(1-\eta)}{(1-\tau)}\left[\frac{mrs}{(1+\phi)} + \frac{b}{h}\right]$, the shift in relative bargaining power brought about by the changes in taxes can be interpreted as follows. Decreases in labour tax rates increase the weight of the worker's contribution to the match (the first square brackets) and decrease the weight of the worker's outside option (the second square brackets) in wage determination. This implies that economic conditions get a larger weight in wage determination, or that the wage becomes more flexible. This is an opposite effect to the small surplus calibration of Hagedorn and Manovskii (2008) where they chose a very small value for the bargaining power parameter and a very large value of the outside option b, a calibration choice that is known to make the wage more rigid.

Indeed, the decrease in distortionary taxes increases the total surplus from a match. As is known from the unemployment volatility puzzle literature, this larger match surplus reacts less to shocks of equal magnitude. Therefore, while countercyclical taxes amplify the positive response of labour markets to technology shocks through the incentive effects of lower taxation on labour supply and vacancy creation, at the same time, they decrease the volatility of labour market variables relative to that of productivity through the "large surplus effect".

In Burda and Weder's (2010) model the volatility of the wage relative to productivity *decreases* as countercyclical taxes are introduced since they start from a situation with no distortionary taxes at all. As a result, the simultaneous effect of switching to a small surplus model outweighs the effect of lower taxes on the division of that surplus. In the more realistic setup of this paper, the steady state of the model is kept the same across simulations while countercyclical taxes are introduced with the help of simple rules.

The other problem with this exercise is that in order to generate the volatilities reported above, we have to assume a sensitivity of tax rates to changes in output that is far too high compared with what we find in the data. More specifically, in the above simulation, a one percent increase in output leads to a one *percentage point* decrease in tax rates, requiring a tax sensitivity coefficient Ω_T of approximately 5 in equation (47). However, in our data set, we only find a negative correlation between the employee's labour taxes and output of -0.2, implying an elasticity of taxes with respect to output of -0.13, and no significant correlation between the employers' social security contributions and output. This implies a much less clear countercyclical pattern of taxes than that found in OECD data by Burda and Weder (2010) for Finland. The latter calculate that the correlation between employers' and employees' payroll taxes and output in Finland is -0.5. Using the data-consistent elasticity of labour taxes with respect to output, the model is unable to generate significantly larger labour market fluctuations than the baseline model.

To see whether the observed countercyclical behaviour of labour taxes in Finland could be at least a complementary explanation to the unemployment volatility puzzle, we combine rigidity in wage formation with countercyclical taxes on labour (fifth panel in Table 6). This is a unique feature of the current model, but a common feature of real economies. The result is that labour market volatilities are only marginally larger than with wage rigidity alone, and, in addition, this comes with the cost of too high volatility of the wage relative to productivity. This is because these two modelling approaches interact, so that the countercyclical behaviour of taxes counteracts the explicitly added wage rigidity.

More specifically, the effect of taxes on the division of match surplus is amplified by the staggered bargaining framework, as explained previously. As tax rates are cut following the positive technology shock, bargaining power shifts from the firms to the workers contemporaneously, but now also the expectation of further tax cuts increases the discounting factor of workers relative to that of the firms further increasing the relative effective bargaining power of workers. This leads to larger wage increases. Moreover, the larger surplus brought about by lower tax rates reacts less to the initial shock, dampening the volatility of labour market variables.

5.2.4 Cyclical job separations

There is recent evidence on the cyclicality of the inflow into unemployment by e.g. Pissarides (2009). In addition, although we do not find a clear cyclical pattern for the separation rate in our ESS data set, there is some independent evidence of countercyclical job separations in Finland from worker and job flows data (see Ilmakunnas and Maliranta, 2008). Moreover, as this issue has gained considerable attention in the unemployment volatility puzzle, we test whether countercyclical job separation could be one alternative or complementary explanation to the Finnish unemployment volatility puzzle.

We introduce countercyclical separations with the help of a simple rule in the same way as we did with countercyclical taxes. We assume, as in Costain and Reiter (2008), that the exogenous job separation rate depends negatively on the aggregate technology shock, i.e. there are less separations in good times. The sensitivity of the separation rate to variations in productivity is calibrated to match the amount of variation in separations relative to productivity found in our 1994-2010 Finnish data.

The introduction of cyclical job separations contributes substantially to the cyclical volatility of unemployment which many models have been unable to replicate. The results are shown in the last panel of Table 6. The model-implied unemployment volatility now fits the data. However, the fit of the model deteriorates in many other ways. Most importantly, cyclical job separations decrease the volatility of vacancies. The reason is that changes in the separation rate do not affect the *relative* value of unemployment and vacancies, leaving the vacancy-unemployment ratio unchanged. As a result, a larger fall in unemployment following the technology shock, brought about by the countercyclical separation rate, is accompanied also by a smaller rise in vacancies. Wage volatility, in turn, increases driving the model further away from the data.

Cyclical job separations also yield counterfactual negative correlations between productivity and vacancies, and productivity and wages. One additional problem resulting from this modeling choice is that it destroys the Beveridge curve. This is a well-known outcome in the literature (see e.g. Costain and Reiter, 2008) The model-implied correlation of vacancies and unemployment is now 0.588 in contrast with the strong and negative correlation found in the data. We conclude that cyclical job separations together with aggregate productivity shocks do not seem to contribute to the explanation of the unemployment volatility puzzle in the present framework.

Costain and Reiter (2008) further find that combining variable separations with *match-specific* productivity, in turn, enables the model to replicate data fairly well. They note, however, that the latter alternative would have the tendency to increase wage volatility. While it does not seem that countercyclical separations would have had a significant role in the Finnish labour market dynamics in 1994-2010, they could have had a role in our earlier data period including the deep recession of the early 1990's. As we report below, in that period, the volatility of the wage relative to productivity was significantly higher than more recently implying that the possible contribution of cyclical separations to labour market volatility in that period can not be excluded.

5.2.5 Sensitivity with respect to key parameters

Price rigidity One reason for the support for models with nominal rigidities is that they are, unlike RBC models, able to reproduce the empirical evidence of e.g. Basu, Fernald and Kimball (2006) that positive technology shocks are followed by a short run decline in employment. In a standard New Keynesian model with imperfect competition and sticky prices, as productivity improves but prices can only adjust partially, the immediate implication is that the same amount of output can be produced with fewer inputs. In the present model, this effect is seen in an initial downward adjustment of hours worked. Because of matching frictions, the number of workers cannot be adjusted on impact.

If, however, prices were more flexible, the negative response of hours would be smaller, vacancies would start rising immediately and unemployment would fall more rapidly. More flexible prices would thus make labour market variables more responsive to the technology shock, bringing the model one step closer to solving the unemployment volatility puzzle. In contrast, Andrés et al. (2006) find that price rigidity is the single most important factor bringing their model closer to the data. This is because they use the conventional closed economy model closed by a Taylor rule on the interest rate. As we explain in more detail in the next section, in that setup the real interest rate falls on impact. With rigid prices the fall in the real interest rate is drawn out, and the implied increase in the present value of vacancies is stronger.

The open-economy dimension The majority of papers concentrating on the unemployment volatility puzzle approaches the issue with closed-economy models which typically feature a Taylor-type monetary policy rule. This has important implications for the effects of technology shocks. When monetary policy is accommodative, a positive technology shock induces a fall in the real interest rate because the initial fall in prices is - by definition - counteracted with a more than one-for-one fall in the nominal interest rate. In the presence of matching frictions, the real interest rate has, in addition to its importance for determining the intertemporal path of consumption, a central role in shaping the responses of labour market variables to shocks. This is because, with matching frictions, the vacancy posting decision of firms, or labour demand, is forward-looking. A falling real interest rate raises the discounted marginal benefit from new vacancies, encouraging vacancy posting.

In contrast, in the small monetary union member state setup of this paper, the nominal interest rate of the common monetary union central bank does not react to changes in the inflation of the small member state. As the present model is closed with the help of the debt-elastic interest rate assumption, the nominal interest rate only reacts to the external indebtedness of the home country. Moreover, the parameter governing the debt-elasticity of the interest rate is typically small, so that the improved foreign asset position brought about by the positive shock to domestic technology, only marginally lowers the nominal interest rate.

In the present baseline model, prices initially fall more than the nominal interest rate. This causes a rise in the real interest rate which discourages vacancy posting, thus dampening employment fluctuations. On the other hand, the improvement in technology increases the expected profits of firms, so that the net effect of the technology shock on vacancies is positive. Changing the debt-elasticity parameter from 0.05 to 0.005, also a typical value in many open-economy models (see e.g. Corsetti, Meier and Müller, 2009), would make the initial increase in the real interest rate large enough to produce a fall in vacancies.

Another crucial parameter shaping the open-economy's response to a technology shock is the trade elasticity parameter, i.e. the extent to which domestic goods can be substituted for foreign goods when their relative prices change. E.g. Collard and Dellas (2007) show that a technology shock can cause a decline in employment if the elasticity of substitution between domestic and foreign goods is low. In the present model, the initial response of total hours is already negative to start with because of the fall in hours per worker explained above, but the negative effect on employment would be amplified if we assumed a lower trade price elasticity than in our baseline model. In that case the response of vacancies would be negative since the deterioration in the terms of trade[17] discourages domestic production so that output increases less than productivity. As a result, total hours would fall more than in the baseline case.

The home bias parameter is 0.75 in the baseline model implying that the share of imported intermediate goods in domestic final goods production is 0.25. This might be somewhat small for the Finnish economy considering that this parameter applies to private final goods, as public final goods are defined separately to use only domestic intermediate goods in their production. However, raising the share of imported intermediates to 0.50 does not significantly affect the results.

The small surplus calibration Finally, we assess the model's performance against changing two key parameters: the worker's bargaining power and the unemployment benefit to values that were found by Hagedorn and Manovskii (HM, 2008) to reconciliate the matching model with the data without having to add wage rigidity. Increasing the relative value of non-work to work activities

[17] In the present framework, the fall in the domestic price level directly implies a deterioration in the terms of trade because we abstract from exchange rate changes and the rest of the currency union pays the domestic price for domestic exports.

to 0.95 as in HM implies an increase of the unemployment benefit to 0.50. This is enough for the relative value of non-work to work activities to rise to 0.95 because, in the present model, it also consists of the disutility of work and distortionary taxes. The worker's fallback position is thus relatively high even in the baseline model. The worker's bargaining power is fixed at 0.05.

As expected, in these conditions, the "Hagedorn-Manovskii" parameterization does not lead to a dramatic rise in labour market volatilities. This is precisely the result of the above mentioned different construction of the marginal value of work which is already in the baseline case 0.73 when realistic assumptions on the value of leisure and distorting labour taxes are included. The value of work decreases with the introduction of taxes, and that of non-work increases when also the value of leisure is accounted for. At the same time the implied level of benefits remains at values corresponding to the data.

5.2.6 Higher labour market volatilities

The Finnish depression of the early 1990's was so severe that it has earned a place on Reinhart and Rogoff's (2008) list of the "Big Five" postwar rich country large-scale financial crises. A distinguishing feature of that depression was the dramatic increase in unemployment which has been studied by e.g. Koskela and Uusitalo (2004). As we pointed out earlier, the labour market reaction during that depression relative to the experienced fall in productivity was many times larger than in the most recent recession, leading to very different volatilities and cross-correlations than in the later data period studied above.

The summary statistics for unemployment, vacancies, labour market tightness, job-finding, wages, and productivity in 1981-1993 are presented in Table 7.

Table 7: Summary statistics and correlation table for Finland 1981Q1-1993Q3

	u	v	v/u	q^W	w	z
Std deviation	0.339	0.443	0.775	0.469	0.032	0.014
/ Productivity	24.214	31.643	55.357	33.500	2.286	1.000
Autocorrelation	0.916	0.914	0.918	0.914	0.954	0.932
			Correlation			
u Unemployment	1.000	−0.963	−0.988	−0.984	−0.785	−0.288
v Vacancies		1.000	0.993	0.977	0.705	0.324
$v/u = \theta$ Tightness			1.000	0.989	0.747	0.311
q^W Job-finding rate				1.000	0.729	0.273
w Wages					1.000	0.351
z Productivity						1.000

The labour market appears much more volatile in the period 1981-1993 than in more recent years. In absolute terms, vacancies, tightness and job-finding are three times more volatile in 1981-1993 than in 1994-2010, whereas unemployment is almost four times as volatile, and the real wage two times as volatile as in the later part of our data. In contrast, fluctuations in productivity are smaller, implying that in relative terms the differences between the two time periods are even greater.

The much smaller correlation between labour market variables and productivity in 1981-1993 compared with more recent data indicates that other driving forces than productivity changes have been important for labour market fluctuations in that period, as expected. E.g. Koskela and Uusitalo (2004) identify as the most important shocks that caused the deep depression, first, the abrupt fall in exports due to the collapse of Soviet trade. Second, the rise in European interest rates following German unification were combined with a very restrictive domestic monetary policy in Finland that led to a dramatic rise in real interest rates. In addition, they emphasize that, owing to the rapid restructuring of the economy, job destruction was very rapid during the first years of the 1990's contributing to the fast rise of unemployment. Moreover, the structural change created a large mismatch problem on the labour market also reflected as an outward movement of the Beveridge curve.

As to institutions, they could not, in Koskela and Uusitalo's (2004) view, have caused the crisis but they are responsible for the slow recovery. They identify as the most important institutions in this respect, distinguishing Finland from other European countries that did not experience a similar deterioration in their labour market situation, the unemployment benefit system and wage bargaining. Concerning the latter, Gorodnichenko et al. (2009) find that during the Finnish "Great Depression" nominal wages were totally downward rigid leading to only small adjustments in the real wage compared to the gravity of the downturn. At that time, collective agreements were typically negotiated for two to three years at a time, though actual wage changes may have occurred more frequently. One specific feature of the unemployment benefit system, in turn, was the generosity of benefits to 55-59 years old. Hakola and Uusitalo (2001) argue that the extended duration of unemployment benefits for older workers was responsible for a large share of the rise in aggregate unemployment.

Most of these features that characterize the Finnish economy in the earlier part of our observation period are also likely to amplify labour market fluctuations in the context of our model economy. We show this for foreign demand shocks and more rigid wages, and discuss the role of interest rates, the unemployment benefit system and larger job destruction.

The monetary policy regime An additional important factor that probably contributes to the magnitude of labour market flows is the monetary policy regime. This is one crucial difference when we compare the Finnish economy before the mid 1990's and after.

In the years 1981-1993, Finland conducted its own monetary policy. In the 1980's monetary policy can be characterised by inflation targeting although Finland did not have an official inflation target until 1993. E.g. Peura (1999) estimates that the path of short term interest rates in that period is consistent with a Taylor-rule type monetary policy rule with an inflation sensitivity coefficient equal to 1.87. During the depression years, however, Finnish monetary policy can hardly be characterised by any standard monetary policy rule. In that period the other monetary policy objective, namely supporting the prevailing fixed exchange rate regime, began to dominate, and consequently the nominal interest rate was kept at high levels despite the fall in output.

To assess the role of the monetary policy regime for our results, we replace the debt-elastic interest rate assumption in our model by a standard Taylor-type monetary policy rule (see e.g. Woodford, 2003). The rule dictates that the nominal interest rate is always adjusted more than one-for-one to changes in inflation. We use a parameterization usual in the existing literature, where the interest rate smoothing parameter is 0.8 and the inflation sensitivity coefficient is equal to 1.5.

We showed earlier that the way the open-economy dimension is modelled and calibrated is central to the outcome of technology shocks because it affects the response of the real interest rate. As we described in the section on the open-economy dimension of the current model, if the nominal interest can not be adjusted to changing inflation, the real interest rate *rises* and dampens the labour market effects of the technology shock.

In contrast, when the nominal rate aims at stabilizing inflation, as when monetary policy follows a Taylor rule, the *fall* in the real interest rate, by the same logic, amplifies the labour market response to the positive technology shock as shown in the third panel of Table 8. This would apply, of course, symmetrically to a negative technology shock. This result lends support to the hypothesis that differences in the monetary policy regime could be one explanation for larger labour market fluctuations in Finland before membership in the Euro area. However, the volatility comparisons under these different regimes show that the effect is tiny compared to the huge volatilities experienced during the depression in Finland. Of course, this sensitivity test only assesses the effect of changing the monetary policy regime keeping the exchange rate fixed. Therefore, in reality, a large currency union member state with influence on the common monetary policy is a closer reference than the small economy not belonging to the currency area. Nevertheless, it shows again how important the determination of the real interest rate is for the response of the economy to shocks.

In addition to its effects on the labour market through the interest rate channel, membership in a monetary union can also in other ways influence the functioning of the labour market. E.g. Calmfors and Johansson (2006) argue that membership in a monetary union implies stronger incentives for nominal wage flexibility in the form of wage indexation and shorter contract length than non-membership, offering an alternative adjustment mechanism that can substitute for national monetary policy. While there could be other factors also affecting contract length, there is both direct evidence of shorter wage adjustment frequency in Finland in the later part of the observation period, and indirect support for increased wage flexibility in the form of less volatile labour market variables.

Wage rigidity To see what the effects of a longer contract length are, we experiment with a wage rigidity parameter $\gamma = 0.83$, implying an average duration of wage contracts of one and a half years. The fourth panel in Table 8 shows the results. While they confirm our earlier conclusion that wage rigidity is an efficient way of increasing the volatility of labour market variables in this kind of framework, this model specification still just accounts for half of the volatilities of vacancies and labour market tightness found in the data and for only a third of the volatilities observed in unemployment and job-finding figures. Also, the relative volatility of the wage to productivity is too low and the correlation of the wage with productivity and other labour market variables too high.

In addition, the model with this much wage rigidity generates a negative effect of vacancies, tightness and unemployment to the technology shock after approximately eight quarters. In particular, as the improvement in technology starts to fade but wages remain above their steady state level, the expected profits of firms turn negative and they start decreasing vacancies. Unemployment increases and labour market tightness decreases. This weakening in the persistence of the labour market response when wages are rigid is stronger than in Gertler and Trigari (2009). One possible reason is the absence, in the present model, of adjustment costs of hiring.

Foreign demand shocks We add a negative shock to foreign demand of domestic intermediate goods C^* to mimic in a simple way the collapse of Soviet trade experienced in Finland in 1991, and combine that shock with a negative technology shock. The shock corresponds to a decrease in foreign export demand of nearly 17 percent. According to Gorodnichenko et al. (2009) the share of Soviet exports to total exports was 18 percent before the collapse.

Just adding the negative export demand shock does not dramatically magnify the fluctuations of labour market variables. The relative standard deviations become approximately twice as large as in the baseline case, a very moderate effect compared with the data. Gorodnichenko et al. (2009) argue that wage rigidity is key to understanding the strong amplification of the Soviet trade collapse shock. Indeed, combining wage rigidity with the export demand shock increases significantly the magnitude of labour market responses to the foreign demand shock in the present model as shown in the fifth panel of Table 8.

While the relative volatilities of labour market variables and productivity are now close to those found in the data, the model understates the magnitudes of the declines in output and employment. This depends partly on the fact that, unlike in the model economy, in reality the foreign demand shock was not temporary and it was not possible to redirect the Soviet trade to other markets because of its special nature. Freystätter (2011) argues that the collapse of Soviet-Finnish trade would be best understood as a capital obsolescence shock as opposed to a conventional trade shock. Combined with balance sheet constraints on firms she shows that the shock leads to a deep and persistent downturn similar to that experienced in Finland in the early 1990's. Moreover, in our model economy, the real interest rate quickly adjusts back to its steady state level whereas in the Finnish case the dramatic and persistent increase in real interest rates played an important role in magnifying the recession.

Other factors E.g. Honkapohja and Koskela (1999) emphasize the negative effect from defending the fixed exchange rate regime with high interest rates. While the modelling of such a monetary policy objective is beyond the scope of this paper, we note in this context our earlier finding that the real interest rate is the key factor that determines, not only the path of consumption, but also has important implications for labour market variables in an environment where labour demand is a forward looking decision. A persistently high real interest rate would decrease the expected value of filled vacancies, discouraging firms from opening new vacancies.

E.g. Koskela and Uusitalo (2004) stress the importance of the unemployment benefit system for the persistence of unemployment in the Finnish depression. The duration of benefits was generally longer for all workers in the

1980's and 1990's than today, and the system was especially favourable to older workers who could enter the so-called unemployment retirement tunnel at the age of 55. In the present model environment it is not possible to compare different benefit durations because benefits are assumed to be paid infinitely, but Costain and Reiter (2008) show in a similar model that longer unemployment benefit duration decreases the employment surplus, and consequently increases the cyclical volatility of unemployment.

Furthermore, the behaviour of job separations could have had a role in shaping labour market dynamics in our earlier data period. Our calculations of the separation rate on the basis of ESS data show that there were more separations and more fluctuation in separations in the earlier data period. We first assess the sensitivity of the results to the calibration of the level of the *constant* separation rate, and raise the separation rate to 0.20, an unconventionally high value in the literature but one that corresponds to the Finnish average quarterly separation rate in the first half of our observation period. A higher separation rate has, expectably, the effect of making the labour market response to the technology shock less favourable. Unemployment initially rises a little but then starts to fall and vacancies fall on impact. A higher equilibrium separation rate implies a shorter average duration of jobs ($\frac{1}{\rho}$). When a positive productivity shock hits the economy, with a shorter average duration of jobs, firms expect to profit from the increased productivity for a shorter time period. Therefore, firms' expected profits rise less than with a longer average job duration, and, as a consequence, they open less vacancies. A higher equilibrium separation rate thus implies less volatile labour market behaviour relative to that of productivity. In the present model, however, the differences to the baseline case are fairly small.

Table 8. Aggregate Statistics

	u	v	v/u	qw	w	z
			FI Economy, 1981-1993			
Relative std. dev.	24.214	31.643	55.357	33.500	2.286	1.000
Autocorrelation	0.916	0.914	0.918	0.914	0.954	0.932
Correlation with z	−0.288	0.324	0.311	0.273	0.351	1.000
			Model Economy, baseline			
Relative std. dev.	1.000	2.045	2.864	1.136	1.318	1.000
Autocorrelation	0.961	0.832	0.919	0.919	0.358	0.890
Correlation with z	−0.735	0.839	0.843	0.843	0.168	1.000
			Model Economy, monetary policy follows Taylor rule			
Relative std. dev.	1.263	2.641	3.682	1.472	1.018	1.000
Autocorrelation	0.954	0.863	0.924	0.924	0.774	0.891
Correlation with z	−0.855	0.966	0.986	0.986	0.756	1.000
			Model Economy, $\gamma = 0.83$			
Relative std. dev.	9.267	17.533	26.133	10.467	0.867	1.000
Autocorrelation	0.984	0.963	0.978	0.978	0.908	0.741
Correlation with z	−0.393	0.710	0.615	0.615	0.917	1.000
			Model Economy, foreign demand shock and $\gamma = 0.66$			
Relative std. dev.	17.955	36.636	51.909	20.773	0.864	1.000
Autocorrelation	0.962	0.768	0.899	0.899	0.980	0.889
Correlation with z	−0.190	−0.216	0.218	0.218	0.840	1.000

As we argued in the previous section, countercyclical separations would increase the cyclical volatility of unemployment significantly. In the evaluation of the 1994-2010 labour market volatilities this explanation was rejected because it generated too much wage volatility and decreased the volatility of vacancies. In the earlier data period, however, the wage was much more volatile relative to productivity than our other model simulations suggest.

6 Concluding remarks

This paper describes Finnish labour market volatilities in a DSGE framework with labour market matching frictions. We contribute with a new data set to the unemployment volatility puzzle discussion initiated by Shimer (2005) and offer some plausible explanations for the Finnish case.

We first describe the time series behaviour of unemployment, vacancies, labour market tightness, the job-finding rate, wages and productivity in Finland in the period from 1981 to 2010. We find that the relative volatility of labour market variables to productivity is roughly twice as high as in the U.S., meaning that fitting a standard search and matching model to this data is a priori likely to be challenging. However, the deep recession of the early 1990's associated with huge fluctuations in employment clearly dominates the behaviour of Finnish labour market series, and therefore requires a separate explanation. As a result, in the subsequent analysis, we concentrate on the period 1994-2010 and establish some benchmark volatilities to be matched that are an order of magnitude smaller than in the longer data sample. We also calculate benchmark cross-correlations which are used to assess the model's ability to match the data.

We find that our baseline search and matching model extended with distortionary labour taxes is not able to reproduce the relatively smooth behaviour of wages and relatively volatile behaviour of employment found in the data. We therefore confirm the presence of an unemployment volatility puzzle in Finland. Although the calibration of distortionary taxes to realistic levels does take the model one step closer to being able to explain the observed labour market fluctuations by decreasing the relative after-tax value of non-work.

We add rigidity in wage determination with the help of the staggered bargaining framework developed by Gertler and Trigari (2009), and find that this is the most promising candidate for explaining Finnish labour market volatilities. While accounting for the countercyclical behaviour of taxes also brings the model-implied volatilities close to the data, they worsen the model in some other dimensions. Likewise, countercyclical job separations increase the volatility of unemployment but fail to replicate some other key volatilities and correlations in the data. The advantage with the model with wage rigidity is precisely that it generates, in addition to volatilities that match the data, realistic comovements between different variables.

We also discuss the much larger labour market volatilities of the period 1981Q1-1993Q4 in Finland. We find that most of the specific features that characterized the Finnish economy in that period are such that they also increase the magnitude of the responses of labour market variables to shocks in our model economy. In that context we consider e.g. lower foreign export demand, a different monetary regime and more rigid wages.

A general consideration raised in the literature and pointed out by e.g. Gertler and Trigari (2009) in the context of the unemployment volatility puzzle is that it does not make sense to try to exactly quantitatively match the model moments and correlations under a single (technology) shock to unconditional data moments. To retain comparability to the unemployment volatility puzzle literature, that is however what we have done in this paper. We have mostly not taken stance on what other possible shocks drive aggregate economic activity in Finland but this is certainly a relevant concern for a small open economy. This is left for future research.

