



Pekka Lauri

HUMAN CAPITAL, DYNAMIC INEFFICIENCY
AND ECONOMIC GROWTH

HELSINKI SCHOOL OF ECONOMICS

ACTA UNIVERSITATIS OECOMICAE HELSINGIENSIS

A-237

Pekka Lauri

HUMAN CAPITAL, DYNAMIC INEFFICIENCY
AND ECONOMIC GROWTH

HELSINKI SCHOOL OF ECONOMICS

ACTA UNIVERSITATIS OECONOMICAE HELSINGIENSIS

A-237

© Pekka Lauri and
Helsinki School of Economics

ISSN 1237-556X
ISBN 951-791-854-2
ISBN 951-791-855-0 (Electronic dissertation)

Helsinki School of Economics -
HeSE print 2004

ACKNOWLEDGEMENTS

Nowadays people have seldom the time or the financial position to carry out an in-depth treatment of comprehensive theoretical issues. I am grateful to the Department of Economics at the Helsinki School of Economics for giving me this opportunity for several years.

I would also like to thank the pre-examiners, Professor Erkki Koskela and Professor Mikko Puhakka for helpful comments and suggestions. Special thanks are due to Helsingin Kauppakorkeakoulun Tukisäätiö for financial support and Liisa Roponen for checking the language.

Helsinki, April 2004

Pekka Lauri

ABSTRACT

We investigate an endogenous growth overlapping generations model, which allows dynamic inefficiency and thereby has a role for the redistribution of resources from children to parents through bubbles, government debt or intergenerational altruistic transfers. The model has two sources of economic growth: human capital accumulation due to education investments and technological progress due to learning-by-doing externalities. Technological progress has two opposite effects in the model, a positive productivity effect on the final goods production and a negative erosion effect on human capital accumulation. These effects allow us to generate new results compared to models where the source of economic growth is technological progress or human capital accumulation alone. In particular, we show that bubbles can have a positive or negative effect on economic growth. We also consider the relationship between bubbles and two-sided intergenerational altruism. We show that bequests, gifts and bubbles cannot be operative in the same steady state if altruism is symmetric and households take the actions of other generations as given. Moreover, altruistic education investments are a perfect substitute for bequests if young agents do not face borrowing constraints. On the other hand, gifts from children to parents are an imperfect substitute for bubbles and bubbles eliminate gifts. In the end, we consider government debt and permanent budget deficits. The deficit has a maximum sustainable upper bound and it decreases the effect of debt on the economy. The calibration of the model to the postwar U.S. data shows that a maximum sustainable deficit/GDP ratio by the Ponzi game debt finance is around 2.1 %, which is slightly higher than the average realized U.S. deficit/GDP ratio.

Keywords: Bubbles, Calibration, Dynamic inefficiency, Government debt, Endogenous growth, Human capital, Intergenerational altruism, Learning-by-doing externalities, Overlapping generations model, Ponzi games, Technological progress.

CONTENTS

1. INTRODUCTION	
1.1 Intertemporal allocation and overlapping generations	7
1.2 Sources of economic growth	9
1.3 Ponzi games and government budget policy.....	10
1.4 Outline of the study	12
2. AN ENDOGENOUS GROWTH MODEL WITH HUMAN CAPITAL AND TECHNOLOGICAL PROGRESS	
2.1 The model.....	13
2.2 Steady states and transitional dynamics.....	19
2.3 Characterization of dynamic inefficiency.....	22
3. INTRINSICALLY USELESS ASSETS	
3.1 The model.....	31
3.2 Steady states and transitional dynamics.....	33
4. TWO-SIDED ALTRUISM	
4.1 The model.....	42
4.2 Steady states and transitional dynamics without intrinsically useless assets.....	47
4.3 Steady states and transitional dynamics with intrinsically useless assets.....	53
5. GOVERNMENT DEBT WITH PERMANENT BUDGET DEFICITS	
5.1 The model.....	56
5.2 Steady states	58
5.3 Calibration of the model.....	60
6. CONCLUSIONS.....	68
REFERENCES.....	71
APPENDIX 1: GENERAL FORMS OF THE PRODUCTION FUNCTIONS AND A CONSTANT ELASTICITY OF SUBSTITUTION UTILITY FUNCTION.....	76
APPENDIX 2: CALIBRATION OF THE MODEL BY MATHCAD	86

1. INTRODUCTION

It is well known that in dynamically inefficient economies where the growth rate of output exceeds the rate of return on physical capital, there is a case for a redistribution of resources from children to parents. It is clear that this type of intergenerational reallocation can have huge welfare effects in the long run. The subject is studied in the exogenous growth overlapping generations (OLG) model by Diamond (1965), Feldstein (1974), Tirole (1985), Abel (1987) and Kimball (1987) among others. The first two papers investigate intergenerational reallocation through government debt and pay-as-you-go social security while the third paper considers it through asset bubbles. The last two papers study intergenerational reallocation through altruistic intergenerational transfers. In the balanced growth equilibrium of the exogenous growth OLG model, the rate of return on physical capital is decreasing and the growth rate of output constant in physical capital. It follows that the elimination of dynamic inefficiency crowds out physical capital. Crowding out increases the welfare of the economy, because the Pareto optimality of the competitive equilibrium only depends on the dynamic inefficiency. However, crowding out does not affect the growth rate of output, because the growth rate of output is exogenous to the model.

Since Romer's (1986) seminal study on endogenous growth, the main interest of the economic growth theory has turned to the analysis of the growth rate of output rather than the level of output. In the Romer's model, the source of output growth is technological progress due to learning-by-doing externalities in the physical capital production. Learning-by-doing externalities imply an additional static inefficiency to the model, which causes physical capital underaccumulation. In the Romer-type of endogenous growth OLG model, the Pareto optimality of the competitive equilibrium only depends on the static inefficiency, because the social rate of return on physical capital always exceeds the growth rate of output in the perpetual growth equilibrium. The redistribution of resources from children to parents is studied in the Romer-type of endogenous growth OLG models by Saint-Paul (1992), Grossman and Yanagawa (1993) and Wigger (2001) among others. The first paper investigates intergenerational reallocation through government debt and pay-as-you-go social security while the second paper considers it through asset bubbles. The last paper studies intergenerational reallocation through altruistic intergenerational transfers. In the balanced growth equilibrium of the Romer-type of endogenous growth OLG model, the rate of return on physical capital is constant and the growth rate of output increasing in physical capital. It follows that the elimination of dynamic inefficiency crowds out physical capital as in the exogenous growth OLG models. Crowding out decreases the welfare of the economy, because the Pareto optimality of the competitive equilibrium only depends on the static inefficiency. Moreover, crowding out decreases also the growth rate of output, because the growth rate of output is increasing in physical capital.

There are also other sources of economic growth in addition to technological progress due to learning-by-doing externalities. Lucas (1988) studies a model where the engine of growth is human capital accumulation due to education investments. In the Lucas-type of endogenous growth OLG models, human capital is transmitted between generations by an

intergenerational externality effect of previous generation human capital (Azariadis and Drazen 1990). Intergenerational externalities imply an additional static inefficiency to the model, which causes underinvestments in education. Moreover, in the Lucas-type of endogenous growth OLG model the Pareto optimality of the competitive equilibrium depends on the static and dynamic inefficiency, because the social rate of return on physical capital does not necessary exceed the growth rate of output in the perpetual growth equilibrium. The redistribution of resources from children to parents is studied in the Lucas-type of endogenous growth OLG model by Michel (1992), Kahn et al. (1997) and Marchand et al. (2003) among others. The first paper studies intergenerational reallocation through asset bubbles while the second paper considers it through government debt. The last paper studies intergenerational reallocation through lump-sum tax-transfers. In the balanced growth equilibrium of the Lucas-type of endogenous growth OLG model, the rate of return on physical capital is decreasing and the growth rate of output is increasing in physical capital due to the trade-off between education investments and investments in physical capital. It follows that the elimination of dynamic inefficiency crowds out physical capital as in the exogenous growth OLG models. Crowding out can decrease or increase the welfare of the economy, because the Pareto optimality of the competitive equilibrium depends on the static and dynamic inefficiency. However, crowding out decreases the growth rate of output, because the growth rate of output is increasing in physical capital.

If the engine of growth is technological progress due to learning-by-doing externalities and human capital accumulation together, the growth and welfare effects of intergenerational reallocation can be different from those in the Romer-and Lucas-types of endogenous growth OLG models. In particular, intergenerational reallocation can have a positive effect on economic growth. There are two reasons for the result. First, learning-by-doing externalities in the final good production function allow the rate of return on physical capital to be increasing in physical capital, which implies that the elimination of dynamic inefficiency can crowd in physical capital and increase economic growth. Second, learning-by-doing externalities in the human capital production function can change the trade-off between education investments and investments in physical capital such that the growth rate of output is decreasing in physical capital, which implies that the elimination of dynamic inefficiency can crowd out physical capital and increase economic growth.

A positive growth effect of government debt, pay-as-you-go social security or asset bubbles is also derived by Zhang (1995), Zhang (1997), Sinn (1998), Forslid (1998), Ferreira (1999), Futagami and Shibata (2000), Kemnitz and Wigger (2000), Lin (2000), Sanched-Losada (2000), Zhang (2001) and Zhang (2003). In these models, however, the positive growth effect is caused by endogenous fertility, by a moral hazard problem between children and parents, by government income transfers, by government productive investments, by an increase in the supply of the asset, by intergenerational externalities or by joy-of-giving altruism. Hence, none of these results rests on the elimination of dynamic inefficiency.

1.1 Intertemporal allocation and overlapping generations

A workhorse of the endogenous growth literature is an infinitely lived representative agent model. This model has a finite number of agents with an infinite planning horizon and an infinite number of dated goods with complete contingency markets. Hence, all goods are traded in a single market where the number of traders is finite, which implies that the present value of aggregate wealth must be finite. It follows that there is no role for intergenerational reallocation and the competitive equilibrium is always dynamically efficient. The intertemporal allocation of the resources in the competitive equilibrium is determined by the Euler equation and the transversality condition. The former defines the optimal path of consumption over time while the latter implies that the present value of aggregate wealth must be finite. Together these conditions imply that in the dynamically efficient stationary allocation, the rate of return on physical capital exceeds the growth rate of output or in the limit they are equal. The limit case maximizes the stationary utility and is usually called the Golden Rule allocation.

In the real world, agents do not have complete contingency markets and all goods are not traded in a single market. Hence, infinitely lived representative agent models are clearly a simplification of the real world. Growth models, which correct this simplification, are called sequential economies. Sequential economies have an infinite sequence of trading opportunities, which implies that the present value of aggregate wealth can be also infinite. The infinite aggregate wealth violates the assumptions of the First Welfare Theorem, which implies that the competitive equilibrium of the sequential economy is not necessary Pareto-optimal and the economy has a role for intergenerational reallocation.

The most well-known sequential economy is an overlapping generations (OLG) model (Samuelson 1958). OLG models have an infinite number of traders and goods, which implies that they have an infinite sequence of trading opportunities. In the OLG models, the intertemporal allocation of resources is determined by a saving function, which breaks the link between the growth rate of output and the rate of return on physical capital defined by the Euler equation and the transversality condition. Hence, OLG models can have dynamically inefficient equilibrium allocations, i.e., allocations where the growth rate of output exceeds the rate of return on physical capital. In these allocations, the productivity of physical capital is low compared to the Golden Rule allocation and there is a case for the redistribution of resources through bubbles, government debt, altruistic intergenerational transfers or some other mechanism of intergenerational reallocation.¹

Bubbles are persistent deviations from the fundamental value of the asset. Examples include fiat money (Samuelson 1958), government Ponzi game debt (O'Connell and Zeldes 1988), asset bubbles (Tirole 1985) or a price of the land (Rhee 1991). The fundamental reason for bubbles in the OLG models is the infinite number of traders and

¹ Another example of sequential economies is Bewley's (1980) monetary model, which have a finite number of traders, but an infinite sequence of trading opportunities due to borrowing constraints. For a more detailed discussion on the sequential economies, see Santos and Woodford (1997).

markets, which allows agents to use the non-fundamental value of the asset as a store of wealth instead of savings in the physical capital. Tirole (1985) shows that the economy can have bubbles and that bubbles increase the welfare of the economy if the economy without bubbles is dynamically inefficient. Hence, a necessary condition for bubbles is dynamic inefficiency.² In Tirole's exogenous growth model, bubbles eliminate low productivity of physical capital due to the overaccumulation of physical capital relative to the Golden Rule. We show that similar results can be derived for the endogenous growth model, where low productivity of physical capital can be caused by over- or underaccumulation of physical capital relative to the Golden Rule.

Government debt can work in the economy in the similar way to bubbles. Diamond (1965) considers a constant debt policy and shows that government debt can eliminate dynamic inefficiency due to OLG-structure of the economy. As an alternative to the constant debt policy, we can have a constant deficit policy. Azariadis (1993, 322) and De la Croix and Michel (2002, 193) show that the economy with a constant deficit policy and permanent budget deficits tends to have two bubble steady states with different types of transitional dynamics if the economy without debt is dynamically inefficient. Moreover, they show that permanent budget deficits have a maximum sustainable upper bound and they decrease the effect of debt on the economy. We show that similar results can be derived for the endogenous growth model.

In the OLG models with altruism, generations are linked to each other by altruistic intergenerational transfers from parents to children (bequests, altruistic education investments) and children to parents (gifts). Barro (1974) and Carmichael (1982) show that as long as intergenerational transfers are positive, i.e., intergenerational transfer motive is operative, government debt is neutral. Moreover, Carmichael (1982) shows that a competitive equilibrium of the economy with an operative bequest motive is dynamically efficient and a competitive equilibrium of the economy with an operative gift motive is dynamically inefficient. Government debt does not eliminate gifts or dynamic inefficiency in Carmichael's model, because lump sum transfers make government debt the perfect substitute for intergenerational transfers. Because the debt neutrality result depends on an operative intergenerational transfer motive, it is important to determine when the transfer motives are operative. Weil (1987b) derives explicit conditions under which the bequest motive is operative. He shows that the bequest motive is inoperative if the agents' altruism is low, i.e., intergenerational discount rate is small. Weil's result is extended to the two-sided altruism by Abel (1987) and Kimball (1987), who show that gift and bequest motives form upper and lower bounds for inoperative transfer motive equilibria. Moreover, Kimball (1987) shows that gift and bequest motives cannot be operative in the same equilibrium if altruism is symmetric and agents take the actions of other generations as given. General conditions for the existence and co-existence of operative and inoperative transfer motive equilibria are derived by Thibault (2000). We show that similar results can be derived for the endogenous growth

² If OLG models have imperfect risk-sharing, bubbles can also arise in the dynamically efficient economies (Bertocchi 1991, Gale 1995 and Blanchard and Weil 2001). In this case, an infinite number of agents and markets allows agents to use the non-fundamental value of the asset as insurance against income fluctuations. Because our model does not have any uncertainty, a necessary condition for bubbles is dynamic inefficiency.

model. Moreover, we show that gifts are an imperfect substitute for bubbles and bubbles eliminate gifts if altruism is symmetric and households take the actions of other generations as given. On the other hand, altruistic education investments are a perfect substitute for bequests if young agents do not face borrowing constraints.

1.2 Sources of economic growth

Endogenous growth models offer three fundamental sources of growth: human capital accumulation due to education investments, technological progress due to R&D investments and technological progress due to learning-by-doing externalities. The first approach is based on Becker's (1964) theory of human capital and the seminal work by Uzawa (1965), who argues that productivity of the economy depends on human capital, which is accumulated through households' education investments. Uzawa's human capital model is extended to the context of the endogenous growth literature by Lucas (1988). The second approach is based on the seminal work by Nelson and Phelps (1966), who argue that productivity of the economy depends on the firms' investments in R&D. Nelson's and Phelps' R&D model is extended to the context of the endogenous growth literature by Romer (1990) and Aghion and Howitt (1992). The third approach is based on the seminal work by Arrow (1962), who argues that productivity of the economy increases, because agents learn better working methods during the production process and because knowledge of the working methods is a public good. Arrow's learning-by-doing model is extended to the context of the endogenous growth literature by Romer (1986).³

Empirical literature on economic growth offers evidence on each of these sources of growth, including standard approaches, Barro and Sala-i-Martin (1995) for education investments, Benhabib and Spiegel (1994) and Coe and Helpman (1995) for R&D investments and Romer (1986) for learning-by-doing externalities. There are also some evidence that economic growth does not depend on either of these sources but merely depends on physical factor accumulation (Mankiw et al. 1992, Jones 1995). However, Bernanke and Gurkaynak (2001) show that long-run growth is significantly correlated with decision and state variables of the economy, which implies that economic growth cannot be explained purely by physical factor accumulation. Moreover, empirical evidence suggests that human capital accumulation or technological progress alone cannot explain the long-run economic development but that it depends on the interaction of these factors (Temple 2000, Acemoglu 2002, Topel 2003).

Usually the endogenous growth models with human capital do not include technological progress (Lucas 1988) and the endogenous growth models with technological progress treat human capital as an exogenous stock (Romer 1986, Romer 1990, Aghion and Howitt 1992). Recently, there have been a few attempts to integrate these approaches. First, Acemoglu (1996) and Redding (1996) investigate the interaction between human capital accumulation and technological progress due to R&D in a search model. These studies imply that technological progress tends to have a positive effect on human capital accumulation, because it increases incentives for education. Second, Eicher (1996), Galor

³ For a more detailed discussion on the differences between education, R&D and learning-by-doing as sources of economic growth, see Cannon (2000) and Storesletten and Zilibotti (2000).

and Weil (2000) and Galor and Moav (2000) investigate the interaction between human capital accumulation and technological progress due to R&D and/or learning-by-doing externalities. These studies imply that technological progress tends to have a negative effect on human capital accumulation, because it absorbs resources from the education sector and/or it decreases the adaptability of human capital.

We focus on the latter type of interaction between human capital accumulation and technological progress. Technological progress depends on learning-by-doing externalities and it has two opposite effects on the model. First, technological progress has a positive productivity effect on final goods production as in Romer (1986). Second, it has a negative erosion effect on human capital accumulation as in Galor and Weil (2000) and Galor and Moav (2000). The erosion effect arises because existing human capital is not completely applicable in the new technological environment, i.e., human capital is technology specific. The overall effect of technological progress on the economy depends on the tradeoff between the productivity and erosion effects, which allows us to generate some new results compared to models where the source of economic growth is technological progress or human capital accumulation alone.

1.3 Ponzi games and government budget policy

In the last twenty years, the U.S. government has run budget deficits and experienced a large increase in public debt. At the same time, the average rate of return on debt has been below the average growth rate of output, which has allowed the government to roll over the debt. This type of government debt finance policy, where old debt is financed by issuing new debt instead of levying taxes, is called a Ponzi game. Ponzi games are an example of bubbles. If the government uses Ponzi game debt finance, the present value of future taxes does not cover the initial value of the debt and the government intertemporal budget constraint does not hold. Hence, Ponzi games are not a neutral debt finance policy, but they violate the Ricardian equivalence by redistributing resources from children to parents. Ponzi games typically make sense in fast growing economies, where children's lifetime incomes are likely to be higher than their parents' lifetime incomes.

Several researchers argue that Ponzi games are not sustainable in the long run (Sargent and Wallace 1981, Abel et al. 1989, Ball et al. 1998). The rationale for these results is based on two suggestions, dynamic efficiency and a perfect risk-sharing between the living generations. If the economy is not affected by production uncertainty, then the rate of return on public debt is equal to the rate of return on physical capital. This implies that in the dynamically efficient economy, the costs of debt grow faster than the output and the government cannot roll over the debt. If the economy has production uncertainty and the incomes of the living generations are perfectly correlated, the debt cannot be used as insurance against income fluctuations. However, it is possible that the average risk-free rate of return on public debt can be lower than the growth rate of output even if the rate of return on physical capital exceeds the growth rate of output. In this case, the government might be able to roll over the debt for some time but not forever, because the costs of the debt would eventually exceed the output increase.

Some researchers examine the issue from the alternative viewpoint, which allows dynamic inefficiency (Bullard and Russell 1999, Chalk 2000) or imperfect risk-sharing (Bertocchi 1991, Blanchard and Weil 2001, Gale 1995). They argue that moderate permanent budget deficits are sustainable by Ponzi game debt finance and public debt can even be welfare improving. However, the results are based on exogenous growth models, where budget deficits and public debt do not affect the long-run growth rate of the economy. This study attempts to fill this gap by considering Ponzi game debt finance in the endogenous growth model, which allows dynamic inefficiency.

Because dynamic inefficiency is a necessary condition for Ponzi games in our model, the question of whether actual economies are dynamically efficient is central to our study. In this discussion, the main issue is the explanation for the fact that the average risk-free rate of return on government debt has been lower than the average growth rate of output in the U.S. economy. This question is also closely related to the difference between the risk-free rate of return and the rate of return on equity, i.e., the Mehra and Prescott (1985) equity premium puzzle. We can find three different explanations for the matter. First, if the reason for the low risk-free rate of return is uncertainty and the economy does not have non-systematic risk between the living generations, a sufficiently high risk aversion would imply that the average risk-free rate of return can be below the growth rate of output in the dynamically efficient economy (Abel et al. 1989). In this case, Ponzi games are not feasible, because incomes of the living generations are perfectly correlated and Ponzi games do not provide insurance against income fluctuations. Second, if the reason for the low risk-free rate of return is uncertainty and the economy has nonsystematic risk between the living generations, Ponzi games may be feasible, but not because of dynamic inefficiency, but rather due to imperfect risk-sharing between the living generations (Blanchard and Weil 2001). Third, if the reason for the low risk-free rate of return is intermediation costs or other market imperfections, the low risk-free rate of return implies dynamic inefficiency and a feasibility of Ponzi games (Bullard and Russell 1999, Bohn 1999). As long as the equity premium puzzle remains unsolved, the comparison of the growth rate of output between the rate of returns cannot give a decisive answer to the empirical relevance of Ponzi games.⁴

Abel et al. (1989) develop an alternative empirical method to study dynamic efficiency, which tries to avoid the equity premium puzzle. This so-called cash flow criterion compares cash flows going in and out of the production sector. The cash flow criterion is a strong argument against the dynamic inefficiency of the U.S. economy, because gross profits of firms have exceeded gross investments in the U.S. in every post-war (1945-2000) year. However, because gross profits and investments of firms are not necessarily equal to the return on physical capital, it is not clear if this criterion is the correct way to measure dynamic efficiency. In particular, gross profits of firms do not include taxes, intermediation costs and other frictions that might reduce the return on physical capital. On the other hand, gross investments of firms do not include the consumption of intermediate and durable goods. If we amend the cash-flow criterion with these costs and

⁴ For a more detailed discussion on the equity premium puzzle, see Kocherlakota (1996).

goods, it is possible that the post-war U.S. economy is dynamical inefficient (Bullard and Russell 1999).

One way to connect dynamic inefficiency to the empirical discussion is calibration. Calibrated versions of dynamically efficient OLG models usually imply unrealistically high risk-free rate of returns and too low growth rates. For example in Auerbach and Kotlikoff (1987) the real risk-free gross rate of return is 1.07 and the gross growth rate of output is 1.01, while the corresponding values in the post-war U.S. data are 1.01 and 1.03, respectively. Bullard and Russell (1999) and Chalk (2000) show that this inconvenience can be corrected if the economy is allowed to be dynamically inefficient and the difference between the risk-free rate of return and the return on equity is corrected with an exogenous equity premium. Moreover, Chalk finds that the U.S. economy can sustain even 5% deficit/GDP ratio with Ponzi game debt finance. However, Bullard and Russell and Chalk only consider exogenous growth OLG models. We study the issue in the endogenous growth OLG model and show that the highest sustainable level of deficit/GDP ratio in the U.S. economy is around 2.1%. Because the realized value of U.S. deficit/GDP ratio has been 2% since 1980, our model implies that the current deficit can be sustained purely by the Ponzi game debt finance.

1.4 Outline of the study

The study is organized in the following way. In Chapter 2 we describe the basic structure of the endogenous growth OLG model, where the source of economic growth is human capital accumulation due to education investments and technological progress due to learning-by-doing externalities. We solve the competitive equilibrium of the model and study the existence of steady states as well as transitional dynamics. Moreover, we also consider the welfare properties of the steady states. In Chapter 3 we add an intrinsically useless asset to the model and study the existence of bubbles. In Chapter 4 we add two-sided altruism to the model. In Chapter 5 we consider government debt with permanent budget deficits. Moreover, we study some empirical implications of the model and calibrate the model to the U.S. data. Finally, in Chapter 6 we make some concluding remarks.

2. AN ENDOGENOUS GROWTH MODEL WITH HUMAN CAPITAL AND TECHNOLOGICAL PROGRESS

In this chapter we present an endogenous growth overlapping generations (OLG) model, where the source of economic growth is human capital accumulation due to education investments and technological progress due to learning-by-doing externalities. The OLG structure of the model allows stationary equilibrium allocations where the growth rate of output exceeds the rate of return on physical capital, i.e., dynamic inefficiency. The model is a variant of the three-period overlapping generations model with human capital by Boldrin and Montes (2002), where young agents' education investments are financed by borrowing. We add to the model technological progress due to learning-by-doing externalities. Technological progress has a positive effect on physical capital accumulation as in Boldrin (1992), Azariadis and Reichlin (1996) and Antinolfi et al.

(2001), but it also erodes human capital as in Galor and Weil (2000) and Galor and Moav (2000).

We focus on a particular class of stationary solutions, which allows perpetual growth in the steady state. These types of solutions are called balanced growth equilibria (BGE). In the BGE, all endogenously accumulated variables grow at the same rate, which implies that every steady state of the economy in terms of effective variables is a BGE. A BGE is a common equilibrium concept in growth models, because it maintains the tractability of the analysis and fulfills some empirical regularities, which are part of the so-called Kaldor's facts (Barro and Sala-i-Martin 1995).

The existence of BGE also imposes some restrictions on the model. In particular, it implies that the utility function must be additive separable and homogenous and production functions must be linearly homogenous (Jones and Manuelli 1990). These restrictions eliminate transitional dynamics in the perpetual growth equilibria if the model does not have human capital (Grossman and Yanagawa 1993) or human capital is accumulated purely by education investments (Caballe 1995, Rangazas 1996). To avoid this inconvenience we assume that human capital is also accumulated by intergenerational externalities as in Azariadis and Drazen (1990).

To satisfy the requirements for the existence of BGE and to maintain the tractability of the analysis, we assume a log-linear utility function and Cobb-Douglas production functions. We show that the model has a unique balanced growth equilibrium and the qualitative properties of the equilibrium depend directly on the degrees of returns to scale in the intensive forms of human capital and final goods production functions. The degrees of returns to scale, on the other hand, are determined by the productivity and erosion effects of technological progress.

2.1 The model

Agents live for three periods and they are identical within generations. The first period (young age) is devoted to education, the second period (adulthood) to employment and the last period (old age) to retirement. There is a single commodity in the economy, which can be either consumed or invested in physical capital or education. When young, agents decide on education investments.⁵ Education investments are financed by borrowing on capital markets, because agents are born without physical endowments. We therefore assume that young agents have perfect access to capital markets and they can borrow against their future income.⁶ Adults supply inelastically one unit of labor, pay

⁵ For simplicity we assume that agents do not consume or work in the first period. Young agents' consumption can be thought of as included in their parents consumption. The tradeoff between working and studying is considered by Azariadis and Drazen (1990) and Michel (1992) among others. In this type of model, agents work in several periods, which complicates households' saving behavior. In particular, the existence of dynamic inefficiency also depends on labor productivity. A sufficiently high labor productivity of old agents can eliminate dynamic inefficiency by maintaining low savings and high rate of return on physical capital (Decreuse and Thibault 2001)

⁶ The simplest way to add education investments in the OLG models is to allow young agents to have perfect access to capital markets and to use human capital as collateral to finance their education spending. Usually human capital

their debt and accrued interest on debt and allocate the rest of labor income between consumption and savings. When old, agents are retired and they consume their savings and accrued interest on savings.

The generation at work in period t and born in period $t-1$ is indexed by t . Periodic budget constraints for generation t are:

$$(1a) \quad c_{1t} + s_t + R_t e_{t-1} = h_t w_t$$

$$(1b) \quad c_{2t+1} = R_{t+1} s_t$$

where c_{1t} is the consumption in adulthood, c_{2t+1} is the consumption in old age, s_t is the savings, $R_{t+1} = 1 + r_{t+1}$ is the gross rate of return on savings, h_t is the amount of human capital (or effective labor supply), w_t is the wage per effective unit of labor and e_{t-1} is the investment in education.

Periodic budget constraints (1a) and (1b) can be combined into a single lifetime budget constraint:

$$(2) \quad c_{1t} + R_t e_{t-1} + c_{2t+1}/R_{t+1} = h_t w_t$$

Final goods are produced by the following C-D production function:

$$(3) \quad Y_t = F(K_t, H_t, \bar{k}_t) = K_t^\alpha H_t^{1-\alpha} \bar{k}_t^\eta \quad 0 < \alpha < 1, \eta > 0$$

where $K_t = N_t k_t$ is the aggregate physical capital, k_t is the physical capital per worker, $H_t = N_t h_t$ is the effective aggregate labor supply (or aggregate human capital), N_t is the size of generation t , $k_t = K_t/H_t$ is the physical capital per effective unit of labor and \bar{k}_t is the average level of k_t and it represents technological progress due to the learning-by-doing externality effect.

Parameter α defines the effect of physical and human capital on the final goods production. Constraint $0 < \alpha < 1$ implies that the final goods production function is increasing and strictly concave in K_t and H_t and it fulfills the Inada conditions for K_t and H_t . Hence, the final goods production function satisfies the standard properties for interior profit maximization.

Parameter η defines the effect of technological progress on the final goods production. Constraint $\eta > 0$ implies that the final goods production function is increasing in \bar{k}_t .

cannot be used as collateral for a loan, but many countries have working student loan markets, which allow young agents to borrow against their future income. OLG models with education investments and perfect access to capital markets are studied by Boldrin and Montes (2002), De La Croix and Michel (2002, 269) and Marchand et al. (2003) among others. Alternative way to finance education investments is parental funding or public education. Parental funding is studied in Chapter 4, where we add altruism to the model. Public education is studied by Glomm and Ravikumar (1992) and Kaganovich and Zilcha (1999) among others.

The idea that technological progress depends on positive spillovers from economy-wide factors of production is developed by Arrow (1962) and Romer (1986). They arise because agents learn better working methods during the production process when they repeatedly deal with the same problems and the knowledge of working methods is a public good. OLG models where the source of economic growth is technological progress due to learning-by-doing externalities are studied by Boldrin (1992), Azariadis and Reichlin (1996) and Antinolfi et al. (2001) among others. The analytical purpose of adding a positive productivity effect of technological progress to the economy is to allow increasing returns to scale in the intensive form of the final goods production function $F(K_t, H_t, \bar{k}_t)/H_t = F(k_t, 1, \bar{k}_t) \equiv f(k_t, \bar{k}_t) = k_t^\alpha \bar{k}_t^\eta$. If f has increasing returns to scale, i.e., $\alpha + \eta > 1$, then the rate of return on physical capital is increasing in physical capital and the properties of the model are different from those in models, where the source of growth is human capital accumulation or technological progress alone.

Markets for physical capital, consumption goods and labor are assumed to be perfectly competitive and the depreciation rate of physical capital is assumed to be one.⁷ It follows that in the competitive equilibrium of the factor markets, factors are paid their private marginal products:

$$(4a) \quad R_t = F_1(K_t, H_t, \bar{k}_t) = \alpha k_t^{\alpha-1} \bar{k}_t^\eta$$

$$(4b) \quad w_t = F_2(K_t, H_t, \bar{k}_t) = (1 - \alpha) k_t^\alpha \bar{k}_t^{\eta-1}$$

Human capital is accumulated by the following Cobb-Douglas (C-D) production function:

$$(5) \quad h_t = G(\bar{h}_{t-1}, e_{t-1}, \bar{k}_t) = \bar{h}_{t-1}^{1-\delta} e_{t-1}^\delta \bar{k}_t^{-\mu} \quad 0 < \delta < 1, 0 < \mu < 1$$

where \bar{h}_{t-1} is the average level of h_{t-1} and represents the intergenerational externality effect.

Parameter δ defines the effect of education investments on human capital accumulation. Constraint $0 < \delta < 1$ implies that the human capital production function is increasing and strictly concave in e_{t-1} and it fulfills the Inada conditions for e_{t-1} . Hence, the human capital production function satisfies the standard properties for interior utility maximization with respect to education investments.

Parameter δ also defines the effect of intergenerational externalities on human capital accumulation. Constraint $0 < \delta < 1$ also implies that the human capital production function is increasing in \bar{h}_{t-1} . The idea that human capital is transmitted between generations by intergenerational externalities is developed by Azariadis and Drazen (1990). Intergenerational externalities arise because children inherit production skills from the previous generation. The analytical purpose of adding them into the economy is to allow

⁷ In calibration a period in a three-period OLG model is usually interpreted to be 20-25 years. From this viewpoint full depreciation of physical capital is empirically a plausible assumption.

transitional dynamics. Without these externalities, the linear homogeneity requirement of the human capital production function tends to eliminate the transitional dynamics of the model in the balanced growth equilibrium as in Caballe (1995) and Rangazas (1996).

Parameter μ defines the effect of technological progress on human capital accumulation. Constraint $0 < \mu < 1$ implies that the human capital production function is decreasing and strictly convex in \bar{k}_t .⁸ Strict convexity implies that technological progress erodes human capital at a decreasing rate such that human capital will not become zero at high rates of technological progress. The idea that technological progress has a negative effect on human capital accumulation is developed by Galor and Weil (2000) and Galor and Moav (2000).⁹ Technological progress erodes human capital, because human capital is technology specific and existing human capital is not completely applicable in the new technological environment. The analytical purpose of adding a negative erosion effect of technological progress to the economy is to allow negative returns to scale in the intensive form of the human capital production function $G(\bar{h}_{t-1}, e_{t-1}, \bar{k}_t) / \bar{h}_{t-1} = G(1, e_{t-1} / \bar{h}_{t-1}, \bar{k}_t) \equiv g(e_{t-1} / \bar{h}_{t-1}, \bar{k}_t) = (e_{t-1} / \bar{h}_{t-1})^\delta \bar{k}_t^{-\mu}$. If g has negative returns to scale, i.e., $\delta - \mu < 0$, then the balanced growth rate is decreasing in physical capital and the properties of the model are different from those in models, where the source of growth is human capital accumulation or technological progress alone.

Households of generation t have the following additive separable logarithmic utility function

$$(6) \quad U(c_{1t}, c_{2t+1}) = u(c_{1t}) + \rho u(c_{2t+1}) \quad \rho > 0$$

$$u(c_{it}) = \ln c_{it} \quad \text{for } i=1,2$$

where ρ is the individual discount factor.

Logarithmic utility and constraint $\rho > 0$ imply that the utility function is increasing, strictly concave and it fulfills the Inada conditions. Hence, it satisfies the standard properties for interior utility maximization with respect to consumption. From the additive separability of U it follows that $U_{12} = U_{21} = 0$, which implies that consumption is a normal good in both periods. The normality assumption allows us to solve the decision functions and to sign certain partial derivatives of them (Azariadis 1993, 198).¹⁰

⁸ Restriction $\mu < 1$ is not necessary for the results of the model, because assumption $\mu > 0$ alone guarantees that the human capital production function is decreasing and strictly convex in \bar{k}_t . The motivation for this restriction is that it simplifies the model to some extent by eliminating cyclical solutions. For details, see section 2.2.

⁹ Galor and Weil (2000) and Galor and Moav (2000) focus on economic development and wage inequality rather than endogenous growth, which makes their models different from our model. However, the idea that technological progress depends on the economy-wide factors of production and that it erodes human capital accumulation is the same.

¹⁰ For the effects of non-separable U on the dynamics of OLG models, see Michel and Venditti (1997).

Before presenting the households' utility maximization problem, we must determine how households form expectations about future prices. We assume that households of generation t foresee future prices w_t , R_t and R_{t+1} correctly, i.e., households have perfect foresight.¹¹

Households choose the optimal amounts of consumption and education investments to maximize utility function (6) subject to lifetime budget constraint (2) and human capital production function (5). Substituting (2) and (5) into the utility function (6) simplifies the households' t utility maximization problem to the following unconstrained problem:

$$(7) \quad \max_{c_{1t}, e_{t-1}} u(c_{1t}) + \rho u[R_{t+1}(G(\bar{h}_{t-1}, e_{t-1}, \bar{k}_t)w_t - c_{1t} - R_t e_{t-1})]$$

The first-order conditions are:

$$(8a) \quad u'(c_{1t}) = R_{t+1} \rho u'(c_{2t+1})$$

$$(8b) \quad R_t = G_2(\bar{h}_{t-1}, e_{t-1}, \bar{k}_t) w_t$$

where $c_{1t}, c_{2t+1}, e_{t-1} > 0$ by Inada-conditions. The first-order conditions are sufficient for the maximum, because objective function (7) is strictly concave in c_{1t} and e_{t-1} by (5) and (6).

By using the explicit forms of production and utility functions, we can solve the savings and consumption from (8a) and (8b) as a linear function of wage income, and education investments from (8b) as a linear function of discounted wage income. Substituting (1a), (1b) and (6) into (8a) and (5) into (8b) implies:

$$(9a) \quad s_t = [\rho / (1 + \rho)] (1 - \delta) h_t w_t$$

$$(9b) \quad c_{1t} = [1 / (1 + \rho)] (1 - \delta) h_t w_t$$

$$(9c) \quad e_{t-1} = \delta h_t w_t / R_t$$

Equations (9a), (9b) and (9c) are identified as saving, consumption and education functions, respectively. These functions imply that decision variables depend linearly on wage income as usual in the economies with a log-linear utility function.

