



Juha Virrankoski

ESSAYS IN SEARCH ACTIVITY

HELSINKI SCHOOL OF ECONOMICS

ACTA UNIVERSITATIS OECONOMICAE HELSINGIENSIS

A-240

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Preface

When I was an undergraduate student, Mikko Puhakka got me interested in studying search models. I thank him for suggesting such a fruitful topic. As a graduate student I continued the theme in my licentiate's thesis which benefited a lot from comments made by Klaus Kultti, Mikko Leppämäki and Hannu Salonen. As search models evolved and they fascinated me more and more, I decided that they will be the topic for my doctoral thesis, too. I thank Pertti Haaparanta for being an encouraging and tolerant supervisor. I am especially grateful to my inspiring advisor and co-author Klaus Kultti. He has given me very constructive advice regarding the first two chapters in this thesis, but even more important, he has patiently guided me into the world of directed search models. One of the best ways of learning how to do research is to write papers with one's advisor, and I have been very lucky in that respect. I thank Klaus also for encouraging me along the way. I thank Matti Liski for suggesting me a joint project of applying search approach to pollution permit market. I am indebted also to Antti Miettunen whose computer skills proved indispensable in deriving some of the results in essay three.

Several other people have made valuable contributions at different stages of the process. I have got good advice especially from Dale Mortensen (regarding chapters one, two and five), Pertti Haaparanta and Pekka Ilmakunnas (regarding chapters one and two), John Kennes and Pieter Gautier (regarding chapter three), and from Robert Shimer (regarding chapter four). Bouwe Dijkstra, Denny Ellerman, Cathrine Hagem, Anthony Hayes, Peter Kennedy and Ottar Maestad have given useful comments on earlier versions of chapter five. Klaus Kultti's and Juuso Välimäki's insightful comments have benefited all the chapters. I also thank the anonymous referees for commenting chapters four and five, as well as the people who have commented my presentations at Helsinki School of Economics seminars, Finnish Doctorate Programme in Economics seminars, Society for Economic Dynamics Meetings, European Economic Association Meetings and European Meetings of Econometric Society.

I thank Dorothea Herreiner and John Kennes for kindly taking the task of reviewing the thesis. I also thank Anna Raikas for correcting the language.

This thesis has mainly been written while I have been at the Department of Economics at Helsinki School of Economics, first as a full-time researcher at Finnish Doctorate Programme in Economics, and then as an assistant and acting senior assistant. Part of the work was done while I worked as a program development coordinator at The Center for Doctoral Program at Helsinki School of Economics. I gratefully acknowledge financial support from Helsinki School of Economics Foundation and Yrjö Jahnsson Foundation.

Finally, I give my warmest thanks to my father Markus and to my sister Anna-Maija and her family for their love and support.

Helsinki, August 2004

Juha Virrankoski

Abstract

A conventional wisdom regarding search models is that multiple unemployment equilibrium may result if the matching function has increasing returns to scale. *Essays one and two* challenge this view in models where projects are homogenous but the job seekers and firms choose their search intensities (essay one), or the job seekers choose their search intensity but firms enter the market according to a zero-profit condition (essay two). Multiple equilibrium may in principle be caused by strategic complementarity or by common pool externality. It turns out that large returns to scale do not endanger uniqueness but rather guarantee it.

In directed search models, the identity of stayers and movers has traditionally been just assumed. In *essay three*, searching and waiting decisions of buyers and sellers are considered. One-sided search is an evolutionarily stable outcome in an economy where each buyer and seller can either search or wait, and where the trading mechanism is auction or bargaining. If the relative number of buyers to sellers increases, the likelihood of all sellers wait and all buyers search -equilibrium increases relative to the likelihood of all buyers wait and all sellers search -equilibrium. In two-sided search, bargaining is more efficient than auction. One-sided search is more efficient than two-sided search. In one-sided search, it is more efficient if the larger pool searches and the smaller pool waits, than vice versa.

We can often see that similar goods are sold for different prices, which violates the law of one price. Up to now, the models resulting in price distribution have assumed imperfect information on prices or ex ante heterogenous agents. In the model of *essay four*, we develop a search model where price distribution results even if all the sellers are symmetric, all the buyers are symmetric, the commodity is homogenous, and all the buyers know all the prices. The sellers are in locations and post prices simultaneously. The buyers observe the prices, and each buyer visits one location. The buyers act independently and employ symmetric mixed strategies. We show that when there are several sellers in a location, the Nash equilibrium features price dispersion, i.e. the sellers post different prices. The equilibrium strategy of the sellers is a non-atomic distribution.

Essay five studies the supply of pollution permits in a labor market -style search model. We look at economic implications of allowing pollution permit trading between developed countries (DC) and less developed countries (LDC). The firms in a given region are similar, but the DC-firms are assumed to have clean technology, whereas the LDC-firms have dirty technology that emits a lot of carbon dioxide. We assume that each firm in both regions is first allocated a pollution cap. However, the permits can be transferred by making an investment which reduces the emission rate for a host firm (in an LDC-country) below the host's permit endowment. The market for these emissions-reducing projects have friction: finding a suitable partner for a project takes time. Once emission reductions are made and verified, the permits are traded in a frictionless permit market. Our model combines the search market for projects with a frictionless permit market to quantify the supply-side frictions in the market. We also decompose the effects of frictions into the effects of search friction, bargaining, and bilateralism. A calibration using previous cost estimates of reductions illustrates changes in cost savings and allocative implications.

Keywords: search, multiple equilibria, directed search, auction, bargaining, price distribution, pollution permits

Essays in Search Activity

Juha Virrankoski

Department of Economics, Helsinki School of Economics

Office: Research Unit on Economic Structures and Growth
Department of Economics
P.O. Box 17 (Arkadiankatu 7)
00014 University of Helsinki
Finland

e-mail juha.virrankoski@helsinki.fi
tel. +358-9-191 28760

The dissertation consists of an introduction and the following essays:

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- II Juha Virrankoski (2004) “Search, Entry, and Unique Equilibrium.” An earlier version of the essay, with the same name, is published in Helsinki School of Economics Working Papers, W-359, October, 2003.
- III Klaus Kultti, Antti Miettunen and Juha Virrankoski (2004) “Physical Search.” An earlier version of the essay, with the same name, is published in University of Helsinki, Faculty of Social Sciences, Department of Economics Discussion Papers, No 573 / August 17, 2003.
- IV Klaus Kultti and Juha Virrankoski (2003) “Price Distribution in a Symmetric Economy.” *Topics in Macroeconomics* 3, No 1, Article 5, <http://www.bepress.com/bejm/topics/vol3/iss1/art5>.
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1 INTRODUCTION

The Walrasian theory assumes that trading happens as if there was a centralized, frictionless market where an auctioneer sets the equilibrium price. Buyers and sellers, or firms and job seekers, gather together without costs and announce their offers and bids to the auctioneer. The auctioneer calculates the equilibrium price - the price that equalizes supply and demand - and announces it to the trading agents, and the transactions are executed. The outcome is Pareto-optimal: it is not possible to increase the utility of any agent without decreasing the utility of at least one agent.

A centralized market with perfect information and zero transaction costs is not very plausible description of trading in general. We often see that buyers and sellers meet each other pairwise or in small groups. Prices may be agreed on in bilateral negotiations where the parties have some degree of monopoly power on each other. In a decentralized market many opportunities are not exploited because agents have not found suitable trading partners. As a result, the volume of trading is smaller than in a Walrasian market. In a labor market, unemployment persists in equilibrium.

Search theory addresses the implications of time and other costs required in finding a trading partner. Job seekers wish to find a job with a wage high enough or with suitable characteristics, firms want to hire workers equipped with skills that match some specific requirements, buyers seek for a price low enough, and so on. Since the 1960's, search models have developed from partial equilibrium models to general equilibrium models, and their scope has widened from labor and product market models to include money and marriage, for example.

Search theory has become a mainstream approach in studying the labor market. Unemployment results partly from frictions in finding an employer or employee, and search models capture the frictions explicitly. Job seekers or firms can decide how intensively to search for each other, or job seekers can decide what is the minimum acceptable wage. Besides attempting to explain the amount of unemployment, search theory can also be used to explain wage dispersion among homogeneous workers, the labor market experiences of different demographic groups, etc. The labor market model can be enriched by

interlinking the decisions of effort during the employment relationship and the labor market frictions, like in the model of Ramey and Watson (2001). In product market context, search models have been used to explain price dispersion of homogeneous goods. The heterogeneity of either buyers or sellers (or of both), combined with imperfect information about prices, can produce price dispersion.

The function of money as a medium of exchange has been a rapidly expanding area of search theory. In a world with perfect markets (including the possibility to make complete contracts), money would not be needed since it could be agreed on who produces what, then production happens, and the goods are delivered according to the contracts. As such contracting is practically impossible, barter would prevail. However, person A's potential trading partner B does not necessarily want the object that A has, even if B has the object that A wants. This is called the 'double coincidence of wants' problem. Introducing money would solve this by making trading indirect: If B has the object that A wants, but B does not like the object that A has, A buys the object from B and gives B a piece of paper called money. Later, B can find someone who has an object that B desires, and B can buy it using money. Money can serve as a substitute for memory (Kocherlakota 1998). The early search models of money (like Diamond 1984) assumed fixed prices; in newer models (see Shi 1997) money and goods can be divisible, which allows studying the determination of prices.

Search theory has also been applied to marriage. A special feature of marriage market is that utility is for the most part non-transferable: The utility of the man from marrying a woman is a function of the characteristics of the woman, and vice versa. Burdett and Coles (1997) study equilibrium sorting, and the result is that men of a certain class (defined by a real-valued interval depicting some characteristics of the man) marry women who belong to a certain class.

1.1 Matching Functions

A central apparatus in search models is a matching function that determines the number of relationships that form per unit of time as a function of the number of searching agents (and perhaps of their search intensities or other choices). The matching function is either

(i) not specified but it is assumed to have certain properties that are ‘realistic’ or make the analysis simple and tractable (For example, the number of matches that form in a labor market may be assumed to be an increasing and concave function of the number of vacancies and unemployed, with constant returns to scale.), or (ii) specified, without necessarily being derived from agents’ behavior in a serious manner. However, a specified matching function is often rationalized by some plausible assumptions on how matches can form. For example, one could assume that matches cannot form without both job seekers and vacancies taking part into the meeting process (Cobb-Douglas matching function has this property). On the other hand, one can think that job seekers can contact firms and firms can contact job seekers, thus matches can form even if one party is passive. This is the story behind additive matching functions, used for example in Mortensen (1982). Pissarides and Petrongolo (2001) have an excellent survey on matching functions.

The search models in the first generation were partial equilibrium models where the behavior of a single searching buyer or job seeker was considered. The pioneering work was done by George Stigler (1961, 1962). In these nonsequential models an agent decides the number of partners he will meet, and having met them all he chooses with whom to trade. A nonsequential strategy is, however, suboptimal relative to an optimal stopping rule, also called reservation price strategy (for example, McCall 1965 and 1970, Gronau 1971): A job-seeking worker collects wage offers sequentially, one per period, or according to a Poisson process. Each offer is a random draw from a commonly known, exogenously given nondegenerate wage distribution. The worker decides on a reservation wage that is the lowest wage he can accept. If the realized wage is less than the reservation wage, he rejects the offer and waits for the next one. If the wage is high enough, the worker accepts it and stops searching. The optimal reservation wage depends on the wage distribution and on the frequency of offers.

The second generation models are general equilibrium models where the behavior of all agents in the market is analyzed. Usually it is assumed that there are two kinds of agents: firms and workers, for example. Each agent wishes to find some agent of the opposite type in order to trade with him, or to carry out a joint project. While

searching the agents have imperfect information on each others' locations or on potential partners' abilities to join a project. Time is usually continuous and meetings are pairwise. When two agents of the opposite types meet, they may create a surplus and divide it according to some trading rule. A much used assumption is that the surplus is divided using the Nash bargaining solution (for example, Diamond 1982b, Hosios 1990, Pissarides 1985a, 1985b). One of the problems in the Nash bargaining solution is that the relative bargaining power is exogenous, usually it is assumed to be one half. The value of the relative bargaining power can however be derived from a sequential strategic model where the negotiating parties make offers and counteroffers (Binmore, Rubinstein, and Wolinsky 1986). This approach requires that one makes very specific assumptions about the matching environment and the negotiation rules.

While searching the agents may choose their search intensity (for example, Howitt and McAfee 1987 and Pissarides 1984), or they may include decisions on reservation wage (Pissarides 2000, pp. 145-165). Alternatively, an agent may invest in acquiring a tradable object before he starts searching (Diamond 1982a). According to Yoon (1981), approximately ninety percent of unemployment duration is due to not having received an offer, only ten percent is due to a rejection of an unfavorable offer. This suggests that the agents' search efforts and decisions on participating have an important role in determining the performance of the market. The trading frictions typically create externalities which in turn may generate inefficiencies and even multiple equilibria.

The third generation of search models consists of directed search models, also called urn-ball models (see for example Montgomery (1991) and Lu and McAfee (1996)). Time is discrete. Firms, for example, are in specific locations, one firm in each location. The job seekers choose locations independently and use a mixed strategy. Some locations receive no visitors, some locations get one applicant, while some locations get several applicants. The number of visitors in a location is a binomial random variable. Usually it is assumed that the number of agents is large, and the binomial distribution is approximated by a Poisson distribution, which simplifies the analyses considerably. The resulting matching function is explicitly derived from an optimal behavior of the agents. Price formation can be modeled by various ways. Firms can post binding wage offers, and job seekers choose

which firm to visit based on offers observed. Wages can also be determined by bargaining. In case of price posting and bargaining, if a firm gets several identical applicants, it chooses one of them randomly. The possibility that a firm gets several applicants allows that wages are determined by auction. If a firm gets just one job seeker, either the firm or the job seeker makes a take-it-or-leave-it offer. If several applicants appear, the applicants engage in auction and they are driven to their reservation values.

1.2 The Questions Answered to in the Present Thesis

The present thesis addresses some fundamental questions analyzed both in the second and third generation models. In all the models, there are two kinds of searching agents: firms and workers, or buyers and sellers, or potential project partners of type A and B. We assume that there is no heterogeneity among agents on a given side of the market. This assumption is, of course, a strong simplification of reality, but it enables us to better extract some basic results. We concentrate on the following problems:

(i) The agents' decisions on search intensities and participating in the market create externalities which may cause multiplicity of unemployment equilibria. These decisions made by searching agents affect the utility and optimal behavior of other searching agents, but the effects are ignored by an individual agent. What is the relation between uniqueness of equilibrium and returns to scale in the matching function? The first two essays study this question in the style of second-generation models.

(ii) Who searches whom? In most models of directed search, it is assumed either that all sellers wait and all buyers search, or vice versa. Could it be the case that both sides in the market are active? How many of the buyers and sellers wait and how many of them search? Or is the equilibrium outcome such that search is one-sided? Casual empiricism indicates that in most markets search is one-sided. For example, in most markets buyers search for sellers who wait for the buyers. This question is addressed in the third essay, using an urn-ball model of the third generation.

(iii) The law of one price does not seem to hold: seemingly similar goods are sold for seemingly different prices. Price distribution can result in search models, but all the models thus far have assumed either ex ante heterogeneity among buyers or sellers, or that

search is costly, which makes otherwise homogenous buyers heterogenous in their information on prices. We present a model where buyers are symmetric, sellers are symmetric, information on prices is costless and perfect, and a uniform price for a homogenous object cannot be an equilibrium. The model belongs to the third generation.

(iv) Besides answering these rather fundamental problems in labor markets and product markets, search theory is applied in a new area: supply of pollution permits. Pollution permit trading is, besides taxes and quotas, one possible means to control pollution. Each firm is assigned a pollution cap, say certain amount of carbon dioxide per year. If a firm wishes to pollute more, it has to buy permits. Firms that want to pollute less than their quota allows can sell some of their permits. As a result, the marginal cost of polluting becomes equal between the firms, and the total cost of having a given amount of pollution is minimized. However, pollution reduction can entail a costly project: Cleaning devices must be installed, or the firm must switch to another technology. Many times, the potential projects are in less developed countries where firms may have difficulties in financing the projects, or they may lack technical expertise. Therefore, they need a partner from a developed country. We model this project market by applying search framework. The permit trading itself happens in a frictionless market, but the amount of permits for sale depends on the number of ongoing pollution-reducing projects. The essay is the first model where frictions in finding pollution-reducing projects are treated in a general equilibrium model. The earlier so called joint implementation models do not extend their analysis into the market level.

1.2.1 Multiple Unemployment Equilibria?

The present knowledge says that multiple equilibria are likely to emerge if the matching function has increasing returns to scale with respect to the number of searching agents. This view originates from a famous model of Diamond (1982a). In the ‘coconut model’, depicted in Figure 1, agents invest in acquiring a tradable object, a coconut. An individual wanders around an island and seeks for coconut trees. The trees vary in height, c , and the maximum acceptable height is c^* . Having found a coconut that is not too high in a tree, the agent exerts effort c and picks it. There is a taboo on the island that no

one is allowed to eat a coconut one has picked by himself. To enjoy a coconut, one must exchange it for an identical coconut picked by someone else. Unfortunately, there is no centralized market place on the island. Instead, the individual wanders around with his coconut until he meets a similar trader. The fraction of people who carry a coconut is e . The arrival of potential trading partners is a Poisson process with arrival rate $b(e)$, and it is assumed that $b'(e) > 0$. The partners exchange their coconuts, eat them, separate, and go to search for suitably low coconut trees again. The more the agents are willing to invest in acquiring the object (that is, the higher trees they are ready to climb to), the more there are agents searching for a partner, willing to exchange their coconuts. Curve $de/dt = 0$ tells the value of c^* that keeps e at a steady state value. The agents' reservation cost c^* is the higher the more there are agents searching (curve $c^*(e)$ in the figure) if $b'(e) > 0$. Assuming $b'(e) > 0$ implies that the meeting rate has increasing returns to scale with respect to e : $[d(eb(e))/de]e = [eb'(e) + b(e)]e > eb(e)$ if $b'(e) > 0$. Returns to scale are decreasing only if $b'(e) < 0$. Because the number of searching agents in a steady state is an increasing function of the reservation cost (curve $de/dt = 0$ is increasing in c^*) and the reservation cost is an increasing function of the number of searchers only if returns to scale are increasing, multiple unemployment equilibria can exist only if the meeting technology has increasing returns to scale with respect to the number of searching agents.

The coconut model is behind the view that multiple unemployment equilibria may result also in labor market search models if the matching function has increasing returns to scale. The coconut model, however, has several features that make it unsuitable to represent a labor market, as noted by Howitt and McAfee (1987). For example, they ask whether searching for a suitably low coconut tree should be interpreted as employment or unemployment. In addition to the notions made by Howitt and McAfee, one should also consider the following: First, in a labor market, there are usually two types of searching agents: job seekers and vacancies. Both can choose their search intensities which affect each other directly. The intensities are strategic complements if the intensity of agents of type A increases the optimal intensity of agents of type B, and vice versa. Multiple equilibria can result because there are two increasing reaction functions. In the coconut

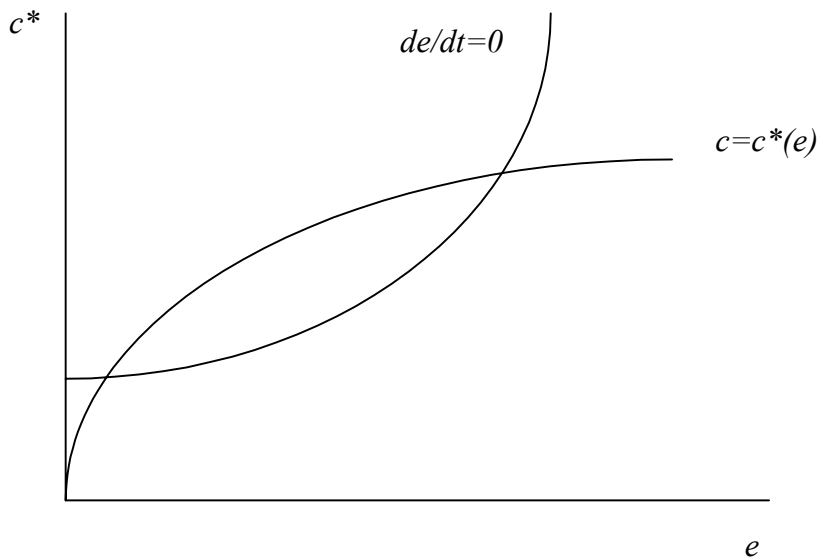


Figure 1: Multiple equilibria in Diamond's coconut model

model the choices of reservation costs do not affect each other in the same way as in the two-sided model because there is no party A whose choices affect the optimal choices of party B. Moreover, if search intensities are the decision variables, it is by no means self-evident that it is increasing returns to scale that makes the reaction functions cross each other several times, as we see in Essay 1 of this thesis.

Second, the search intensities are affected by the number of searching agents on both sides. This is called ‘common-property externality’ by Howitt and McAfee; in many instances it is also called ‘common pool externality’. In the coconut model, the reservation cost increases with the number of searchers only if the matching function has increasing returns to scale, and the steady state pool of searchers is the larger the higher the reservation cost is. Thus, common pool externality can cause multiple equilibria only if returns to scale are increasing. In a labor market where vacancies and job seekers choose their search intensities (but where the potential projects are homogenous), the search intensities increase with the number of searching agents if and only if the matching function has increasing returns to scale (as demonstrated in Essay 1 of this thesis), but the steady state pool of searchers decreases if intensities increase. This implies that common

pool externality can cause multiple equilibria only if returns to scale are decreasing¹.

In addition, the lowest possible cost in the coconut model is larger than zero. The curvature of $c^*(e)$, which clearly depends on the returns to scale of the matching function, has therefore a minor role in producing multiple equilibria. If instead it is assumed that the minimum cost is zero, we would have to pay more attention to returns to scale. Howitt and McAfee have a similar kind of assumption (the marginal cost of a firm's recruiting effort is larger than zero at recruiting intensity of zero), which is partly responsible for multiple equilibria in their search intensity model. Essays 1 and 2 of the present thesis assume that marginal costs are zero if search intensity or recruiting intensity is zero, so the role of returns to scale in producing multiple equilibria is clearer.

The first two essays study the sufficient conditions for a unique unemployment equilibrium in a labor market where homogenous job seekers choose their search intensities in order to match with homogenous vacancies that either choose their recruiting intensities (Essay 1) or enter the market with zero profit condition while keeping the recruiting intensity at a constant level (Essay 2).

ESSAY 1 *Search Intensities, Returns to Scale, and the Uniqueness of Equilibrium in the Labor Market*

A standard result in search models is that multiple unemployment equilibrium may result if the matching function has increasing returns to scale. This result originates from models where the agents choose reservation productivities. This essay reconsiders this result in models where job seekers and firms choose their search intensities but all projects are the same. Three modeling specifications are considered. In each model the numbers of firms and workers are fixed, and wages are assumed to be fixed, determined from outside the model. The first one is a symmetric model with a multiplicative (Cobb-Douglas) matching function, the second one is a symmetric model with an additive matching function. The third model is a generalization of a model of Howitt and McAfee (1987). In all these models we assume that the flow product is split equally between the firm and the worker because (i) in the symmetric model where the basic

¹In a reservation wage model (which is outside the scope of this thesis) it still seems to hold that multiple equilibria can result if returns are increasing.

nature of strategic complementarity and common pool externality are most clearly presented, a symmetric Nash bargaining solution would also split the flow product equally, (ii) we want to keep the modified Howitt-McAfee model as close to their original model as possible. We allow variable returns to scale in the matching function and also assume that firms' marginal recruiting cost is zero at recruiting cost of zero. The basic difference between the first and third model is that the third model is asymmetric: the number of firms may be different from the number of workers. In none of these models strategic complementarity can cause multiple equilibria whatever the returns to scale. Having an asymmetric number of firms and workers in the first model, combined with a fixed wage or Nash bargaining, or assuming Nash bargaining in the modified Howitt-McAfee model could perhaps change this result. Also, in all these models common-pool externality can cause multiple equilibria only if the scale parameters of the matching function are low enough. In this case, increasing the pool of searchers decreases the optimal search intensities. This is often called 'discouraged worker effect'.