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A Appendix

A.1 Steady state of the model economy

Euler equation

$$\beta = \frac{1}{R}$$

Marginal utility of consumption

$$\lambda = (C - \varkappa C)^{-\varrho}$$

Marginal utilility of wealth

$$\Lambda = \frac{\lambda}{(1 + \tau^c)}$$

Interest rate on foreign bonds

$$R = R^*$$

FOC of retail firm

$$x = \frac{1}{\mu} = \frac{\varepsilon - 1}{\varepsilon}$$

Matches

$$m = \sigma_m u^\sigma v^{1-\sigma}$$

Employment

$$\rho n = m$$

Unemployment

$$u = 1 - n$$

Probability of finding a worker

$$q^F = \frac{m}{v}$$

Probability of finding a job

$$q^W = \frac{m}{u}$$

Labour market tightness

$$\theta = \frac{v}{u}$$

FOC for hours

$$(1-\tau)\, x^H mpl = (1+s)\, mrs(1+\tau^c)$$

where

$$mpl = \alpha z h^{\alpha-1} \text{ and } mrs = \frac{\delta h^{\phi}}{\lambda} \text{ and } x^H = x$$

Economy-wide resource constraint

$$\begin{aligned} Y &= (1-W)\, C + WC^{*} + G + \kappa v \\ &= C + G + \kappa v, \text{ in the symmetric steady state} \end{aligned}$$

Government budget constraint

$$TR = nwh(\tau + s) + \tau^c C - G - bu$$

Market clearing / aggregate output

$$Y = nzh^{\alpha}$$

Wage

$$w = \frac{\eta}{(1+s)} \left[\frac{xmpl}{\alpha} + \frac{\kappa\theta}{h}\right] + \frac{(1-\eta)}{(1-\tau)} \left[\frac{mrs(1+\tau^c)}{(1+\phi)} + \frac{b}{h}\right]$$

Job creation condition

$$\kappa{=}q^F \beta J$$

where the firm surplus

$$J = \frac{1}{1-\beta\,(1{-}\rho)} \left[xzh^{\alpha} - wh\,(1+s)\right]$$

Worker surplus

$$H = \frac{1}{1-\beta\,(1-\rho-q^W)} \left[wh\,(1-\tau) - \frac{mrsh(1+\tau^c)}{(1+\phi)} - b\right]$$

Worker discount factor

$$\overline{\Delta} = \frac{\overline{h}\,(1-\overline{\tau})}{1-\overline{\beta}\,(1-\overline{\rho})\,\gamma}$$

Firm discount factor

$$\overline{\Sigma} = \frac{\overline{h}\,(1+s)}{1-\overline{\beta}\,(1-\overline{\rho})\,\gamma}$$

A.2 Model dynamics

The dynamics of the model are obtained by taking a log-linear approximation around a deterministic steady state.

Euler equation

$$\widehat{\Lambda}_t = E_t \left(\widehat{\Lambda}_{t+1} + \widehat{R}_t - \widehat{\pi}_{t+1} \right)$$

Shadow value of wealth

$$\widehat{\Lambda}_t = \widehat{\lambda}_t - \frac{\overline{\tau}^c}{(1+\overline{\tau}^c)} \widehat{\tau}_t^c$$

Marginal utility of consumption

$$\widehat{\lambda}_t = -\frac{\varrho}{(1-\varkappa)} \left(\widehat{C}_t - \varkappa \widehat{C}_{t-1} \right)$$

Interest rates

$$\widehat{R}_t = \widehat{R}_t^* - \gamma_{b^*} \widehat{b}_t^*$$

Matching function

$$\hat{m}_t = \sigma \hat{u}_t + (1-\sigma) \hat{v}_t$$

Employment dynamics

$$\hat{n}_t = (1-\rho) \hat{n}_{t-1} + \frac{\bar{m}}{\bar{n}} \hat{m}_{t-1}$$

Unemployment

$$\widehat{u}_t = -\frac{1-\overline{u}}{\overline{u}} \hat{n}_t$$

Transition probabilities

$$\hat{q}_t^F = \hat{m}_t - \hat{v}_t$$

$$\hat{q}_t^W = \hat{m}_t - \hat{u}_t$$

labour market tightness

$$\widehat{\theta}_t = \hat{v}_t - \hat{u}_t$$

FOC for hours worked

$$\hat{x}_t^H = m\hat{r}s_t - m\hat{p}l_t + \frac{\overline{\tau}}{(1-\overline{\tau})}\widehat{\tau}_t + \frac{\overline{s}}{(1+\overline{s})}\widehat{s}_t + \frac{\overline{\tau}^c}{(1+\overline{\tau}^c)}\widehat{\tau}_t^c$$

where

$$\hat{x}_t^H = \widehat{x}_t + \widehat{P}_{H,t} - \widehat{P}_t$$

$$m\hat{p}l_t = \hat{z}_t - (1-\alpha)\,\hat{h}_t$$

and

$$m\hat{r}s_t = \phi\hat{h}_t - \hat{\lambda}_t$$

New Keynesian Phillips Curve

$$\widehat{\pi}_{H,t} = \nu\widehat{x}_t + \beta E_t\widehat{\pi}_{H,t+1}$$

where $\widehat{\pi}_{H,t} = \widehat{P}_{H,t} - \widehat{P}_{H,t-1}$ is domestic inflation

First order condition for wage setting

$$\widehat{J}_t(w_t^*) + \widehat{\Delta}_t = \widehat{H}_t(w_t^*) + \widehat{\Sigma}_t$$

Firm surplus

$$\begin{aligned}
\widehat{J}_t(w_t^*) \;=\; & \frac{\overline{xmplh}}{\alpha\overline{J}}\left(\widehat{x^H}_t + \widehat{mpl}_t + \widehat{h}_t\right) - \frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}\left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t\right) - \frac{\overline{wh}\overline{s}}{\overline{J}}\widehat{s}_t \\
& + \overline{\beta}\,(1-\rho)\,E_t\left(\widehat{J}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) \\
& - \frac{\overline{\beta}\,(1-\rho)\,\gamma}{1-\overline{\beta}\,(1-\rho)\,\gamma}\frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}E_t\left(\widehat{w}_t^* - \widehat{w}_{t+1}^*\right)
\end{aligned}$$

Worker discount factor

$$\widehat{\Delta}_t = (1-\iota)\,\widehat{h}_t - \frac{(1-\iota)\,\overline{\tau}}{(1-\overline{\tau})}\widehat{\tau}_t + \iota E_t\left(\widehat{\beta}_{t,t+1} + \widehat{\Delta}_{t+1}\right)$$

Worker surplus

$$\begin{aligned}
\widehat{H}_t(w_t^*) \;=\; & \frac{\overline{wh}\,(1-\overline{\tau})}{\overline{H}}\left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t\right) - \frac{\overline{wh}\overline{\tau}}{\overline{H}}\widehat{\tau}_t - \frac{\overline{mrs}\overline{h}\overline{\tau}^c}{(1+\phi)\,\overline{H}}\widehat{\tau}_t^c \\
& - \frac{1}{1+\phi}\frac{\overline{mrs}\overline{h}\,(1+\overline{\tau}^c)}{\overline{H}}\left(\widehat{mrs}_t + \widehat{h}_t\right) - \overline{\beta}\overline{q}^W E_t\left(\hat{q}_t^W + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1}\right) \\
& + \overline{\beta}\,(1-\rho)\,E_t\left(\widehat{H}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) \\
& + \frac{\overline{\beta}\,(1-\rho)\,\gamma}{1-\overline{\beta}\,(1-\rho)\,\gamma}\frac{\overline{wh}\,(1-\overline{\tau})}{\overline{H}}E_t\left(\widehat{w}_t^* - \widehat{w}_{t+1}^*\right)
\end{aligned}$$

Firm discount factor

$$\widehat{\Sigma}_t = (1-\iota)\,\widehat{h}_t + \frac{(1-\iota)\,\overline{s}}{(1+\overline{s})}\widehat{s}_t + \iota E_t\left(\widehat{\beta}_{t,t+1} + \widehat{\Sigma}_{t+1}\right)$$

Optimal contract wage

$$\widehat{w}_t^* = [1-\iota]\,\widehat{w}_t^0\,(r) + \iota E_t \widehat{w}_{t+1}^*$$

Target wage

$$\widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H \Gamma E_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]$$

Spillover-free target wage

$$\begin{aligned}\widehat{w}_t^0 \;\; = \;\; & \varphi_x\left(\widehat{x^H}_t + \widehat{mpl}_t\right) + \varphi_m \widehat{mrs}_t + \varphi_H E_t\left(\widehat{q}_t^W + \widehat{H}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) \\ & -\varphi_h \widehat{h}_t - \varphi_s \widehat{s}_t + \varphi_\tau \widehat{\tau}_t + \varphi_{\tau^c}\widehat{\tau}_t^c + \varphi_D E_t\left[\widehat{\Sigma}_{t+1} - \widehat{\Delta}_{t+1}\right] + \widehat{P}_t\end{aligned}$$

Average wage

$$\widehat{w}_t = (1-\gamma)\,\widehat{w}_t^* + \gamma \widehat{w}_{t-1}$$

or

$$\widehat{w}_t = \lambda_b \widehat{w}_{t-1} + \lambda_0 \widehat{w}_t^0 + \lambda_f E_t \widehat{w}_{t+1}$$

Vacancy posting condition

$$-\hat{q}_t^F = E_t\left(\widehat{J}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) + \frac{\gamma}{1-\iota}\frac{\overline{wh}\,(1+\overline{s})}{\overline{J}} E_t\left(\widehat{w}_{t+1}^* - \widehat{w}_t\right)$$

Trade balance

$$\widehat{TB}_t = \widehat{Y}_t - \overline{C}\widehat{C}_t + W\overline{C}\left(\widehat{P}_{H,t} - \widehat{P}_t^*\right) - \overline{G}\widehat{G}_t - \kappa\overline{v}\,(\widehat{v}_t)$$

Economy-wide resource constraint

$$\begin{aligned}\widehat{Y}_t \;\; = \;\; & (1-W)\,\overline{C}\widehat{C}_t + W\overline{C}^*\left(\widehat{C}_t^* + \varpi \widehat{P}_t^*\right) \\ & -\left[(1-W)\,\overline{C}\varpi W\right]\left(\widehat{P}_{H,t} - \widehat{P}_t^*\right) - \overline{C}\varpi W \widehat{P}_{H,t} + \overline{G}\widehat{G}_t + \kappa\overline{v}\,(\widehat{v}_t)\end{aligned}$$

Consumer price index

$$\widehat{P}_t = (1 - W)\,\widehat{P}_{H,t} + W\widehat{P}_t^*$$

Government budget constraint

$$\begin{aligned}
\overline{TR}(\widehat{TR}_t - \widehat{P}_t) &= \overline{nw}\overline{h}\,(\overline{\tau} + \overline{s})\,(\widehat{n}_t + \widehat{w}_t - \widehat{P}_t + \widehat{h}_t) + \overline{nw}\overline{h}\overline{\tau}\widehat{\tau}_t \\
&\quad + \overline{nw}\overline{h}\overline{s}\widehat{s}_t + \overline{\tau}^c\overline{C}\left(\widehat{\tau}_t^c + \widehat{C}_t\right) - \overline{Rb}(\widehat{R}_{t-1} + \widehat{b}_{t-1} - \widehat{\pi}_t) \\
&\quad - \overline{G}\left(\widehat{P}_{H,t} - \widehat{P}_t + \widehat{G}_t\right) - b\overline{u}\widehat{u}_t
\end{aligned}$$

Market clearing / aggregate output

$$\widehat{Y}_t = \widehat{n}_t + \hat{z}_t + \alpha\hat{h}_t$$

A.3 Data appendix

In this appendix we compare and explain the most important differences between the data set we use: the Employment Service Statistics which covers registered vacancies and unemployment, and the labour Force Survey.

One implication of the limited coverage of the ESS is, for example, that the implied proportion of vacancies to job seekers, i.e. labour market tightness is very low. Compared to Statistics Finland's data the ESS data set records more unemployed and less vacancies. It seems, however, that the differences between the two series are mostly differences in levels which do not significantly change over time. Moreover, the cyclical properties for the series are similar irrespective of the source of the data, and should therefore not affect the calculated volatilities.

A.3.1 Unemployment

A comparison of the ESS data on unemployed job seekers with Statistics Finland's corresponding measure shows (see Figure A1) that there is a level difference in the amount of unemployed in favour of the ESS data (solid line). This is mainly because the criteria for being classified as unemployed in the labour Force Survey are tighter. They require that the person has been actively seeking a job within the previous four weeks. Therefore many, e.g. elderly unemployed waiting for retirement or students that are not actively searching for a job are not classified as unemployed in the LFS. On the other hand, a student about to enter the labour market within a few weeks may be classified as unemployed in the LFS.

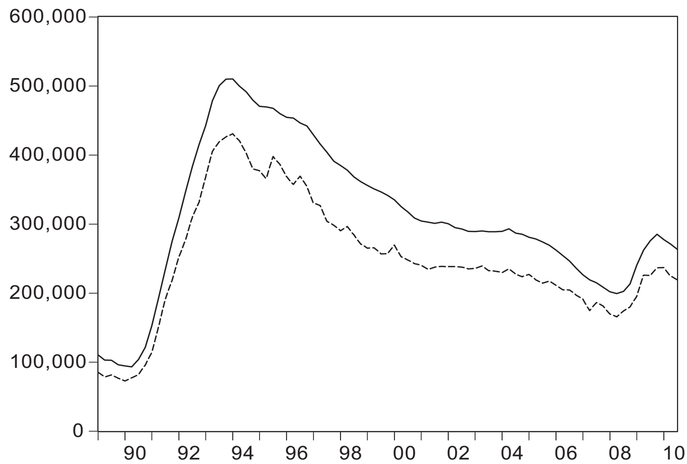

Figure A1. Quarterly seasonally adjusted unemployment, 1989Q1-2010Q3.

A.3.2 Vacancies

The ESS data on open vacancies cover only a part of the labour market and the coverage may vary according to the sector and the business cycle. Statistics Finland, in turn, publishes quarterly figures for vacancies based on a survey on working aged (15 to 74 years) Finns. The advantage with this figure would be that it includes, not only registered vacancies, but also open positions which are advertised in the internet or major newspapers, and vacancies which firms fill using their own contacts. Unfortunately, the latter data source has only been available since 2003 and does not, therefore, allow for systematic analysis of that time series.

To confirm our conclusions on the volatility of vacancies, Figure A2 plots the time series for both available vacancy measures from 2003 forward. As with unemployment figures, the vacancy series of the Employment Service Statistics (solid line) is somewhat smoother than that of Statistics Finland (dotted line). A comparison of the cyclical properties of the two series shows, however, that their volatility is very similar, at least for the short time period the measure for all vacancies has been recorded. The log-deviations from H-P trend are 0.12 for the ESS series and 0.13 for that of the LFS.

Figure A2. Quarterly seasonally adjusted vacancies, 2003Q1 - 2010Q3.

A.4 Period-by-period Nash bargaining

In the standard MP model, it is assumed that total match surplus, $S_t = (W_t - U_t) + (J_t - V_t)$, the sum of the worker and firm surpluses is shared according to efficient Nash bargaining where wages and hours are negotiated simultaneously. The firm and the worker choose the wage and the hours of work to maximize the weighted product of the worker's and the firm's net return from the match.

$$\max_{w,h} (H_t)^{\eta} (J_t)^{1-\eta}$$

where $0 \leq \eta \leq 1$ is the relative measure of workers' bargaining strength.

The worker surplus gets the following form.

$$H_t = W_t - U_t = \frac{w_t^*}{P_t} h_t (1 - \tau_t) - \frac{g(h_t)}{\Lambda_t} - b + E_t \beta_{t,t+1} \left(1 - \rho - q_t^W\right) H_{t+1}$$

and the firm surplus is (after taking into account the free entry condition $V_t = 0$)

$$J_t = x_t^H f(h_t) - \frac{w_t^*}{P_t} h_t (1 + s_t) + E_t \beta_{t,t+1} (1 - \rho) J_{t+1}$$

The first-order condition for wage-setting is

$$\begin{aligned} \eta \frac{\partial H_t}{\partial w_t} J_t &= (1 - \eta) \frac{\partial J_t}{\partial w_t} H_t \\ &\iff \eta (1 - \tau_t) J_t = (1 - \eta)(1 + s_t) H_t \end{aligned}$$

which would, without taxes, correspond to the simple surplus splitting result

where the total surplus from the match is shared according to the bargaining power parameter η.

The optimality condition for wage-setting can be rewritten as a wage equation that includes only contemporaneous variables by substituting the value equations into the Nash FOC, and making use of the expressions for the production and utility functions.

$$\begin{aligned} \frac{w_t^*}{P_t} &= \frac{\eta}{(1 + s_t)} \left[\frac{x_t^H mpl_t}{\alpha} - \frac{r_t^k k_t}{h_t} \right] \\ &+ \frac{(1 - \eta)}{(1 - \tau_t)} \left[\frac{mrs_t (1 + \tau_t^c)}{(1 + \phi)} + \frac{b}{h_t} + \frac{q_t^W}{h_t} E_t \beta_{t,t+1} H_{t+1} \right] \\ &+ \frac{\eta}{h_t} E_t \beta_{t,t+1} \left(1 - \rho_{t+1}\right) J_{t+1} \left[\frac{1}{(1 + s_t)} - \frac{(1 - \tau_{t+1})}{(1 - \tau_t)} \frac{1}{(1 + s_{t+1})} \right] \end{aligned}$$

where w_t is the nominal hourly wage in a match.

The wage equation is a convex combination of what the worker contributes to the match (the first square brackets) and what he has to give up in terms of disutility from supplying hours of work plus a term that accounts for possible changes in tax rates over time. Since workers and firms are homogeneous and all matches adjust their wages every period, they will all choose the same wage. The economy's wage bill is this wage rate times the total number of hours worked in the economy. It is clear from the wage equation that the introduction of taxes works to decrease the worker's relative effective bargaining power from η to $\frac{\eta}{(1+s_t)}$. Consequently, economic conditions get a smaller weight in wage determination.

A.5 Dynamics with wage rigidity

The derivation of the wage under staggered contracting follows Gertler, Sala and Trigari (GST) (2008). The Nash first order condition is in this case

$$\eta \Delta_t J_t\left(w_t^*\right) = (1-\eta)\, \Sigma_t H_t\left(w_t^*\right)$$

where the effect of a rise in the *real* wage on the worker's surplus is

$$\begin{aligned}
\Delta_t &= P_t \frac{\partial H_t\left(w_t\right)}{\partial w_t} \\
&= h_t\left(1-\tau_t\right) + E_t \beta_{t,t+1}(1-\rho)\, \gamma P_{t+1} \frac{\partial H_{t+1}\left(w_t\right)}{\partial w_t} \\
&= h_t\left(1-\tau_t\right) \\
&\quad + E_t \beta_{t,t+1}(1-\rho)\, \gamma \left[h_{t+1}\left(1-\tau_{t+1}\right) + E_t \beta_{t+1,t+2}(1-\rho)\, \gamma P_{t+2} \frac{\partial H_{t+2}\left(w_t\right)}{\partial w_t}\right] \ldots \\
&= E_t \sum_{s=0}^{\infty} \beta_{t,t+s}(1-\rho)^s \gamma^s h_{t+s}\left(1-\tau_{t+s}\right)
\end{aligned}$$

$$\Longleftrightarrow \Delta_t = h_t\left(1-\tau_t\right) + E_t \beta_{t,t+1}(1-\rho)\, \gamma \Delta_{t+1}$$

And similarly for the firm

$$\Sigma_t = -P_t \frac{\partial J_t\left(w_t\right)}{\partial w_t} = h_t\left(1+s_t\right) + E_t \beta_{t,t+1}(1-\rho)\, \gamma \Sigma_{t+1}$$

The dynamic contract wage equation is solved by first linearizing the FOC

for wage setting, and then substituting the linearized worker and firm surplus equations as well as the above discount factors in their loglinearized form (see GST (2008) for more details).

First order condition

$$\widehat{J}_t(w_t^*) + \widehat{\Delta}_t = \widehat{H}_t(w_t^*) + \widehat{\Sigma}_t$$

where the loglinear forms of the discount factors are

$$\widehat{\Delta}_t = \left[1-\overline{\beta}(1-\rho)\gamma\right] \widehat{h}_t - \frac{\left[1-\overline{\beta}(1-\rho)\gamma\right] \overline{\tau}}{(1-\overline{\tau})} \widehat{\tau}_t + \overline{\beta}(1-\rho)\, \gamma E_t\left(\widehat{\beta}_{t,t+1} + \widehat{\Delta}_{t+1}\right)$$

$$\widehat{\Sigma}_t = \left[1 - \overline{\beta}(1-\rho)\gamma\right]\widehat{h}_t + \frac{\left[1 - \overline{\beta}(1-\rho)\gamma\right]\overline{s}}{(1+\overline{s})}\widehat{s}_t + \overline{\beta}(1-\rho)\gamma E_t\left(\widehat{\beta}_{t,t+1} + \widehat{\Sigma}_{t+1}\right)$$

and the expressions for $\widehat{J}_t(w_t^*)$ and $\widehat{H}_t(w_t^*)$ can be found as follows

Worker surplus The worker surplus can be written as

$$\begin{aligned} H_t(w_t^*) &= \frac{w_t^*}{P_t}h_t(1-\tau_t) - \left[\frac{g(h_t)}{\Lambda_t} + b + E_t\beta_{t,t+1}q_t^W H_{x,t+1}\right] \\ &\quad + E_t\beta_{t,t+1}(1-\rho)H_{t+1}(w_{t+1}^*) \\ &\quad + \gamma E_t\beta_{t,t+1}(1-\rho)\left[H_{t+1}(w_t^*) - H_{t+1}(w_{t+1}^*)\right] \end{aligned}$$

In the last term, evaluate the expression $E_t\left[H_{t+1}(w_t^*) - H_{t+1}(w_{t+1}^*)\right]$

$$\begin{aligned} & E_t\left[H_{t+1}(w_t^*) - H_{t+1}(w_{t+1}^*)\right] \\ = \; & E_t\left(\frac{w_t^*}{P_{t+1}} - \frac{w_{t+1}^*}{P_{t+1}}\right)h_{t+1}(1-\tau_{t+1}) \\ & + (1-\rho)\gamma E_t\beta_{t+1,t+2}\left[H_{t+2}(w_t^*) - H_{t+2}(w_{t+1}^*)\right] \end{aligned}$$

When linearized, this expression gets the following form

$$\begin{aligned} & E_t\left[\widehat{H}_{t+1}(w_t^*) - \widehat{H}_{t+1}(w_{t+1}^*)\right] \\ = \; & \frac{\overline{wh}(1-\overline{\tau})}{\overline{H}}E_t\left(\widehat{w}_t^* - \widehat{w}_{t+1}^*\right) + \overline{\beta}(1-\rho)\gamma E_t\left[\widehat{H}_{t+2}(w_t^*) - \widehat{H}_{t+2}(w_{t+1}^*)\right] \end{aligned}$$

Iterating forward this can be further simplified to yield

$$E_t\left[\widehat{H}_{t+1}(w_t^*) - \widehat{H}_{t+1}(w_{t+1}^*)\right] = \frac{1}{1-\overline{\beta}(1-\rho)\gamma}\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}}E_t\left(\widehat{w}_t^* - \widehat{w}_{t+1}^*\right)$$

With the help of the above expression, the loglinear formulation of the worker surplus is found to be

$$\begin{aligned}
\widehat{H}_t &= \frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}\left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t\right) - \frac{\overline{wh}\overline{\tau}}{\overline{H}}\widehat{\tau}_t \\
&-\frac{1}{1+\phi}\frac{\overline{mrs}\overline{h}\left(1+\overline{\tau}^c\right)}{\overline{H}}\left(\widehat{mrs}_t + \widehat{h}_t\right) \\
&-\frac{\overline{mrs}\overline{h}\overline{\tau}^c}{\left(1+\phi\right)\overline{H}}\widehat{\tau}_t^c - \overline{\beta}\overline{q}^W E_t\left(\hat{q}_t^W + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1}\right) \\
&+\overline{\beta}\left(1-\rho\right)E_t\left(\widehat{H}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) \\
&+\frac{\overline{\beta}\left(1-\rho\right)\gamma}{1-\overline{\beta}\left(1-\rho\right)\gamma}\frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}E_t\left(\widehat{w}_t^* - \widehat{w}_{t+1}^*\right)
\end{aligned}$$

Firm surplus The firm surplus can be written as

$$\begin{aligned}
J_t(w_t^*) &= x_t^H f\left(h_t\right) - \frac{w_t^*}{P_t}h_t\left(1+s_t\right) + E_t\beta_{t,t+1}\left(1-\rho\right)J_{t+1}(w_{t+1}^*) \\
&+\gamma E_t\beta_{t,t+1}\left(1-\rho\right)\left[J_{t+1}(w_t^*) - J_{t+1}(w_{t+1}^*)\right]
\end{aligned}$$

In the last term, evaluate the expression $E_t\left[J_{t+1}(w_t^*) - J_{t+1}(w_{t+1}^*)\right]$

$$\begin{aligned}
& E_t\left[J_{t+1}(w_t^*) - J_{t+1}(w_{t+1}^*)\right] \\
=& -E_t\left(\frac{w_t^*}{P_{t+1}} - \frac{w_{t+1}^*}{P_{t+1}}\right)h_{t+1}\left(1+s_{t+1}\right) \\
& +\left(1-\rho\right)\gamma E_t\beta_{t+1,t+2}\left[J_{t+2}(w_t^*) - J_{t+2}(w_{t+1}^*)\right]
\end{aligned}$$

When linearized this expression gets the following form

$$\begin{aligned}
& E_t\left[\widehat{J}_{t+1}(w_t^*) - \widehat{J}_{t+1}(w_{t+1}^*)\right] \\
=& -\frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}}E_t\left[\widehat{w}_t^* - \widehat{w}_{t+1}^*\right] + \overline{\beta}\left(1-\rho\right)\gamma E_t\left[\widehat{J}_{t+2}(w_t^*) - \widehat{J}_{t+2}(w_{t+1}^*)\right]
\end{aligned}$$