Asset and goods market clearing conditions are:

$$(10) \quad N_t s_t = K_{t+1} + N_{t+1} e_t$$

$$(11) \quad F(K_t, H_t, \bar{k}_t) = N_t c_{1t} + N_{t-1} c_{2t} + N_{t+1} e_t + K_{t+1}$$

¹¹ For alternative assumptions of expectations in the OLG models, see for example Evens and Honkapohja (1999).

The equilibrium of the economy is defined as:

DEFINITION 1: *A competitive equilibrium of the economy is a sequence*

$\{c_{1t}, c_{2t+1}, e_t, h_t, k_{t+1}\}_{t=0}^{\infty}$ *such that*

(i) c_{1t} , c_{2t+1} and e_t *maximize utility (6) subject to budget constraint (2) and human capital production function (5) under given factor prices and external effects*

(ii) *factors are paid their marginal products (4)*

(iii) *budget constraints (1) and market clearing conditions (10) and (11) are satisfied*

(iv) $k_0 > 0$, $h_0 > 0$

(v) $\bar{k}_t = k_t$ and $\bar{h}_t = h_t$

We focus on a particular class of competitive equilibria called balanced growth equilibria, which allows perpetual growth in a steady state: ¹²

DEFINITION 2: *A balanced growth equilibrium is a competitive equilibrium in which c_{1t} , c_{2t} , e_t , h_t and k_t grow at the same endogenous gross growth rate γ .*

In the balanced growth equilibrium, all endogenously accumulating variables grow at the same rate. This type of stationary equilibrium is convenient for two reasons. First, it maintains the tractability of the analysis in models with perpetual endogenous growth. ¹³ Second, along the balanced growth equilibrium path, the rate of return on physical capital and the growth rate of per worker output are constants, which fulfills part of the so-called Kaldor's facts (Barro and Sala-i-Martin 1995).

Equations (4), (5), (9), (10) and (11) define the competitive equilibrium of the economy:

$$(12a) \quad n\gamma k_{t+1} + ne_t = [\rho/(1+\rho)](1-\delta)w_t$$

$$(12b) \quad c_{1t} = [1/(1+\rho)](1-\delta)w_t$$

$$(12c) \quad k_t^{\alpha+\eta} = c_{1t} + c_{2t}/n + ne_t + n\gamma k_{t+1}$$

$$(12d) \quad e_t = [\delta(1-\alpha)/\alpha]^{1/(1-\delta)} k_{t+1}^{(1-\mu)/(1-\delta)} \equiv e(k_{t+1})$$

$$(12e) \quad \gamma_t = [\delta(1-\alpha)/\alpha]^{\delta/(1-\delta)} k_{t+1}^{(\delta-\mu)/(1-\delta)} \equiv \gamma(k_{t+1})$$

$$(12f) \quad w_t = (1-\alpha)k_t^{\alpha+\eta} \equiv w(k_t)$$

¹² The existence of balanced growth paths requires that the utility function is additive separable and homogenous and production functions are linearly homogenous (Jones and Manuelli 1990). These requirements are satisfied by functions (3), (5) and (6).

¹³ If the economy does not have a balanced growth equilibrium, steady state equilibria of the economy do not sustain perpetual endogenous growth. In this case, endogenous growth equilibria are unstable equilibrium trajectories, which converge to the infinity. The analysis of the unstable equilibrium trajectories is much more complicated than the analysis of the steady state equilibria. For the analysis of the unstable equilibrium trajectories, see Boldrin (1992), Azariadis and Reichlin (1996) and Antinolfi et al. (2001).

$$(12g) \quad R_t = \alpha k_t^{\alpha+\eta-1} \equiv R(k_t)$$

where $n = N_{t+1}/N_t$ is the gross growth rate of population, $c_{1t} = c_{1t}/h_t$ and $c_{2t} = c_{2t}/h_t$ are the consumption per effective unit of labor, $e_t = e_t/h_t$ is the education per effective unit of labor, $\gamma_t = h_{t+1}/h_t$ is the gross growth rate per worker and (12d)-(12g) define e_t , γ_t , w_{t+1} and R_{t+1} as functions of k_{t+1} .

Definition 2 implies that every steady state of system (12) is a balanced growth equilibrium, because in the steady state $c_1 = c_{1t}/h_t$, $e = e_t/h_t$ and $k = k_t/h_t$, i.e., c_{1t} , c_{2t} , e_t , h_t and k_t grow at the same endogenously determined gross growth rate $\gamma(k) = h_{t+1}/h_t$.

If the economy does not have human capital accumulation, i.e., $\delta = 0$ and $\mu = 0$, then $e_t = 0$ and $\gamma_t = 1$ by (12d) and (12e). In this case, the source of economic growth is technological progress alone and the economy has a balanced growth equilibrium only if $\alpha + \eta = 1$, which implies that the gross growth rate is $k_{t+1}/k_t = [\rho/(1+\rho)](1-\alpha)/n$ by (12a). Because $K_{t+1}/K_t = nk_{t+1}/k_t = [\rho/(1+\rho)](1-\alpha) < 1$, the economy disappears in time. This can be corrected by adding a scale factor to the production function, i.e., $F(K_t, H_t, \bar{k}_t) = \Lambda K_t^\alpha H_t^{1-\alpha} \bar{k}_t^\eta$, where $\Lambda > 1/[\rho/(1+\rho)](1-\alpha)$. This type of model is studied by Saint-Paul (1992), Grossman and Yanagawa (1993) and King and Ferguson (1993) among others. If $\delta = 0$ and $\alpha + \eta > 1$, then the economy does not have a balanced growth equilibrium, but it has a non-balanced growth equilibrium, which displays perpetual endogenous growth. This type of model is studied by Boldrin (1992), Azariadis and Reichlin (1996) and Antinolfi et al. (2001) among others.

If the economy does not have technological progress, i.e., $\eta = 0$ and $\mu = 0$, then the source of economic growth is human capital accumulation alone and $\gamma' > 0$ and $R' < 0$. This type of model is studied by Azariadis and Drazen (1990), Michel (1992), Kahn et al. (1997), Boldrin and Montes (2002) and Marchand et al. (2003) among others.

2.2 Steady states and transitional dynamics

In this section we consider the existence of non-trivial steady states and transitional dynamics in the economy. Galor and Ryder (1989) show that exogenous growth OLG models with a log-linear utility function and C-D production functions have a unique non-trivial steady state, which is globally stable in the forward dynamics. We show that this result also holds in our model if the effects of technological progress are weak. If the productivity effect of technological progress is sufficiently strong, it can make the non-trivial steady state globally unstable. A similar result holds in the endogenous growth OLG models, where the source of growth is technological progress due to learning-by-doing externalities alone (Boldrin 1992, Azariadis and Reichlin 1996, Antinolfi et al. 2001). However, the interpretation of the unstable steady state in these models is different from that in our model, because steady states do not sustain perpetual growth in the models. If the erosion effect of technological progress is sufficiently strong, it can also make the non-trivial steady state globally unstable.

Substituting (12d-f) into (12a) simplifies system (12) to the following scalar system:

$$(13) \quad n[1+\delta(1-\alpha)/\alpha]\gamma(k_{t+1})k_{t+1}=[\rho/(1+\rho)](1-\delta)w(k_t)$$

Equation (13) can be solved in the backward or forward dynamics. The dynamic equilibrium of the production economy is usually solved in the forward dynamics, because the initial value of k_0 is historical data and is given by the definition of the competitive equilibrium.

Equation (13) defines the following mapping in the forward dynamics:

$$(14) \quad k_{t+1}=\Phi k_t^{(\alpha+\eta)(1-\delta)/(1-\mu)}\equiv\phi(k_t)$$

where $\Phi=[\rho/(1+\rho)](1-\delta)(1-\alpha)/n[\delta(1-\alpha)/\alpha]^{\delta/(1-\delta)}[1+\delta(1-\alpha)/\alpha]^{(1-\delta)/(1-\mu)}>0$ by $0<\delta<1$, $\rho>0$ and $0<\alpha<1$.

From (14) it follows that

$$(15) \quad \phi'=[(\alpha+\eta)(1-\delta)/(1-\mu)]k_{t+1}/k_t$$

where $\phi'>0$ by $0<\delta<1$, $\eta>0$, $0<\alpha<1$ and $0<\mu<1$.¹⁴

If the human capital production function does not have intergenerational externalities, i.e., if $\delta=1$, then $k_{t+1}=[\alpha/(1-\alpha)]^{1/(1-\mu)}$ by (14), which implies that the economy does not have transitional dynamics. With intergenerational externalities, i.e., when $\delta<1$, we cannot make this type of simplification and the model has transitional dynamics.

By using (14) and (15) we can show:

- PROPOSITION 1:** (i) *The economy has a trivial steady state $k=0$. If $(\alpha+\eta)(1-\delta)/(1-\mu)\neq 1$, then the economy also has a unique non-trivial steady state.*
(ii) *If $(\alpha+\eta)(1-\delta)/(1-\mu)<1$, then the non-trivial steady state is globally stable.*
(iii) *If $(\alpha+\eta)(1-\delta)/(1-\mu)>1$, then the non-trivial steady state is globally unstable.*

PROOF: (i) Difference equation (14) has a trivial solution $\phi(0)=\Phi 0^{(\alpha+\eta)(1-\delta)/(1-\mu)}=0$, because $(\alpha+\eta)(1-\delta)/(1-\mu)>0$. Moreover, it has a unique non-trivial steady state $k=\phi(k)$ as long as $\phi'(k)=(\alpha+\eta)(1-\delta)/(1-\mu)\neq 1$, because mapping $\phi(k_t)$ is an exponential function.¹⁵ If

¹⁴ If $\mu>1$, then $\phi'<0$. In this case, the economy has a unique non-trivial steady state. If $(\alpha+\eta)(1-\delta)/(1-\mu)<-1$, then the non-trivial steady state is an unstable spiral. If $(\alpha+\eta)(1-\delta)/(1-\mu)=-1$, then the non-trivial steady state has an infinite number of 2-period cycles around it. If $-1<(\alpha+\eta)(1-\delta)/(1-\mu)<0$, then the non-trivial steady state is a stable spiral.

¹⁵ Exponential function $\phi(k)=k^a$ satisfies $\phi'>0, \phi''<0$, $\lim_{k\rightarrow 0} \phi'(k)=\infty$ and $\lim_{k\rightarrow\infty} \phi'(k)=0$ if $0<a<1$ and $\phi'>0, \phi''>0$, $\lim_{k\rightarrow 0} \phi'(k)=0$ and $\lim_{k\rightarrow\infty} \phi'(k)=\infty$ if $a>1$, which are necessary and sufficient conditions for the existence of a unique non-trivial steady state.

$(\alpha+\eta)(1-\delta)/(1-\mu)=1$, then equation (14) has zero or infinite number of non-trivial steady states.

(ii-iii) The local dynamics of the non-trivial steady state in system (14) is topologically equivalent to the dynamics of the non-trivial steady state in linearized system by the Hartman-Grobman theorem if $\phi(k)$ is invertible and the non-trivial steady state is hyperbolic (Azariadis 1993, 59). These conditions are satisfied if $\phi'(k) \neq 0$ and $\phi''(k) \neq 1$. Because $(\alpha+\eta)(1-\delta)/(1-\mu) > 0$, the Hartman-Grobman theorem applies as long as $(\alpha+\eta)(1-\delta)/(1-\mu) \neq 1$. The global dynamics of the non-trivial steady state in system (14) is topologically equivalent to the local dynamics of the steady state if mapping ϕ does not have other stable non-trivial invariant sets in addition to the non-trivial steady state (Azariadis 1993, 85-92). This is true as long as $(\alpha+\eta)(1-\delta)/(1-\mu) \neq 1$, because $k = \phi^2(k) = \Phi(\Phi k^{(\alpha+\eta)(1-\delta)/(1-\mu)})^{(\alpha+\eta)(1-\delta)/(1-\mu)}$ does not have other non-trivial solutions than $k = \phi(k) = \Phi k^{(\alpha+\eta)(1-\delta)/(1-\mu)}$. Hence, if $\phi'(k) = (\alpha+\eta)(1-\delta)/(1-\mu) < 1$, then the non-trivial steady state is a stable node. If $\phi'(k) = (\alpha+\eta)(1-\delta)/(1-\mu) > 1$, then the non-trivial steady state is an unstable node. Q.E.D.

Proposition 1 implies that the economy has a unique non-trivial steady state except in the special case $(\alpha+\eta)(1-\delta)/(1-\mu)=1$, where it has zero or infinite number of non-trivial steady states. Moreover, it implies that the stability of the non-trivial steady state depends on the degree of returns to scale in production functions g and f , i.e., on the strength of the productivity and erosion effects of technological progress.

If g and f have decreasing returns to scale, then the non-trivial steady state is stable, because $\delta-\mu > 0$ and $\alpha+\eta < 1 \Rightarrow (\alpha+\eta-1)(1-\delta) < \delta-\mu \Rightarrow (\alpha+\eta)(1-\delta)/(1-\mu) < 1$. This result is sensible, because the same is true for the exogenous growth OLG models with a log-linear utility function and C-D production functions (Galor and Ryder 1989).

If f has increasing returns to scale ($\alpha+\eta > 1$), then it is possible that $(\alpha+\eta-1)(1-\delta) > \delta-\mu \Rightarrow (\alpha+\eta)(1-\delta)/(1-\mu) > 1$. In this case, the non-trivial steady state is unstable. A similar result holds for the endogenous growth OLG models, where the source of growth is technological progress due to learning-by-doing externalities alone (Boldrin 1992, Azariadis and Reichlin 1996, Antinolfi et al. 2001). The interpretation of the unstable steady state in these models is different from that in our model. They are not balanced growth equilibrium models, which implies that they do not sustain perpetual growth in the steady state. Hence, the instability of the non-trivial steady state is a necessary condition for the perpetual growth in these models. Because our model sustains perpetual growth in the steady state, the instability of the non-trivial steady state is not required for perpetual growth.

If g has negative returns to scale ($\delta-\mu < 0$), then it is also possible that $(\alpha+\eta-1)(1-\delta) > \delta-\mu \Rightarrow (\alpha+\eta)(1-\delta)/(1-\mu) > 1$. Hence, the erosion effect of technological progress is an additional reason for the instability of the non-trivial steady state.

A log-linear utility function and C-D production functions are not necessary for the existence of a balanced growth equilibrium in the model. However, if we allow more general forms of utility and production functions than (3), (5) and (6), then the dynamics of the economy can be more complicated than Proposition 1 indicates and the stability of the non-trivial steady state also depends on model properties other than the degrees of returns to scale in the production functions g and f . For example, with a constant elasticity of substitution (CES) utility function, it is possible that ϕ is a multivalued mapping and the economy can have multiple non-trivial steady states, periodic solutions and indeterminacy. The same is also true if we have production functions, which allow indirect effects of technological progress through technological complementarities.¹⁶

The global dynamics of system (14) can be also analyzed qualitatively by phase-diagrams in the (k_{t+1}, k_t) space. For this purpose, let us consider time paths, which satisfy $k_{t+1} \geq k_t$. Equation (14) implies that

$$(16) \quad k_t \leq \phi(k_t) \quad \text{if } k_{t+1} \geq k_t$$

From (16) it follows that k_t is increasing when ϕ lies above the 45° line and decreasing when ϕ lies below the 45° line. Hence, the global dynamics of the non-trivial steady state can be demonstrated by the following phase-diagrams:

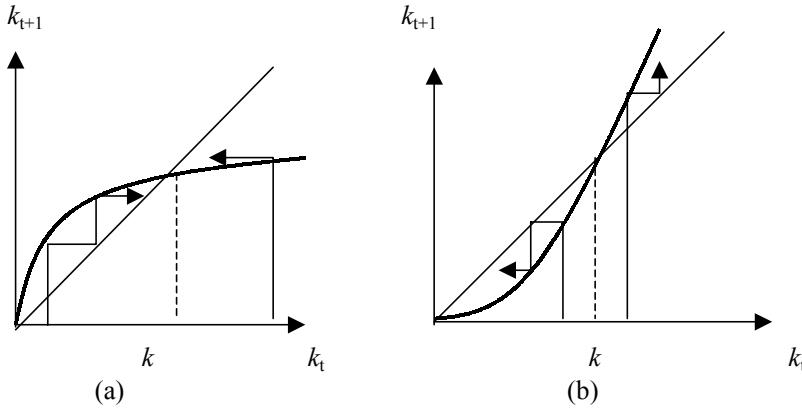


FIGURE 1 Global dynamics of the non-trivial steady state when (a) $(\alpha + \eta)(1 - \delta) / (1 - \mu) < 1$
(b) $(\alpha + \eta)(1 - \delta) / (1 - \mu) > 1$

2.3 Characterization of dynamic inefficiency

In this section we examine the welfare properties of the competitive equilibrium. In particular, we consider so called dynamic inefficiency. Usually, dynamic efficiency is meant to convey productive efficiency, which is connected to the physical capital

¹⁶ For the analysis of the model with general forms of production functions and a constant elasticity of substitution utility function, see Appendix 1.

overaccumulation relative to the Golden Rule (Cass 1972). In the endogenous growth OLG models, productive efficiency is not an appropriate method to evaluate dynamic efficiency, because it is not able to indicate all allocations, where intergenerational reallocation is feasible. Hence, we adapt a stronger measure of dynamic efficiency than Cass (1972) and show that dynamic inefficiency can be connected to physical capital over- or underaccumulation relative to the Golden Rule allocation.

To characterize the social optimum, we assume a social planner, who maximizes the sum of discounted utilities over generations subject to the resource constraints of the economy. The competitive equilibria of the economy may differ from the social optimum in two ways. First, the OLG-structure of the model may cause a failure in the households' saving behavior. Because this inefficiency is connected to the allocation of resources between generations, it is called dynamic inefficiency. Second, intergenerational and learning-by-doing externalities cause a difference between the private and the social returns from education and physical capital investments. The welfare loss due to externalities is called static inefficiency.

Both inefficiencies affect the accumulation of physical capital and economic growth, which makes the welfare analysis of the model difficult. To maintain the tractability of the analysis, we conduct the welfare analysis in the second-best setting, where the social planner does not internalize externalities.¹⁷ The second-best problem typically occurs when the economy lacks suitable instruments to fulfill some part of the optimality conditions (Lipsey and Lancaster 1956).

Social planner has the following utility function:

$$(17) \quad W = u(c_{20}) + \sum_{t=0}^{\infty} \omega^t [u(c_{1t}) + \rho u(c_{2t+1})] \quad \omega > 0$$

where ω is the social discount factor.

Utility function W is a weighted sum of generations' utilities alive at date 0 or later. Hence, we can obtain all Pareto optimal allocations by varying the weights ω^t in the feasible range of the parameter space. To ensure that the objective function of the planner is finite, we must set some restrictions (transversality conditions) on the feasible values of the weights. These restrictions define the feasible range of Pareto optimal allocations.

The resource constraints of the economy are:

$$(18a) \quad F(k_t, h_t, \bar{k}_t) = c_{1t} + c_{2t}/n + ne_t + nk_{t+1}$$

$$(18b) \quad h_{t+1} = G(\bar{h}_t, e_t, \bar{k}_{t+1})$$

¹⁷ In the endogenous growth OLG models the welfare loss due to externalities can be eliminated by distortionary subsidies and taxes. For the elimination of static inefficiency, see Saint-Paul (1992), Caballe (1995) and Marchand et al. (2003).

$$(18c) \quad h_t = \bar{h}_t$$

$$(18d) \quad k_t = \bar{k}_t$$

where constraints (18c) and (18d) imply that the planner internalizes externalities.

In the first-best problem the planner chooses the optimal amounts of consumption, education investments, human capital and physical capital to maximize utility function (17) subject to (18a-d). Substituting (18c-d) into (18a-b) and (18a) into the utility function (17) simplifies the planner's utility maximization to the following Lagrangean:

$$(19) \quad \max_{\{c_{2t+1}, e_t, h_{t+1}, k_{t+1}\}_{t=0}^{\infty}} u(c_{20}) + \sum_{t=0}^{\infty} \omega^t \{u[F^{\wedge}(k_t, h_t) - c_{2t}/n - n e_t - n k_{t+1}] + \rho u(c_{2t+1})\} + \lambda_t [h_{t+1} - G^{\wedge}(h_t, e_t, k_{t+1})]$$

where $F^{\wedge}(k_t, h_t) \equiv F(k_t, h_t, k_t/h_t)$ and $h_{t+1} = G^{\wedge}(h_t, e_t, k_{t+1})$ is defined implicitly by $h_{t+1} = G(h_t, e_t, k_{t+1}/h_{t+1})$.

The first-order conditions are:

$$(20a) \quad \omega(1/n)u'(c_{1t+1}) = \rho u'(c_{2t+1})$$

$$(20b) \quad nu'(c_{1t}) + \lambda_t G^{\wedge}_2(h_t, e_t, k_{t+1}) = 0$$

$$(20c) \quad \omega F^{\wedge}_2(k_{t+1}, h_{t+1})u'(c_{1t+1}) + \lambda_t = \omega \lambda_{t+1} G^{\wedge}_1(h_{t+1}, e_{t+1}, k_{t+2})$$

$$(20d) \quad \omega F^{\wedge}_1(k_{t+1}, h_{t+1})u'(c_{1t+1}) = nu'(c_{1t}) + \lambda_t G^{\wedge}_3(h_t, e_t, k_{t+1})$$

where $c_{1t}, c_{2t+1} > 0$ by Inada-conditions.

The transversality conditions are:¹⁸

$$(20e) \quad \lim_{t \rightarrow \infty} \omega^t u'(c_{1t}) k_t = 0$$

$$(20f) \quad \lim_{t \rightarrow \infty} \omega^t u'(c_{1t}) e_t = 0$$

$$(20g) \quad \lim_{t \rightarrow \infty} \omega^t u'(c_{1t}) h_t = 0$$

From conditions (20b-d) it follows that:

$$(21a) \quad R^*_{t+1} = G^{\wedge}_2(h_t, e_t, k_{t+1}) w^*_{t+1}$$

¹⁸ The transversality condition ensures that the objective function is finite for all feasible allocations. For details, see De La Croix and Michel (2002, 103).

$$(21b) \quad nu'(c_{1t})/u'(c_{1t+1})=\omega R^*_{t+1}$$

where $R^*_{t+1}=F^*_1(k_{t+1},h_{t+1})/[1-G^*_3(h_t,e_t,k_{t+1})/G^*_2(h_t,e_t,k_{t+1})]$ and $w^*_{t+1}=F^*_2(k_{t+1},h_{t+1})-nG^*_1(h_{t+1},e_{t+1},k_{t+2})/G^*_2(h_{t+1},e_{t+1},k_{t+2})$.

Equation (21a) determines the optimal allocation of education investments. It is equal to the households' first-order condition (8b) except that the private gross rate of return on physical capital R_{t+1} is replaced by the social gross rate of return on physical capital R^*_{t+1} and the private wage rate w_{t+1} is replaced by the social wage rate w^*_{t+1} . Equation (21b) determines the optimal allocation of physical capital and it is usually called the Euler equation. In the planner's problem, the Euler equation replaces the households' saving function (9a).

If externalities are sufficiently strong, then the planner's production possibility set is non-convex. From non-convexity it follows that the Kuhn-Tucker Theorem does not apply and the first-order conditions of the social planner's utility maximization problem are not sufficient conditions for the maximum.

Besides the fact that the planner's problem with externalities may be ill-defined, the effects of externalities may conflict with each other and their overall effect is ambiguous. First, learning-by-doing externalities in the final goods production function cause underaccumulation of physical capital relative to the social optimum. Second, learning-by-doing externalities in the human capital production function cause overaccumulation of physical capital relative to the social optimum. Third, intergenerational externalities in the human capital production function implies underinvestments in education, which can cause over- or underaccumulation of physical capital relative to the social optimum. Moreover, the effects of externalities may conflict with the effect of dynamic inefficiency.

To avoid these problems, we consider a second-best problem, where the planner does not internalize externalities. Hence, externalities form an additional distortion to the planner's problem. Usually second-best analysis is used in the public finance literature to determine welfare implications of distortionary taxation. However, Lipsey and Lancaster (1956) show that the second-best setting applies to a much wider class of problems. More recently, Kehoe et al. (1992) use a second-best analysis in model with externalities and non-convexity.

In the second-best problem the planner chooses the optimal amounts of consumption, education investments, human capital and physical capital to maximize utility function (17) subject to (18a-b). Substituting (18a-b) into the utility function (17) simplifies the planner's utility maximization to the following unconstrained problem:

$$(22) \quad \max_{\{c_{2t+1}, e_t, k_{t+1}\}_{t=0}^{\infty}} \omega^t \{u[F(k_t, G(\bar{h}_{t-1}, e_{t-1}, \bar{k}_t), \bar{k}_t) - c_{2t}/n - ne_t - nk_{t+1}] + \rho u(c_{2t+1})\}$$

The first-order conditions are:

$$(23a) \quad \omega(1/n)u'(c_{1t+1})=\rho u'(c_{2t+1})$$

$$(23b) \quad nu'(c_{1t})=\omega G_2(\bar{h}_t, e_t, \bar{k}_{t+1})F_2(k_{t+1}, h_{t+1}, \bar{k}_{t+1})u'(c_{1t+1})$$

$$(23c) \quad nu'(c_{1t})=\omega F_1(k_{t+1}, h_{t+1}, \bar{k}_{t+1})u'(c_{1t+1})$$

where $c_{1t}, c_{2t+1}, e_t, k_{t+1} > 0$ by Inada-conditions.

The transversality conditions are:

$$(23d) \quad \lim_{t \rightarrow \infty} \omega^t u'(c_{1t})k_t = 0$$

$$(23e) \quad \lim_{t \rightarrow \infty} \omega^t u'(c_{1t})e_t = 0$$

Conditions (23) together with (18c) and (18d) are sufficient for the maximum, because objective function (22) is strictly concave in $\{c_{2t+1}, e_t, k_{t+1}\}_{t=0}^{\infty}$ by (3), (5) and (6).

From conditions (23b), (23c), (18c) and (18d) it follows that:

$$(24a) \quad R_{t+1} = G_2(h_t, e_t, k_{t+1})w_{t+1}$$

$$(24b) \quad nu'(c_{1t})/u'(c_{1t+1}) = \omega R_{t+1}$$

where $R_{t+1} = F_1(k_{t+1}, h_{t+1}, k_{t+1})$ and $w_{t+1} = F_2(k_{t+1}, h_{t+1}, k_{t+1})$ as in the competitive equilibrium. It follows that the solution of the planner's second-best problem is similar to the competitive equilibrium except that the accumulation of physical capital is determined by the Euler equation (24b) instead of the saving function (9a).

To clarify what we mean by dynamic inefficiency, let us define:

DEFINITION 3: *If a competitive equilibrium solves the planner's second-best problem (22) for some $\omega > 0$, then it is said to be dynamically efficient. If a competitive equilibrium does not solve the planner's second-best problem (22) for any $\omega > 0$, then it is said to be dynamically inefficient. If a competitive equilibrium satisfies $R_{t+1} = R^*_{t+1}$ and $w_{t+1} = w^*_{t+1}$ for all k_t , then it is said to be statically efficient. If a competitive equilibrium has $R_{t+1} \neq R^*_{t+1}$ or $w_{t+1} \neq w^*_{t+1}$ for some k_t , then it is said to be statically inefficient.*

Definition 3 implies that dynamic inefficiency is a welfare loss due to OLG-structure of the economy while static inefficiency is a welfare loss due to externalities. By using Definition 3 we can show:

PROPOSITION 2: *If $n\gamma > R$ in the non-trivial steady state, then it is dynamically inefficient. If $n\gamma \leq R$ in the non-trivial steady state, then it is dynamically efficient.*

PROOF: Substituting utility function (6) into the Euler equation (24b) implies that $n\gamma c_{1t+1}/c_{1t} = \omega R_{t+1}$, where γ_t is defined implicitly by (18b) and (24a) as in the competitive equilibrium. It follows that a dynamically efficient steady state allocation must satisfy $n\gamma = \omega R$. Substituting (6) into transversality condition (23d) implies that $\lim_{t \rightarrow \infty} \omega^t k/c_1 = 0$, which is satisfied when $0 < \omega < 1$. The limit case $\omega = 1$ does not satisfy the transversality condition (23d). In this case, however, we can obtain the solution of the planner's problem by modifying the planner's objective to $u(c_{20}) - u(c_2^*) + \sum_{t=0}^{\infty} \omega^t [u(c_{1t}) + \rho u(c_{2t+1}) - u(c_1^*) - \rho u(c_2^*)]$, where $u(c_1^*) + \rho u(c_2^*)$ is the maximum stationary utility level (De la Croix and Michel 2002, 92). Hence, feasible values of weights are $0 < \omega \leq 1$. It follows that if $n\gamma = \omega R > R$ in the non-trivial steady state, then the planner can increase the welfare of the economy for all feasible ω and the allocation must be dynamically inefficient. On the other hand, if $n\gamma < R$ in the non-trivial steady state, then the planner cannot increase the welfare of the economy for some feasible ω and the allocation must be dynamically efficient. Q.E.D.

Proposition 2 implies a simple condition for the dynamic inefficiency of the non-trivial steady states. From equations (12f), (12g) and (13) it follows that in the competitive equilibrium $n\gamma/R = [(1-\alpha)/\alpha] n\gamma k/w = [\rho/(1+\rho)] / [\alpha(1-\alpha)(1-\delta) + \delta/(1-\delta)]$, where $[\rho/(1+\rho)] / [\alpha(1-\alpha)(1-\delta) + \delta/(1-\delta)]$ can be smaller or higher than unity by $0 < \rho < 1$, $0 < \alpha < 1$ and $0 < \delta < 1$. Hence, we have the following:

COROLLARY 1: *If $[\rho/(1+\rho)] / [\alpha(1-\alpha)(1-\delta) + \delta/(1-\delta)] \leq 1$, then the non-trivial steady state of the competitive economy is dynamically efficient. If $[\rho/(1+\rho)] / [\alpha(1-\alpha)(1-\delta) + \delta/(1-\delta)] > 1$, then the non-trivial steady state of the competitive economy is dynamically inefficient.*

Corollary 1 implies that dynamic inefficiency depends on the discount factor ρ and the productivity of physical capital α as in exogenous growth OLG model with a log-linear utility and C-D production functions (De la Croix and Michel 2002, 80). Moreover, it also depends on the productivity of education δ . However, it does not depend on the productivity and erosion effects of technological progress μ and η . This result is sensible, because μ and η define the effect of production externalities on the model and thereby they do not have a direct influence on intertemporal allocation.

Proposition 2 is consistent with Cass's (1972) condition for dynamic inefficiency, which implies that a growth path is dynamically inefficient if the terms of trade from the present to the future decrease sufficiently fast in time, i.e., $\sum_{t=0}^{\infty} \prod_{s=0}^t R_s/n\gamma_s < \infty$. However, Cass defines dynamic inefficiency as productive inefficiency, which is always connected by definition to the physical capital overaccumulation relative to some reference allocation.¹⁹ In our model it is possible that allocations which satisfy Cass's condition for

¹⁹ Cass (1972) defines path $\{K_t\}$ to be dynamically inefficient if there exists another path $\{K_t'\}$, which in each period provides at least as much consumption, and in some period more, i.e., if $\exists \{K_t'\}$ such that $F(K_t', H_t) - K_{t+1}' \geq F(K_t, H_t) - K_{t+1}$ for all t , $F(K_t', H_t) - K_{t+1}' > F(K_t, H_t) - K_{t+1}$ for some t and $K_t = K_t'$ for $t \leq t^*$ and $K_t \neq K_t'$ for $t > t^*$. Because $F(K_t, H_t) - K_{t+1}' < F(K_t, H_t) - K_{t+1}$ for $K_{t+1}' > K_{t+1}$, it must be true that path $\{K_t\}$ can be dynamically inefficient only if $K_t \geq K_t'$ for all t . It follows that inefficiency is always connected to physical capital overaccumulation relative to some reference allocation (for example Golden Rule) and any allocation that is underaccumulated compared to the

dynamic inefficiency can also have physical capital underaccumulation relative to some reference allocation.²⁰ Hence, even if Cass's condition for dynamic inefficiency is consistent with Proposition 2, the argument for the condition is different from the argument for Proposition 2.

To show that physical capital underaccumulation can be dynamically inefficient, let us consider a limit case of the planner's second-best problem, where the planner treats all generations equally, i.e., $\omega=1$. This allocation satisfies in the steady state $n\gamma=R$, which is the analogue of the Golden Rule allocation in our model. Hence, we denote this allocation by k^{GR} . The economy has a unique $k^{GR}>0$, because equation $n\gamma(k)=R(k)$ defines implicitly an exponential function for k by (12d) and (12f). From the uniqueness of $k^{GR}>0$ it follows that $R<n\gamma$ for $k>k^{GR}$ if $R'(k^{GR})<n\gamma'(k^{GR})$ and $R<n\gamma$ for $k<k^{GR}$ if $R'(k^{GR})>n\gamma'(k^{GR})$. Hence, dynamic inefficiency implies physical capital overaccumulation relative to the Golden Rule if $k^{GR}R'/R<nk^{GR}\gamma'/n\gamma$, i.e., $(\alpha+\eta)(1-\delta)/(1-\mu)<1$. This case also includes Cass's (1972) model without externalities, where $R'(k^{GR})<0$ and $n\gamma'(k^{GR})=0$. On the other hand, dynamic inefficiency implies physical capital underaccumulation relative to the Golden Rule if $k^{GR}R'/R>nk^{GR}\gamma'/n\gamma$, i.e., $(\alpha+\eta)(1-\delta)/(1-\mu)>1$. This discussion can be summarized the following corollary:

COROLLARY 2: *If $(\alpha+\eta)(1-\delta)/(1-\mu)<1$, then dynamically inefficient steady states satisfy $k>k^{GR}$. If $(\alpha+\eta)(1-\delta)/(1-\mu)>1$, then dynamically inefficient steady states satisfy $k<k^{GR}$.*

Corollary 2 implies that dynamic inefficiency can be associated with physical capital over- or underaccumulation relative to the Golden Rule. The result depends on the degree of returns to scale in production functions g and f , i.e., on the strength of the productivity and erosion effects of technological progress.

If g and f have decreasing returns to scale, then dynamic inefficiency implies overaccumulation relative to the Golden Rule, because $\delta-\mu>0$ and $\alpha+\eta<1 \Rightarrow (\alpha+\eta-1)(1-\delta)<\delta-\mu \Rightarrow (\alpha+\eta)(1-\delta)/(1-\mu)<1$. This result is sensible, because the same is true for the exogenous growth OLG models without externalities (Cass 1972). If g has negative returns to scale ($\delta-\mu<0$) or f has increasing returns to scale ($\alpha+\eta>1$), then it is possible that $(\alpha+\eta)(1-\delta)/(1-\mu)>1 \Leftrightarrow (\alpha+\eta-1)(1-\delta)>\delta-\mu$. In this case dynamic inefficiency implies underaccumulation relative to the Golden Rule.

The intuition for the result is the following. The Euler equation and the transversality condition imply that a social planner can increase the welfare of the economy if the productivity of physical capital is low such that the growth rate of output exceeds the rate

reference allocation is dynamically efficient by definition. Hence, to consider dynamic inefficiency in the case of physical capital underaccumulation requires a different type of analysis. In particular, it requires that we can compare a decrease of consumption in one period to an increase of consumption in another period, i.e., a social welfare function.

²⁰ Another example of a model where dynamic inefficiency is connected to physical capital underaccumulation relative to some reference allocation is Gutierrez's (2003) production OLG model with negative pollution externalities.

of return on physical capital. If production functions g and f have decreasing returns to scale, then the rate of return on physical capital is decreasing and the growth rate of output increasing in physical capital. This implies that the growth rate of output only exceeds the rate of return on physical capital when physical capital is overaccumulated relative to the Golden Rule. If f has increasing returns to scale, then the rate of return on physical capital is increasing in physical capital. If g has negative returns to scale, then the growth rate of output is decreasing in physical capital. In these cases, the growth rate of output can also exceed the rate of return on physical capital when physical capital is underaccumulated relative to the Golden Rule.