ESSAY 2 *Search, Entry, and Unique Equilibrium*

The model differs from those studied in Essay 1 in two respects: The number of firms adjusts according to a zero-profit condition, and wages are determined by Nash bargaining. The sufficient conditions for a unique unemployment equilibrium are solved. The model is an application of a model of Pissarides (2000, pp. 123-143). In contrast to Pissarides's model, the present essay assumes variable returns to scale instead of constant returns to scale, and it treats the individual search intensity choices more carefully. Uniqueness of equilibrium is guaranteed if returns to scale are high enough and the workers' bargaining power in the Nash bargaining solution is high enough, and if the job seekers' search costs increase fast enough with search intensity. If these conditions hold, multiplicity of equilibria cannot emerge from strategic complementarity nor from common pool externality.

1.2.2 Who Searches Whom?

Casual empiricism indicates that in most markets search is one-sided in the sense that buyers, for example, search for sellers who wait for the buyers to appear. Two-sided search where agents on both sides of the market search and wait is quite rare. The number of trades, that is, the efficiency of the economy, depends on how the pool of agents is decomposed into those who wait and those who search. The decomposition is usually postulated, with few exceptions. Burdett, Coles, Kiyotaki and Wright (1995) have tackled the question in a model where some agents are sellers with one object each, and the rest are buyers with money. Their results depend on the magnitude of moving costs for buyers and sellers. Herreiner (1999) studies the question of who searches, but she ignores price formation. In both the above mentioned models, stayers can meet only movers, but movers may meet stayers or other movers.

In Herreiner's model, each agent of type A likes to meet an agent of type B, and vice versa. In a meeting, the agents swap their goods and get a fixed utility. Each agent can either stay in a location and wait for visitors, or he can move and visit staying agents. Some stayers will not get any visitors, some stayers get one visitor, and some stayers get several visitors, but only one visitor is chosen to trade with the stayer. The number of agents on either side is finite. (An infinite approximation is also considered) Because of finite number of agents, several small markets produce more matches (is more efficient) than one large market. That is, the matching process has decreasing returns to scale. A market where all the agents of the long side move and all the agents of the short side stay is more efficient than a market where the short side moves. She also considers a case where a fraction of both types search and the rest stay. Stayers can meet movers, and movers can meet stayers and other movers. The model has many Nash equilibria with different equilibrium fractions of stayers and movers.

ESSAY 3 *Physical Search*

Searching and waiting decisions of buyers and sellers are considered in an infinite-horizon urn-ball model where the agents have no explicit search costs. We assume that there are infinitely many buyers and sellers but their ratio is fixed. Each seller has one

indivisible object which he values at zero, and each buyer wants to buy one object which he values at one. Each agent has a mixed strategy that tells the likelihood of searching or waiting. Those who stay can meet only those who move, and movers can meet stayers only. The likelihood of staying depends on the relative size of buyer and seller pools and the trading mechanism (auction or bargaining).

We solve the fractions of stayers and movers explicitly in case of auction, and in case of bargaining we solve numerically the change in the fraction of movers on both sides when the ratio of buyers to sellers changes. With both trading mechanisms, increasing the relative number of buyers to sellers increases the fraction of buyers who wait and the fraction of sellers who search. Bargaining leads to more trades than auction: that is, bargaining is the more efficient trading mechanism. The stability of two-sided search - where a fraction of both buyers and sellers move and the rest of them stay - is studied by applying simple replicatory dynamics. A market with two-sided search is not evolutionary stable, whereas a market with one-sided search is. In case of auction and bargaining, the basin of attraction for the market where all the sellers wait and all the buyers search grows when the ratio of buyers to sellers grows. That is, if in market A the ratio of buyers to sellers is larger than in market B, then converging to an equilibrium where all sellers stay is more probable in market A than in market B. We compare the efficiency of various equilibria. The most efficient equilibrium is that where all the members of the long side of the market move and the short side waits. The second-most efficient equilibrium is that where all the members of the short side of the market move and the long side waits. The most inefficient equilibrium is that with search and waiting on both sides, trades being consummated by auction; the second-most inefficient case is the one where there is search and waiting on both sides, trades being consummated by bargaining. We also compare the efficiency of different one-sided search equilibria to different two-sided equilibria. The essay provides a rationale for one-sided search which is prevalent in many markets.

1.2.3 Price Distribution

We can often see that similar goods are sold for different prices, which is against the ‘law of one price’. Up to now, the models resulting in price distribution have assumed that information on prices is imperfect, or that the agents are otherwise heterogeneous ex ante. In models of the former type, buyers, for example, get to know the price charged by any individual seller only by visiting the sellers sequentially (Diamond 1971, Salop and Stiglitz 1982). In these models ex ante identical buyers know the price distribution, but they do not know ex ante the price charged by any individual seller. Alternatively, buyers can sample price information before visiting any seller. In the model of Butters (1977), sellers choose the number of costly advertisements to be sent to randomly selected buyers. In Burdett and Judd (1983), buyers choose how much to invest in getting price quotations. A similar type of model is presented in Kandel and Simhon (2002). In these three models, the buyers know the price quotations of some sellers, thus the buyers are heterogeneous in their information prior to meeting a seller.

A price distribution can be generated also by assuming ex ante heterogeneous agents. Consumers may differ in their willingness to pay (Diamond 1987), or firms differ in their costs of production, and consumers differ in their search costs (Carlson and McAfee 1983). In these two models, buyers select the sellers randomly as they do in the models of Diamond, and Salop and Stiglitz, mentioned above. This imperfection is absent in the wage-posting model of Montgomery (1991). Firms who want to hire labor post wage offers without cost, and job seekers observe all the offers without cost. Job seekers then choose which firm to visit on the basis of observed wage offers. Wage distribution results if firms value the vacancies differently. Montgomery’s model has a typical property of ‘directed search’: buyers are directed to sellers by the latter’s price quotations.

In discrete-time random matching models it is easy to get an ex post price distribution if one thinks that trades are consummated in auctions; the price will differ depending on how many buyers come to a seller (Lu and McAfee 1996). But here it is a little misleading to call the outcomes prices. They are rather terms of trade. To talk about price distribution one should have meaningful prices, i.e. terms of trade that the buyers

can observe in advance and base their decision to visit any particular seller on this information.

ESSAY 4 *Price Distribution in a Symmetric Economy*

We develop a search model where price distribution results even if all the sellers are symmetric, all the buyers are symmetric, the commodity is homogenous, and all the buyers know all the prices. There are a large number of sellers who are grouped in locations that can be thought of as physical sites like stores. We assume that there are two sellers in each location. Each seller has one indivisible object that he values at zero. First, the sellers post prices simultaneously. There are a large number of buyers, and we assume that each buyer observes all the prices. Second, the buyers use a mixed strategy to choose which location to visit, based on the prices. Third, trades are made. Some sellers and some buyers will not trade because the buyers do not coordinate on their choices. There does not exist a pure strategy equilibrium for the sellers. Instead, there exists a symmetric mixed strategy equilibrium which is a non-atomic distribution of prices with a connected support. Consequently, price distribution is a necessary feature in equilibrium.

The result is based on two main assumptions. First, when there are several sellers in a location the sellers compete both with the sellers in the same location and with the other locations. As buyers can visit just one location they choose the location with low prices more probably than a location with high prices. If the number of buyers that come to a particular location is less than the number of sellers, the high price sellers do not manage to sell their goods as buyers buy the low price goods. Second, the sellers are capacity constrained (each of them has only one indivisible object for sale), and the buyers have unit demands.

1.2.4 Supply of Pollution Permits

Climate change, especially global warming, has been a controversial topic for a couple of decades. It has been argued that the temperature of the Earth has risen, and that the warming partly results from using fossil fuels. Using fossil fuels releases carbon dioxide

into the atmosphere. Carbon dioxide acts as a ‘greenhouse gas’: it prevents heat from escaping into the space, making the Earth warmer. Warming is harmful because it tends to cause desertification, it melts the ice caps on poles, etc.: it generally worsens the living conditions of man. Therefore a relatively large consensus has been formed that emissions of greenhouse gases have to be controlled and reduced. There are many ways to achieve this goal: by setting quotas to greenhouse-gas emitting firms, by taxing the emissions, or by allowing pollution permit trading between firms while maintaining a global emission cap. The latter is often argued to be a cost-effective way to control pollution.

ESSAY 5 *Frictions in Project-Based Supply of Permits*

In this essay we study the supply of pollution permits in a labor market -style search model. It should be noted that we do not say anything about whether the Earth is warming or not; neither do we claim that if warming has happened, it is because of human-based acts. We look at economic implications of allowing pollution permit trading between developed countries (DC) and less-developed countries (LDC). The firms in a given region are similar, but their technology differs between the regions. The DC-firms are assumed to have clean technology, whereas the LDC-firms have dirty technology that emits a lot of carbon dioxide. We assume that each firm in both regions is first allocated a pollution cap. However, the permits can be transferred by making an investment which reduces the emission rate for a host firm (in an LDC-country) below the host’s permit endowment. The market for these emissions-reducing projects has friction: finding a suitable partner for a project takes time. Once emission reductions are made and verified, the permits are traded in a frictionless permit market. Our model combines a search market for projects with a frictionless permit market to quantify the supply-side frictions in the market. We also decompose the effects of frictions into the effects of search friction, bargaining, and bilateralism. A calibration using previous cost estimates of reductions illustrates changes in cost savings and allocative implications.

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Search Intensities, Returns to Scale, and Uniqueness of Unemployment Equilibrium

Juha Virrankoski*

University of Helsinki

Abstract

We study the robustness of conditions for a unique unemployment equilibrium to modifications in search environment. Multiple equilibria may in principle be caused by strategic complementarity or by common-pool externality. It turns out that large returns to scale do not endanger uniqueness but rather guarantee it.

Keywords: search, multiple equilibria

JEL classification: C78, E24, J64

1 Introduction

One of the important questions in labor market search models is the possible existence of multiple steady state unemployment equilibria. Strategic complementarity - positive interaction between the levels of choice variables (like search intensities) - is a potential source of multiple equilibria (Cooper and John 1988, Drazen 1987). Another potential

*Research Unit on Economic Structures and Growth, Department of Economics, P.O. Box 17 (Arkadiankatu 7), FIN-00014 University of Helsinki, Finland. Telephone:+358-9-191 28760, e-mail: juha.virrankoski@helsinki.fi. This paper was mainly written when I was at Department of Economics at Helsinki School of Economics. I thank Dale Mortensen, Robert Shimer, and other participants in Society for Economics Dynamics meetings and European Economic Association meetings, as well as participants in Helsinki School of Economics research seminar for useful comments. I gratefully acknowledge financial support from Helsinki School of Economics Foundation and Yrjö Jahnsson Foundation.

source of multiplicity is common property externality: the optimal value of a choice variable depends on the number of searching agents (Howitt and McAfee 1987). Multiple equilibria may in principle arise because of either or both of these externalities. The existence of multiple equilibria depends e.g. on the nature of the choice variables and on the properties of the meeting technology. The existence of multiple equilibria has implications for welfare and economic policy. Equilibria are often Pareto-rankable because externalities tend to make them inefficient (Cooper and John, *ibid.*). Optimal economic policy actions are different in a unique-equilibrium case from the case of multiple equilibria. In the former case, the policymaker can induce small changes to move the equilibrium into a preferred direction, while in the latter case the economy must be shifted from an inefficient equilibrium to a less inefficient one (Diamond, 1982a).

The present knowledge says that multiple equilibria are likely to emerge if the matching function has increasing returns to scale with respect to the number of searching agents. This view originates from a famous model of Diamond (1982a). In this ‘coconut model’, agents invest to acquire a tradable object (coconut). Having found a coconut that is not too high in a tree, the agent exerts some effort and picks it. There is a taboo on the island that no one is allowed to eat a coconut one has picked by himself. To enjoy a coconut, one must trade it for an identical coconut picked by someone else. Unfortunately, there is no centralized market place on the island. Instead, a man with a coconut wanders around until he meets a similar trader. They exchange their coconuts, eat them, separate, and go on to find a suitably low coconut trees again. The more the agents invest to acquire the object (that is, the higher trees they are ready to climb to), the more there are agents searching for a partner, willing to exchange their coconuts. If the meeting technology has increasing returns to scale with respect to the number of searchers, then the agents’ reservation cost in acquiring a coconut to sell is the higher the more there are agents searching. (This is how the common property externality works in the model.) Because the number of searching agents is an increasing function of the reservation cost, and the reservation cost is an increasing function of the number of searchers only if returns to scale are increasing, then there exist multiple unemployment equilibria only if the meeting technology has increasing returns to scale with respect to the number of searching

agents. However, the result on multiplicity does not hold if firms and job-seekers can choose their search intensity as we show in Section 2. For other search models including reservation output see also e.g. Diamond (1981), Hosios (1990), Mortensen (1982a), and Pissarides (1984a and 2000). Alternatively, agents could control their search intensity (see Hosios 1990, Howitt and McAfee 1987, Mortensen 1982a, 1982b, Pissarides 1984b, 1986, 2000). Search intensity means the effort exerted in finding a trading opportunity.

In the present paper we derive sufficient conditions for a unique steady state unemployment equilibrium in three closely related models. In all models, there is a fixed-size pool of identical and infinitely-living firms and a fixed-size pool of identical and infinitely-living workers. Job seekers and firms choose how intensively to search for a production partner. When a firm and a worker meet, they agree on the division of the flow product during the life of the match. They produce until the match dissolves for exogenous reasons. (The machinery breaks down, the worker dies etc.) After separation, the worker and the firm start to search for a partner again. The more intensively the firms and unemployed search for each other, the faster they find partners, and the lower is the steady state level of unemployment. The number of job seekers and recruiting firms affect the optimal search intensities. This is called common pool externality. (See Howitt and McAfee 1987 who name the effect as common-property externality.) On the other hand, the search intensities of firms and workers affect each other. This is called strategic complementarity or strategic substitutability¹. The former (latter) means that higher search intensity on one side of the market increases (decreases) the optimal search intensity of the other side of the market. We analyze both cases.

The general message of the present article is that models with multiplicative matching functions can in principle produce multiple steady state unemployment equilibria via strategic complementarity or common pool externality. However, the former will not happen in the two models presented, and the latter can happen only if returns to scale are sufficiently low. If the matching function is additive, returns must be low to guarantee

¹See Bulow, Geanakoplos and Klemperer (1985), from which the terms originate. See also Cooper and John (1988). Instead of ‘strategic complementarity’, Drazen (1987) uses term ‘reciprocal externality’, and Howitt and McAfee (1987) uses ‘thin-market externality’.

the existence of equilibrium, but they must be high to exclude multiple equilibria caused by common pool effect.

In order to consider returns to scale, the matching function is assumed to be of form $m = m(\alpha u, \beta v)$, where m is the aggregate matching rate, α and β are the job seekers' and vacancies' search and recruiting intensities, respectively, and u and v are the number of unemployed and vacancies. The aggregate search and recruiting efforts are then αu and βv , and returns to scale means how the aggregate matching rate changes when these aggregate inputs change. In Sections 2-4 we use specific multiplicative and additive functions for tractability. Assuming specific functional forms for the matching technology - without deriving them rigorously from the agents' environment and actions - is of course not satisfactory from a theoretical perspective, but the specific functions we use anyhow capture some essential features of matching processes. Moreover, the Cobb-Douglas function we use in Sections 2 and 4 is widely used in empirical research.

The rest of the paper is organized as follows: In Section 2 we analyze a model where each firm can employ at most one worker at a time, and the matching function is multiplicative, leading to strategic complementarity. The model in Section 3 is otherwise the same as the model in Section 2 except that the matching function is additive, leading to strategic substitutability. In Section 4 we build and analyze a modification of a model of Howitt and McAfee (1987). Each firm can have as many workers as it wants to. The variable called vacancy is not needed, the firms just decide how much to invest in recruiting to get more workers. The way Howitt and McAfee analyze their model leaves the role of returns to scale in producing multiple equilibria rather unclear. In the present paper we analyze their model by looking at strategic complementarity and common pool externality and allowing variable returns to scale in the matching function. Section 5 concludes. Section 6 is an appendix that contains the derivation of some of the results.

2 A Model with a Multiplicative Matching Function

In this section the simplest possible model is analyzed. Everything is made symmetric to extract the basic results. There is a large number L of identical and infinitely living

workers. The number of unemployed workers is u , and the number of employed workers is $L - u$. We make the model very simple to highlight the results, and we assume that the economy is symmetric: the number of firms is also L .² Firms, too, are identical and live forever. Each worker and firm discount future at rate r . The number of vacancies is v . Each firm employs at most one worker at a time, so the number of vacancies equals the number of unemployed. An unemployed worker i chooses search intensity α_i in order to find a firm. The flow cost of searching is $c(\alpha)$, and we assume $c(\alpha) = \alpha^a$, where $a > 1$. Likewise, a vacant firm chooses recruiting intensity β_i to find a worker, and we assume $g(\beta) = \beta^b$, where $b > 1$.

The aggregate matching rate is assumed to depend on the number of job seekers and vacancies, and their search intensities. We assume that the matching function is of Cobb-Douglas form³:

$$m = (\alpha u)^\phi (\beta v)^\psi. \quad (1)$$

The aggregate search effort of job seekers is αu , and the aggregate search effort of vacancies is βv . These aggregate efforts are then transformed into matching rate by using non-negative scale parameters ϕ and ψ . As we see later, this formulation allows that multiple equilibria can in principle be caused by strategic complementarity or by common pool externality.

Each match dissolves at rate λ , so the aggregate number of matches that break down per unit of time is $\lambda(L - u)$. After separation, the worker joins the pool of unemployed, and the firm joins the pool of vacancies, and they start to search again. In a steady state equilibrium, the number of breakdowns equals the number of matches that form:

$$m = \lambda(L - u). \quad (2)$$

Equation (2) determines a steady state curve in $(u, \alpha^\phi \beta^\psi)$ -plane. Each worker-firm relationship produces a flow output of $2f$ which is exogenously given.

²Instead of speaking of firms and workers, we could as well speak of two pools consisting of agents called A and B, who have to find an agent from the opposite pool in order to form a bilateral partnership. Look at Virrankoski (2003) where the uniqueness conditions are solved in a model where firms enter according to zero profit condition.

³The Cobb-Douglas form is widely used in empirical search models, and it is easy to handle.

For simplicity we assume that the matching technology is symmetric, which means that $\phi = \psi$. We also assume symmetric costs: $a = b$. The symmetry assumptions imply that if we used the Nash bargaining solution to determine wages, the result would be that worker and firm share the flow output evenly: both of them get f . The symmetry assumptions yield equal values for optimal α and β . However, to keep track on what happens, we use notation u, v, ϕ, ψ, α and β , and we make the corresponding variables equal at a very last stage of the analysis. Then, we use u for u and v, ϕ for ϕ and ψ , and α for α and β . Also, while solving the individual intensities α_i and β_i , we use the subscripts to distinguish between individual decisions and the choices of all the other agents.

The value functions are

$$rU_i = -c(\alpha_i) + p_i(W - U_i), \quad (3)$$

$$rW = f - \lambda(W - U), \quad (4)$$

$$rV_i = -g(\beta_i) + q_i(J - V_i), \quad (5)$$

$$rJ = f - \lambda(J - V). \quad (6)$$

In equation (3), an unemployed worker chooses search intensity α_i and finds a vacancy at rate p_i which is defined below. The surplus from the match is denoted by $W - U_i$. In equation (4), a worker in match receives f as long as the the relationship goes on. A vacancy chooses β_i and finds a worker at rate q_i , defined below, and gets surplus $J - V_i$. A producing firm receives f until the match breaks down. Note that in equations (4) and (6) U and V do not have a subscript i . This is because while in a match, the workers and firms take the future values U and V as given. The matching rates are:

$$p_i = \frac{\alpha_i}{\alpha} p = \alpha_i (\alpha u)^{\phi-1} (\beta v)^\psi, \quad (7)$$

$$q_i = \frac{\beta_i}{\beta} q = \beta_i (\alpha u)^\phi (\beta v)^{\psi-1}. \quad (8)$$

The individual matching rate is thus proportional to individual search intensity. If a job seeker searches for example twice as intensively as all the other job seekers, he finds vacancies twice as fast. This is a reasonable assumption, because the number of agents

in the economy is assumed to be very large. Any individual agent's decision has no effect on the rest of the economy.

Job seekers choose α_i to maximize the right-hand side of (3), and vacancies choose β_i to maximize the right-hand side of (5). The first-order conditions are

$$-c'(\alpha_i) + \frac{\partial p_i}{\partial \alpha_i} (W - U_i) = 0, \quad (9)$$

$$-g'(\beta_i) + \frac{\partial q_i}{\partial \beta_i} (J - V_i) = 0. \quad (10)$$

Because the model is symmetric, it suffices to study α_i only. We look at how the optimal search intensity of an unmatched agent on one side of the market is affected by the search intensity of all the agents on the other side of the market and by the numbers of searching agents on both sides. The derivatives of the matching rate p_i are presented in Appendix 1.

The two sources that may be responsible for multiple equilibria in this model are *strategic complementarity* and *common pool externality*. Strategic complementarity means that the optimal value of the choice variable (search intensities in this case) of a member of one party is an increasing function of the value of the respective choice variable of the members of the opposing party, and that this reciprocity holds for both parties. Thus, the reaction functions $\alpha(\beta)$ and $\beta(\alpha)$ are increasing, and in principle they may intersect more than once. Common pool externality is in question when the optimal value of choice variable (search intensity in this case) of a member of either party depends on the number of agents in the market. For example, more unemployed job seekers may discourage each job seeker from searching, but it may make the vacancies to recruit more actively. The steady state equilibrium is unique if the optimal search intensity is an increasing function of the number of searching agents and if there is only one curve for optimal α . This situation is depicted in Figure 1.

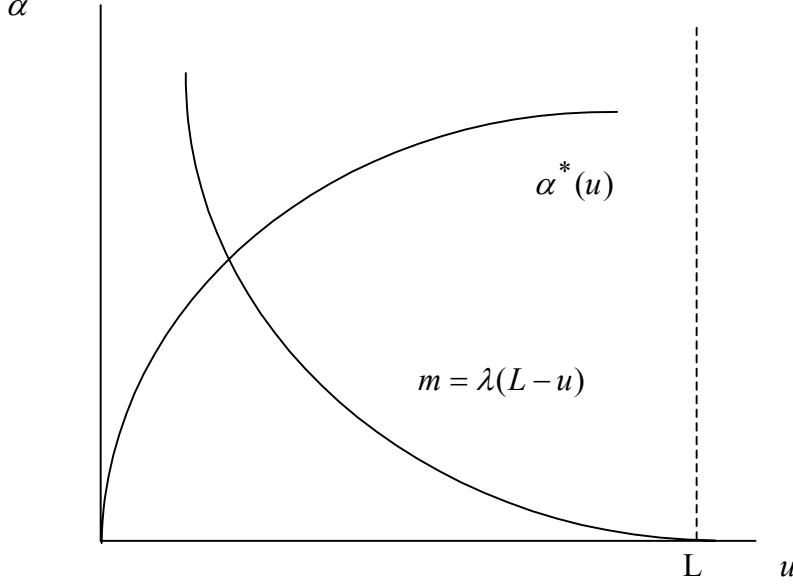


Figure 1: Unique equilibrium

2.1 Strategic Complementarity

The reaction curves may intersect more than once. We will find sufficient condition for the scale parameter ϕ such that we have a unique non-zero equilibrium value for search intensity at each value of u , like in Figure 2. Note that in the figure we have separated the search intensities of the two sides of the market. If the reaction functions intersect for example twice, we would have a situation depicted in Figure 3 in (α, u) -plane.