Iterating forward this can be further simplified to yield

$$E_t\left[\widehat{J}_{t+1}(w_t^*) - \widehat{J}_{t+1}(w_{t+1}^*)\right] = -\frac{1}{1-\overline{\beta}\left(1-\rho\right)\gamma}\frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}}E_t\left[\widehat{w}_t^* - \widehat{w}_{t+1}^*\right]$$

Finally, as with the worker surplus, the loglinear formulation of the firm surplus can be found with the help of the above expression

$$
\begin{aligned}
\widehat{J}_t &= \frac{\overline{xmplh}}{\alpha \overline{J}}\left(\widehat{x}_t^H + \widehat{mpl}_t + \widehat{h}_t\right) - \frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}\left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t\right) - \frac{\overline{wh}\overline{s}}{\overline{J}}\widehat{s}_t \\
&\quad + \overline{\beta}\,(1-\rho)\,E_t\left(\widehat{J}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) \\
&\quad + \frac{\overline{\beta}\,(1-\rho)\,\gamma}{1-\overline{\beta}\,(1-\rho)\,\gamma}\frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}E_t\left(\widehat{w}_{t+1}^* - \widehat{w}_t^*\right)
\end{aligned}
$$

The Contract wage Inserting the expressions for the worker and firm surpluses, as well as those for the discount factors, into the linearized FOC yields (after collecting the wage terms to the left-hand side and using the Nash FOC for next period)

$$
\begin{aligned}
\Longrightarrow &\quad \left[\frac{\overline{wh}\,(1-\overline{\tau})}{\overline{H}} + \frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}\right]\widehat{w}_t^* \\
&\quad + \frac{\iota}{(1-\iota)}\left[\frac{\overline{wh}\,(1-\overline{\tau})}{\overline{H}} + \frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}\right]E_t\left(\widehat{w}_t^* - \widehat{w}_{t+1}^*\right) \\
= &\quad \frac{\overline{xmplh}}{\alpha \overline{J}}\left(\widehat{x}_t^H + \widehat{mpl}_t\right) + \frac{1}{1+\phi}\frac{\overline{mrsh}\,(1+\overline{\tau}^c)}{\overline{H}}(\widehat{mrs}_t) + \frac{\overline{mrsh}\overline{\tau}^c}{(1+\phi)\,\overline{H}}\widehat{\tau}_t^c \\
&\quad + \overline{\beta}\,(1-\rho)\,E_t\left[\widehat{J}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right] \\
&\quad - \overline{\beta}\,(1-\rho)\,E_t\left[\widehat{J}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1} + \widehat{\Delta}_{t+1} - \widehat{\Sigma}_{t+1}\right] \\
&\quad + \overline{\beta}\overline{q}^W E_t\left(\widehat{q}_t^W + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1}\right) \\
&\quad - \left\{\left[\frac{\overline{wh}\,(1-\overline{\tau})}{\overline{H}} + \frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}\right] - \frac{\overline{xmplh}}{\alpha \overline{J}} - \frac{\overline{mrs}}{1+\phi}\frac{\overline{h}\,(1+\overline{\tau}^c)}{\overline{H}}\right\}\widehat{h}_t \\
&\quad - \left[\frac{\overline{whs}}{\overline{J}} + \frac{(1-\iota)\,\overline{s}}{(1+\overline{s})}\right]\widehat{s}_t + \left[\frac{\overline{wh\tau}}{\overline{H}} - \frac{(1-\iota)\,\overline{\tau}}{(1-\overline{\tau})}\right]\widehat{\tau}_t \\
&\quad + \left[\overline{\beta}\,(1-\rho)\,\gamma\right]E_t\widehat{\Delta}_{t+1} - \left[\overline{\beta}\,(1-\rho)\,\gamma\right]E_t\widehat{\Sigma}_{t+1} \\
&\quad + \left[\frac{\overline{wh}\,(1-\overline{\tau})}{\overline{H}} + \frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}\right]\widehat{P}_t
\end{aligned}
$$

where $\iota = \overline{\beta}\,(1-\rho)\,\gamma$. Dividing by the term $\left[\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}} + \frac{\overline{wh}(1+\overline{s})}{\overline{J}}\right] = \frac{\overline{wh}(1+\overline{s})}{\eta\overline{J}} = \frac{\overline{wh}(1-\overline{\tau})}{(1-\eta)\overline{H}}$, and using the steady state equations for $\overline{\Delta}$ and $\overline{\Sigma}$, and for the Nash FOC allows us to rewrite the contract wage equation in the following simpler form

$$
\begin{aligned}
\Longrightarrow &\quad \widehat{w}_t^* + \frac{\iota}{1-\iota}E_t\left(\widehat{w}_t^* - \widehat{w}_{t+1}^*\right) \\
= &\quad \varphi_x\left(\widehat{x}_t^H + \widehat{mpl}_t\right) + \varphi_m\widehat{mrs}_t + \varphi_H E_t\left(\widehat{q}_t^W + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1}\right) \\
&\quad - \varphi_h\widehat{h}_t - \varphi_s\widehat{s}_t + \varphi_\tau\widehat{\tau}_t + \varphi_{\tau^c}\widehat{\tau}_t^c + \varphi_D E_t\left[\widehat{\Sigma}_{t+1} - \widehat{\Delta}_{t+1}\right] + \widehat{P}_t \\
= &\quad \widehat{w}_t^0(r)
\end{aligned}
$$

where $\widehat{w}_t^0(r)$ is the target wage in the bargain, and its coefficients are combinations of deep parameters and model-implied steady state values

$$
\begin{aligned}
\varphi_x &= \frac{\overline{xmpl}\eta}{\alpha\overline{w}\left(1+\overline{s}\right)}, \quad \varphi_m = \frac{\overline{mrs}\left(1-\eta\right)\left(1+\overline{\tau}^c\right)}{\left(1+\phi\right)\overline{w}\left(1-\overline{\tau}\right)}, \quad \varphi_H = \frac{\left(1-\eta\right)\overline{H}}{\overline{wh}\left(1-\overline{\tau}\right)}\overline{\beta}\overline{q}^W, \\
\varphi_{\tau^c} &= \frac{\overline{mrs}\left(1-\eta\right)\overline{\tau}^c}{\left(1+\phi\right)\overline{w}\left(1-\overline{\tau}\right)}, \quad \varphi_h = \left\{1-\frac{\overline{xmpl}\eta}{\alpha\overline{w}\left(1+\overline{s}\right)}-\frac{\overline{mrs}\left(1-\eta\right)\left(1+\overline{\tau}^c\right)}{\left(1+\phi\right)\overline{w}\left(1-\overline{\tau}\right)}\right\}, \\
\varphi_s &= \frac{\eta\overline{s}}{\left(1+\overline{s}\right)}\left[1+\frac{\left(1-\iota\right)\overline{J}}{\overline{wh}\left(1+\overline{s}\right)}\right], \quad \varphi_\tau = \frac{\left(1-\eta\right)\overline{\tau}}{\left(1-\overline{\tau}\right)}\left[1-\frac{\left(1-\iota\right)\overline{H}}{\overline{wh}\left(1-\overline{\tau}\right)}\right], \\
\text{and } \varphi_D &= \left[\overline{\beta}\left(1-\rho\right)\left(1-\gamma\right)\frac{\eta\overline{J}}{\overline{wh}\left(1+\overline{s}\right)}\right]
\end{aligned}
$$

The target wage $\widehat{w}_t^0(r)$ is of the same form than the period-by-period negotiated wage, adjusted for the new bargaining weights. The equation for the contract wage can be further rewritten as

$$
\frac{1}{\left(1-\iota\right)}\widehat{w}_t^* = \widehat{w}_t^0\left(r\right) + \frac{\iota}{\left(1-\iota\right)}E_t\widehat{w}_{t+1}^*
$$

$$
\Longleftrightarrow \widehat{w}_t^* = \left[1-\iota\right]\widehat{w}_t^0\left(r\right) + \iota E_t\widehat{w}_{t+1}^*
$$

This is the optimal contract wage set at time t *by all matches that are allowed to renegotiate their wage.* As is usual with Calvo-type contracting, it depends on a wage target $w_t^0(r)$ and next period's optimal wage.

The spillover effect To derive the spillover effect, consider the worker surplus with *optimal (contract) wage* versus the expected *average market wage* in the same way as above

$$
E_t\widehat{H}_{t+1}(w_{t+1}) = E_t\widehat{H}_{t+1}(w_{t+1}^*) + \frac{\overline{wh}\left(1-\overline{\tau}\right)}{\left(1-\iota\right)\overline{H}}E_t\left(\widehat{w}_{t+1}-\widehat{w}_{t+1}^*\right)
$$

Denoting $\frac{\overline{wh}(1-\overline{\tau})}{(1-\iota)\overline{H}} = \Gamma$ and substituting the above expression in the target wage equation gives

$$
\begin{aligned}
\widehat{w}_t^0(r) &= \varphi_x\left(\widehat{x}_t^H + \widehat{mpl}_t\right) + \varphi_m\widehat{mrs}_t \\
&\quad + \varphi_H E_t\left(\widehat{q}_t^W + \widehat{H}_{t+1}(w_{t+1}^*) + \widehat{\beta}_{t,t+1} + \Gamma E_t\left[\widehat{w}_{t+1}-\widehat{w}_{t+1}^*\right]\right) \\
&\quad + \varphi_h\widehat{h}_t - \varphi_s\widehat{s}_t + \varphi_\tau\widehat{\tau}_t + \varphi_{\tau^c}\widehat{\tau}_t^c + \varphi_D E_t\left[\widehat{\Sigma}_{t+1}-\widehat{\Delta}_{t+1}\right] + \widehat{P}_t
\end{aligned}
$$

$$
\Longleftrightarrow \widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H\Gamma E_t\left[\widehat{w}_{t+1}-\widehat{w}_{t+1}^*\right]
$$

where the target wage $\widehat{w}_t^0(r)$ - the wage the firm and its worker would agree to if they are allowed to renegotiate, *and if firms and workers elsewhere* *remain on staggered multiperiod wage contracts* - is a sum of the wage that would arise *if all matches were negotiating wages period-by-period* $\widehat{w}_t^0$ and the spillover effect $\varphi_H \Gamma E_t \left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]$.

Evolution of the average wage To derive the appropriate loglinear expression for the evolution of the average wage, first collect the necessary elements from previous calculations

1) The contract wage

$$\widehat{w}_t^* = [1-\iota]\, \widehat{w}_t^0(r) + \iota E_t \widehat{w}_{t+1}^*$$

2) The average wage

$$\widehat{w}_t = (1-\gamma)\, \widehat{w}_t^* + \gamma \widehat{w}_{t-1}$$

3) The target wage

$$\widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H \Gamma E_t \left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]$$

First, insert the target wage in the contract wage equation

$$\widehat{w}_t^* = [1-\iota] \left(\widehat{w}_t^0 + \varphi_H \Gamma E_t \left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]\right) + \iota E_t \widehat{w}_{t+1}^*$$

Then update the average wage equation by one period and take expectations

$$E_t \widehat{w}_{t+1} = (1-\gamma)\, E_t \widehat{w}_{t+1}^* + \gamma \widehat{w}_t$$

$$\Longleftrightarrow E_t \widehat{w}_{t+1}^* = \frac{1}{(1-\gamma)} \left(E_t \widehat{w}_{t+1} - \gamma \widehat{w}_t\right)$$

Use this expression to eliminate $E_t \widehat{w}_{t+1}^*$ from the contract wage equation

$$\begin{aligned} \widehat{w}_t^* &= [1-\iota] \left(\widehat{w}_t^0 + \varphi_H \Gamma E_t \widehat{w}_{t+1} - \varphi_H \Gamma \left[\frac{1}{(1-\gamma)} \left(E_t \widehat{w}_{t+1} - \gamma \widehat{w}_t\right)\right]\right) \\ &\quad + \iota \left[\frac{1}{(1-\gamma)} \left(E_t \widehat{w}_{t+1} - \gamma \widehat{w}_t\right)\right] \end{aligned}$$

$$\begin{aligned} \widehat{w}_t^* &= (1-\iota)\, \widehat{w}_t^0 + (1-\iota)\, \varphi_H \Gamma E_t \widehat{w}_{t+1} - (1-\iota)\, \varphi_H \Gamma \frac{1}{(1-\gamma)} E_t \widehat{w}_{t+1} \\ &\quad + [1-\iota]\, \varphi_H \Gamma \frac{\gamma}{(1-\gamma)} \widehat{w}_t + \frac{\iota}{(1-\gamma)} E_t \widehat{w}_{t+1} - \frac{\iota\gamma}{(1-\gamma)} \widehat{w}_t \end{aligned}$$

$$\iff \widehat{w}_t^* = (1-\iota)\,\widehat{w}_t^0 + \left[(1-\iota)\,\varphi_H \Gamma \frac{\gamma}{(1-\gamma)} - \iota \frac{\gamma}{(1-\gamma)}\right] \widehat{w}_t$$
$$+ \left[(1-\iota)\,\varphi_H \Gamma - (1-\iota)\,\varphi_H \Gamma \frac{1}{(1-\gamma)} + \iota \frac{1}{(1-\gamma)}\right] E_t \widehat{w}_{t+1}$$

Denote $\zeta = (1-\iota)\,\varphi_H \Gamma$, and use the above equation to eliminate $\widehat{w}_t^*$ from the average wage equation (equation 2)

$$\widehat{w}_t = (1-\gamma)\,(1-\iota)\,\widehat{w}_t^0 + (\zeta\gamma - \iota\gamma)\,\widehat{w}_t + [(1-\gamma)\,\zeta - \zeta + \iota]\, E_t \widehat{w}_{t+1} + \gamma \widehat{w}_{t-1}$$

$$[1-\gamma\,(\zeta-\iota)]\,\widehat{w}_t = (1-\gamma)\,(1-\iota)\,\widehat{w}_t^0 + [(1-\gamma)\,\zeta - \zeta + \iota]\, E_t \widehat{w}_{t+1} + \gamma \widehat{w}_{t-1}$$

Finally, after dividing by $[1-\gamma\,(\zeta-\iota)]$, the dynamic average wage equation can be expressed as

$$\iff \widehat{w}_t = \lambda_b \widehat{w}_{t-1} + \lambda_0 \widehat{w}_t^0 + \lambda_f E_t \widehat{w}_{t+1}$$

$$\begin{aligned}
\text{where } \lambda_b &= \frac{\gamma}{[1-\gamma\,(\zeta-\iota)]},\ \lambda_0 = \frac{(1-\gamma)\,(1-\iota)}{[1-\gamma\,(\zeta-\iota)]},\ \text{and } \lambda_f = \frac{\iota - \gamma\zeta}{[1-\gamma\,(\zeta-\iota)]}, \\
\text{with } \zeta &= (1-\iota)\,\varphi_H \Gamma,\ \iota = \overline{\beta}\,(1-\rho)\,\gamma,\ \Gamma = \frac{\overline{wh}\,(1-\overline{\tau})}{(1-\iota)\,\overline{H}}, \\
\text{and } \varphi_H &= \frac{(1-\eta)\,\overline{H}\overline{\beta}\overline{q}^W}{\overline{wh}\,(1-\overline{\tau})},\ \text{as previously denoted.}
\end{aligned}$$

A.6 Match surplus and reservation wages

Total match surplus is the sum of the worker surplus and the firm surplus

$$\begin{aligned}
S_t &= H_t + J_t \\
&= \frac{w_t^*}{P_t} h_t\,(1-\tau_t) - \frac{g\,(h_t)}{\Lambda_t} - b \\
&\quad + E_t \beta_{t,t+1}\,(1-\rho)\left[\gamma H_{t+1}(w_t^*) + (1-\gamma)\,H_{t+1}(w_{t+1}^*)\right] \\
&\quad - q_t^W E_t \beta_{t,t+1} H_{x,t+1} + x_t^H f\,(h_t) - \frac{w_t^*}{P_t} h_t\,(1+s_t) \\
&\quad + E_t \beta_{t,t+1}\,(1-\rho)\left[\gamma J_{t+1}(w_t^*) + (1-\gamma)\,J_{t+1}(w_{t+1}^*)\right]
\end{aligned}$$

$$\begin{aligned}
\iff\ & S_t = x_t^H z {h_t}^\alpha - \frac{w_t^*}{P_t} h_t\,(\tau_t + s_t) - \frac{mrs_t h_t\,(1+\tau_t^c)}{(1+\phi)} - b \\
& + E_t \beta_{t,t+1}\,(1-\rho)\left[\gamma S_{t+1}(w_t^*) + (1-\gamma)\,S_{t+1}(w_{t+1}^*)\right] \\
& - q_t^W E_t \beta_{t,t+1} H_{x,t+1}
\end{aligned}$$

If wages are renegotiated each period, the surplus equation can be further written as follows

$$\begin{aligned}\iff\quad S_t &= x_t^H z h_t{}^{\alpha} - \frac{w_t^*}{P_t} h_t\left(\tau_t + s_t\right) - \frac{mrs_t h_t\left(1+\tau_t^c\right)}{(1+\phi)} - b \\ &+ E_t\beta_{t,t+1}\left(1-\rho\right) S_{t+1}(w_{t+1}^*) - \frac{\eta}{(1-\eta)}\kappa\theta_t E_t \frac{(1-\tau_{t+1})}{(1+s_{t+1})}\end{aligned}$$

where the last term is derived using the Nash FOC equation and the vacancy posting condition

$$\begin{aligned} q_t^W E_t\beta_{t,t+1} H_{t+1}\left(w_{t+1}^*\right) &= q_t^W E_t\beta_{t,t+1}\frac{\eta}{(1-\eta)}\frac{(1-\tau_{t+1})}{(1+s_{t+1})} J_{t+1}\left(w_{t+1}^*\right) \\ &= \theta_t q_t^F E_t\beta_{t,t+1}\frac{\eta}{(1-\eta)}\frac{(1-\tau_{t+1})}{(1+s_{t+1})}\frac{\kappa}{q_t^F} \\ &= \frac{\eta}{(1-\eta)}\kappa\theta_t E_t\frac{(1-\tau_{t+1})}{(1+s_{t+1})}\end{aligned}$$

The loglinear version of this surplus equation is

$$\begin{aligned}\widehat{S}_t &= \frac{\overline{xzh}^{\alpha}}{\overline{S}}\left(\widehat{x}_t^H + \alpha\widehat{h}_t\right) - \frac{\overline{wh}\left(\overline{\tau}+\overline{s}\right)}{\overline{S}}\left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t\right) \\ &-\frac{\overline{mrsh}\left(1+\overline{\tau}^c\right)}{(1+\phi)\overline{S}}\left(\widehat{mrs}_t + \widehat{h}_t\right) - \frac{\overline{mrsh}\overline{\tau}^c}{(1+\phi)\overline{S}}\widehat{\tau}_t^c - \frac{\overline{wh\tau}}{\overline{S}}\widehat{\tau}_t \\ &-\frac{\overline{whs}}{\overline{S}}\widehat{s}_t + \overline{\beta}\left(1-\rho\right)E_t\left[\widehat{S}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right] \\ &+\frac{\eta\left(1-\overline{\tau}\right)}{(1-\eta)\left(1+\overline{s}\right)}\frac{\kappa\overline{\theta}}{\overline{S}}\left[\widehat{\theta}_t + E_t\left(\widehat{\tau}_{t+1} + \widehat{s}_{t+1}\right)\right]\end{aligned}$$

In the presence of staggered bargaining, the dynamic surplus equation is, in turn

$$\begin{aligned}\widehat{S}_t &= \frac{\overline{xzh}^{\alpha}}{\overline{S}}\left(\widehat{x}_t^H + \alpha\widehat{h}_t\right) - \frac{\overline{wh}\left(\overline{\tau}+\overline{s}\right)}{\overline{S}}\left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t\right) \\ &-\frac{\overline{mrsh}\left(1+\overline{\tau}^c\right)}{(1+\phi)\overline{S}}\left(\widehat{mrs}_t + \widehat{h}_t\right) - \frac{\overline{mrsh}\overline{\tau}^c}{(1+\phi)\overline{S}}\widehat{\tau}_t^c - \frac{\overline{wh\tau}}{\overline{S}}\widehat{\tau}_t \\ &-\frac{\overline{whs}}{\overline{S}}\widehat{s}_t - \overline{\beta}\overline{q}^W E_t\left(\hat{q}_t^W + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1}\right) \\ &+\overline{\beta}\left(1-\rho\right)E_t\left[\widehat{S}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right] \\ &+\frac{\overline{\beta}\left(1-\rho\right)\gamma}{1-\overline{\beta}\left(1-\rho\right)\gamma}\frac{\overline{wh}\left(\overline{\tau}+\overline{s}\right)}{\overline{S}}E_t\left(\widehat{w}_{t+1}^* - \widehat{w}_t^*\right)\end{aligned}$$

The reservation wages of the worker and of the firm are, respectively, the wages at which the value of employment is exactly equal to the value of unemployment, i.e. $H_t\left(\underline{w}_t\right) = 0$, or the value of a filled vacancy is exactly equal to the value of an open vacancy, i.e. $J_t\left(\overline{w}_t\right) = 0$.

$$H_t\left(\underline{w}_t\right) = 0$$

$$
\begin{aligned}
0 &= \frac{\underline{w}_t}{P_t} h_t (1-\tau_t) - \frac{g(h_t)}{\Lambda_t} - b \\
&\quad + E_t \beta_{t,t+1} (1-\rho) \left[\gamma H_{t+1}(w_t^*) + (1-\gamma) H_{t+1}(w_{t+1}^*) \right] - q_t^W E_t \beta_{t,t+1} H_{x,t+1}
\end{aligned}
$$

$$
\begin{aligned}
\iff \underline{w}_t = & \frac{P_t}{h_t (1-\tau_t)} \left[\frac{g(h_t)}{\Lambda_t} + b \right] \\
& - \frac{P_t}{h_t (1-\tau_t)} E_t \beta_{t,t+1} (1-\rho) \left[\gamma H_{t+1}(w_t^*) + (1-\gamma) H_{t+1}(w_{t+1}^*) \right] \\
& + \frac{P_t}{h_t (1-\tau_t)} q_t^W E_t \beta_{t,t+1} H_{x,t+1}
\end{aligned}
$$

$$
\begin{aligned}
J_t(\overline{w}_t) &= 0 \\
0 &= x_t^H f(h_t) - \frac{\overline{w}_t}{P_t} h_t (1+s_t) \\
&\quad + E_t \beta_{t,t+1} (1-\rho) \left[\gamma J_{t+1}(w_t^*) + (1-\gamma) J_{t+1}(w_{t+1}^*) \right] \\
\overline{w}_t &= \frac{P_t}{h_t (1+s_t)} x_t^H f(h_t) \\
&\quad + E_t \beta_{t,t+1} (1-\rho) \left[\gamma J_{t+1}(w_t^*) + (1-\gamma) J_{t+1}(w_{t+1}^*) \right]
\end{aligned}
$$

The steady state equations for the reservation wages are

$$
\begin{aligned}
\underline{w} &= \frac{1}{h(1-\tau)} \left[\frac{g(h)}{\Lambda} + b - \beta \left(1 - \rho - q^W\right) H \right] \\
&= \frac{1}{h(1-\tau)} \left[\frac{mrsh(1+\tau^c)}{(1+\phi)} + b - \beta \left(1 - \rho - q^W\right) H \right]
\end{aligned}
$$

$$
\overline{w} = \frac{1}{h(1+s)} \left[xf(h) + \beta(1-\rho) J \right] = \frac{1}{h(1+s)} \left[\frac{xmplh}{\alpha} + \beta(1-\rho) J \right]
$$

III

Labour market frictions and wage rigidity in an estimated DSGE model of the Finnish economy

Abstract

This paper is a first attempt to estimate a DSGE model with labour market frictions and wage rigidity for the Finnish economy. The contribution is twofold. First, we obtain estimates of nominal rigidities and wage indexation that are well in line with the existing literature and give an economically plausible picture of the Finnish economy. We establish that wage rigidity is empirically relevant for the business cycle, and provide a detailed assessment of the degree of wage rigidity and indexation in different model specifications. Second, our estimation approach sheds more light on the magnitude and effects of labour market shocks. We find that labour market shocks importantly drive fluctuations of labour market variables but they also account for a non-trivial share of output fluctuations.

1 Introduction

As a member of the Euro area, Finland misses two channels that help a standard small open economy to adjust to shocks. It has practically no control over the common monetary policy, and the nominal exchange rate is fixed. In the preparation phase for EMU, accordingly, the most central question that was raised was the ability of other adjustment channels to ensure the stability of the Finnish economy in the face of asymmetric shocks. There was wide agreement that, in this environment, in addition to the mobility of factors of production and the role of fiscal policy, the functioning of labour markets including the wage bargaining system will become more important.

After that the relevance of frictions on the labour market and of wage rigidity has received a lot of attention in the recent macroeconomic literature. It has been shown that models with these features are better equipped to reproduce empirically observed business cycles (see Shimer, 2010 and Gertler and Trigari, 2009). The typical approach in this branch of the literature has been to match DSGE model-implied moments to data moments assuming that technology shocks drive economic activity. Following this approach we found, in Obstbaum (2011), that a calibrated model for the Finnish economy with labour market matching and moderate rigidity in wage determination provided model moments that best corresponded to data moments for the time period 1994-2010.

While this moment comparison approach has become the typical one in the literature, recent advances in Bayesian estimation techniques offer the possibility to analyze the complete macroeconomic framework, and empirically investigate the significance of different mechanisms such as wage rigidity for model performance. This full information procedure also permits to account for a complete range of shocks that hit the economy.

In this paper we empirically analyze the role of labour market frictions and wage rigidity in shaping Finnish business cycle fluctuations, and the role of different shocks, including labour market shocks, in driving the Finnish economy. For this purpose, we estimate the New Keynesian DSGE model built in Obstbaum (2011) using Bayesian Maximum Likelihood methods. The objective is to find a model that best describes Finnish business cycles, in particular the behaviour of labour markets. The focus is on the mechanisms that affect the dynamic behaviour of the economy which are at the core of New Keynesian macroeconomics. We specifically assess the role of wage rigidities.