If we allow more general forms of utility and production functions than (3), (5) and (6), then dynamic inefficiency may also depend on model properties other than the productivity of physical capital, education and technological progress. Moreover, the relationship between dynamic inefficiency and over- or underaccumulation does not necessarily depend directly on the degrees of returns to scale in production functions g and f . Furthermore, it is possible that the economy has multiple non-trivial stationary solutions for the planner's utility maximization problem and the economy has multiple k^{GR} . In this case, it is unclear which one is social optimum and we cannot conduct the efficiency analysis by comparing the equilibrium allocations to the social optimum.²¹

In the end we make some notes about the connection between static and dynamic inefficiency. First, King and Ferguson (1993) show that endogenous growth OLG models without externalities are always dynamically efficient. This result also holds in our model. If the economy does not have externalities, i.e., $\delta=1$ and $\mu=\eta=0$, then the competitive equilibrium is equal to the social optimum by (14), (21a) and (24a). Hence, the existence of dynamic inefficiency in the endogenous growth models requires static inefficiency even if the fundamental reason for dynamic inefficiency is the OLG structure of the model. Second, Saint-Paul (1992) and Grossman and Yanagawa (1993) show that if the source of growth is technological progress due to learning-by-doing externalities, then the Pareto optimality of the competitive equilibrium depends only on static inefficiency and the elimination of dynamic inefficiency decreases the overall welfare of the economy. This type of situation is also possible in our model. To see this, let us consider a special case of the model, where the final goods production function f has non-decreasing returns to scale ($\alpha+\eta\geq 1$) and human capital production function does not have learning-by-doing externalities ($\mu=0$). From market clearing (18a) it follows that $R^*=(\alpha+\eta)k^{\alpha+\eta-1}\geq k^{\alpha+\eta} = f(k)/k=c_1/k+c_2/nk+n\gamma>n\gamma$, which implies that the balanced growth equilibrium satisfies the planner's first-best problem for some social discount factor. Hence, the Pareto optimality of the competitive equilibrium only depends on static inefficiency and the elimination of dynamic inefficiency decreases the overall welfare of the economy if static and dynamic inefficiency conflicts with each other. In Saint-Paul's (1992) and Grossman and Yanagawa's (1993) model, we have $\alpha+\eta=1$, $\delta=1$ and $\mu=0$, which implies that static inefficiency causes underaccumulation of physical capital and dynamic inefficiency causes overaccumulation of physical capital. Hence, static and

²¹ For the analysis of the model with general forms of production functions and a constant elasticity of substitution utility function, see Appendix 1.

dynamic inefficiency conflicts with each other, which implies that the elimination of dynamic inefficiency decreases the overall welfare of the economy. If the model has intergenerational externalities ($\delta < 1$), the result may not hold, because intergenerational externalities may cause physical capital overaccumulation. Moreover, if the final goods production function f has decreasing returns to scale ($\alpha + \eta < 1$) or human capital production function has learning-by-doing externalities ($\mu > 0$), then it is possible that $R^* < n\gamma$, which implies that the Pareto optimality of the competitive equilibrium also depends on dynamic inefficiency as in Michel (1992), Kahn et al. (1997) and Marchand et al. (2003).

3. INTRINSICALLY USELESS ASSETS

In this chapter we consider the redistribution of resources from children to parents through bubbles. We add to the model an intrinsically useless infinitely lived asset and show that it can eliminate dynamic inefficiency due to OLG-structure of the model as in Tirole (1985).

Intrinsically useless assets do not pay any dividends or rents, which implies that the fundamental value of the intrinsically useless assets is zero.²² Examples of intrinsically useless assets are fiat money and government Ponzi game debt. If an intrinsically useless asset has a positive value, it is called a bubble. Because intrinsically useless assets are owned by old agents and sold to the next generation, bubbles move resources from children to parents. Hence, a necessary condition for bubbles is that the economy without intrinsically useless assets is dynamically inefficient such that there is a case for reallocation of resources from children to parents.

If the source of economic growth is endogenous technological progress, then bubbles crowd out savings in physical capital and the growth rate of output depends positively on physical capital (Grossman and Yanagawa 1993, Azariadis and Reichlin 1996). Hence, bubbles cannot increase economic growth in these models. The same is true if the source of economic growth is human capital accumulation alone (Michel 1992). If the source of economic growth is human capital accumulation and technological progress together, the situation is different. Bubbles can crowd out or crowd in savings in physical capital and the balanced growth rate of output can depend positively or negatively on physical capital, which implies that bubbles can increase or decrease economic growth.

²² The fundamental value of an asset is the present discounted value of its future dividends or rents. If assets pay dividends or rents, they are intrinsically useful. Examples of intrinsically useful assets are land, natural resource or stock market shares of firms. Usually intrinsically useful assets only have fundamental value. If dividends or rents grow at a lower rate than the rest of the economy, intrinsically useful assets can also have non-fundamental value. The same is true if the aggregate stock of assets grows in time and new assets are not a gift to the owners of the old assets. In this case, the dividends or the rents of new assets cannot be discounted and valued before the actual creation of assets, i.e., the dividends or the rents are non-capitalized. Examples of assets with non-capitalized dividends are future inventions or stock market shares of firms in the industry with free entry. On the other hand, examples of assets with capitalized dividends are land and stock market shares of firms in the industry with blockaded entry. (Tirole 1985, Rhee 1991, Grossman and Yanagawa 1993, Femminis 1999)

3.1 The model

The model is the same as in Chapter 2 except that the economy has intrinsically useless assets, which can be used as a substitute for savings in physical capital. Because intrinsically useless assets do not affect periodic budget constraints (1a) and (1b), the households' utility maximization problem (7) and first-order conditions (8) hold.

The free disposability of assets implies that the value of intrinsically useless assets is non-negative:

$$(25) \quad B_t \geq 0$$

where B_t is the value of intrinsically useless assets, i.e., the bubble.

Non-arbitrage between savings in the intrinsically useless assets and physical capital implies that:

$$(26) \quad R_{t+1}^B \leq R_{t+1} \quad (\text{if } B_t > 0)$$

where R_{t+1}^b is the gross rate of return on intrinsically useless assets.

Suppose that intrinsically useless assets are owned by old agents of generation $t-1$. They sell intrinsically useless assets to the adults of generation t , who pay these assets B_t , because they believe that they can sell them again in the next period to the adults of the following generation and receive $R_{t+1}^B B_t$. It follows that

$$(27) \quad B_{t+1} = R_{t+1}^B B_t$$

Intrinsically useless assets in (25)-(27) can be interpreted as fiat money or government Ponzi game debt. In the former case, let $M > 0$ be the aggregate amount of money and p_t the price of money in terms of consumption goods at time t . It follows that we obtain (25)-(27) by setting $p_{t+1}/p_t \equiv R_{t+1}^B$ and $p_t M \equiv B_t$. In the latter case, let $A_{t+1} = R_{t+1}^A A_t$ and $A_t \geq 0$ be the government budget constraint. It follows that we obtain (25)-(27) by setting $R_{t+1}^A \equiv R_{t+1}^B$ and $A_t \equiv B_t$.²³

By using (27) we can rewrite the non-arbitrage condition (26):

$$(28) \quad n b_{t+1}/b_t \leq R_{t+1} \quad (\text{if } b_t > 0)$$

where $b_t = B_t/N_t$.

²³ An increase in the aggregate money supply M would decrease the effect of bubbles on the economy (Ferreira 1999, Futagami and Shibata 2000). The same is true for permanent budget deficits. For details, see Chapter 5.

Condition (28) determines when the economy has a bubble and when the non-negativity constraint for bubbles is binding. If the economy sustains a bubble, condition (28) defines a link between the rate of return on physical capital and the growth rate of the bubble.

Assets market clearing condition is

$$(29) \quad N_t s_t = K_{t+1} + B_t + N_{t+1} e_t$$

Perfect foresight implies that households of generation t foresee the future value of the bubble b_{t+1} correctly. Hence, agents' beliefs on the bubble are non-stochastic and the probability that the bubble will collapse is zero.²⁴

The equilibrium of the economy is defined as:

DEFINITION 4: *A competitive equilibrium of the economy with intrinsically useless assets is a sequence $\{b_t, c_{1t}, c_{2t+1}, e_t, h_t, k_{t+1}\}_{t=0}^{\infty}$ such that*

(i) c_{1t}, c_{2t+1} and e_t maximize utility (6) subject to budget constraint (2) and human capital production function (5) under given factor prices and external effects

(ii) factors are paid their marginal products (4)

(iii) budget constraints (1) and market clearing conditions (11) and (29) are satisfied

(iv) $\bar{k}_0 > 0, \bar{h}_0 > 0$

(v) $\bar{k}_t = k_t$ and $\bar{h}_t = h_t$

(vi) non-arbitrage condition (28) is satisfied

The definition of the competitive equilibrium of the economy with intrinsically useless assets is similar to the definition of the competitive equilibrium of the economy without intrinsically useless assets except that the economy has an additional state variable b_t , which satisfies non-arbitrage condition (28). Moreover, the economy satisfies asset market clearing condition (29) instead of (10).

Equations (4),(5),(9),(11),(28) and (29) define the competitive equilibrium of the economy with intrinsically useless assets:

$$(30a) \quad n\gamma_t k_{t+1} + n e_t + b_t = [\rho/(1+\rho)](1-\delta)w_t$$

$$(30b) \quad c_{1t} = [1/(1+\rho)](1-\delta)w_t$$

$$(30c) \quad n\gamma_t b_{t+1}/b_t \leq R_{t+1} \quad (= \text{if } b_t > 0)$$

$$(30d) \quad k_t^{\alpha+\eta} = c_{1t} + c_{2t}/n + n e_t + n\gamma_t k_{t+1}$$

$$(30e) \quad e_t = [\delta(1-\alpha)/\alpha]^{1/(1-\delta)} k_{t+1}^{(1-\mu)/(1-\delta)} \equiv e(k_{t+1})$$

²⁴ A positive probability of collapsing would decrease the effect of bubbles on the economy (Weil 1987a, Bertocchi and Wang 1993).

$$(30f) \quad \gamma_t = [\delta(1-\alpha)/\alpha]^{\delta/(1-\delta)} k_{t+1}^{(\delta-\mu)/(1-\delta)} \equiv \gamma(k_{t+1})$$

$$(30g) \quad w_t = (1-\alpha)k_t^{\alpha+\eta} \equiv w(k_t)$$

$$(30h) \quad R_t = \alpha k_t^{\alpha+\eta-1} \equiv R(k_t)$$

where $b_t = B_t/H_t$ is the bubble per effective unit of labor and (30e)-(30h) define e_t , γ_t , w_{t+1} and R_{t+1} as functions of k_{t+1} .

System (30) can have two types of equilibria. If $b_t > 0$, they are called bubble equilibria, and if $b_t = 0$, they are called bubbleless equilibria.

3.2 Steady states and transitional dynamics

In this section we consider the existence of non-trivial steady states and transitional dynamics in the economy with intrinsically useless assets. Tirole (1985) shows that exogenous growth OLG models with intrinsically useless assets have a unique globally saddle-path stable bubble steady state if the economy without intrinsically useless assets has a unique dynamically inefficient globally stable non-trivial steady state. We show that this result also holds in our model if the effects of technological progress are weak. If the productivity effect of technological progress is sufficiently strong, it can make the bubble steady state globally unstable and cause oscillations around the steady state. A similar result holds in the endogenous growth OLG models where the source of growth is technological progress due to learning-by-doing externalities alone (Azariadis and Reichlin 1996). However, the interpretation of the unstable steady state in Azariadis and Reichlin's model is different from that in our model, because steady states do not sustain perpetual growth in the model. If the erosion effect of technological progress is sufficiently strong, it can also make the non-trivial steady state unstable and cause oscillations.

Moreover, we show that bubbles can decrease or increase economic growth. This result is different from those in models, where the source of growth is technological progress (Grossman and Yanagawa 1993, Azariadis and Reichlin 1996) or human capital accumulation alone (Michel 1992).

Substituting (30e-h) into (30a) and (30c) simplifies system (30) to the following planar system:

$$(31a) \quad n[1+\delta(1-\alpha)/\alpha]\gamma(k_{t+1})k_{t+1} = [\rho/(1+\rho)](1-\delta)w(k_t) - b_t$$

$$(31b) \quad n\gamma(k_{t+1})b_{t+1}/b_t \leq R(k_{t+1}) \quad (= \text{if } b_t > 0)$$

If $b_t=0$, system (31) is equal to the scalar system (13) in the economy without intrinsically useless assets. If $b_t > 0$, equations (31a) and (31b) define the following mappings in the forward dynamics:

$$(32a) \quad k_{t+1} = \Omega[(\rho/(1+\rho))(1-\delta)w(k_t) - b_t]^{(1-\delta)/(1-\mu)} \equiv \varphi^1(k_t, b_t)$$

$$(32b) \quad b_{t+1} = R[\varphi^1(k_t, b_t)]b_t/\gamma[\varphi^1(k_t, b_t)] \equiv \varphi^2(k_t, b_t)$$

where $\Omega = \{n[\delta(1-\alpha)/\alpha]^{\delta/(1-\delta)}[1+\delta(1-\alpha)/\alpha]\}^{-(1-\delta)/(1-\mu)} > 0$ by $0 < \delta < 1$, $\rho > 0$ and $0 < \alpha < 1$.

From (32) it follows that

$$(33a) \quad \varphi^1_1 = [(\alpha+\eta)(1-\delta)/(1-\mu)](k_{t+1}/k_t)[\rho/(1+\rho)](1-\delta)w_t/[(\rho/(1+\rho))(1-\delta)w_t - b_t]$$

$$(33b) \quad \varphi^1_2 = -[(1-\delta)/(1-\mu)]k_{t+1}/[(\rho/(1+\rho))(1-\delta)w_t - b_t]$$

$$(33c) \quad \varphi^2_1 = (R'k_{t+1}/R_{t+1} - \gamma'k_t/\gamma_t)(b_{t+1}/k_{t+1})\varphi^1_1 = [\alpha+\eta-(1-\mu)/(1-\delta)](b_{t+1}/k_{t+1})\varphi^1_1$$

$$(33d) \quad \varphi^2_2 = R_{t+1}/n\gamma_t + (R'k_{t+1}/R_{t+1} - \gamma'k_{t+1}/\gamma_{t+1})(b_{t+1}/k_{t+1})\varphi^1_2 =$$

$$R_{t+1}/n\gamma_t + [\alpha+\eta-(1-\mu)/(1-\delta)](b_{t+1}/k_{t+1})\varphi^1_2$$

where $\varphi^1_1 > 0$ and $\varphi^1_2 < 0$ by $0 < \mu < 1$, $0 < \delta < 1$, $\rho > 0$, $0 < \alpha < 1$ and $\eta > 0$.

Let us denote a non-trivial steady state of the economy without intrinsically useless assets by k^D , i.e., $\phi(k^D) = k^D > 0$. By using (32) and (33) we can show:

PROPOSITION 3: (i) *The economy with intrinsically useless assets has a trivial bubbleless steady state $k = 0$. Moreover, if $(\alpha+\eta)(1-\delta)/(1-\mu) \neq 1$ and $n\gamma(k^D) > R(k^D)$, then the economy has a unique bubble steady state. If $(\alpha+\eta)(1-\delta)/(1-\mu) \neq 1$ and $n\gamma(k^D) \leq R(k^D)$, then the economy has a unique non-trivial bubbleless steady state.*

(ii) *If $(\alpha+\eta)(1-\delta)/(1-\mu) < 1$, then the bubble steady state is globally saddle-path stable. Moreover, the non-trivial bubbleless steady state is globally stable.*

(iii) *If $(\alpha+\eta)(1-\delta)/(1-\mu) > 1$, then the bubble steady state is globally saddle-path stable, or it is unstable and it can have oscillations around it. Moreover, the non-trivial bubbleless steady state is globally unstable.*

PROOF: (i) System of difference equation (32) has a trivial solution $b=k=0$, because $(\alpha+\eta)(1-\delta)/(1-\mu) > 0$. Moreover, it has a bubble steady state if $b = [\rho/(1+\rho)](1-\delta)w(k) - n[1+\delta(1-\alpha)/\alpha]\gamma(k)k > 0$ and $n\gamma(k) = R(k)$. The former equation is a function in the (k, b) space, which satisfies $b > 0$ if $\phi(k) > k$, where $\phi(k) = \varphi^1(k, 0)$ by (14) and (32). The latter equation is a vertical line in the (k, b) space. It follows that the economy has a bubble steady state if $n\gamma = R$ for some $\phi(k) > k$. On the other hand, the economy has a unique bubbleless steady state by Proposition 1 if $n\gamma \leq R$ for some $\phi(k) = k$.

If $\phi'(k^D) < 1$, i.e., $(\alpha + \eta)(1 - \delta)/(1 - \mu) < 1$, then $\phi(k) > k$ for $k < k^D$ and $\phi(k) < k$ for $k > k^D$. Moreover, equation (33c) implies that $R'(k)/R(k) < n\gamma'(k)/n\gamma(k)$, i.e., curve $n\gamma(k)$ crosses curve $R(k)$ from below and these curves have a unique strictly positive crossing point. Hence, if $n\gamma(k^D) > R(k^D)$, then there exists a unique $k > 0$ such that $n\gamma = R$ and $\phi(k) > k$, but we cannot find $k > 0$ such that $n\gamma \leq R$ and $\phi(k) = k$. If $n\gamma(k^D) \leq R(k^D)$, then there exists a unique $k > 0$ such that $n\gamma \leq R$ and $\phi(k) = k$, but we cannot find $k > 0$ such that $n\gamma = R$ and $\phi(k) > k$.

If $\phi'(k^D) > 1$, i.e., $(\alpha + \eta)(1 - \delta)/(1 - \mu) > 1$, then $\phi(k) < k$ for $k < k^D$ and $\phi(k) > k$ for $k > k^D$. Moreover, equation (33c) implies that $R'(k)/R(k) > n\gamma'(k)/n\gamma(k)$, i.e., curve $n\gamma(k)$ crosses curve $R(k)$ from above and these curves have a unique strictly positive crossing point. Hence, if $n\gamma(k^D) > R(k^D)$, then there exists a unique $k > 0$ such that $n\gamma = R$ and $\phi(k) > k$, but we cannot find $k > 0$ such that $n\gamma \leq R$ and $\phi(k) = k$. If $n\gamma(k^D) \leq R(k^D)$, then there exists a unique $k > 0$ such that $n\gamma \leq R$ and $\phi(k) = k$, but we cannot find $k > 0$ such that $n\gamma = R$ and $\phi(k) > k$.

(ii-iii) Because bubble steady states and non-trivial bubbleless steady states cannot co-exist by (i), the dynamics of the non-trivial bubbleless steady state is defined by Proposition 1.

The local dynamics of the bubble steady state in system (32) is topologically equivalent to the dynamics of the bubble steady state in linearized system by the Hartman-Grobman theorem if $D\phi(k, b)$ is invertible and the bubble steady state is hyperbolic (Azariadis 1993, 59). To show that these conditions are satisfied when $(\alpha + \eta)(1 - \delta)/(1 - \mu) > 0$ and $(\alpha + \eta)(1 - \delta)/(1 - \mu) \neq 1$, let us consider the Jacobian matrix of partial derivatives of the system at the bubble steady state

$$J \equiv D\phi(k, b) = \begin{pmatrix} \phi^1_1 & \phi^1_2 \\ [\alpha + \eta - (1 - \mu)/(1 - \delta)](b/k)\phi^1_1 & 1 + [\alpha + \eta - (1 - \mu)/(1 - \delta)](b/k)\phi^1_2 \end{pmatrix}$$

From (33) it follows that $\det J = \lambda_1 \lambda_2 = \phi^1_1 > 0$ and $\text{tr} J = \lambda_1 + \lambda_2 = 1 + \phi^1_1 + [\alpha + \eta - (1 - \mu)/(1 - \delta)](b/k)\phi^1_2$, where λ_1 and λ_2 are eigenvalues of J . Moreover, $1 - \text{tr} J + \det J = -[\alpha + \eta - (1 - \mu)/(1 - \delta)](b/k)\phi^1_2$ and $1 + \text{tr} J + \det J = 2(1 + \phi^1_1) + [\alpha + \eta - (1 - \mu)/(1 - \delta)](b/k)\phi^1_2$. Because $\det J > 0$, J is invertible. Moreover, if $(\alpha + \eta)(1 - \delta)/(1 - \mu) < 1$, then $1 - \text{tr} J + \det J < 0$, and if $(\alpha + \eta)(1 - \delta)/(1 - \mu) > 1$, then $\det J = \phi^1_1 > \phi^1_2 > 1$ by (15) and (33a). It follows that if $(\alpha + \eta)(1 - \delta)/(1 - \mu) \neq 1$, then eigenvalues of J do not have modulus 1 and the bubble steady state is hyperbolic. Hence, as long as $(\alpha + \eta)(1 - \delta)/(1 - \mu) \neq 1$, the Hartman-Grobman theorem applies.

The dynamics of the linearized system can be analyzed by studying the properties of the eigenvalues of J (Azariadis 1993, 62). If $(\alpha + \eta)(1 - \delta)/(1 - \mu) < 1$, then $1 - \text{tr} J + \det J < 0$ and $1 + \text{tr} J + \det J > 0$. It follows that the bubble steady state is locally a saddle.

If $(\alpha + \eta)(1 - \delta)/(1 - \mu) > 1$, then $1 - \text{tr} J + \det J > 0$ and $\det J > 1$. However, the sign of $1 + \text{tr} J + \det J$ is ambiguous. If $1 + \text{tr} J + \det J < 0$, then the bubble steady state is locally a saddle. If $1 + \text{tr} J + \det J > 0$ for $k > 0$ and $b > 0$, then the bubble steady state is locally an unstable node or spiral.

The global dynamics of the bubble steady state in system (32) is topologically equivalent to the local dynamics of the steady state if mapping ϕ does not have other stable invariant sets with bubbles in addition to the bubble steady state. This is true by the Stable Manifold theorem if $\phi(k_t, b_t)$ is diffeomorphism and the bubble steady state is hyperbolic (Galor 1992). Because $\phi^1(k_t, b_t) > 0$, it must be true that $\det D\phi(k_t, b_t) = \phi^1 R_{t+1} / n\gamma_t > 0$, which implies ϕ is a diffeomorphism. Hence, as long as $(\alpha + \eta)(1 - \delta) / (1 - \mu) \neq 1$, the Stable Manifold Theorem applies. Q.E.D.

Proposition 3 is similar to Proposition 1 in the economy without intrinsically useless assets except that the non-trivial steady state is a bubble equilibrium if the economy without intrinsically useless assets has a dynamically inefficient non-trivial steady state. Hence, the reason for bubbles is dynamic inefficiency as in Tirole (1985). The stability of the bubble steady state depends on the degrees of returns to scale in production functions g and f as in the model without intrinsically useless assets, i.e., on the strength of the productivity and erosion effects of technological progress.

If g and f have decreasing returns to scale, then the bubble steady state is saddle-path stable, because $\delta - \mu > 0$ and $\alpha + \eta < 1 \Rightarrow (\alpha + \eta - 1)(1 - \delta) < \delta - \mu \Rightarrow (\alpha + \eta)(1 - \delta) / (1 - \mu) < 1$. This result is sensible, because the same is true for the exogenous growth OLG models with a log-linear utility function and C-D production functions (Tirole 1985).

If f has increasing returns to scale ($\alpha + \eta > 1$), then it is possible that $(\alpha + \eta - 1)(1 - \delta) > \delta - \mu \Rightarrow (\alpha + \eta)(1 - \delta) / (1 - \mu) > 1$. In this case, the bubble steady state is unstable and can have oscillations around it. A similar result holds for the endogenous growth OLG models, where the source of growth is technological progress due to learning-by-doing externalities alone (Azariadis and Reichlin 1996). The interpretation of the unstable steady state in Azariadis and Reichlin's model is different from that in our model, because the model does not sustain perpetual growth in the steady state.

If g has negative returns to scale ($\delta - \mu < 0$), then it is also possible that $(\alpha + \eta - 1)(1 - \delta) > \delta - \mu \Rightarrow (\alpha + \eta)(1 - \delta) / (1 - \mu) > 1$. Hence, the erosion effect of technological progress is an additional reason for the instability of the non-trivial steady state and oscillations.

It follows that dynamic properties of the model are almost analogous to the model without intrinsically useless assets in Chapter 2. The only difference is that in the model with intrinsically useless assets, technological progress can also cause explosive oscillations. There are two reasons for this similarity. First, the second state variable is a forward-looking variable without any initial condition, which implies that a saddle is a stable steady state in the planar system. Second, the economy has C-D production functions and a log-linear utility function, which eliminates local indeterminacy and complex dynamics from the model.²⁵

²⁵ In general, the dynamics of planar systems can be much more complicated than the dynamics of scalar systems. For details, see for example Azariadis (1993).

The global dynamics of the system (32) can be also analyzed qualitatively by the phase-diagrams in the (k_t, b_t) space. For this purpose, let us consider time paths, which satisfy $k_{t+1} \geq k_t$ and $b_{t+1} \geq b_t$. Equations (32) imply that

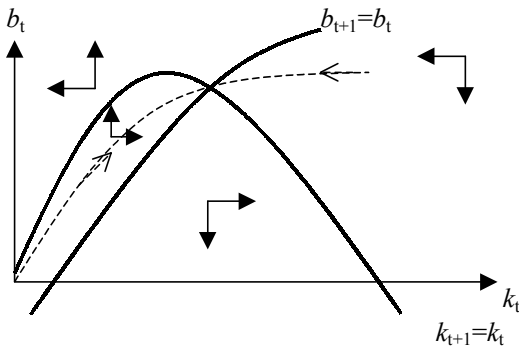
$$(34a) \quad \varphi^1(k_t, b_t) \geq k_t \text{ if } k_{t+1} \geq k_t$$

$$(34b) \quad \varphi^2(k_t, b_t) \leq b_t \text{ if } b_{t+1} \geq b_t$$

From (34a) it follows that $b_t = [\rho/(1+\rho)](1-\delta)(1-\alpha)k_t^{\delta+\eta} - n(1+\delta(1-\alpha)/\alpha)[\delta(1-\alpha)/\alpha]^{\delta/(1-\delta)} k_t^{(1-\mu)/(1-\delta)}$ when $k_{t+1} = k_t$ by (31a). Hence, locus $k_{t+1} = k_t$ is an humped-shaped $[(\alpha+\eta)(1-\delta)/(1-\mu) < 1]$ or j-shaped curve $[(\alpha+\eta)(1-\delta)/(1-\mu) > 1]$ in the (k_t, b_t) space, which starts from the trivial steady state $(0,0)$ and crosses the horizontal axis at the non-trivial bubbleless steady state $\varphi^1(k_t, 0) = \phi(k_t) = k_t$. Moreover, k_t is increasing below and decreasing above the locus $k_{t+1} = k_t$, because $\varphi^1_2 < 0$.

From (34b) it follows that $\partial b_t / \partial k_t = \varphi^2_1 / (1 - \varphi^2_2) = -\varphi^1_1 / \varphi^1_2 = (\alpha+\eta)(\rho/(1+\rho))(1-\delta)w_t/k_t > 0$ when $b_{t+1} = b_t$. Hence, locus $b_{t+1} = b_t$ is an increasing curve in the (k_t, b_t) space, which crosses the horizontal axis at $n\gamma[\phi(k_t)] = R[\phi(k_t)]$. This point is lower (higher) than the non-trivial bubbleless steady state $\phi(k_t) = k_t$ if $(\alpha+\eta)(1-\delta)/(1-\mu) < 1$ $[(\alpha+\eta)(1-\delta)/(1-\mu) > 1]$, because $n\gamma[\phi(k_t)] > R[\phi(k_t)]$ at $\phi(k_t) = k_t$. Hence locus $b_{t+1} = b_t$ crosses the horizontal axis at a lower (higher) point than locus $k_{t+1} = k_t$ if $(\alpha+\eta)(1-\delta)/(1-\mu) < 1$ $[(\alpha+\eta)(1-\delta)/(1-\mu) > 1]$. Moreover, b_t is decreasing (increasing) below and increasing (decreasing) above the locus $b_{t+1} = b_t$ if $(\alpha+\eta)(1-\delta)/(1-\mu) < 1$ $[(\alpha+\eta)(1-\delta)/(1-\mu) > 1]$, because $\varphi^2_1 = (b_{t+1}/k_{t+1})(\alpha+\eta)[(\alpha+\eta)(1-\delta)/(1-\mu) - 1](k_{t+1}/k_t)(\rho/(1+\rho))(1-\delta)w_t / [(\rho/(1+\rho))(1-\delta)w_t - b_t]$.

It follows that the global dynamics of the bubble steady state can be demonstrated by the following phase-diagrams:



(a)

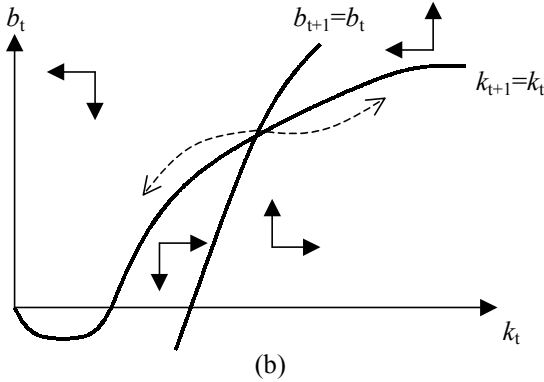


FIGURE 2 Global dynamics of the bubble steady state when (a) $(\alpha+\eta)(1-\delta)/(1-\mu)<1$ (b) $(\alpha+\eta)(1-\delta)/(1-\mu)>1$

When $(\alpha+\eta)(1-\delta)/(1-\mu)>1$, the qualitative dynamics implies that the bubble steady state is an unstable spiral or node and it eliminates the possibility that the bubble steady state is a saddle. This additional information is due to the fact that the crossing point of the horizontal axis and locus $b_{t+1}=b_t$ is lower (higher) than the crossing point of the horizontal axis and locus $k_{t+1}=k_t$ when $(\alpha+\eta)(1-\delta)/(1-\mu)<1$ [$(\alpha+\eta)(1-\delta)/(1-\mu)>1$].

Let us next consider the growth effects of bubbles. By using Proposition 3 we can show:

PROPOSITION 4: *If $(\alpha+\eta-1)(1-\delta)<\delta-\mu<0$ or $0<\delta-\mu<(\alpha+\eta-1)(1-\delta)$, then bubbles increase the growth rate of the economy. Otherwise bubbles decrease the growth rate of the economy.*

PROOF: Bubbles increase γ if $\gamma'<0$ and the bubble steady state is lower than the non-trivial bubbleless steady state or if $\gamma'>0$ and the bubble steady state is higher than the non-trivial bubbleless steady state.

From Proposition 3 it follows that the economy has a unique bubble steady state if the non-trivial bubbleless steady state satisfies $R<n\gamma$. Because in the bubble steady state $R=n\gamma$, bubbles increase growth if and only if $R'<n\gamma'<0$ or $R'>n\gamma'>0$ in the bubble steady state, i.e., if and only if $kR'/R<k\gamma'/\gamma<0$ or $kR'/R>k\gamma'/\gamma>0$. These conditions imply that $\alpha+\eta-1<(\delta-\mu)/(1-\delta)<0 \Leftrightarrow (\alpha+\eta-1)(1-\delta)<\delta-\mu<0$ or $\alpha+\eta-1>(\delta-\mu)/(1-\delta)>0 \Leftrightarrow 0<\delta-\mu<(\alpha+\eta-1)(1-\delta)$ by (30f) and (30h). Q.E.D.

Proposition 4 implies that the growth effect of bubbles depends on the degree of returns to scale in production functions g and f , i.e., on the strength of the productivity and erosion effects of technological progress. If g and f have decreasing returns to scale, then bubbles decrease growth, because $\delta-\mu>0$ and $\alpha+\eta<1 \Rightarrow (\alpha+\eta-1)(1-\delta)<\delta-\mu>0$. This result is sensible, because the same is true for the exogenous growth OLG models (Tirole 1985). If g has negative returns to scale ($\delta-\mu<0$) or f has increasing returns to scale

$(\alpha+\eta>1)$, then it is possible that $(\alpha+\eta-1)(1-\delta)<\delta-\mu<0$ or $0<\delta-\mu<(\alpha+\eta-1)(1-\delta)$, which implies that bubbles increase growth. In the former case, the bubble steady state is a saddle-path stable. In the latter case, the bubble steady state is an unstable spiral or node. Hence, the positive growth effect of bubbles can be connected to stable or unstable transitional dynamics.

The intuition for the result is the following. The effect of bubbles on the economy is that they eliminate dynamically inefficient steady states, i.e., steady states where the growth rate of output exceeds the rate of return on physical capital. If production functions g and f have decreasing returns to scale, then the rate of return on physical capital is decreasing and the growth rate of output increasing in physical capital. This implies that bubbles crowd out physical capital and decrease growth. If f has increasing returns to scale, then the rate of return on physical capital is increasing in physical capital. This implies that bubbles can crowd in physical capital and increase economic growth if the growth rate of output is sufficiently increasing in physical capital. If g has negative returns to scale, then the growth rate of output is decreasing in physical capital. This implies that bubbles can crowd out physical capital and increase growth if the rate of return on physical capital is sufficiently decreasing in physical capital.

If we allow more general forms of production functions than (3) and (5), then the growth effect of bubbles is not necessarily defined directly by the degree of returns to scale in production functions g and f . In particular, it is possible that technological progress and education investments are complements as in Galor and Weil (2000) and Galor and Moav (2000), which implies that technological progress raises the return on education investments. If this indirect effect of technological progress is sufficiently high, then negative returns to scale in g do not necessarily imply $\gamma<0$. Moreover, it is possible that mapping γ is a non-monotone function and/or the model has multiple bubble steady states, which makes the growth effect of bubbles ambiguous.²⁶

The effect of bubbles is different from those in models, where the source of economic growth is endogenous technological progress or human capital accumulation alone. The reason for the different result is the following. If the source of economic growth is endogenous technological progress and the perpetual growth equilibrium is a BGE, then the rate of return on physical capital is constant and the balanced growth rate of output is increasing in physical capital (Grossman and Yanagawa 1993). If the source of economic growth is human capital accumulation, then the rate of return on physical capital is decreasing and the growth rate of output is increasing in physical capital (Azariadis and Drazen 1990, Michel 1992). Hence, dynamically inefficient steady states cannot have a lower growth rate of output than bubble equilibria in these models, which implies that bubbles cannot increase economic growth. If the source of economic growth is endogenous technological progress and the perpetual growth equilibrium is a non-BGE, then the rate of return on physical capital can be increasing in physical capital. However, because perpetual growth equilibria are unstable equilibrium trajectories, they are eliminated by bubble steady states (Azariadis and Reichlin 1996). If the source of

²⁶ For the analysis of the model with general forms of production functions, see Appendix 1.

economic growth is human capital accumulation and technological progress together, then the rate of return on physical capital can be increasing or the balanced growth rate of output decreasing in physical capital, which implies that bubbles can also increase economic growth.

Government debt, fiat money and pay-as-you-go social security are examples of government policies, which work in the economy in a similar way to bubbles, i.e., transferring resources from children to parents. The literature on government budget policy offers examples, where debt, money or pay-as-you-go social security can increase economic growth. However, the reason for the positive growth effect of government policy in these models differs from the reason for the positive growth effect of bubbles in our model. A positive effect of government debt on economic growth is derived by Zhang (1997), Forslid (1998), Lin (2000) and Zhang (2003). Forslid (1998) and Lin (2000) show that government debt can increase economic growth if debt is used to finance income transfers or productive expenditures. Zhang (1997) and Zhang (2003) show that government debt can increase economic growth by reducing fertility. A positive effect of fiat money on economic growth is derived by Ferreira (1999) and Futagami and Shibata (2000). They show that the growth rate of the economy depends positively on the supply rate of money. A positive effect of pay-as-you-go social security on economic growth is derived by Zhang (1995,2001), Sinn (1998), Kemnitz and Wigger (2000) and Sanchez-Losada (2000). They show that pay-as-you-go social security can increase economic growth by reducing fertility, by eliminating the moral hazard problem between children and parents or by eliminating inefficiency due to intergenerational externalities or joy-of-giving altruism. Hence, none of these results is based on the elimination of dynamic inefficiency.

4. TWO-SIDED ALTRUISM

In this chapter we consider the redistribution of resources from children to parents through altruistic intergenerational transfers. We add to the model two-sided symmetric altruism and assume that agents take the actions of other generations as given. We show that gifts from children to parents can eliminate some part of the dynamic inefficiency due to OLG-structure of the model as in Kimball (1987).

OLG models with altruism (OLGA models) are different from OLG models without altruism, because generations are linked to each other by altruistic intergenerational transfers. Two-sided altruism implies that the model can have transfers from parents to children (bequests, altruistic education investments) as well as from children to parents (gifts). The advantage of the two-sided altruism is that we analyse bequest and gift motives in the same model instead of two different models with one-sided altruism. Symmetric two-sided altruism means that bequest, gift and altruistic education motives are treated similarly in the utility maximization. Altruism can be also asymmetric, which would change some implications of the model. There is, however, no a priori reason why bequest and altruistic education motives should be treated differently from the gift motive besides mathematical problems in solving the double recursion (Kimball 1987).

Usually in the OLGA models, it is assumed that households take the actions of other generations as given (Abel 1987). O'Connell and Zeldes (1992) consider a possibility that agents take into account the actions of other generations. They show that this type of strategic interaction does not affect the bequest motive, but it can increase the gift motive, which would change some implications of the model. However, O'Connell and Zeldes ignore the effect of the non-negativity constraint for gifts on the agents' strategic behavior. If we add the non-negativity constraint to the agents' reaction functions, the effect of the strategic interactions on the gift motive is eliminated in many cases (Lagerlöf 1997).

Barro (1974) and Carmichael (1982) investigate an exogenous growth OLG model with one-sided altruism and show that as long as intergenerational transfers are positive, i.e., intergenerational transfer motive is operative, government debt is neutral. Moreover, Carmichael (1982) shows that a competitive equilibrium of the economy with an operative bequest motive is dynamically efficient and a competitive equilibrium of the economy with an operative gift motive is dynamically inefficient. Government debt does not eliminate gifts or dynamic inefficiency in the Carmichael's model, because lump sum transfers make government debt a perfect substitute for intergenerational transfers.