Totally differentiating the first-order condition of α_i gives

$$\begin{aligned} & \left[-c''(\alpha_i) + \frac{\partial^2 p_i}{\partial \alpha_i^2} (W - U_i) - \frac{\partial p_i}{\partial \alpha_i} \frac{\partial U_i}{\partial \alpha_i} \right] d\alpha_i + \\ & \left[\frac{\partial^2 p_i}{\partial \alpha_i \partial \beta} (W - U_i) + \frac{\partial p_i}{\partial \alpha_i} \frac{\partial (W - U_i)}{\partial \beta} \right] d\beta = 0, \end{aligned} \quad (11)$$

where $\frac{\partial^2 p_i}{\partial \alpha_i^2} = 0$ by (7) and $\frac{\partial U_i}{\partial \alpha_i} = 0$ by maximization. Equation (11) gives, using $c''(\alpha_i) = a(a-1)\alpha_i^{a-2}$ and the properties of the matching function, that

$$\frac{d\alpha_i}{d\beta} = \frac{\frac{\psi p}{\alpha \beta} (W - U_i) + \frac{p}{\alpha} \frac{\partial (W - U_i)}{\partial \beta}}{a(a-1)\alpha_i^{a-2}}, \quad (12)$$

where

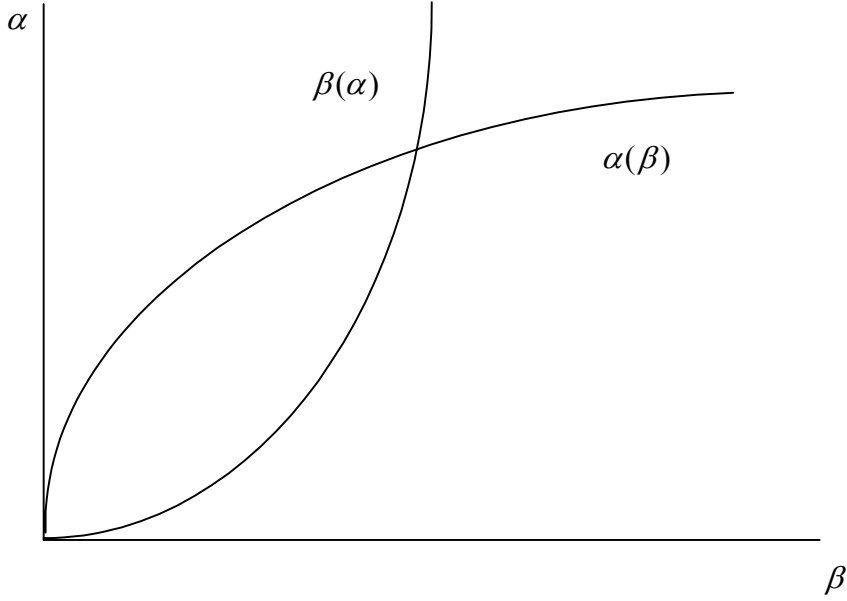


Figure 2: Unique and stable (α, β) - equilibrium

$$W - U_i = \frac{f - \lambda(W - U) + c(\alpha_i)}{r + p_i} \quad (13)$$

from (3) and (4). We get $W - U$ by setting $\alpha_i = \alpha$ and $p_i = p$:

$$W - U = \frac{f + c(\alpha)}{r + \lambda + p}. \quad (14)$$

Differentiating $W - U_i$ with respect to β gives

$$\frac{\partial(W - U_i)}{\partial\beta} = \frac{-\lambda(r + p_i) \frac{\partial(W - U)}{\partial\beta} - [f - \lambda(W - U) + c(\alpha_i)] \frac{\partial p_i}{\partial\beta}}{(r + p_i)^2}, \quad (15)$$

where

$$\frac{\partial(W - U)}{\partial\beta} = \frac{-(f + c(\alpha)) \frac{\partial p}{\partial\beta}}{(r + \lambda + p)^2} = \frac{-(W - U) \psi p}{(r + \lambda + p) \beta}. \quad (16)$$

Using (15) and (16) in (12), and utilizing symmetry ($\alpha_i = \alpha$ and $p_i = p$) we get (see Appendix 2)

$$\frac{d\alpha^*}{d\beta} = \frac{\psi p (W - U_i) (r + \lambda)}{\alpha \beta a (a - 1) \alpha^{a-2} (r + \lambda + p)}. \quad (17)$$

In (17), we still use $(W - U_i)$ instead of $(W - U)$, so that we can use the first-order condition of α_i and the search cost function to give

$$W - U_i = \frac{c'(\alpha_i)}{\partial p_i / \partial \alpha_i} = \frac{a \alpha_i^{a-1} \alpha}{p}. \quad (18)$$

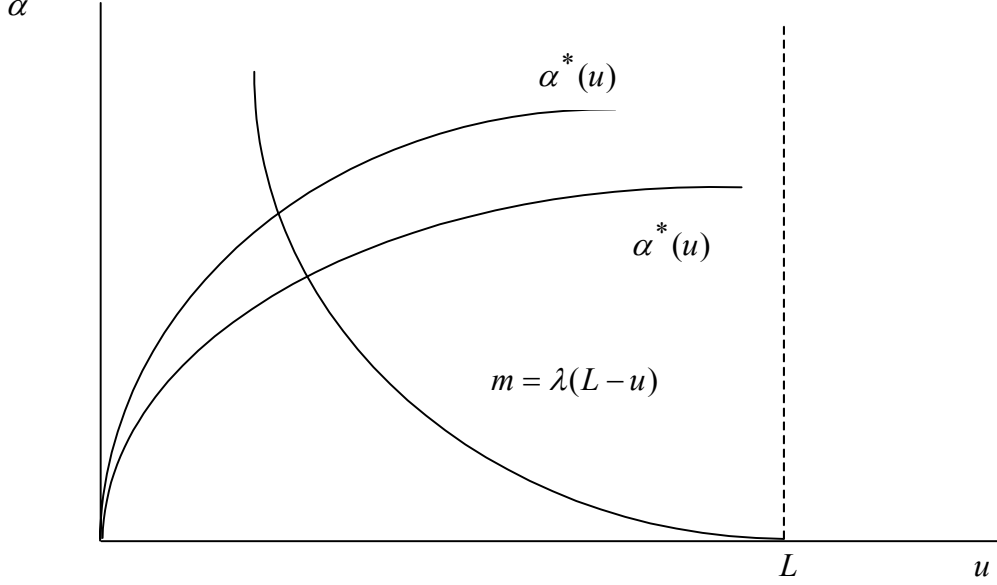


Figure 3: Multiple equilibria caused by strategic complementarity

Plugging (18) into (17) and then setting $\alpha_i = \alpha$ yields

$$\frac{d\alpha^*}{d\beta} = \frac{\phi(r + \lambda)}{(a - 1)(r + \lambda + p)}. \quad (19)$$

Expression (19) gives the slope of reaction function $\alpha(\beta)$ in an intersection of the reaction functions. By symmetry, $\alpha = \beta$ in an intersection.

Proposition 1 *The symmetric model does not have multiple equilibria caused by strategic complementarity.*

Proof. Consider equation (19). The symmetry assumption implies that $\alpha = \beta$ in an intersection of the reaction functions. The meeting rate p is equal to $\alpha^{2\phi}u^{2\phi-1}$, and we see that $\partial p/\partial \alpha > 0$ (with a fixed value of u). In consecutive equilibria in (α, β) -plane, $d\alpha^*/d\beta$ has ever-decreasing values. Assume first that $\phi \leq a - 1$. If $\alpha = \beta = 0$, then $p = 0$, and $\frac{d\alpha^*}{d\beta} = \frac{\phi}{a - 1} \leq 1$ in the origin. But then a non-degenerate equilibrium cannot exist because in the first non-degenerate equilibrium $d\alpha^*/d\beta$ must be larger than one. This is not possible since $d\alpha^*/d\beta$ is decreasing in p . Then assume that $\phi > a - 1$. Then $d\alpha^*/d\beta > 1$ in the origin, and in the next intersection of the reaction curves $d\alpha^*/d\beta$ is smaller than one. But this is the only non-degenerate equilibrium, because in the next

equilibrium (if it existed) $d\alpha^*/d\beta$ is larger than one, but such an equilibrium cannot exist because in consecutive equilibria $d\alpha^*/d\beta$ is decreasing in α . ■

The above proof implies the following result:

Remark 1 *A necessary condition for the existence of a nondegenerate equilibrium in (α, β) -plane in the symmetric model is that $\phi > a - 1$.*

The scale parameter must be large enough compared to the convexity of the cost function in order to lift the search intensities from zeroes. A nondegenerate equilibrium in (α, β) -plane is stable.

2.2 Common-Pool Externality

A change in u will change the position of the reaction functions $\alpha(\beta)$ and $\beta(\alpha)$. The standard way to solve how α and β will change is to use Jacobians. Because we have a symmetric model, both reaction functions behave in the same way, and Jacobians are not needed. If the optimal search intensity decreases as the number of searching agents increase, multiple equilibria may result, as depicted by Figure 4.

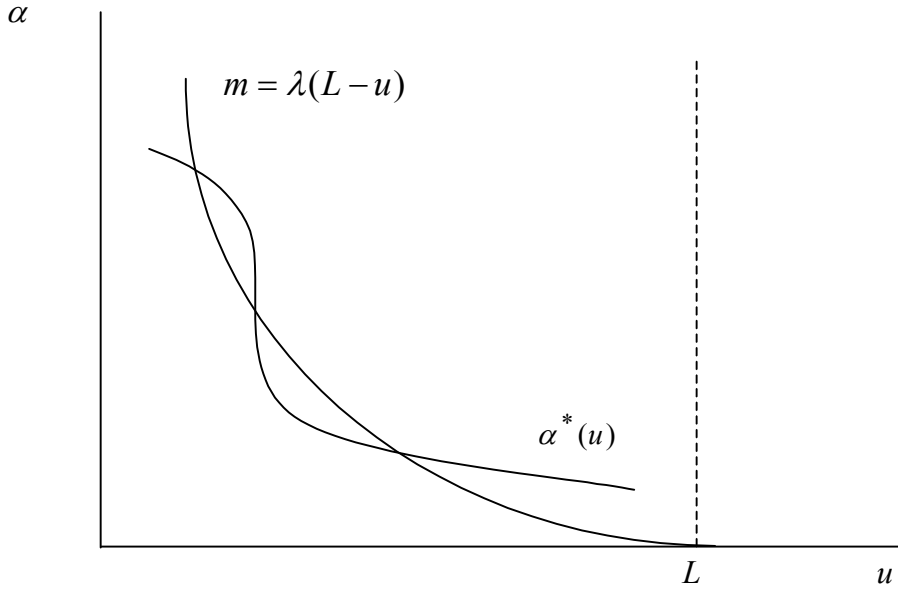


Figure 4: Multiple equilibria caused by common-pool externality

Totally differentiating the first-order condition for α_i with respect to α_i and u yields

$$\frac{d\alpha_i}{du} = \frac{\partial^2 p_i}{\partial \alpha_i \partial u} (W - U_i) + \frac{\partial p_i}{\partial \alpha_i} \frac{\partial (W - U_i)}{\partial u}. \quad (20)$$

Differentiating $W - U_i$ with respect to u we get

$$\frac{\partial (W - U_i)}{\partial u} = \frac{(r + p_i) \left[-\lambda \left(\frac{\partial (W - U)}{\partial u} \right) \right] - [f + c(\alpha_i) - \lambda (W - U)] \frac{\partial p_i}{\partial u}}{(r + p_i)^2}, \quad (21)$$

where $W - U = \frac{f + c(\alpha)}{r + \lambda + p}$ from equations (3) and (4) when $\alpha_i = \alpha$, and $\frac{\partial (W - U)}{\partial u} = \frac{-(W - U) \frac{\partial p}{\partial u}}{r + \lambda + p}$. After some manipulation (see Appendix 3) we get

$$\frac{d\alpha^*}{du} = \frac{(2\phi - 1) p (W - U) (r + \lambda)}{a(a - 1) \alpha_i^{a-2} \alpha u (r + \lambda + p)}, \quad (22)$$

and using $W - U = a\alpha^a/p$ given by the first-order condition of α_i , and letting $\alpha_i = \alpha$ (see equation (18)) we obtain

$$\frac{d\alpha^*}{du} = \frac{(2\phi - 1) \alpha (r + \lambda)}{(a - 1) u (r + \lambda + p)}. \quad (23)$$

Remark 2 *There are no multiple equilibria caused by common-pool externality if $\phi \geq 1/2$.*

Increasing returns to scale guarantee that the optimal search intensity increases when the number of searching agents increases. If $\phi < 1/2$, multiplicity of equilibria is possible but can be ruled out if ϕ is large enough:

Proposition 2 *In the symmetric model, there are no multiple equilibria caused by common-pool externality if $\phi \geq 1 - \frac{a}{2}$.*

Proof. We compare the slopes of $\alpha^*(u)$ and the steady-state curve $m = \lambda(L - u)$ in an intersection of the curves. If the derivative of $\alpha^*(u)$ is larger than the derivative of the steady state curve in an intersection of the curves, then $\alpha^*(u)$ and the steady state curve cannot have more than one intersection. In the symmetric case the matching function is $m = (\alpha u)^{2\phi}$, and differentiating the steady state condition with

respect to α and u gives $\frac{d\alpha}{du} = -\frac{2\phi p + \lambda}{2\phi m/\alpha}$. The difference of the slopes is equal to $\frac{(2\phi - 1)\alpha(r + \lambda)}{(a - 1)u(r + \lambda + p)} + \frac{2\phi p + \lambda}{2\phi m/\alpha}$, which is positive if (remembering that $pu \equiv m$ by pairwise matches) $(r + \lambda)[2\phi m(2\phi + a - 2) + (a - 1)\lambda u] + (a - 1)(2\phi p + \lambda)m > 0$, which holds if $\phi \geq 1 - \frac{a}{2}$. ■

We can also note the following by using equation (23):

Remark 3 *If the matching function is of Cobb-Douglas form, the aggregate search effort αu is increasing in u if $\phi \geq 1 - \frac{a}{2}$.*

2.3 Condition for Unique Equilibrium with Multiplicative Matching Function

Combining Proposition 1, Remark 1 and Proposition 2, we obtain

Theorem 1 *The symmetric model with multiplicative matching function has a non-degenerate equilibrium only if $\phi > a - 1$, and the equilibrium is unique if $\phi \geq 1 - \frac{a}{2}$.*

The result is in a striking contrast with the conventional wisdom, according to which increasing returns to scale is necessary for multiple equilibria. The scale parameter ϕ must be large enough for the equilibrium to exist at all, and if ϕ is not large enough, the optimal search intensity may decrease with unemployment in such a way that multiple equilibria emerge via common pool externality. The appropriate parameter values are depicted in Figure 5. With constant returns to scale ($\phi = 1/2$), multiple equilibria cannot exist because $a > 1$ by assumption, and with decreasing returns, multiple equilibria may exist if $a \in (1, 2)$. One should notice that this result partly relies on the assumption of perfect symmetry.

Diamond's coconut model resembles this model very closely. In his model, multiplicity of equilibria results also from common pool externality, but why does his model have multiple equilibria if returns to scale are *increasing*, while the present labor market model can have multiple equilibria only if the returns to scale are *low*? In both models, the more there are potential trading partners (people carrying coconuts, or unemployed) the

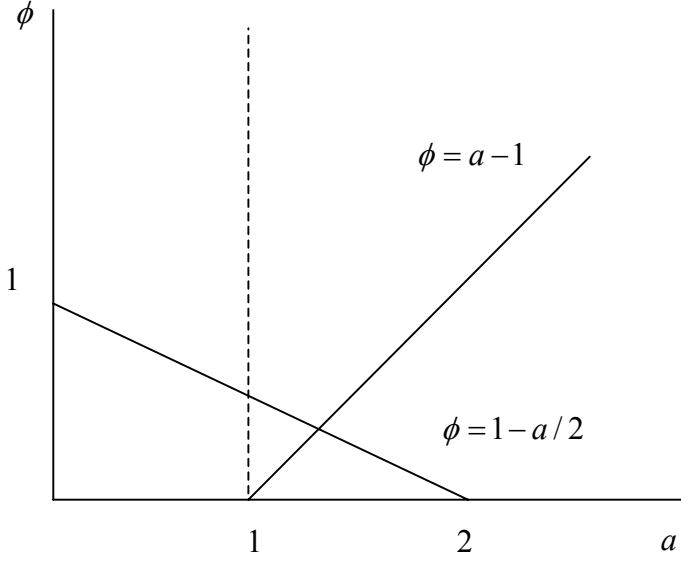


Figure 5: Parameter values for unique equilibrium in the symmetric model with multiplicative matching function: $\phi > a - 1$ and $\phi \geq 1 - a/2$.

higher the optimal value of choice variable (maximum acceptable height of a coconut tree, or search intensity, or the number of vacancies) is if returns to scale are high. But in the coconut model, the larger the value of the choice variable is, the *more* there are potential trading partners, whereas in the labor market model the larger the value of the choice variable, the *smaller* the pool of potential partners. The results of Diamond's model have thus been misinterpreted in the labor market context.

3 Additive Matching Function

The Cobb-Douglas matching function implies that matches are not formed unless both sides of the market put effort into searching. However, if matching function is additive, matches can form even if one side of the market is totally passive and just waits for contacts from the other side (see for example Diamond 1982b and Mortensen 1982a, 1982b). We specify an additive matching function that can have different levels of returns to scale:

$$m = (\alpha u)^\phi + (\beta v)^\psi. \quad (24)$$

The formulation implies that the amount of contacts initiated by job seekers can be separated from the contacts initiated by the vacancies. This formulation is chosen to highlight the difference between two types of matching ‘processes’ and to preserve the property that returns to scale can be determined. Search cost function is assumed to be $c(\alpha) = \alpha^a$, and recruiting cost function is $g(\beta) = \beta^b$, where $a > 1$ and $b > 1$. As in the previous section, we assume perfect symmetry: $u = v$, $\phi = \psi$ and $a = b \Rightarrow \alpha^* = \beta^*$. In the sequel we analyse the behavior of job seekers only; the behavior of vacancies is similar to that of job seekers.

Job seekers, when all of them search at intensity α , find vacancies at rate $p \equiv m/u = \alpha^\phi u^{\phi-1} + (\beta v)^\psi u^{-1}$. A job seeker who searches at intensity α_i finds a vacancy at rate $p_i = \alpha_i (\alpha u)^{\phi-1} + (\beta v)^\psi u^{-1}$. Note that the contact-taking rate of a job seeker is proportional on α_i (the first term) but his rate of receiving contacts (the second term) is directly independent of α_i . However, a change in the search intensity of all job seekers affects the recruiting intensity of all firms and vice versa, so the job seekers’ rate of receiving contacts depends on their search intensity.

3.1 Reaction Functions

The value functions are the same as equations (3)-(6) above. The first-order condition of a job seeker gives $\frac{d\alpha_i}{d\beta} = \frac{\frac{\partial p_i}{\partial \alpha_i} \left(\frac{\partial (W - U_i)}{\partial \beta} \right)}{c''(\alpha_i)}$ (look at equation (11)). Proceeding as in the case of multiplicative matching function, and using $\frac{\partial p_i}{\partial \alpha_i} = (\alpha u)^{\phi-1}$ and $c''(\alpha_i) = a(a-1)\alpha_i^{a-2}$, we have

$$\frac{d\alpha_i}{d\beta} = \frac{-(W - U) (\alpha u)^{\phi-1} \psi \beta^{\psi-1} v^\psi}{a(a-1)\alpha_i^{a-2} (r + \lambda + p) u} < 0. \quad (25)$$

From the first-order condition of α_i we get, using $c(\alpha) = \alpha^a$, that $W - U = \frac{a\alpha^{a-1}}{(\alpha u)^{\phi-1}}$. Plugging that into $d\alpha_i/d\beta$ and using symmetry ($u = v$, $\phi = \psi$, $\alpha_i = \alpha = \beta$) we get

$$\frac{d\alpha^*}{d\beta} = \frac{-\phi \alpha^\phi u^{\phi-1}}{(a-1)(r + \lambda + p)}. \quad (26)$$

The symmetric model has a stable intersection of reaction functions if $\left| \frac{d\alpha^*}{d\beta} \right| < 1$ (see Figure 6).

Proposition 3 *The symmetric model has a stable and unique intersection of reaction functions if $\phi \leq 2(a - 1)$.*

Proof. The equilibrium in (α, β) -plane is on the 45-degree line. It is stable if $\left| \frac{d\alpha^*}{d\beta} \right| < 1$, which holds if $\phi \alpha^\phi u^{\phi-1} < (a - 1)(r + \lambda + p)$. By symmetry, $p = 2\alpha^\phi u^{\phi-1}$, and the preceding inequality holds for all positive values of r and λ if $\phi \leq 2(a - 1)$. The search intensities are strategic substitutes, and therefore the reaction functions can have only one intersection in the symmetric model. ■

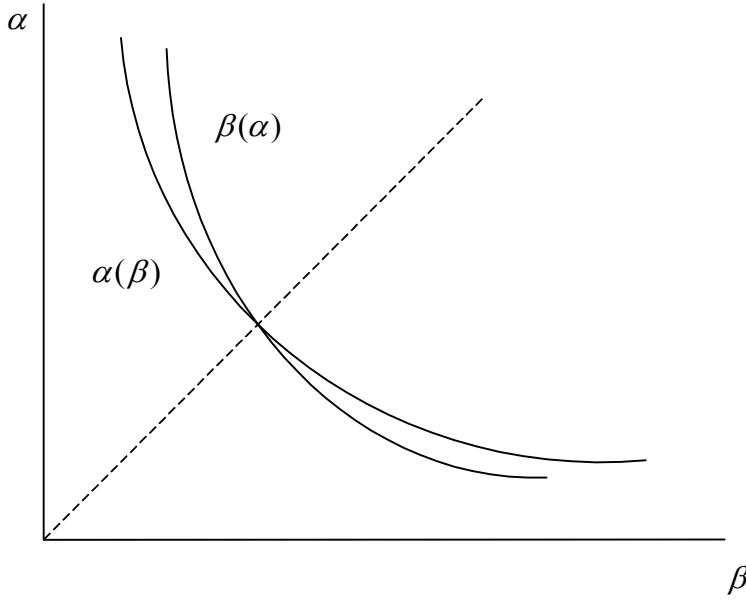


Figure 6: A symmetric (α, β) -equilibrium with additive matching function

By comparing Remark 1 and Proposition 3 we note the following:

Remark 4 *The multiplicative matching function and the additive matching function have opposing requirements for ϕ , regarding the existence of a stable equilibrium in (α, β) -plane.*

3.2 Common-Pool Externality

Increasing the number of searching agents on both sides of the market affects the optimal search intensities as follows:

$$\frac{d\alpha_i}{du} = \frac{\frac{\partial^2 p_i}{\partial \alpha_i \partial u} (W - U_i) + \frac{\partial p_i}{\partial \alpha_i} \frac{\partial (W - U_i)}{\partial u}}{c''(\alpha_i)}, \quad (27)$$

where $\frac{\partial^2 p_i}{\partial \alpha_i \partial u} = (\phi - 1) \alpha^{\phi-1} u^{\phi-2}$ and $\frac{\partial (W - U_i)}{\partial u} = \frac{2(1 - \phi) \alpha^\phi u^{\phi-2} (W - U)}{r + p}$. We get, using $c(\alpha) = \alpha^a$, symmetry and result $W - U = \frac{a\alpha^{a-1}}{(\alpha u)^{\phi-1}}$ from the first-order condition of α , that

$$\frac{d\alpha^*}{du} = \frac{(\phi - 1) \alpha (r + \lambda)}{(a - 1) (r + \lambda + p) u}. \quad (28)$$

If $\phi \geq 1$, common-pool externality does not cause multiple equilibria. We have a stronger result:

Proposition 4 *If the matching function is additive, the common pool externality does not cause multiple equilibria if $\phi \geq 2 - a$.*

Proof. As in the case of multiplicative matching function, we solve whether the steady state curve $m = \lambda(L - u)$ and the curve for optimal α can intersect more than once. Along the steady state curve, we have $\frac{d\alpha}{du} = -\frac{\phi m/u + \lambda}{\phi m/\alpha}$. The curves cannot intersect more than once if $\frac{(\phi - 1) \alpha (r + \lambda)}{(a - 1) (r + \lambda + p) u} + \frac{\phi m/u + \lambda}{\phi m/\alpha} > 0$, which holds if $(r + \lambda) [\phi(\phi - 1)m + (a - 1)u(\phi p + \lambda)] + (a - 1)m(\phi p + \lambda) > 0$, which holds if $\phi \geq 2 - a$. ■

The following results straightforwardly:

Remark 5 *If the matching function is additive, the aggregate search effort αu is increasing in u if $\phi \geq 2 - a$.*

3.3 Condition for Unique Equilibrium with Additive Matching Function

The search intensities are strategic substitutes, and low enough returns are required to guarantee that the intersection of the reaction functions is stable, whereas high enough

returns are needed to prevent multiplicity caused by common pool externality. The appropriate parameter region is depicted in Figure 7.