There are only a few previous estimation studies that feature a similar labour market structure and rigidity in wage setting. First, Gertler, Sala and Trigari (2008), which is the closest reference for the present approach, develop and estimate a medium-scale DSGE model that allows for labour market frictions and staggered nominal wage contracting. They find that the model with wage rigidity provides a better description of the data than a flexible wage version. We employ a similar labour market structure but modify the model to account for monetary union membership and for distortionary taxes that we believe may importantly determine the reaction of the Finnish economy to different kinds of disturbances.

Second, we follow Christoffel, Kuester and Linzert (2009) in including labour market data into our estimation procedure. They argue that despite possible measurement problems related to labour market data these series are at the core of the search and matching literature and should therefore be included in the set of observable time series in order to help the identification of parameters. As they do, we also add labour market shocks and analyze their significance for shaping the cyclical behaviour of selected key variables.

In addition, Faccini, Millard and Zanetti (2010) estimate a New Keynesian model with matching frictions and nominal wage rigidities on UK data but they do not include labour market shocks or data. Their staggered bargaining framework is a simplified version of Gertler, Sala and Trigari (2008). They establish that while wage rigidity enables the model to fit the data more closely, the model is unable to precisely identify the frequency of wage adjustment.

While these studies are, to our knowledge, the only other papers that share a similar labour market structure and wage determination model, there is, of course, a wide range of other New Keynesian DSGE models that have been estimated with different data sets and different end use purposes. To mention a few of these which are close to our approach, the Smets and Wouters (2003) model is certainly a starting point for the majority of these studies, including the labour market extension of Gertler, Sala and Trigari (2008).

Adolfson, Laséen, Lindé and Villani (2007), in turn, has been a useful reference because it is a small open-economy model estimated on Swedish data. Sweden and Finland share some important economic features including many labour market structures and wage negotiation traditions but these are not at the centre of the analysis by Adolfson et al.. Their model features a monopolistic labour market typical in New Keynesian models. Adding labour market frictions and staggered wage bargaining has, however, clear advantages compared to the more conventional setup, including the possibility to single out the source of imperfections and rigidities. Almeida (2009) estimates a New

Keynesian DSGE model for the Portuguese economy which, like Finland, is a small Euro area member state.

All the above estimation approaches provide posterior distributions for the nominal rigidity parameters, but several report difficulties in identifying them properly. While sticky prices and wages are key contributions of the New Keynesian model, there is still uncertainty concerning the degree of rigidities and the right combination of rigity and indexation. We follow Del Negro and Schorfheide (2008) and Rabanal and Rubio-Ramirez (2003) in providing a comparison of models with high or low wage rigidity, as well as sensitivity analysis with respect to different priors on rigidity parameters but also parameters of shock processes.

There is a relatively long tradition of DSGE modelling in Finland. The Bank of Finland's AINO model (see Kilponen and Ripatti, 2006) was one of the first models of that kind that was operative for both policy simulations and forecasting. More recently Freystätter (2010) has estimated a New Keynesian DSGE model with financial frictions with Finnish data. Differently from these studies, however, we are the first to estimate a model with labour market frictions and wage rigidity on the Finnish economy.

2 The model

2.1 General features

The model considers a small monetary union member state and builds in this respect on Galí and Monacelli (2008). As suggested by Schmitt-Grohé and Uribe (2003), however, we close the model by assuming a debt-elastic interest rate instead of complete asset markets. The home country is modelled along standard New-Keynesian practise comprising households, firms and a public sector. Following Gertler, Sala and Trigari (2008) we incorporate labour market frictions and, staggered Nash wage bargaining. But differently from their approach, in the present setup, firms only employ one worker, employment can be adjusted along both the extensive and the intensive margin (hours of work), and there are distortionary labour taxes as in Obstbaum (2011).

One advantage of the staggered bargaining approach is that wage rigidity gets the explicit interpretation of longer wage contracts. Lengthening the duration of wage contracts makes wages in each period less responsive to economic conditions, and shifts adjustment to the labour quantity side.

The wage and price setting decisions are separated from each other. labour market frictions arise in the intermediate good sector. The wholesale firms, in turn, buy intermediate goods and re-sell them to the final goods sector. They operate under monopolistic competition and set prices subject to Calvo rigidities. Final goods are produced from domestic and imported intermediate inputs under perfect competition.

The government's policy instruments include a lump-sum tax, a proportional wage tax paid by the employees, wage taxes paid by the employers in the form of social security contributions, unemployment benefits and other government transfers as well as a consumption tax.

2.2 Preferences

As in similar kinds of models, we adopt the representative or large household interpretation. This implies perfect consumption insurance, a key assumption needed to embed the MP model in a GE framework. Household members perfectly insure each other against variations in labour income due to their labour market status. This tackles the problem whereby households are identical but not all of their members are employed. As a result, the employment and unemployment rates are identical at the household level and across the population at large (see e.g. Merz, 1995).

The representative household maximizes the expected lifetime utility of its members

$$\int_0^1 E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_{i,t} - \varkappa C_{t-1})^{1-\varrho}}{1-\varrho} - \delta n_t \frac{(h_{i,t})^{1+\phi}}{1+\phi} \right] \right\} di \tag{1}$$

where $C_{i,t}$ is final good consumption in period t by household member i, $\varkappa \in (0,1)$ indicates an external habit motive, C_{t-1} stands for aggregate consumption in the previous period, $h_{i,t}$ are hours worked by household member i, and δ is a scaling parameter for the disutility of work. The inverses of ϱ and ϕ are the elasticities of intertemporal substitution and of labour supply respectively. The household's (real) budget constraint is

$$\begin{aligned} & (1+\tau_t^c) C_t + I_t + A(\nu_t) K_{t-1}^p + \frac{B_t^*}{P_t} \\ = \; & n_t \frac{w_t}{P_t} h_t (1-\tau_t) + (1-n_t) b + \frac{TR_t}{P_t} \\ & + R_{t-1}^* p\left(b_{t-1}^*\right) \frac{B_{t-1}^*}{P_t} + r_t^k \nu_t K_{t-1}^p + \frac{D_t}{P_t} \end{aligned} \tag{2}$$

The left-hand side of the equation describes the expenditures of the household. Consumption C_t is subject to a proportional tax τ_t^c. As an alternative to consumption, the household may choose to invest I_t. There are only consumer goods and therefore investment is in units of consumption good (see e.g. Hall, 2009 for a similar assumption). As households own the capital stock, they also bear the cost of capital utilization $A(\nu_t) K_{t-1}^p$. In addition, the household can buy nominal one-period foreign bonds B_t^* which are denominated in the monetary union currency.

The right hand side describes the household's income sources which consist of after-tax real wage $n_t \frac{w_t}{P_t} h_t (1-\tau_t)$, unemployment benefits $(1-n_t) b$, lump-sum transfers $\frac{TR_t}{P_t}$, capital rental payments from firms $r_t^k \nu_t K_{t-1}^p$and profit from firm ownership $\frac{D_t}{P_t}$. Income is also received in the form of repayment of last period's foreign bond purchases. The interest rate paid or earned on foreign bonds by domestic households $R_t = R_{t-1}^* p\left(b_{t-1}^*\right)$ consists of the common currency union gross interest rate R_{t-1}^* which, for the small member state is taken to be exogenous, and a country-specific risk premium $p\left(b_{t-1}^*\right)$. The risk premium is assumed to be increasing in the aggregate level of foreign real debt as a share of domestic output $(-b_t^* = -\frac{B_t^*}{P_t Y_t})$.[1]

[1] This is the debt-elastic interest rate assumption which is one of the mechanisms suggested by Schmitt-Grohé and Uribe (2003) to close a small open economy model. Note that

The capital utilization rate ν_t transforms physical capital into effective capital according to

$$K_t = \nu_t K^p_{t-1} \tag{3}$$

Effective capital is rented to firms at the rate r^k_t. The cost of capital utilization per unit of physical capital is $A(\nu_t)$. It is assumed that in the steady state $\nu_t = 1$ and $A(1) = 0$. $A'(1)/A''(1) = \eta_\nu$. The capital accumulation equation is

$$K^p_t = \left(1 - \delta^k\right) K^p_{t-1} + \epsilon^I_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{4}$$

where δ^k is the capital depreciation rate and ϵ^I_t is an investment specific shock. The investment adjustment cost function $S(\cdot)$ is assumed to have the following properties in the steady state: $S(\gamma_z) = S'(\gamma_z) = 0$ and $S''(\gamma_z) \equiv \eta_z > 0$, where η_z is the economy's steady state growth rate. We assume ϵ^I_t follows the exogenous stochastic process

$$\log(\epsilon^I_t) = (1 - \rho_I)\log\left(\epsilon^I\right) + \rho_I \log\left(\epsilon^I_{t-1}\right) + \varsigma^I_t$$

where $\rho_I \in (0,1)$, and $\varsigma^I_t \overset{iid}{\sim} N\left(0, \sigma^2_I\right)$. We leave aside for a moment the labour supply decision, which will be dealt with in the section describing the labour market, below. Optimal allocations of consumption and financial assets are characterized by the following conditions

$$\Lambda_t = \frac{\lambda_t}{(1 + \tau^c_t)} \tag{5}$$

$$\Lambda_t = \beta E_t \left[\Lambda_{t+1} \frac{R^*_{t-1} p\left(b^*_{t-1}\right)}{\pi_{t+1}}\right] \tag{6}$$

where $\lambda_t = (C_t - \varkappa C_{t-1})^{-\varrho}$ is the marginal utility of consumption and $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ is CPI inflation.

The nominal rate of return in the domestic economy is $R_t = R^*_t p\left(b^*_t\right)$, where the risk premium on foreign bond holdings $p\left(b^*_t\right)$ follows the function

$$p\left(b^*_t\right) = \exp\left[-\gamma_{b^*}\left(b^*_t - \bar{b}\right)\right], \text{ with } \gamma_{b^*} > 0 \tag{7}$$

This should ensure the stability and determinacy of equilibrium in a small member state of the monetary union model [2]. In the steady state, the risk premium is assumed to be equal to one, and the domestic and foreign interest rates are the same. After loglinearization the above arbitrage relation gets the form

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with the current notation a negative (positive) deviation of the stock of foreign bonds from the steady state zero level implies that the home country as a whole becomes a net borrower (lender), and faces a positive (negative) risk premium.

[2] As Galí and Monacelli (2008) point out, along with accession to the monetary union the small member state no longer meets the Taylor principle since variations in its inflation that result from idiosyncratic shocks will have an infinitesimal effect on union-wide inflation, and will thus induce little or no response from the union's central bank. According to the Taylor principle, in order to guarantee the uniqueness of the equilibrium, the central bank would have to adjust the nominal interest rates more than one-for-one with changes in inflation (see e.g. Woodford, 2003)

$$\widehat{R}_t = \widehat{R}^*_t - \gamma_{b^*}\widehat{b}^*_t$$

Furthermore, the first order conditions for capital utilization ν_t, investment I_t, and physical capital K^p_t are respectively

$$r^k_t = A'(\nu_t) \tag{8}$$

$$Q_t\epsilon^I_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] = Q_t\epsilon^I_t S'(\frac{I_t}{I_{t-1}})(\frac{I_t}{I_{t-1}}) - \beta E_t Q_{t+1}\frac{\Lambda_{t+1}}{\Lambda_t}\epsilon^I_{t+1}S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 + 1 \tag{9}$$

$$Q_t = \beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\left[\left(1-\delta^k\right)Q_{t+1} + r^k_{t+1}\nu_{t+1} - A\left(\nu_{t+1}\right)\right] \tag{10}$$

where Q_t is Tobin's Q, the present value of an additional unit of capital divided by the cost of acquiring one unit of capital, or the ratio of the Lagrange multipliers for the capital accumulation equation and for the consumer budget constraint. The discount factor is the same for all optimizing agents in the economy and is hereafter defined throughout the paper as $\beta_{t,t+s} = \beta^s\frac{\Lambda_{t+s}}{\Lambda_t}$.

2.3 The labour market

The labour market brings together workers and intermediate good firms.

2.3.1 Unemployment, vacancies and matching

The measure of successful matches m_t is given by the matching function

$$m_t(u_t, v_t) = \sigma_m u^\sigma_t v^{1-\sigma}_t \tag{11}$$

where u_t and v_t are the aggregate measures of unemployed workers and vacancies. m_t is the flow of matches during a period t, and u_t and v_t are the stocks at the beginning of the period. The matching function is, as usual, increasing in both vacancies and unemployment, concave, and homogeneous of degree one. The Cobb-Douglas form implies that σ is the elasticity of matching with respect to the stock of unemployed people, and σ_m represents the efficiency of the matching process. The probabilities that a vacancy will be filled and that the unemployed person finds a job are respectively

$$q^F_t = q^F_t(\theta_t) = \frac{m_t}{v_t} = \sigma_m\theta^{-\sigma}_t \tag{12}$$

$$q^W_t = \theta_t q^F_t(\theta_t) = \frac{m_t}{u_t} = \sigma_m\theta^{1-\sigma}_t \tag{13}$$

and the inverse of these probabilities is the mean duration of vacancies and unemployment.

$\theta_t = \frac{v_t}{u_t}$ is labour market tightness. The tighter the labour market is, or the less there are unemployed people relative to the number of open vacancies (i.e. larger θ_t), the smaller the probability that the firm succeeds in filling the vacancy and the larger the probability that the unemployed person finds a job. Similarly, a decrease in the number of vacancies relative to unemployment (smaller θ_t) implies that the unemployed person has a smaller probability to find a job.

In the beginning of each period, a fraction of matches will be terminated with an exogenous probability $\rho_t \in (0,1)$. The separation rate evolves according to the autoregressive process

$$\log(\rho_t) = \left(1-\rho_\rho\right)\log(\rho) + \rho_\rho \log\left(\rho_{t-1}\right) + \epsilon_t^\rho$$

where $\rho_\rho \in (0,1)$, and $\epsilon_t^\rho \overset{iid}{\sim} N\left(0,\sigma_\rho^2\right)$. Labour market participation is characterised as follows. The size of the labour force is normalised to one. The number of employed workers at the beginning of each period is

$$n_t = (1-\rho_t)\, n_{t-1} + m_{t-1} \tag{14}$$

where the first term on the right hand side represents those workers who were employed already in the previous period and whose jobs have survived beginning-of-period job destruction, and the second term covers those workers who got matched in the previous period and become productive in the current period. After the exogenous separation shock, the separated workers return to the pool of unemployed workers and start immediately searching for a job. The number of unemployed is $u_t = 1 - n_t$.

In the steady state an equal amount of jobs are created and destructed:

$$JC = JD \Longleftrightarrow m = \rho n \tag{15}$$

2.3.2 Wage bargaining

Job creation takes place when a worker and a firm meet and agree to form a match at a negotiated wage. The wage that the firm and the worker choose must be high enough that the worker wants to work in the job, and low enough that the employer wants to hire the worker. These requirements define a range of wages that are acceptable to both the firm and the worker. The unique equilibrium wage.is, however, the outcome of a bargain between the worker and the firm.

The structure of the staggered multiperiod contracting model follows Gertler, Sala and Trigari (2008) but includes also the intensive margin of adjustment of the labour input (hours worked per worker) as well as distortionary taxes. For comparison, the period-by-period bargaining outcome is presented in the appendix. The idea of staggered wage bargaining is analogous to Calvo price setting. Rigidity is created by assuming that a fraction γ of firms are not allowed to renegotiate their wage in a given period. As a result, all workers in those firms receive the wage paid the previous period w_{t-1} partially indexed to inflation. The constant probability that firms are allowed to renegotiate the wage is labeled $1-\gamma$. Accordingly, $\frac{1}{1-\gamma}$ is the average duration of a wage contract. Thus, the combination of wage bargaining and Calvo price setting allows to give an intuitive interpretation to the source of wage rigidity instead of more or less ad hoc formulations. Period-by-period bargaining corresponds to the special case of $\gamma = 0$.

As in the standard Mortensen-Pissarides model, it is assumed that match surplus, the sum of the worker and firm surpluses, is shared according to efficient Nash bargaining. In the baseline model, wages and hours are negotiated simultaneously. The firm and the worker choose the nominal wage and the hours of work to maximize the weighted product of their net return from the match. When wages are rigid, it is assumed that as new matches become productive, they enter the same Calvo scheme for wage-setting than existing matches. This is an important assumption for wage rigidity to have an effect on job creation. Gertler and Trigari (2009) argue that after controlling for compositional effects there are no differences empirically in the flexibility of new and existing worker's wages.[3]

The contract wage w_t^* is chosen to solve

$$\max \left[H_t\left(r\right)\right]^{\eta_t}\left[J_t\left(r\right)\right]^{1-\eta_t} \tag{16}$$

subject to the random renegotiation probability. $H_t(r)$ and $J_t(r)$ are the matching surpluses of renegotiating workers and firms respectively, and $0 \leq \eta_t \leq 1$ is the relative measure of workers' bargaining strength. The value equations describing the worker's and the firm's surplus from employment are the key determinants of the outcome of the wage bargain. We assume that the bargaining power of workers η_t is subject to shocks ϵ_t^η as follows

$$\log(\eta_t) = \left(1-\rho_\eta\right)\log\left(\eta\right) + \rho_\eta \log\left(\eta_{t-1}\right) + \epsilon_t^\eta$$

where $\rho_\eta \in (0,1)$, and $\epsilon_t^\eta \overset{iid}{\sim} N\left(0,\sigma_\eta^2\right)$

Workers The value to the renegotiating worker of being employed consists of after-tax labour income, the disutility from working, expressed in marginal utility terms, and the expected present value of his situation in the next period. In the case of non-renegotiation, the past nominal wage is partially indexed to CPI inflation $\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]$ as in Smets and Wouters (2003) or Christoffel, Kuester and Linzert (2009).

$$\begin{aligned} W_t(r) = {} & \frac{w_t^*}{P_t} h_t\left(1-\tau_t\right) - \frac{g\left(h_t\right)}{\Lambda_t} \\ & + E_t\beta_{t,t+1}\left(1-\rho_{t+1}\right)\left[\gamma W_{t+1}\left(w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]\right) + \left(1-\gamma\right)W_{t+1}(w_{t+1}^*)\right] \\ & + E_t\beta_{t,t+1}\rho_{t+1}U_{t+1} \end{aligned} \tag{17}$$

The value to the worker of being unemployed is

$$U_t(r) = b + E_t\beta_{t,t+1}\left[q_t^W W_{x,t+1} + \left(1-q_t^W\right)U_{t+1}\right] \tag{18}$$

where the first term on the RHS is the value of the outside option to the worker, i.e. the unemployment benefit b, and the second term gives the expected present value of either working or being unemployed in the following period. Unemployed workers do not need to take into account the probability of job destruction even if they get matched because of the timing assumption. A

[3] E.g. Pissarides (2009) argues the opposite: that wages of newly hired workers are volatile unlike wages for ongoing job relationships. This would mean that there is wage rigidity, but not of the kind that leads to more volatility in unemployment fluctuations.

match that has not yet become productive cannot be destroyed. Note that the value for the worker who is currently unemployed to move from unemployment to employment next period is $W_{x,t+1}$, the expected *average* value of being employed. New matches are subject to the same bargaining scheme as existing matches, and therefore the new worker does not have a priori knowledge of whether the firm he will start working for will be allowed to renegotiate its wage[4].

Combining these value equations gives the expression for the renegotiating worker's surplus

$$\begin{aligned} H_t(r) &= W_t(r) - U_t(r) \\ &= \frac{w_t^*}{P_t} h_t (1-\tau_t) - \frac{g(h_t)}{\Lambda_t} - b \\ &\quad + E_t \beta_{t,t+1} (1-\rho_{t+1}) \left[\gamma H_{t+1}(w_t^* \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w} \right) \right]) + (1-\gamma) H_{t+1}(w_{t+1}^*) \right] \\ &\quad - q_t^W E_t \beta_{t,t+1} H_{x,t+1} \qquad (19) \end{aligned}$$

Intermediate firms For the renegotiating firm, the value of an occupied job is equal to the profit of the firm in the current period net of payroll taxes s_t and capital rental payments $r_t^k k_t$, and the expected future value of the job

$$\begin{aligned} J_t(r) &= x_t^H f(k_t, h_t) - \frac{w_t^*}{P_t} h_t (1+s_t) - r_t^k k_t \qquad (20) \\ &\quad + E_t \beta_{t,t+1} (1-\rho_{t+1}) \left[\gamma J_{t+1}(w_t^* \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w} \right) \right]) + (1-\gamma) J_{t+1}(w_{t+1}^*) \right] \end{aligned}$$

where $x_t^H = \frac{P_{H,t}}{P_t} x_t$ is the relative price of the intermediate sector's good in terms of the CPI[5]. As in Gertler, Sala and Trigari (2008) capital is introduced into the model assuming that there is a perfect rental market for capital goods. In the present framework with only one worker per firm, as capital is costly, the firm only rents it when the job becomes active (the firm finds a worker). The capital rented by firms becomes a part of the value of an occupied job (see Pissarides, 2000, ch. 1). $f(k_t, z_t h_t) = k_t^{1-\alpha} (z_t h_t)^{\alpha}$ is the *per worker* production function, where $k_t = \frac{K_t}{n_t}$ is the capital-labour ratio. The corresponding aggregate production function is $f(K_t, n_t z_t h_t)$. The marginal product of an extra hour of work in the match is $mpl_t = \alpha k_t^{1-\alpha} (z_t h_t)^{\alpha-1} = \alpha \frac{f(k_t, z_t h_t)}{h_t}$, and the marginal product of capital is $mpk_t = (1-\alpha) \frac{f(k_t, z_t h_t)}{k_t}$. The firm rents as

[4] Accordingly, the average surplus from working is $H_{x,t+1} = \gamma H_{t+1}(w_t \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w} \right) \right]) + (1-\gamma) E_t H_{t+1}(w_{t+1}^*)$. If the worker starts working in a firm that is not allowed to renegotiate, he will get last period's *average* wage partially indexed to inflation. This is because in the one firm - one worker setup of this paper also firms in new matches are new, they cannot have negotiated a contract wage in the previous period.

[5] The firm surplus is here expressed in terms of consumer prices, as opposed to producer prices, to avoid confusion in computing the wage bargaining solution. As a result, the intermediate firms get the nominal price $P_{H,t} x_t$ for their product while they value this revenue in terms of the CPI. This creates a channel for CPI-PPI differences to enter the wage bargaining problem.

much capital as is necessary to maximize the value of the job. The maximization of $J_t(r)$ w.r.t. k_t yields the equilibrium condition for an individual firm's capital stock

$$x_t^H mpk_t = r_t^k \tag{21}$$

Labour-augmenting productivity z_t is identical for all matches and follows

$$\log(z_t) = (1-\rho_z)\log(z) + \rho_z \log(z_{t-1}) + \epsilon_t^z$$

where $\rho_z \in (0,1)$, and $\epsilon_t^z \overset{iid}{\sim} N(0,\sigma_z^2)$. The value to the firm of an open vacancy is

$$\begin{aligned} V_t &= -\kappa_t + E_t\beta_{t,t+1} q_t^F \left[\gamma J_{t+1}(w_t \left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]) + (1-\gamma) J_{t+1}(w_{t+1}^*)\right] \\ &\quad + E_t\beta_{t,t+1}\left(1-q_t^F\right)V_{t+1} \end{aligned} \tag{22}$$

The value of a vacancy consists of an exogenous hiring cost κ_t, and of the expected value from future matches. Vacancy costs are subject to shocks

$$\log(\kappa_t) = (1-\rho_\kappa)\log(\kappa) + \rho_\kappa \log(\kappa_{t-1}) + \epsilon_t^\kappa$$

where $\rho_\kappa \in (0,1)$, and $\epsilon_t^\kappa \overset{iid}{\sim} N(0,\sigma_\kappa^2)$. The introduction of capital does not affect the value of a vacant job V_t since firms only rent capital upon finding a worker. In equilibrium, all profit opportunities from new jobs are exploited so that the equilibrium condition for the supply of vacant jobs is $V_t = 0$. With each firm having only one job, profit maximization is equivalent to this zero-profit condition for firm entry. Setting the equation for V_t as zero in every period gives:

$$\frac{\kappa_t}{q_t^F} = E_t\beta_{t,t+1}\left[\gamma J_{t+1}(w_t\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]) + (1-\gamma) J_{t+1}(w_{t+1}^*)\right] \tag{23}$$

This vacancy posting condition equates the marginal cost of adding a worker (real cost times mean duration of vacancy) to the discounted marginal benefit from a new worker. After taking into account the free entry condition, the firm surplus reduces to J_t.

Multiperiod bargaining set up Unlike with period-to-period bargaining, in the presence of staggered contracting, firms and workers have to take into account the impact of the contract wage on the expected future path of firm and worker surplus. Accordingly, the first order condition for wage-setting is given by:

$$\eta_t \Delta_t J_t(r) = (1-\eta_t)\Sigma_t H_t(r) \tag{24}$$

where the partial derivatives of the surplus equations w.r.t. the wage $\Delta_t = P_t\frac{\partial H_t(r)}{\partial w_t}$ and $\Sigma_t = -P_t\frac{\partial J_t(r)}{\partial w_t}$ denote the effect of a rise in the *real* wage on the worker surplus and (minus) the effect of a rise in the real wage on the firm's surplus respectively (see Appendix for details).

$$\Delta_t = h_t (1 - \tau_t) + E_t \beta_{t,t+1} \left(1 - \rho_{t+1}\right) \gamma \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right] \pi_{t+1}^{-1} \Delta_{t+1} \tag{25}$$

$$\Sigma_t = h_t (1 + s_t) + E_t \beta_{t,t+1} \left(1 - \rho_{t+1}\right) \gamma \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right] \pi_{t+1}^{-1} \Sigma_{t+1} \tag{26}$$

These expressions can be interpreted as the discounting factors for the worker and the firm (respectively) for evaluating the value of the future stream of wage payments. As wage contracts extend over multiple periods, agents have to take into account also the *future* probabilities of not being allowed to renegotiate the wage, or of not surviving exogenous destruction. In the one firm - one worker setup, used in this paper, the discounting factors would be equal across agents unless distortionary taxes were breaking this symmetry[6]. With staggered bargaining, labour taxes enter the discounting factor equations of the agents implying that workers and firms also take into account the future path of taxation in their negotiating behaviour. As is apparent from the loglinearized forms of the discounting factors, presented in the Appendix, both the worker's and the firm's marginal tax rate effectively reduce the worker's relative bargaining power, and consequently his share of the surplus. This effect on the division of match surplus is amplified by staggered bargaining. In the limiting case of period-by-period bargaining, $\gamma = 0$, the partial derivatives of the surpluses w.r.t. the wage reduce to $\Delta_t = h_t (1 - \tau_t)$, and $\Sigma_t = h_t (1 + s_t)$, and the first order condition accordingly reduces to its period-by-period counterpart $\eta (1 - \tau_t) J_t = (1 - \eta) (1 + s_t) H_t$.