Because the debt neutrality result depends on an operative intergenerational transfer motive, it is important to determine when the altruistic transfer motives are operative. Weil (1987b) derives explicit conditions under which the bequest motive is operative. He shows that the bequest motive is inoperative in the exogenous growth OLG model with one-sided altruism if the agents' altruism is low, i.e., the intergenerational discount rates are small. Weil's result is extended to the two-sided asymmetric altruism by Abel (1987) and to the two-sided symmetric altruism by Kimball (1987). They show that gift and bequest motives form upper and lower bounds for inoperative transfer motive equilibria. Moreover, Kimball (1987) shows that gift and bequest motives cannot be operative in the same equilibrium if altruism is symmetric. General conditions for the existence and co-existence of operative and inoperative transfer motive equilibria in the exogenous growth OLG model with one-sided altruism are derived by Thibault (2000).

We show that similar results can be obtained for the endogenous growth OLG model with two-sided symmetric altruism, where agents take the actions of other generations as given. In particular, we show that a necessary condition for an operative gift motive is that the economy without altruism is dynamically inefficient such that there is a case for reallocation of resources from children to parents. On the other hand, a necessary condition for an operative bequest or altruistic education motive is that the economy without altruism is dynamically efficient such that there is a case for reallocation of resources from parents to children. Moreover, we show that altruistic education investments are a perfect substitute for bequests if the young agents do not face a borrowing constraint, while gifts are an imperfect substitute for bubbles and bubbles eliminate gifts.

4.1 The model

The model is the same as in Chapters 2 and 3 except that agents are altruistic. It follows that periodic budget constraints (1a) and (1b) are replaced by the following constraints

$$(35a) \quad c_{1t} + s_t + R_t e_{0t-1} + n e_{1t} + j_t = h_t w_t + q_t$$

$$(35b) \quad c_{2t+1} + n q_{t+1} = R_{t+1} s_t + n j_{t+1}$$

$$(35c) \quad j_t \geq 0$$

$$(35d) \quad q_{t+1} \geq 0$$

where q_t is the bequest given by generation $t-1$ to generation t , j_t is the gift given by generation t to generation $t-1$, e_{0t-1} is the investment in own education and e_{1t} is the investment in children's education.

Non-negativity constraints (35c) and (35d) prevent negative bequests and gifts. These constraints play a crucial role in the analysis of altruistic intergenerational transfers. However, they are well-founded, because legal restrictions usually forbid parents to take away resources from their children and vice versa.

Periodic budget constraints (1a) and (1b) can be combined into a single lifetime budget constraint:

$$(35e) \quad c_{1t} + R_t e_{0t-1} + n e_{1t} + j_t + (c_{2t+1} + n q_{t+1}) / R_{t+1} = h_t w_t + q_t + n j_{t+1} / R_{t+1}$$

Households of generation t have the following utility function

$$(36) \quad V_t = U(c_{1t}, c_{2t+1}) + \Psi V_{t-1} + B V_{t+1} \quad \Psi > 0, B > 0, \Psi + B < 1$$

where Ψ and B are the weights on ancestors' and descendants' utility.

Constraints $\Psi > 0$ and $B > 0$ imply that altruism cannot cause disutility. When $\Psi = B = 0$, the economy simplifies to the OLG model without altruism. Constraint $\Psi + B < 1$ implies that the sum of households' weights on the ancestors' and descendants' utility must be lower than the weight on own utility, which is needed for the finiteness of V_t .²⁷

Kimball (1987) has shown that utility function (36) can be represented by the following utility function

$$(37) \quad V_t = U(c_{1t}, c_{2t+1}) + \sum_{i=1}^{\infty} \Psi^i [U(c_{1t-i}, c_{2t+1-i})] + \sum_{i=1}^{\infty} \beta^i [U(c_{1t+i}, c_{2t+1+i})] \quad 0 < \Psi < 1, 0 < \beta < 1$$

²⁷ In the steady state it must be true that $U = (1 - \Psi - B)V$, which can be true for positive U and V only if $\Psi + B < 1$.

where ψ and β are the intergenerational degrees of altruism and constraints $0 < \psi < 1$ and $0 < \beta < 1$ are needed for the finiteness of V_t .²⁸

Equation (37) implies that two-sided altruism is symmetric. This property has some important implications for the model. In particular, it sets restrictions on the degrees of altruism and thereby it affects the operative transfer motive equilibria. An alternative formulation of two-sided altruism is the Buiter-Carmichael-Burbridge utility function $V_t = U(c_{1t}, c_{2t+1}) + \psi[U(c_{1t+i}, c_{2t+1+i})] + \sum_{i=1}^{\infty} \beta^i [U(c_{1t+i}, c_{2t+1+i})]$, where the altruism is asymmetric. Both utility functions lead to the same first-order conditions, but in the Buiter-Carmichael-Burbridge utility function we do not face restriction $\psi < 1$ due to the finiteness of V_t (Abel 1987).

Perfect foresight implies that households of generation t must foresee future prices w_{t+i} , R_{t+i} and R_{t+i+i} for all $i \geq 0$. Hence, the assumption of perfect foresight is stronger than in the model without altruism. Moreover, households also form expectations about the actions of their parents and children. We assume that households take the actions of other generations as given as in Abel (1987). A competitive equilibrium of the economy with altruism where households take the actions of other generations as given corresponds to a Nash equilibrium. An alternative to the Nash approach is the Stackelberg approach, where households take into account the actions of other generations (O'Connell and Zeldes 1993). These approaches tend to imply similar results in the case of bequest motive, but they can lead to a different result in the case of gift motive. In particular, if households behave as Stackelberg leaders, they realize that children respond to higher savings by reducing gifts. In the steady state this causes a similar effect on the economy as an increase of the degree of altruism ψ . Whether this effect is strong enough to overcome the effects of restriction $\psi < 1$ depends on the form of the utility function and the definition of the game.

Households choose the optimal amounts of consumption, education investments and intergenerational transfers to maximize utility function (37) subject to budget constraints (35c-e) and human capital production function (5). We can ignore the inequality constraints (35c) and (35d) from the maximization problem by studying the Kuhn-Tucker conditions. Substituting (35e) and (5) into the utility function (37) simplifies the households' t utility maximization problem to the following unconstrained problem:

$$(38) \quad \max_{c_{1t}, e_{0t-1}, e_{1t}, q_{t+1}, j_t} u(c_{1t}) + \rho u[R_{t+1}(G(\bar{h}_{t-1}, e_{t-1}, \bar{k}_t)w_t + q_t - c_{1t} - R_t e_{0t-1} - n e_{1t} - j_t) + n j_{t+1} - n q_{t+1}] +$$

$$\sum_{i=1}^{\infty} \psi^i \{u(c_{1t+i}) + \rho u[R_{t+1+i}(G(\bar{h}_{t-1+i}, e_{t-1+i}, \bar{k}_{t+i})w_{t+i} + q_{t+i} - c_{1t+i} - R_{t+i} e_{0t-1+i} - n e_{1t+i} - j_{t+i}) + n j_{t+1+i} - n q_{t+1+i}]\} +$$

²⁸ The original result was discovered by Kimball (1987). Our presentation follows Bergstrom (1999). He shows that utility function (36) can be represented in the form of utility function (37), where $\psi = [1 - (1 - 4\Psi B)^{1/2}] / 2B$ and $\beta = [1 - (1 - 4\Psi B)^{1/2}] / 2\Psi$. From $\Psi > 0, B > 0$ and $\Psi + B < 1$ it follows that $\Psi B < B(1 - B) \Rightarrow (1 - 2B)^2 = 1 - 4B(1 - B) < 1 - 4\Psi B \Rightarrow 1 - 2B < (1 - 4\Psi B)^{1/2} \Rightarrow \psi = [1 - (1 - 4\Psi B)^{1/2}] / 2B < 1 \Rightarrow 0 < \psi < 1$ and $\Psi B < \Psi(1 - \Psi) \Rightarrow (1 - 2\Psi)^2 = 1 - 4\Psi(1 - \Psi) < 1 - 4\Psi B \Rightarrow 1 - 2\Psi < (1 - 4\Psi B)^{1/2} \Rightarrow \beta = [1 - (1 - 4\Psi B)^{1/2}] / 2\Psi < 1 \Rightarrow 0 < \beta < 1$.

$$\Sigma_{i=1}^{\infty} \beta^i \{ u(c_{1t+i}) + \rho u[R_{t+1+i}(G(\bar{h}_{t-1+i}, \bar{e}_{t-1+i}, \bar{k}_{t+i})w_{t+i} + q_{t+i} - c_{1t+i} - R_{t+i}e_{0t-1+i} - ne_{1t+i} - j_{t+i}) + n_{j_{t+1+i}} - nq_{t+1+i}] \}$$

where $e_t = e_{0t} + e_{1t}$.

The first-order conditions are:

$$(39a) \quad u'(c_{1t}) = R_{t+1} \rho u'(c_{2t+1})$$

$$(39b) \quad R_t \geq G_2(\bar{h}_{t-1}, \bar{e}_{t-1}, \bar{k}_t) w_t \quad (= \text{if } e_{0t-1} > 0)$$

$$(39c) \quad n R_{t+1} u'(c_{2t+1}) \geq \beta G_2(\bar{h}_t, \bar{e}_t, \bar{k}_{t+1}) w_{t+1} R_{t+2} u'(c_{2t+2}) \quad (= \text{if } e_{1t} > 0)$$

$$(39d) \quad n u'(c_{2t+1}) \geq \beta R_{t+2} u'(c_{2t+2}) \quad (= \text{if } q_{t+1} > 0)$$

$$(39e) \quad R_{t+1} u'(c_{2t+1}) \geq \psi n u'(c_{2t}) \quad (= \text{if } j_t > 0)$$

where $c_{1t}, c_{2t+1}, e_t > 0$ by Inada-conditions and (39a) is an equality by the Kuhn-Tucker conditions.²⁹

The transversality conditions are:

$$(39f) \quad \lim_{t \rightarrow \infty} \beta^t u'(c_{1t}) q_t = 0$$

$$(39g) \quad \lim_{t \rightarrow \infty} \beta^t u'(c_{1t}) e_{1t} = 0$$

$$(39h) \quad \lim_{t \rightarrow \infty} (1/\psi)^t u'(c_{1t}) j_t = 0$$

Conditions (39) are sufficient for the maximum, because objective function (38) is strictly concave in $c_{1t}, e_{0t-1}, e_{1t}, q_{t+1}$ and j_t by (5) and (6). Because transversality conditions (39f-h) are satisfied in the balanced growth equilibrium by a log-linear utility function and $0 < \psi < 1$ and $0 < \beta < 1$, they do not play any role in the analysis of the model and we can ignore them.³⁰

Conditions (39) implies

$$(40a) \quad R_{t+1} \geq G_2(\bar{h}_t, \bar{e}_t, \bar{k}_{t+1}) w_{t+1} \quad (= \text{if } e_{0t} > 0)$$

²⁹ Kuhn-Tucker conditions imply $\{ \partial u / \partial x \leq 0, (\partial u / \partial x)_x = 0 \text{ and } x \geq 0 \} \Leftrightarrow \partial u / \partial x \leq 0 \text{ (if } x > 0) \Rightarrow \partial u / \partial x = 0 \text{ if } x > 0$.

³⁰ Transversality conditions ensure that the objective function is finite for all feasible allocations. For details, see De La Croix and Michel 2002, 245). With a log-linear utility function transversality conditions imply that $\lim_{t \rightarrow \infty} \beta^t u'(c_{1t}) q_t = \lim_{t \rightarrow \infty} \beta^t q_t / c_{1t} = 0$, $\lim_{t \rightarrow \infty} \beta^t u'(c_{1t}) e_{1t} = \lim_{t \rightarrow \infty} \beta^t e_{1t} / c_{1t} = 0$ and $\lim_{t \rightarrow \infty} (1/\psi)^t u'(c_{1t}) j_t = \lim_{t \rightarrow \infty} (1/\psi)^t j_t / c_{1t} = 0$. These conditions are always satisfied in the steady state by $0 < \beta < 1$ and $0 < \psi < 1$.

$$(40b) \quad nu'(c_{1t})/u'(c_{1t+1}) \geq \beta G_2(\bar{h}_t, e_t, \bar{k}_{t+1}) w_{t+1} \quad (=if \ e_{1t} > 0)$$

$$(40c) \quad nu'(c_{1t})/u'(c_{1t+1}) \geq \beta R_{t+1} \quad (=if \ q_{t+1} > 0)$$

$$(40d) \quad nu'(c_{1t})/u'(c_{1t+1}) \leq (1/\psi) R_{t+1} \quad (=if \ j_{t+1} > 0)$$

where (40a) is obtained by writing (39b) for $t+1$, (40b) and (40c) by substituting (39a) into (39c) and (39d), and (40d) by writing (39e) for $t+1$ and substituting (39a) into it.

Because $e_t = e_{0t} + e_{1t} > 0$, conditions (40a-c) can be satisfied only if (40a) is an equality. Moreover, if (40a) is an equality, then conditions (40b) and (40c) are equal. Hence, we can rewrite conditions (40) in the following form:

$$(41a) \quad R_{t+1} = G_2(\bar{h}_t, e_t, \bar{k}_{t+1}) w_{t+1}$$

$$(41b) \quad \psi nu'(c_{1t})/u'(c_{1t+1}) \leq R_{t+1} \leq nu'(c_{1t})/\beta u'(c_{1t+1}) \quad (=if \ j_{t+1} > 0, =if \ q_{t+1} + e_{1t} > 0)$$

Equation (41a) is equal to the households' first-order condition (8b) in the model without altruism. Condition (41b) determines when altruistic transfers j_{t+1} and $q_{t+1} + e_{1t}$ are positive and when the non-negativity constraints for altruistic transfers are binding. If altruistic transfers are positive, condition (41b) is equal to the Euler equation. Moreover, condition (41b) implies that altruistic education investments and bequests are equivalent ways to transfer resources across generations, i.e., altruistic education investments are a perfect substitute for bequests.

If young agents do not have access to capital markets, then $e_{0t} = 0$ and all education investments are financed by parents. OLGA models, where young agents face a borrowing constraint and cannot borrow against their future income, are studied by Drazen (1978), Caballe (1995) and Rangazas (1996) among others.³¹ In these models, education investments depend on altruism, but altruistic education investments are not a perfect substitute for bequests. In particular, if $e_{0t} = 0$, then $e_{1t} > 0$ by Inada conditions and equation (41a) only holds when $q_{t+1} > 0$. If $j_{t+1} > 0$, then equation (41a) is replaced by $R_{t+1} = \beta \psi G_2 w_{t+1}$, which implies that $R_{t+1} < G_2 w_{t+1}$. If $b_t > 0$, then equation (41a) is replaced by $R_{t+1} = \beta G_2 w_{t+1}$, which implies that $R_{t+1} < G_2 w_{t+1}$. If $q_{t+1} = j_{t+1} = b_t = 0$, then it must be true that $R_{t+1} < G_2 w_{t+1}$. Hence, if $q_{t+1} = 0$, then a borrowing constraint causes underinvestments in education and overinvestments in physical capital compared to the model without a borrowing constraint. If $q_{t+1} > 0$, then the economy with a borrowing constraint is equal to the economy without a borrowing constraint, i.e., an operative bequest motive (or a reallocation of resources from parents to children) eliminates the effect of the borrowing constraint to the economy.³²

³¹ Boldrin and Montes (2002) study the effects of the borrowing constraint in the model without altruism.

³² Besides the fact that the borrowing constraint causes underinvestments in education, it changes some implications of the model. In particular, if the economy has the borrowing constraint and $q_{t+1} = j_{t+1} = b_t = 0$, then the balanced growth rate of output is defined by (40b) instead of (41a). This implies that $ne_t = \beta \delta w_t$ and $\gamma_t = [\beta \delta (1-\alpha)/n]^\delta k_t^{\delta(\alpha+n)} k_{t+1}^\mu$.

By using the explicit forms of the production and utility functions, we can solve savings and consumption from (39a) as a linear function of the adults' and old agents' incomes, and education investments from (41a) as a linear function of discounted wage income. Substituting (35a),(35b) and (6) into (39a) and (5) into (41a) implies:

$$(42a) \quad s_t = [\rho/(1+\rho)][(1-\delta)h_t w_t + q_t + R_t e_{1,t-1} - n e_{1,t} j_t] - [1/(1+\rho)]n(j_{t+1} - q_{t+1})/R_{t+1}$$

$$(42b) \quad c_{1t} = [1/(1+\rho)][(1-\delta)h_t w_t + q_t + R_t e_{1,t-1} - n e_{1,t} j_t + n(j_{t+1} - q_{t+1})/R_{t+1}]$$

$$(42c) \quad e_{t-1} = \delta h_t w_t / R_t$$

Equations (42a) and (42b) are the counterparts of the savings and consumption functions (9a) and (9b) in the economy without altruism. They are not ordinary decision functions like (9a) and (9b), because endogenous variables $e_{0,t-1}$, e_{1t} , j_t and q_{t+1} are not eliminated from them.³³ Equation (42c) defines education function, which is equal to the education function in the economy without altruism (9c).

Asset and goods market clearing conditions are:

$$(43a) \quad N_t s_t = K_{t+1} + B_t + N_{t+1} e_{0t}$$

$$(43b) \quad F(K_t, H_t, \bar{k}_t) = N_t c_{1t} + N_{t-1} c_{2t} + N_{t+1} e_t + K_{t+1}$$

The equilibrium of the economy is defined as:

DEFINITION 5: *A competitive equilibrium of the economy with altruism and intrinsically useless assets is a sequence $\{b_t, c_{1t}, c_{2t+1}, e_{0t}, e_{1t}, j_t, h_t, k_{t+1}, q_{t+1}\}_{t=0}^{\infty}$ such that*

- (i) c_{1t} , c_{2t+1} , e_{0t-1} , e_{1t} , j_t and q_{t+1} maximize utility (37) subject to budget constraints (35c-e) and human capital production function (5) under given factor and asset prices and external effects
- (ii) factors are paid their marginal products (4)
- (iii) budget constraints (35) and market clearing conditions (43) are satisfied
- (iv) $k_0 > 0$, $h_0 > 0$, $q_0 \geq 0$, $j_0 \geq 0$
- (v) $\bar{k}_t = k_t$ and $\bar{h}_t = h_t$
- (vi) non-arbitrage condition (29) is satisfied

The definition of the competitive equilibrium of the economy with altruism and intrinsically useless assets is similar to the definition of the competitive equilibrium of the economy without altruism and intrinsically useless assets except that the economy has

Hence, if $q_{t+1} = j_{t+1} = b_t = 0$, then the borrowing constraint breaks the link between the returns to scale in human capital production ($\delta + \mu$) and the sign of $\partial \gamma / \partial k$ implied by (41a).

³³ Ordinary decision functions for j_t and q_{t+1} would be quite complicated and they are not usually solved explicitly in the OLG models. See for example Abel (1987) and Thibault (2000).

four additional decision variables b_t , e_{1t} , j_t and q_{t+1} , which satisfy initial conditions $q_0 \geq 0$ and $j_0 \geq 0$ and non-arbitrage condition (28). Moreover, the economy satisfies budget constraints (35) and market clearing conditions (43) instead of budget constraints (1) and market clearing conditions (10) and (11).

Equations (4), (5), (29), (42) and (43) define the competitive equilibrium of the economy with altruism and intrinsically useless assets:

$$(44a) \quad n\gamma_t k_{t+1} + ne_t + b_t = [\rho/(1+\rho)][(1-\delta)w_t + q_t + R_t e_{1t-1}/\gamma_{t-1} - j_t] - [1/(1+\rho)][n\gamma_t(j_{t+1} - q_{t+1})/R_{t+1} - ne_{1t}]$$

$$(44b) \quad c_{1t} = [1/(1+\rho)][(1-\delta)w_t + q_t + R_t e_{1t-1}/\gamma_{t-1} - ne_{1t} - j_t + n\gamma_t(j_{t+1} - q_{t+1})/R_{t+1}]$$

$$(44c) \quad \psi n\gamma_t c_{1t+1}/c_{1t} \leq R_{t+1} \leq (1/\beta)n\gamma_t c_{1t+1}/c_{1t} \quad (= \text{if } j_{t+1} > 0, = \text{if } q_{t+1} + e_{1t} > 0)$$

$$(44d) \quad n\gamma_t b_{t+1}/b_t \leq R_{t+1} \quad (= \text{if } b_t > 0)$$

$$(44e) \quad k_t^{\alpha+\eta} = c_{1t} + c_{2t}/n + ne_t + n\gamma_t k_{t+1}$$

$$(44f) \quad e_t = [\delta(1-\alpha)/\alpha]^{1/(1-\delta)} k_{t+1}^{(1-\mu)/(1-\delta)} \equiv e(k_{t+1})$$

$$(44g) \quad \gamma_t = [\delta(1-\alpha)/\alpha]^{\delta/(1-\delta)} k_{t+1}^{(\delta-\mu)/(1-\delta)} \equiv \gamma(k_{t+1})$$

$$(44h) \quad w_t = (1-\alpha)k_t^{\alpha+\eta} \equiv w(k_t)$$

$$(44i) \quad R_t = \alpha k_t^{\alpha+\eta-1} \equiv R(k_t)$$

where $j_t = j_t/h_t$ is the gift per effective unit of labor, $q_t = q_t/h_t$ is the bequest per effective unit of labor and (44f-i) define e_t , γ_t , w_{t+1} and R_{t+1} as a function of k_{t+1} .

System (44) can have four types of equilibria. If $j_{t+1} > 0$ or $q_{t+1} + e_{1t} > 0$, they are called operative transfer motive equilibria, and if $j_{t+1} = 0$, $q_{t+1} + e_{1t} = 0$, they are called inoperative transfer motive equilibria. If $b_t > 0$, they called bubble equilibria, and if $b_t = 0$, they are called bubbleless equilibria.

4.2 Steady states and transitional dynamics without intrinsically useless assets

In this section we consider the existence of operative transfer motive steady states and transitional dynamics in the economy without intrinsically useless assets. Abel (1987) and Weil (1987b) show that OLG models have an operative transfer motive steady state if the intergenerational degrees of altruism are sufficiently high, and operative transfer motive steady states form lower and upper bounds for the inoperative transfer motive steady states. Moreover, Kimball (1987) shows that bequest and gift motives cannot be operative in the same steady state if altruism is symmetric and agents take the actions of other generations as given. Furthermore, Thibault (2000) shows that operative and

inoperative transfer motive steady states cannot co-exist if the non-trivial steady state equilibrium without altruism is unique and determinate. We show that these results also hold in our model.

Nourry and Venditti (2001) show that exogenous growth OLGA models tend to have a unique and globally saddle-path stable operative transfer motive steady state. We show that this result also holds in our model if the effects of technological progress are weak. If the productivity effect of technological progress is sufficiently strong, it can make the operative transfer motive steady state globally unstable and cause oscillations around the steady state. A similar result holds in the endogenous growth OLGA models, where the source of growth is technological progress due to learning-by-doing externalities alone (Venditti 2003). However, interpretation of the unstable steady state in Venditti's model is different from that in our model, because steady states do not sustain perpetual growth in the models. If the erosion effect of technological progress is sufficiently strong, it can also make the operative transfer motive steady state globally unstable.

Substituting (44f-i) into (44a), (44c) and (44e) and ignoring b_t simplifies system (44) to the following system:

$$(45a) \quad n[1+\delta(1-\alpha)/\alpha]\gamma(k_{t+1})k_{t+1}=[\rho/(1+\rho)][(1-\delta)w(k_t)+q_t+R(k_t)e_{1t-1}/\gamma(k_t)-j_t]- \\ [1/(1+\rho)][n\gamma(k_{t+1})(j_{t+1}-q_{t+1})/R(k_{t+1})-ne_{1t}]$$

$$(45b) \quad \psi n\gamma(k_{t+1})c_{1t+1}/c_{1t} \leq R(k_{t+1}) \leq (1/\beta)n\gamma(k_{t+1})c_{1t+1}/c_{1t} \quad (= \text{if } j_{t+1} > 0, = \text{if } q_{t+1}+e_{1t} > 0)$$

$$(45c) \quad k_t^{\alpha+\eta} = c_{1t} + \rho R(k_t)c_{1t-1}/n\gamma(k_t) + n[1+\delta(1-\alpha)/\alpha]\gamma(k_{t+1})k_{t+1}$$

where $c_{2t}/n = \rho R(k_t)c_{1t-1}/n\gamma(k_t)$ by (39a).

If $j_{t+1}=q_{t+1}=e_{1t}=0$, system (45) is equal to the scalar system (13) in the economy without altruism. If $j_{t+1}>0$ or $q_{t+1}+e_{1t}>0$, system (45) simplifies to the following planar system:

$$(46a) \quad n[1+\delta(1-\alpha)/\alpha]\gamma(k_{t+1})k_{t+1} = f(k_t) - (1+\rho/\kappa)c_{1t}$$

$$(46b) \quad n\gamma(k_{t+1})c_{1t+1}/c_{1t} = \kappa R(k_{t+1})$$

where $\kappa = \beta$ if $q_{t+1}+e_{1t} > 0$ and $\kappa = 1/\psi$ if $j_{t+1} > 0$.

Equations (46a) and (46b) define the following mappings in the forward dynamics:

$$(47a) \quad k_{t+1} = \Omega[f(k_t) - (1+\rho/\kappa)c_{1t}]^{(1-\delta)/(\delta+\mu)} \equiv \chi^1(k_t, c_{1t})$$

$$(47b) \quad c_{1t+1} = \kappa R[\chi^1(k_t, c_{1t})]c_{1t}/n\gamma[\chi^1(k_t, c_{1t})] \equiv \chi^2(k_t, c_{1t})$$

where $\Omega = \{n[\delta(1-\alpha)/\alpha]^{\delta/(1-\delta)}[1+\delta(1-\alpha)/\alpha]\}^{-(1-\delta)/(1-\mu)} > 0$ by $0 < \delta < 1$, $\rho > 0$ and $0 < \alpha < 1$.

From (47) it follows that

$$(48a) \quad \chi^1_1 = [(\alpha + \eta)(1 - \delta)/(1 - \mu)](k_{t+1}/k_t)f(k_t)/[f(k_t) - (1 + \rho/\kappa)c_{1t}]$$

$$(48b) \quad \chi^1_2 = -[(1 - \delta)/(1 - \mu)](1 + \rho/\kappa)k_{t+1}/[f(k_t) - (1 + \rho/\kappa)c_{1t}]$$

$$(48c) \quad \chi^2_1 = (R'k_{t+1}/R_{t+1} - \gamma k_t/\gamma_t)(c_{1t+1}/k_{t+1})\chi^1_1 = [\alpha + \eta - (1 - \mu)/(1 - \delta)](c_{1t+1}/k_{t+1})\chi^1_1$$

$$(48d) \quad \chi^2_2 = \kappa R_{t+1}/n\gamma_t + (R'k_{t+1}/R_{t+1} - \gamma k_t/\gamma_t)(c_{1t+1}/k_{t+1})\chi^1_2 =$$

$$\kappa R_{t+1}/n\gamma_t + [\alpha + \eta - (1 - \mu)/(1 - \delta)](c_{1t+1}/k_{t+1})\chi^1_2$$

where $\chi^1_1 > 0$ and $\chi^1_2 < 0$ by $0 < \mu < 1$, $0 < \delta < 1$, $\rho > 0$, $0 < \alpha < 1$ and $\eta > 0$.

Let us denote a non-trivial steady state of the economy without altruism and intrinsically useless assets by k^D , i.e., $\phi(k^D) = k^D > 0$. By using (47) and (48) we can show:

PROPOSITION 5: (i) *The economy with altruism and without intrinsically useless assets has a trivial inoperative transfer motive steady state $k = 0$. Moreover, if $(\alpha + \eta)(1 - \delta)/(1 - \mu) \neq 1$ and $\beta > n\gamma(k^D)/R(k^D)$ or $1/\psi < n\gamma(k^D)/R(k^D)$, then the economy has a unique operative transfer motive steady state. If $(\alpha + \eta)(1 - \delta)/(1 - \mu) \neq 1$ and $\beta \leq n\gamma(k^D)/R(k^D) \leq 1/\psi$, then the economy has a unique inoperative transfer motive steady state.*

(ii) *If $(\alpha + \eta)(1 - \delta)/(1 - \mu) < 1$, then the operative transfer motive steady state is globally saddle-path stable. Moreover, the non-trivial inoperative transfer motive steady state is globally stable.*

(iii) *If $(\alpha + \eta)(1 - \delta)/(1 - \mu) > 1$, then the operative transfer motive steady state is globally saddle-path stable, or it is unstable and it can have oscillations around it. Moreover, the non-trivial inoperative transfer motive steady state is globally unstable.*

PROOF: Notice first that it is not possible that $e_1 + q > 0$ and $j > 0$ in the same steady state equilibrium by (45b), $0 < \beta < 1$ and $0 < \psi < 1$. Hence, gifts and bequests or altruistic education motives cannot be operative in the same steady state.

(i) System of difference equations (47) has a trivial solution $q = j = k = c_1 = 0$, because $(\alpha + \eta)(1 - \delta)/(1 - \mu) > 0$. Moreover, it has an operative bequest or altruistic education motive steady state if $q + ne_1/\beta = [(1 + \rho)/(\rho + \beta)] [n(1 + \delta(1 - \alpha)/\alpha)\gamma(k)k - (\rho/(1 + \rho))(1 - \delta)w(k)] > 0$ and $n\gamma(k) = \beta R(k)$. The former equation is a function in the $(k, q + ne_1/\beta)$ space, which satisfies $q > 0$ if $\phi(k) < k$ by (13). The latter equation is a vertical line in the $(k, q + ne_1/\beta)$ space. It follows that the economy has an operative bequest or altruistic education motive steady state if $n\gamma = \beta R$ for some $\phi(k) < k$. Furthermore, it has an operative gift motive steady state if $j = [(1 + \rho)/(\rho + 1/\psi)] [(p/(1 + \rho))(1 - \delta)w(k) - n(1 + \delta(1 - \alpha)/\alpha)\gamma(k)k] > 0$ and $n\gamma(k) = (1/\psi)R(k)$. The former equation is a function in the (k, j) space, which satisfies $j > 0$ if $\phi(k) > k$ by (13). The latter equation is a vertical line in the (k, j) space. It follows that the economy has an

operative gift motive steady state if $n\gamma=(1/\psi)R$ for some $\phi(k)>k$. On the other hand, the economy has a unique inoperative transfer motive steady state by Proposition 1 if $\beta R \leq n\gamma \leq (1/\psi)R$ for some $\phi(k)=k$.

If $\phi'(k^D) < 1$, i.e., $(\alpha+\eta)(1-\delta)/(1-\mu) < 1$, then $\phi(k) > k$ for $k < k^D$ and $\phi(k) < k$ for $k > k^D$. Moreover, then $R'k/R(k) < n\gamma k/n\gamma(k)$ by (48c), i.e., curve $n\gamma(k)$ crosses curve $R(k)$ from below and these curves have a unique strictly positive crossing point. Hence, if $n\gamma(k^D) > (1/\psi)R(k^D)$, then there exists a unique $k > 0$ such that $n\gamma=R$ and $\phi(k) > k$, but we cannot find $k > 0$ such that $n\gamma=\beta R$ and $\phi(k) < k$ or $\beta R \leq n\gamma \leq (1/\psi)R$ and $\phi(k)=k$. If $n\gamma(k^D) < \beta R(k^D)$, then there exists a unique $k > 0$ such that $n\gamma=\beta R$ and $\phi(k) < k$, but we cannot find $k > 0$ such that $n\gamma=(1/\psi)R$ and $\phi(k) > k$ or $\beta R \leq n\gamma \leq (1/\psi)R$ and $\phi(k)=k$. If $\beta R(k^D) \leq n\gamma(k^D) \leq (1/\psi)R(k^D)$, then there exists a unique $k > 0$ such that $\beta R \leq n\gamma \leq (1/\psi)R$ and $\phi(k)=k$, but we cannot find $k > 0$ such that $n\gamma=\beta R$ and $\phi(k) < k$ or $n\gamma=(1/\psi)R$ and $\phi(k) > k$.

If $\phi'(k^D) > 1$, i.e., $(\alpha+\eta)(1-\delta)/(1-\mu) > 1$, then $\phi(k) < k$ for $k < k^D$ and $\phi(k) > k$ for $k > k^D$. Moreover, then $R'k/R(k) < n\gamma k/n\gamma(k)$ by (48c), i.e., curve $n\gamma(k)$ crosses curve $R(k)$ from above and these curves have a unique strictly positive crossing point. Hence, if $n\gamma(k^D) > (1/\psi)R(k^D)$, then there exists a unique $k > 0$ such that $n\gamma=(1/\psi)R$ and $\phi(k) > k$, but we cannot find $k > 0$ such that $n\gamma=\beta R$ and $\phi(k) < k$ or $\beta R \leq n\gamma \leq (1/\psi)R$ and $\phi(k)=k$. If $n\gamma(k^D) < \beta R(k^D)$, then there exists a unique $k > 0$ such that $n\gamma=\beta R$ and $\phi(k) < k$, but we cannot find $k > 0$ such that $n\gamma=(1/\psi)R$ and $\phi(k) > k$ or $\beta R \leq n\gamma \leq (1/\psi)R$ and $\phi(k)=k$. If $\beta R(k^D) \leq n\gamma(k^D) \leq (1/\psi)R(k^D)$, then there exists a unique $k > 0$ such that $\beta R \leq n\gamma \leq (1/\psi)R$ and $\phi(k)=k$, but we cannot find $k > 0$ such that $n\gamma=\beta R$ and $\phi(k) < k$ or $n\gamma=(1/\psi)R$ and $\phi(k) > k$.

(ii-iii) Because different types of non-trivial steady states cannot co-exist by (i), the dynamics of the inoperative transfer motive steady states is defined by Proposition 1 and 3. The dynamics of the operative transfer motive steady state is similar to the dynamics of bubble steady state in Proposition 3, because the qualitative properties of the Jacobian matrix depend in both systems on $R'k/R(k)$ - $n\gamma k/n\gamma(k)$. Q.E.D.

Proposition 5 is similar to Proposition 1 in the economy without altruism and intrinsically useless assets except that the non-trivial steady state has an operative transfer motive if the economy without altruism and intrinsically useless assets has a non-trivial steady state with sufficiently low or high $n\gamma/R$. It follows that different types of steady states cannot co-exist. Hence, Proposition 5 is consistent with Thibault (2000), who shows that different types of steady states cannot co-exist if the non-trivial steady state in the economy without altruism and intrinsically useless assets is unique and determinate. Moreover, the steady state can have an operative transfer motive, but both transfer motives cannot be operative in the same steady state. Hence, Proposition 5 is consistent with Kimball (1987), who shows that bequest and gift motives cannot be operative in the same steady state if altruism is symmetric and agents take the actions of other generations as given.³⁴

³⁴ With asymmetric two-sided altruism we do not face $\psi < 1$ restriction due to the finiteness of V_t (Abel 1987, Wigger 2001). If $\psi \geq 1$, it is possible that $\psi\beta=1$ and $\psi=1$, which implies that both gift and bequest motives can be operative in the same steady state. If agents take into account the actions of other generations, condition (45b) includes an

To clarify Proposition 5, let us consider the existence condition for the operative transfer motive more carefully. From equations (12f), (12g) and (13) it follows that $n\gamma(k^D)/R(k^D) = [(1-\alpha)/\alpha]n\gamma(k^D)k^D/w(k^D) = [\rho/(1+\rho)]/[\alpha/(1-\alpha)(1-\delta)+\delta/(1-\delta)]$. Let us denote $\beta^{\wedge} = [\rho/(1+\rho)]/[\alpha/(1-\alpha)(1-\delta)+\delta/(1-\delta)]$ and $\psi^{\wedge} = [\alpha/(1-\alpha)(1-\delta)+\delta/(1-\delta)]/[\rho/(1+\rho)] = 1/\beta^{\wedge}$, where β^{\wedge} and ψ^{\wedge} can be smaller or higher than unity by $0 < \rho < 1$, $0 < \alpha < 1$ and $0 < \delta < 1$. From Proposition 5 it follows that:

COROLLARY 3: If $\beta > \beta^{\wedge}$, then bequest motive is operative in the non-trivial steady state, and if $\beta \leq \beta^{\wedge}$, then bequest motive is inoperative in the non-trivial steady state. If $\psi > \psi^{\wedge}$, then gift motive is operative in the non-trivial steady state, and if $\psi \leq \psi^{\wedge}$, then gift motive is inoperative in the non-trivial steady state.

Corollary 3 implies the non-trivial steady state has an operative transfer motive if β or ψ is sufficiently high. Hence, the economy has an operative transfer motive if the degrees of altruism are sufficiently high as in Abel (1987) and Weil (1987b). The threshold values of β and ψ depend on the discount factor ρ and the productivity of physical capital α as in the exogenous growth OLGA model with a log-linear utility and C-D production functions (De la Croix and Michel 2002, 253). Moreover, they also depend on the productivity of education δ . However, they do not depend on the productivity of technological progress μ and η . This result is sensible, because μ and η define the effect of production externalities on the model and thereby they do not have a direct influence on intertemporal allocation.

Dynamic properties of the model are analogous to the model with intrinsically useless assets in the previous chapter, because the second state variable (c_{1t}) is a forward looking variable without any initial condition. The stability of the operative transfer motive steady state depends on the degrees of returns to scale in production functions g and f , i.e., on the strength of the productivity and erosion effects of technological progress.

If g and f have decreasing returns to scale, then the operative transfer motive steady state is saddle-path stable, because $\delta - \mu > 0$ and $\alpha + \eta < 1 \Rightarrow (\alpha + \eta - 1)(1 - \delta) < \delta - \mu \Rightarrow (\alpha + \eta)(1 - \delta)/(1 - \mu) < 1$. This result is sensible, because the same is true for the exogenous growth OLGA models with a log-linear utility function and C-D production functions (Nourry and Venditti 2001).