Theorem 2 *If the matching function is additive, the model has a unique equilibrium if $2 - a \leq \phi \leq 2(a - 1)$.*

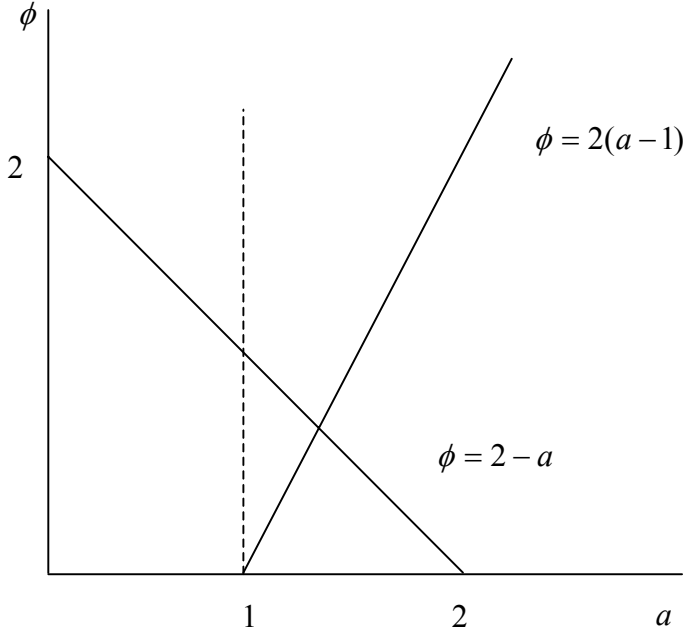


Figure 7: Parameter values for unique equilibrium in the symmetric model with additive matching function: $2 - a \leq \phi \leq 2(a - 1)$.

4 A Modified Howitt-McAfee Model

Howitt and McAfee (1987) present a model that is otherwise the same as the one with multiplicative matching function presented in Section 2, except that

- (i) The matching function is $m = \alpha\beta un$, where n is the number of firms.
- (ii) Each firm can have as many workers as it wants to, and each worker has a constant productivity. An increase of unemployment by one person does not necessarily mean that there is one more vacancy in the economy. In fact, the variable called vacancy is not

needed in the analysis, the firms just decide how much to invest in recruiting to get more workers.

(iii) Search cost functions are not specified, but it is assumed that job seekers' search cost function $c(\alpha)$ satisfies $c(0) = c'(0) = 0$, $c''(\alpha) > 0 \forall \alpha \geq 0$, and $c'(\alpha) \rightarrow \infty$ as $\alpha \rightarrow \infty$; firms' recruiting cost function $g(\beta)$ satisfies $g'(\beta) > 0 \forall \beta \geq 0$ and $g''(\beta) > 0 \forall \beta \geq 0$.

(iv) Workers die at rate λ , employed and unemployed alike. Each employed worker works until he dies, and there are no separations for other reasons. New, searching workers enter the economy at rate L . These assumptions are equivalent to assuming that the worker-firm relationships end at rate λ and the worker joins the pool of unemployed and starts to search again. The first-order conditions for the job seekers are the same in these two cases.

(v) Wages are fixed but wage does not necessarily equal profit.

Item (ii) above is what really makes this model different from the first one in the present article. In the first model, when there are more unemployed, the number of vacancies is larger as well, and that alleviates the discouraged-worker effect. (More unemployment decreases the search effort of every unemployed worker, if returns to scale are decreasing. See Pissarides, 2000, pp. 169 and 172-174.) In Howitt and McAfee's model that mirror effect is absent, and the discouraged worker effect is thus stronger.

In item (iii) assumption $g'(\beta) > 0$ for all $\beta \geq 0$ is partly responsible for the existence of multiple equilibria. Because of this assumption, it is not clear what is really the role of returns to scale in producing multiple equilibria. It is the same kind of assumption that Diamond used in his 'coconut model', where the lowest cost in investing for the tradable object is greater than zero. In order to get insight to the role of returns to scale, we assume $g'(0) = 0$.

In Howitt and McAfee's model, the wage is assumed to be fixed. This assumption could be criticized, and we could use Nash bargaining. However, we want to keep the present model as close to the Howitt and McAfee's model as possible in order to find out the role of returns to scale in producing multiple equilibria. For simplicity, we assume that the worker and firm split the product of their relationship equally.

In the present model the matching function is assumed to be $m = (\alpha u)^\phi (\beta n)^\psi$ where n is the fixed number of firms. The individual matching rates for job seekers and firms are denoted by p_i and q_i , respectively. The steady state condition is

$$m = L - \lambda u. \quad (29)$$

Workers enter the labor market at rate L , and the job seekers exit the labor market via matching or dying. Each relationship produces a flow product $2f$ as long as the worker lives. The value functions for workers are

$$rU_i = -c(\alpha_i) + p_i(W - U_i) - \lambda U_i, \quad (30)$$

$$rW = f - \lambda W, \quad (31)$$

thus the surplus from getting a job is equal to $W - U_i = \frac{f + c(\alpha_i)}{r + \lambda + p_i}$. The first-order conditions for job seekers and firms are

$$-c'(\alpha_i) + \frac{\partial p_i}{\partial \alpha_i}(W - U_i) = 0, \quad (32)$$

$$-g'(\beta_i) + \frac{\partial q_i}{\partial \beta_i} \frac{f}{r + \lambda} = 0. \quad (33)$$

Equation (33) implies that the firms are not alternating between a vacancy and a filled firm; a firm has an indeterminate number of workers and it just chooses how eagerly it wants to get another worker. We assume that the search and recruiting cost functions are $c(\alpha) = \alpha^a$, and $g(\beta) = \beta^b$, where $a > 1$ and $b > 1$.

4.1 Howitt and McAfee's Method of Analysis

The analysis used by Howitt and McAfee is best explained by using Figure 8. The same figure (except its notation) appears in their article. Their argument regarding multiple equilibria goes as follows: As firms increase their recruiting effort β , job seekers increase their optimal search effort α , because the marginal benefit of α in producing contacts increases. This results from multiplicative matching function. An increase in α makes recruiting effort more profitable in turn because the marginal benefit of recruiting increases. This effect is depicted by the increasing part of curve $\frac{\partial q_i}{\partial \beta} \frac{f}{r + \lambda}$.

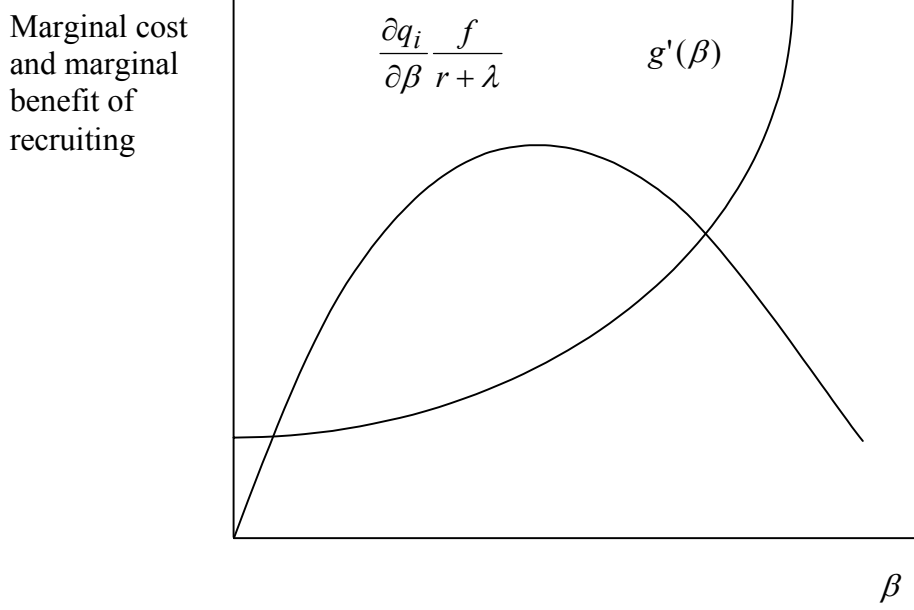


Figure 8: Multiple equilibria in Howitt and McAfee's model

But the larger the search and recruiting intensities are, the smaller will be the pool of unemployed from which the firms recruit. This common-pool effect decreases the marginal benefit of recruiting. The marginal cost of recruiting is increasing. The result is that the economy has multiple equilibria. However, the model does not necessarily have multiple equilibria if $g'(0) = 0$. The authors recognize it, though, and they provide a condition to have multiple equilibria: if $g'(0) = 0$ but $g''(0) > \frac{\partial^2 q_i}{\partial \beta^2} \frac{f}{r + \lambda}$, the value of the latter being evaluated at $\beta = 0$, multiple equilibria would emerge. Whether it is assumed that $g'(0) > 0$ or $g'(0) = 0$, their model would have multiple equilibria even without the common-pool effect, as long as $\frac{\partial q_i}{\partial \beta}$ is an increasing and linear, or increasing and concave function of β . Howitt and McAfee's argument about multiple equilibria is in fact verbal, and the link between multiplicity and returns to scale is vague. Allowing variable returns to scale, assuming $g'(0) = 0$, and cutting the problem into pieces of strategic complementarity and common-pool externality will reveal how multiple equilibria and returns to scale are connected. This will be done in the following two subsections.

4.2 Strategic Complementarity in the Modified Model

Differentiating the first-order conditions (with respect to α_i and β , and with respect to β_i and α), using the cost functions $c(\alpha) = \alpha^a$ and $g(\beta) = \beta^b$, where $a > 1$ and $b > 1$, then letting $\alpha_i = \alpha$ and $\beta_i = \beta$, and using the first-order condition for β_i again, (see Appendix 4) we get the reaction functions:

$$\frac{d\alpha^*}{d\beta} = \frac{\psi\alpha(r+\lambda)}{(a-1)\beta(r+\lambda+p)}, \quad (34)$$

$$\frac{d\beta^*}{d\alpha} = \frac{\phi\beta}{(b-1)\alpha}. \quad (35)$$

The reaction functions have no more than one stable intersection if for any intersection of $\alpha^*(\beta)$ and $\beta^*(\alpha)$, where α and β are strictly positive, we have $\frac{d\alpha^*}{d\beta} < \left(\frac{d\beta^*}{d\alpha}\right)^{-1}$. The latter condition can be written as

$$\frac{\alpha}{\beta} \left[\frac{\psi(r+\lambda)}{(a-1)(r+\lambda+p)} - \frac{b-1}{\phi} \right] < 0. \quad (36)$$

Proposition 5 *The modified Howitt-McAfee -model has no multiple equilibria caused by strategic complementarity.*

Proof. In criterion (36) the term in brackets is decreasing in p , and $\partial p/\partial \alpha > 0$ and $\partial p/\partial \beta > 0$. That is, the term in brackets is decreasing in α and β . If $\phi\psi < (a-1)(b-1)$, the only equilibrium is in the origin, because then $\frac{d\alpha^*}{d\beta} < \left(\frac{d\beta^*}{d\alpha}\right)^{-1}$ in the origin, and $\frac{d\alpha^*}{d\beta} > \left(\frac{d\beta^*}{d\alpha}\right)^{-1}$ in the first non-degenerate equilibrium, but the latter is not possible because the term in brackets in criterion (36) is decreasing in p . The first non-degenerate equilibrium (if the equilibrium exists) has thus $\frac{d\alpha^*}{d\beta} < \left(\frac{d\beta^*}{d\alpha}\right)^{-1}$. No other non-degenerate equilibria can exist because the second consecutive equilibrium would have $\frac{d\alpha^*}{d\beta} > \left(\frac{d\beta^*}{d\alpha}\right)^{-1}$, but that is not possible because $\frac{\psi(r+\lambda)}{(a-1)(r+\lambda+p)} - \frac{b-1}{\phi}$ decreases as we move away from the origin. ■

The proof above implies the following:

Remark 6 *A necessary condition for the existence of a nondegenerate equilibrium in the modified Howitt-McAfee -model is that $\phi\psi > (a-1)(b-1)$.*

For the rest of Section 4 we assume that $\phi\psi > (a-1)(b-1)$, unless otherwise mentioned.

4.3 Common-Pool Externality in the Modified Model

An increase in unemployment may discourage the job seekers from searching. On the other hand, this is at least partly compensated by the increased recruiting effort of firms. The equilibrium effect of increase in unemployment on the search and recruiting intensities is analyzed by using Jacobians. Let

$$F = -c'(\alpha_i) + \frac{\partial p_i}{\partial \alpha_i}(W - U_i), \quad (37)$$

$$G = -g'(\beta_i) + \frac{\partial q_i}{\partial \beta_i} \frac{f}{r + \lambda} 0. \quad (38)$$

Along optimal $\alpha^\phi \beta^\psi$ we have $F = G = 0$. In the standard manner, $\frac{d\alpha^*}{du} = \frac{|A_\alpha|}{|A|}$ and

$$\frac{d\beta^*}{du} = \frac{|A_\beta|}{|A|}, \text{ where } |A| \equiv \begin{vmatrix} \frac{\partial F}{\partial \alpha} & \frac{\partial F}{\partial \beta} \\ \frac{\partial G}{\partial \alpha} & \frac{\partial G}{\partial \beta} \end{vmatrix}, |A_\alpha| \equiv \begin{vmatrix} -\frac{\partial F}{\partial u} & \frac{\partial F}{\partial \beta} \\ -\frac{\partial G}{\partial u} & \frac{\partial G}{\partial \beta} \end{vmatrix} \text{ and } |A_\beta| \equiv \begin{vmatrix} \frac{\partial F}{\partial \alpha} & -\frac{\partial F}{\partial u} \\ \frac{\partial G}{\partial \alpha} & -\frac{\partial G}{\partial u} \end{vmatrix}.$$

The partial derivatives of F and G are solved in the Appendix 5. We then have

$$|A_\alpha| = \frac{ab\alpha^{a-1}\beta^{b-2}[\phi\psi - (b-1)(1-\phi)](r+\lambda)}{(r+\lambda+p)u}, \quad (39)$$

$$|A_\beta| = \frac{ab\alpha^{a-2}\beta^{b-1}\phi[(r+\lambda)(a+\phi-2) + (a-1)p]}{(r+\lambda+p)u}, \quad (40)$$

$$|A| = \frac{ab\alpha^{a-2}\beta^{b-2} \left[(r+\lambda)[(a-1)(b-1) - \phi\psi] + (a-1)(b-1)p \right]}{r+\lambda+p}. \quad (41)$$

In a non-degenerate equilibrium in (α, β) -plane we have $\frac{d\alpha^*}{d\beta} < \left(\frac{d\beta^*}{d\alpha}\right)^{-1}$ if and only if $\phi\psi > (a-1)(b-1)$. (See the proof of Proposition 5.) That is, if and only if $\phi\psi > (a-1)(b-1)$, we have $|A| < 0$ in the origin. Then $|A| > 0$ in the first non-degenerate equilibrium in (α, β) -plane. We can find a condition for discouraged-worker effect, by looking at the sign of $|A_\alpha|$, and the same kind of result for recruiting intensity:

Remark 7 *The effects of a change in unemployment on search intensities are $\frac{d\alpha^*}{du} < 0$ if and only if $\phi < \frac{b-1}{\psi+b-1}$, and $\frac{d\beta^*}{du} > 0$ if $\phi \geq 2-a$.*

If $\frac{\partial (\alpha^\phi \beta^\psi)}{\partial u} > 0$, the possible discouraged worker effect is more than compensated by an increase in firms' recruiting effort, and there are no multiple equilibria caused by the common-pool externality. Plugging in the determinants we have

$$\begin{aligned} \frac{d(\alpha^\phi \beta^\psi)}{du} &= |A|^{-1} [\alpha^\phi \psi \beta^{\psi-1} |A_\beta| + \phi \alpha^{\phi-1} \beta^\psi |A_\alpha|] \\ &= \frac{\left\{ \begin{aligned} &(r + \lambda) \phi a b \alpha^{a+\phi-2} \beta^{b+\psi-2} [(a + \phi - 2) \psi + (\phi \psi - (b - 1)(1 - \phi))] \\ &+ a(a - 1) b \phi \psi \alpha^{a+\phi-2} \beta^{b+\psi-2} p \end{aligned} \right\}}{|A| (r + \lambda + p) u} \end{aligned} \quad (42)$$

> 0 if $(a + \phi - 2) \psi + (\phi \psi - (b - 1)(1 - \phi)) > 0$ (assuming that $|A| > 0$). Rearranging, we get

Lemma 1 *The modified Howitt-McAfee -model has no multiple equilibria caused by common-pool externality if $\phi \geq 1 - \frac{a\psi}{2\psi + b - 1}$.*

The larger ϕ and ψ are, the less likely it is that multiple equilibria emerge. Like in the two previous models, we can find a condition that rules out multiple equilibria even if the optimal $\alpha^\phi \beta^\psi$ -curve is decreasing in u :

Proposition 6 *The modified Howitt-McAfee -model does not have multiple equilibria caused by common-pool externality if $\phi \geq 2 - a$.*

Proof. Along the steady state-curve, $\frac{d(\alpha^\phi \beta^\psi)}{du} = -\frac{\phi \alpha^\phi u^{\phi-1} (\beta n)^\psi + \lambda}{u^\phi n^\psi} = -\frac{\phi p + \lambda}{u^\phi n^\psi}$.

Manipulating (42) a little we get that along the optimal $\alpha^\phi \beta^\psi$ -curve $\frac{d(\alpha^\phi \beta^\psi)}{du} = \frac{\alpha^\phi \beta^\psi \phi \{(r + \lambda) [\psi(a + \phi - 2) + \phi \psi - (b - 1)(1 - \phi)] + \psi(a - 1)p\}}{\{(r + \lambda) [(a - 1)(b - 1) - \phi \psi] + (a - 1)(b - 1)p\} u}$. The curves cannot have more than one intersection if

$$\frac{\alpha^\phi \beta^\psi \phi \{(r + \lambda) [2\phi \psi + \psi(a - 2) - (b - 1)(1 - \phi)] + \psi(a - 1)p\}}{\{(r + \lambda) [(a - 1)(b - 1) - \phi \psi] + (a - 1)(b - 1)p\} u} + \frac{\phi p + \lambda}{u^\phi n^\psi} \geq 0. \quad (43)$$

Inequality (43) holds if $\phi \psi > (a - 1)(b - 1)$ (then the denominator of the left-hand side of (43) is positive) and if

$$\begin{aligned} &m\phi \{(r + \lambda) [2\phi \psi + \psi(a - 2) - (b - 1)(1 - \phi)] + \psi(a - 1)p\} \\ &+ (\phi m + \lambda u) \{(r + \lambda) [(a - 1)(b - 1) - \phi \psi] + (a - 1)(b - 1)p\} \\ &\geq 0, \end{aligned} \quad (44)$$

which holds if

$$\begin{aligned}
& m\phi(r + \lambda) \{2\phi\psi + \psi(a - 2) - (b - 1)(1 - \phi) + (a - 1)(b - 1) - \phi\psi\} \quad (45) \\
& + m\phi p(a - 1)(\psi + b - 1) \\
& + \lambda u \{(r + \lambda)[(a - 1)(b - 1) - \phi\psi] + (a - 1)(b - 1)p\} \\
& \geq 0,
\end{aligned}$$

which holds if the first line of (45) is non-negative (note that the third line of (45) is positive at a nondegenerate equilibrium, see the argument immediately after expression (41)), which is true if $(\psi + b - 1)(\phi + a - 2) \geq 0$. Because we have assumed that $b > 1$, the latter holds if $\phi \geq 2 - a$. ■

If $\phi \geq 2 - a$, the discouraged-worker effect is thus compensated by an increase in β such that even if the optimal $\alpha^\phi \beta^\psi$ is decreasing in u , it cannot cross the steady state curve more than once. In fact, $d\beta/du > 0$ (which holds if $\phi \geq 2 - a$) suffices to compensate for the discouraged-worker effect. Like in the preceding models, we can state the following:

Remark 8 *In the modified Howitt-McAfee model the aggregate search effort αu is increasing in u if $\phi \geq 2 - a$ and $\phi\psi > (a - 1)(b - 1)$.*

Proof. We have $\frac{d(\alpha^* u)}{du} = \frac{|A_\alpha| u + |A| \alpha}{|A|} \geq 0$ if $|A| > 0$ (which holds if $\phi\psi > (a - 1)(b - 1)$) and if $\phi \geq 2 - a$. ■

4.4 Condition for Unique Equilibrium in the Modified Howitt-McAfee Model

Combining Proposition 5, Remark 6 and Proposition 6 we end up with

Theorem 3 *The modified Howitt-McAfee Model has a non-degenerate equilibrium only if $\phi\psi > (a - 1)(b - 1)$, and the equilibrium is unique if $\phi \geq 2 - a$.*

This result conforms with Theorem 1, and it shows that the basic results regarding multiplicity are not sensitive to symmetry assumptions made in Section 2.

5 Conclusion

We have shown that a basic search model with a Cobb-Douglas matching function can in principle produce multiple unemployment equilibria via strategic complementarity or common pool externality. In the simple models presented here, strategic complementarity does not, however, cause multiple equilibria, regardless of returns to scale. Common pool externality can cause multiplicity only if the returns to scale are *low* and if the search cost function is not very convex. These results contradict the present view according to which increasing returns to scale is a necessary condition for multiple equilibria. In the end of Section 2 we showed that the implications of Diamond’s ‘coconut model’ must be reversed in order to apply in a labor market model where vacancies and job seekers choose their search intensities.

The symmetry assumption in the first two models of the present article is partly responsible for the results, and the results might change a little bit if asymmetry was introduced. Then it would be natural to endogenize wages and profits by applying the Nash bargaining solution. The results regarding the model of Howitt and McAfee would also change if we allowed Nash bargaining. The exogenously given wage was chosen because we wanted to keep the model close to Howitt and McAfee’s original model.

A few empirical papers have estimated returns to scale in labor market. Pissarides (1986, 1988) finds evidence for constant returns to scale in Britain in years 1967 - 1983, with scale parameters $\phi = 0.7$ and $\psi = 0.3$. He says that “A necessary, though not sufficient, condition for multiplicity is that the search technology ... should have increasing returns to scale.” (Pissarides 1988) On the other hand, in his theoretical model on which he bases his judgement the projects are of equal value but the number of vacancies adjusts according to zero-profit condition $V = 0$. That kind of model is outside the scope of the present paper, but it is considered in Virrankoski (2003). Coles and Smith (1996) estimated the returns to scale using a cross-section of city-level data on unemployment, vacancies and job placings in the United Kingdom in March 1987. They found the same parameter values as Pissarides did. They refer to Diamond (1982a) when saying that with increasing returns to scale it is possible to have multiple equilibria. Baker, Hogan

and Ragan (1996) find “significant increasing returns to labour-market matching”. They refer to Pissarides (1986) and Diamond (1982a), hinting that increasing returns can produce multiple equilibria. Warren (1996) estimated returns to scale in U.S. manufacturing sector from 1969 to 1973 and he found that the scale elasticity is 1.332 or 1.536, depending on the estimated model. He concludes that the finding (increasing returns to scale with respect to the number of searchers) is consistent with multiple equilibria, referring for example to Diamond (1982a), Mortensen (1982a), Pissarides (1984b), Diamond and Fudenberg (1989), and to Howitt and McAfee (1987). The empirical papers’ suggestions of the existence of multiple equilibria are based on theoretical models where the agents control their reservation values but not search intensities (Howitt and McAfee (1987) is an exception), or on models where the agents control their search intensities but where the number of firms is determined by a zero-profit condition.