Given that the probability of wage adjustment is i.i.d., and all matches at renegotiating firms end up with the same wage w_t^*, the evolution of the nominal *average hourly* wage in the economy can be expressed as a convex combination of the contract wage and the average wage across the matches that do not renegotiate, after taking into account the indexation scheme.

$$w_{t+1} = (1 - \gamma) w_{t+1}^* + \gamma \int_0^{n_t} \frac{w_{it}}{n_t} \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right] di \tag{27}$$

Wage dynamics The staggered bargaining framework has implications on the behaviour of workers and firms. To describe wage dynamics in the presence of staggered contracting, we will develop loglinear expressions for the relevant wage equations in the same way as in Gertler, Sala and Trigari (2008). The contract wage is solved by first linearizing the first order condition

$$\widehat{J}_t(r) + \widehat{\Delta}_t = \widehat{H}_t(r) + \widehat{\Sigma}_t - \frac{1}{1-\eta} \widehat{\eta}_t \tag{28}$$

and then plugging into the FOC the value equations and discounting factors for the worker and the firm respectively in their loglinearized form. The latter,

[6] In Gertler and Trigari (2009), this is not the case. Differences in the worker's and the firm's optimization perspectives, a "horizon effect", arises because large firms take into account possible changes in future hiring rates. The effect of distortionary taxes is different. Proportional tax rates influence the *division* of the total surplus from a job in equilibrium, irrespective of the bargaining horizon (see Pissarides, 2000, chapter 9).

as well as the derivation of the contract wage, are presented in detail in the Appendix. The resulting contract wage is

$$\widehat{w}_t^* = [1-\iota]\,\widehat{w}_t^0\,(r) + \iota E_t\left(\widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t\right) + \iota E_t \widehat{w}_{t+1}^* \tag{29}$$

where $\iota = \overline{\beta}\,(1-\overline{\rho})\,\gamma$. This is the optimal wage set at time t *by all matches that are allowed to renegotiate their wage.* As is usual with Calvo contracting, it depends on a wage target $w_t^0(r)$ and next period's optimal wage. As the probability of not being able to renegotiate the wage approaches zero $\gamma \longrightarrow 0$, $\iota \longrightarrow 0$, and the contract wage, w_t^*, approaches the period-by-period Nash wage.

Unlike in the more conventional set up of New Keynesian models, where Calvo wage contracting is combined with a monopolistic supplier of labour, the target wage here also includes a spillover effect that brings about additional rigidity on top of that implied by the Calvo scheme alone. Gertler and Trigari (2009) show how these spillover effects result from wage bargaining. The target wage can then be decomposed into two parts

$$\widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H \Gamma E_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right] \tag{30}$$

where $\varphi_H\Gamma = \frac{(1-\eta)\beta q^w}{(1-\iota)}$ is the spillover effect[7]. The spillover coefficient is positive, indicating that whenever the expected average market wage $E_t\widehat{w}_{t+1}$ is higher than the expected contract wage $E_t\widehat{w}_{t+1}^*$, (reflecting unusually good labour market conditions), this raises the target wage in the negotiations. Thus, wage rigidity and the resulting employment dynamics are not only a product of the Calvo-type rigidity in wage setting, but also of the spillover effects from the Nash bargaining process.

The spillover-free component of the target wage is of the same form than the period-by-period negotiated wage, adjusted for the multiperiod discounting factors.

$$\begin{aligned}
\widehat{w}_t^0 \;=\; & \varphi_x\left(\widehat{x}_t^H + \widehat{mpl}_t\right) - \varphi_k(\widehat{r}_t^k + \widehat{k}_t) + \varphi_m \widehat{mrs}_t - \varphi_h \widehat{h}_t \\
& + \varphi_H E_t\left(\widehat{q}_t^W + \widehat{H}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) \\
& - \varphi_s \widehat{s}_t + \varphi_\tau \widehat{\tau}_t + \varphi_{\tau^c}\widehat{\tau}_t^c + \varphi_D E_t\left[\widehat{\Sigma}_{t+1} - \widehat{\Delta}_{t+1}\right] + \varphi_\eta \widehat{\eta}_t + \widehat{P}_t
\end{aligned} \tag{31}$$

Increases in the productivity of the match, in the marginal rate of substitution of the worker/consumer, in the value of the worker's outside option, in the labour tax of the worker or the consumption tax, and in the bargaining power of the worker raise the target wage in the negotiations, whereas, increases in the cost of capital, in working hours per worker and in the firms' social security contributions lower the target wage. Finally, combining all the relevant elements of the wage bargaining outcome yields a second-order difference equation for the evolution of the average wage (see Appendix for detailed derivation)

[7] In Gertler and Trigari's (2006) original framework, there is also an indirect spillover effect because the expected hiring rate of the large renegotiating firm affects the bargaining outcome. In the present one worker per firm setup that effect disappears.

$$\widehat{w}_t = \lambda_b \left(\widehat{w}_{t-1} + \varepsilon_w \widehat{\pi}_{t-1} - \widehat{\pi}_t\right) + \lambda_0 \widehat{w}_t^0 + \lambda_f E_t \left(\widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t\right) \tag{32}$$

Due to staggered contracting, $\widehat{w}_t$ depends on the lagged wage $\widehat{w}_{t-1}$, the spillover-free target wage $\widehat{w}_t^0$, and the expected future wage $E_t\widehat{w}_{t+1}$.

2.3.3 Determining hours of work

While matches are restrained to renegotiate the wage only with a given exogenous probability, hours per worker can be *renegotiated at each point in time.* With efficient Nash bargaining, optimal hours of work can be found from the following first order condition obtained by differentiating the Nash maximand w.r.t hours

$$(1-\tau_t)\, x_t^H f_{h,t} = (1+s_t)\, \frac{g'(h_t)}{\Lambda_t}$$

where $f_{h,t}$ is, as before, the marginal product of the labour input i.e. hours, and which, using the expressions for the production and utility functions, can be written as

$$(1-\tau_t)\, x_t^H mpl_t = (1+s_t)\, mrs_t \left(1+\tau_t^c\right) \tag{33}$$

This optimality condition equates the value of marginal product to the marginal rate of substitution between work and leisure, and resembles, thus, to the corresponding condition in a competitive labour market. However, with labour market frictions, while the hourly wage is such that the marginal cost to the worker from working is equal to the marginal gain to the firm, neither of these measures needs to be equal to the wage. It is important to observe that the optimality condition for hours determines the optimal hours per worker, i.e. the intensive margin of labour adjustment. This individual labour input of a worker is determined *irrespective of the wage.* But the model also allows for labour adjustment in the number of workers, as defined by the vacancy posting condition and the matching function.

2.4 Final good firms

There are two types of final goods firms. One produces private consumption goods and the other type of final goods firm produces public consumption goods[8].

2.4.1 Private consumption good

The private consumption good is a composite of intermediate goods distributed by a continuum of monopolistically competitive wholesale firms at home and abroad. Wholesale firms, their products and prices are indexed by $i \in [0,1]$. Final good firms operate under perfect competition and purchase both domestically produced intermediate goods $y_{H,t}(i)$ and imported intermediate goods

[8] This is a standard assumption in New Open Economy Macro Models that assess fiscal policy. E.g. in Obstfeld and Rogoff's (1996) extension of the Redux model, government spending is introduced as a basket of public consumption goods aggregated in the same way as for private consumption.

$y_{F,t}(i)$. They minimize expenditure subject to the following aggregation technology

$$C_t = \left[(1-W)^{\frac{1}{\varpi}} \left(\left[\int_0^1 y_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \right)^{\frac{\varpi-1}{\varpi}} + W^{\frac{1}{\varpi}} \left(\left[\int_0^1 y_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \right)^{\frac{\varpi-1}{\varpi}} \right]^{\frac{\varpi}{\varpi-1}} \tag{34}$$

where ϖ measures the trade price elasticity, or elasticity of substitution between domestically produced intermediate goods and imported intermediate goods in the production of final goods for given relative prices, and W is the weight of imports in the production of final consumption goods.The parameter $\varepsilon > 1$ is the elasticity of substitution across the differentiated intermediate goods produced and distributed within a country.

The optimization problem determining the allocation of expenditure between the individual varieties of domestic and foreign intermediate goods yields the following demand curves facing each wholesale firm

$$y_{H,t}(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_{H,t} \tag{35}$$

$$y_{F,t}(i) = \left(\frac{p_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} Y_{F,t} \tag{36}$$

where $P_{H,t}$ and $P_{F,t}$ are the aggregate price indexes for the domestic and foreign intermediate goods respectively

$$P_{H,t} = \left[\int_0^1 p_{H,t}(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \tag{37}$$

$$P_{F,t} = \left[\int_0^1 p_{F,t}(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \tag{38}$$

To determine the optimal allocation between the domestic and imported intermediate goods, the final good firm minimizes costs $P_{H,t}Y_{H,t} + P_{F,t}Y_{F,t}$ subject to its production function or aggregation constraint. This yields the demands for the domestic and foreign intermediate good *bundles* by domestic final good producers

$$Y_{H,t} = (1-W) \left(\frac{P_{H,t}}{P_t} \right)^{-\varpi} C_t \tag{39}$$

$$Y_{F,t} = W \left(\frac{P_{F,t}}{P_t} \right)^{-\varpi} C_t \tag{40}$$

where P_t is the home country's aggregate price index, or consumption price index

$$P_t = \left((1-W) P_{H,t}^{1-\varpi} + W P_{F,t}^{1-\varpi} \right)^{\frac{1}{1-\varpi}} \tag{41}$$

At the level of individual intermediate goods the law of one price holds[9]. That, together with the assumption that the weight of the home country good in the foreign consumer price index is infinitesimally small, implies that $P_{F,t}$ is equal to the foreign CPI P_t^* (see Galí-Monacelli, 2008).

2.4.2 Public consumption good

The public consumption good is composed of only domestic intermediate goods $g_t(i)$. This assumption implies, contrary to e.g. the Redux model, full home bias in government spending. This simplifying assumption can be supported by the observation from input-output tables that the use of foreign intermediate goods in government spending is significantly lower than in private consumption.

$$G_t = \left[\int_0^1 g_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{42}$$

Each wholesale firm i selling intermediate goods to the public consumption good producer faces the following demand schedule

$$g_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} G_t \tag{43}$$

2.5 Wholesale firms and price setting

The wholesale firms buy the homogeneous intermediate goods at nominal price $p_{H,t}x_t$ per unit and transform them one-to-one into the differentiated product. As in most models that incorporate labour market matching into the NK framework, the price setting decision is separated from the wage setting decision to maintain the tractability of the model. Price rigidities arise at the wholesale level while search frictions and wage rigidity only affect directly the intermediate goods sector.

There is Calvo-type stickiness in price-setting and the relative price of intermediate goods x_t coincides with the real marginal cost faced by wholesale firms. In each period, the wholesale firm can adjust its price with a constant probability $1-\xi$ which implies that prices are fixed on average for $\frac{1}{1-\xi}$ periods. The wholesale firm's optimization problem is to maximize expected future discounted profits by choosing the sales price $p_{H,t}(i)$, taking into account the pricing frictions and the demand curve they face. It is assumed that the wholesale firm sells the home-country intermediate goods for the same price for domestic and foreign final goods producers, and for the domestic government.

The expected future discounted profit of a wholesale firm that reoptimizes at t is

$$E_t \sum_{s=0}^{\infty} \xi^s \beta_{t,t+s} \left[\left(\frac{p_{H,t}(i)}{P_{H,t+s}} \right) y_{t+s}(i) - x_{t+s} y_{t+s}(i) \right] = 0 \tag{44}$$

[9] Note, however, that due to home bias in consumption the basket of consumed goods may differ in the two areas, and therefore purchasing power parity does not hold.

where $y_t(i)$ is the demand of firm i's product by domestic private consumption good firms, foreign private consumption good firms and the domestic government, as outlined in the previous section

$$y_t(i) = y_{H,t}(i) + y^*_{H,t}(i) + g_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_t^D$$

where Y_t^D stands for total demand for domestic intermediate goods. All wholesale firms are identical except that they may have set their current price at different dates in the past. However, in period t, if they are allowed to reoptimize their price, they all face the same decision problem and choose the same optimal price $p^*_{H,t}$. Using the definition of the discount factor and rearranging, the FOC can be rewritten as

$$E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[(1-\varepsilon)\left(\frac{p^*_{H,t}}{P_{H,t+s}}\right) + \varepsilon x_{t+s}\right] \left(\frac{1}{p^*_{H,t}}\right) \left(\frac{p^*_{H,t}}{P_{H,t+s}}\right)^{-\varepsilon} Y^D_{t+s} = 0 \tag{45}$$

which can be solved for $\frac{p^*_{H,t}}{P_{H,t}}$ to yield the following pricing equation

$$\frac{p^*_{H,t}}{P_{H,t}} = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} x_{t+s} \left(\frac{P_{H,t+s}}{P_{H,t}}\right)^{\varepsilon} Y^D_{t+s}}{E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left(\frac{P_{H,t+s}}{P_{H,t}}\right)^{\varepsilon-1} Y^D_{t+s}} \tag{46}$$

where $\frac{\varepsilon}{\varepsilon-1} = \mu$ is the flexible-price markup. This is the standard Calvo result.

In the absence of price rigidity, the optimal price would reduce to a constant markup over marginal costs. Log-linearizing the FOC around the steady state yields the New Keynesian Phillips Curve where domestic inflation depends on marginal costs and expected future inflation

$$\widehat{\pi}_{H,t} = \nu \widehat{x}_t + \beta E_t \widehat{\pi}_{H,t+1} \tag{47}$$

where $\nu = \frac{(1-\xi)(1-\xi\beta)}{\xi}$.

2.6 Fiscal policies

The public sector's role in this economy is to collect taxes and use them to finance unemployment benefits and lump-sum transfers as well as government spending G_t. We make the simplifying assumption that the government budget is balanced in each period with the help of lumpsum transfers. In real terms, the government budget constraint thus reads as

$$\frac{TR_t}{P_t} = n_t \frac{w_t}{P_t} h_t(\tau_t + s_t) + \tau^c_t C_t - \frac{P_{H,t}}{P_t} G_t - b u_t \tag{48}$$

The per period lumpsum transfers thus depend positively on tax revenue from labour taxes or consumption taxes. On the other hand, transfers have to be cut if government spending or unemployment benefit expenditure increase.

Government spending is subject to shocks

$$\begin{aligned}\log(G_t) &= (1-\rho_G)\log(\overline{G}) + \rho_G \log(G_{t-1}) + \epsilon_t^G,\\ \text{where } \rho_G &\in (0,1),\ \epsilon_t^G \overset{iid}{\sim} N\left(0,\sigma_G^2\right)\end{aligned}$$

2.7 Equilibrium

For each intermediate good, supply must equal total demand. The demand for good i is, as shown previously, $y_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_t^D$, where Y_t^D is total demand for domestic intermediate goods by domestic and foreign final goods firms and the domestic government. Using the expressions for the demands for domestic intermediate good *bundles* derived previously, this can be written as

$$y_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left\{(1-W)\left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t + W\left(\frac{P_{H,t}}{P_t^*}\right)^{-\varpi} C_t^* + G_t\right\} \tag{49}$$

Following Galí and Monacelli (2008) defining an index for aggregate domestic demand $Y_t^D = \left[\int_0^1 y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ allows us to rewrite this as

$$Y_t^D = (1-W)\left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t + W\left(\frac{P_{H,t}}{P_t^*}\right)^{-\varpi} C_t^* + G_t$$

Aggregate demand for domestic intermediate goods production has to equal their aggregate supply minus the resources lost to vacancy posting and the costs of capital utilization, leading to the home economy's aggregate resource constraint

$$Y_t = (1-W)\left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t + W\left(\frac{P_{H,t}}{P_t^*}\right)^{-\varpi} C_t^* + G_t + \frac{P_t}{P_{H,t}} A(\nu_t) K_{t-1}^p + \kappa_t v_t \tag{50}$$

where the demand for domestic intermediate goods by foreign final goods firms follows the AR(1) process

$$\begin{aligned}\log(C_t^*) &= (1-\rho_{C^*})\log(\overline{C^*}) + \rho_{C^*}\log(C_{t-1}^*) + \epsilon_t^{C^*},\\ \text{where } \rho_{C^*} &\in (0,1),\ \epsilon_t^{C^*} \overset{iid}{\sim} N\left(0,\sigma_{C^*}^2\right)\end{aligned}$$

While the above resource constraint equation states that in equilibrium domestic output has to equal its usage as consumption, exports and government spending, market-clearing in the intermediate good sector also requires

$$Y_t = K_t^{1-\alpha} \left(n_t z_t h_t \right)^{\alpha} \tag{51}$$

The net foreign asset position (in real CPI terms) is determined by the trade balance - the difference between domestic output and domestic consumption.

$$\frac{B_t^*}{P_t} - R_{t-1}^* p \left(b_{t-1}^* \right) \frac{B_{t-1}^*}{P_t} = \frac{P_{H,t}}{P_t} Y_t - C_t - I_t - \frac{P_{H,t}}{P_t} G_t - A\left(\nu_t\right) K_{t-1}^p - \frac{P_{H,t}}{P_t} \kappa_t v_t \tag{52}$$

This relation is obtained by combining the consumers' budget constraint, the government's budget constraint and the economy's aggregate resource constraint as well as the equation for total real dividends accrued to households, i.e. the sum of the profits in the intermediate and wholesale sectors

$$\frac{D_t}{P_t} = \frac{P_{H,t}}{P_t} Y_t - n_t \frac{w_t^*}{P_t} h_t \left(1 + s_t\right) - r_t^k K_t - \frac{P_{H,t}}{P_t} \kappa_t v_t \tag{53}$$

3 Model estimation

We estimate the model in log-linearized form using Bayesian Maximum Likelihood methods described in e.g. An and Schorfheide (2007)[10].

For the parameters that control the steady state we assign values that match as closely as possible sample averages in Finnish data or relevant microeconomic evidence. The parameters that affect the dynamics of the model are then estimated with Bayesian methods, with an emphasis on the parameters determining the degree of nominal rigidities in the Finnish economy.

The closest reference for the chosen approach is Gertler, Sala and Trigari (GST, 2008) who estimate a medium-sized monetary DSGE model with labour market frictions and staggered wage bargaining using the same seven quarterly data series as e.g. Smets and Wouters (2003). Unlike GST, but following Christoffel, Kuester and Linzert (CKL, 2009), we include additional labour market information in the form of time series collected from the Finnish Employment Service Statistics. Christoffel, Kuester and Linzert (2009), argue that it is important to include variables characterizing the labour market because they are at the core of the search and matching literature and are crucial for the identification of parameters. Other series that we use in our estimation procedure are from the Finnish national accounts database. All in all our dataset is composed of quarterly series for 1) output, 2) investment 3) hourly nominal wages, 4) unemployment, 5) productivity, 6) government consumption, 7) the interest rate for Finnish 10 year government bonds, and 8) vacancies. The filtered time series are displayed in the Appendix.

[10]The estimation is performed with Dynare/Matlab that uses the Kalman filter to find the likelihood function. We introduce priors on estimated parameters to update the likelihood, and to find the posterior distribution of parameters Dynare runs Metropolis-Hastings simulations. The posterior simulation is done via a MH algorithm on five chains of 500 000 draws.

To avoid problems of stochastic singularity which forms an obstacle to likelihood estimation, we add structural shocks to the model so that the number of shocks equals the number of observable variables. While this is the minimum requirement, the number of shocks could be even larger in which case the model would be over-identified. Our model includes the following shock processes: a technology shock, an investment-specific shock, a government spending shock, a risk premium shock, a shock to the foreign consumption of home produced goods, as well as three labour market shocks: a shock to the bargaining power of workers, a shock to the job destruction rate and a shock to vacancy posting costs. We follow CKL in including also these labour market shocks and assess their relative importance for fluctuations in central macroeconomic variables. CKL find that while other labour market shocks have some influence on the fluctuation of labour market variables, the most important labour market shock in this respect is the shock to the bargaining power of workers which also importantly seems to affect the behaviour of output and inflation over the business cycle.

The bargaining power shock accounts for any exogenous variations that affect the wage bargaining process over and above those captured by the model.

Likewise, as we abstract from endogenous job separations in our model framework, a shock to the separation rate is added to account for variations in the separation margin. Whether the flow of workers into unemployment follows a cyclical pattern is a central question in the search and matching literature. According to one view unemployment fluctuations are mostly driven by changes in the outflow of workers from unemployment, i.e. the hiring - or job creation - margin. As a consequence, it is a common practice to assume an exogenous job destruction rate in matching models. More recently, however, several authors have pointed out to evidence showing that also job separations are cyclical (see e.g. Pissarides, 2009). There is also evidence from Finnish job and worker flow data that the job separation rate has tended to be higher in bad times, indicating a possible countercyclical pattern (see Ilmakunnas and Maliranta, 2008).

The vacancy posting cost shock, in turn, captures sources of variation in hiring incentives other than wage changes

The most important difference from both GST and CKL is that we operate with an open-economy model displaying the economic mechanisms of a small monetary union member state. The open-economy set up is, however, quite simplified and abstracts from exchange rate and monetary policy considerations. Therefore, we add two open-economy shocks to account for possibly important exogenous variation in the international economy that our model is not able to fully characterize.

3.1 Calibrated parameters

A number of parameters are kept fixed throughout the estimation procedure. This applies primarily to steady state related parameters which are taken from the calibrated version of the model used in Obstbaum (2011). The fixed parameters are, accordingly, either matched to the sample means of steady state values of the observed variables using Finnish data in the period 1994Q1-2010Q3 ("great ratios") or taken from evidence found in micro level studies. The parameters for which such empirical observations are not available, are fixed using standard ranges of parameter values in the business cycle and labour market matching literature. Capital-related parameters are new to the estimated version of the model, and the choice of their values is explained below. All fixed parameter values are summarized in Table 1.

Table 1. Fixed parameter values

Parameter	$Value$	Explanation
Preferences		
β	.992	Time-discount factor
ϕ	10	Labour supply (Frish) elasticity of 0.1
ϱ	1.5	Risk aversion
$\varkappa$	0.6	External habit persistence
labour market		
α	0.66	Labour elasticity of production
σ	0.6	Elasticity of matches w.r.t. unemployment
σ_m	0.6	Efficiency of matching
ρ	0.06	Exogenous quarterly job destruction rate
η	0.6	Bargaining power of workers
b	0.4	Unemployment benefits
z	1.1	Technology, targets output $Y = 1$
Capital		
δ_k	0.025	Capital depreciation rate
η_k	2.4	Capital adjustment cost elasticity
Wholesale sector		
ε	6	Elasticity of substitution, markup 20 percent
Final goods sector		
ϖ	6	Trade price elasticity

The quarterly discount factor is $\beta = 0.992$ which corresponds to an annual interest rate of $3,3\%$. The labour supply, or Frish elasticity $(\frac{1}{\phi})$, is set to 0.1. This is in the lower range of values implied by most microeconomic studies, which estimate this elasticity to be between 0 and 0.5. The risk aversion and habit persistence parameters are standard values in the literature, and used in a similar model by e.g. Christoffel, Kuester and Linzert (2009).

The quarterly separation rate is calibrated at $\rho = 0.06$, corresponding to the average Finnish separation rate in 1994-2010. The elasticities of labour and capital in production are set to $\alpha = 0.67$ and $1 - \alpha = 0.33$ respectively. The unemployment benefit parameter is calibrated at $b = 0.4$, and generates a net replacement rate of 66 percent, defined as the ratio of unemployment benefits to average net (after-tax) income from work $\frac{b}{wh(1-\tau)}$. The OECD's "Benefits and Wages 2007" publication suggests an average net replacement rate over 60 months of unemployment of 70 percent for Finland, averaging over four different family types. The replacement rate of 66 percent corresponds to a relative value of non-work to work activities of 73 percent. This latter value is known to be a key determinant of labour market volatility in matching models of the business cycle[11]. The parameters of the aggregate matching function and the bargaining power of workers are all set to 0.6 reflecting the review of most plausible values by Petrongolo and Pissarides (2001).

[11] In Obstbaum (2011) we show in detail how this value is calibrated and what its significance for the steady state properties and dynamics of the model is.

The sensitivity of the capital utilization rate with respect to changes in the rental rate η_ν is initially fixed at 0.43 and the capital adjustment cost elasticity η_k at 2.4. These are their estimated values fromn Gertler, Sala and Trigari (2008) but we also attempt to estimate them in the current model framework using different data. The same applies for open-economy parameters. We initially fix the share of imported intermediate goods in the production of final goods at 0.25 and trade price elasticity to 6 but we also attempt to estimate them. The results are reported below.

The steady state values of key model variables implied by the current parameterization can be found in Table 2. The steady state equations of the model are, in turn, provided in Appendix A.1. In the steady state, output is normalized to one, so that GDP components can be interpreted directly as percent shares of GDP. Government spending includes both government consumption and government investment and investment refers to private investment which has accounted on average for 17 percent of GDP in Finland. The share of private consumption in the model, 0.57, is slightly higher than the sample mean. This is because in the model a symmetric open economy steady state is assumed where consumption levels are initially the same at home and abroad, and both the trade balance and net foreign asset holdings are zero. In reality, Finland experienced a prolonged period of positive net trade over the whole sample period until very recent years.