If f has increasing returns to scale ($\alpha + \eta > 1$), then it is possible that $(\alpha + \eta - 1)(1 - \delta) > \delta - \mu \Rightarrow (\alpha + \eta)(1 - \delta)/(1 - \mu) > 1$. In this case, the operative transfer motive steady state is unstable and it can have oscillations around it. A similar result holds for the endogenous growth OLGA models, where the source of growth is technological progress due to learning-by-doing externalities alone (Venditti 2003). The interpretation of the unstable steady state in Venditti's model is different from that in our model, because the model does not sustain perpetual growth in the steady state.

additional term due to the reaction of children to an increase in parents' saving (O'Connell and Zeldes 1993, Lagerlöf 1997). In this case, it is possible that both gift and bequest motives can be operative in the same steady state.

If g has negative returns to scale ($\delta - \mu < 0$), then it is also possible that $(\alpha + \eta - 1)(1 - \delta) > \delta - \mu \Rightarrow (\alpha + \eta)(1 - \delta)/(1 - \mu) > 1$. Hence, the erosion effect of technological progress is an additional reason for the instability of the non-trivial steady state and oscillations.

To consider the relationship between different types of steady state equilibria, let us denote an operative gift motive steady state by k^j , an operative bequest or altruistic education motive steady state by k^q and a non-trivial inoperative transfer motive steady state by k^0 . By using these notations we can show:

PROPOSITION 6: *In the economy with altruism and without intrinsically useless assets, operative bequest and gift motive steady states form an upper and lower bound for the non-trivial inoperative transfer motive steady state such that $k^q < k^j$ and $k^q \leq k^0 \leq k^j$ if $(\alpha + \eta)(1 - \delta)/(1 - \mu) < 1$, and $k^j < k^q$ and $k^j \leq k^0 \leq k^q$ if $(\alpha + \eta)(1 - \delta)/(1 - \mu) > 1$.*

PROOF: Constraints $0 < \psi < 1$ and $0 < \beta < 1$ together with (45b) imply that $R(k^q) > n\gamma(k^q)$ and $R(k^j) < n\gamma(k^j)$. Moreover, curves $R(k)$ and $n\gamma(k)$ has a unique crossing point $k^{GR} > 0$ by (44g) and (44i). From the uniqueness of $k^{GR} > 0$, it follows that $R < n\gamma$ for $k > k^{GR}$ and $R > n\gamma$ for $k < k^{GR}$ if $R'(k^{GR}) < n\gamma'(k^{GR})$ and $R < n\gamma$ for $k < k^{GR}$ and $R > n\gamma$ for $k > k^{GR}$ if $R'(k^{GR}) > n\gamma'(k^{GR})$. It follows that $k^q < k^j$ if $k^{GR}R'/R < nk^{GR}\gamma'/n\gamma$, i.e., $(\alpha + \eta)(1 - \delta)/(1 - \mu) < 1$. Moreover, it is not possible that $k^0 < k^q$ or $k^0 > k^j$, because $\beta R(k^0) > n\gamma(k^0)$ and $(1/\psi)R(k^0) < n\gamma(k^0)$ are not feasible equilibria by (45b). Hence, if k^0 exists, it must satisfy $k^q \leq k^0 \leq k^j$. On the other hand, $k^q > k^j$ if $k^{GR}R'/R > nk^{GR}\gamma'/n\gamma$, i.e., $(\alpha + \eta)(1 - \delta)/(1 - \mu) > 1$. Moreover, it is not possible that $k^0 > k^q$ or $k^0 < k^j$, because $\beta R(k^0) > n\gamma(k^0)$ and $(1/\psi)R(k^0) < n\gamma(k^0)$ are not feasible equilibria by (45b). Hence, if k^0 exists, it must satisfy $k^j \leq k^0 \leq k^q$. Q.E.D.

Proposition 6 implies that the relationship between k^q and k^j depends on the degree of returns to scale in production functions g and f , i.e., on the strength of the productivity and erosion effects of technological progress. If g and f have decreasing returns to scale ($\delta - \mu > 0$, $\alpha + \eta < 1$), then $(\alpha + \eta)(1 - \delta)/(1 - \mu) < 1 \Leftrightarrow (\alpha + \eta - 1)(1 - \delta) < \delta - \mu$ and $k^j < k^q$. This result is sensible, because the same is true for the exogenous growth OLGA models (Abel 1987). If g has negative returns to scale ($\delta - \mu < 0$) or f has increasing returns to scale ($\alpha + \eta > 1$), then it is possible that $(\alpha + \eta)(1 - \delta)/(1 - \mu) > 1 \Leftrightarrow (\alpha + \eta - 1)(1 - \delta) > \delta - \mu$ and $k^q < k^j$.

Moreover, Proposition 6 implies that operative transfer motive steady states k^q and k^j restrict the feasible values of the non-trivial inoperative transfer motive steady state k^0 such that k^q and k^j form an upper and lower bound for k^0 . The strength of these bounds depends on the intergenerational degrees of altruism such that $\lim_{\beta \rightarrow 0} k^q = 0$ and $\lim_{\psi \rightarrow 0} k^j = \infty$ if $(\alpha + \eta)(1 - \delta)/(1 - \mu) < 1$ and $\lim_{\beta \rightarrow 0} k^j = 0$ and $\lim_{\psi \rightarrow 0} k^q = \infty$ if $(\alpha + \eta)(1 - \delta)/(1 - \mu) > 1$. It follows that the type of the steady state depends on the size of the intergenerational degrees of altruism α and ψ . When α and/or ψ increases, the feasible range of k^0 decreases. If α and/or ψ is sufficiently high, it is possible that k^0 does not exist. Hence, with a low level of altruism, the economy tends to have inoperative transfer motive and with high level of altruism the economy tends to have operative transfer motive.

If we allow more general forms of utility and production functions than (3), (5) and (6), then the threshold values of the degrees of altruism may also depend on model properties other than the productivity of physical capital, education and technological progress. Moreover, the relationship between k^q and k^j does not necessarily depend directly on the degree of returns to scale in production functions g and f . Furthermore, it is possible that the economy has multiple k^{GR} , k^q and k^j . In this case, the relationship between k^q , k^j and k^0 is ambiguous.³⁵

4.3 Steady states and transitional dynamics with intrinsically useless assets

In this section we consider the existence of operative transfer motive steady states and transitional dynamics in the economy with intrinsically useless assets. The analysis is similar to the economy without intrinsically useless assets except that the gift motive is replaced by the non-arbitrage condition of bubbles, because bubbles dominate gifts in the steady state.

Substituting (44f-i) into (44a), (44c-e) simplifies system (44) to the following system:

$$(49a) \quad n[1+\delta(1-\alpha)/\alpha]\gamma(k_{t+1})k_{t+1}=[\rho/(1+\rho)][(1-\delta)w(k_t)+q_t+R(k_t)e_{1t-1}/\gamma(k_t)-j_t]- \\ [1/(1+\rho)][n\gamma(k_{t+1})(j_{t+1}-q_{t+1})/R(k_{t+1})-ne_{1t}]-b_t$$

$$(49b) \quad \psi n\gamma(k_{t+1})c_{1t+1}/c_{1t} \leq R(k_{t+1}) \leq (1/\beta)n\gamma(k_{t+1})c_{1t+1}/c_{1t} \quad (=if \ j_{t+1} > 0, =if \ q_{t+1}+e_{1t} > 0)$$

$$(49c) \quad n\gamma(k_{t+1})b_{t+1}/b_t \leq R(k_{t+1}) \quad (=if \ b_t > 0)$$

$$(49d) \quad k_t^{\alpha+\eta}=c_{1t}+\rho R(k_t)c_{1t-1}/n\gamma(k_t)+n[1+\delta(1-\alpha)/\alpha]\gamma(k_{t+1})k_{t+1}$$

where $c_{2t}/n=\rho R(k_t)c_{1t-1}/n\gamma(k_t)$ by (39a).

If $j_{t+1}=q_{t+1}=e_{1t}=0$, system (49) is equal to the planar system (31) in the economy with intrinsically useless assets and without altruism. If $j_{t+1}>0$ or $q_{t+1}+e_{1t}>0$, system (49) simplifies to the planar system (46).

By using (47) and (48) we can show:

PROPOSITION 7: *(i) The economy with altruism and intrinsically useless assets has a trivial inoperative transfer motive bubbleless steady state $k=0$. Moreover, if $(\alpha+\eta)(1-\delta)/(1-\mu)\neq 1$ and $\beta>n\gamma(k^D)/R(k^D)$, then the economy has a unique operative transfer motive bubbleless steady state. If $(\alpha+\eta)(1-\delta)/(1-\mu)\neq 1$ and $1>n\gamma(k^D)/R(k^D)$, then the economy has a unique inoperative transfer motive bubble steady state. If $(\alpha+\eta)(1-\delta)/(1-\mu)\neq 1$ and*

³⁵ For the analysis of the model with general forms of production functions and a constant elasticity of substitution utility function, see Appendix 1.

$\beta \leq n\gamma(k^D)/R(k^D) \leq 1$, then the economy has a unique inoperative transfer motive bubbleless steady state.

(ii) If $(\alpha+\eta)(1-\delta)/(1-\mu) < 1$, then the operative transfer motive and bubble steady states are globally saddle-path stable. Moreover, the non-trivial inoperative transfer motive bubbleless steady state is globally stable.

(iii) If $(\alpha+\eta)(1-\delta)/(1-\mu) > 1$, then the operative transfer motive and bubble steady states are globally saddle-path stable, or they are unstable and they can have oscillations around them. Moreover, the non-trivial inoperative transfer motive bubbleless steady state is globally unstable.

PROOF: Notice first that it is not possible that $e_1+q>0, j>0$ or $b>0$ in the same steady state by (49b), (49c), $0<\beta<1$ and $0<\psi<1$. Hence, an operative transfer motive steady state does not have bubbles and a bubble steady state does not have an operative transfer motive. Moreover, $j=0$ in the steady state by (49b), (49c) and $0<\psi<1$. Hence, the economy cannot have an operative gift motive in the steady state.

(i-ii) The proof follows as in Proposition 5 by noticing that bubble steady states dominate operative gift motive steady states, but not operative bequest or altruistic education motive steady states. Q.E.D.

Proposition 7 is similar to Proposition 5 in the economy with altruism and without intrinsically useless assets except that gifts are replaced by bubbles. The economy may have an operative bequest motive in the non-trivial steady state or the non-trivial steady state may be a bubble steady state, but the gift motive is never operative in the non-trivial steady state. This result follows, because $0<\beta<1$ and $0<\psi<1$ and the non-trivial steady state is a bubble steady state if $\beta^>1$ by Proposition 3. Hence, altruism does not eliminate bubbles, but bubbles eliminate gifts.³⁶

To consider the relationship between different types of steady state equilibria, let us denote a bubble steady state by k^b , an operative bequest or altruistic education motive steady state by k^a and a non-trivial inoperative transfer motive bubbleless steady state by k^0 . By using these notations we can show:

PROPOSITION 8: *The operative bequest motive steady state and the bubble steady state form an upper and lower bound for the non-trivial inoperative transfer motive steady state such that $k^a < k^b$ and $k^a \leq k^0 \leq k^b$ if $(\alpha+\eta)(1-\delta)/(1-\mu) < 1$, and $k^b < k^a$ and $k^b \leq k^0 \leq k^a$ if $(\alpha+\eta)(1-\delta)/(1-\mu) > 1$.*

³⁶ This results follow from the assumptions that altruism is symmetric and agents take the actions of other generations as given as in proposition 5. With asymmetric two-sided altruism we do not face $\psi < 1$ restriction due to finiteness of V_t (Abel 1987, Wigger 2001). If $\psi \geq 1$, it is possible that $\psi\beta=1$ and $\psi=1$, which implies that gift motive can be operative in the bubble steady state. Moreover, if $\psi > 1$, then condition (49b) implies that a steady state must satisfy $n\gamma \leq (1/\psi)R < R$, which eliminates bubble steady states $n\gamma=R$. If agents take into account the actions of other generations, condition (49b) includes an additional term due to the reaction of children to an increase in parents' saving (O'Connell and Zeldes 1993, Lagerlöf 1997). In this case, it is possible that gift motive can be operative in the bubble steady state.

PROOF: The proof follows as in Proposition 6 by noticing that $k^{\text{GR}}=k^{\text{m}}$. Q.E.D.

Proposition 8 is similar to Proposition 6 in the economy with altruism and without intrinsically useless assets except that gifts are replaced by bubbles. Because k^{b} forms a stronger bound for k^0 than the operative gift motive steady state k^{g} , gifts are an imperfect substitute for bubbles. Moreover, the strength of this bound does not depend on the degree of altruism Ψ , but it depends on the relationship between the growth rate of output and the rate of return on physical capital, i.e., on the dynamic efficiency. In particular, because k^{b} satisfies the Golden Rule, it is dynamically efficient and eliminates dynamically inefficient k^0 .

5. GOVERNMENT DEBT AND PERMANENT BUDGET DEFICITS

In this chapter we consider the redistribution of resources from children to parents through government debt and permanent budget deficits. We add to the model government debt and non-productive government expenditures, and consider a constant deficit policy and permanent budget deficits as in Azariadis (1993, 322) and De la Croix and Michel (2002, 193).

Government debt can work in the economy in a way similar to bubbles. Diamond (1965) considers a constant debt policy and shows that government debt can eliminate dynamic inefficiency due to OLG-structure of the economy. Azariadis (1993, 322) and De la Croix and Michel (2002, 193) extend Diamond's result to a constant deficit policy.

To keep things simple, we assume that government debt and expenditures are financed by issuing new debt instead of future tax payments. It follows that the present value of future taxes does not cover the initial value of the debt and the government intertemporal budget constraint does not hold. This type of debt finance is called a Ponzi game. Ponzi games are an example of bubbles, which were studied in chapter 3.³⁷ Hence, a necessary condition for the Ponzi game debt finance is that the economy without bubbles is dynamically inefficient. Otherwise the analysis of government debt differs somewhat from the analysis of bubbles, because government expenditures cause a permanent government budget deficit. Budget deficits increase the amount of debt, which implies that they decrease the effect of debt on the economy. Moreover, the economy with government debt and permanent budget deficits tends to have two bubble steady states

After the theoretical discussion of the government debt and permanent budget deficits, we connect the model to the empirical discussion on dynamic inefficiency and sustainability of the Ponzi game debt finance by calibrating a stationary version of the model for the U.S. data.

³⁷ For the discussion on the connection between Ponzi games and bubbles, see O'Connell and Zeldes (1988) and De La Croix and Michel (2002, 211). For the effects of taxation in the endogenous growth OLG models with human capital, see Caballe (1995, 1998), Ihori (2001) and Yakita (2003).

5.1. The model

The model is the same as in Chapter 2 except that the economy has government debt and non-productive government expenditures. The economy does not have taxes, which implies that the government debt is a Ponzi game and the government has permanent budget deficits. Government debt can be used as a substitute for savings in physical capital in a manner similar to intrinsically useless assets. Because government debt does not affect periodic budget constraints (1a) and (1b), the households' utility maximization problem (7) and first-order conditions (8) hold.

The government budget constraint is:

$$(50) \quad A_{t+1} = R_{t+1}^A A_t + D_{t+1} \quad A_t \geq 0, D_t \geq 0$$

where A_t is the government debt, R_{t+1}^A is the gross rate of return on debt and D_t is the government budget deficit, which is used for government non-productive expenditures G_t (wasted consumption). Because the model does not have taxes, we have $G_t = D_t \geq 0$. Constraint $A_t \geq 0$ implies that the model does not have public production and the government cannot save in private capital.³⁸

Non-arbitrage between savings in the government debt and physical capital implies that:

$$(51) \quad R_{t+1}^A \leq R_{t+1} \quad (\text{if } A_t > 0)$$

By using (51) we can rewrite government budget constraint (50):

$$(52) \quad n a_{t+1}/a_t \leq R_{t+1} + n d_{t+1}/a_t \quad (\text{if } a_t > 0)$$

where $a_t = A_t/N_t$ and $d_{t+1} = D_{t+1}/N_{t+1}$.

Condition (52) determines when the economy can sustain Ponzi games and when the non-negativity constraint for government debt is binding. If the economy sustains Ponzi games, condition (52) defines a link between the rate of return on debt, government deficit and the growth rate of debt.

To close the model, we assume that government runs a constant deficit budget policy:

$$(53) \quad d_{t+1} = d$$

where $d_{t+1} = D_{t+1}/H_{t+1}$ is the deficit per effective unit of labor.

³⁸ If the model has public production or the government can save in private capital, then A_t can be interpreted as government net debt and it can be also negative (Farmer 1986).

From (53) it follows that $D_{t+1}/D_t = H_{t+1}/H_t$, i.e., the growth rate of the deficit is equal to the growth rate of the economy in the balanced growth equilibrium. If the growth rate of the deficit were higher than the growth rate of the economy, government budget constraint (52) would eventually violate periodic budget constraint (1a) in the balanced growth equilibrium. If the growth rate of the deficit were lower than the growth rate of the economy, then d_{t+1} would eventually vanish in the balanced growth equilibrium.

Asset and goods market clearing conditions are:

$$(54a) \quad N_t s_t = K_{t+1} + A_t + N_{t+1} e_t$$

$$(54b) \quad F(K_t, H_t, \bar{k}_t) = N_t c_{1t} + N_{t-1} c_{2t} + N_{t+1} e_t + D_t + K_{t+1}$$

The equilibrium of the economy is defined as:

DEFINITION 6: *A competitive equilibrium of the economy with government debt and permanent budget deficits is a sequence $\{a_t, c_{1t}, c_{2t+1}, d_{t+1}, e_t, h_t, k_{t+1}\}_{t=0}^{\infty}$ such that*

- (i) c_{1t} , c_{2t+1} and e_{t-1} maximize utility (6) subject to budget constraint (2) and human capital production function (5) under given factor and asset prices and external effects
- (ii) factors are paid their marginal products (4)
- (iii) budget constraints (1) and market clearing conditions (54) are satisfied
- (iv) $k_0 > 0$, $h_0 > 0$
- (v) $\bar{k}_t = k_t$ and $\bar{h}_t = h_t$
- (vi) government budget constraint (52) and policy conditions (53) are satisfied

The definition of the competitive equilibrium of the economy with government debt and permanent budget deficits is similar to the definition of the competitive equilibrium of the economy without government debt except that the economy has two additional state variables a_t and d_{t+1} , which satisfy government budget constraint (52) and policy condition (53). Moreover, the economy satisfies market clearing conditions (54) instead of market clearing conditions (10) and (11).

Equations (4), (5), (9), (52), (53) and (54) define the competitive equilibrium of the economy with government debt and permanent budget deficits:

$$(55a) \quad n\gamma_t k_{t+1} + ne_t + a_t = [\rho/(1+\rho)](1-\delta)w_t$$

$$(55b) \quad c_{1t} = [1/(1+\rho)](1-\delta)w_t$$

$$(55c) \quad n\gamma_t a_{t+1}/a_t \leq R(k_{t+1}) + n\gamma_t d/a_t \quad (= \text{if } a_t > 0)$$

$$(55d) \quad k_t^{\alpha+\eta} = c_{1t} + c_{2t}/n + ne_t + d + n\gamma_t k_{t+1}$$

$$(55e) \quad e_t = [\delta(1-\alpha)/\alpha]^{1/(1-\delta)} k_{t+1}^{(1-\mu)/(1-\delta)} \equiv e(k_{t+1})$$

$$(55f) \quad \gamma_t = [\delta(1-\alpha)/\alpha]^{\delta/(1-\delta)} k_{t+1}^{(\delta-\mu)/(1-\delta)} \equiv \gamma(k_{t+1})$$

$$(55g) \quad w_t = (1-\alpha)k_t^{\alpha+\eta} \equiv w(k_t)$$

$$(55h) \quad R_t = \alpha k_t^{\alpha+\eta-1} \equiv R(k_t)$$

where $a_t = a_t/h_t$ is the debt per effective unit of labor and (55e-h) define e_t , γ_t , w_{t+1} and R_{t+1} as a function of k_{t+1} .

System (55) can have two types of equilibria. If $a_t > 0$, they are called bubble equilibria, and if $a_t = 0$, they are called bubbleless equilibria.

5.2 Steady states

In this section we consider the existence of non-trivial steady states in the economy with government debt and permanent budget deficits. Azariadis (1993, 322) and De la Croix and Michel (2002, 203) show that exogenous growth OLG models with government debt and permanent budget deficits tend to have two bubble steady states with different types of transitional dynamics if the non-trivial steady state of the economy without debt is dynamically inefficient. Moreover, they show that permanent budget deficits have a maximum sustainable upper bound and that they decrease the effect of debt on the economy. We show that these results also hold in our model.

Multiple bubble steady states complicate the dynamics of the model and tend to cause indeterminacy if some part of the debt is infinitely lived, i.e., the initial value of debt is not predetermined.³⁹ The dynamic analysis of the model with complex dynamics and indeterminacy is beyond the scope of this study. Hence, we focus on the stationary solutions.

Substituting (55e-h) into (55a) and (55c) simplifies system (55) to the following planar system:

$$(56a) \quad n[1+\delta(1-\alpha)/\alpha]\gamma(k_{t+1})k_{t+1} = [\rho/(1+\rho)](1-\delta)w(k_t) - a_t$$

$$(56b) \quad n\gamma(k_{t+1})a_{t+1}/a_t \leq R(k_{t+1}) + n\gamma(k_{t+1})d/a_t \quad (= \text{if } a_t > 0)$$

If $d=0$, system (56) is equal to the planar system (31) in the economy with intrinsically useless assets.

³⁹ Government debt can be finitely or infinitely lived (O'Connell and Zeldes 1988). A finitely lived debt always has an initial value, which is defined by the present discounted value of the aggregate debt payments. Hence, finitely lived debt is a predetermined state variable like physical capital while infinitely lived debt is a forward-looking variable like bubbles without any initial conditions. This implies that a planar system for physical capital and debt can have local indeterminacy if some part of the debt is infinitely lived and the model has a stable invariant set.

Let us denote a non-trivial steady state of the economy without debt by k^D , i.e., $\phi(k^D)=k^D>0$. By using (56) we can show:

PROPOSITION 8: *If $(\alpha+\eta)(1-\delta)/(1-\mu)\neq 1$, $0<d<d^\wedge$ and $n\gamma(k^D)>R(k^D)$, then the economy with government debt and permanent budget deficits has two bubble steady states. If $(\alpha+\eta)(1-\delta)/(1-\mu)\neq 1$, $d=d^\wedge$ and $n\gamma(k^D)>R(k^D)$, then the economy has a unique bubble steady state. If $(\alpha+\eta)(1-\delta)/(1-\mu)\neq 1$ and $d>d^\wedge$ or $n\gamma(k^D)\leq R(k^D)$, then the economy has a unique non-trivial bubbleless steady state.*

PROOF: System of difference equation (56) has a bubble steady state if $a=[\rho/(1+\rho)](1-\delta)w(k)-n[1+\delta(1-\alpha)/\alpha]\gamma(k)k>0$ and $a=n\gamma(k)d/[n\gamma(k)-R(k)]$. The former equation is a function in the (k,a) space, which satisfies $a>0$ if $\phi(k)>k$. The latter equation is a decreasing $[(\alpha+\eta)(1-\delta)/(1-\mu)<1]$ or increasing $[(\alpha+\eta)(1-\delta)/(1-\mu)>1]$ hyperbola in the (k,a) space, which approaches to the vertical and horizontal asymptotes $n\gamma(k)=R(k)$ and $k=0$ as $d\rightarrow 0$. It follows that the economy has a two (one) bubble steady states if $n\gamma=R$ for some $\phi(k)>k$ and $0<d<d^\wedge$ ($d=d^\wedge$), where

$$d^\wedge = \underset{k}{\operatorname{argmax}} \{ [\rho(1-\delta)w(k)/(1+\rho) - n(1+\delta(1-\alpha)/\alpha)\gamma(k)k] [1 - R(k)/n\gamma(k)] \}$$

On the other hand equations (56a) and (56b) imply that a bubbleless steady state must satisfy $n\gamma\leq R$ for some $\phi(k)=k$ or $d>d^\wedge$ and $\phi(k)=k$.

The rest of the proof follows as in Proposition 3(i). Q.E.D.

Proposition 8 implies that the economy with government debt and permanent budget deficits tends to have two bubble steady states as in Azariadis (1993, 322) and De la Croix and Michel 2002, 203). Moreover, it implies that deficit decreases the effect of debt on the economy and can even eliminate it if the size of the deficit is sufficiently high. Hence, the deficit has a maximum sustainable upper bound.

If the deficit is small, then the local dynamics of the first one bubble steady state is similar to the dynamics of the bubble steady state in the model with intrinsically useless assets, because $n\gamma$ is close to R in this steady state. However, the dynamics of the second bubble steady state tends to be different, because $n\gamma$ is higher than R in this steady state. In particular, the second bubble steady state can be locally stable and thereby locally indeterminate if some part of the debt is infinitely lived. On the other hand, the global dynamics of the economy can be indeterminate, because equilibrium mapping can have multiple stable manifolds, which overlap (Galor 1992). Hence, we ignore the dynamic analysis of the model and just note that the permanent budget deficits can cause local as well as global indeterminacy even with a log-linear utility function and C-D production functions.

5.3 Calibration of the model

In this section we consider the empirical relevance of the model by calibrating a stationary version of the economy with government debt and permanent budget deficits for the U.S. post-war data (1945-2000). Ponzi game debt finance in the U.S. economy is studied by Bullard and Russell (1999) and Chalk (2000) in the calibrated exogenous growth OLG model. They argue that moderate permanent budget deficits are sustainable by Ponzi game debt finance, because the average rate of return on the U.S. government debt has been lower than the average growth rate of output in the U.S. economy. We show that the same is true for our model, but the sustainable levels of deficit and debt are lower than in the exogenous growth OLG models.

The empirical discussion on dynamic inefficiency and sustainability of Ponzi game debt finance is concentrated on the U.S. economy. The reason for this is that it does not make very much sense to study Ponzi game debt finance and dynamic inefficiency in the small open economy such as in Finland, where the rate of return on physical capital is determined by international capital markets and the government can use external Ponzi game debt finance. Moreover, the U.S. economy has grown faster than other developed countries, which makes the case for redistributing resources from children to parents through the Ponzi game debt finance more relevant.

In general, we cannot test directly if government runs a Ponzi game, because agents' expectations and future values of the fundamentals are non-observable. However, we can find four types of indirect evidence on Ponzi games.

First, we can test the time series properties of the historical debt data. Hamilton and Flavin (1986) find evidence on the fact that the Ponzi game term is not significant in the U.S. government debt. After the seminal study by Hamilton and Flavin a number of studies have concluded in contradictory results. For example, Wilcox (1989) finds evidence on the non-stationarity in the U.S. debt, which implies that the possibility of Ponzi game debt finance cannot be rejected. On the other hand, Bohn (1995,1998) considers a stochastic version of the government intertemporal budget constraint. He finds evidence on the fact that the U.S. debt/GDP ratio displays mean-reversion, which implies that the Ponzi game term tends to be insignificant. Hence, we can conclude that time series tests give mixed evidence on Ponzi games.⁴⁰

Second, we can consider the possibility of dynamic inefficiency through bequest and gift motives. Overall transfers from parents to children are usually positive in all developed economies. However, these transfers are seldom motivated by altruism. They may be motivated by uncertain lifetime (Abel 1985), strategic exchange motive (Bernheim et al. 1985) or factor complementarities in production (Boldrin 1994). If we eliminate non-altruistically motivated transfers from overall transfers, the remaining transfers tend to be zero (Bernheim et al. 1985, Cox 1987, Altonji et al. 1997). Hence, we can find evidence

⁴⁰ For a more detailed discussion on time series evidence on Ponzi games, see a survey by Chalk and Hemming (2000).

for the inoperative altruistic transfer motive from parents to children, which implies a possibility of dynamic inefficiency.

Third, we can consider the possibility of dynamic inefficiency through the rate of return on physical capital and the growth rate of output. In this literature the main issue is the question of why the average risk-free rate of return in the U.S. economy has been lower than the average growth rate of output. This question is also closely related to the difference between the risk-free rate of return and the rate of return on equity, i.e., the Mehra and Prescott (1985) equity premium puzzle. If the reason for the low risk-free rate of return is uncertainty, a sufficiently high risk aversion would imply that the average risk-free rate of return can be below the growth rate of output in the dynamically efficient economy (Abel et al. 1989). In this case, Ponzi games are not feasible or the reason for them is not dynamic inefficiency but imperfect risk-sharing (Bertocchi 1991, Gale 1995, Blanchard and Weil 2001). On the other hand, if the reason for the low risk-free rate of return is intermediation costs or other market imperfections, the low risk-free rate of return implies dynamic inefficiency and the feasibility of Ponzi games (Bullard and Russell 1999, Bohn 1999). Because the equity premium puzzle still remains unsolved (Kocherlakota 1996), the feasibility of Ponzi games from this viewpoint remains an open question.

Fourth, we can simulate a parametric version of the model and check if the model resembles the empirical features of the U.S. economy with realistic parameter values of the model. Bullard and Russell (1999) and Chalk (2000) have calibrated a dynamically inefficient exogenous growth n -period OLG model with government debt to U.S. data. Bullard and Russell show that dynamic inefficiency fits well in the U.S. data and solves the problem of too low savings rates in the earlier models. Chalk considers explicitly the Ponzi game debt finance in the U.S. economy and finds that the maximum sustainable debt deficit/GNP ratio is about 5%.

In the calibration we use a stationary version of the model with government debt and permanent budget deficits. The transitional dynamics of the steady states is ignored in the calibration, because it would complicate the analysis significantly. Moreover, we add to the model exogenous scale factors and an exogenous equity premium. The scale factors are needed to match the rate of return on physical capital and the growth rate of output to the observed values. The equity premium corrects the observed difference between the rate of return on government debt and the rate of return on physical capital. A similar type of exogenous equity premium is used by Bullard and Russell (1999) and Chalk (2000). The equity premium can be viewed as compensation for risk, intermediation costs or some other market imperfection. The explicit modeling of the equity premium would cause some major changes to the model and is beyond the scope of this study.⁴¹

We can rewrite the government budget constraint by using the deficit/GDP ratio and the exogenous equity premium in the following form:

⁴¹ For endogenous equity premium and Ponzi games, see Bertocchi (1991), Gale (1995), Bohn (1999) and Blanchard and Weil (2001).

$$(57) \quad n\gamma a_{t+1} = \pi_{t+1} R_{t+1} a_t + \tilde{d}_{t+1} f(k_{t+1}) \quad 0 \leq \tilde{d}_{t+1} < 1 \quad 0 < \pi_{t+1} < 1$$

where $\tilde{d}_{t+1} = D_{t+1}/Y_{t+1}$ is a deficit/GDP ratio and $1/\pi_{t+1}$ is a proportional equity premium between the gross rate of return on physical capital and government debt. Because $\pi_{t+1} < 1$, the gross rate of return on government debt $\pi_{t+1} R_{t+1}$ is always smaller than the gross rate of return on physical capital R_{t+1} .

Equations (55a), (55e-h) and (57) imply that the calibrated model is defined as follows:

$$(58a) \quad a = [\rho/(1+\rho)](1-\delta)w(k) - n[1+\delta(1-\alpha)/\alpha]\gamma(k)k \equiv K(k)$$

$$(58b) \quad a = \tilde{d}f(k)/[1-\pi R(k)/n\gamma(k)] \equiv A(k)$$

$$(58c) \quad \gamma(k) = \Lambda_1^{1/(1-\delta)} [\delta(1-\alpha)/\alpha]^{\delta/(1-\delta)} k^{(\delta-\mu)/(1-\delta)}$$

$$(58d) \quad w(k) = (1-\alpha)\Lambda_2 k^{\alpha+\eta}$$

$$(58e) \quad R(k) = \alpha\Lambda_2 k^{\alpha+\eta-1}$$

$$(58f) \quad f(k) = \Lambda_2 k^{\alpha+\eta}$$

where Λ_1 and Λ_2 are exogenous scale factors.

To conduct the calibration exercise, we must next define the parameter values of the model. The choice of the parameter values is a central part of the calibration exercise. While there is some agreement in the calibration literature on the form and the parameter values of the physical capital production function and the utility function, one cannot find a similar consensus on the form or the parameter values of the human capital production function (Docquier and Michel 1999, Fougere and Merette 1999, Rangazas 2000, Bouzahzah et al. 2002). The main reason for this is that empirical evidence on human capital is not so clear, because empirical interpretation of the theoretical concept of human capital is often broad and less established than empirical interpretation of physical capital. Usually empirical evidence on human capital is based mostly on the empirical estimates of skills acquired through schooling (Temple 2000, Topel 2003) while the concept of human capital in theoretical models is often more closely related to knowledge, which also includes other types of skills.

We divide the parameters of the model into two classes according to the above distinction. The first class encompasses the parameters of the production functions and the utility function, on which there, more or less, is a consensus in the literature. Their values are fixed in all variations of the model. The second class is the parameters of the production functions, on which there is no consensus in the literature. Their values depend on the particular scenario of the model. Moreover, within the given scenario, they are chosen to match, in the best possible way, with the observed values of the variables.

The model is calibrated to match the facts of the post-war U.S. economy. The values of the parameters and variables are more or less standard values of the U.S. economy used in the calibration literature. To match the three-period model to the annual data, we assume that the length of the time period is 25 years. Hence, annual values of the parameters and variables are raised to the power of 25. This type of transformation is used by Pemberton (1994), Rangazas (1996,2000) and Lord and Rangazas (1998) among others.

The parameter and variables of the model are represented in the following tables:

Table 1a *Fixed parameters*

Parameters		Annual value	25-year period value
Physical capital share of income	α	0.25	0.25
Private discount factor	ρ	0.96	0.36
Scale parameter in human capital production function	Λ_1	1.02	1.64
Scale parameter in physical capital production function	Λ_2	1.14	26.5
Gross growth rate of population	n	1.01	1.28

Table 1b *Scenario parameters*

Parameters		Scen. 1	Scen. 2	Scen. 3	Scen. 4
Education parameter	δ	0	0.1	0.1	0.1
Erosion parameter	μ	0	0	0.2	0
Productivity parameter	η	0	0	0	2

Table 1c *Observed variables*

Variables		Annual value	25-year period value
Gross return on equity	R	1.07	5.43
Gross return on government debt	πR	1.01	1.28
Inverse of the equity premium	π	0.94	0.24
Gross growth rate of per worker GDP	γ	1.02	1.64

Source: Chalk (2000), Rangazas (2000)

The values of the physical capital share of income α and the discount factor ρ are important to the model, because they have a direct effect on dynamic inefficiency and the sustainable level of the deficit. The value of the physical capital share of income 0.25 is based on data and it is used generally in all calibrated models. The empirical estimate for the private discount factor is less clear. The value for the annual discount factor in the infinitely lived representative agent models is usually 0.96, while in the OLG models it is usually 0.99 (Auerbach and Kotlikoff 1987, Chalk 2000). The reason for higher ρ in the OLG models is that it increases the savings rate, which is otherwise too low due to low estimated value of the elasticity of intertemporal substitution. With a log-linear utility function the elasticity of intertemporal substitution is unity and we do not face this problem. Hence, we can use the lower value of ρ .

The values of the scale parameters Λ_1 and Λ_2 are not very important to the model, because they have little effect on dynamic inefficiency and the sustainable level of the deficit. The annual value of the scale parameter in the human capital production function 1.02 is chosen to match the per worker growth rate in case of zero education investments ($\delta=0$). The annual value of the scale parameter in the physical capital production function 1.14 is chosen to match, in the best possible way, with the observed returns on government debt and equity in the steady state with the highest sustainable level of the budget deficit.

The values of the parameters δ , μ and η depend on the particular scenario of the model and they are chosen to match in the best possible way to the observed values of the returns and the growth rate in the steady state with the highest sustainable level of the budget deficit. Empirical estimates for δ can be formed by using education spending shares of the income and they vary between 0.1 and 0.3 (Rangazas 2000, Bouzahzah et al. 2002). We choose $\delta=0.1$, because higher value of δ would imply too low balanced growth rate of output. In the third and fourth scenario, values of μ and η are chosen to be 0.2 and 2 to demonstrate the possibility of the positive growth effect of debt.

The values of the gross return on equity R , the gross return on government debt πR , the inverse of the equity premium π and the gross growth rate of GDP $n\gamma$ are based on data and are generally used in all calibrated models. The annual value of the inverse of the equity premium is $1.01/1.07 \approx 0.94$ by the observed annual values of the gross return on government debt 1.01 and the gross return on equity 1.07. A change in the risk premium would only have a small effect on the dynamic inefficiency of the economy and the sustainable level of the deficit, because the change would be compensated for by a corresponding adjustment in the scale parameter Λ_2 .⁴² In the endogenous growth version of the model, the annual gross growth rate of GDP 1.03 consists of the exogenous annual gross growth rate of the labor force 1.01 and the annual endogenous gross growth rate of per worker GDP 1.02. In the exogenous growth version of the model, the annual gross growth rate of per worker GDP 1.03 consists of the exogenous gross growth rate of the labor force 1.01 and the exogenous technological progress 1.02 defined by the scale factor Λ_1 .

⁴² Similar effect is also found by Chalk (2000).