In the light of the present paper and of Virrankoski (2003), the above speculations go in the wrong direction. Large returns to scale do not endanger uniqueness but guarantee it.

6 Appendix

1. Derivatives of matching rates resulting from matching function $m = (\alpha u)^\phi (\beta v)^\psi$:

$$\frac{\partial p_i}{\partial \alpha_i} = (\alpha u)^{\phi-1} (\beta v)^\psi = \frac{p}{\alpha}, \quad \frac{\partial p_i}{\partial \beta} = \alpha_i (\alpha u)^{\phi-1} \psi \beta^{\psi-1} v^\psi, \quad \frac{\partial^2 p_i}{\partial \alpha_i \partial \beta} = (\alpha u)^{\phi-1} \psi \beta^{\psi-1} v^\psi = \frac{\psi p}{\alpha \beta}.$$

When calculating $\frac{\partial p_i}{\partial u}$, v changes by the same amount as u , because we have assumed that the total number of agents on both sides is fixed (In the symmetric model, $u = v$, but for tractability, let us first denote u and v as separate variables.):

$$\frac{\partial p_i}{\partial u} = \alpha_i \alpha^{\phi-1} (\phi-1) u^{\phi-2} (\beta v)^\psi + \alpha_i (\alpha u)^{\phi-1} \beta^\psi \psi v^{\psi-1} \frac{\partial v}{\partial u}, \text{ and using } \alpha_i = \alpha, v = u, \\ \text{and } \phi = \psi \text{ we get } \frac{\partial p_i}{\partial u} = \frac{p(2\phi-1)}{u}. \text{ We also have } \frac{\partial^2 p_i}{\partial \alpha_i \partial u} = \alpha^{\phi-1} (\phi-1) u^{\phi-2} (\beta v)^\psi + \\ (\alpha u)^{\phi-1} \beta^\psi \psi v^{\psi-1} \frac{\partial v}{\partial u} = \frac{p(2\phi-1)}{\alpha u}.$$

2. Derivation of equation (17)

Equation (15) can be written, using (16), and substituting α for α_i and p for p_i , as

$$\frac{\partial(W - U_i)}{\partial\beta} = \frac{\frac{\lambda\psi p(r+p)(W-U)}{(r+\lambda+p)\beta} + \frac{\lambda\psi p(W-U)}{\beta} - \frac{\psi p(f+c(\alpha))}{\beta}}{(r+p)^2} \quad (\text{A1})$$

$$= \frac{(W-U) \left[\frac{\lambda\psi p(r+p)}{(r+\lambda+p)\beta} + \frac{\lambda\psi p}{\beta} - \frac{(r+\lambda+p)\psi p}{\beta} \right]}{(r+p)^2}. \quad (\text{A2})$$

Then use (A2) and (16) to get (17).

3. Derivation of equation (22)

Equation (21) can be written as

$$\frac{\partial(W - U_i)}{\partial u} = \frac{\left\{ \frac{\lambda p(r+p)(2\phi-1)(W-U)}{(r+\lambda+p)u} + \frac{\lambda p(2\phi-1)(W-U)}{u} - \frac{(r+\lambda+p)p(W-U)}{u} \right\}}{(r+p)^2}, \quad (\text{A3})$$

and using $\frac{\partial p_i}{\partial \alpha_i} = \frac{p}{\alpha}$ and $\frac{\partial^2 p_i}{\partial \alpha_i \partial u} = \frac{p(2\phi-1)}{\alpha u}$, (20) can then be written as

$$\begin{aligned} \frac{d\alpha_i}{du} &= \frac{p(2\phi-1)(W-U)}{a(a-1)\alpha_i^{a-2}\alpha u} \left[1 + \frac{\lambda p}{(r+p)(r+\lambda+p)} + \frac{\lambda p}{(r+p)^2} - \frac{(r+\lambda+p)p}{(r+p)^2} \right] \\ &= \frac{p(2\phi-1)(W-U)(r+\lambda)}{a(a-1)\alpha_i^{a-2}\alpha u(r+\lambda+p)}. \end{aligned} \quad (\text{A4})$$

4. Derivation of equations (34) and (35)

We have

$$\frac{d\alpha_i}{d\beta} = \frac{\frac{\partial^2 p_i}{\partial \alpha_i \partial \beta}(W - U_i) + \frac{\partial p_i}{\partial \alpha_i} \frac{\partial(W - U_i)}{\partial \beta}}{c''(\alpha_i)}, \quad (\text{A5})$$

plug in $\partial p_i / \partial \alpha_i = p / \alpha$, $\frac{\partial^2 p_i}{\partial \alpha_i \partial \beta} = \frac{\psi p}{\alpha \beta}$, $W - U_i = \frac{a\alpha^a}{p}$ (from the first-order condition of α_i), and proceed as in deriving (17), and (34) results (Note than in (17) we used assumption $\phi = \psi$).

Totally differentiating a firm's first-order condition and then letting $\beta_i = \beta$ yields

$$\begin{aligned} \frac{d\beta_i}{d\alpha} &= \frac{\phi \alpha^{\phi-1} u^\phi(\beta n)^{\psi-1} \left(\frac{f}{r+\lambda} \right)}{g''(\beta)} \\ &= \frac{\phi m f}{\alpha \beta n g''(\beta)(r+\lambda)}. \end{aligned} \quad (\text{A6})$$

Then use the first-order condition of β_i and $g''(\beta) = b(b-1)\beta^{b-2}$ to get $\frac{f}{r+\lambda} = \frac{b\beta^b n}{m}$, and (35) follows.

5. Partial derivatives of Jacobians in the generalized Howitt-McAfee model:

Equations (37) and (38) give

$$\frac{\partial F}{\partial \alpha_i} = -c''(\alpha_i) + \frac{\partial^2 p_i}{\partial \alpha_i^2} (W - U_i) + \frac{\partial p_i}{\partial \alpha_i} \frac{\partial (W - U_i)}{\partial \alpha_i} = -a(a-1)\alpha_i^{a-2}, \text{ because } \frac{\partial^2 p_i}{\partial \alpha_i^2} = \frac{\partial (W - U_i)}{\partial \alpha_i} = 0.$$

Assuming symmetry we have $\alpha_i = \alpha$, and we can write $\frac{\partial F}{\partial \alpha} = -a(a-1)\alpha^{a-2}$. We also have $\frac{\partial F}{\partial \beta} = \frac{\psi a \alpha^{a-1} (r + \lambda)}{\beta (r + \lambda + p)}$; $\frac{\partial G}{\partial \alpha} = \frac{\phi q f}{\alpha \beta (r + \lambda)}$, and using the first-order condition of β_i and $g''(\beta) = b(b-1)\beta^{b-2}$ to get $\frac{f}{r+\lambda} = \frac{b\beta^b n}{m}$, we get $\frac{\partial G}{\partial \alpha} = \frac{\phi b \beta^{b-1}}{\alpha}$; $\frac{\partial G}{\partial \beta} = -b(b-1)\beta^{b-2}$. We also have $\frac{\partial F}{\partial u} = \frac{(\phi-1)p(W-U)(r+\lambda)}{\alpha u (r + \lambda + p)}$, and using $W - U = a\alpha^a/p$ (from the first-order condition of α) we get $\frac{\partial F}{\partial u} = \frac{(\phi-1)a\alpha^{a-1}(r+\lambda)}{u(r+\lambda+p)}$. We get $\frac{\partial G}{\partial u} = \frac{\phi b \beta^{b-1}}{u}$ by replacing $\frac{f}{r+\lambda}$ by $\frac{b\beta^b n}{m}$.

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Search, Entry, and Unique Equilibrium

Juha Virrankoski*

University of Helsinki

Abstract

We study a labor market where projects are homogenous, firms can enter the market, and wages are determined by Nash bargaining. Multiplicity of unemployment equilibria caused by strategic complementarity is distinguished from that caused by common pool externality. The main result is that high scale elasticities of aggregate search efforts of workers and firms are associated with unique equilibrium.

Keywords: search, multiple equilibria

JEL classification: C78, E24, J64

1 Introduction

A widely accepted view in search literature is that multiple unemployment equilibria may emerge if the matching function has increasing returns to scale with respect to aggregate search efforts on the vacancy and job seeker sides of the labor market. This view originates for example from Diamond (1982), Howitt and McAfee (1987), and Pissarides (1986).

*Research Unit on Economic Structures and Growth, Department of Economics, P.O. Box 17 (Arkadiankatu 7), FIN-00014 University of Helsinki, Finland. Telephone: +358-9-191 28760, e-mail: juha.virrankoski@helsinki.fi. This paper was mainly written when I was at Department of Economics at Helsinki School of Economics. I thank Dale Mortensen, Robert Shimer, and other participants in Society for Economics Dynamics meetings and European Economic Association meetings, as well as participants in Helsinki School of Economics research seminar for useful comments. I gratefully acknowledge financial support from Helsinki School of Economics Foundation and Yrjö Jahnsson Foundation.

In Diamond's 'coconut model', agents decide on the reservation cost of a project. The higher the reservation cost, the larger the pool of potential trading partners. The larger the pool, the larger in turn the reservation cost. In Howitt and McAfee's labor market model, firms and workers decide how intensively to search for partners. In Pissarides's (1986) model, projects are homogenous, output is shared according to Nash bargaining solution, and the number of vacancies is determined by zero-profit condition. If the matching function has increasing returns to scale, more unemployment can reduce the number of vacancies that enter, leading to multiple employment equilibria, because the Beveridge curve that relates the number of unemployed and vacancies in a steady state is decreasing. Pissarides (2000, 123-143) considers an otherwise similar model as Pissarides (1986) except that the matching function is assumed to have constant returns to scale. A unique employment equilibrium emerges.

A usual reason for multiplicity of equilibria in economic models is that various externalities are present. In search literature, concepts like strategic complementarity, congestion externality, common pool externality (also called 'common property' externality), thin-market externality, and so on, are mentioned as generators of multiplicity. The number of job seekers and recruiting firms affect the optimal search intensities. This is called common pool externality. (See Howitt and McAfee (1987) who actually name the effect as common property externality.) On the other hand, the search intensities of firms and workers affect each other. This is called strategic complementarity if the effect is positive on both sides (see Bulow, Geanakoplos, and Klemperer, 1985). Note that strategic complementarity (or substitutability) requires that there are large pools of 'opposite' sides in the market whose behavior affects the optimal behavior of the members of the other party. Therefore, the coconut model where agents on only one side of the market make decisions (the other behaves mechanistically) has no strategic complementarity. Multiple equilibria in the coconut model results from common pool externality, where the size of the pool of potential partners interacts with willingness to produce.

The role of returns to scale in the above mentioned models in producing multiple equilibria is, however, not very clear. In some models like Diamond (1982) and Howitt and McAfee (1987), specific assumptions on search costs seem to be responsible for multiple

equilibria, whether the returns to scale are increasing or not. In Pissarides (1986), like in the model of Howitt and McAfee (1987), the analysis of multiple equilibria suffers from treating strategic complementarity and common pool externality together. Moreover, in Pissarides (1986) the analysis of job seekers' search intensity is made at the aggregate level and not by solving the Nash equilibrium of search intensities.

In the present article we solve the sufficient conditions for a unique unemployment equilibrium. We specify a model where job seekers and vacancies choose their search intensities and where the number of firms is determined by a zero profit condition. (A variety of models with a fixed number of firms are treated in Virrankoski, 2003.) We use Cobb-Douglas matching function, and a power function for search costs for tractability. Multiple equilibria can in principle result from strategic complementarity or common pool externality, and these effects are studied in turn. The main result is that if the scale parameters of the matching function are large enough, and if the workers' bargaining power is large enough, and if the job seekers' search costs increase fast enough with search intensity, the equilibrium is unique. Large values of those parameters prevent both strategic complementarity and common pool externality from causing multiplicity of equilibria. The rest of the paper is organized as follows: Section 2 presents the model, and in Section 3 we solve the equilibrium conditions. Section 4 studies whether strategic complementarity or common pool externality can cause multiple equilibria. Section 5 concludes.

2 The Model

The model is a very standard job market search model with endogenous number of vacancies (see for example Pissarides, 2000). Conditions for unique equilibrium have, however, features not reported before. Time is continuous and extends to infinity. The analysis is confined to a steady state. All agents live forever, and their rate of time preference is r . There is a large number L of workers. They are identical but u of them are unemployed at each moment, the rest are employed. While unemployed, worker i chooses search intensity α_i in order to find a firm. The flow cost of searching is $c(\alpha_i)$,

and for tractability we assume that $c(\alpha_i) = \alpha_i^a$, where $a > 1$. A job seeker finds a firm with rate p_i . This rate depends positively on α_i , and it depends also on other variables specified later. The number of firms in the market is endogenously determined so that the value of a vacancy is zero. A firm, whether it is a vacancy or a producing firm, pays a capital cost flow k if it is in the market. The number of vacancies at each moment is v . Each vacancy i chooses recruiting intensity β_i to find a worker and pays recruiting cost flow $g(\beta_i) = \beta_i^b$, where $b > 1$. A vacancy meets a worker at rate q_i which depends positively on β_i . Having met, the worker and the firm start a project if they have agreed upon sharing the outcome flow. The output flow is constant f , the same in all worker-firm relationships, and it is shared according to the Nash bargaining solution. The length of the project is stochastic with an exogenous death rate λ . When the project ends, the firm and the worker separate and both start to look for a new partner.

We assume that the matching function is of Cobb-Douglas form:

$$m = (\alpha u)^\phi (\beta v)^\psi, \quad (1)$$

where αu and βv are the aggregate search efforts of job seekers and firms, respectively, and ϕ and ψ are non-negative scale parameters. In equilibrium the matches dissolve at the same rate as they are formed, thus the steady state satisfies

$$m = \lambda (L - u), \quad (2)$$

where $L - u$ is the number of ongoing firm-worker relationships.

3 Equilibrium

We first define the steady state equilibrium:

Definition 1 *The steady state equilibrium of the model is tuple (α, β, u, v) where α maximizes the value of job search for each job seeker for given (β, u, v) ; β maximizes the value of each vacancy for given (α, u, v) ; v is the amount of vacancies that gives zero value to each vacancy for given (α, β, u) ; and u is the amount of unemployed workers that satisfies steady state condition (2), given the values for α , β , and v .*

The value functions are

$$rU_i = -c(\alpha_i) + p_i(W - U_i), \quad (3)$$

$$rW = w - \lambda(W - U), \quad (4)$$

$$rV_i = -k - g(\beta_i) + q_i(J - V_i), \quad (5)$$

$$rJ = \pi - \lambda(J - V), \quad (6)$$

where U_i is the life-time value for job seeker i who chooses search intensity α_i , W is a worker's value for having a job, V_i is the value of a vacancy that searches with intensity β_i , and J is the value of a firm when it is producing. A producing worker and a producing firm take values U and V , respectively, as given. Wage is denoted by w , and profit π is determined as $\pi \equiv f - k - w$. The individual matching rates are defined as $p_i \equiv \frac{\alpha_i m}{\alpha u}$ for a job seeker, $q_i \equiv \frac{\beta_i m}{\beta v}$ for a vacancy, and the average matching rates are $p = m/u$ and $q = m/v$. If an individual's search effort is twice as large as the average search effort, he finds a partner twice as quickly as the average individual.

Setting $V = 0$ in (5) gives $J = (k + g(\beta_i)) / q(\beta_i)$, and setting $V = 0$ in (6) gives $J = \pi / (r + \lambda)$. Together they imply that a vacancy's value V is zero if (assuming that all vacancies recruit with effort β)

$$k + g(\beta) = \frac{q(\beta) \pi}{r + \lambda}. \quad (7)$$

Letting the right-hand side of (5) to be zero, the number of vacancies that gives $V = 0$ is equal to

$$v = \left[\frac{\beta_i (\alpha u)^\phi \pi}{(k + g(\beta_i)) (r + \lambda)} \right] \frac{1}{1 - \psi}, \quad (8)$$

and we note the following:

Lemma 1 *There are no vacancies in the market if job seekers' search intensity is zero.*

Each vacancy chooses β_i to maximize the right-hand-side of (5). The first-order condition for β_i is

$$-g'(\beta_i) + \frac{\partial q_i}{\partial \beta_i} (J - V_i) - q_i \frac{\partial V_i}{\partial \beta_i} = 0 \quad (9)$$

where $V_i = 0$, $\frac{\partial V_i}{\partial \beta_i} = 0$ by definition, and $J = \frac{k + g(\beta_i)}{q_i}$. In a symmetrical equilibrium $q_i = q$, and we have

$$\beta^* g'(\beta^*) = k + g(\beta^*). \quad (10)$$

We see that the optimal recruiting intensity does not depend on the number of unemployed or vacancies nor the search intensity of job seekers (This type of result is also derived in Pissarides 2000, 133). We will not analyze changes in k , therefore we can normalize $\beta = 1$.

Each job seeker chooses α_i to maximize the right-hand side of (3). The first-order condition for α_i is

$$-c'(\alpha_i) + \frac{\partial p_i}{\partial \alpha_i} (W - U_i) - p_i \frac{\partial U_i}{\partial \alpha_i} = 0, \quad (11)$$

where $\frac{\partial U_i}{\partial \alpha_i} = 0$ by definition. Equations (3) and (4) give $W - U_i = \frac{w - \lambda(W - U) + c(\alpha_i)}{r + p_i}$ and $W - U = \frac{w + c(\alpha)}{r + \lambda + p}$.

Lemma 2 *The optimal search intensity of job seekers is zero if there are no vacancies in the market.*

Wages are determined by Nash bargaining rule:

$$\max_w (W_i - U)^\gamma J_i^{1-\gamma}, \quad (12)$$

where $\gamma \in [0, 1]$ is the bargaining power of the worker. The problem is equal to

$$\begin{aligned} & \max_w [\gamma \ln(W_i - U) + (1 - \gamma) \ln J_i], \text{ and the first-order condition is } \frac{\gamma \frac{\partial (W_i - U)}{\partial w}}{W_i - U} + \\ & \frac{(1 - \gamma) \frac{\partial J_i}{\partial w}}{J_i} = 0. \text{ The maximization gives } \gamma J_i = (1 - \gamma) (W_i - U) \\ & \Rightarrow w = \frac{\gamma(r + \lambda + p)(f - k) - (1 - \gamma)(r + \lambda)c}{r + \lambda + \gamma p} \text{ and } \pi = \frac{(1 - \gamma)(r + \lambda)(f - k + c)}{r + \lambda + \gamma p}. \end{aligned}$$

Equating the profit from condition $V = 0$ (from equation (5)) and Nash bargaining we get

$$(1 - \gamma)(f - k + c)q = (r + \lambda + \gamma p)(k + g), \quad (13)$$

which is called *job creation condition* (JC -condition). The JC -condition implicitly determines the number of vacancies so that the value of a vacancy is zero. We can write

wage and profit by using (13):

$$w = -(1 - \gamma)c + \gamma[f - k + \theta(k + g)], \quad (14)$$

$$\pi = (1 - \gamma)(f - k + c) - \gamma\theta(k + g), \quad (15)$$

where $\theta \equiv v/u$.

4 Uniqueness of Equilibrium

This far we have described a very standard Mortensen-Pissarides -type search model. The rest of the paper is devoted to analyzing the sufficient conditions for a unique steady state equilibrium. We will see that a careful analysis of optimal search intensities and the number of vacancies leads to results that are surprising as contrasted to the present knowledge.

4.1 Unique Number of Vacancies

First we determine a condition for a unique number of vacancies, everything else as given. That is, we have to find a condition for a unique value for v in JC -condition (13). The right-hand side of JC -condition is increasing in v (because $\frac{\partial p}{\partial v} = \alpha^\phi u^{\phi-1} \beta^\psi \psi v^{\psi-1} > 0$), whereas the left-hand-side is decreasing in v if and only if q is decreasing in v , which holds if $\psi \leq 1$.

Lemma 3 *A sufficient condition for the existence of a unique v is that $\psi \leq 1$, everything else given.*

In the sequel we assume that $\psi \leq 1$. The model has a unique unemployment equilibrium if there is only one $(\alpha^*)^\phi (v^*)^\psi$ -curve (where α^* is the optimal value of each job seeker's search intensity, and v^* gives $V = 0$) and if it crosses the steady state curve once. The situation is depicted by Figure 1. We also derive conditions for a unique equilibrium in case where $(\alpha^*)^\phi (v^*)^\psi$ is decreasing in u .

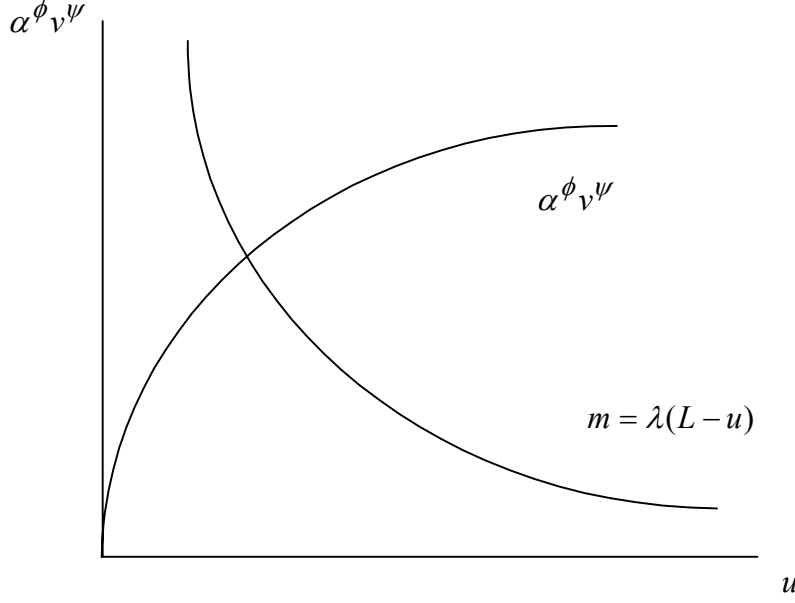


Figure 1: Unique equilibrium

4.2 Strategic Complementarity

The form of the matching function implies that α and v are strategic complements, that is, when there are more vacancies, job seekers put more effort in search, and more effort in search lures more vacancies in the market. Before analyzing the reaction functions $\alpha(v)$ and $v(\alpha)$, let us make two definitions:

Definition 2 *An (α, v) -equilibrium is an intersection of reaction functions $\alpha(v)$ and $v(\alpha)$ so that along $\alpha(v)$, α is the optimal response of every job seeker to a given v , given the value of u and all the parameters; and along $v(\alpha)$, v is the number of vacancies as a response to α so that $V = 0$, given the value of u and all the parameters.*

Definition 3 *Locus (α^*, v^*) consists of values of α and v that satisfy the job creation -condition and the first-order condition for α .*

First we solve sufficient conditions for scale parameters ϕ and ψ and the search cost parameter a , so that the reaction functions $\alpha(v)$ and $v(\alpha)$ intersect only once in the strictly positive quadrant of (α, v) -plane. Solving $dv^*/d\alpha$ from job creation function

(13) yields (see Appendix 1):

$$\frac{dv^*}{d\alpha} = \frac{(k+g)(r+\lambda)\phi v + (1-\gamma)qva\alpha^\alpha}{(k+g)\alpha[(r+\lambda)(1-\psi) + \gamma\theta q]}. \quad (16)$$

A job seeker's surplus from finding a vacancy is $W - U_i = \frac{(r+p)w - \lambda c(\alpha) + (r+\lambda+p)c(\alpha_i)}{(r+p_i)(r+\lambda+p)}$ by using (3) and (4). The first-order condition for α_i can then be written as

$$-(r+p_i)(r+\lambda+p)c'(\alpha_i) + \frac{\partial p_i}{\partial \alpha_i}[(r+p)w - \lambda c(\alpha) + (r+\lambda+p)c(\alpha_i)] = 0. \quad (17)$$

Notice that the choice of α_i has no effect on wage because each job seeker is a small agent. Using symmetry ($\alpha_i = \alpha$), $c(\alpha) = \alpha^a$, and the equilibrium wage from equation (14) we obtain

$$(r+\lambda+p)a\alpha^a = p\gamma[f - k + \alpha^a + \theta(k+g)]. \quad (18)$$

Plugging (18) into JC-condition (13) gives us locus (α^*, v^*) which satisfies (see Appendix 2)

$$(1-\gamma)a\alpha^a = \theta\gamma(k+g). \quad (19)$$

Differentiating (19) gives $\frac{dv}{d\alpha} = \frac{(1-\gamma)a^2\alpha^{a-1}u}{\gamma(k+g)}$, and using (19) gives

$$\frac{dv}{d\alpha} = \frac{av}{\alpha} \quad (20)$$

along (α^*, v^*) . Using (19) in $dv/d\alpha$ of JC-condition, in equation (16), gives

$$\frac{dv}{d\alpha} = \frac{(r+\lambda)\phi v + \gamma p v}{\alpha[(r+\lambda)(1-\psi) + \gamma p]}, \quad (21)$$

which holds on JC-curve at a point where α is optimal. The difference between $\frac{dv}{d\alpha}$ given by JC-condition and $\frac{dv}{d\alpha}$ along locus (α^*, v^*) , in their intersection, is equal to

$$\frac{v}{\alpha} \left[\frac{(r+\lambda)\phi + \gamma p}{(r+\lambda)(1-\psi) + \gamma p} - a \right]. \quad (22)$$

Next we solve if it is possible that the JC-curve and locus (α^*, v^*) intersect more than once in the strictly positive quadrant of (α, v) -plane. That is, we solve under what conditions strategic complementarity may or may not lead to multiple equilibria.