Table 2. Steady state ratios

Variable	Value	Description
Y	1	Output
C	0.57	Private consumption
I	0.17	Private investment
G	0.25	Government spending
κv	0.01	Total vacancy costs
u	0.1	Unemployment rate
n	0.9	Employment
qw	0.6	Probability of finding a job
qf	0.7	Probability of finding a worker
$b/(wh(1-\tau))$	0.65	Net replacement rate
nwh	0.5	Wage bill
τ^C	0.11	Consumption tax
τ	0.15	labour tax rate on employee
s	0.12	Employers' social security contribution
TR / τ^{LS}	0.09	Lump-sum transfers

The labour force is normalised to one, and the steady state unemployment level is 10 percent, corresponding to the average Finnish unemployment rate in 1994-2010.

The steady state tax rates for labour and consumption are computed as ten year historical averages of corresponding tax rates in Finland times the model-implied tax base for each tax category. Accordingly, labour taxes for the employee and the employer respectively amount to 30 percent and 25 percent times the wage bill and the consumption tax rate corresponds to an average of 19 percent times the size of private consumption.

3.2 Prior distributions of the estimated parameters

We estimate all in all 22 parameters of which 16 are the autoregressive coefficients of the shock processes and the standard deviations of shocks.

Like Adolfson et al. (2008), we use the beta distribution for all parameters bounded between 0 and 1, i.e. the nominal rigidity parameters, wage indexation and the persistence parameters of the shock processes.

In choosing prior values, we emphasize values fixed in the calibration procedure of Obstbaum (2011) and, when there is no specific indication on values pertaining to the Finnish data, we use standard ranges of parameter values in the literature. All prior distributions are collected in Table 3.

The prior mean of the wage rigidity parameter is set at 0.75 because, on the one hand, a value of 0.6 was found in the moment comparison analysis done with the calibrated version of the model (see Obstbaum, 2011) to generate fluctuations in labour market and other variables that most closely matched the Finnish time series data. On the other hand, the Confederation of Finnish Industries calculates an average frequency of wage adjustments of slightly more than one year based on wage contracts negotiated by their member companies. The prior for wage indexation is set at 0.5 because we do not have strong prior beliefs on either high or low degree of indexation. As with other estimated parameters, we also assign sufficient prior uncertainty to allow the data to have a nontrivial influence on parameter estimates.

The prior mean for price rigidity is set at 0.5 following Vilmunen and Laakkonen's (2004) estimates from Finnish micro-level CPI data, meaning that prices are believed to be changed once in two quarters. These prior values for the nominal rigidities are somewhat smaller than those most often used in similar estimated business cycle models but they are firmly based on micro-level or other evidence from Finnish data. We will discuss the significance of changing these prior values for our results in the section on nominal rigidities.

For all the shock processes' autoregressive parameters, the prior mean of the chosen beta distribution is set at the same value of 0.75 reflecting our lack of knowledge on the individual persistence pattern of each shock. The value of 0.75, in turn, is based on previous experience from the literature that the shock processes tend to show some persistence. These rather informative priors for the autoregressive parameters are standard in the literature. As to the prior means of the shocks' standard deviation parameters, an inverse gamma distribution is assumed with a prior mean of 0.01 for the technology shock, the government spending shock and the risk premium shock, and a prior mean of 0.05 for all the other shock innovations.

The final three parameters we estimate are the sensitivity of the capital utilization rate with respect to changes in the rental rate of capital, the import content of final goods production and the debt-elasticity of interest rates.

3.3 Posterior distributions of parameters and model fit

Table 3 reports the prior and posterior distributions for each parameter which are also plotted in Figure 2 as well as 90% confidence intervals. For almost all parameters, data is reasonably informative about the parameter as reflected in prior and posterior densities that sufficiently differ from each other and in variances of posterior distributions that are lower than those of their priors.

Table 3. Prior and Posterior distribution of estimated parameters

Parameter		Prior	Posterior distribution		
		Distribution	Mean	5%	95%
Calvo wage parameter	γ	Beta (0.75, 0.1)	0.83	0.68	0.95
Calvo price parameter	ξ	Beta (0.5, 0.2)	0.45	0.42	0.49
Wage indexing parameter	ε_w	Beta (0.5,0.2)	0.72	0.59	0.84
Sensitivity of capital utilization rate	η_ν	Beta (0.5,0.1)	0.67	0.55	0.79
Import content of final goods production	W	Beta (0.25,0.2)	0.41	0.33	0.49
Debt-elasticity of interest rates	γ_{b^*}	IGamma (0.05,inf)	0.018	0.011	0.024
AR coefficients of shocks					
Technology	ρ_z	Beta (0.75, 0.1)	0.66	0.48	0.82
Investment	ρ_I	Beta (0.75, 0.1)	0.41	0.27	0.55
Bargaining power	ρ_η	Beta (0.75, 0.1)	0.23	0.13	0.32
Job destruction	ρ_ρ	Beta (0.75, 0.1)	0.72	0.60	0.82
Government spending	ρ_G	Beta (0.75, 0.1)	0.70	0.59	0.82
Foreign export demand	ρ_{C^*}	Beta (0.75, 0.1)	0.65	0.60	0.70
Risk premium	ρ_R	Beta (0.75, 0.1)	0.82	0.75	0.90
Vacancy costs	ρ_κ	Beta (0.75, 0.1)	0.77	0.67	0.86
Std. deviations of shocks					
Technology	ϵ_z	IGamma (0.015,0.2)	0.013	0.011	0.015
Investment	ϵ_I	IGamma (0.05,0.2)	0.084	0.069	0.097
Bargaining power	ϵ_η	IGamma (0.05,0.2)	0.414	0.351	0.485
Job destruction	ϵ_ρ	IGamma (0.05,0.2)	0.031	0.027	0.035
Government spending	ϵ_G	IGamma (0.01,0.2)	0.009	0.008	0.010
Foreign export demand	ϵ_{C^*}	IGamma (0.05,0.2)	0.138	0.114	0.163
Risk premium	ϵ_R	IGamma (0.01,0.2)	0.002	0.001	0.002
Vacancy costs	ϵ_κ	IGamma (0.05,0.2)	0.096	0.077	0.114

The estimate for the wage rigidity parameter is 0.83 corresponding to an average frequency of wage adjustment of 6 quarters, i.e. higher than our prior. In both Gertler, Sala and Trigari (2008) and Christoffel, Kuester and Linzert (2009) the estimate for wage rigidity is updated downwards by the data using the same prior than we do. The high wage rigidity implied by our model is indeed slightly surprising taking into account that the wage data we use includes, in addition to contract wages, also more flexible wage elements such as performance pay. The wage rigidity parameter seems to be fairly well identified but we will perform some sensitivity analysis with respect to different priors for this key parameter in the next section.

The estimate for price rigidity, in turn, is 0.45. This implies a lower degree of price rigidity than in many other estimated New Keynesian DSGE models and is very close to our prior. The latter feature could be a sign of the results being mostly led by the prior and only marginally by the data, but this does not seem to be the case here, since estimated price rigidity is still 0.46 when we change the prior to the more conventional 0.75. Also another recent study based on Finnish data by Freystätter (2010) estimates a value of 0.48 for the sticky price parameter in a similar DSGE model with financial frictions instead of labour market frictions.

Our fairly uninformative prior of 0.5 for the wage indexing parameter is updated upwards by the data to 0.72 implying a high degree of indexing of wages to past inflation. As Gertler, Sala and Trigari (2008) note, this suggests a high degree of effective *real* wage rigidity, well in line with the empirical results of Böckerman, Laaksonen and Vainiomäki (2006) from Finnish micro-level data, and international comparisons of wage rigidities (see Dickens et al., 2007).

As to the shock processes, starting with the same prior for all shocks' autoregressive coefficients gives us significantly different posterior values across different shocks. The most persistent shocks appear to be the ones to vacancy posting costs, to the risk premium and to job destruction. On the other hand, the shock to the bargaining power of workers is much less persistent than our prior, the posterior mean for the AR-coefficient is just 0.23. The largest shocks, in turn, seem to be the bargaining power shock, followed by the shock to export demand, vacancy costs and investment. The result that bargaining power shocks are large but not persistent is consistent with the findings of both GST and CKL. We will explore the significance of this result for the nominal rigidities in the next section.

Figure 3. shows the time series of the estimated shock processes. They show, for example, that the international financial crisis shows up in the Finnish economy as a huge negative shock to export demand, combined with a smaller negative technology shock, and accompanied by an additional negative shock in the form of an increased job separation rate. This picture seems to square well with the fact that the recent recession in Finland was clearly export-led. The increased job separation rate also fits to available evidence from Finland by Ilmakunnas and Maliranta (2008) that separations tend to increase in bad times.

One problem of contemporary macroeconomic models that has been pointed out in recent years is that they cannot explain the observed smooth behaviour of wages over time together with the volatile behaviour of employment, the so-called unemployment volatility puzzle. However, Shimer (2010) argues that labour market frictions together with wage rigidity solve the puzzle. Gertler and Trigari (2009) show that this is the case with moment comparisons in a model with staggered wage bargaining. Table 4 shows the standard deviations implied by the model along with the sample standard deviations based on the observed data over the estimation period.

Table 4. Relative standard deviations of selected variables

Variable	Estimated model	Data
Unemployment	4.944	4.067
Vacancies	10.944	7.000
Tightness	13.944	10.200
Wage	1.055	0.867
Consumption	0.778	0.733
Investment	5.689	3.467
Productivity	1.000	1.000

The relative standard deviations implied by the model are very close to those found in the data, reflecting the fact that our model specification with labour market frictions and wage rigidity is able to solve the unemployment volatility puzzle fairly well. The model even slightly overpredicts the volatility of labour market variables and also that of investment.

3.4 The assessment of nominal rigidities

Canova and Sala (2009) argue that to address identification issues in the Bayesian estimation of DSGE models every model should at least be tested for the sensitivity with respect to different priors. Del Negro and Schorfheide (2008) have specifically pointed out the significance of the choice of priors for assessing nominal rigidities. In the Bayesian framework, the question whether a flexible or sticky price or wage model fits to the data, can be evaluated by comparing posterior odds associated with *different* priors on the rigidities. The problem that Del Negro and Schorfheide observe is that priors for the *other* parameters, especially those describing exogenous processes, affect this comparison.

For the shock processes that can be observed, one could measure their persistence and standard deviation but very often shock processes are latent making the choice of priors for exogenous processes' parameters more difficult. In our model this is the case with labour market shocks. Starting with the same priors, the posterior distributions point to large but not persistent bargaining power shocks and to persistent but smaller vacancy cost and job destruction shocks.

To see what the interaction between the shock processes and nominal rigidity in this model is, we re-estimate the model with different priors for wage rigidity and for the bargaining power shock's parameters. In each panel of Table 5, we consider a high and a low prior for wage rigidity and assess the respective models' relative fit to the data by comparing their marginal likelihoods. We report the posterior means and confidence intervals of the nominal rigidity parameters and wage indexation which are key parameters of the current model, as well as the parameters of the bargaining power shock. We also assess the sensitivity of results to changes in the other labour market shocks' priors.

The estimation of the baseline model with different priors for wage rigidity shows, in line with Del Negro and Schorfheide's (2008) results, that the posterior estimate of the Calvo wage parameter is significantly driven by its prior. Depending on the prior, the posterior estimate of wage rigidity ranges from 0.46 to 0.8, and the marginal likelihoods of the different specifications are close to each other. As we reported in the section on posterior distributions, the estimate of price rigidity is not similarly sensitive to changing its prior.

Rabanal and Rubio-Ramirez (2003) compared New Keynesian models of the Euro area which comprised of different combinations of nominal rigidities and indexation. They found problems in simultaneously identifying the Calvo price parameter and the mean of the price markup, as well as the Calvo wage parameter and the elasticity of substitution between different types of labour in their monopolistic labour market specification. Similarly, Gertler, Sala and Trigari (2008) point to the possibility that, in the labour market matching model with staggered wage bargaining, it might be difficult to separately identify the bargaining power parameter and the flow value of unemployment which are known to be key determinants of effective wage rigidity in this kind of models (see Hagedorn and Manovski, 2008). Moreover, Rabanal and Rubio-Ramirez find that estimated price rigidity is the lower the lower is the estimated standard deviation of the price markup shock.

The above considerations suggest that the assessment of nominal rigidities may importantly depend on other parameters. More specifically, in the current model, there might be interaction between wage rigidity and the bargaining power shock that could also lead to identification issues. To investigate this issue, we change the prior for the bargaining power shock's autoregressive parameter to 0.2 reflecting the estimation results of Gertler, Sala and Trigari (2008) and of Christoffel, Kuester and Linzert (2009) that the shock in question is almost white noise.

The second panel of Table 5 shows that in both the high and low rigidity specifications the posterior estimate of wage rigidity is significantly updated downwards when the bargaining power shock is not assumed to be as persistent as before. This indeed indicates that in previous specifications, including our baseline model, part of the estimated wage rigidity inherits the properties of the bargaining power shock. Rabanal and Rubio-Ramirez (2008) make the same point concerning price rigidity and price markup shocks. They argue that markup shocks should be modelled as *iid* shocks instead of AR(1) processes for inflation persistence to be explained endogenously by the model. The marginal likelihoods of the low rigidity and high rigidity specifications are again close to each other.

Table 5. Sensitivity of rigidity estimates w.r.t to different priors

	high rigidity $\gamma = 0.8$			low rigidity $\gamma = 0.5$		
Baseline	mean	5%	95%	mean	5%	95%
rho_eta, beta $(0.75, 0.1)$	0.38	0.28	0.49	0.28	0.21	0.37
epsilon_eta, inv.gamma $(0.05, inf.)$	0.18	0.15	0.21	0.19	0.16	0.22
wage rigidity	0.8	0.63	0.96	0.46	0.28	0.64
price rigidity, beta $(0.5, 0.1)$	0.38	0.34	0.41	0.37	0.34	0.39
wage indexation, beta $(0.5, 0.2)$	0.45	0.31	0.60	0.38	0.26	0.50
log marginal density		1512.00			1512.84	
less shock persistence						
rho_eta, beta $(0.2, 0.1)$	0.30	0.18	0.43	0.27	0.17	0.36
epsilon_eta, inv.gamma $(0.05, inf.)$	0.20	0.15	0.25	0.18	0.14	0.21
wage rigidity	0.71	0.48	0.93	0.36	0.20	0.55
price rigidity, beta $(0.5, 0.1)$	0.35	0.30	0.41	0.33	0.28	0.38
wage indexation, beta $(0.5, 0.2)$	0.49	0.26	0.74	0.34	0.11	0.54
log marginal density		1499.27			1501.86	
higher shock variance						
rho_eta, beta $(0.75, 0.1)$	0.24	0.12	0.32	0.42	0.32	0.50
epsilon_eta, inv.gamma $(0.2, 0.2)$	0.60	0.52	0.68	0.15	0.12	0.18
wage rigidity	0.90	0.85	0.98	0.45	0.28	0.62
price rigidity, beta $(0.5, 0.1)$	0.48	0.45	0.51	0.31	0.27	0.33
wage indexation, beta $(0.5, 0.2)$	0.79	0.68	0.90	0.27	0.13	0.42
log marginal density		1510.24			1499.27	

Next, we assess the role of the prior on shock variance for estimated rigidities. We choose a higher and tighter prior for the standard deviation of the bargaining power shock, again leaning on GST and CKL estimation results, with different data though, that bargaining power shocks are large. In the high rigidity specification, the posterior of the bargaining power shock is updated upwards implying huge exogenous disturbances to the wage negotiation process, and the wage rigidity and indexation parameters get very high posterior mean values. In the low rigidity specification, in turn, the posterior means of key parameters are not significantly different from previous results.

In addition, we test the baseline model with a wage rigidity prior of 0.75 under a scenario where we keep the high prior for the standard deviation of the bargaining power shock but shut off the persistence of that shock as Rabanal and Rubio-Ramirez (2008) suggest should be done with markup shocks. This turns out to be our most successful model on the basis of marginal likelihoods, in the model comparison exercise. All the parameter estimates, including price rigidity and wage indexation, are similar to the baseline model except for wage rigidity which is now only 0.4. The estimated standard deviation of the bargaining power shock is now smaller than in the previous alternatives where the shock was allowed to have persistence. These results imply that although nominal wage rigidity might not be that important, wages are in practise rather tightly indexed to past inflation, a feature that translates into a relatively high effective real wage rigidity. This is what Böckerman et al. (2006) suggest was the case in Finland at least in the latter part of the 1990's.

Finally, we consider for the baseline model how similar changes to other labour market shocks' parameters affects the assessment of nominal rigidities. We find that changes in neither the vacancy cost shock's or the job destruction rate shock's persistence parameters or standard deviation parameters significantly affect estimated wage rigidity. For all the alternatives we test, the posterior mean of the Calvo wage parameter stays between 0.81 and 0.83.

To sum up, this sensitivity analysis with respect to different priors shows that estimated wage rigidity importantly depends on its own prior but also on the bargaining power shock process' priors. On one hand, it would seem important to include bargaining power shocks in the model to account for exogenous variations present in the negotiation process, but on the other hand, these variations are extremely hard to quantify and our prior beliefs on them heavily influences our estimation results on wage rigidity. Marginal likelihood comparisons between the low rigidity and high rigidity specification cannot unambiguously tell which specification is preferred by the data. However, as we estimate the model with both wage rigidity and wage indexation shut off, the posterior odds comparison clearly favours the specifications where they are included. The most successful model is the one where the bargaining power shock is modelled with no persistence.

In addition to the sensitivity of our results with respect to different priors, we have also tried different combinations of data series and shock processes but concluded that the present one still gave the most reliable estimation results.

3.5 The importance of labour market shocks

Next, we assess the relative importance of shocks originating from the labour market in driving Finnish business cycles. If these shocks turn out to be important determinants of output fluctuations, this would give policy makers valuable information on which features of the labour market should be monitored more closely.

Table 6 shows the contribution of all the shocks included in our estimated model to the forecast error variance of selected variables for two different horizons. Output fluctuations are mainly driven by technology shocks and shocks to foreign export demand but the effect of labour market shocks is also non-trivial. The bargaining power shock and the vacancy posting cost shock both account for approximately five percent of output fluctuations. Taken together, almost twelve percent of output fluctuations are caused by shocks originating from the labour market. This is a considerable amount and suggests that also policies targeted to the labour market may have significant effects on output fluctuations.

While investment specific shocks are the most important driving force of consumption fluctuations[12], and technology and foreign demand shocks also play a major role, labour market shocks do have an effect on consumption fluctuations as well, although to a lesser extent than on output fluctuations.

Unemployment and vacancy fluctuations seem to be, in turn, mostly driven by labour market shocks. The bargaining power shock is the most important of these shocks while the contribution of the job destruction rate shock is not very significant on other variables than unemployment. Compared to Christoffel, Kuester and Linzert's (2009) results, the bargaining power shock is much more important for unemployment fluctuations in the current framework which is probably due to our assumption of efficient, as opposed to right-to-manage, bargaining. This would imply in practice that the labour market organizations that negotiate the wage contracts are also importantly responsible for labour market outcomes. This responsibility is further emphasized in an environment with no possibility to conduct independent monetary policy. Another difference to CKL is that vacancy posting cost shocks play a more important role in our model. This is understandable, since they are estimated to be one of the largest and most persistent shocks in the current framework.

Wage fluctuations are also importantly determined by disturbances in the wage negotiation process but the contribution of bargaining power shocks is less important than in CKL's model for the Euro area. The importance of bargaining power shocks to the forecast error variance of wages is comparable to the importance of wage markup shocks for wage fluctuations in more conventional New Keynesian DSGE models featuring monopolistic labour suppliers (see e.g. Smets and Wouters, 2003). In our small monetary union member state framework, however, shocks to foreign export demand determine as much as half of the wage fluctuations. This is because, in our estimated model, a shock

[12] Due to the assumption of consumers owning the capital stock, investment-specific shocks directly affect the consumption-investment trade-off of the representative agent. Hornstein, Krusell and Violante (HKV, 2007) argue that when capital is introduced into the matching model in the standard way as in this paper, the assumption that the capital stock in matched firm-worker pair is continuously adjustable limits the impact of investment-specific technical change. Accordingly, in the standard model, changes in the price of capital affect labour-market outcomes only through their impact on net labour productivity whereas in HKV's vintage model, capital price changes also represent shocks to vacancy posting costs.

to foreign export demand directly feeds through to the demand of domestic intermediate firms which also conduct the wage negotiations with workers. The importance of foreign export demand shocks has also been reported in e.g. Almeida (2009).

Table 6. Variance decomposition of estimated model

						Labour mkt shocks		
	z	I	G	C^*	R	η	ρ	κ
On impact								
Y	47.9	10.3	0.2	30.0	0.1	5.2	1.7	4.6
C	20.5	36.9	1.3	28.9	6.7	1.7	1.2	2.9
u	5.6	1.2	0.1	9.7	0.2	36.3	13.9	33.1
v	6.7	0.9	0.1	14.1	0.3	37.2	3.8	36.8
w	4.6	2.7	0.2	51.6	0.2	40.2	0.3	0.2
At 5 years								
Y	48.8	9.0	0.2	30.3	0.1	5.2	1.7	4.6
C	28.0	18.3	1.7	34.2	11.0	1.9	1.5	3.4
u	5.6	1.0	0.1	9.7	0.2	36.4	13.9	33.1
v	6.7	0.9	0.1	14.1	0.3	37.3	3.8	36.8
w	4.4	0.8	0.2	52.5	0.2	41.6	0.3	0.2

4 Concluding remarks

This paper is a first attempt to estimate a DSGE model for the Finnish economy that incorporates labour market frictions and wage rigidity. We slighty modify the staggered wage bargaining framework of Gertler and Trigari (2009) to describe the rigidity in wage determination.

Our contribution is twofold. First, we obtain estimates of nominal rigidities and wage indexation that are in line with the existing literature and give an economically plausible picture of the Finnish economy. We establish that wage rigidity is empirically relevant for the Finnish business cycle, and provide a detailed assessment of the significance of prior beliefs on the degree of wage rigidity. We find that the data seems to support a relatively wide range of different degrees of wage rigidity conditional on the assumption of exogenous disturbances to wage negotiations. In particular, it appears that the nature of shocks to the bargaining power of workers, similarly to wage markup shocks in the conventional New Keynesian model with monopolistic labour suppliers, importantly affect the conclusions that can be made on wage rigidity.

While the exact degree of rigidity of nominal wages is left unclear, the indexation of wages to past inflation seems to be an important feature of the Finnish economy. As this can be translated into relatively high effective real wage rigidity, it corresponds well to previous evidence on the high degree of real wage rigidity in Finland. Price rigidity is estimated to be lower than in many other similar models but is again well in line with Finnish evidence.

Second, our estimation approach sheds more light on the magnitude and effects of different shocks in the Finnish economy. Of the conventional shocks, that to export demand is especially important as expected for a small open economy. We also add labour market shocks to account for exogenous disturbances originating in the labour market that our model is not able to fully capture although the labour market extension we apply is much more detailed than that of the standard New Keynesian model. We find that labour market shocks importantly drive fluctuations of labour market variables (unemployment and vacancies) but they also account for a non-trivial share of output fluctuations.

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A Appendix

A.1 Steady state of the model economy

Euler equation

$$\beta = \frac{1}{R}$$

Marginal utility of consumption

$$\lambda = (C - \varkappa C)^{-\varrho}$$

Marginal utilility of wealth

$$\Lambda = \frac{\lambda}{(1 + \tau^c)}$$

Interest rate on foreign bonds

$$R^* = R$$

Capital rental rate

$$r^k = xmpk = (R - 1) + \delta^k$$

where

$$mpk = (1 - \alpha)\, k^{-\alpha}\, (zh)^{\alpha} = (1 - \alpha)\, \frac{y}{k}$$

Tobin's Q

$$Q = 1$$

FOC of retail firm

$$x = \frac{1}{\mu} = \frac{\varepsilon - 1}{\varepsilon}$$

Matches

$$m = \sigma_m u^{\sigma} v^{1-\sigma}$$

Employment

$$\rho n = m$$

Unemployment

$$u = 1 - n$$

Probability of finding a worker

$$q^F = \frac{m}{v}$$

Probability of finding a job

$$q^W = \frac{m}{u}$$

Labour market tightness

$$\theta = \frac{v}{u}$$

FOC for hours

$$(1-\tau)\, x^H mpl = (1+s)\, mrs(1+\tau^c)$$

where

$$mpl = \alpha z h^{\alpha-1} \text{ and } mrs = \frac{\delta h^\phi}{\lambda} \text{ and } x^H = x$$

Economy-wide resource constraint

$$Y = C + I + G + \kappa v, \text{ in the symmetric steady state}$$

Government budget constraint

$$TR = nwh(\tau + s) + \tau^c C - G - bu$$

Market clearing / aggregate output

$$Y = K^{1-\alpha}\,(nzh)^\alpha$$

Wage

$$w = \frac{\eta}{(1+s)}\left[\frac{xmpl}{\alpha} - \frac{r^k k}{h}\right] + \frac{(1-\eta)}{(1-\tau)}\left[\frac{mrs\,(1+\tau^c)}{(1+\phi)} + \frac{b}{h} + \beta\frac{q^w}{h}H\right]$$

Job creation condition

$$\kappa = q^F \beta J$$

where the firm surplus

$$J = \frac{1}{1-\beta\,(1-\rho)}\left[xk^{1-\alpha}\,(zh)^\alpha - wh\,(1+s) - r^k k\right]$$

Worker surplus

$$H = \frac{1}{1-\beta\,(1-\rho-q^W)}\left[wh\,(1-\tau) - \frac{mrsh\,(1+\tau^c)}{(1+\phi)} - b\right]$$

Worker discount factor

$$\overline{\Delta} = \frac{\overline{h}\,(1 - \overline{\tau})}{1 - \overline{\beta}\,(1 - \overline{\rho})\,\gamma}$$

Firm discount factor

$$\overline{\Sigma} = \frac{\overline{h}\,(1 + s)}{1 - \overline{\beta}\,(1 - \overline{\rho})\,\gamma}$$

A.2 Model dynamics

The dynamics of the model are obtained by taking a log-linear approximation around a deterministic steady state.