The first scenario is an exogenous growth model, where education investments are zero ($\delta=0$) and there is no endogenous technological progress ($\mu=\eta=0$). The annual exogenous growth rate of the economy is $n\gamma=n\Lambda_1\approx 1.03$. The highest sustainable deficit/GDP ratio is 4.1%. This is illustrated in Figure 3, where we have drawn curves $K(k)$ (equation 58a) and $A(k)$ (equation 58b) and the gross growth rate of the economy $n\gamma(k)$ (equation 58c) in the (k,a) space. The highest sustainable deficit is a tangency point for the curves $K(k)$ and $A(k)$.

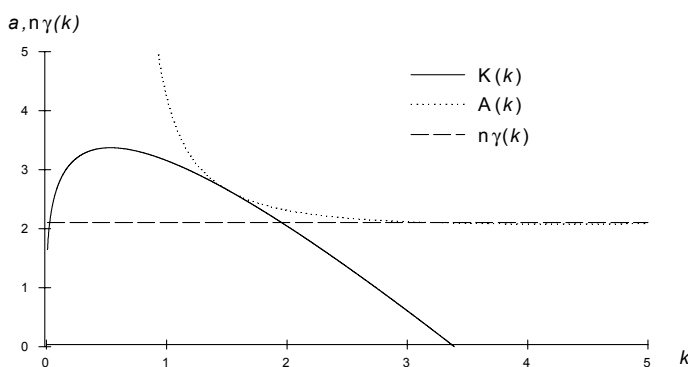


FIGURE 3 Exogenous growth model (full calibrated model in Appendix 2)

In the exogenous growth model, the highest sustainable level of deficit/GDP ratio is 4.1%, which is close to Chalk's (2000) results in the n -period exogenous growth OLG model (4.4% with $\rho=0.96$, 5.2% with $\rho=0.99$). Compared to the post-war U.S. deficit/GDP ratio 2%, this model implies a very high sustainable level of deficit.

The second scenario is an endogenous growth model with human capital accumulation ($\delta=0.1$) and without technological progress ($\mu=\eta=0$).

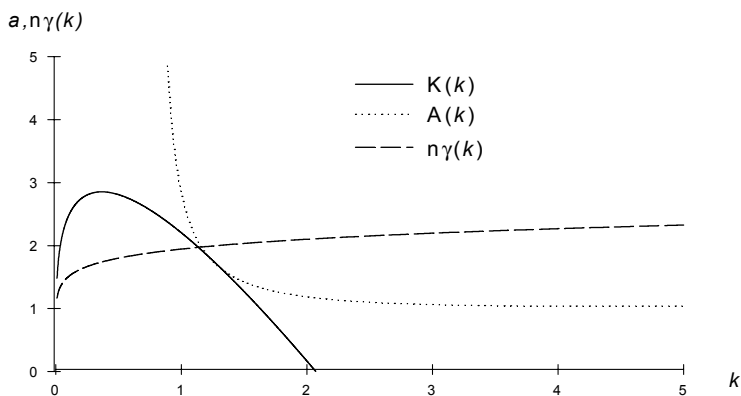


FIGURE 4 Endogenous growth model without technological progress (full calibrated model in Appendix 2)

In the endogenous growth model without technological progress, the highest sustainable level of deficit/GDP ratio is 2.1 %, which is lower than in the exogenous growth model and close to the post-war U.S. deficit/GDP ratio 2%. The effect of debt on the growth rate of the economy is negative, as can be seen from Figure 4.

The third scenario is an endogenous growth model with human capital accumulation ($\delta=0.1$) and a negative erosion effect of technological progress ($\mu=0.2, \eta=0$).

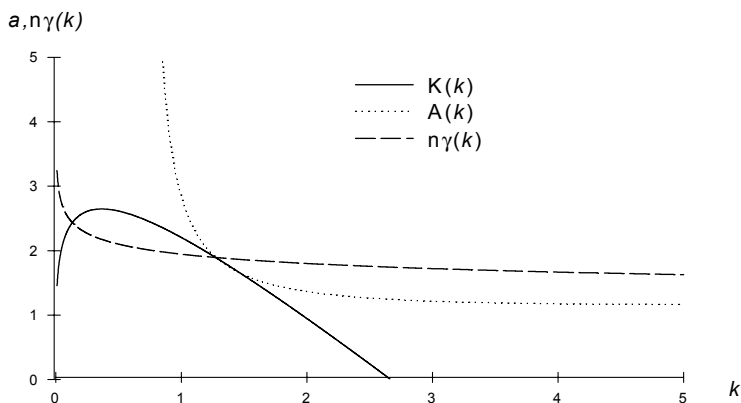


FIGURE 5 Endogenous growth model with a negative erosion effect of technological progress (full calibrated model in Appendix 2)

In the endogenous growth model with the erosion effect, the highest sustainable level of deficit/GDP ratio is 2.1 %, which is lower than in the exogenous growth model and close

to the post-war U.S. deficit/GDP ratio 2%. The effect of debt on the growth rate of the economy is positive, as can be seen from Figure 5.

The fourth scenario is an endogenous growth model with human capital accumulation ($\delta=0.1$) and a positive productivity effect of technological progress ($\mu=0$, $\eta=2$).

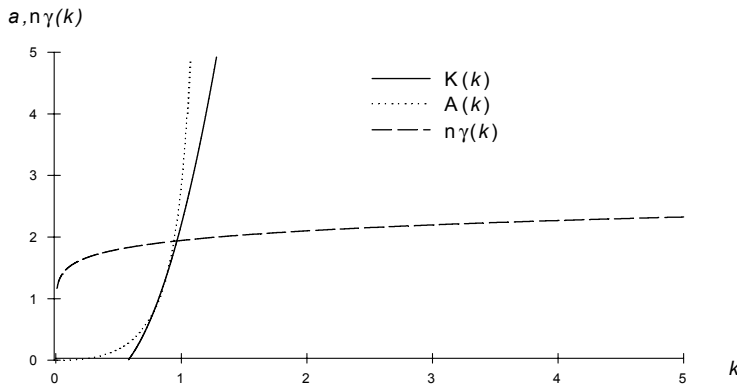


FIGURE 6 Endogenous growth model with a positive productivity effect of technological progress (full calibrated model in Appendix 2)

In the endogenous growth model with the productivity effect, the highest sustainable level of deficit/GDP ratio is 2.1 %, which is lower than in the exogenous growth model and close to the post-war U.S. deficit/GDP ratio 2%. The effect of debt on the growth rate of the economy is positive, as can be seen from Figure 6.

The results of the calibration exercise are summarized in the following table.

Table 2 *Results from alternative scenarios*

Variables		Scen. 1	Scen. 2	Scen. 3	Scen. 4
Highest sustainable deficit/GDP ratio	D/Y	0.041	0.021	0.021	0.021
Highest sustainable debt/GDP ratio	A/Y	2.2	1.4	1.5	1.5

From Table 2 we can make two conclusions. First, in the endogenous growth models with human capital, the highest sustainable deficit/GDP ratio is around 2.1%, which implies that the highest sustainable debt/GDP ratio is around 150 %. The realized U.S. government budget deficit/GDP ratio has been 2 % since 1980. Hence, our model implies that the current U.S. deficit could be sustained purely by the Ponzi game debt finance. The current net U.S. debt/GDP ratio is around 50 %. Hence, our model implies that the U.S.

economy is still well below the highest sustainable debt/GDP ratio. Second, the highest sustainable deficit/GDP ratio in the endogenous growth model is much lower than the highest sustainable deficit/GDP ratio in the exogenous growth model. Hence, if the reason for economic growth is endogenous human capital accumulation rather than exogenous technological progress, the sustainable levels of deficit and debt are much lower than Bullard and Russel's (1999) and Chalk's (2000) results indicate.

6. CONCLUSIONS

We have investigated the existence of steady states and transitional dynamics in the endogenous growth OLG model with a log-linear utility function and C-D production functions, where the source of economic growth is human capital accumulation due to education investments and technological progress due to learning-by-doing externalities. Technological progress has two opposite effects in the model, a positive productivity effect on final goods production and a negative erosion effect on human capital accumulation. We have shown that the properties of the model depend on these two effects.

First, we show that if the effects of technological progress are weak, the model has a unique globally stable non-trivial steady state, as usual in exogenous growth OLG models with a log-linear utility function and C-D production functions (Galor and Ryder 1989). If the productivity effect of technological progress is sufficiently strong, the non-trivial steady state is globally unstable, as usual in endogenous growth OLG models where the source of growth is technological progress alone (Boldrin 1992, Azariadis and Reichlin 1996, Antinolfi et al. 2001). The same is true if the erosion effect of technological progress is sufficiently strong. Hence, the erosion effect is an additional reason for the instability of the non-trivial steady state.

Second, we show that the model can have stationary equilibrium allocations, where the growth rate of output exceeds the rate of return on physical capital. These types of allocations violate the link between the growth rate of output and the rate of return on physical capital defined by the Euler equation and the transversality condition. Hence, the economy can be dynamically inefficient and there is a case for the redistribution of resources from children to parents. To consider intergenerational reallocation, we add intrinsically useless assets and two-sided altruism to the model. We show that bequests, gifts and bubbles cannot be operative in the same steady state if altruism is symmetric and agents take the actions of other generations as given. On the other hand, altruistic education investments are a perfect substitute for bequests if young agents do not face borrowing constraints. Moreover, gifts are an imperfect substitute for bubbles and bubbles eliminate gifts. The dynamic properties of the model with intrinsically useless assets and/or altruism are almost analogous to the model without intrinsically useless assets and altruism. The only difference is that in the model with intrinsically useless assets and/or altruism, technological progress can also cause explosive oscillations.

Third, we show that bubbles can have a positive or negative effect on economic growth. Hence, the effect of bubbles is different from those in models, where the source of

economic growth is endogenous technological progress (Grossman and Yanagawa 1993, Azariadis and Reichlin 1996) or human capital accumulation (Michel 1992) alone. The reason for the different result is the erosion and productivity effects of technological progress, which allow the growth rate of output to be decreasing in physical capital or the rate of return on physical capital to be increasing in physical capital.

Fourth, we show that government can sustain permanent budget deficits by Ponzi game debt finance. Ponzi game debt is an example of bubbles. Feasibility of Ponzi games implies that government's budget policy is not neutral, i.e., Ricardian equivalence does not hold. This non-neutrality result has two interesting policy conclusions. First, saving in government debt instead of physical capital can increase economic growth. Government debt also increases growth in Forslid (1998), Lin (2000), Zhang (1997) and Zhang (2003). However, in their models the positive growth effect of debt is based on government income transfers, productive government expenditures or endogenous fertility rather than the elimination of dynamic inefficiency. Second, government non-productive expenditures can be sustained in the market economy. The government can run a permanent budget deficit and roll over the resulting government debt forever. However, budget deficits have a maximum sustainable upper bound and they decrease the welfare and growth effects of debt.

We also consider the empirical relevance of the Ponzi game debt finance. We cannot test directly if the government runs a Ponzi game, because expectations and future fundamentals of the economy are non-observable. Hence, the empirical discussion of Ponzi games is concentrated on the feasibility of Ponzi games, i.e., on the difference between the rate of return on the debt and the growth rate of output. The average rate of return on the U.S. government debt has been lower than the average growth rate of output. This does not necessarily indicate dynamic inefficiency or the feasibility of Ponzi game debt finance if the reason for the low return on government debt is production uncertainty (Abel et al. 1989, Blanchard and Weil 2001). However, Mehra and Prescott (1985) show that risk-premium alone is not sufficient to explain the difference between the return on government debt and equity, which implies that low risk-free rate of returns cannot be explained purely by production uncertainty. Hence, some researchers (Bullard and Russell 1999, Bohn 1999) have argued that the reason for the low risk-free rate of return is intermediation costs or other market imperfections, which indicates the possibility of dynamic inefficiency and the feasibility of Ponzi game debt finance. Because the equity premium puzzle still remains unsolved, the final question regarding the empirical feasibility of Ponzi games due to dynamic inefficiency remains unanswered.

An alternative way to consider Ponzi game debt finance in real economies is calibration. Bullard and Russell (1999) and Chalk (2000) calibrate the n -period exogenous growth OLG model to the post-war U.S. data and they find support for an empirically plausible model of Ponzi game debt finance. Moreover, Chalk finds that the maximal sustainable deficit/GDP ratio by Ponzi game debt finance is around 5 %. We calibrate a three-period endogenous growth OLG model to the postwar U.S. data and show that the sustainable deficit/GDP ratio by Ponzi game debt finance is around 2.1 %, which is slightly higher

than the average realized U.S. deficit/GDP ratio since 1980 (2%). Hence, calibration results indicate that the current permanent U.S. government budget deficit can be sustained by the Ponzi game debt finance.

The feasibility of Ponzi games does not necessarily mean that the government can use Ponzi game debt finance if agents' confidence on the value of government debt is low. Weil (1987a) and Bertocchi and Wang (1995) have shown that the effect of the bubble on the economy depends on the agents' expectations on the probability that the bubble will persist and the sustainability of the bubble requires some minimum amount of confidence. Hence, we can ask if the U.S. government can achieve the minimum amount of confidence. It certainly was not achieved by the Italian immigrant, named Charles Ponzi, who made a quick fortune in the 1920s by using chain letters. He was sent to prison for a decade and died poor.

Despite the fact that Ponzi game debt finance can fail if agents lose their confidence in the government, it offers an alternative to the tax finance. This alternative becomes more relevant in situations where the government's capacity to tax is constrained. One example of this type of situation is globalization, causing the factors of production to be more mobile and thereby more difficult to tax. Hence, we can expect that the use of the Ponzi game debt finance becomes more common as the world economy becomes more integrated.

Although we have conducted our analysis in a simple parametric OLG model, our conclusions would hold in the more general context. First, they can be extended to OLG models with general forms of production functions and a constant elasticity of substitution utility function.⁴³ Hence, our results are not tied to C-D production functions and a log-linear utility function. Second, the results can be extended to the Bewley (1980) type of sequential economies, which have a finite number of traders, but an infinite sequence of trading opportunities due to borrowing constraints. Hence, our results are not tied to the infinite number of traders. Third, the results can be extended to the Bertocchi (1991), Gale (1995) and Blanchard and Weil (2001) type of stochastic OLG models, where the reason for Ponzi games is not dynamic inefficiency but imperfect risk-sharing between the living generations. Hence, our results are not tied to the infinite sequence of trading opportunities. Fourth, the results can be extended to the OLG models with pay-as-you-go social security. Hence, our results are not tied to the intrinsically useless assets or altruism. Feldstein (1974) and Wigger (2001) show that the pay-as-you-go social security works in the dynamically inefficient economy in a way similar to gifts or bubbles.⁴⁴

⁴³ For the analysis of the model with general forms of production functions and a constant elasticity of substitution utility function, see Appendix 1.

⁴⁴ For the relationship between pay-as-you-go social security and intergenerational transfers, see Wigger (2002). For the relationship between pay-as-you-go social security and government Ponzi game debt, see Ono (2003).

REFERENCES

- ABEL, A., 1985, Precautionary saving and accidental bequests, *American Economic Review* 75, 777-91.
- ABEL, A., 1987, Operative gift and bequest motives, *American Economic Review* 77, 1037-47.
- ABEL, A., MANKIW, G., SUMMERS, L. AND R. ZECKHAUSER, 1989, Assessing dynamic efficiency: theory and evidence, *Review of Economic Studies* 56, 1-20.
- ACEMOGLU, D., 1996, A microfoundation for social increasing returns in human capital accumulation, *Quarterly Journal of Economics* 111, 779-804.
- ACEMOGLU, D., 2002, Technical change, inequality and the labor market, *Journal of Economic Literature* 40, 7-72.
- AGHION, P. AND P. HOWITT, 1992, A model of growth through creative destruction, *Econometrica* 60, 323-51
- ALTONJI, J., HAYASHI, F. AND L. KOTLIKOFF, 1997, Parental altruism and inter vivos transfers: theory and evidence, *Journal of Political Economy* 105, 1121-1166.
- ANTINOLFI, G., KEISTER, T. AND K. SHELL, 2001, Growth dynamics and returns to scale: bifurcation analysis, *Journal of Economic Theory* 96, 70-96.
- ARROW, K., 1962, The economic implications of learning by doing, *Review of Economic Studies* 29, 155-73.
- AUERBACH, A. AND L. KOTLIKOFF, 1987, *Dynamic Fiscal Policy*, Cambridge University Press.
- AZARIADIS, C., 1993, *Intertemporal Macroeconomics*, Blackwell.
- AZARIADIS, C. AND A. DRAZEN, 1990, Threshold externalities in economic development, *Quarterly Journal of Economics* 105, 501-526.
- AZARIADIS, C. AND P. REICHLIN, 1996, Increasing returns and crowding out, *Journal of Economic Dynamics and Control* 20, 847-877.
- BARRO, R., 1974, Are government bonds net wealth? *Journal of Political Economy* 82, 1095-1118.
- BARRO, R. AND X. SALA-I-MARTIN, 1995, *Economic Growth*, McGraw-Hill.
- BECKER, G., 1964, *Human Capital*, Columbia University Press.
- BENHABIB, J. AND M. SPIEGEL, 1994, The role of human capital in economic development: evidence from aggregate cross-country data, *Journal of Monetary Economics* 34, 143-74.
- BERGSTROM, T., 1999, Systems of benevolent utility functions, *Journal of Public Economic Theory* 1, 71-100.
- BERNANKE, B. AND R. GURKAYNAK, 2001, Is growth exogenous? Taking Mankiw, Romer and Weil seriously, NBER 8365.
- BERNHEIM, D., SCHLEIFER, A. AND L. SUMMERS, 1985, The strategic bequest motive, *Journal of Political Economy* 93, 1045-76.
- BERTOCCHI, G., 1991, Bubbles and inefficiencies, *Economic Letters* 35, 117-22.
- BERTOCCHI, G. AND Y. WANG, 1995, The real value of money under endogenous beliefs, *Journal of Economic Theory* 67, 205-222.
- BEWLEY, T., 1980, The optimum quantity of money, in *Models of Monetary Economies*, ed. by Karaken, J., and N. Wallace, Minneapolis: Federal Reserve Bank.

- BLANCHARD, O. AND P. WEIL, 2001, Dynamic efficiency, the riskless rate, and debt Ponzi games under uncertainty, *Advances in Macroeconomics* 1, no. 2, article 3.
<http://www.bepress.com/bejm/advances/vol1/iss2/art3>
- BOHN, H., 1995, The sustainability of the budget deficits in a stochastic economy, *Journal of Money, Credit and Banking* 27, 257-72.
- BOHN, H., 1998, The behavior of U.S. public debt and deficits, *Quarterly Journal of Economics* 113, 949-63.
- BOHN, H., 1999, Fiscal policy and the Mehra-Prescott puzzle, *Journal of Money, Credit and Banking* 31, 1-13.
- BOLDRIN, M., 1992, Dynamic externalities, multiple equilibria and growth, *Journal of Economic Theory* 1992, 198-218.
- BOLDRIN, M., 1994, Public education and capital accumulation, C.M.S.E.M.S. discussion paper 1017, Northwestern University.
- BOLDRIN, M. AND A. MONTES, 2002, The intergenerational state. Education and pensions, mimeo, University of Minnesota and Universidad de Murcia.
- BOSE, A., AND D. RAY, 1993, Monetary equilibrium in an overlapping generations model with productive capital, *Economic Theory* 3, 697-716.
- BOUZAHZAH, M., DE LA CROIX, D. AND F. DOCQUIER, 2002, Policy reforms and growth in computable OLG economies, *Journal of Economic Dynamics & Control*, 26, 2093-2113.
- BULLARD, J., AND S. RUSSELL, 1999, An empirical plausible model of low real gross interest rate and unbacked government debt, *Journal of Monetary Economics* 44, 477-508.
- CABALLE, J., 1995, Endogenous growth, human capital, and bequests in a life-cycle model, *Oxford Economic Papers* 47, 156-81.
- CABALLE, J., 1998, Growth effects of taxation under altruism and low elasticity of intertemporal substitution, *Economic Journal* 108, 92-104.
- CANNON, E., 2000, Human capital: level versus growth effects, *Oxford Economic Papers* 52, 670-76.
- CARMICHAEL, J., 1982, On Barro's theorem of debt neutrality: the irrelevance of net wealth, *American Economic Review* 72, 202-13.
- CASS, D., 1972, On capital overaccumulation in the aggregative, neoclassical model of economic growth, *Journal of Economic Theory* 4, 200-23.
- CHALK, N., 2000, The sustainability of bond financed deficits: an overlapping generations approach, *Journal of Monetary Economics* 45, 293-328.
- CHALK, N. AND R. HEMMING, 2000, Assessing fiscal sustainability in theory and practice, IMF WORKING PAPER/00/81.
- COE, D., AND E. HELPMAN, 1995, International R&D spillovers, *European Economic Review* 39, 859-87.
- COX, D., 1987, Motives for private income transfers, *Journal of Political Economy* 95, 508-46.
- DECREUSE, B. AND E. THIBAUT, 2001, Labor productivity and dynamic efficiency, *Economics Bulletin* 4, 1-6.
- DE LA CROIX, D. AND P. MICHEL, 2002, *A Theory of Economic Growth*, Cambridge University Press.
- DIAMOND, P., 1965, National debt in a neoclassical growth model, *American Economic Review* 55, 1126-50.

- DOCQUIER, F., AND P. MICHEL, 1999, Education subsidies, social security and growth: the implications of a demographic shock, *Scandinavian Journal of Economics* 101, 425-440.
- DRAZEN, A., 1978, Government debt, human capital and bequests in a life-cycle model, *Journal of Political Economy* 86, 505-16.
- EICHER, T., 1996, Interaction between endogenous human capital and technological change, *Review of Economic Studies* 63, 127-144.
- EVANS, G. AND S. HONKAPOHJA, 1999, Learning dynamics, in *Handbook of Macroeconomics* vol. 1A, ed. by Taylor, J. and M. Woodford, 449-542.
- FARMER, R., 1986, Deficits and cycles, *Journal of Economic Theory* 40, 77-88.
- FELDSTEIN, M., 1974, Social security, induced retirement and aggregate capital accumulation, *Journal of Political Economy* 82, 905-26.
- FEMMINIS, G., 1999, Monopolistic competition, dynamic inefficiency and asset bubbles, CEPR 2272.
- FERREIRA, C., 1999, Inflationary financing of public investment an economic growth, *Journal of Economic Dynamics and Control* 23, 539-63.
- FORSLID, R., 1998, External debt and Ponzi-Games in a small open economy with endogenous growth, *Journal of Macroeconomics* 20, 341-49.
- FOUGERE, M. AND M. MERETTE, 1999, Population ageing and economic growth in seven OECD countries, *Economic Modelling* 16, 411-27.
- FUTAGAMI, K., AND A. SHIBATA, 2000, Growth effects of bubbles in an endogenous growth model, *Japanese Economic Review* 51, 221-35.
- GALE, D., 1995, The efficient design of public debt, in *Financial Innovation and Risk-sharing*, ed. by Allen F. and D. Gale, MIT Press, 271-307.
- GALOR, O., 1992, A two-sector overlapping-generations model: a global characterization of the dynamical system, *Econometrica* 60, 1351-86.
- GALOR, O. AND O. MOAV, 2000, Ability-biased technological transition, wage inequality and economic growth, *Quarterly Journal of Economics* 115, 469-97.
- GALOR, O. AND H. RYDER, 1989, Existence, uniqueness, and stability of equilibrium in an overlapping-generations model with productive capital, *Journal of Economic Theory* 49, 360-75.
- GALOR, O. AND D. WEIL, 2000, Population, technology and growth: From the Malthusian stagnation to the demographic transition and beyond, *American Economic Review* 90, 806-29.
- GLOMM, G., AND B. RAVIKUMAR, 1992, Public versus private investments in human capital, *Journal of Political Economy* 100, 818-834.
- GROSSMAN, G. AND N. YANAGAWA, 1993, Asset bubbles and endogenous growth, *Journal of Monetary Economics* 31, 3-19.
- GUTIERREZ, M., 2003, Dynamic inefficiency in an overlapping generation economy with pollution health costs, Universidad del Pais Vasco working paper.
- HAMILTON, J. AND M. FLAVIN, 1986, On the limitations of government borrowing: a framework for empirical testing, *American Economic Review* 76, 809-19.
- IHORI, T., 2001, Wealth taxation and economic growth, *Journal of Public Economics* 79, 129-48.
- JONES, C., 1995, Time series test of endogenous growth models, *Quarterly Journal of Economics* 110, 495-525.

- JONES, L. AND R. MANUELLI, 1990, A convex model of equilibrium growth: theory and policy implications, *Journal of Political Economy* 98, 1008-38.
- JULLIEN, B., 1988, Competitive business cycles in the overlapping generations economy with productive investments, *Journal of Economic Theory* 46, 45-65.
- KAGANOVICH, M. AND I. ZILCHA, 1999, Education, social security and growth, *Journal of Public Economics* 71, 289-309.
- KAHN, J., LIM, J. AND C. RHEE, 1997, Birth, death, and dynamic inefficiency in an endogenous growth model, *International Economic Review* 11, 85-102.
- KEMNITZ A., AND B. WIGGER, 2000, Growth and social security: the role of human capital, *European Journal of Political Economy* 16, 673-83.
- KIMBALL, M., 1987, Making sense of two-sided altruism, *Journal of Monetary Economics* 20, 301-26.
- KING, I. AND D. FERGUSON, 1993, Dynamic inefficiency, endogenous growth and Ponzi games, *Journal of Monetary Economics* 32, 79-104.
- KOCHERLAKOTA, N., 1996, The equity premium: It's still a puzzle, *Journal of Economic Literature* 34, 42-71.
- KEHOE, T., LEVINE, D. AND P. ROMER, 1992, On characterizing equilibria of economics with externalities and taxes as solutions to optimization problems, *Economic Theory* 2, 43-68.
- LAGERLÖF, N., 1997, Strategic saving and non-negative gifts, Stockholm University discussion paper S-106.
- LIN, S., 2000, Government debt and economic growth in an overlapping generations model, *Southern Economic Journal* 66, 754-63.
- LIPSEY, R. AND K. LANCASTER, 1956, The general theory of the second best, *Review of Economic Studies* 24, 11-32.
- LORD, W. AND P. RANGAZAS, 1998, Capital accumulation and taxation in a general equilibrium model with risky human capital, *Journal of Macroeconomics* 20, 509-31.
- LUCAS, R., 1988, On the mechanism of economic development, *Journal of Monetary Economics* 21, 3-42.
- MANKIW, N., ROMER, D. AND D. WEIL, 1992, A contribution to the empirics of economic growth, *Quarterly Journal of Economics* 107, 407-37.
- MARCHAND, M., MICHEL, P., PADDISON, O., AND P. PESTIEAU, 2003, Optimal education subsidy and taxes in an endogenous growth model with human capital, CORE working paper 2003/19.
- MEHRA, R. AND E. PRESCOTT, 1985, Equity premium puzzle, *Journal of Monetary Economics* 145-161.
- MICHEL, P., 1992, Bubbles slowing down economic growth, CORE discussion paper 9201.
- MICHEL, P. AND A. VENDITTI, 1997, Optimal growth and cycles in overlapping generations models, *Economic Theory* 9, 511-28.
- NELSON R. AND E. PHELPS, 1966, Investments in humans, technological diffusion and economic growth, *American Economic Review* 56, 69-75.
- NOURRY, C. AND A. VENDITTI, 2001, Determinacy of equilibrium in a overlapping generations model with heterogeneous agents, *Journal of Economic Theory* 96, 230-255.
- O'CONNELL, S., AND S. ZELDES, 1988, Rational ponzi games, *International Economic Review* 3, 431-50.

- O'CONNELL, S., AND S. ZELDES, 1993, Dynamic efficiency in the gifts economy, *Journal of Monetary Economics* 31, 363-79.
- ONO, T., 2003, Social security policy with public debt in an aging economy, *Journal of Population Economics* 16, 363-87.
- PEMBERTON, J., 1994, Ricardian equivalence: Are the required assumptions defensible?, *Economic Letters* 46, 223-227.
- RANGAZAS, P., 1996, Fiscal policy and endogenous growth in a bequest-constrained economy, *Oxford Economic Papers* 48, 52-74.
- RANGAZAS, P., 2000, Schooling and economic growth: A King-Rebelo experiment with human capital, *Journal of Monetary Economics* 46, 397-416.
- REDDING, S., 1996, The low-skill, low-quality trap: strategic complementarities between human capital and R&D, *Economic Journal* 106, 458-70.
- RHEE, C., 1991, Dynamic inefficiency in an economy with land, *Review of Economic Studies* 58, 791-97.
- ROMER, P., 1986, Increasing returns and long-run growth, *Journal of Political Economy* 94, 1002-35.
- ROMER, P., 1990, Endogenous technological change, *Journal of Political Economy* 98, S57-102.
- SAINT-PAUL, J., 1992, Fiscal policy in an endogenous growth model, *Quarterly Journal of Economics* 57, 1243-59.
- SAMUELSON, P., 1958, An exact consumption-loan model of interest with or without the social contrivance of money, *Journal of Political Economy* 467-82.
- SANCHEZ-LOSADA, F., 2000, Growth effects of an unfunded social security system when there is altruism and human capital, *Economic Letters*, 69, 95-99.
- SANTOS, M. AND M. WOODFORD, 1997, Rational asset pricing bubbles, *Econometrica* 65, 19-57.
- SARGENT, T. AND N. WALLACE, 1981, Some unpleasant monetarist arithmetic, *Federal Reserve Bank of Minneapolis Quarterly Review* 5, 1-17.
- SINN, H., 1998, The pay-as-you-go pension system as a fertility insurance and enforcement device, CEPR working paper 2023.
- STORESLETTEN, K. AND F. ZILIBOTTI, 2000, Education, educational policy and growth, *Swedish Economic Policy Review* 7, 39-70.
- TEMPLE, J., 2000, Growth effects of education and social capital in the OECD, OECD working paper.
- THIBAUT, E., 2000, Existence of equilibrium in an OLG model with production and altruistic preferences, *Economic Theory* 15, 709-15.
- TIROLE, J., 1985, Asset bubbles and overlapping generations, *Econometrica* 53, 1499-1528.
- TOPEL, R., 2003, Labor markets and economic growth, in *Handbook of Labor Economics* vol. 3c, 2943-84, Elsevier Science.
- UZAWA, H., 1965, Optimum technological change in an aggregative model of economic growth, *International Economic Review* 6, 18-31.
- VENDITTI, A., 2003, Altruism and determinacy of equilibria in overlapping generations models with externalities, *Japanese Economic Review* 54, 179-202.
- WEIL, P., 1987a, Confidence and the real value of money in an overlapping generations economy, *Quarterly Journal of Economics* 52, 1-22.

- WEIL, P., 1987b, Love thy children. Reflections on the Barro debt neutrality theorem, *Journal of Monetary Economics* 19, 377-91.
- WILCOX, D., 1989, Sustainability of government deficit: implications of the present-value borrowing constraint, *Journal of Money, Credit and Banking*, 21, 291-306.
- WIGGER, B., 2001, Gifts, bequests and growth, *Journal of Macroeconomics* 23, 121-29.
- WIGGER, B., 2002, Social security and growth in an altruistic economy, *German Economic Review* 3(1), 53-80.
- YAKITA, A., 2003, Taxation and growth with overlapping generations, *Journal of Public Economics* 87, 467-87.
- ZHANG, J., 1995, Social security and endogenous growth, *Journal of Public Economics* 58, 185-213.
- ZHANG, J., 1997, Government debt, human capital and endogenous growth, *Southern Economic Journal* 64, 281-92.
- ZHANG, J., 2001, Long-run implications of social security taxation for growth and fertility, *Southern Economic Journal* 67, 713-24.
- ZHANG, J., 2003, Optimal debt, endogenous fertility, and human capital externalities in a model with altruistic bequests, *Journal of Public Economics* 87, 1825-35.

APPENDIX 1:

GENERAL FORMS OF THE PRODUCTION FUNCTIONS AND A CONSTANT ELASTICITY OF SUBSTITUTION UTILITY FUNCTION

In this appendix we consider the analysis of the model with utility and production functions, which satisfy standard requirements for the utility and profit maximization and are consistent with the existence of the balanced growth equilibrium. The last requirement implies that the utility function must be a constant elasticity of substitution (CES) function, but it is consistent with a wide class of production functions.

If we allow more general forms of utility and production functions than (3), (5) and (6), then the qualitative properties of the model also depend on properties of the model other than the effects of technological progress on the degrees of returns to scale in production functions f and g . In particular, technological progress can have indirect effects on the model through complementarity as in Galor and Weil (2000) and Galor and Moav (2000). With C-D production functions, technological progress is a scale factor, which eliminates this types of indirect effects.

Moreover, the qualitative properties of the model can be more complicated than Propositions 1, 3 and 5 indicate. It is well known that OLG models can have multiple non-trivial steady states, periodic solutions and global indeterminacy even without externalities if the elasticity of substitution between the factors of production and the elasticity of intertemporal substitution are lower than unity (Azariadis 1993, 198-204). With C-D production functions and a log-linear utility function, these elasticities are unity, which eliminates this possibility. On the other hand, OLG models with externalities can have multiple non-trivial steady states, periodic solutions and global indeterminacy even with C-D production functions if the elasticity of intertemporal

substitution is higher than unity (Boldrin 1992). With a log-linear utility function the elasticity of intertemporal substitution is unity, which eliminates this possibility.

Human capital production function is:

$$(A1) \quad h_t = G(\bar{h}_{t-1}, e_{t-1}, \bar{k}_t)$$

We assume that G satisfies the following regularity conditions:

ASSUMPTION A1: *Function $G: \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$ is smooth in all arguments and linearly homogenous, increasing and strictly concave in the first two arguments. Moreover, $G_3 < 0$, $G_{33} > 0$, $\lim_{e \rightarrow 0} G_2 = \infty$ and $\lim_{e \rightarrow \infty} G_2 = 0$.*

Assumption A1 implies that G satisfies the standard properties for interior utility maximization with respect to education investments and G is consistent with the balanced growth equilibrium, and technological progress erodes human capital in the decreasing rate. From linear homogeneity of G it follows that $G(\bar{h}_{t-1}, e_{t-1}, \bar{k}_t) = \bar{h}_{t-1} G(1, e_{t-1}/\bar{h}_{t-1}, \bar{k}_t) \equiv \bar{h}_{t-1} g(e_{t-1}/\bar{h}_{t-1}, \bar{k}_t)$, where $g_1 > 0$, $g_2 < 0$, $g_{11} < 0$, $g_{22} > 0$ for $e/\bar{h} > 0$, $\lim_{e/\bar{h} \rightarrow 0} g_1 = \infty$ and $\lim_{e/\bar{h} \rightarrow \infty} g_1 = 0$.

Final goods production function is:

$$(A2) \quad Y_t = F(K_t, H_t, \bar{k}_t)$$

We assume that F satisfies the following regularity conditions:

ASSUMPTION A2: *Function $F: \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$ is smooth in all arguments and linearly homogenous, increasing and strictly concave in the first two arguments. Moreover, $F_3 > 0$.*

Assumption A2 implies that F satisfies the standard properties for the profit maximization, F is consistent with balanced growth equilibrium and technological progress increases productivity in the final goods sector. From linear homogeneity of F it follows that $F(K_t, H_t, \bar{k}_t) = H_t F(k_t, 1, \bar{k}_t) \equiv H_t f(k_t, \bar{k}_t)$, where $f_1 > 0$, $f_{11} < 0$ and $f_2 > 0$ for $k_t > 0$.

Profit maximization of competitive firms implies that these factors are paid their private marginal products:

$$(A3a) \quad R_t = f_1(k_t, \bar{k}_t)$$

$$(A3b) \quad w_t = f(k_t, \bar{k}_t) - k_t f_1(k_t, \bar{k}_t)$$

The existence of balanced growth paths requires that the utility function must be additive separable and homogenous (Jones and Manuelli 1990). Equivalently to the additive

separability and homogeneity we could assume that households have the following CES utility function over their own consumption

$$(A4) \quad U(c_{1t}, c_{2t+1}) = c_{1t}^{\theta/\rho} + \rho c_{2t+1}^{\theta/\rho} \quad (\text{for } \theta \neq 0) \quad \theta < 1, \rho > 0$$

$$U(c_{1t}, c_{2t+1}) = \ln c_{1t} + \rho \ln c_{2t+1} \quad (\text{for } \theta = 0)$$

where θ is the degree of homogeneity of U and the special case $\theta=0$ is a log-linear utility function. Constraints $\rho>0$ and $\theta<1$ imply that U is increasing, strictly concave and it fulfills the Inada conditions. Hence, U satisfies the standard properties for interior utility maximization with respect to consumption.