Lemma 4 *Strategic complementarity does not cause multiple equilibria if (i) $\phi > a(1 - \psi)$ and $\phi + \psi \geq 1$, or (ii) if $\phi < a(1 - \psi)$ and $\phi + \psi \leq 1$.*

Proof. If the reaction functions had multiple intersections, the sign of expression (22) would alternate between positive and negative as we look at consecutive intersections. In consecutive intersections both α and v get larger and larger values, and consequently p increases as we move away from the origin. If the sign of expression (22) is positive (negative) in the origin and if its derivative with respect to p is non-negative (non-positive), then expression (22) can change its sign at most once, thus no more than one intersection in the strictly positive quadrant of (α, v) -plane can exist. The derivative of the term in brackets with respect to p is non-negative (non-positive) if $\phi + \psi \geq (\leq) 1$. In the origin the JC -curve is steeper than locus (α^*, v^*) (the sign of (22) is then positive) if and only if $\phi > a(1 - \psi)$. ■

The case where $\phi > a(1 - \psi)$ and $\phi + \psi \geq 1$ is presented in Figure 2.

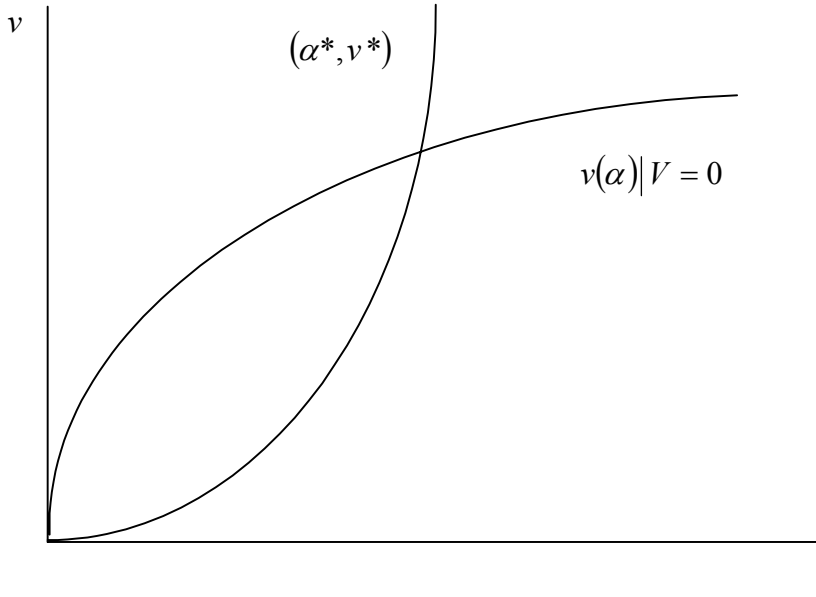


Figure 2: Unique non-degenerate (α, v) -equilibrium

Pissarides (2000, 131-135) also studies the possibility of multiple equilibria in this way. He assumes that the matching function has constant returns to scale with respect to job

seekers' and vacancies' aggregate search efforts. In his analysis, there is job seekers' search intensity in the horizontal axis and labor market tightness v/u in the vertical axis. He concludes that the curves can intersect only once with positive values of search intensity and tightness, and that results from constant returns to scale. However, he does not consider the position of the curves in the origin, which is crucial in determining whether the intersection with positive values of search intensity and tightness is stable. Likewise, Lemma 4 seems to indicate that constant returns to scale exclude multiple equilibria. Lemma 4 does not, however, tell us whether the unique equilibrium is stable or not. What we want to do is to solve conditions for *unique and stable equilibrium* in (α, v) -plane.

The above analysis is not suitable to solve the stability of equilibrium, because the (α^*, v^*) -locus does not depict the optimal response of α to a given v but gives the values that α and v must satisfy in *equilibrium*. In a stable equilibrium, illustrated by Figure 3, the response of vacancies to search intensity, $v(\alpha)$, given by job creation function (13) cuts $\alpha^*(v)$ given by first-order condition of α from above.

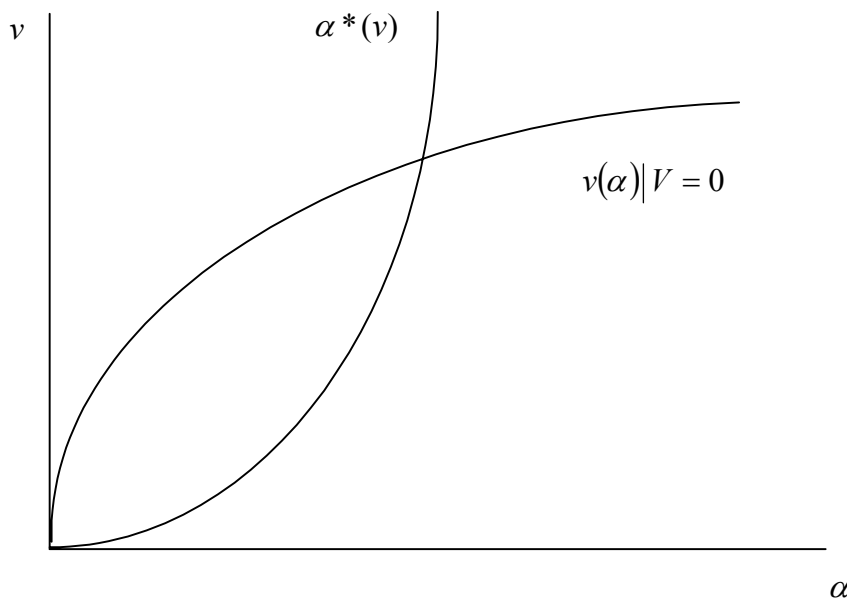


Figure 3: Unique and stable (α, v) -equilibrium

Multiple intersections of the reaction functions would imply multiple optimal $\alpha^\phi v^\psi$ -curves, depicted by Figure 4.

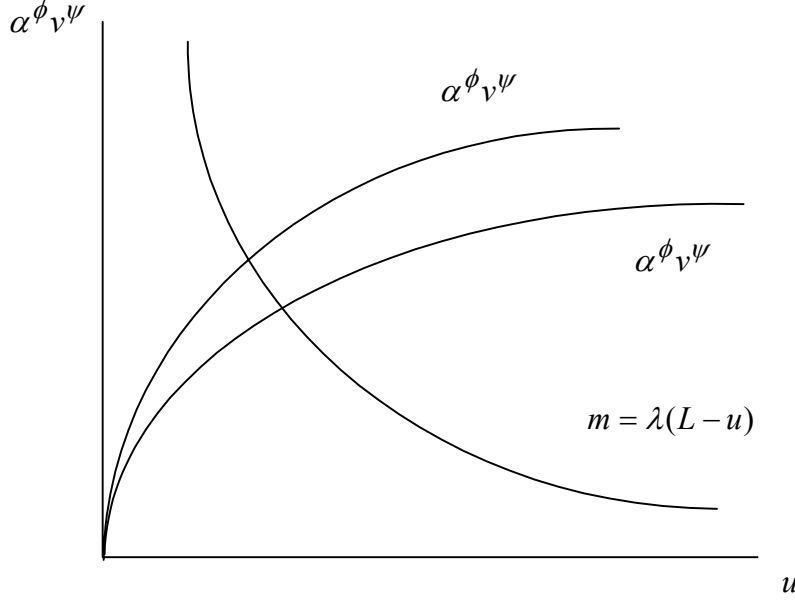


Figure 4: Multiple equilibria caused by strategic complementarity

The first-order condition for α_i , when using $c(\alpha) = \alpha^a$, can be written as (use (17) and the fact $\frac{\partial p_i}{\partial \alpha_i} = \frac{p}{\alpha}$):

$$(r + \lambda + p) (ra\alpha_i^{a-1} + p(a-1)\alpha_i^a) - p(r+p)w + p\lambda\alpha^a = 0. \quad (23)$$

Differentiating (23) with respect to α_i and v , taking into account that the choice of α_i does not affect the equilibrium wage, and using the wage equation (14) after differentiation gives (see Appendix 3)

$$\frac{d\alpha^*}{dv} = \frac{\alpha [(r + \lambda)\psi + (1 - \gamma)p]}{(a - 1)(r + \lambda + p)v}. \quad (24)$$

Next we compare $\frac{dv}{d\alpha}$ given by the JC -condition, equation (21), and $\frac{dv}{d\alpha}$ given by (24), thus we solve the sign of

$$\frac{(r + \lambda)\phi + \gamma p}{[(r + \lambda)(1 - \psi) + \gamma p]} - \frac{(a - 1)(r + \lambda + p)}{[(r + \lambda)\psi + (1 - \gamma)p]}. \quad (25)$$

We want to find such conditions that no more than one (α, v) -equilibrium exists and that the equilibrium is stable. For the unique strictly positive (α, v) -equilibrium to be

stable, the (α, v) -equilibrium in the origin must be unstable. In the origin $p = 0$, and we obtain

Lemma 5 *The (α, v) -equilibrium in the origin is unstable if and only if*

$$\phi\psi > (a - 1)(1 - \psi).$$

If $\phi\psi > (a - 1)(1 - \psi)$, the first non-degenerate equilibrium (if it exists) is stable. Then we have to find conditions such that no other non-degenerate equilibria can exist. The argument is similar to that in Lemma 4, and the result is somewhat surprising:

Proposition 1 *(i) A non-degenerate (α, v) -equilibrium is stable and unique (there are no multiple equilibria caused by strategic complementarity) if $\phi\psi > (a - 1)(1 - \psi)$, $\phi + \psi \geq 1$, and $\gamma + \psi \geq 1$, and if at least one of the latter inequalities is strict. (ii) A necessary condition for the existence of a non-degenerate (α, v) -equilibrium is that $\gamma > 2 - a$.*

Proof. (i) The first non-degenerate (α, v) -equilibrium is stable if and only if the sign of expression (25) is negative, which holds if the equilibrium in the origin is unstable, which holds if $\phi\psi > (a - 1)(1 - \psi)$, by Lemma 5. In that case, in the second consecutive non-degenerate equilibrium expression (25) has a positive sign. But if the sign of expression (25) is negative in the first non-degenerate equilibrium and if its derivative with respect to p is negative (the latter holds if $\phi + \psi \geq 1$ and $\gamma + \psi \geq 1$, and if at least one of these inequalities is strict), only one non-degenerate equilibrium can exist. The sign of this derivative with respect to p is solved in Appendix 4. (ii) If the conditions in part (i) above hold and if $p > 0$, expression (25) is negative only if $\gamma > 2 - a$ (look at Appendix 4). ■

In part (ii) of the Proposition, a large value of γ (workers' bargaining power) implies that the number of vacancies does not increase too fast when job seekers increase their search intensity, and a large value of a (exponent in the search cost function) means that job seekers' search intensity does not respond too much to a larger number of vacancies. In general, strategic complementarity does not cause multiple equilibria if the scale parameters of the matching function are large enough and if the workers' bargaining

power is also large enough. This result is in a striking contrast with the prevailing view (see the Introduction).

Applying Proposition 1 shows that constant returns to scale do not guarantee uniqueness:

Corollary 1 *If the matching function has constant returns to scale, a non-degenerate (α, v) -equilibrium is (i) unique, (ii) stable if $\phi < 2 - a$; (iii) A stable non-degenerate (α, v) -equilibrium exists only if $\gamma > \phi$ and $\gamma > 2 - a$.*

If $\phi + \psi = 1$, then dB/dp in Appendix 4 is either positive everywhere in (α, v) -plane (if $\phi > \gamma$), or negative (if $\phi < \gamma$), or equal to zero, so multiple (α, v) -equilibria cannot exist if returns to scale are constant. Further, (a) if $\phi > 2 - a$ and $\gamma > \phi$, the equilibrium in the origin is stable, and a non-degenerate equilibria does not exist, (b) if $\phi > 2 - a$ and $\gamma < \phi$, there is a stable equilibrium in the origin, and in addition to that, one unstable non-degenerate equilibrium might exist, (c) if $\phi < 2 - a$ and $\gamma < \phi$, there is an unstable equilibrium in the origin, and no non-degenerate equilibria exist. In short, a necessary condition for the existence of a stable non-degenerate equilibrium in (α, v) -plane is that $\phi < 2 - a < \gamma$.

Burdett and Wright (1998) have a model of two-sided search with non-transferable utility, where multiple equilibria may also emerge due even if the meeting technology has constant returns to scale. Employers' and workers' reservation utilities are strategic substitutes, and the reservation utility functions can cross several times. They provide conditions for uniqueness also if returns to scale are increasing. In the present model, constant returns to scale guarantee that reaction functions cannot cross more than once in the strictly positive quadrant of (α, v) -plane. However, multiple equilibria caused by common pool externality are not ruled out as we will see.

4.3 Common Pool Externality

Common pool externality can be defined as the effect of a change in the size of the pool of unemployed on optimal search intensity and the number of vacancies. That is, we have to find out how the reaction functions $\alpha(v)$ and $v(\alpha)$ move if u changes. As a

result, a new equilibrium pair (α, v) emerges. The analysis of common pool externality is facilitated by letting the state variable u be on the horizontal axis and function $\alpha^\phi v^\psi$ of choice variables to be on the vertical axis. If the optimal $\alpha^\phi v^\psi$ is a decreasing function of u , then common pool externality may cause multiple equilibria, because the steady state curve is decreasing in u . The idea is best depicted by Figure 5.

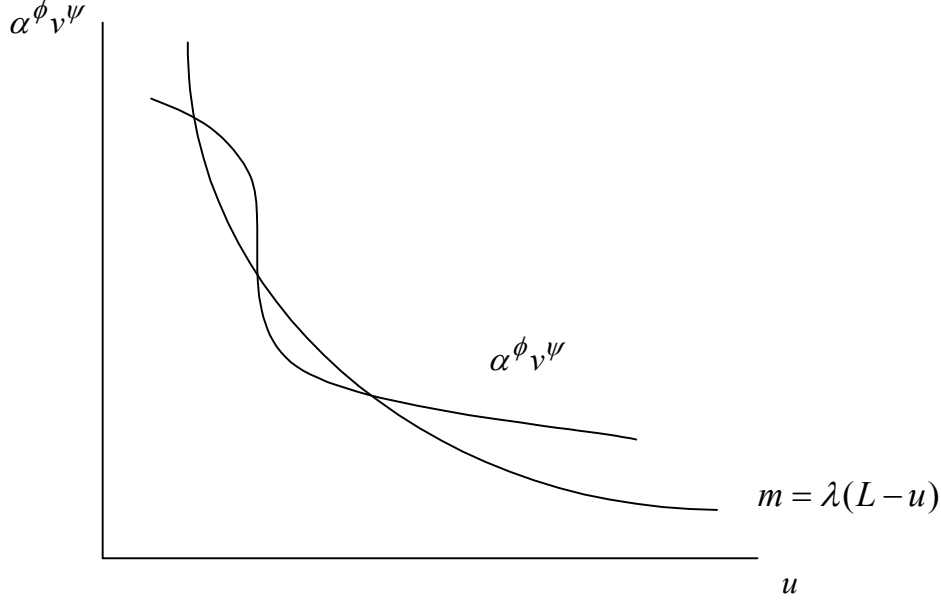


Figure 5: Multiple equilibria caused by common-pool externality

Curve $m = \lambda(L - u)$ gives the value of $\alpha^\phi v^\psi$ that is needed to keep unemployment at a steady state level. The curve $\alpha^\phi v^\psi$ gives the optimal response of vacancies and search intensities to a change in u . Denote the left-hand side of JC-equation by F and the first-order condition of α_i by G :

$$F \equiv (1 - \gamma)(f - k + c)q - (r + \lambda + \gamma p)(k + g). \quad (26)$$

$$G \equiv -(r + p_i)(r + \lambda + p)a\alpha_i^{a-1} + \frac{\partial p_i}{\partial \alpha_i}[(r + p)w - \lambda\alpha^a + (r + \lambda + p)\alpha_i^a] \quad (27)$$

Along optimal $\alpha^\phi v^\psi$ we have $F = G = 0$. Derivatives $\frac{d\alpha}{du}$ and $\frac{dv}{du}$ are solved in the standard manner by using Jacobians.

$$\text{Denote } |A| \equiv \begin{vmatrix} \frac{\partial F}{\partial \alpha} & \frac{\partial F}{\partial v} \\ \frac{\partial G}{\partial \alpha} & \frac{\partial G}{\partial v} \end{vmatrix}, |A_\alpha| \equiv \begin{vmatrix} -\frac{\partial F}{\partial u} & \frac{\partial F}{\partial v} \\ -\frac{\partial G}{\partial u} & \frac{\partial G}{\partial v} \end{vmatrix}, \text{ and } |A_v| \equiv \begin{vmatrix} \frac{\partial F}{\partial \alpha} & -\frac{\partial F}{\partial u} \\ \frac{\partial G}{\partial \alpha} & -\frac{\partial G}{\partial u} \end{vmatrix}.$$

The partial derivatives of F and G are derived in Appendices 5 and 6. The determinants are (see Appendix 7)

$$|A| = \left[\frac{-(k+g)^2(r+p)\gamma}{(1-\gamma)\alpha u} \right] \left[\begin{array}{c} (r+\lambda)^2(\phi\psi - (a-1)(1-\psi)) \\ + (r+\lambda)p \left[\begin{array}{c} \phi(1-\gamma) + \gamma\psi \\ + (a-1)(\psi - \gamma - 1) \end{array} \right] \\ + \gamma p^2(2 - \gamma - a) \end{array} \right], \quad (28)$$

$$|A_\alpha| = \left[\frac{(k+g)^2(r+p)\gamma}{(1-\gamma)u^2} \right] (r+\lambda)(r+\lambda+p)(\phi+\psi-1), \quad (29)$$

$$|A_v| = \left[\frac{(k+g)^2(r+p)\gamma v}{(1-\gamma)\alpha u^2} \right] \left[\begin{array}{c} (r+\lambda)^2\phi(a+\phi-2) \\ + (r+\lambda)p[(a-2)(\gamma+\phi) + 2\gamma\phi] \\ + \gamma p^2(a+\gamma-2) \end{array} \right]. \quad (30)$$

Let us first determine the sign of $|A|$. In the origin of (α, v) -plane $p = 0$, and $|A| < 0$ if $\phi\psi > (a-1)(1-\psi)$. In this case, $V = 0$ -curve is steeper than $\alpha(v)$ -curve in the origin, and the equilibrium in the origin is unstable. In the next equilibrium $|A| > 0$ because then $\alpha(v)$ -curve is steeper than $V = 0$ -curve. If the conditions presented in Proposition 1 are fulfilled, then there cannot be more than one non-degenerate equilibrium in (α, v) -plane, and $|A| > 0$ in the unique non-degenerate equilibrium. From now on, we assume that $|A| > 0$ unless otherwise mentioned.

The responses of search intensity and the number of vacancies to a change in unemployment are $\frac{d\alpha}{du} = \frac{|A_\alpha|}{|A|}$ and $\frac{dv}{du} = \frac{|A_v|}{|A|}$, and we see from (29) that when unemployment increases, the job seekers respond by searching less (more) actively if the matching function has decreasing (increasing) returns to scale. Less active searching when unemployment increases is often called as ‘discouraged worker effect’ (see for example Pissarides 2000, p. 169, 172-174). The larger the pool of unemployed is, the more there are vacancies at a given level of unemployment if $a > 2 - \phi$, $a > 2 - \gamma$, and $a > 2 - \frac{2\gamma\phi}{\gamma + \phi}$ (see (30)). These hold if $a \geq 2$, and then an increase in unemployment lures more vacancies into the market. The combined effect of an increase in unemployment on the optimal

curve $\alpha^\phi v^\psi$ is:

$$\begin{aligned} \frac{d(\alpha^\phi v^\psi)}{du} &= \alpha^\phi \psi v^{\psi-1} \frac{dv}{du} + \phi \alpha^{\phi-1} v^\psi \frac{d\alpha}{du} \\ &= |A|^{-1} [\alpha^\phi \psi v^{\psi-1} |A_v| + \phi \alpha^{\phi-1} v^\psi |A_\alpha|], \end{aligned} \quad (31)$$

where

$$\begin{aligned} &\alpha^\phi \psi v^{\psi-1} |A_v| + \phi \alpha^{\phi-1} v^\psi |A_\alpha| \\ &= \left[\frac{(k+g)^2 (r+p) \gamma \alpha^{\phi-1} v^\psi}{(1-\gamma) u^2} \right] \times \left[\begin{aligned} &(r+\lambda)^2 \phi [\psi (a+\phi-2) + \phi + \psi - 1] \\ &+ (r+\lambda) p \left[\begin{aligned} &\psi (a-2) (\gamma + \phi) \\ &+ 2\gamma \phi \psi + \phi (\phi + \psi - 1) \end{aligned} \right] \\ &+ \gamma p^2 (a + \gamma - 2) \end{aligned} \right] \end{aligned} \quad (32)$$

which is positive if $\phi + \psi \geq 1$ and $a \geq 2$. The following lemma gives sufficient conditions for optimal $\alpha^\phi v^\psi$ -curve to be non-decreasing in u :

Lemma 6 *Common-pool externality does not cause multiple equilibria if*

$\phi\psi > (a-1)(1-\psi)$, $a \geq 2$, $\phi + \psi \geq 1$, $\gamma + \psi \geq 1$, and if at least one of the latter two inequalities is strict.