Euler equation

$$\widehat{\Lambda}_t = E_t\left(\widehat{\Lambda}_{t+1} + \widehat{R}_t - \widehat{\pi}_{t+1}\right)$$

Shadow value of wealth

$$\widehat{\Lambda}_t = \widehat{\lambda}_t - \frac{\overline{\tau}^c}{(1+\overline{\tau}^c)}\widehat{\tau}_t^c$$

Marginal utility of consumption

$$\widehat{\lambda}_t = -\frac{\varrho}{(1-\varkappa)}\left(\widehat{C}_t - \varkappa\widehat{C}_{t-1}\right)$$

Interest rates

$$\widehat{R}_t = \widehat{R}_t^* - \gamma_{b^*}\widehat{b}_t^*$$

Capital utilization

$$\widehat{\nu}_t = \eta_\nu \widehat{r}_t^k$$

Investment

$$\widehat{I}_t = \frac{1}{(1+\beta)}\widehat{I}_{t-1} + \frac{1/\eta_k}{(1+\beta)}\left(\widehat{Q}_t + \widehat{\epsilon}_t^I\right) + \frac{\beta}{(1+\beta)}E_t\widehat{I}_{t+1}$$

Capital accumulation

$$\widehat{K}_t^p = \left(1-\delta^k\right)\widehat{K}_{t-1}^p + \frac{\overline{I}}{\overline{K}^p}\left(\widehat{I}_t + \widehat{\epsilon}_t^I\right)$$

Effective capital

$$\widehat{K}_t = \widehat{\nu}_t + \widehat{K}_{t-1}^p$$

Tobin's Q

$$\widehat{Q}_t = \beta\left(1-\delta^k\right)E_t\widehat{Q}_{t+1} + \left[1-\beta\left(1-\delta^k\right)\right]E_t\left(\widehat{r}_{t+1}^k + \widehat{\nu}_{t+1}\right) - \left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right)$$

Matching function

$$\hat{m}_t = \sigma\hat{u}_t + (1-\sigma)\hat{v}_t$$

Employment dynamics

$$\hat{n}_t = (1-\bar{\rho})\hat{n}_{t-1} + \frac{\bar{m}}{\bar{n}}\hat{m}_{t-1} - \bar{\rho}\hat{\rho}_t$$

Unemployment

$$\widehat{u}_t = -\frac{1-\overline{u}}{\overline{u}}\hat{n}_t$$

Transition probabilities

$$\hat{q}_t^F = \hat{m}_t - \hat{v}_t$$

$$\hat{q}_t^W = \hat{m}_t - \hat{u}_t$$

Labour market tightness

$$\widehat{\theta}_t = \hat{v}_t - \hat{u}_t$$

FOC for hours worked

$$\hat{x}_t = m\hat{r}s_t - m\hat{p}l_t + \frac{\overline{\tau}}{(1-\overline{\tau})}\widehat{\tau}_t + \frac{\overline{s}}{(1+\overline{s})}\widehat{s}_t + \frac{\overline{\tau}^c}{(1+\overline{\tau}^c)}\widehat{\tau}_t^c$$

where

$$\hat{x}_t^H = \widehat{x}_t + \widehat{P}_{H,t} - \widehat{P}_t$$

$$m\hat{p}l_t = \hat{z}_t - (1-\alpha)\,\hat{h}_t$$

and

$$m\hat{r}s_t = \phi\hat{h}_t - \hat{\lambda}_t$$

Capital rental rate

$$\widehat{r}_t^k = \hat{x}_t^H + m\hat{p}k_t$$

New Keynesian Phillips Curve

$$\widehat{\pi}_{H,t} = \nu\widehat{x}_t + \beta E_t\widehat{\pi}_{H,t+1}$$

where $\widehat{\pi}_{H,t} = \widehat{P}_{H,t} - \widehat{P}_{H,t-1}$ is domestic inflation

First order condition for wage setting

$$\widehat{J}_t(w_t^*) + \widehat{\Delta}_t = \widehat{H}_t(w_t^*) + \widehat{\Sigma}_t - \frac{1}{1-\eta}\widehat{\eta}_t$$

Firm surplus

$$\begin{aligned}
\widehat{J}_t(w_t^*) \quad = \quad & \frac{\overline{x}\overline{mplh}}{\alpha \overline{J}} \left(\widehat{x}_t^H + \widehat{mpl}_t + \widehat{h}_t \right) - \frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}} \left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t \right) - \frac{\overline{wh}\overline{s}}{\overline{J}} \widehat{s}_t \\
& - \frac{\overline{r}^k \overline{k}}{\overline{J}} \left(\widehat{r}_t^k + \widehat{k}_t \right) - \overline{\beta}\overline{\rho} E_t \widehat{\rho}_{t+1} + \overline{\beta}\left(1-\overline{\rho}\right) E_t \left(\widehat{J}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1} \right) \\
& - \frac{\overline{\beta}\left(1-\overline{\rho}\right)\gamma}{1-\overline{\beta}\left(1-\overline{\rho}\right)\gamma} \frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}} E_t \left(\widehat{w}_t^* + \varepsilon_w \widehat{\pi}_t - \widehat{w}_{t+1}^* - \widehat{\pi}_{t+1} \right)
\end{aligned}$$

Worker discount factor

$$\widehat{\Delta}_t = \left(1-\iota\right) \widehat{h}_t - \frac{\left(1-\iota\right)\overline{\tau}}{\left(1-\overline{\tau}\right)} \widehat{\tau}_t + \iota E_t \left(\widehat{\beta}_{t,t+1} + \varepsilon_w \widehat{\pi}_t - \widehat{\pi}_{t+1} + \widehat{\Delta}_{t+1} \right) - \overline{\beta}\overline{\rho}\gamma E_t \widehat{\rho}_{t+1}$$

Worker surplus

$$\begin{aligned}
\widehat{H}_t(w_t^*) \quad = \quad & \frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}} \left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t \right) - \frac{\overline{wh}\overline{\tau}}{\overline{H}} \widehat{\tau}_t - \frac{\overline{mrs}\overline{h}\left(1+\overline{\tau}^c\right)}{\left(1+\phi\right)\overline{H}} \left[\widehat{mrs}_t + \widehat{h}_t \right] \\
& - \frac{\overline{mrs}\overline{h}\overline{\tau}^c}{\left(1+\phi\right)\overline{H}} \widehat{\tau}_t^c - \overline{\beta}\overline{q}^W E_t \left(\hat{q}_t^W + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1} \right) \\
& + \overline{\beta}\left(1-\overline{\rho}\right) E_t \left(\widehat{H}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1} \right) - \overline{\beta}\overline{\rho} E_t \widehat{\rho}_{t+1} \\
& + \frac{\overline{\beta}\left(1-\overline{\rho}\right)\gamma}{1-\overline{\beta}\left(1-\overline{\rho}\right)\gamma} \frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}} E_t \left(\widehat{w}_t^* + \varepsilon_w \widehat{\pi}_t - \widehat{w}_{t+1}^* - \widehat{\pi}_{t+1} \right)
\end{aligned}$$

Firm discount factor

$$\widehat{\Sigma}_t = \left(1-\iota\right) \widehat{h}_t + \frac{\left(1-\iota\right)\overline{s}}{\left(1+\overline{s}\right)} \widehat{s}_t + \iota E_t \left(\widehat{\beta}_{t,t+1} + \varepsilon_w \widehat{\pi}_t - \widehat{\pi}_{t+1} + \widehat{\Sigma}_{t+1} \right) - \overline{\beta}\overline{\rho}\gamma E_t \widehat{\rho}_{t+1}$$

Optimal contract wage

$$\widehat{w}_t^* = \left[1-\iota\right] \widehat{w}_t^0\left(r\right) + \iota E_t \left(\widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t \right) + \iota E_t \widehat{w}_{t+1}^*$$

Target wage

$$\widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H \Gamma E_t \left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^* \right]$$

Spillover-free target wage

$$\begin{aligned}
\widehat{w}_t^0 \quad = \quad & \varphi_x \left(\widehat{x}_t^H + \widehat{mpl}_t \right) - \varphi_k (\widehat{r}_t^k + \widehat{k}_t) + \varphi_m \widehat{mrs}_t - \varphi_h \widehat{h}_t \\
& + \varphi_H E_t \left(\widehat{q}_t^W + \widehat{H}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1} \right) - \varphi_s \widehat{s}_t + \varphi_\tau \widehat{\tau}_t + \varphi_{\tau^c} \widehat{\tau}_t^c \\
& + \varphi_D E_t \left[\widehat{\Sigma}_{t+1} - \widehat{\Delta}_{t+1} \right] + \varphi_\eta \widehat{\eta}_t + \widehat{P}_t
\end{aligned}$$

Average wage

$$\widehat{w}_t = (1-\gamma)\,\widehat{w}_t^* + \gamma\left(\widehat{w}_{t-1} - \widehat{\pi}_t + \varepsilon_w \widehat{\pi}_{t-1}\right)$$

or

$$\widehat{w}_t = \lambda_b \widehat{w}_{t-1} + \lambda_0 \widehat{w}_t^0 + \lambda_f E_t \widehat{w}_{t+1}$$

Vacancy posting condition

$$\begin{aligned} \widehat{\kappa}_t - \hat{q}_t^F &= E_t\left(\widehat{J}_{t+1}(r) + \widehat{\beta}_{t,t+1}\right) \\ &\quad + \frac{\gamma}{1-\iota}\frac{\overline{wh}\,(1+\overline{s})}{\overline{J}} E_t\left(\widehat{w}_{t+1}^* + \widehat{\pi}_{t+1} - \widehat{w}_t - \varepsilon_w \widehat{\pi}_t\right) \end{aligned}$$

Trade balance

$$\widehat{TB}_t = \widehat{Y}_t - \overline{C}\widehat{C}_t - \overline{I}\widehat{I}_t - \overline{G}\widehat{G}_t - \overline{r}^k\overline{K}\widehat{\nu}_t - \overline{\kappa v}\left(\widehat{\kappa}_t + \widehat{v}_t\right) + \left(\overline{C} + \overline{I}\right)\left(\widehat{P}_{H,t} - \widehat{P}_t\right)$$

Economy-wide resource constraint

$$\widehat{Y}_t = (1-W)\,\overline{C}\widehat{C}_t + W\overline{C}^*\widehat{C}_t^* + \overline{G}\widehat{G}_t + \overline{\kappa v}\left(\widehat{\kappa}_t + \widehat{v}_t\right) - \left[(2-W)\,\overline{C}\varpi W\right]\left(\widehat{P}_{H,t} - \widehat{P}_t\right)$$

Consumer price index

$$\widehat{P}_t = (1-W)\,\widehat{P}_{H,t} + W\widehat{P}_t^*$$

Government budget constraint

$$\begin{aligned} \overline{TR}(\widehat{TR}_t - \widehat{P}_t) &= \overline{nw}\overline{h}\left(\overline{\tau} + \overline{s}\right)\left(\widehat{n}_t + \widehat{w}_t - \widehat{P}_t + \widehat{h}_t\right) + \overline{nw}\overline{h}\overline{\tau}\widehat{\tau}_t \\ &\quad + \overline{nw}\overline{h}\overline{s}\widehat{s}_t + \overline{\tau}^c\overline{C}\left(\widehat{\tau}_t^c + \widehat{C}_t\right) - b\overline{u}\widehat{u}_t \\ &\quad - \overline{Rb}(\widehat{R}_{t-1} + \widehat{b}_{t-1} - \widehat{\pi}_t) - \overline{G}\left(\widehat{P}_{H,t} - \widehat{P}_t + \widehat{G}_t\right) \end{aligned}$$

Market clearing / aggregate output

$$\widehat{Y}_t = (1-\alpha)\,\widehat{K}_t + \alpha\left(\widehat{n}_t + \hat{z}_t + \hat{h}_t\right)$$

A.3 Period-by-period Nash bargaining

In the standard MP model, it is assumed that total match surplus, $S_t = (W_t - U_t) + (J_t - V_t)$, the sum of the worker and firm surpluses is shared according to efficient Nash bargaining where wages and hours are negotiated simultaneously. The firm and the worker choose the wage and the hours of work to maximize the weighted product of the worker's and the firm's net return from the match.

$$\max_{w,h} (H_t)^{\eta} (J_t)^{1-\eta}$$

where $0 \leq \eta \leq 1$ is the relative measure of workers' bargaining strength.

The worker surplus gets the following form.

$$H_t = W_t - U_t = \frac{w_t^*}{P_t} h_t (1 - \tau_t) - \frac{g(h_t)}{\Lambda_t} - b + E_t \beta_{t,t+1} \left(1 - \rho_{t+1} - q_t^W\right) H_{t+1}$$

and the firm surplus is (after taking into account the free entry condition $V_t = 0$)

$$J_t = x_t^H f(k_t, h_t) - \frac{w_t^*}{P_t} h_t (1 + s_t) - r_t^k k_t + E_t \beta_{t,t+1} \left(1 - \rho_{t+1}\right) J_{t+1}$$

The first-order condition for wage-setting is

$$\begin{aligned} \eta \frac{\partial H_t}{\partial w_t} J_t &= (1 - \eta) \frac{\partial J_t}{\partial w_t} H_t \Longleftrightarrow \\ \eta (1 - \tau_t) J_t &= (1 - \eta)(1 + s_t) H_t \end{aligned}$$

which would, without taxes, correspond to the simple surplus splitting result where the total surplus from the match is shared according to the bargaining power parameter η.

The optimality condition for wage-setting can be rewritten as a wage equation that includes only contemporaneous variables by substituting the value equations into the Nash FOC, and making use of the expressions for the production and utility functions.

$$\begin{aligned} \frac{w_t^*}{P_t} &= \frac{\eta}{(1 + s_t)} \left[\frac{x_t^H mpl_t}{\alpha} - \frac{r_t^k k_t}{h_t} \right] \\ &+ \frac{(1 - \eta)}{(1 - \tau_t)} \left[\frac{mrs_t (1 + \tau_t^c)}{(1 + \phi)} + \frac{b}{h_t} + \frac{q_t^W}{h_t} E_t \beta_{t,t+1} H_{t+1} \right] \\ &+ \frac{\eta}{h_t} E_t \beta_{t,t+1} \left(1 - \rho_{t+1}\right) J_{t+1} \left[\frac{1}{(1 + s_t)} - \frac{(1 - \tau_{t+1})}{(1 - \tau_t)} \frac{1}{(1 + s_{t+1})} \right] \end{aligned}$$

where w_t is the nominal hourly wage in a match.

A.4 Dynamics with wage rigidity

The derivation of the wage under staggered contracting follows Gertler, Sala and Trigari (GST) (2008). The Nash first order condition is in this case

$$\eta_t \Delta_t J_t\left(w_t^*\right) = \left(1-\eta_t\right) \Sigma_t H_t\left(w_t^*\right)$$

where the effect of a rise in the *real* wage on the worker's surplus is

$$\begin{aligned}
\Delta_t &= P_t \frac{\partial H_t\left(w_t\right)}{\partial w_t} \\
&= h_t\left(1-\tau_t\right) + E_t \beta_{t,t+1}\left(1-\rho_{t+1}\right) \gamma P_{t+1} \frac{\partial H_{t+1}\left(w_t\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]\right)}{\partial w_t} \\
&= h_t\left(1-\tau_t\right) + E_t \beta_{t,t+1}\left(1-\rho_{t+1}\right) \gamma(\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right] h_{t+1}\left(1-\tau_{t+1}\right) \\
&\quad + E_t \beta_{t+1,t+2} \varsigma_{t+1,t+2} \gamma P_{t+2} \frac{\partial H_{t+2}\left(w_t\left[\pi_{t+1}^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]\right)}{\partial w_t})\ldots \\
&= E_t \sum_{s=0}^{\infty} \beta_{t,t+s} \varsigma_{t,t+s} \gamma^s \left[\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)^s\right] h_{t+s}\left(1-\tau_{t+s}\right)
\end{aligned}$$

$$\Longleftrightarrow \Delta_t = h_t\left(1-\tau_t\right) + E_t \beta_{t,t+1}\left(1-\rho_{t+1}\right) \gamma \left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right] \pi_{t+1}^{-1} \Delta_{t+1}$$

where $\varsigma_{t,t+s}$ denotes the probability of match survival from period t to period $t+s$.

And similarly for the firm

$$\Sigma_t = -P_t \frac{\partial J_t\left(w_t\right)}{\partial w_t} = h_t\left(1+s_t\right) + E_t \beta_{t,t+1}\left(1-\rho_{t+1}\right) \gamma \left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right] \pi_{t+1}^{-1} \Sigma_{t+1}$$

The dynamic contract wage equation is solved by first linearizing the FOC for wage setting, and then substituting the linearized worker and firm surplus equations as well as the above discount factors in their loglinearized form (see GST, 2008 for more details).

First order condition

$$\widehat{J}_t(w_t^*) + \widehat{\Delta}_t = \widehat{H}_t(w_t^*) + \widehat{\Sigma}_t - \frac{1}{1-\eta}\widehat{\eta}_t$$

where the loglinear forms of the discount factors are

$$\widehat{\Delta}_t = (1-\iota)\,\widehat{h}_t - \frac{(1-\iota)\,\overline{\tau}}{(1-\overline{\tau})}\widehat{\tau}_t + \iota E_t\left(\widehat{\beta}_{t,t+1} + \varepsilon_w \widehat{\pi}_t - \widehat{\pi}_{t+1} + \widehat{\Delta}_{t+1}\right) - \overline{\beta}\overline{\rho}\gamma E_t \widehat{\rho}_{t+1}$$

$$\widehat{\Sigma}_t = (1-\iota)\,\widehat{h}_t + \frac{(1-\iota)\,\overline{s}}{(1+\overline{s})}\widehat{s}_t + \iota E_t\left(\widehat{\beta}_{t,t+1} + \varepsilon_w \widehat{\pi}_t - \widehat{\pi}_{t+1} + \widehat{\Sigma}_{t+1}\right) - \overline{\beta}\overline{\rho}\gamma E_t \widehat{\rho}_{t+1}$$

and the expressions for $\widehat{J}_t(w_t^*)$ and $\widehat{H}_t(w_t^*)$ can be found as follows

A.4.1 Worker surplus

The worker surplus can be written as

$$
\begin{aligned}
H_t(w_t^*) \;=\; & \frac{w_t^*}{P_t} h_t (1-\tau_t) - \left[\frac{g(h_t)}{\Lambda_t} + b + q_t^W E_t \beta_{t,t+1} H_{x,t+1}\right] \\
& + E_t \beta_{t,t+1} \left(1-\rho_{t+1}\right) H_{t+1}(w_{t+1}^*) \\
& + \gamma E_t \beta_{t,t+1} \left(1-\rho_{t+1}\right) \left[H_{t+1}(w_t^* \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right]) - H_{t+1}(w_{t+1}^*)\right]
\end{aligned}
$$

In the last term, evaluate the expression $E_t \left[H_{t+1}(w_t^* \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right]) - H_{t+1}(w_{t+1}^*)\right]$

$$
\begin{aligned}
& E_t \left[H_{t+1}(w_t^* \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right]) - H_{t+1}(w_{t+1}^*)\right] \\
= \; & E_t \left[\frac{w_t^* \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right]}{P_{t+1}} - \frac{w_{t+1}^*}{P_{t+1}}\right] h_{t+1} (1-\tau_{t+1}) \\
& + \gamma E_t \beta_{t+1,t+2} \varsigma_{t+1,t+2} \left[H_{t+2}(w_t^* \left[\pi_{t+1}^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right) \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right]\right])\right] \\
& - \gamma E_t \beta_{t+1,t+2} \varsigma_{t+1,t+2} \left[H_{t+2}(w_{t+1}^* \left[\pi_{t+1}^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right])\right]
\end{aligned}
$$

When linearized, this expression gets the following form

$$
\begin{aligned}
& E_t \left[\widehat{H}_{t+1}(w_t^* \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right]) - \widehat{H}_{t+1}(w_{t+1}^*)\right] \\
= \; & \frac{\overline{wh} (1-\overline{\tau})}{\overline{H}} E_t \left[\widehat{w}_t^* + \varepsilon_w \widehat{\pi}_t - \widehat{w}_{t+1}^* - \widehat{\pi}_{t+1}\right] \\
& + \overline{\beta} (1-\overline{\rho}) \gamma E_t \left[\widehat{H}_{t+2}(w_t^* \left[\pi_{t+1}^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right) \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right]\right])\right] \\
& - \overline{\beta} (1-\overline{\rho}) \gamma E_t \left[\widehat{H}_{t+2}(w_{t+1}^* \left[\pi_{t+1}^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right])\right]
\end{aligned}
$$

Iterating forward this can be further simplified to yield

$$
\begin{aligned}
& E_t \left[\widehat{H}_{t+1}(w_t^* \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right]) - \widehat{H}_{t+1}(w_{t+1}^*)\right] \\
= \; & \frac{1}{1-\overline{\beta}(1-\overline{\rho})\gamma} \frac{\overline{wh} (1-\overline{\tau})}{\overline{H}} E_t \left[\widehat{w}_t^* + \varepsilon_w \widehat{\pi}_t - \widehat{w}_{t+1}^* - \widehat{\pi}_{t+1}\right]
\end{aligned}
$$

With the help of the above expression, the loglinear formulation of the worker surplus is found to be

$$
\begin{aligned}
\widehat{H}_t &= \frac{\overline{wh}\,(1-\overline{\tau})}{\overline{H}}\left(\widehat{w}_t^* - \widehat{P}_t + \widehat{h}_t\right) - \frac{\overline{wh}\overline{\tau}}{\overline{H}}\widehat{\tau}_t - \frac{\overline{mrs}\overline{h}\,(1+\overline{\tau}^c)}{(1+\phi)\,\overline{H}}\left[\widehat{mrs}_t + \widehat{h}_t\right] \\
&\quad - \frac{\overline{mrs}\overline{h}\overline{\tau}^c}{(1+\phi)\,\overline{H}}\widehat{\tau}_t^c - \overline{\beta}\overline{q}^W E_t\left(\hat{q}_t^W + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1}\right) \\
&\quad + \overline{\beta}\,(1-\overline{\rho})\,E_t\left(\widehat{H}_{t+1}\left(w_{t+1}^*\right) + \widehat{\beta}_{t,t+1}\right) - \overline{\beta}\overline{\rho}E_t\widehat{\rho}_{t+1} \\
&\quad + \frac{\overline{\beta}\,(1-\overline{\rho})\,\gamma}{1-\overline{\beta}\,(1-\overline{\rho})\,\gamma}\frac{\overline{wh}\,(1-\overline{\tau})}{\overline{H}}E_t\left(\widehat{w}_t^* + \varepsilon_w\widehat{\pi}_t - \widehat{w}_{t+1}^* - \widehat{\pi}_{t+1}\right)
\end{aligned}
$$

A.4.2 Firm surplus

The firm surplus can be written as

$$
\begin{aligned}
J_t(w_t^*) &= x_t^H f\,(h_t) - \frac{w_t^*}{P_t}h_t\,(1+s_t) - r_t^k k_t + E_t\beta_{t,t+1}\left(1-\rho_{t+1}\right)J_{t+1}(w_{t+1}^*) \\
&\quad + \gamma E_t\beta_{t,t+1}\left(1-\rho_{t+1}\right)\left[J_{t+1}(w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]) - J_{t+1}(w_{t+1}^*)\right]
\end{aligned}
$$

In the last term, evaluate the expression
$E_t\left[J_{t+1}(w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]) - J_{t+1}(w_{t+1}^*)\right]$

$$
\begin{aligned}
& E_t\left[J_{t+1}(w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]) - J_{t+1}(w_{t+1}^*)\right] \\
=\; & -E_t\left[\frac{w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]}{P_{t+1}} - \frac{w_{t+1}^*}{P_{t+1}}\right]h_{t+1}\,(1+s_{t+1}) \\
& + \gamma E_t\beta_{t+1,t+2}\varsigma_{t+1,t+2}\left[J_{t+2}(w_t^*\left[\pi_{t+1}^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right])\right] \\
& - \gamma E_t\beta_{t+1,t+2}\varsigma_{t+1,t+2}\left[J_{t+2}(w_{t+1}^*\left[\pi_{t+1}^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right])\right]
\end{aligned}
$$

When linearized this expression gets the following form

$$
\begin{aligned}
& E_t\left[\widehat{J}_{t+1}(w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]) - \widehat{J}_{t+1}(w_{t+1}^*)\right] \\
=\; & -\frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}E_t\left[\widehat{w}_t^* + \varepsilon_w\widehat{\pi}_t - \widehat{w}_{t+1}^* - \widehat{\pi}_{t+1}\right] \\
& + \overline{\beta}\,(1-\overline{\rho})\,\gamma E_t\left[\widehat{J}_{t+2}(w_t^*\left[\pi_{t+1}^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right])\right] \\
& - \overline{\beta}\,(1-\overline{\rho})\,\gamma E_t\left[\widehat{J}_{t+2}(w_{t+1}^*\left[\pi_{t+1}^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right])\right]
\end{aligned}
$$

Iterating forward this can be further simplified to yield

$$
\begin{aligned}
& E_t\left[\widehat{J}_{t+1}(w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right]) - \widehat{J}_{t+1}(w_{t+1}^*)\right] \\
=\; & -\frac{1}{1-\overline{\beta}\,(1-\overline{\rho})\,\gamma}\frac{\overline{wh}\,(1+\overline{s})}{\overline{J}}E_t\left[\widehat{w}_t^* + \varepsilon_w\widehat{\pi}_t - \widehat{w}_{t+1}^* - \widehat{\pi}_{t+1}\right]
\end{aligned}
$$

Finally, as with worker surplus, the following loglinear formulation of the renegotiating firm's surplus can be found with the help of the above expression

$$\begin{aligned}\widehat{J}_t &= \frac{\overline{x}\overline{mplh}}{\alpha\overline{J}}\left(\widehat{x}_t^H+\widehat{mpl}_t+\widehat{h}_t\right)-\frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}}\left(\widehat{w}_t^*-\widehat{P}_t+\widehat{h}_t\right)-\frac{\overline{wh}\overline{s}}{\overline{J}}\widehat{s}_t \\ &\quad -\frac{\overline{r}^k\overline{k}}{\overline{J}}\left(\widehat{r}_t^k+\widehat{k}_t\right)-\overline{\beta}\overline{\rho}E_t\widehat{\rho}_{t+1}+\overline{\beta}\left(1-\overline{\rho}\right)E_t\left(\widehat{J}_{t+1}\left(w_{t+1}^*\right)+\widehat{\beta}_{t,t+1}\right) \\ &\quad +\frac{\overline{\beta}\left(1-\overline{\rho}\right)\gamma}{1-\overline{\beta}\left(1-\overline{\rho}\right)\gamma}\frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}}E_t\left(\widehat{w}_{t+1}^*+\widehat{\pi}_{t+1}-\widehat{w}_t^*-\varepsilon_w\widehat{\pi}_t\right)\end{aligned}$$