With CES utility function (A4) and assumption A1, households' objective (38) is strictly concave in $c_{1t}, e_{0t-1}, e_{1t}, q_{t+1}$ and j_t and first-order and transversality conditions (39) are sufficient for the maximum. Hence, the competitive equilibrium of the economy with general forms of production functions and CES utility function is defined as follows:

$$(A5a) \quad n\gamma_t k_{t+1} + ne_t + b_t = z(R_{t+1}) [w_t + q_t - R_t e_{t-1} / \gamma_{t-1} + R_t e_{1t-1} / \gamma_{t-1} - j_t] -$$

$$[1 - z(R_{t+1})] [n\gamma_t (j_{t+1} - q_{t+1}) / R_{t+1} - ne_{1t}]$$

$$(A5b) \quad c_{1t} = [1 - z(R_{t+1})] [w_t + q_t - R_t e_{t-1} / \gamma_{t-1} + R_t e_{1t-1} / \gamma_{t-1} - ne_{1t} - j_t + n\gamma_t (j_{t+1} - q_{t+1}) / R_{t+1}]$$

$$(A5c) \quad \psi n(\gamma_t c_{1t+1} / c_{1t})^{1-\theta} \leq R_{t+1} \leq (1/\beta) n(\gamma_t c_{1t+1} / c_{1t})^{1-\theta} \quad (= \text{if } j_{t+1} > 0, = \text{if } q_{t+1} + e_{1t} > 0)$$

$$(A5d) \quad n\gamma_t b_{t+1} / b_t \leq R_{t+1} \quad (= \text{if } b_t > 0)$$

$$(A5e) \quad f(k_t, k_t) = c_{1t} + c_{2t} / n + ne_t + n\gamma_t k_{t+1}$$

$$(A5f) \quad R_{t+1} = g_1(e_t, k_{t+1}) w_{t+1}$$

$$(A5g) \quad \gamma_t = g(e_t, k_{t+1})$$

$$(A5h) \quad w_t = f(k_t, k_t) - k_t f_1(k_t, k_t) \equiv w(k_t)$$

$$(A5i) \quad R_t = f_1(k_t, k_t) \equiv R(k_t)$$

$$(A5j) \quad z(R_{t+1}) = \begin{cases} [1 + \rho^{1/(\theta-1)} R_{t+1}^{\theta/(\theta-1)}]^{-1} & \text{for } \theta \neq 0 \\ \rho / (1 + \rho) & \text{for } \theta = 0 \end{cases}$$

$$(A5k) \quad \lim_{t \rightarrow \infty} \beta^t (q_t / c_{1t}) (c_{10} \Pi_{i=0}^{t-1} \gamma_i)^\theta = 0$$

$$(A5l) \quad \lim_{t \rightarrow \infty} \beta^t (e_{1t} / c_{1t}) (c_{10} \Pi_{i=0}^{t-1} \gamma_i)^\theta = 0$$

$$(A5m) \lim_{t \rightarrow \infty} (1/\psi)^t (j_{1t}/c_{1t}) (c_{10} \prod_{i=0}^{t-1} \gamma_i)^0 = 0$$

where (A5h) and (A5i) define w_t and R_t as a function of k_t and (A5j) defines z_t as a function of R_{t+1} .

Implicit differentiation of (A5f) and (A5g) yields

$$(A6) \begin{pmatrix} g_{11}w_{t+1} & 0 \\ -g_1 & 1 \end{pmatrix} \begin{pmatrix} \partial e_t \\ \partial \gamma_t \end{pmatrix} = \begin{pmatrix} R_1 - g_1 w' - g_{12} w_{t+1} \\ g_2 \end{pmatrix} \partial k_{t+1}$$

and

$$(A7a) \quad e_t = e(k_{t+1})$$

$$(A7b) \quad \gamma_t = \gamma(k_{t+1})$$

where mappings $e: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ and $\gamma: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ are continuous functions by the implicit function theorem, because $g_{11}w_{t+1} \neq 0$ for $k_{t+1} > 0$.

$$(A8a) \quad e'(k_{t+1}) = (R' - g_1 w' - g_{12} w_{t+1}) / g_{11} w_{t+1}$$

$$(A8b) \quad \gamma'(k_{t+1}) = [g_1(R' - g_1 w') - (g_1 g_{12} - g_{11} g_2) w_{t+1}] / g_{11} w_{t+1}$$

It follows that the sign of γ' depends on terms $R' - g_1 w' = (f_{11} + f_{12} - f_1 f_2 / f) / (1 - k_{t+1} f_1 / f)$ and $(g_1 g_{12} - g_{11} g_2) w_{t+1}$. With C-D production functions $f(k_t, \bar{k}_t) = k_t^\alpha \bar{k}_t^{1-\alpha}$ and $g(e_{t-1}, \bar{k}_t) = e_{t-1}^\delta \bar{k}_t^{1-\delta}$ it follows that $R' - g_1 w' = -R_{t+1} / k_{t+1} < 0$ and $g_1 g_{12} - g_{11} g_2 = g_1 g_2 / e_t < 0$, which implies that $g_1(R' - g_1 w') - (g_1 g_{12} - g_{11} g_2) w_{t+1} = -(g_1 e_t / g + g_2 k_{t+1} / g) g R_{t+1} / k_{t+1} e_t = -(\delta - \mu) g R_{t+1} / k_{t+1} e_t$. Hence, mapping γ is a monotone function and the sign of γ' depends on $\delta - \mu$. In general, it is possible that $R' - g_1 w' > 0$ if the technology-capital complementarity $f_{12} > 0$ is sufficiently strong. Moreover, it is possible that $g_1 g_{12} - g_{11} g_2 > 0$ if the technology-skill complementarity $g_{12} > 0$ is sufficiently strong. Hence, the sign of γ' also depends on the indirect effects of technological progress, and negative returns to scale in function g do not necessarily imply that $\gamma' < 0$. Moreover, mapping γ can be a non-monotone function due to indirect effects of technological progress.

Equilibrium without intrinsically useless assets and altruism

Without intrinsically useless assets and altruism, system (A5) simplifies to the following scalar system:

$$(A9) \quad n[\gamma(k_{t+1})k_{t+1} + e(k_{t+1})] = z[R(k_{t+1})][w(k_t) - R(k_t)e(k_t)/\gamma(k_t)]$$

Equation (A9) defines implicitly the following mapping in the forward dynamics:

$$(A10) \quad k_{t+1} = \phi(k_t)$$

From (A10) it follows that

$$(A11) \quad \phi'(k_t) = z_t [w' - R'e_{t-1}/\gamma_{t-1} - R_{t-1}e'/\gamma_{t-1} + R_{t-1}e_{t-1}\gamma'/\gamma_{t-1}^2] / [n\gamma_t + n\gamma'k_{t+1} + ne' - z'R'(w_t - R_{t-1}e_{t-1}/\gamma_{t-1})]$$

It follows that the properties of mapping $\phi(k_t)$ depend on mappings $R(k_t)$, $R(k_{t+1})$, $w(k_t)$, $e(k_t)$, $e(k_{t+1})$, $\gamma(k_t)$, $\gamma(k_{t+1})$ and $z(R_{t+1})$. With C-D production functions $f(k_t, \bar{k}_t) = k_t^\alpha \bar{k}_t^{1-\alpha}$ and $g(e_{t-1}, \bar{k}_t) = e_{t-1}^\delta \bar{k}_t^{1-\mu}$ it follows that $\phi' = z_t(1-\delta)w' / [n\gamma_t(1+\delta(1-\alpha)/\alpha)(1-\mu)/(1-\delta) - z'R'(1-\delta)w_t]$. If $\theta=0$, then $z'=0$, which implies that ϕ is a monotone increasing ($0 < \mu < 1$) or decreasing ($\mu > 1$) function and it has a unique non-trivial steady state. If $\theta \neq 0$, then it is possible that $[n\gamma_t(1+\delta(1-\alpha)/\alpha)(1-\mu)/(1-\delta) - z'R'(1-\delta)w_t] = 0$ for some $k_t > 0$, which implies that ϕ is a multi-valued mapping. Hence, the economy can have multiple non-trivial steady states, periodic solutions and indeterminacy even with C-D production functions. The same is also true if $\theta=0$ and production functions are not C-D form. In this case, it is possible that mapping γ is non-monotone as shown above. If technological complementarities are sufficiently strong, then it is possible that $n\gamma_t + n\gamma'k_{t+1} + ne' = 0$, which implies that ϕ is a multi-valued mapping. Hence, the economy can have multiple non-trivial steady states, periodic solutions and indeterminacy even with a log-linear utility function.

With general forms of the production functions, the efficiency analysis of steady states can be more complicated than Proposition 2 indicates. In particular, if the economy has multiple non-trivial stationary solutions for the planner's utility maximization problem, it is unclear which one is the social optimum. On the other hand, if the economy has a unique non-trivial stationary solution for the planner's utility maximization problem, we can show that

PROPOSITION A1: *Suppose that the economy has a unique non-trivial steady state solution for the planner's second-best utility maximization problem. If $n\gamma > R$ in the non-trivial steady state, then it is dynamically inefficient. If $n\gamma \leq R$ in the non-trivial steady state, then it is dynamically efficient.*

PROOF: From the transversality condition (23d) it follows that a dynamically efficient steady state allocation must satisfy $\lim_{t \rightarrow \infty} \omega^t (k/c_1)(c_{10}\gamma^t)^\theta = 0$, which implies that $\omega\gamma^\theta < 1$. The limit case $\omega\gamma^\theta = 1$ does not satisfy the transversality condition (23d). In this case, we can obtain the solution of the planner's problem by modifying the planner's objective to $u(c_{20}) - u(c_2^\wedge) + \sum_{t=0}^{\infty} \omega^t [u(c_{1t}) + \rho u(c_{2t+1}) - u(c_1^\wedge) - \rho u(c_2^\wedge)]$, where $u(c_1^\wedge) + \rho u(c_2^\wedge)$ is the maximum stationary utility level (De la Croix and Michel 2002, 92). Hence, feasible values of weights are $0 < \omega\gamma^\theta \leq 1$. Substituting this constraint to Euler equation (24b) gives $n\gamma^{1-\theta} = \omega R \Rightarrow n\gamma = \omega\gamma^\theta R \leq R$. Hence, if $n\gamma > R$ in the non-trivial steady state, the planner can increase the welfare of the economy for all feasible ω and the allocation must be

inefficient. On the other hand, if $n\gamma \leq R$ in the non-trivial steady state, the planner cannot increase the welfare of the economy for some feasible ω and the allocation must be dynamically efficient. Q.E.D.

COROLLARY A1: *Suppose that the economy has a unique Golden Rule allocation $n\gamma(k^{GR})=R(k^{GR})$. If $R'(k^{GR}) < n\gamma'(k^{GR})$, then dynamically inefficient steady states satisfy $k > k^{GR}$ and if $R'(k^{GR}) > n\gamma'(k^{GR})$, then dynamically inefficient steady states satisfy $k < k^{GR}$.*

Proposition 2 and Corollary 2 in Chapter 2 are special cases of Proposition A1 and Corollary A1. With C-D production functions $f(k_t, \bar{k}_t) = k_t^\alpha \bar{k}_t^{1-\alpha}$ and $g(e_{t-1}, \bar{k}_t) = e_{t-1}^\delta \bar{k}_t^{1-\delta}$ the planner's utility maximization problem has a unique non-trivial steady state solution. Hence, the economy has a unique $k^{GR} > 0$. Moreover, condition $R'(k^{GR}) < n\gamma'(k^{GR})$ implies that $0 < (\alpha + \eta)(1 - \delta)/(1 - \mu) < 1$ and condition $R'(k^{GR}) > n\gamma'(k^{GR})$ implies that $(\alpha + \eta)(1 - \delta)/(1 - \mu) > 1$ [or $(\alpha + \eta)(1 - \delta)/(1 - \mu) < 0$ if we allow $\mu > 1$].

Equilibrium without altruism

Without altruism system (A5) simplifies to the following planar system:

$$(A12a) \quad n[\gamma(k_{t+1})k_{t+1} + e(k_{t+1})] + b_t = z[R(k_{t+1})][w(k_t) - R(k_t)e(k_t)/\gamma(k_t)]$$

$$(A12b) \quad n\gamma(k_{t+1})b_{t+1}/b_t \leq R(k_{t+1}) \quad (\text{if } b_t > 0)$$

If $b_t = 0$, system (A12) simplifies to the scalar system (A9). If $b_t > 0$, equations (A12a) and (A12b) define implicitly the following mappings in the forward dynamics:

$$(A13a) \quad k_{t+1} = \varphi^1(k_t, b_t)$$

$$(A13b) \quad b_{t+1} = R[\varphi^1(k_t, b_t)]b_t/\gamma[\varphi^1(k_t, b_t)] \equiv \varphi^2(k_t, b_t)$$

From (A13) it follows that

$$(A14a) \quad \varphi^1_1 = z_t[w' - R'e_{t-1}/\gamma_{t-1} - R_{t-1}e'/\gamma_{t-1} + R_{t-1}e_t\gamma'/\gamma_{t-1}^2]/$$

$$[n\gamma_{t+1} + n\gamma'k_{t+1} + ne' - z'R'(w_t - R_{t-1}e_{t-1}/\gamma_{t-1})]$$

$$(A14b) \quad \varphi^1_2 = -1/[n\gamma_{t+1} + n\gamma'k_{t+1} + ne' - z'R'(w_t - R_{t-1}e_{t-1}/\gamma_{t-1})]$$

$$(A14c) \quad \varphi^2_1 = (R'k_{t+1}/R_{t+1} - \gamma'k_{t+1}/\gamma_t)(b_{t+1}/k_{t+1})\varphi^1_1$$

$$(A14d) \quad \varphi^2_2 = R_{t+1}/n\gamma_t + (R'k_{t+1}/R_{t+1} - \gamma'k_{t+1}/\gamma_{t+1})(b_{t+1}/k_{t+1})\varphi^1_2$$

It follows that the properties of mapping $\varphi(k_t, b_t)$ depend on mappings $R(k_t)$, $R(k_{t+1})$, $w(k_t)$, $e(k_t)$, $e(k_{t+1})$, $\gamma(k_t)$, $\gamma(k_{t+1})$ and $z(R_{t+1})$ as in the model without intrinsically useless

assets. The economy can have multiple bubble steady states if mappings $R(k)$ and $n\gamma(k)$ have multiple crossing points. On the other hand, if the economy has multiple non-trivial non-monetary steady states, it is possible that the economy does not have a bubble steady state even if it has a dynamically inefficient bubbleless steady state (Bose and Ray 1993). Moreover, the bubble equilibria can display periodic solutions even if the bubbleless equilibria were stationary (Jullien 1988). With C-D production functions $f(k_t, \bar{k}_t) = k_t^\alpha \bar{k}_t^{1-\alpha}$ and $g(e_{t-1}, \bar{k}_t) = e_{t-1}^\delta \bar{k}_t^{1-\mu}$ mappings $R(k) = \alpha k^{\alpha+\eta-1}$ and $n\gamma(k) = n[\delta(1-\alpha)/\alpha]^{\delta/(1-\delta)} k^{(\delta-\mu)/(1-\delta)}$ have a unique crossing point, but the dynamics of the economy can be indeterminate or display periodic solutions if $\theta \neq 0$.

With general forms of the production functions the growth effect of bubbles can be more complicated than Proposition 3 indicates. In particular, if the economy has multiple bubble steady states, the growth effect of bubbles may be ambiguous. On the other hand, if the economy has a unique bubble steady states, we can show that:

PROPOSITION A2: *Suppose that the economy has a unique bubble steady state and mapping $\gamma(k)$ is a monotone function. If $R' < n\gamma' < 0$ or $R' > n\gamma' > 0$ in the bubble steady state, then bubbles increase the growth rate of the economy. Otherwise bubbles decrease the growth rate of the economy.*

PROOF: Bubbles increase γ if $\gamma' < 0$ and the bubble steady state is lower than the non-trivial bubbleless steady state or if $\gamma' > 0$ and the bubble steady state is higher than the non-trivial bubbleless steady state.

Equation (A12b) implies that bubbles eliminate non-trivial bubbleless steady states, which satisfy $n\gamma > R$, and the bubble steady satisfies $n\gamma = R$. Hence, if the economy has a unique bubble steady state, bubbles increase growth if and only if $R' < n\gamma' < 0$ or $R' > n\gamma' > 0$ in the bubble steady state. Q.E.D.

Proposition 4 in Chapter 3 is a special case of Proposition A2. With C-D production functions $f(k_t, \bar{k}_t) = k_t^\alpha \bar{k}_t^{1-\alpha}$ and $g(e_{t-1}, \bar{k}_t) = e_{t-1}^\delta \bar{k}_t^{1-\mu}$ the bubble steady state is unique and mapping $\gamma(k)$ is a monotone function. Moreover, conditions $R' < n\gamma' < 0$ and $R' > n\gamma' > 0$ imply that $(\alpha+\eta-1)(1-\delta) < \delta-\mu < 0$ or $0 < \delta-\mu < (\alpha+\eta-1)(1-\delta)$.

Equilibrium without intrinsically useless assets

Without intrinsically useless assets, system (A5) simplifies to the following system:

$$(A15a) \quad n\gamma(k_{t+1})k_{t+1} + ne(k_{t+1}) = z[R(k_{t+1})][w(k_t) + q_t - R(k_t)e(k_t)/\gamma(k_t) + R(k_t)e_{1t-1}/\gamma(k_t) - j_t] -$$

$$[1 - z(R(k_{t+1}))][n\gamma(k_{t+1})(j_{t+1} - q_{t+1})/R(k_{t+1}) - ne_{1t}]$$

$$(A15b) \quad \psi n[\gamma(k_{t+1})c_{1t+1}/c_{1t}]^{1-\theta} \leq R(k_{t+1}) \leq (1/\beta)n[\gamma(k_{t+1})c_{1t+1}/c_{1t}]^{1-\theta} \quad (= \text{if } j_{t+1} > 0, = \text{if } q_{t+1} + e_{1t} > 0)$$

$$(A15c) \quad f(k_t, k_t) = c_{1t} + [\rho R(k_t)]^{1/(1-\theta)} c_{1t-1} / n\gamma(k_t) + n\gamma(k_{t+1})k_{t+1} + ne(k_{t+1})$$

where $c_{2t}=[\rho R(k_t)]^{1/(1-\theta)}c_{1t-1}/\gamma(k_t)$ by (38a).

If $j_{t+1}=q_{t+1}=e_{1t}=0$, system (A15) simplifies to the scalar system (A9). If $j_{t+1}>0$ or $q_{t+1}+e_{1t}>0$, system (A15) simplifies to the following planar system:

$$(A16a) \quad n\gamma(k_{t+1})k_{t+1}+ne(k_{t+1})=f(k_t, k_t)-[1+(n\rho/\kappa)^{1/(1-\theta)}/n]c_{1t}$$

$$(A16b) \quad n[\gamma(k_{t+1})c_{1t+1}/c_{1t}]^{1-\theta}=\kappa R(k_{t+1})$$

where $\kappa=\beta$ if $q_{t+1}+e_{1t}>0$ and $\kappa=1/\psi$ if $j_{t+1}>0$.

Equations (A16) define implicitly the following mappings in the forward dynamics:

$$(A17a) \quad k_{t+1}=\chi^1(k_t, c_{1t})$$

$$(A17b) \quad c_{1t+1}=\kappa^{1/(1-\theta)}R[\chi^1(k_t, c_{1t})]^{1/(1-\theta)}c_{1t}/n\gamma[\chi^1(k_t, c_{1t})]\equiv\chi^2(k_t, c_{1t})$$

From (A17) it follows that

$$(A18a) \quad \chi^1_1=(f_1+f_2)/(n\gamma k_{t+1}+n\gamma_t+ne')$$

$$(A18b) \quad \chi^1_2=-[1+(n\rho/\kappa)^{1/(1-\theta)}/n]/(n\gamma k_{t+1}+n\gamma_t+ne')$$

$$(A18c) \quad \chi^2_1=[R'k_{t+1}/(1-\theta)R_{t+1}-\gamma'k_{t+1}/\gamma_t](c_{1t+1}/k_{t+1})\chi^1_1$$

$$(A18d) \quad \chi^2_2=\kappa^{1/(1-\theta)}R_{t+1}^{1/(1-\theta)}/n\gamma_t+[R'k_{t+1}/(1-\theta)R_{t+1}-\gamma'k_{t+1}/\gamma_t](c_{1t+1}/k_{t+1})\chi^1_2$$

It follows that the properties of mapping $\chi(k_t, c_{1t})$ depend on mappings $R(k_{t+1})$, $e(k_{t+1})$ and $\gamma(k_{t+1})$. The economy can have multiple operative transfer motive steady states if mappings $\kappa R(k)$ and $n\gamma(k)^{1-\theta}$ have multiple crossing points. With C-D production functions $f(k_t, \bar{k}_t)=k_t^\alpha \bar{k}_t^{1-\alpha}$ and $g(e_{t-1}, \bar{k}_t)=e_{t-1}^\delta \bar{k}_t^{1-\mu}$ mappings $\kappa R(k)=\kappa\alpha k^{\alpha+\eta-1}$ and $n\gamma(k)^{1-\theta}=n[\delta(1-\alpha)/\alpha]^{\delta(1-\theta)/(1-\delta)}k^{(\delta-\mu)(1-\theta)/(1-\delta)}$ have a unique crossing point, but the dynamics of the economy can be indeterminate or display periodic solutions if $\theta \neq 0$.

With general forms of the production functions and CES utility function, it is possible that operative and inoperative transfer motive steady state equilibria can co-exist as shown by Thibault (2000). To consider the co-existence of different types of steady state equilibria, let us denote an operative gift motive steady state by k^g , an operative bequest or altruistic education motive steady state by k^q , a non-trivial inoperative transfer motive steady state by k^0 , a non-trivial steady state in the economy without altruism and intrinsically useless assets by k^D and a non-trivial steady state solution for system (A16) by k^{inf} . By using this notation we can show:

PROPOSITION A3: *Suppose that economy has $p \geq 0$ non-trivial k^D , $n > 0$ non-trivial k^{inf} for $\kappa = \beta$ and $m > 0$ non-trivial k^{inf} for $\kappa = 1/\psi$. Competitive equilibrium of the economy has at least one non-trivial steady state equilibrium and at most $p+m+n$. The non-trivial steady state equilibrium is unique if*

- (i) $m=1, n=1$ and $p=0$
- (ii) $m=1, n=1$ and $\beta\gamma^\theta R > n\gamma$ or $(1/\psi)\gamma^\theta R < n\gamma$ for all k^D
- (iii) $\beta\gamma^\theta R \leq n\gamma \leq (1/\psi)\gamma^\theta R$ for a unique k^D and
 $n\gamma \geq [z(w-Re/\gamma)-ne]/k$ for all k^{inf} with $\kappa = 1/\psi$ and
 $n\gamma \leq [z(w-Re/\gamma)-ne]/k$ for all k^{inf} with $\kappa = \beta$

PROOF: Suppose that $\exists k_1^D > 0, \dots, k_p^D > 0, k_1^{\text{inf}} > 0, \dots, k_n^{\text{inf}} > 0$ for $\kappa = \beta$ and $k_1^{\text{inf}} > 0, \dots, k_m^{\text{inf}} > 0$ for $\kappa = 1/\psi$, where $p \geq 0, m > 0$ and $n > 0$. From (A16) it follows that $k^{\text{inf}} = k^q$ if and only if $[z(w-Re/\gamma)-ne]/k < n\gamma = \beta\gamma^\theta R$, $k^{\text{inf}} = k^j$ if and only if $[z(w-Re/\gamma)-ne]/k > n\gamma = (1/\psi)\gamma^\theta R$ and $k^D = k^0$ if and only if $\beta\gamma^\theta R \leq n\gamma = [z(w-Re/\gamma)-ne]/k \leq (1/\psi)\gamma^\theta R$.

By using these conditions we can divide the existence proof of the non-trivial steady state into four cases:

(1) Suppose that $p=0$. Equation (A9) implies that $n\gamma(k) > [z(w-Re/\gamma)-ne]/k$ or $n\gamma(k) < [z(w-Re/\gamma)-ne]/k$ for all k , which implies that for all $k^{\text{inf}} = k^q$ or for all $k^{\text{inf}} = k^j$. Hence, the economy has n non-trivial steady states $k^q > 0$ or m non-trivial steady states $k^j > 0$.

(2) Suppose that $p > 0$ and for all $k^D \neq k^0$. It can be show by contradiction that $k^{\text{inf}} = k^q$ or $k^{\text{inf}} = k^j$ for at least one $k > 0$. Suppose that $k^{\text{inf}} \neq k^q$ and $k^{\text{inf}} \neq k^j$, i.e., $[z(w-Re/\gamma)-ne]/k \geq n\gamma = \beta\gamma^\theta R$ and $[z(w-Re/\gamma)-ne]/k \leq n\gamma = (1/\psi)\gamma^\theta R$. It follows by continuity that \exists at least one $k > 0$ such that $\beta\gamma^\theta R \leq [z(w-Re/\gamma)-ne]/k = n\gamma \leq (1/\psi)\gamma^\theta R$, i.e., $k^D = k^0 > 0$, which contradicts the original assumption. Hence, the economy has at least one trivial steady states $k^q > 0$ or $k^j > 0$.

(3) Suppose that $p > 0$ and for all $k^{\text{inf}} \neq k^q$ and for all $k^{\text{inf}} \neq k^j$. It follows by continuity that the economy has at least one $k^0 > 0$.

(4) Suppose that $p > 0$ and $k_i^D = k_i^0$ for $i \in \{1, \dots, z \mid z \leq p\}$, $k_i^{\text{inf}} = k_i^q$ for $i \in \{1, \dots, x \mid x \leq n\}$ and $k_i^{\text{inf}} = k_i^j$ for $i \in \{1, \dots, y \mid y \leq m\}$. It follows that the economy has z non-trivial steady states $k^0 > 0$, x non-trivial steady states $k^q > 0$ and y non-trivial steady states $k^j > 0$.

From (1)-(4) it follows that the economy has at least one non-trivial steady state and at most $p+m+n$. Moreover, it follows that:

- (i) If $p=0, m=1$ and $n=1$, then the economy has a unique $k^q > 0$ or $k^j > 0$ by case (1).
- (ii) If $p > 0, m=1, n=1$ and for all $k^D \neq k^0$, i.e., $\beta\gamma(k^D)^\theta R(k^D) > n\gamma(k^D)$ or $(1/\psi)\gamma(k^D)^\theta R(k^D) < n\gamma(k^D)$, then the economy has a unique $k^q > 0$ or $k^j > 0$ by case (2).
- (iii) If $p > 0, k_i^D = k_i^0$ for one $i \in \{1, \dots, p\}$, for all $k^{\text{inf}} \neq k^q$ and for all $k^{\text{inf}} \neq k^j$, i.e., $\beta\gamma^\theta R \leq n\gamma \leq (1/\psi)\gamma^\theta R$ for a unique $k^D > 0$ and $n\gamma = (1/\psi)\gamma^\theta R \geq [z(w-Re/\gamma)-ne]/k^{\text{inf}}$ and

$n\gamma = \beta\gamma^\theta R \leq [z(w-Re/\gamma) - ne]/k^{\text{inf}}$, then the economy has a unique $k^0 > 0$ by case (3).
Q.E.D.

Proposition A3 is an extension of the results by Thibault (2000) in the exogenous growth model with one-sided altruism to the endogenous growth model with human capital and two-sided altruism. Proposition 5(i) in Chapter 4 is a special case of Proposition A3. With C-D production functions $f(k_t, \bar{k}_t) = k_t^\alpha \bar{k}_t^\eta$ and $g(e_{t-1}, \bar{k}_t) = e_{t-1}^\delta \bar{k}_t^{-\mu}$ and a log-linear utility function, we have $m=1$, $n=1$ and $p=1$ and condition (ii) or (iii) is always satisfied.

With general forms of the production functions and CES utility function, the relationship between different types of steady state equilibria can be more complicated than Proposition 6 indicates. In particular, if the economy has multiple Golden Rule allocations, the relationship between operative and inoperative transfer motive steady states is ambiguous. On the other hand, if the economy has a unique Golden Rule allocation, we can show that

PROPOSITION A4: *Suppose that the economy has a unique Golden Rule allocation $n\gamma(k^{\text{GR}}) = R(k^{\text{GR}})$. If $R'(k^{\text{GR}}) < n\gamma'(k^{\text{GR}})$, then $k^q \leq k^0 \leq k^j$. If $R'(k^{\text{GR}}) > n\gamma'(k^{\text{GR}})$, then $k^j \leq k^0 \leq k^q$.*

PROOF: From the transversality condition (A5k-m) it follows that an operative transfer motive steady state allocation must satisfy $\lim_{t \rightarrow \infty} \beta^t (q/c_1)(c_{10}\gamma^t)^0 = 0$, $\lim_{t \rightarrow \infty} \beta^t (e_1/c_1)(c_{10}\gamma^t)^0 = 0$ or $\lim_{t \rightarrow \infty} (1/\psi)^t (j/c_1)(c_{10}\gamma^t)^0 = 0$, which implies that $\beta\gamma^\theta < 1$ or $(1/\psi)\gamma^\theta > 1$. Substituting this to the Euler equation (A16b) gives $n\gamma(k^q)^{1-\theta} = \beta R(k^q) \Rightarrow n\gamma(k^q) = \beta\gamma(k^q)^\theta R(k^q) < R(k^q)$ and $n\gamma(k^j)^{1-\theta} = (1/\psi)R(k^j) \Rightarrow n\gamma(k^j) = (1/\psi)\gamma(k^j)^\theta R(k^j) > R(k^j)$.

From the uniqueness of k^{GR} it follows that $R < n\gamma$ for $k > k^{\text{GR}}$ and $R > n\gamma$ for $k < k^{\text{GR}}$ if $R'(k^{\text{GR}}) < n\gamma'(k^{\text{GR}})$ and $R < n\gamma$ for $k < k^{\text{GR}}$ and $R > n\gamma$ for $k > k^{\text{GR}}$ if $R'(k^{\text{GR}}) > n\gamma'(k^{\text{GR}})$. It follows that $k^q < k^j$ if $R'(k^{\text{GR}}) < n\gamma'(k^{\text{GR}})$. Moreover, it is not possible that $k^0 < k^q$ or $k^0 > k^j$, because $\beta R(k^0) > n\gamma(k^0)$ and $(1/\psi)R(k^0) < n\gamma(k^0)$ are not feasible equilibria by (A15b). Hence, if k^0 exists, it must satisfy $k^q \leq k^0 \leq k^j$. On the other hand, $k^q > k^j$ if $R'(k^{\text{GR}}) > n\gamma'(k^{\text{GR}})$. Moreover, it is not possible that $k^0 > k^q$ or $k^0 < k^j$, because $\beta R(k^0) > n\gamma(k^0)$ and $(1/\psi)R(k^0) < n\gamma(k^0)$ are not feasible equilibria by (A15b). Hence, if k^0 exists, it must satisfy $k^j \leq k^0 \leq k^q$.
Q.E.D.

Proposition 6 in Chapter 4 is a special case of Proposition A4. With C-D production functions $f(k_t, \bar{k}_t) = k_t^\alpha \bar{k}_t^\eta$ and $g(e_{t-1}, \bar{k}_t) = e_{t-1}^\delta \bar{k}_t^{-\mu}$ and a log-linear utility function, the economy has a unique $k^{\text{GR}} > 0$. Moreover, condition $R'(k^{\text{GR}}) < \gamma'(k^{\text{GR}})$ implies that $(\alpha + \eta)(1 - \delta)/(1 - \mu) < 1$ and condition $R'(k^{\text{GR}}) > \gamma'(k^{\text{GR}})$ implies that $(\alpha + \eta)(1 - \delta)/(1 - \mu) > 1$ [or $(\alpha + \eta)(1 - \delta)/(1 - \mu) < 0$ if we allow $\mu > 1$].

Equilibrium with intrinsically useless assets and altruism

It is straight forward to show that similar results as Proposition A3 and A4 hold, except that operative gift motive steady states are dominated by bubble steady states.

APPENDIX 2: CALIBRATION OF THE MODEL BY MATCAD

Exogenous growth model (figure 3)

Parameters

$$\alpha := 0.25 \quad \Lambda_2 := 1.14^{25} \quad \rho := 0.96^{25} \quad n := 1.01^{25}$$

$$\Lambda_1 := 1.02^{25} \quad \mu := 0 \quad \eta := 0 \quad \delta := 0 \quad \pi := \left(\frac{1.01}{1.07}\right)^{25}$$

Highest sustainable deficit/GDP ratio

$$d := 0.041$$

Equations

$$k := 0.01, 0.02.. 5$$

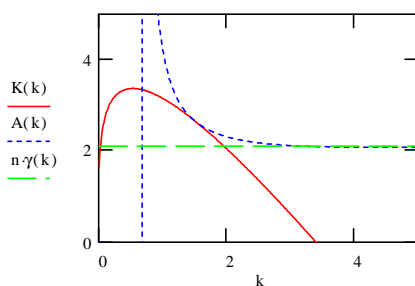
$$R(k) := \alpha \cdot \Lambda_2 \cdot k^{\alpha + \eta - 1}$$

$$w(k) := (1 - \alpha) \cdot \Lambda_2 \cdot k^{\alpha + \eta}$$

$$\gamma(k) := \left(\Lambda_1\right)^{\frac{1}{1 - \delta}}$$

$$K(k) := (1 - \delta) \cdot \frac{\rho}{1 + \rho} \cdot w(k) - n \cdot \left[1 + (1 - \alpha) \cdot \frac{\delta}{\alpha}\right] \cdot \gamma(k) \cdot k$$

$$A(k) := \frac{d \cdot \Lambda_2 \cdot k^{\alpha}}{1 - \pi \cdot \frac{R(k)}{n \cdot \gamma(k)}}$$



Observed variables

$$\sqrt[25]{\pi \cdot R(1.46)} = 1.007$$

$$\sqrt[25]{R(1.46)} = 1.066$$

$$\sqrt[25]{n \cdot \gamma(1.46)} = 1.03$$

Tangency point

$$K(1.46) = 2.708$$

$$A(1.46) = 2.707$$

Highest sustainable debt/GDP ratio

$$\frac{A(1.46)}{1.14 \cdot 1.46^{\alpha + \eta}} = 2.16$$

k =	K(k) =	A(k) =	n·γ(k) =
0.01	1.642	-0.015	2.104
0.02	1.935	-0.031	2.104
0.03	2.125	-0.049	2.104
0.04	2.267	-0.066	2.104
0.05	2.381	-0.085	2.104
0.06	2.476	-0.105	2.104
0.07	2.557	-0.125	2.104
0.08	2.628	-0.146	2.104
0.09	2.69	-0.169	2.104
0.1	2.746	-0.192	2.104
0.11	2.796	-0.216	2.104
0.12	2.842	-0.242	2.104
0.13	2.884	-0.268	2.104
0.14	2.922	-0.295	2.104
0.15	2.956	-0.324	2.104
0.16	2.989	-0.354	2.104

Endogenous growth model without technological progress (figure 4)

Parameters

$$\alpha := 0.25 \quad \Lambda_2 := 1.14^{25} \quad \rho := 0.96^{25} \quad n := 1.01^{25}$$

$$\Lambda_1 := 1.02^{25} \quad \mu := 0 \quad \eta := 0 \quad \delta := 0.1 \quad \pi := \left(\frac{1.01}{1.07}\right)^{25}$$

Highest sustainable deficit/GDP ratio

$$d := 0.021$$

Equations

$$k := 0.01, 0.02.. 5$$

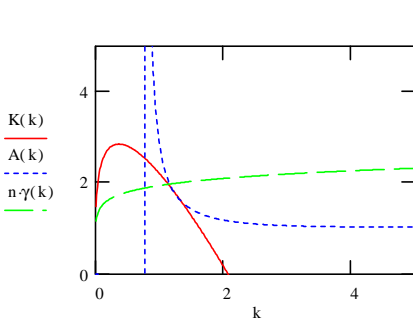
$$R(k) := \alpha \cdot \Lambda_2 \cdot k^{\alpha + \eta - 1}$$

$$w(k) := (1 - \alpha) \cdot \Lambda_2 \cdot k^{\alpha + \eta}$$

$$\gamma(k) := \left(\Lambda_1\right)^{\frac{1}{1-\delta}} \cdot \left[(1 - \alpha) \cdot \frac{\delta}{\alpha} \right]^{\frac{\delta}{1-\delta}} \cdot k^{\frac{\delta - \mu}{1-\delta}}$$

$$K(k) := (1 - \delta) \cdot \frac{\rho}{1 + \rho} \cdot w(k) - n \cdot \left[1 + (1 - \alpha) \cdot \frac{\delta}{\alpha} \right] \cdot \gamma(k) \cdot k$$

$$A(k) := \frac{d \cdot \Lambda_2 \cdot k^\alpha}{1 - \pi \cdot \frac{R(k)}{n \cdot \gamma(k)}}$$



Observed variables

Tangency point

$$\sqrt[25]{\pi \cdot R(1.3)} = 1.01$$

$$K(1.3) = 1.669$$

$$\sqrt[25]{R(1.3)} = 1.07$$

$$A(1.3) = 1.654$$

$$\sqrt[25]{n \cdot \gamma(1.3)} = 1.028$$

Highest sustainable deficit/GDP ratio

$$\frac{A(1.3)}{1.14 \cdot 1.3^{\alpha + \eta}} = 1.359$$

k =	K(k) =	A(k) =	n·γ(k) =
0.01	1.481	4·10 ⁻³	1.166
0.02	1.747	3·10 ⁻³	1.259
0.03	1.918	-0.015	1.317
0.04	2.045	-0.021	1.36
0.05	2.147	-0.027	1.394
0.06	2.231	-0.034	1.423
0.07	2.302	-0.041	1.447
0.08	2.364	-0.049	1.469
0.09	2.418	-0.056	1.488
0.1	2.465	-0.065	1.506
0.11	2.508	-0.073	1.522
0.12	2.545	-0.082	1.536
0.13	2.579	-0.091	1.55
0.14	2.61	-0.101	1.563
0.15	2.638	-0.111	1.575
0.16	2.663	-0.121	1.586

Endogenous growth model with the erosion effect of technological progress (figure 5)

Parameters

$$\alpha := 0.25 \quad \Lambda_2 := 1.14^{25} \quad \rho := 0.96^{25} \quad n := 1.01^{25}$$

$$\Lambda_1 := 1.02^{25} \quad \mu := 0.2 \quad \eta := 0 \quad \delta := 0.1 \quad \pi := \left(\frac{1.01}{1.07}\right)^{25}$$