Proof. Determinant $|A|$ is positive at a non-degenerate equilibrium if $\phi\psi >$

$(a-1)(1-\psi)$, $\gamma > 2-a$, $\phi + \psi \geq 1$ and $\gamma + \psi \geq 1$, and if at least one of the latter inequalities is strict (see Proposition 1); $[\alpha^\phi \psi v^{\psi-1} |A_v| + \phi \alpha^{\phi-1} v^\psi |A_\alpha|] > 0$ if $\phi + \psi \geq 1$ and $a \geq 2$. ■

The result in Lemma 6 is perhaps surprising, and it is somewhat contradictory to the result in Pissarides (1986, 554-556). In Pissarides's model, unemployment rate is on the horizontal axis and vacancy rate is on the vertical axis. The steady state curve (also called as Beveridge curve) is downward sloping. The optimal response of vacancies to a change in unemployment rate can be negative (giving possibility to multiple equilibria) only if the matching function has increasing returns to scale. In Pissarides's model firms can have many jobs, but that does not seem to be the crucial reason for the discrepancy between the results in his model and the present one. Pissarides does not specify a

matching function, but he assumes that the number of matches is a function of the number of vacancies and of the amount of workers' aggregate search intensity. Then he makes some assumptions about the behavior of optimal search intensity and about the matching function. His analysis treats the interaction of search intensity and vacancies at the same time as it treats the response of search intensities and vacancies to changes in unemployment. As a contrast, the present article treats strategic complementarity and common pool externality separately. Moreover, Pissarides does not consider the optimal response of *individual* job seekers' search intensity to changes in the vacancy rate and unemployment rate and then make the symmetry assumption that every job seeker behaves similarly, as we do here. That is, the equilibrium in our model is a Nash equilibrium, whereas in Pissarides's model it is not necessarily so.

One can find less restrictive conditions for values of a , γ , ϕ , and ψ so that the optimal curve $\alpha^\phi v^\psi$ is non-decreasing in u . Instead of writing them, we note that even if the optimal curve $\alpha^\phi v^\psi$ is decreasing in u , we can find conditions for the above-mentioned parameters so that the optimal curve $\alpha^\phi v^\psi$ crosses the steady state curve at most once.

Proposition 2 *Common-pool externality does not cause multiple equilibria if the conditions presented in Proposition 1 hold and $\gamma \geq 2 - a$ and $\gamma\phi \geq 2 - a$.*

Proof. Using the expressions for determinants $|A|$, $|A_\alpha|$, and $|A_v|$, along the optimal $\alpha^\phi v^\psi$ -curve we have

$$\frac{d(\alpha^\phi v^\psi)}{du} = \frac{\alpha^\phi v^\psi \left[(r + \lambda)^2 \phi [\psi(a + \phi - 2) + \phi + \psi + 1] + \gamma p^2 (a + \gamma - 2) + (r + \lambda) p [\psi(a - 2)(\gamma + \phi) + 2\gamma\phi\psi + \phi(\phi + \psi + 1)] \right]}{u \left[(r + \lambda)^2 [(a - 1)(1 - \psi) - \phi\psi] + \gamma p^2 (a + \gamma - 2) - (r + \lambda) p [\phi(1 - \gamma) + \gamma\psi + (a - 1)(\psi - \gamma - 1)] \right]}. \quad (33)$$

Along the steady state curve we have

$$\frac{d(\alpha^\phi v^\psi)}{du} = -\frac{\phi p + \lambda}{u^\phi}. \quad (34)$$

At $u = L$, the optimal value of $\alpha^\phi v^\psi$ is strictly positive. Then the steady state equilibrium with the largest value of u is such that derivative of the optimal $\alpha^\phi v^\psi$ -curve with respect

to u is larger than the derivative of the steady state curve with respect to u . The curves can intersect more than once only if there is also an equilibrium where the derivative of the optimal $\alpha^\phi v^\psi$ -curve with respect to u is smaller than the derivative of the steady state curve. (That is, where the optimal $\alpha^\phi v^\psi$ -curve is steeper than the steady state curve.) Assume that the conditions presented in Proposition 1 hold, which implies that $|A| > 0$, which in turn implies that the denominator of the right-hand side of (33) is positive. The right-hand side of (33) is larger than the right-hand side of (34) if

$$\begin{aligned}
& (r + \lambda)^2 \phi p \{ \psi (a + \phi - 2) + \phi + \psi + 1 + (a - 1) (1 - \psi) - \phi \psi \} \\
& + (r + \lambda) p^2 \left[\begin{array}{l} \psi (a - 2) (\gamma + \phi) + 2\gamma \phi \psi + \phi (\phi + \psi + 1) \\ -\phi [\phi (1 - \gamma) + \gamma \psi + (a - 1) (\psi - \gamma - 1)] \end{array} \right] \\
& + \gamma p^3 (a + \gamma - 2) (1 + \phi) \\
> & -\lambda \left[\begin{array}{l} (r + \lambda)^2 [(a - 1) (1 - \psi) - \phi \psi] + \gamma p^2 (a + \gamma - 2) \\ - (r + \lambda) p [\phi (1 - \gamma) + \gamma \psi + (a - 1) (\psi - \gamma - 1)] \end{array} \right].
\end{aligned} \tag{35}$$

The right-hand side of (35) is negative if the conditions presented in Proposition 1 hold. The left-hand side is equal to

$$\begin{aligned}
& (r + \lambda)^2 \phi p (a + \phi - 2) + \gamma p^3 (a + \gamma - 2) (1 + \phi) \\
& + (r + \lambda) p^2 [(a - 1) \gamma \phi + (a + \phi - 2) \gamma \psi + (a + \gamma \phi - 2) \phi],
\end{aligned} \tag{36}$$

which is non-negative if $a \geq 1$, $a \geq 2 - \phi$, $a \geq 2 - \gamma$ and $a \geq 2 - \gamma \phi$. We have earlier assumed that $a \geq 1$, and because $\gamma \leq 1$, condition $a \geq 2 - \phi$ is satisfied if $a \geq 2 - \gamma \phi$. ■

In the Introduction we mentioned that multiplicity of equilibria in Diamond's model (1982) results from common pool externality. Why is it then the case that the coconut model can have multiple equilibria with *increasing* returns to scale, while the present labor market model can have multiple equilibria (because of common pool effect) if the scale returns to job seekers' aggregate search effort are *small*? In both models, the more there are potential trading partners (people carrying coconuts, or unemployed), the higher the optimal value of choice variable (maximum acceptable height of a coconut tree, or search intensity, or the number of vacancies) if returns to scale are high. But in the coconut model, the larger the value of the choice variable is the *more* there are potential

trading partners, whereas in the present model the larger the value of the choice variable, the *smaller* the pool of potential partners. The results of Diamond's model have been largely misinterpreted in a labor market context.

Constant returns to scale are claimed to prevent the emergence of multiple equilibria (Pissarides 2000, p. 133, Pissarides 1986). This holds in the present model but only as far as we consider strategic complementarity.

Proposition 3 *If the matching function has constant returns to scale, all the sufficient conditions for unique equilibrium cannot hold.*

Proof. If $\phi + \psi = 1$, the conditions in Lemma 6 can be written as $\phi < 2 - a$, $\phi < \gamma$, and $a \geq 2$. Obviously, $\phi < 2 - a$ and $a \geq 2$ cannot both hold as $\phi > 0$ by assumption. This implies that all the sufficient conditions for $\alpha^\phi v^\psi$ to be non-decreasing in u cannot hold. What if $\alpha^\phi v^\psi$ is decreasing in u ? As noted in Corollary 1, a necessary condition for the existence of a stable nondegenerate equilibrium in (α, v) -plane is that $\phi < 2 - a < \gamma$. By Proposition 2, multiple equilibria caused by common pool externality cannot be ruled out if $\gamma\phi < 2 - a$. However, this inequality holds if $\phi < 2 - a$ because $\gamma \leq 1$ by definition. As a result, the optimal $\alpha^\phi v^\psi$ -curve might cross the steady state curve more than once.

■

In the light of the present paper and in contrast with the conventional view, multiple equilibria can emerge with homogenous workers and homogenous firms, even if returns to scale are constant and utility is transferable, that is, if worker and firm share the surplus of the match.

5 Conclusion

We have solved sufficient conditions for a unique unemployment equilibrium in a search model where job seekers and vacancies choose their search intensities and where the number of firms is determined by a zero profit condition. The rather surprising result is that large returns to scale are associated with unique equilibrium. If the scale parameters of the matching function are small, it is possible that strategic complementarity

or common-pool externality (or both) causes multiple equilibria. We assumed a Cobb-Douglas matching function and analyzed the Nash equilibrium of the model.

The scale returns have been estimated in empirical literature, where also the plausibility of multiple equilibria is discussed in the light of theoretical results. Pissarides (1986, 1988) finds evidence for constant returns to scale in Britain in years 1967 - 1983, with scale parameters $\phi = 0.7$ and $\psi = 0.3$. He says: "A necessary, though not sufficient, condition for multiplicity is that the search technology...should have increasing returns to scale." (Pissarides 1988). Coles and Smith (1996) find constant returns to scale and conclude that there is a unique employment equilibrium. Baker, Hogan and Ragan (1996) find "significant increasing returns to labour-market matching". They refer to Pissarides (1986) and Diamond (1982), hinting that increasing returns can produce multiple equilibria. Warren (1996) estimated returns to scale in U.S. manufacturing sector from 1969 to 1973, and he found that the scale elasticity is 1.332 or 1.536 (which means increasing returns to scale) depending on the estimated model. He concludes that the finding is consistent with multiple equilibria, referring for example to Diamond (1982), Mortensen (1982), Pissarides (1984), Diamond and Fudenberg (1989), and to Howitt and McAfee (1987).

In the light of the present article, the above conclusions must be treated with skepticism. The theoretical models that give support to the arguments in the empirical articles are mostly unsatisfactorily executed, like Howitt and McAfee (1987) and Pissarides (1986); or in the case of Diamond (1982), their basic result is misinterpreted in a labor market environment where job seekers choose their search intensities and firms choose whether to enter the market or not.

6 Appendix

This appendix contains the derivations of some of the results in subsections 4.2 and 4.3.

1. Derivation of equation (16)

Write the JC-equation as $(1 - \gamma)(f - k + c)(\alpha u)^\phi v^{\psi-1} =$

$(r + \lambda)(k + g) + (k + g)\gamma\alpha^\phi u^{\phi-1} v^\psi$, and differentiate with respect to α and v :

$$\begin{aligned} & \left[\begin{aligned} & (1 - \gamma)(f - k + c)v^{\psi-1}u^\phi\phi\alpha^{\phi-1} + (1 - \gamma)(\alpha u)^\phi v^{\psi-1}c'(\alpha) \\ & - (k + g)\gamma v^\psi u^{\phi-1}\phi\alpha^{\phi-1} \end{aligned} \right] d\alpha \quad (\text{A1}) \\ & = \left[(k + g)\gamma\alpha^\phi u^{\phi-1}\psi v^{\psi-1} - (1 - \gamma)(f - k + c)(\alpha u)^\phi(\psi - 1)v^{\psi-2} \right] dv. \end{aligned}$$

Plugging in JC-equation and using $p/\alpha = \alpha^{\phi-1}u^{\phi-1}v^\psi$ and $p/v = \alpha^\phi u^{\phi-1}v^{\psi-1}$, and $c'(\alpha) = a\alpha^{a-1}$, we have equation (16).

2. Derivation of equation (19)

We can use identity $pu \equiv qv$, or $p \equiv \theta q$, and write the first-order condition for α and JC-equation, respectively, as

$$(r + \lambda + \theta q)a\alpha^a = \theta q\gamma[f - k + \alpha^a + \theta(k + g)], \quad (\text{A2})$$

$$(1 - \gamma)(f - k + c)q = (r + \lambda + \gamma\theta q)(k + g). \quad (\text{A3})$$

From (A2), solve $(f - k + \alpha^a)q = \frac{(r + \lambda + \theta q)a\alpha^a - \theta^2 q\gamma(k + g)}{\theta\gamma}$, and plug the right-hand-side into (A3) to get

$$(1 - \gamma)a\alpha^a(r + \lambda + \theta q) - (1 - \gamma)\theta^2 q\gamma(k + g) - (r + \lambda + \gamma\theta q)(k + g)\theta\gamma = 0, \quad (\text{A4})$$

which is equal to $[(1 - \gamma)a\alpha^a - (k + g)\gamma\theta](r + \lambda + \theta q) = 0$, which gives equation (19).

3. Derivation of equation (24)

We use the following results: $\frac{\partial p}{\partial \alpha_i} = 0$ (the average matching rate of a job seeker does not depend on any individual job seeker's search intensity), $\frac{\partial p}{\partial v} = \alpha^\phi u^{\phi-1}\psi v^{\psi-1} = \frac{\psi p}{v}$, $\frac{\partial w}{\partial \alpha_i} = 0$ (since each job seeker is small compared to the mass of all job seekers, an individual job seeker's search intensity has no effect on his wage), and $\frac{\partial w}{\partial v} = \frac{\gamma(k + g)}{u}$

from wage equation (14). Differentiating (23) with respect to α_i and v gives

$$\frac{d\alpha_i}{dv} = - \frac{\frac{\psi}{v} \left[\frac{(r + \lambda + p)(a - 1)\alpha_i^a p + r a \alpha_i^{a-1} p + (a - 1)\alpha_i^a p^2}{-p(r + p)w + -wp^2 + \lambda \alpha_i^a p} \right] - \frac{p(r + p)\gamma(k + g)}{u}}{(r + \lambda + p)a(a - 1)(r a \alpha_i^{a-2} + \alpha_i^{a-1} p)}. \quad (\text{A5})$$

Then use the first-order condition of α_i , (23), and then let $\alpha_i = \alpha$ to obtain

$$\frac{d\alpha}{dv} = \frac{\frac{\psi}{v} a \alpha^a r (r + \lambda) - \frac{\psi p^2}{v} [(a - 1)\alpha^a - w] + p(r + p) \frac{\gamma(k + g)}{u}}{(r + \lambda + p)(r + p)a(a - 1)\alpha^{a-1}}. \quad (\text{A6})$$

Then use the equilibrium wage given by (14) to get $(a - 1)\alpha^a - w =$

$$a\alpha^a - \gamma[f - k + \alpha^a + \theta(k + g)], \text{ and use (A2) to write } (a - 1)\alpha^a - w = -\frac{(r + \lambda)a\alpha^a}{p},$$

and using that in (A6) yields

$$\frac{d\alpha}{dv} = \frac{\frac{a\alpha^a \psi (r + \lambda)}{v} + \frac{\gamma(k + g)p}{u}}{(r + \lambda + p)a(a - 1)\alpha^{a-1}}. \quad (\text{A7})$$

In an intersection of (α^*, v^*) and $V = 0$ we have $(1 - \gamma)a\alpha^a = \theta\gamma(k + g)$, and using this in (A7) yields equation (24).

4. Here we derive the conditions presented in Proposition 1 for expression (25) to have a negative sign for all strictly positive values of p .

Defining $B \equiv \frac{(r + \lambda)\phi + \gamma p}{[(r + \lambda)(1 - \psi) + \gamma p]} - \frac{(a - 1)(r + \lambda + p)}{[(r + \lambda)\psi + (1 - \gamma)p]}$ we have

$$\begin{aligned} & \{(r + \lambda)\gamma(1 - \phi - \psi)[(r + \lambda)\psi + (1 - \gamma)p]^2 \\ \frac{dB}{dp} = & \frac{-(r + \lambda)(a - 1)(\gamma + \psi - 1)[(r + \lambda)(1 - \psi) + \gamma p]^2}{[(r + \lambda)\psi + (1 - \gamma)p]^2 [(r + \lambda)(1 - \psi) + \gamma p]^2}, \end{aligned} \quad (\text{A8})$$

which is negative if $\phi + \psi \geq 1$ and $\gamma + \psi \geq 1$ and if at least one of the inequalities is strict, and it is positive if $\phi + \psi \leq 1$ and $\gamma + \psi \leq 1$ and if at least one of the inequalities is strict. We have $B < 0$ if

$$\begin{aligned} & (r + \lambda)^2 (\phi\psi - (a - 1)(1 - \psi)) \\ & + (r + \lambda)p \left[\begin{aligned} & \phi(1 - \gamma) + \gamma\psi \\ & + (a - 1)(\psi - \gamma - 1) \end{aligned} \right] \\ & + \gamma p^2 (2 - \gamma - a) < 0, \end{aligned} \quad (\text{A9})$$

and if $\phi\psi > (a-1)(1-\psi)$ and $\gamma + \psi \geq 1$, then $B < 0$ only if $\gamma > 2-a$. Thus, $\gamma > 2-a$ is a necessary condition for the existence of a non-degenerate (α, v) -equilibrium.

5. Derivation of partial derivatives $\frac{\partial F}{\partial \alpha}$, $\frac{\partial F}{\partial u}$, and $\frac{\partial F}{\partial v}$.

We use the following results from the matching function: $\frac{\partial q}{\partial \alpha} = \phi\alpha^{\phi-1}u^\phi\beta^\psi v^{\psi-1} = \frac{\phi q}{\alpha}$, $\frac{\partial q}{\partial v} = (\alpha u)^\phi \beta^\psi (\psi-1)v^{\psi-2} = \frac{(\psi-1)q}{v}$, $\frac{\partial q}{\partial u} = \alpha^\phi \phi u^{\phi-1} \beta^\psi v^{\psi-1} = \frac{\phi q}{u}$, $\frac{\partial p}{\partial \alpha} = \phi\alpha^{\phi-1}u^{\phi-1}(\beta v)^\psi = \frac{\phi p}{\alpha}$, $\frac{\partial p}{\partial v} = \alpha^\phi u^{\phi-1} \beta^\psi \psi v^{\psi-1} = \frac{\psi p}{v}$, and $\frac{\partial p}{\partial u} = \alpha^\phi (\phi-1)u^{\phi-2}(\beta v)^\psi = \frac{(\phi-1)p}{u}$, $\frac{\partial p_i}{\partial \alpha_i} = (\alpha u)^{\phi-1}v^\psi = \frac{p}{\alpha}$, and $\frac{\partial^2 p_i}{\partial \alpha_i \partial v} = (\alpha u)^{\phi-1} \psi v^{\psi-1} = \frac{\psi p}{v}$. The Nash bargaining solution gives $\frac{\partial w}{\partial \alpha_i} = 0$, $\frac{\partial w}{\partial v} = \frac{\gamma(k+g)}{u}$, and $\frac{\partial w}{\partial u} = -\frac{\gamma(k+g)v}{u^2}$.

Differentiating F respect to α gives

$$\frac{\partial F}{\partial \alpha} = (1-\gamma)(f-k+c)\frac{\phi q}{\alpha} + (1-\gamma)qa\alpha^{a-1} - (k+g)\frac{\gamma\phi p}{\alpha}. \quad (\text{A10})$$

Using F in (A10) yields

$$\frac{\partial F}{\partial \alpha} = (k+g)(r+\lambda)\frac{\phi}{\alpha} + (1-\gamma)qa\alpha^{a-1}, \quad (\text{A11})$$

and plugging in equation (19) (we look at the intersection of the reaction functions) we have

$$\frac{\partial F}{\partial \alpha} = \frac{k+g}{\alpha} [(r+\lambda)\phi + \gamma p]. \quad (\text{A12})$$

Differentiating F with respect to u gives

$$\frac{\partial F}{\partial u} = (1-\gamma)(f-k+c)\frac{\phi q}{u} - (k+g)\frac{\gamma(\phi-1)p}{u}, \quad (\text{A13})$$

and using F gives

$$\frac{\partial F}{\partial u} = \frac{k+g}{u} [(r+\lambda)\phi + \gamma p]. \quad (\text{A14})$$

Differentiating F with respect to v gives

$$\frac{\partial F}{\partial v} = (1-\gamma)(f-k+c)\frac{(\psi-1)q}{v} - (k+g)\frac{\gamma\psi p}{v}, \quad (\text{A15})$$

and using F gives

$$\frac{\partial F}{\partial v} = \frac{k+g}{v} [(r+\lambda)(\psi-1) - \gamma p]. \quad (\text{A16})$$

6. Derivation of partial derivatives $\frac{\partial G}{\partial \alpha}$, $\frac{\partial G}{\partial u}$, and $\frac{\partial G}{\partial v}$.

We have

$$\frac{\partial G}{\partial \alpha} = (r + \lambda + p) (r + p) a (a - 1) \alpha^{a-1}, \quad (\text{A17})$$

which results when after differentiating G with respect to α_i , we replace α_i with α . That is, we assume that all job seekers choose the same search intensity.

Differentiating G with respect to v gives

$$\begin{aligned} \frac{\partial G}{\partial v} = & (r + \lambda + p) (a - 1) \alpha_i^a \frac{\psi p}{v} + [ra \alpha_i^{a-1} \alpha + p (a - 1) \alpha_i^a] \frac{\psi p}{v} \\ & - p (r + p) \frac{\partial w}{\partial v} - w \left[\frac{r \psi p}{v} + \frac{2 \psi p^2}{v} \right] + \lambda \alpha^a \frac{\psi p}{v}. \end{aligned} \quad (\text{A18})$$

Letting $\alpha_i = \alpha$ and using $\frac{\partial w}{\partial v} = \frac{\gamma (k + g)}{u}$ and the expression for G (where $\alpha_i = \alpha$), we have

$$\frac{\partial G}{\partial v} = \frac{\psi}{v} [p^2 ((a - 1) \alpha^a - w) - (r + \lambda) ra \alpha^a] - \frac{p (r + p) \gamma (k + g)}{u}. \quad (\text{A19})$$

Then use the wage given by Nash bargaining (equation (14)) and use F (we look at the intersection of reaction functions) to get

$$\begin{aligned} \frac{\partial G}{\partial v} = & \frac{\psi}{v} \left\{ \frac{p^2}{(1 - \gamma) q} [(1 - \gamma) qa \alpha^a - \gamma (r + \lambda + p) (k + g)] - (r + \lambda) ra \alpha^a \right\} \\ & - \frac{p (r + p) \gamma (k + g)}{u}. \end{aligned} \quad (\text{A20})$$

Then plug (19) into (A20) to obtain

$$\frac{\partial G}{\partial v} = -\frac{\psi}{v} \left[\frac{pv \gamma (k + g) (r + \lambda)}{(1 - \gamma) u} + (r + \lambda) ra \alpha^a \right] - \frac{p (r + p) \gamma (k + g)}{u}, \quad (\text{A21})$$

and after some manipulation we have

$$\frac{\partial G}{\partial v} = \frac{-\gamma (k + g) (r + p) [(r + \lambda) \psi + (1 - \gamma) p]}{(1 - \gamma) u}. \quad (\text{A22})$$

We get $\frac{\partial G}{\partial u}$ by proceeding as in case of $\frac{\partial G}{\partial v}$:

$$\frac{\partial G}{\partial u} = \frac{\gamma (k + g) (r + p) v [(r + \lambda) (1 - \phi) + (1 - \gamma) p]}{(1 - \gamma) u^2}. \quad (\text{A23})$$

7. Determinant $|A|$

Using the partial derivatives of F and G we get

$$|A| = \left[\frac{-(k+g)(r+p)}{(1-\gamma)\alpha u \theta} \right] \left\{ \begin{array}{l} [(r+\lambda)\phi + \gamma p] \gamma \theta (k+g) \\ \times [(r+\lambda)\psi + (1-\gamma)p] \\ + [(r+\lambda)(\psi-1) - \gamma p] \times \\ (r+\lambda+p)(1-\gamma)a(a-1)\alpha^a \end{array} \right\}, \quad (\text{A24})$$

and using (19), after some manipulation we get

$$|A| = \left[\frac{-(k+g)^2(r+p)\gamma}{(1-\gamma)\alpha u} \right] \left\{ \begin{array}{l} (r+\lambda)^2 [\phi\psi - (a-1)(1-\psi)] \\ + \gamma p^2 (2-\gamma-a) \\ + (r+\lambda)p \left[\begin{array}{l} \phi(1-\gamma) + \gamma\psi \\ + (a-1)(\psi-\gamma-1) \end{array} \right] \end{array} \right\}. \quad (\text{A25})$$

Determinant $|A_\alpha|$ results straightforwardly.