A.4.3 The Contract wage

Inserting the expressions for the worker and firm surpluses, as well as those for the discount factors, into the FOC yields (after collecting the wage terms to the left-hand side and using the Nash FOC for next period)

$$\begin{aligned}\Longrightarrow\; & \left[\frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}+\frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}}\right]\widehat{w}_t^* \\ & +\frac{\iota}{\left(1-\iota\right)}\left[\frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}+\frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}}\right]E_t\left(\widehat{w}_t^*+\varepsilon_w\widehat{\pi}_t-\widehat{w}_{t+1}^*-\widehat{\pi}_{t+1}\right) \\ = \; & \frac{\overline{x}\overline{mplh}}{\alpha\overline{J}}\left(\widehat{x}_t^H+\widehat{mpl}_t\right)-\frac{\overline{r}^k\overline{k}}{\overline{J}}\left(\widehat{r}_t^k+\widehat{k}_t\right)+\frac{\overline{mrs}\overline{h}\left(1+\overline{\tau}^c\right)}{\left(1+\phi\right)\overline{H}}\left(\widehat{mrs}_t\right) \\ & +\frac{\overline{mrs}\overline{h}\overline{\tau}^c}{\left(1+\phi\right)\overline{H}}\widehat{\tau}_t^c+\overline{\beta}\left(1-\overline{\rho}\right)E_t\left[\widehat{J}_{t+1}\left(w_{t+1}^*\right)+\widehat{\beta}_{t,t+1}\right] \\ & -\overline{\beta}\left(1-\overline{\rho}\right)E_t\left[\widehat{J}_{t+1}\left(w_{t+1}^*\right)+\widehat{\beta}_{t,t+1}+\widehat{\Delta}_{t+1}-\widehat{\Sigma}_{t+1}+\frac{1}{1-\eta}\widehat{\eta}_t\right] \\ & +\overline{\beta}\overline{q}^W E_t\left(\widehat{q}_t^W+\widehat{H}_{x,t+1}+\widehat{\beta}_{t,t+1}\right) \\ & -\left\{\left[\frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}+\frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}}\right]-\frac{\overline{x}\overline{mplh}}{\alpha\overline{J}}-\frac{\overline{mrs}}{1+\phi}\frac{\overline{h}\left(1+\overline{\tau}^c\right)}{\overline{H}}\right\}\widehat{h}_t \\ & -\left[\frac{\overline{whs}}{\overline{J}}+\frac{\left(1-\iota\right)\overline{s}}{\left(1+\overline{s}\right)}\right]\widehat{s}_t+\left[\frac{\overline{wh\tau}}{\overline{H}}-\frac{\left(1-\iota\right)\overline{\tau}}{\left(1-\overline{\tau}\right)}\right]\widehat{\tau}_t \\ & +\iota E_t\widehat{\Delta}_{t+1}-\iota E_t\widehat{\Sigma}_{t+1}+\frac{1}{1-\eta}\widehat{\eta}_t+\left[\frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}+\frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}}\right]\widehat{P}_t\end{aligned}$$

where $\iota=\overline{\beta}\left(1-\overline{\rho}\right)\gamma$. Dividing by the term $\left[\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}}+\frac{\overline{wh}(1+\overline{s})}{\overline{J}}\right]=\frac{\overline{wh}(1+\overline{s})}{\eta\overline{J}}=\frac{\overline{wh}(1-\overline{\tau})}{(1-\eta)\overline{H}}$, and using the steady state equations for $\overline{\Delta}$ and $\overline{\Sigma}$, and for the Nash FOC allows us to rewrite the contract wage equation in the following simpler form

$$\begin{aligned}
&\Longrightarrow \widehat{w}_t^* + \frac{\iota}{1-\iota} E_t\left(\widehat{w}_t^* + \varepsilon_w \widehat{\pi}_t - \widehat{w}_{t+1}^* - \widehat{\pi}_{t+1}\right) \\
&= \varphi_x \left(\widehat{x}_t^H + \widehat{mpl}_t\right) - \varphi_k \left(\widehat{r}_t^k + \widehat{k}_t\right) + \varphi_m \widehat{mrs}_t \\
&\quad + \varphi_H E_t\left(\widehat{q}_t^W + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1}\right) - \varphi_h \widehat{h}_t - \varphi_s \widehat{s}_t \\
&\quad + \varphi_\tau \widehat{\tau}_t + \varphi_{\tau^c} \widehat{\tau}_t^c + \varphi_D E_t\left(\widehat{\Delta}_{t+1} - \widehat{\Sigma}_{t+1}\right) + \varphi_\eta \widehat{\eta}_t + \widehat{P}_t \\
&= \widehat{w}_t^0(r)
\end{aligned}$$

where $\widehat{w}_t^0(r)$ is the target wage in the bargain, and its coefficients are

$$\begin{aligned}
\varphi_x &= \frac{\overline{x}\overline{mpl}\eta}{\alpha\overline{w}\left(1+\overline{s}\right)},\ \ \varphi_k = \frac{\overline{r}^k\overline{k}\eta}{\overline{wh}\left(1+\overline{s}\right)},\ \ \ \varphi_m = \frac{\overline{mrs}\left(1-\eta\right)\left(1+\overline{\tau}^c\right)}{\left(1+\phi\right)\overline{w}\left(1-\overline{\tau}\right)}, \\
\varphi_H &= \frac{\left(1-\eta\right)\overline{H}}{\overline{wh}\left(1-\overline{\tau}\right)}\overline{\beta}\overline{q}^W,\ \ \ \varphi_h = \left\{1 - \frac{\overline{x}\overline{mpl}\eta}{\alpha\overline{w}\left(1+\overline{s}\right)} - \frac{\overline{mrs}\left(1-\eta\right)\left(1+\overline{\tau}^c\right)}{\left(1+\phi\right)\overline{w}\left(1-\overline{\tau}\right)}\right\}, \\
\varphi_s &= \frac{\eta\overline{s}}{\left(1+\overline{s}\right)}\left[1 + \frac{\left(1-\iota\right)\overline{J}}{\overline{wh}\left(1+\overline{s}\right)}\right],\ \ \ \varphi_\tau = \frac{\left(1-\eta\right)\overline{\tau}}{\left(1-\overline{\tau}\right)}\left[1 - \frac{\left(1-\iota\right)\overline{H}}{\overline{wh}\left(1-\overline{\tau}\right)}\right], \\
\varphi_{\tau^c} &= \frac{\overline{mrs}\left(1-\eta\right)\overline{\tau}^c}{\left(1+\phi\right)\overline{w}\left(1-\overline{\tau}\right)}, ...\varphi_D = \frac{\left[\iota - \overline{\beta}\left(1-\overline{\rho}\right)\right]\eta\overline{J}}{\overline{wh}\left(1+\overline{s}\right)}, \\
\text{and } \varphi_\eta &= \frac{\left[1-\overline{\beta}\left(1-\overline{\rho}\right)\right]\overline{H}}{\overline{wh}\left(1-\overline{\tau}\right)}
\end{aligned}$$

The target wage $\widehat{w}_t^0(r)$ is of the same form than the period-by-period negotiated wage, adjusted for the new bargaining weights. The equation for the contract wage can be further rewritten as

$$\frac{1}{(1-\iota)}\widehat{w}_t^* = \widehat{w}_t^0(r) + \frac{\iota}{(1-\iota)}E_t\left(\widehat{\pi}_{t+1} - \varepsilon_w\widehat{\pi}_t\right) + \frac{\iota}{(1-\iota)}E_t\widehat{w}_{t+1}^*$$

$$\Longleftrightarrow \widehat{w}_t^* = [1-\iota]\,\widehat{w}_t^0(r) + \iota E_t\left(\widehat{\pi}_{t+1} - \varepsilon_w\widehat{\pi}_t\right) + \iota E_t\widehat{w}_{t+1}^*$$

This is the optimal contract wage set at time t *by all matches that are allowed to renegotiate their wage.* As is usual with Calvo-type contracting, it depends on a wage target $w_t^0(r)$ and next period's optimal wage.

A.4.4 The spillover effect

To derive the spillover effect, consider the worker surplus with *optimal (contract) wage* versus the expected *average market wage* in the same way as above

$$E_t\widehat{H}_{t+1}(w_{t+1}) = E_t\widehat{H}_{t+1}(w_{t+1}^*) + \frac{\overline{wh}\left(1-\overline{\tau}\right)}{\left(1-\iota\right)\overline{H}}E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)$$

Denoting $\frac{\overline{wh}(1-\overline{\tau})}{(1-\iota)\overline{H}} = \Gamma$ and substituting the above expression in the target wage equation gives

$$
\begin{aligned}
\widehat{w}_t^0(r) &= \varphi_x\left(\widehat{x}_t^H + \widehat{mpl}_t\right) - \varphi_k\left(\widehat{r}_t^k + \widehat{k}_t\right) + \varphi_m\widehat{mrs}_t \\
&+\varphi_H E_t\left(\widehat{q}_t^W + \widehat{H}_{t+1}(w_{t+1}^*) + \widehat{\beta}_{t,t+1} + \Gamma E_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]\right) \\
&+\varphi_h\widehat{h}_t - \varphi_s\widehat{s}_t + \varphi_\tau\widehat{\tau}_t + \varphi_{\tau^c}\widehat{\tau}_t^c + \varphi_D E_t\left[\widehat{\Sigma}_{t+1} - \widehat{\Delta}_{t+1}\right] + \varphi_\eta\widehat{\eta}_t + \widehat{P}_t
\end{aligned}
$$

$$
\Longleftrightarrow \widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H\Gamma E_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]
$$

where the target wage $\widehat{w}_t^0(r)$ - the wage the firm and its worker would agree to if they are allowed to renegotiate, *and if firms and workers elsewhere remain on staggered multiperiod wage contracts* - is a sum of the wage that would arise *if all matches were negotiating wages period-by-period* $\widehat{w}_t^0$ and the spillover effect $\varphi_H\Gamma E_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]$.

A.4.5 Evolution of the average wage

To derive the appropriate loglinear expression for the evolution of the average wage, first collect the necessary elements from previous calculations

1) The contract wage

$$
\widehat{w}_t^* = [1-\iota]\,\widehat{w}_t^0(r) + \iota E_t\left(\widehat{\pi}_{t+1} - \varepsilon_w\widehat{\pi}_t\right) + \iota E_t\widehat{w}_{t+1}^*
$$

2) The average wage

$$
\widehat{w}_t = (1-\gamma)\,\widehat{w}_t^* + \gamma\left(\widehat{w}_{t-1} - \widehat{\pi}_t + \varepsilon_w\widehat{\pi}_{t-1}\right)
$$

3) The target wage

$$
\widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H\Gamma E_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]
$$

First, insert the target wage in the contract wage equation

$$
\widehat{w}_t^* = [1-\iota]\left(\widehat{w}_t^0 + \varphi_H\Gamma E_t\left[\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right]\right) + \iota E_t\left(\widehat{\pi}_{t+1} - \varepsilon_w\widehat{\pi}_t\right) + \iota E_t\widehat{w}_{t+1}^*
$$

Then update the average wage equation by one period and take expectations

$$
E_t\widehat{w}_{t+1} = (1-\gamma)\,E_t\widehat{w}_{t+1}^* + \gamma\left(\widehat{w}_t - E_t\widehat{\pi}_{t+1} + \varepsilon_w\widehat{\pi}_t\right)
$$

$$
\Longleftrightarrow E_t\widehat{w}_{t+1}^* = \frac{1}{(1-\gamma)}\left(E_t\widehat{w}_{t+1} - \gamma\left(\widehat{w}_t - E_t\widehat{\pi}_{t+1} + \varepsilon_w\widehat{\pi}_t\right)\right)
$$

Use this expression to eliminate $E_t\widehat{w}_{t+1}^*$ from the contract wage equation

$$
\begin{aligned}
\widehat{w}_t^* &= [1-\iota]\left(\widehat{w}_t^0 + \varphi_H \Gamma E_t \widehat{w}_{t+1}\right) \\
&\quad - [1-\iota]\left(\varphi_H \Gamma \left[\frac{1}{(1-\gamma)}\left(E_t \widehat{w}_{t+1} - \gamma \widehat{w}_t + \gamma E_t\left(\widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t\right)\right)\right]\right) \\
&\quad + \iota E_t\left(\widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t\right) \\
&\quad + \iota\left[\frac{1}{(1-\gamma)}\left(E_t \widehat{w}_{t+1} - \gamma \widehat{w}_t + \gamma E_t\left(\widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t\right)\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
\widehat{w}_t^* &= (1-\iota)\,\widehat{w}_t^0 + (1-\iota)\,\varphi_H \Gamma E_t \widehat{w}_{t+1} - (1-\iota)\,\varphi_H \Gamma \frac{1}{(1-\gamma)} E_t \widehat{w}_{t+1} \\
&\quad + [1-\iota]\,\varphi_H \Gamma \frac{\gamma}{(1-\gamma)}\left(\widehat{w}_t - E_t \widehat{\pi}_{t+1} + \varepsilon_w \widehat{\pi}_t\right) + \iota\left(E_t \widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t\right) \\
&\quad + \frac{\iota}{(1-\gamma)} E_t \widehat{w}_{t+1} - \frac{\iota\gamma}{(1-\gamma)}\left(\widehat{w}_t - E_t \widehat{\pi}_{t+1} + \varepsilon_w \widehat{\pi}_t\right)
\end{aligned}
$$

$$
\begin{aligned}
\Longleftrightarrow \quad & \widehat{w}_t^* = (1-\iota)\,\widehat{w}_t^0 + \\
& \left[(1-\iota)\,\varphi_H \Gamma \frac{\gamma}{(1-\gamma)} - \iota \frac{\gamma}{(1-\gamma)}\right]\left(\widehat{w}_t - E_t \widehat{\pi}_{t+1} + \varepsilon_w \widehat{\pi}_t\right) \\
& + \iota\left(E_t \widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t\right) \\
& + \left[(1-\iota)\,\varphi_H \Gamma - (1-\iota)\,\varphi_H \Gamma \frac{1}{(1-\gamma)} + \iota \frac{1}{(1-\gamma)}\right] E_t \widehat{w}_{t+1}
\end{aligned}
$$

Denote $\zeta = (1-\iota)\,\varphi_H \Gamma$, and use the above equation to eliminate $\widehat{w}_t^*$ from the average wage equation (equation 2)

$$
\begin{aligned}
\widehat{w}_t &= (1-\gamma)(1-\iota)\,\widehat{w}_t^0 + (\zeta\gamma - \iota\gamma)\left(\widehat{w}_t - E_t \widehat{\pi}_{t+1} + \varepsilon_w \widehat{\pi}_t\right) \\
&\quad + (1-\gamma)\,\iota\left(E_t \widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t\right) \\
&\quad + \left[(1-\gamma)\,\zeta - \zeta + \iota\right] E_t \widehat{w}_{t+1} + \gamma\left(\widehat{w}_{t-1} - \widehat{\pi}_t + \varepsilon_w \widehat{\pi}_{t-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
\left[1-\gamma(\zeta-\iota)\right]\widehat{w}_t &= (1-\gamma)(1-\iota)\,\widehat{w}_t^0 - \gamma(\zeta-\iota)\left(E_t \widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t\right) \\
&\quad + (1-\gamma)\,\iota\left(E_t \widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t\right) \\
&\quad + \left[(1-\gamma)\,\zeta - \zeta + \iota\right] E_t \widehat{w}_{t+1} + \gamma\left(\widehat{w}_{t-1} - \widehat{\pi}_t + \varepsilon_w \widehat{\pi}_{t-1}\right)
\end{aligned}
$$

Finally, after dividing by $\left[1-\gamma(\zeta-\iota)\right]$, the dynamic average wage equation can be expressed as

$$
\Longleftrightarrow \widehat{w}_t = \lambda_b\left(\widehat{w}_{t-1} - \widehat{\pi}_t + \varepsilon_w \widehat{\pi}_{t-1}\right) + \lambda_0 \widehat{w}_t^0 + \lambda_f E_t\left(\widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \varepsilon_w \widehat{\pi}_t\right)
$$

$$
\begin{aligned}
\text{where } \lambda_b &= \frac{\gamma}{\left[1-\gamma(\zeta-\iota)\right]},\ \lambda_0 = \frac{(1-\gamma)(1-\iota)}{\left[1-\gamma(\zeta-\iota)\right]},\ \text{and } \lambda_f = \frac{\iota-\gamma\zeta}{\left[1-\gamma(\zeta-\iota)\right]}, \\
\text{with } \zeta &= (1-\iota)\,\varphi_H \Gamma,\ \iota = \overline{\beta}\,(1-\overline{\rho})\,\gamma,\ \Gamma = \frac{\overline{wh}\,(1-\overline{\tau})}{(1-\iota)\,\overline{H}}, \\
\text{and } \varphi_H &= \frac{(1-\eta)\,\overline{H}\overline{\beta}\overline{q}^W}{\overline{wh}\,(1-\overline{\tau})},\ \text{as previously denoted.}
\end{aligned}
$$

B Appendix

B.1 Figures

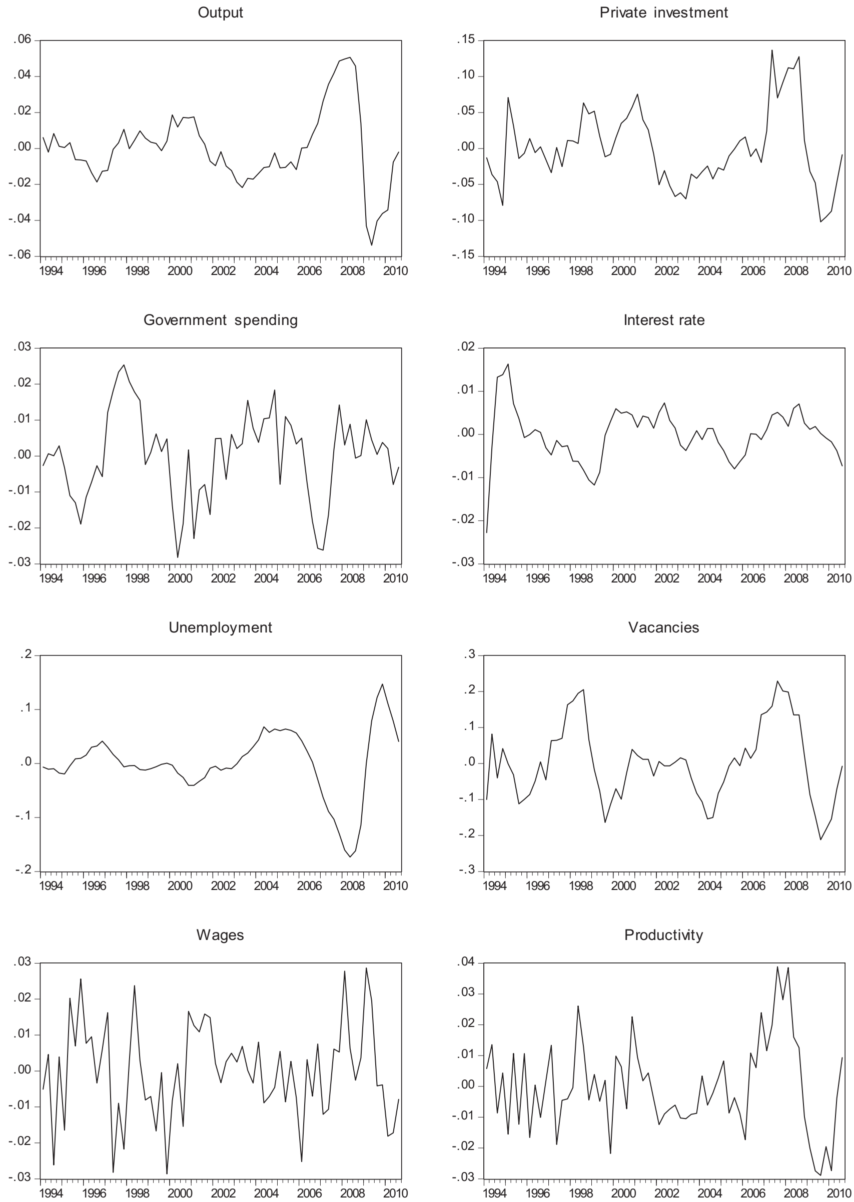


Figure 1. Filtered data series

Figure 2. Priors and posteriors of estimated parameters

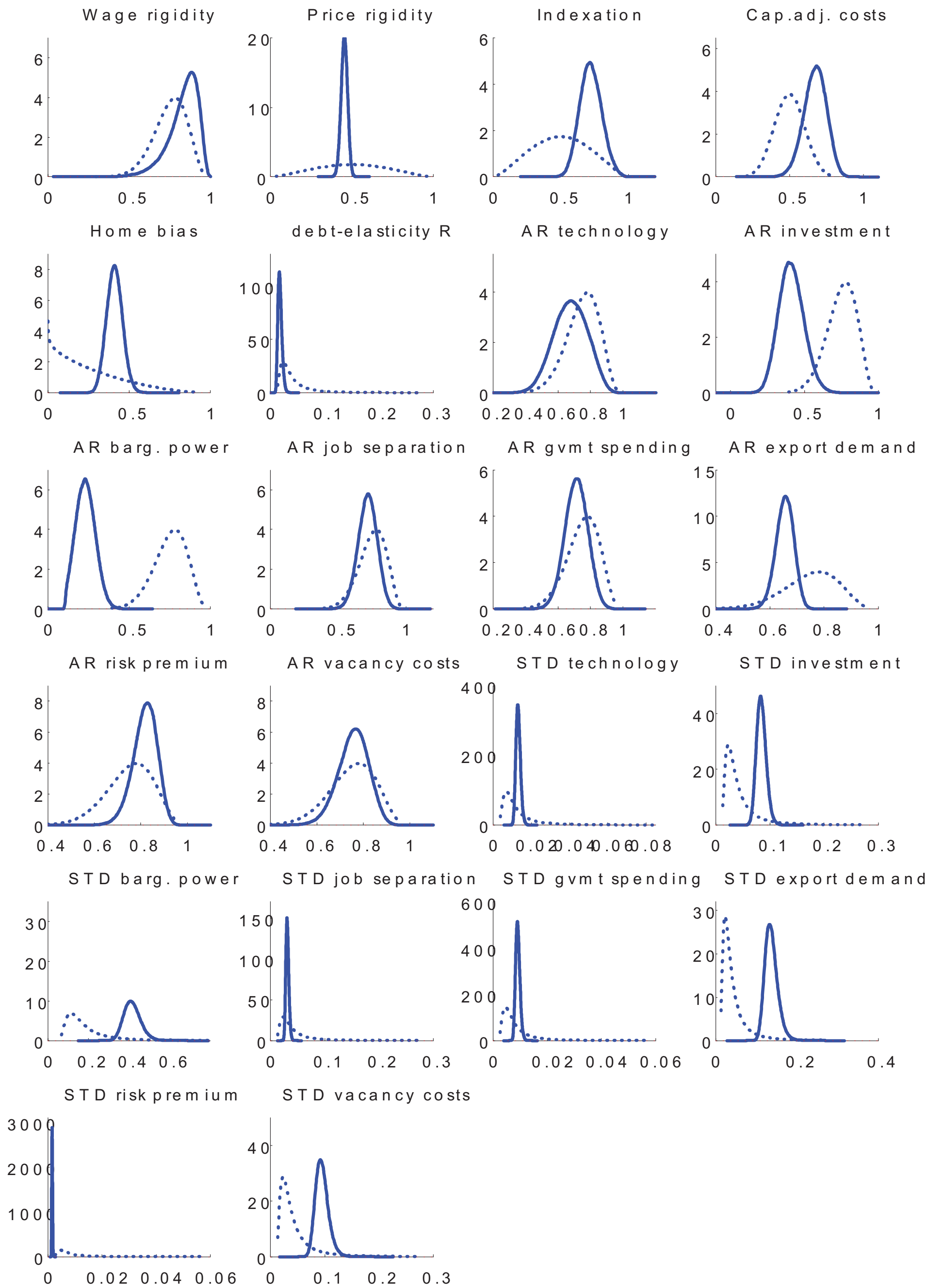
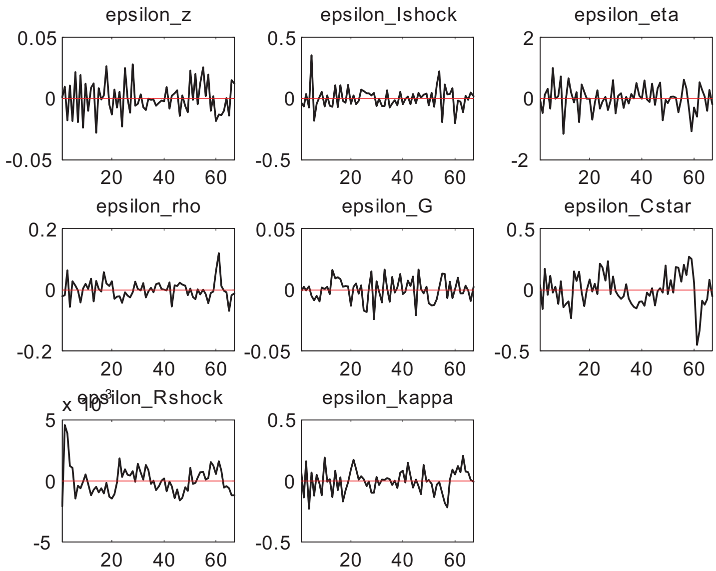


Figure 3. Estimated shocks



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