Highest sustainable deficit/GDP ratio

$$d := 0.021$$

Equations

$$k := 0.01, 0.02.. 5$$

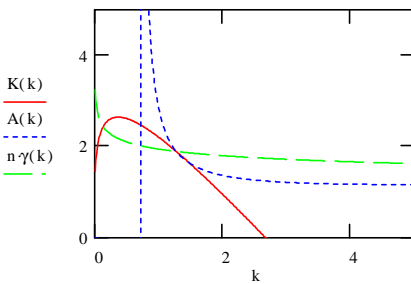
$$R(k) := \alpha \cdot \Lambda_2 \cdot k^{\alpha + \eta - 1}$$

$$w(k) := (1 - \alpha) \cdot \Lambda_2 \cdot k^{\alpha + \eta}$$

$$\gamma(k) := (\Lambda_1)^{\frac{1}{1-\delta}} \cdot \left[(1 - \alpha) \cdot \frac{\delta}{\alpha} \right]^{\frac{\delta}{1-\delta}} \cdot k^{\frac{\delta - \mu}{1-\delta}}$$

$$K(k) := (1 - \delta) \cdot \frac{\rho}{1 + \rho} \cdot w(k) - n \cdot \left[1 + (1 - \alpha) \cdot \frac{\delta}{\alpha} \right] \cdot \gamma(k) \cdot k$$

$$A(k) := \frac{d \cdot \Lambda_2 \cdot k^\alpha}{1 - \pi \cdot \frac{R(k)}{n \cdot \gamma(k)}}$$



Observed variables

Tangency point

$$\sqrt[25]{\pi \cdot R(1.3)} = 1.01$$

$$K(1.3) = 1.861$$

$$A(1.3) = 1.853$$

$$\sqrt[25]{R(1.3)} = 1.07$$

Highest sustainable
debt/GDP ratio

$$\sqrt[25]{n \cdot \gamma(1.3)} = 1.026$$

$$\frac{A(1.3)}{1.14 \cdot 1.3^{\alpha + \eta}} = 1.522$$

k =	K(k) =	A(k) =	n · γ(k) =
0.01	1.454	-0.012	3.244
0.02	1.701	-0.024	3.003
0.03	1.857	-0.035	2.871
0.04	1.972	-0.047	2.781
0.05	2.061	-0.059	2.713
0.06	2.135	-0.071	2.658
0.07	2.196	-0.084	2.613
0.08	2.249	-0.097	2.575
0.09	2.294	-0.111	2.541
0.1	2.334	-0.125	2.512
0.11	2.37	-0.14	2.485
0.12	2.401	-0.155	2.461
0.13	2.429	-0.17	2.439
0.14	2.454	-0.186	2.419
0.15	2.477	-0.203	2.401
0.16	2.497	-0.221	2.384

Endogenous growth model with the productivity effect of technological progress (figure 6)

Parameters

$$\alpha := 0.25 \quad \Lambda_2 := 1.14^{25} \quad \rho := 0.96^{25} \quad n := 1.01^{25}$$

$$\Lambda_1 := 1.02^{25} \quad \mu := 0 \quad \eta := 2 \quad \delta := 0.1 \quad \pi := \left(\frac{1.01}{1.07}\right)^{25}$$

Highest sustainable deficit/GDP ratio

$$d := 0.021$$

Equations

$$k := 0.01, 0.02 \dots 5$$

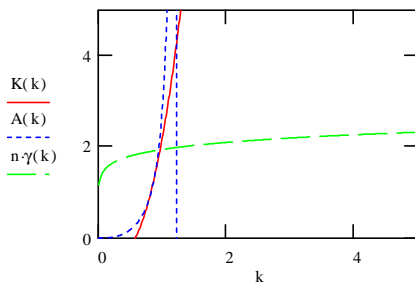
$$R(k) := \alpha \cdot \Lambda_2 \cdot k^{\alpha + \eta - 1}$$

$$w(k) := (1 - \alpha) \cdot \Lambda_2 \cdot k^{\alpha + \eta}$$

$$\gamma(k) := \left(\Lambda_1\right)^{\frac{1}{1-\delta}} \cdot \left[(1 - \alpha) \cdot \frac{\delta}{\alpha} \right]^{\frac{1}{1-\delta}} \cdot k^{\frac{\delta - \mu}{1-\delta}}$$

$$K(k) := (1 - \delta) \cdot \frac{\rho}{1 + \rho} \cdot w(k) - n \cdot \left[1 + (1 - \alpha) \cdot \frac{\delta}{\alpha} \right] \cdot \gamma(k) \cdot k$$

$$A(k) := \frac{d \cdot \Lambda_2 \cdot k^{\alpha + \eta}}{1 - \pi \cdot \frac{R(k)}{n \cdot \gamma(k)}}$$



Observed variables

Tangency point

$$K(0.85) = 1.172$$

$$A(0.85) = 1.161$$

$$\sqrt[25]{\pi \cdot R(0.85)} = 1.01$$

$$\sqrt[25]{R(0.85)} = 1.07$$

$$\sqrt[25]{n \cdot \gamma(0.85)} = 1.026$$

Highest sustainable debt/GDP ratio

$$\frac{A(0.85)}{1.14 \cdot 0.85^{\alpha + \eta}} = 1.468$$

k =	K(k) =	A(k) =	n·γ(k) =
0.01	-0.015	5·10 ⁻⁵	1.166
0.02	-0.032	8·10 ⁻⁵	1.259
0.03	-0.05	3·10 ⁻⁴	1.317
0.04	-0.067	6·10 ⁻⁴	1.36
0.05	-0.085	8·10 ⁻⁴	1.394
0.06	-0.103	2·10 ⁻³	1.423
0.07	-0.12	5·10 ⁻³	1.447
0.08	-0.137	8·10 ⁻³	1.469
0.09	-0.153	6·10 ⁻³	1.488
0.1	-0.169	9·10 ⁻³	1.506
0.11	-0.185	2·10 ⁻²	1.522
0.12	-0.2	4·10 ⁻²	1.536
0.13	-0.214	2·10 ⁻²	1.55
0.14	-0.228	6·10 ⁻²	1.563
0.15	-0.241	6·10 ⁻²	1.575
0.16	-0.253	4·10 ⁻²	1.586

A-SARJA: VÄITÖSKIRJOJA - DOCTORAL DISSERTATIONS. ISSN 1237-556X.

- A:197. ANSSI ÖÖRNI: Consumer Search in Electronic Markets. 2002. ISBN 951-791-680-9.
- A:198. ARI MATIKKA: Measuring the Performance of Owner-Managed Firms: A systems approach. 2002. ISBN 951-791-685-X.
- A:199. RIITTA KOSONEN: Governance, the Local Regulation Process, and Enterprise Adaptation in Post-Socialism. The Case of Vyborg. 2002. ISBN 951-791-692-2.
- A:200. SUSANNE SUHONEN: Industry Evolution and Shakeout Mechanisms: The Case of the Internet Service Provider Industry. 2002. ISBN 951-791-693-0.
- A:201. MATTI TUOMINEN: Market-Driven Capabilities and Operational Performance. Theoretical Foundations and Managerial Practices. 2002. ISBN 95-791-694-9.
- A:202. JUSSI KARHUNEN: Essays on Tender Offers and Share Repurchases. 2002. ISBN 951-791-696-5.
- A:203. HENNAMARI MIKKOLA: Empirical Studies on Finnish Hospital Pricing Methods. 2002. ISBN 951-791-714-7.
- A:204. MIKA KORTELAINEN: EDGE: a Model of the Euro Area with Applications to Monetary Policy. 2002. ISBN 951-791-715-5.
- A:205. TOMI LAAMANEN: Essays on Technology Investments and Valuation. 2002. ISBN 951-791-716-3.
- A:206. MINNA SÖDERQVIST: Internationalisation and its Management at Higher-Education Institutions. Applying Conceptual, Content and Discourse Analysis. 2002. ISBN 951-791-718-X.
- A:207. TARJA PIETILÄINEN: Moninainen yrittäminen. Sukupuoli ja yrittäjänäisten toimintatila tietoteollisuudessa. 2002. ISBN 951-791-719-8.
- A:208. BIRGIT KLEYMANN: The Development of Multilateral Alliances. The Case of the Airline Industry. 2002. ISBN 951-791-720-1.
- A:209. MIKAEL EPSTEIN: Risk Management of Innovative R&D Project. Development of Analysis Model. A Systematic Approach for the Early Detection of Complex Problems (EDCP) in R&D Projects in Order to Increase Success in Enterprises. 2002. ISBN 951-791-717-9.
- A:210. SAMI KAJALO: Deregulation of Retail Hours in Finland: Historical and Empirical Perspectives. 2002. ISBN 951-791-734-1.
- A:211. TOMMI KASURINEN: Exploring Management Accounting Change in the Balanced Scorecard Context. Three Perspectives. 2003. ISBN 951-791-736-8.

- A:212. LASSE NIEMI: Essays on Audit Pricing. 2003. ISBN 951-791-751-1.
- A:213. MARKKU KAUSTIA: Essays on Investor Behavior and Psychological Reference Prices. 2003. ISBN 951-791-754-6.
- A:214. TEEMU YLIKOSKI: Access Denied: Patterns of Consumer Internet Information Search and the Effects of Internet Search Expertise. 2003. ISBN 951-791-755-4.
- A:215. PETRI HALLIKAINEN: Evaluation of Information System Investments. 2003. ISBN 951-791-758-9.
- A:216. PETRI BÖCKERMAN: Empirical Studies on Working Hours and Labour Market Flows. 2003. ISBN 951-791-760-0.
- A:217. JORMA PIETALA: Päivittäistavara-kaupan dynamiikka ja ostokäyttäytyminen Pääkaupunkiseudulla. 2003. ISBN 951-791-761-9.
- A:218. TUOMAS VÄLIMÄKI: Central Bank Tenders: Three Essays on Money Market Liquidity Auctions. 2003. ISBN 951-791-762-7.
- A:219. JUHANI LINNAINMAA: Essays on the Interface of Market Microstructure and Behavioral Finance. 2003. ISBN 951-791-783-X.
- A:220. MARKKU SALIMÄKI: Suomalaisen design-teollisuuden kansainvälinen kilpailukyky ja kansainvälistyminen. Strateginen ryhmä – tutkimus design-aloilta. 2003 ISBN 951-791-786-4.
- A:221. HANNU KAHRA: Consumption, Liquidity and Strategic Asset Allocation. 2003. ISBN 951-791-791-0.
- A:222. TONI RIIPINEN: The Interaction of Environmental and Trade Policies. 2003. ISBN 951-791-797-X.
- A:223. MIKKO SYRJÄNEN: Data Envelopment Analysis in Planning and Heterogeneous Environments. 2003. ISBN 951-791-806-2.
- A:224. ERKKI HÄMÄLÄINEN: Evolving Logistic Roles of Steel Distributors. 2003. ISBN 951-791-807-0.
- A:225. SILJA SIITONEN: Impact of Globalisation and Regionalisation Strategies on the Performance of the World's Pulp and Paper Companies. 2003. ISBN 951-791-808-9.
- A:226. EIREN TUUSJÄRVI: Multifaceted Norms in SMC Export Cooperation: A Discourse Analysis of Normative Expectations. 2003. ISBN 951-791-812-7.
- A:227. MIKA MALIRANTA: Micro Level Dynamics of Productivity Growth. An Empirical Analysis of the Great Leap in Finnish Manufacturing Productivity in 1975-2000. 2003. ISBN 951-791-815-1.
- A:228. NINA KOISO-KANTTILA: Essays on Consumers and Digital Content. 2003. ISBN 951-791-816-X.
- A:229. PETER GABRIELSSON: Globalising Internationals: Product Strategies of ICT Companies. 2004. ISBN 951-791-825-9, ISBN 951-791-826-7 (Electronic dissertation).

- A:230. SATU NURMI: Essays on Plant Size, Employment Dynamics and Survival. 2004. ISBN 951-791-829-1, ISBN 951-791-830-5 (Electronic dissertation).
- A:231. MARJA-LIISA KURONEN: Vakuutusehtotekstin uudistamisprosessi, matkalla alamaaisesta asiakkaaksi. 2004. ISBN 951-791-833-X, ISBN 951-791-834-8 (Electronic dissertation).
- A:232. MIKA KUISMA: Eri-laistuminen vai samanlaistuminen? Vertaileva tutkimus paperiteollisuusyhtiöiden ympäristöjohtamisesta. 2004. ISBN 951-791-835-6, ISBN 951-791-836-4 (Electronic dissertation).
- A:233. ANTON HELANDER: Customer Care in System Business. 2004. ISBN 951-791-838-0.
- A:234. MATTI KOIVU: A Stochastic Optimization Approach to Financial Decision Making. 2004. ISBN 951-791-841-0, ISBN 951-791-842-9 (Electronic dissertation).
- A:235. RISTO VAITTINEN: Trade Policies and Integration – Evaluations with CGE -models. 2004. ISBN 951-791-843-7, ISBN 951-791-844-5 (Electronic dissertation).
- A:236. ANU VALTONEN: Rethinking Free Time: A Study on Boundaries, Disorders, and Symbolic Goods. 2004. ISBN 951-791-848-8, ISBN 951-791-849-6 (Electronic dissertation).
- A:237. PEKKA LAURI: Human Capital, Dynamic Inefficiency and Economic Growth. 2004. ISBN 951-791-854-2, ISBN 951-791-855-0 (Electronic dissertation).

A-SARJA: MUITA JULKAISUJA - OTHER PUBLICATIONS

ANNE HERBERT: The Paradoxes of Action Learning: An Interpretive and Critical Inquiry into Vocational Educators' Professional Development. 2002. ISBN 951-791-684-1.

B-SARJA: TUTKIMUKSIA - RESEARCH REPORTS. ISSN 0356-889X.

- B:38. KRISTIINA KORHONEN(ed.): Current Reflections on the Pacific Rim. 2002. ISBN 951-791-661-2.
- B:39. RISTO Y. JUURMAA: Performance and International Competitiveness of Listed Metal and Telecommunication Industry Groups 1992 - 2000. Finland vs Sweden and Germany. 2002. ISBN 951-791-668-X.
- B:40. KAIJA TUOMI – SINIKKA VANHALA (toim.): Yrityksen toiminta, menestyminen ja henkilöstön hyvinvointi. Seurantatutkimus metalliteollisuudessa ja vähittäiskaupan alalla. 2002. ISBN 951-791-674-4.
- B:41. ANNE ÄYVÄRI: Verkottuneen pienyrityksen markkinointikyvykkyys. 2002. ISBN 951-791-682-5.
- B:42. RIKU OKSMAN: Intohimoa ja ammattitaitoa: puheenvuoroja tuottajan työstä. 2002. ISBN 951-791-700-7.

- B:43. RISTO TAINIO – KARI LILJA – TIMO SANTALAINEN: Organizational Learning in the Context of Corporate Growth and Decline: A Case Study of a Major Finnish Bank. 2002. ISBN 951-791-717-1
- B:44. ELINA HENTTONEN – PÄIVI ERIKSSON – SUSAN MERILÄINEN: Teknologiyrittämisen sukupuoli. Naiset miesten maailmassa. 2003. ISBN 951-791-737-6.
- B:45. KIRSI KORPIAHO: "Kyllä siinä pitää elää mukana!" Kirjanpitäjien tarinoita työstä, osaamisesta ja oppimisesta työyhteisönäkökulmasta analysoituna. 2003. ISBN 951-791-742-2.
- B:46. NIILLO HOME (toim.): Puheenvuoroja ECR-toiminnasta. Discussions on ECR – Summaries. 2003. ISBN 951-791-749-X.
- B:47. PÄIVI KARHUNEN – RIITTA KOSONEN – MALLA PAAJANEN: Gateway-käsitteen elinkaari Venäjän-matkailussa. Etelä-Suomi Pietarin-matkailun väylänä. 2003. ISBN 951-791-756-2.
- B:48. ANNELI KAUPPINEN – ANNE ARANTO – SATU RÄMÖ (toim.): Myyttiset markkinat. 2003. ISBN 951-791-771-6.
- B:49. MIKKO SAARIKIVI – SIMO RIIHONEN: Suomen puuteollisuuden kilpailukyvyyn parantaminen ja kansainvälistyminen piha- ja ympäristörakentamisessa. 2003. ISBN 951-791-779-1.
- B:50. KATARIINA KEMPPAINEN – ARI P.J. VEPSÄLÄINEN – JUKKA KALLIO – TIMO SAARINEN – MARKKU TINNILÄ: From Supply Chain to Networks: A Study of SCM Practices in Finnish Industrial Companies. 2003. ISBN 951-791-780-5.
- B:51. SAMI SARPOLA: Enterprise Resource Planning (ERP) Software Selection and Success of Acquisition Process in Wholesale Companies. 2003. ISBN 951-791-802-X.
- B:52. MATTI TUOMINEN (ed.): Essays on Capabilities Based Marketing and Competitive Superiority. Fimac II - Research: Mai Anttila, Saara Hyvönen, Kristian Möller, Arto Rajala, Matti Tuominen. 2003. ISBN 951-791-814-3.
- B:53. PÄIVI KARHUNEN – RIITTA KOSONEN – ANTTI LEIVONEN: Osaamisen siirtyminen Suomalais-venäläisissä tuotantoalliansseissa. Tapaustutkimuksia pietarista ja leningradin alueelta. 2003. ISBN 951-791-820-8.
- B:54. JARMO ERONEN: Kielten välinen kilpailu: Taloustieteellis-sosiolingtivistinen tarkastelu. 2004. ISBN 951-791-828-3.
- B:47. PÄIVI KARHUNEN – RIITTA KOSONEN – MALLA PAAJANEN: Gateway-käsitteen elinkaari Venäjän-matkailussa. Etelä-Suomi Pietarin-matkailun väylänä. 2004. ISBN 951-791-846-1, korjattu painos.

CKIR-SARJA: HELSINKI SCHOOL OF ECONOMICS. CENTER FOR KNOWLEDGE AND INNOVATION RESEARCH. CKIR WORKING PAPERS. ISSN 1458-5189.

- CKIR:1. SATINDER P. GILL: The Engagement Space and Parallel Coordinated Movement: Case of a Conceptual Drawing Task. 2002. ISBN 951-791-660-4.
- CKIR:2 PEKKA ISOTALUS – HANNI MUUKKONEN: How Do Users of Pda's React to an Animated Human Character in Online News? 2002. ISBN 951-791-664-7.

E-SARJA: SELVITYKSIÄ - REPORTS AND CATALOGUES. ISSN 1237-5330.

- E:100. JUHA KINNUNEN: Opiskelijoiden valikoituminen pääaineisiin Helsingin kauppakorkeakoulussa. Pääainetoiveita ja niihin vaikuttavia tekijöitä kartoittava kyselytutkimus vuosina 1995-2000 opintonsa aloittaneista. 2002. ISBN 951-791-669-8.
- E:101. Research Catalogue 2000 – 2002. Projects and Publications. 2002. ISBN 951-791-670-1.
- E:102. DAN STEINBOCK: The U.S. CIBER Experience: The Centers for International Business Education and Research (CIBERs). 2003. ISBN 951-791-781-3.

N-SARJA: HELSINKI SCHOOL OF ECONOMICS. MIKKELI BUSINESS CAMPUS PUBLICATIONS. ISSN 1458-5383

- N:6 JUHA SIIKAVUO: Taloushallinnon opas alkavalle yrittäjälle. 2002. ISBN 951-791-686-8.
- N:7. JOHANNA NISKANEN: Etelä-Savon pk-yritysten vienti Tanskaan: ulkomaankaupan erityisraportti 2001. 2002. ISBN 951-791-687-6.
- N:8. MIKKO NUMMI: Etelä-Savon pk-yritysten vienti Saksaan: ulkomaankaupan erityisraportti 2001. 2002. ISBN 951-791-688-4.
- N:9. NOORA RUOHONEN – RIIKKA OLLI: Etelä-Savon pk-yritysten vienti Tsekkiin: ulkomaankaupan erityisraportti 2001. 2002. ISBN 951-791-689-2.
- N:10. ANNA HÄKKINEN – ESKO LÄIKKÖ: Etelä-Savon pk-yritysten vientikohteena USA: ulkomaankaupan erityisraportti 2001. 2002. ISBN 951-791-690-6.
- N:11. JUHA SIIKAVUO: Verkko-oppimisympäristön kehittäminen. Esimerkinä HA Boctok Venäjänkaupan erikoistumisopintojen yksi moduuli, vuosi: 2002. 2002. ISBN 951-791-695-7.
- N:12. JUHO PETTER PUHAKAINEN: German Venture Capitalists' Decision Criteria in New Venture Evaluation. 2002. ISBN 951-791-650-7.
- N:13. MILJA LEMMETYINEN: Suomalaisyrityksen etabloituminen Saksaan. Ulkomaankaupan erityisraportti 2002. 2002. ISBN 951-791-730-9.
- N:14. TAPIO PALLASVIRTA: Pk-yritysten vienti Espanjaan. Ulkomaankaupan erityisraportti 2002. 2002. ISBN 951-791-731-7.
- N:15. ELINA HAVERINEN: Etelä-Savon pk-yritysten Viron kauppa. Ulkomaankaupan erityisraportti 2003. ISBN 951-791-732-5.
- N:16. REETA RÖNKKÖ: Latinalainen Amerikka markkina-alueena Argentiina ja Brasilia. Ulkomaankaupan erityisraportti 2003. ISBN 951-791-733-3.
- N:17. JAAKKO VARVIKKO – JUHA SIIKAVUO: Koulutus, oppiminen ja akateeminen yrittäjyys. 2003. ISBN 951-791-745-7.

- N:18. ANNE GUSTAFSSON-PESONEN – SATU SIKANEN: Yrittäjäkoulutuksesta yrittäjäksi. 2003 ISBN 951-791-763-5.
- N:19. TOIVO KOSKI: Impact of a venture capitalists' value added on value of a venture. 2003. ISBN 951-791-764-3.
- N:20. LAURA HIRVONEN: Itävalta suomalaisyritysten markkina-alueena. 2003. ISBN 951-791-765-1.
- N:21. LAURA MALIN: Etelä-Savon pk-yritysten vienti Belgiaan. 2003. ISBN 951-791-766-X.
- N:22. JUKKA PREPULA: Ranska suomalaisten pk-yritysten vientikohteena. 2003. ISBN: 951-791-767-8.
- N:23. HENNA HUCZKOWSKI: Pk-yritysten perustaminen Puolaan. 2003. ISBN 951-791-768-6.
- N:24. HENNA KATAJA – LEENA MÄÄTTÄ: Kiina suomalaisen pk-yrityksen vientikohteena. 2003. ISBN: 951-791-769-4.
- N:25. KAROLIINA IJÄS: Etelä-Savon pk-yritysten vienti Puolaan. 2003. ISBN: 951-791-770-8.
- N:26. MARJO VAHLSTEN: Matkailupalvelujen markkinoinnin kehittäminen verkkoyhteistyön avulla. 2003. ISBN: 951-791-792-9.
- N:27. TUULI SAVOLAINEN: Slovakia suomalaisten pk-yritysten markkina-alueena. 2003. ISBN: 951-791-793-7.
- N:28. HARRY MAASTOVAARA: Etelä-Savon yritysten ulkomaankauppa 2001. 2003. ISBN: 951-791-794-5.
- N:31. HANNA PERÄLÄ: Etelä-Savon pk-yritysten vienti Ruotsiin. 2003. ISBN: 951-791-799-6.
- N:34. TOIVO KOSKI – ANTTI EKLÖF: Uudenmaan yrityshautomoista irtaantuneiden yritysten menestyminen, Yrittäjien näkemyksiä yrityshautomotoiminnasta sekä selvitys ”yrittämisestä Työtä 2000” –projektin asiakkaiden yritystoiminnasta. 2003. ISBN 951-791-805-4.

W-SARJA: TYÖPAPEREITA - WORKING PAPERS . ISSN 1235-5674.

- W:304. PETRI BÖCKERMAN – KARI HÄMÄLÄINEN – MIKA MALIRANTA: Explaining Regional Job and Worker Flows. 2002. ISBN 951-791.662-0.
- W:305. PEKKA KORHONEN – MIKKO SYRJÄNEN: Evaluation of Cost Efficiency in Finnish Electricity Distribution. 2002. ISBN 951-791-663-9.
- W:306. SATU NURMI: The Determinants of Plant Survival in Finnish Manufacturing. 2002. ISBN 951-791-665-5.
- W:307. JUSSI KARHUNEN: Taking Stock of Themselves. An Analysis of the Motives and the Market Reaction in Finnish Share Repurchase Programs 2002. ISBN 951-791-666-3.

- W:308. PEKKA ILMAKUNNAS – HANNA PESOLA: Matching Functions and Efficiency Analysis. 2002. ISBN 951-791-671-X.
- W:309. MARKKU SÄÄKSJÄRVI: Software Application Platforms: From Product Architecture to Integrated Application Strategy. 2002. ISBN 951-791-672-8.
- W:310. MILLA HUURROS – HANNU SERISTÖ: Alliancing for Mobile Commerce: Convergence of Financial Institutions and Mobile Operators. 2002. ISBN 951-791-673-6.
- W:311. ANSSI ÖÖRNI: Objectives of Search and Combination of Information Channels in Electronic Consumer Markets: An Explorative Study. 2002. ISBN 951-791-675-2.
- W:312. ANSSI ÖÖRNI: Consumer Search in Electronic Markets: Experimental Analysis of Travel Services. 2002. ISBN 951-791-676-0.
- W:313. ANSSI ÖÖRNI: Dominant Search Pattern in Electronic Markets: Simultaneous or Sequential Search. 2002. ISBN 951-791-677-9.
- W:314. ANSSI ÖÖRNI: The Amount of Search in Electronic Consumer Markets. 2002. ISBN 951-791-678-7.
- W:315. KLAUS KULTTI – TUOMAS TAKALO – TANJA TANAYAMA: R&d Spillovers and Information Exchange: A Case Study. 2002. ISBN 951-791-681-7.
- W:316. OLLI TAHVONEN: Timber Production v.s. Old Growth Conservation with Endogenous Prices and Forest Age Classes. 2002. ISBN 951-791-691-4.
- W:317. KLAUS KULTTI – JUHA VIRRANKOSKI: Price Distribution in a Symmetric Economy. 2002. ISBN 951-791-697-3.
- W:318. KLAUS KULTTI – TONI RIIPINEN: Multilateral and Bilateral Meetings with Production Heterogeneity. 2002. ISBN 951-791-698-1.
- W:319. MARKKU KAUSTIA: Psychological Reference Levels and IPO Stock Returns. 2002. ISBN 951-791-699-X.
- W:320. MERVI LINDQVIST: Possible Research Issues in Management and Control of New Economy Companies. 2002. ISBN 951-791-701-5.
- W:321. MARKO LINDROOS: Coalitions in Fisheries. 2002. ISBN 951-791-702-3.
- W:322. MIKKO SYRJÄNEN: Non-discretionary and Discretionary Factors and Scale in Data Envelopment Analysis. 2002. ISBN 951-791-705-8.
- W:323. KLAUS KULTTI – HANNU VARTIAINEN: VonNeumann-Morgenstern Solution to the Cake Division Problem. 2002. ISBN 951-791-708-2.
- W:324. TOM LAHTI: A Review of the Principal-agent Theory and the Theory of Incomplete Contracts: An Examination of the Venture Capital Context. 2002. ISBN 951-791-709-0.
- W:325. KRISTIAN MÖLLER – PEKKA TÖRRÖNEN: Business Suppliers' Value-Creation Potential: A Capability-based Analysis. 2002. ISBN 951-791-710-4.
- W:326. KRISTIAN MÖLLER – ARTO RAJALA – SENJA SVAHN: Strategic Business Nets – Their Types and Management. 2002. ISBN 951-791-711-2.

- W:327. KRISTIAN MÖLLER – SENJA SVAHN – ARTO RAJALA: Network Management as a Set of Dynamic Capabilities. 2002. ISBN 951-791-712-0.
- W:328. PANU KALMI: Employee Ownership and Degeneration. Evidence from Estonian case studies. 2002. ISBN 951-791-713-9.
- W:329. ANNELI NORDBERG: Yrittäjyys, johtajuus ja johtaminen – uuden talouden innovatiivisia haasteita. 2002. ISBN 951-791-721-X.
- W:330. LEENA LOUHIHALA-SALMINEN: Communication and language use in merged corporations: Cases Stora Enso and Nordea. 2002. ISBN 951-791-722-8.
- W:331. TOMMI KASURINEN: Conceptualising the Encoding Process Related to Institutionalisation in Organisations. From Key Performance Indicator Scorecard to a Strategic Balanced Scorecard. 2002. ISBN 951-791-723-6.
- W:332. PEKKA KORHONEN – HELENA TOPDAGI: Performance of the AHP in Comparison of Gains and Losses. 2002. ISBN 951-791-724-4.
- W:333. TARJA JORO – PEKKA KORHONEN – STANLEY ZIONTS: An Interactive Approach to Improve Estimates of Value Efficiency in Data Envelopment Analysis. 2002. ISBN 951-791-725-2.
- W:334. JUHA-PEKKA TOLVANEN – JEFF GRAY – MATTI ROSSI (edit.): Proceedings of the Second-Domain Specific Modeling Languages Workshop. 2002. ISBN 951-791-726-0.
- W:335. SATU NURMI: Sectoral Differences In Plant Start-up Size. 2003. ISBN 951-791-738-4.
- W:336. SATU NURMI: Plant Size, Age And Growth In Finnish Manufacturing. 2003. ISBN 951-791-739-2.
- W:337. PETRI HALLIKAINEN – HANNU KIVIJÄRVI: Appraisal of Strategic it Investments: Payoffs And Tradeoffs. 2003. ISBN 951-791-740-6.
- W:338. SENJA SVAHN: Knowledge Creation in Business Networks – A Dynamic-capability Perspective. 2003. ISBN 951-791-743-0.
- W:339. KRISTIAN MÖLLER – SENJA SVAHN: Role of Knowledge in the Value Creation in Business Nets. 2003. ISBN 951-791-744-9.
- W:340. ELI MOEN – KARI LILJA: European Works Councils in M-Real and Norske Skog: The Impact of National Traditions in Industrial Relations. 2003. ISBN 951-791-750-3.
- W:341. KJELD MÖLLER: Salatulla "arvopaperistamisella" tuhottiin yrittäjyyttä. 2003. ISBN 951 791-752-X
- W:342. ATSO ANDERSEN: Competition Between European Stock Exchanges. 2003. ISBN 951-791-753-8.
- W:343. MARKO MERISAVO: The Effects of Digital Marketing on Customer Relationships. 2003. ISBN 951-791-757-0.
- W:344. KLAUS KULTTI – JUHA VIRRANKOSKI: Price Distribution in a Random Matching Model. 2003. ISBN 951-791-759-7.

- W:345. PANU KALMI: The Rise and Fall of Employee Ownership in Estonia, 1987-2001. 2003. ISBN 951-791-772-4.
- W:346. SENJA SVAHN: Managing in Networks: Case Study of Different Types of Strategic Nets. 2003. ISBN 951-791-774-0.
- W:347. KRISTIAN MÖLLER – SENJA SVAHN: Crossing East-West Boundaries: Knowledge Sharing in Intercultural Business Networks. 2003. ISBN 951-791-775-9.
- W:348. KRISTIAN MÖLLER – SENJA SVAHN: Managing in Emergence: Capabilities for Influencing the Birth of New Business Fields. 2003. ISBN 951-791-776-7.
- W:349. TOM RAILIO: The Taxation Consequences of Scandinavian Mutual Fund Investments and After-Tax Performance Evaluation. 2003. ISBN 951-791-777-5.
- W:350. KIRSI LAPOINTE: Subjektiiivinen ura työurien tutkimuksessa ja teorioissa. 2003. ISBN 951-791-778-3.
- W:351. PANU KALMI: The Study of Co-operatives in Modern Economics: A Methodological Essay. 2003. ISBN 951-791-783-X.
- W:352. MARJA TAHVANAINEN: Short-term International Assignments: Popular Yet Largely Unknown Way Of Working Abroad. 2003. ISBN 951-791-784-8.
- W:353. MARKKU KUULA – ANTOINE STAM: An Interior Point Method for Multi-party Negotiation Support. 2003. ISBN 951-791-787-2.
- W:354. JOUKO KINNUNEN: Quantification of Ordered-level Business Sentiment Survey Forecasts by Means of External Validation Data. 2003. ISBN 951-791-790-2.
- W:355. TOM RAILIO: The Nature of Disagreements and Comparability Between Finnish Accumulating Mutual Funds and Voluntary Pension Insurances. 2003. ISBN 951-791-798-8.
- W:356. JUKKA JALAVA: 'Has Our Country the Patience Needed to Become Wealthy?' Productivity in the Finnish Manufacturing Industry, 1960-2000. 2003. ISBN 951-791-803-8.
- W:357. JARI VESANEN: Breaking Down Barriers for Personalization – A Process View. 2003. ISBN 951-791-804-6.
- W:358. JUHA VIRRANKOSKI: Search Intensities, Returns to Scale, and Uniqueness of Unemployment Equilibrium. 2003. ISBN 951-791-809-7.
- W:359. JUHA VIRRANKOSKI: Search, Entry, and Unique Equilibrium. 2003. ISBN 951-791-810-0.
- W:360. HANNA KALLA: Exploration of the Relationship Between Knowledge Creation, Organisational Learning, and Social Capital: Role of Communication. 2003. ISBN 951-791-813-5.
- W:361. PEKKA SÄÄSKILAHTI: Strategic R&D and Network Compatibility. 2003. ISBN 951-791-817-8.
- W:362. MAIJU PERÄLÄ: Allyn Young and the Early Development Theory. 2003. ISBN 951-791-818-6.
- W:363. OSSI LINDSTRÖM – ALMAS HESHMATI: Interaction of Real and Financial Flexibility: An Empirical Analysis. 2004. ISBN 951-791-827-5 (Electronic working paper).

- W:364. RAIMO VOUTILAINEN: Quantitative Methods in Economics and Management Science. 2004. ISBN 951-791-832-1 (Electronic working paper).
- W:365. MATTIKELOHARJU – SAMULI KNÜPFER – SAMI TORSTILA: Retail Incentives in Privatizations: Anti-Flipping Devices or Money Left on the Table? 2004. ISBN 951-791-839-9 (Electronic working paper).
- W:366. JARI VESANEN – MIKA RAULAS: Building Bridges for Personalization – A Process View. 2004. ISBN 951-791-840-2 (Electronic working paper).
- W:367. MAIJU PERÄLÄ: Resource Flow Concentration and Social Fractionalization: A Recipe for A Curse? 2004. ISBN 951-791-845-3 (Electronic working paper).
- W:368. PEKKA KORHONEN – RAIMO VOUTILAINEN: Finding the Most Preferred Alliance Structure between Banks and Insurance Companies. 2004. ISBN 951-791-847-X (Electronic working paper).
- W:369. ANDRIY ANDREEV – ANTTI KANTO: A Note on Calculation of CVaR for Student's Distribution. 2004. ISBN 951-791-850-X (Electronic working paper).
- W:370. ILKKA HAAPALINNA – TOMI SEPPÄLÄ – SARI STENFORS – MIKKO SYRJÄNEN – LEENA TANNER : Use of Decision Support Methods in the Strategy Process – Executive View. 2004. ISBN 951-791-853-4 (Electronic working paper).

Y-SARJA: HELSINKI SCHOOL OF ECONOMICS.
CENTRE FOR INTERNATIONAL BUSINESS RESEARCH. CIBR RESEARCH PAPERS.
ISBN 1237-394X.

- Y:7. ZUHAIR AL-OBAIDI – MIKA GABRIELSSON: Multiple Sales Channel Strategies in Export Marketing of High Tech SMEs. 2002. ISBN 951-791-703-1.
- Y:8. REIJO LUOSTARINEN – MIKA GABRIELSSON: Globalization and Marketing Strategies of Born Globals in SMOPECS. 2004. ISBN 951-701-851-8.

Z-SARJA: HELSINKI SCHOOL OF ECONOMICS.
CENTRE FOR INTERNATIONAL BUSINESS RESEARCH. CIBR WORKING PAPERS. ISSN 1235-3931.

- Z:9. V.H. MANEK KIRPALANI – MIKA GABRIELSSON: Worldwide Evolution of Channels Policy. 2002. ISBN 951-791-704-X.
- Z:10. V.H. MANEK KIRPALANI – MIKA GABRIELSSON: Need for International Intellectual Entrepreneurs and How Business Schools Can Help. 2004. ISBN 951-791-852-6.

Kaikkia Helsingin kauppakorkeakoulun julkaisusarjassa ilmestyneitä julkaisuja voi tilata osoitteella:

KY-Palvelu Oy
Kirjakauppa
Runeberginkatu 14-16
00100 Helsinki
Puh. (09) 4313 8310, fax (09) 495 617
Sähköposti: kirjak@kyppari.hkkk.fi

Helsingin kauppakorkeakoulu
Julkaisutoimittaja
PL 1210
00101 Helsinki
Puh. (09) 4313 8579, fax (09) 4313 8305
Sähköposti: julkaisu@hkkk.fi

All the publications can be ordered from

Helsinki School of Economics
Publications officer
P.O.Box 1210
FIN-00101 Helsinki
Phone +358-9-4313 8579, fax +358-9-4313 8305
E-mail: julkaisu@hkkk.fi