Determinant $|A_v|$

We have

$$|A_v| = \left[\frac{(k+g)(r+p)}{(1-\gamma)\alpha u} \right] \left\{ \begin{array}{l} [(r+\lambda)\phi + \gamma p] (r+\lambda+p) \times \\ a(a-1)\alpha^a(1-\gamma) \\ - [(r+\lambda)\phi + \gamma p] \gamma (k+g)\theta \times \\ [(r+\lambda)(1-\phi) + (1-\gamma)p] \end{array} \right\}, \quad (\text{A26})$$

and using (19) results in $|A_v|$ given by (30).

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Physical Search^{*}

Klaus Kultti[†]

Antti Miettunen

University of Helsinki

Helsinki School of Economics

Juha Virrankoski[‡]

University of Helsinki

Abstract

One-sided search is an evolutionarily stable outcome in an economy where each buyer and seller can either search or wait, and where the trading mechanism is auction or bargaining. If the relative number of buyers to sellers increases, the likelihood of all sellers wait and all buyers search -equilibrium increases relative to the likelihood of all buyers wait and all sellers search -equilibrium. In two-sided search, bargaining is more efficient than auction. One-sided search is more efficient than two-sided search. In one-sided search, it is more efficient if the larger pool searches and the smaller pool waits, than vice versa.

JEL Classification: J41, J64, C78, D44, D83

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[†]Department of Economics, P.O. Box 17 (Arkadiankatu 7), 00014 University of Helsinki, Finland. Tel: +358-9-191 28738. e-mail: klaus.kultti@helsinki.fi

[‡]Research Unit on Economic Structures and Growth, Department of Economics, P.O. Box 17 (Arkadiankatu 7), 00014 University of Helsinki, Finland. Telephone: +358-9-191 28760, e-mail: juha.virrankoski@helsinki.fi.

1 Introduction

Search theoretic models are widely used in labor markets, and recently so called urn-ball models have been successfully applied to study the coordination problems inherent in those markets. This approach has two fruitful features. First, it yields a well-defined equilibrium matching function, and second, it is possible to study different ways of determining wages. The economy consists of employers and workers. Production requires one employer and one worker. The workers contact the employers, and if an employer meets at least one worker, a pair is formed. The model has plenty of equilibria: In pure strategy equilibria a worker goes to a certain employer with probability one, and in equilibrium everyone has correct expectation about each other's choice. Especially in large markets, like labor markets, pure strategy equilibria are not regarded as plausible since they seem to involve a lot of coordination. For this reason, the focus is on a symmetric mixed strategy equilibrium in which each worker randomizes over which employer to visit. This is exactly the coordination problem which, coupled with the employers' capacity constraint, leads to inefficiency as some employers may not meet any workers and some employers may meet several workers. We call this the one-sided coordination problem.

There is no difference between employers and workers in a sense that we could as well think that employers contact workers. This choice is made in Julien, Kennes and King (2000). It is clear that postulating this kind of market structure already involves plenty of coordination. If employers and workers are treated symmetrically, both of them must be allowed to contact the other side, and we are faced with a two-sided coordination problem. To study this, it is helpful to think that each agent is assigned a location. This assignment is public knowledge. In a two-sided coordination problem the agents have two decisions to make. First, they decide whether to contact an agent of the opposite type or not. In case they contact, they choose a mixed strategy over which location to contact, like in a one-sided coordination problem. If, say, a worker contacts employers, he must leave his own location and go to some location assigned to an employer (this could be called 'physical search'). But now it is possible that there is no one in the location; this happens if the employer has decided to contact workers. In the sequel we say that

those who try to contact the opposite type search, or move, and the others wait, or stay. We call the agents buyers and sellers, and each buyer and seller can search or wait.

The main aim of this study is to explain one-sided search as an evolutionary stable equilibrium in case the trading mechanism is auction or bargaining, and to solve how the probability of ending up to either equilibrium (buyers search for sellers or vice versa) depends on the relative size of the buyer and seller pools. We also study the relative efficiency of the two trading mechanisms in two-sided search equilibrium. Further, the relative efficiency of one-sided search and two-sided search is compared for both trading mechanisms. In addition, we solve which one of the one-sided search equilibria is more efficient (produces more matches): the equilibrium where all sellers wait and all buyers search, or the equilibrium where all buyers wait and all sellers search.

There is quite a lot of related labor market literature, the seminal contribution being Montgomery (1991) where one-sided coordination problem is studied with employers posting wages. Acemoglu and Shimer (1999a, 1999b) also use price posting, while Julien, Kennes, and King (2000, 2001, 2002) have models with both price posting and auction. Burdett, Coles, Kiyotaki and Wright (1995) emphasize the question of who searches and who waits, but they do not stress the coordination problems as such. Herreiner (1999) studies also the question about who searches, but she ignores price formation. Kultti, Miettunen, Takalo and Virrankoski (2004) study the same question, paying explicit attention to different ways of determining prices. As opposed to the present study, they assume that search is one-sided at the outset, and they study if a given search pattern is an equilibrium.

The most important results of the present paper are: 1) Independently of the trading mechanism (auction or bargaining) i) the market where both buyers and sellers use mixed strategies over the wait and move decisions (that is, search is two-sided) is not evolutionarily stable, and ii) the market with one-sided coordination problem is evolutionarily stable. 2) For both auction and bargaining, the basin of attraction for the market where sellers wait grows when the ratio of buyers to sellers grows. 3) In the two-sided coordination problem, bargaining is more efficient than auction. 4) One-sided search - whether all buyers search and all sellers wait, or vice versa - is more efficient than two-sided search.

This holds for auction as well as for bargaining. 5) A one-sided search equilibrium where all the members of the larger pool search and all the members of the smaller pool wait is more efficient than the equilibrium where all the members of the larger pool wait and all the members of the smaller pool search.

A two-sided coordination equilibrium is evolutionarily unstable and also inefficient compared to one-sided coordination equilibrium. Solving for the two-sided equilibrium is still important, because we want to find out how the relative likelihood of the two stable (one-sided) equilibria changes when the ratio of buyers to sellers changes. The instability accompanied with inefficiency perhaps explains why it is difficult to find real-life examples that fit well into the two-sided framework. With a little stretching one can think that taxi markets and markets where one has to phone to the other party to establish a contact have some features of two-sided search. One can also think of marriage markets from this point of view. But there the idea would be to explain institutions like intermediaries arising from the problems of two-sided coordination.

The rest of the paper is organized as follows. In Section 2 we present the basic features of the model: the matching process and the trading mechanisms. In Section 3 we solve the search/wait decisions when trades are consummated by auction, and Section 4 deals with the same problem when the trading mechanism is bargaining. Section 5 analyzes the relative efficiency of different equilibria. Section 6 studies the evolutionary stability of two-sided search. Section 7 concludes. Section 8 is an appendix that contains derivations of some of the results, and all the figures.

2 The Model

There are B buyers, each with a unit demand, and S sellers, each with one indivisible object for sale. Let $\theta \equiv B/S$. The buyers get utility normalized to one from consuming the object, and the sellers get utility normalized to zero from consuming it. The economy extends to infinity, and time proceeds in discrete periods. The agents discount future with factor $\delta \in (0, 1)$. When the agents trade they exit the economy and are replaced by identical agents who are not yet matched. This means that the ratio of buyers to sellers

remains the same in every period.

To model the meeting process we use a familiar urn-ball model. In a basic urn-ball model, the agents who decide to wait are in fixed positions, and the agents who decide to search are randomly allocated on the waiting agents. This meeting technology is well defined and tractable. Further, since multiple meetings are possible, one can meaningfully study a variety of trading mechanisms. If w agents wait and m agents move, the number of agents a waiting agent meets is a binomial random variable with parameters m and $1/w$. Tractability is achieved by assuming that w and m are large, since in this case one can approximate the binomial with a Poisson distribution with parameter m/w . Then the probability that a waiting agent meets exactly k moving agents is $\frac{(m/w)^k}{k!}e^{-m/w}$. Assuming that m and w approach infinity, $e^{-m/w}$ is the probability that a waiting agent does not meet any moving agent, $1 - e^{-m/w}$ is the probability that a waiting agent meets at least one moving agent, and with probability $1 - e^{-m/w} - \frac{m}{w}e^{-m/w}$ a waiting agent meets at least two moving agents.

In the present model, each buyer and each seller has his own location. Then, a fraction $\beta \in [0, 1]$ of buyers remain in their locations, waiting for sellers to visit them, and the rest of the buyers, fraction $1 - \beta$, go to sellers' locations, leaving their own locations empty. Each seller, too, makes a decision whether to stay or go. A fraction $\sigma \in [0, 1]$ of the sellers remain to wait for moving buyers, and fraction $1 - \sigma$ go to buyers' locations. The equilibrium values of these fractions will be solved. Each agent knows where each location is, but a moving agent does not know whether the particular location he is going to will have an occupant or not. However, the agents are assumed to know the equilibrium fractions of empty locations. As in the basic model, waiters have a risk that nobody comes to their location, and movers may end up with competing with other movers who have chosen to go to the same location. In addition, a mover has the risk of entering an empty location, whose occupant has chosen to move. The Poisson parameter that governs the arrival of buyers to a seller's location is thus

$$\gamma \equiv (1 - \beta) \theta, \tag{1}$$

and the Poisson parameter that governs the arrival of sellers to a buyer's location is

$$\varphi \equiv \frac{(1 - \sigma)}{\theta}. \quad (2)$$

We investigate two trading mechanisms: auction and bargaining. In auction, if a stayer meets at least two movers, the movers are assumed to engage in a Bertrand-type bidding. All the movers bid the same, and one of them trades. Each mover - whether he trades or not - gets his own reservation value, since the movers are indifferent between trading and waiting for an opportunity to trade in the next period. The stayer gets his own reservation value plus the whole surplus of the match, in other words, the stayer gets one minus a mover's reservation value. If a stayer meets just one mover, we assume that the mover makes a take-it-or-leave-it offer. As a result, the stayer gets his own reservation value, and the mover gets his own reservation value plus the whole surplus of the match, that is, the mover gets one minus the stayer's reservation value. Assuming the take-it-or-leave-it offer gives the movers, too, a positive probability to get the whole surplus of a trade, treating the movers and stayers as equally as possible. Bargaining is always pairwise, and if a stayer meets several movers he just picks one of them randomly for his trading partner. To make things simple we assume that the stayer and mover just split the available surplus in half.

3 Auction

Denote the life-time values of waiting buyers and sellers as V_b^w and V_s^w , the respective values for moving agents are V_b^m and V_s^m . The Poisson parameters determining the arrival rates faced by a buyer's and seller's location are φ and γ , respectively. The value functions are

$$V_b^w = \delta [e^{-\varphi} (1 + \varphi) V_b^w + (1 - e^{-\varphi} - \varphi e^{-\varphi}) (1 - V_s^m)], \quad (3)$$

$$V_b^m = \delta [(1 - \sigma) V_b^m + \sigma e^{-\gamma} (1 - V_s^w) + \sigma (1 - e^{-\gamma}) V_b^m], \quad (4)$$

$$V_s^w = \delta [e^{-\gamma} (1 + \gamma) V_s^w + (1 - e^{-\gamma} - \gamma e^{-\gamma}) (1 - V_b^m)], \quad (5)$$

$$V_s^m = \delta [(1 - \beta) V_s^m + \beta e^{-\varphi} (1 - V_b^w) + \beta (1 - e^{-\varphi}) V_s^m]. \quad (6)$$

For example, in equation (5), a waiting seller has a probability $e^{-\gamma}(1 + \gamma)$ that no buyer or just one buyer arrives, and in both cases the seller gets his reservation value. With probability $1 - e^{-\gamma} - \gamma e^{-\gamma}$ at least two buyers arrive, and the seller gets one minus a buyer's reservation value. In equation (6), a moving seller arrives at an empty location with probability $1 - \beta$ and gets his reservation value. With probability $\beta e^{-\varphi}$ the seller arrives at a non-empty location, no other sellers arrive, and the seller makes a take-it-or-leave-it offer to the buyer and gets one minus the buyer's reservation value. With probability $\beta(1 - e^{-\varphi})$ the seller arrives at a non-empty location but other sellers arrive, too, and the seller receives his reservation value.

Solving from (3) and (6) gives

$$V_b^w = \frac{\delta(1 - e^{-\varphi} - \varphi e^{-\varphi})}{1 - \delta e^{-\varphi}(1 + \varphi - \beta)}, \quad (7)$$

$$V_s^m = \frac{\delta \beta e^{-\varphi}}{1 - \delta e^{-\varphi}(1 + \varphi - \beta)}, \quad (8)$$

and solving from (4) and (5) gives

$$V_b^m = \frac{\delta \sigma e^{-\gamma}}{1 - \delta e^{-\gamma}(1 + \gamma - \sigma)}, \quad (9)$$

$$V_s^w = \frac{\delta(1 - e^{-\gamma} - \gamma e^{-\gamma})}{1 - \delta e^{-\gamma}(1 + \gamma - \sigma)}. \quad (10)$$

In equilibrium, staying and moving give equal life-time utilities: $V_b^w = V_b^m$, and $V_s^w = V_s^m$. If both of these equations hold, then

$$\sigma e^{-\gamma} = 1 - e^{-\varphi} - \varphi e^{-\varphi}, \quad (11)$$

$$\beta e^{-\varphi} = 1 - e^{-\gamma} - \gamma e^{-\gamma}, \quad (12)$$

where (11) results from equating the right-hand sides of (3) and (4), given that $V_s^m = V_s^w$, and (12) results from equating the right-hand sides of (5) and (6), given that $V_b^m = V_b^w$. Solving (11) and (12) gives the equilibrium fractions of staying buyers and sellers (see the Appendix):

$$\beta = 1 - \frac{1}{\theta} \ln(1 + \theta), \quad (13)$$

$$\sigma = 1 - \theta \ln\left(1 + \frac{1}{\theta}\right). \quad (14)$$

Proposition 1 *If trades are consummated by auction, the proportion of moving sellers increases and the proportion of moving buyers decreases if B/S increases.*

Proof. Equation (13) gives $\frac{d\beta}{d\theta} = \frac{1}{\theta} \left[\frac{1}{\theta} \ln(1 + \theta) - \frac{1}{1 + \theta} \right] \geq 0$ because $\ln(1 + \theta) = \frac{\theta}{1 + \theta}$ if $\theta = 0$, and $\ln(1 + \theta) > \frac{\theta}{1 + \theta}$ if $\theta > 0$. The latter holds because $\frac{\partial \left[\ln(1 + \theta) - \frac{\theta}{1 + \theta} \right]}{\partial \theta} = \frac{1}{1 + \theta} - \frac{1}{(1 + \theta)^2} > 0 \forall \theta > 0$. Equation (14) gives $\frac{d\sigma}{d\theta} = \frac{1/\theta}{1 + 1/\theta} - \ln(1 + 1/\theta) < 0 \forall \theta > 0$, because $\frac{x}{1 + x} - \ln(1 + x) < 0 \forall x > 0$. ■

That is, the larger the relative size of the population, the larger the fraction of waiting agents and the smaller the fraction of moving agents.

One might ask whether making the present model static will change the results in any way. The answer is no. In a one-period model, the reservation values in the right-hand sides of value functions (3) - (6) are zeroes, and the equilibrium conditions that result from solving the new value functions are just (13) and (14). This holds in the case of bargaining, too.

We solve the matching function, that is, the aggregate number of trades, M_a , that will emerge per period:

Remark 1 *The matching function implied by two-sided search when the trading mechanism is auction, is $M_a = \frac{\theta S}{1 + \theta} \left(2 - \theta \ln \left(1 + \frac{1}{\theta} \right) - \frac{1}{\theta} \ln(1 + \theta) \right)$.*

Proof. Let us add together the number of matches that waiting agents make. The probability that a waiting seller gets matched is equal to $1 - e^{-(1-\beta)\theta}$. The number of waiting sellers is σS , so waiting sellers form $\sigma S(1 - e^{-(1-\beta)\theta})$ matches. Similarly, waiting buyers form $\beta B(1 - e^{-(1-\sigma)/\theta})$ matches. Using the equilibrium values of β and σ gives $1 - e^{-(1-\beta)\theta} = \frac{B}{B + S}$, and $1 - e^{-(1-\sigma)/\theta} = \frac{S}{B + S}$. We get $M_a = \beta B \left(\frac{S}{B + S} \right) + \sigma S \left(\frac{B}{B + S} \right) = \frac{BS}{B + S} (\beta + \sigma)$. Using (13) and (14) again we get M_a . ■

Remark 2 *Matching function M_a has constant returns to scale.*

Proof. A matching function has constant returns to scale if individual matching probabilities do not depend on the absolute number of agents but on their relative num-

ber. Using $M_a = \frac{BS}{B+S}(\beta + \sigma)$, we have $\frac{M_a}{B} = \frac{1}{1+\theta}(\beta + \sigma)$, and $\frac{M_a}{S} = \frac{\theta}{1+\theta}(\beta + \sigma)$.

■

4 Bargaining

If at least one mover comes to a stayer, the stayer picks one mover at random, and the two split the available surplus in half. The life-time values of waiting buyers and sellers are W_b^w and W_s^w , and the respective values for moving agents are W_b^m and W_s^m . The Poisson parameters are φ and γ for buyers' and sellers' locations, respectively. The value functions are

$$W_b^w = \delta \left\{ e^{-\varphi} W_b^w + (1 - e^{-\varphi}) \left[W_b^w + \frac{1}{2} (1 - W_b^w - W_s^m) \right] \right\}, \quad (15)$$

$$W_s^m = \delta \left\{ (1 - \beta) W_s^m + \frac{\beta(1 - e^{-\varphi})}{\varphi} \left[W_s^m + \frac{1}{2} (1 - W_b^w - W_s^m) \right] + \frac{\beta(\varphi - 1 + e^{-\varphi})}{\varphi} W_s^m \right\}, \quad (16)$$

$$W_s^w = \delta \left\{ e^{-\gamma} W_s^w + (1 - e^{-\gamma}) \left[W_s^w + \frac{1}{2} (1 - W_s^w - W_b^m) \right] \right\}, \quad (17)$$

$$W_b^m = \delta \left\{ (1 - \sigma) W_b^m + \frac{\sigma(1 - e^{-\gamma})}{\gamma} \left[W_b^m + \frac{1}{2} (1 - W_s^w - W_b^m) \right] + \frac{\sigma(\gamma - 1 + e^{-\gamma})}{\gamma} W_b^m \right\}. \quad (18)$$

In equation (15), a waiting buyer meets no seller with probability $e^{-\varphi}$ and continues to the next period. With probability $1 - e^{-\varphi}$ he meets at least one seller and gets his reservation value plus one half of the surplus. In equation (16), a moving seller comes to an empty location with probability $1 - \beta$. With probability β the seller arrives at an occupied location, trades with probability $(1 - e^{-\varphi})/\varphi$ (see the Appendix), and does not trade with probability $(\varphi - 1 + e^{-\varphi})/\varphi$.

Solving from (15) and (16) yields

$$W_b^w = \frac{\delta \varphi (1 - e^{-\varphi})}{\delta (1 - e^{-\varphi}) (\beta + \varphi) + 2(1 - \delta) \varphi}, \quad (19)$$

$$W_s^m = \frac{\delta \beta (1 - e^{-\varphi})}{\delta (1 - e^{-\varphi}) (\beta + \varphi) + 2(1 - \delta) \varphi}, \quad (20)$$

and solving from (17) and (18) gives

$$W_s^w = \frac{\delta \gamma (1 - e^{-\gamma})}{\delta (1 - e^{-\gamma}) (\sigma + \gamma) + 2 (1 - \delta) \gamma}, \quad (21)$$

$$W_b^m = \frac{\delta \sigma (1 - e^{-\gamma})}{\delta (1 - e^{-\gamma}) (\sigma + \gamma) + 2 (1 - \delta) \gamma}. \quad (22)$$

Using the familiar conditions for equal utilities for moving and waiting ($W_b^w = W_b^m$ and $W_s^w = W_s^m$), gives

$$(1 - e^{-\varphi}) \gamma = \sigma (1 - e^{-\gamma}), \quad (23)$$

$$(1 - e^{-\gamma}) \varphi = \beta (1 - e^{-\varphi}), \quad (24)$$

where (23) is the equilibrium condition for the buyers, assuming that $W_s^w = W_s^m$ for the sellers, and (24) is the equilibrium condition for the sellers, assuming that $W_b^w = W_b^m$ for the buyers. We get $1 - e^{-\varphi} = \frac{\sigma (1 - e^{-\gamma})}{\gamma}$ from (23), and using this in (24) we have $\beta \sigma = \gamma \varphi \iff \beta + \sigma = 1$. Plugging this into (23) yields

$$1 - \theta - e^{-\sigma \theta} + \theta e^{\frac{\sigma - 1}{\theta}} = 0 \quad (25)$$

which tells us the equilibrium fraction σ of waiting sellers as a function of B/S . There is a unique equilibrium σ for any given θ . Write (25) as

$$1 - \theta - e^{-\sigma \theta} = -\theta e^{\frac{\sigma - 1}{\theta}}. \quad (26)$$

Fix θ and differentiate both sides of (26) with respect to σ . Then $\frac{\partial (1 - \theta - e^{-\sigma \theta})}{\partial \sigma} =$

$\theta e^{-\sigma \theta} > 0$ and $\frac{\partial \left(-\theta e^{\frac{\sigma - 1}{\theta}} \right)}{\partial \sigma} = -e^{\frac{\sigma - 1}{\theta}} < 0$. That is, σ is a function of θ . We can conclude that there exists a unique equilibrium pair (β, σ) for every value of θ .

Proposition 2 *If trades are consummated by bargaining, the proportion of moving sellers increases and the proportion of moving buyers decreases if B/S increases.*

Proof. By numerically solving $d\sigma/d\theta$ from (25) ■

The decisions to stay or move show a similar pattern as in the case of auction: the larger the relative size of a population is, the larger the fraction of stayers is in that population and the larger the fraction of movers is in the opposite population.

Remark 3 *The matching function implied by two-sided search when the trading mechanism is bargaining is $M_b = S(1 - e^{-\sigma\theta})$, where σ and θ satisfy equation (25).*

Proof. Add together the matches made by waiting agents. We have $M_b = \sigma S(1 - e^{-(1-\beta)\theta}) + \beta B(1 - e^{-(1-\sigma)/\theta})$. Then use $\beta + \sigma = 1$ and condition (25). ■

Remark 4 *Matching function M_b has constant returns to scale.*

Proof. The argument is similar as in the case of M_a . We have $M_b/S = (1 - e^{-\sigma\theta})$ and $M_b/B = \frac{1}{\theta}(1 - e^{-\sigma\theta})$, and because σ is a function of θ only, the individual matching probabilities depend only on θ . ■

5 Efficiency

We determine efficiency simply as the number of trades made per period. It is conceivable that different trading mechanisms lead to different number of trades made: the number of trades depends on how many agents on each side of the market move and how many agents stay, and the trading mechanism affects the incentives of moving and staying. In this section we first study the relative efficiency of auction and bargaining when search is two-sided. Then we compare the efficiency of one-sided and two-sided search. Finally, we compare the two alternative patterns in one-sided search.

5.1 Relative Efficiency of Auction and Bargaining in Two-Sided Search

Letting $E = M_a/M_b$, we have

$$E = \frac{\frac{\theta}{1+\theta} \left(2 - \theta \ln \left(1 + \frac{1}{\theta} \right) - \frac{1}{\theta} \ln(1+\theta) \right)}{1 - e^{-\sigma\theta}}, \quad (27)$$

where the denominator satisfies condition (25).

Proposition 3 *In two-sided coordination equilibrium, bargaining is more efficient than auction.*

Proof. By numerically solving the value of the right-hand side of (27) we see that $E < 1$ with all positive values of θ . ■

In Figure 1, the horizontal axis is for θ , and the vertical axis is for E . We see that $\partial E / \partial \theta < 0$ if $\theta < 1$, and $\partial E / \partial \theta > 0$ if $\theta > 1$. In other words, auction performs worst with respect to bargaining when $B = S$. Plugging $\theta = 1$ into (13) and (14) we get $\beta = \sigma = 1 - \ln 2 \approx 0.3$. That is, if there are equally many buyers and sellers and the trading mechanism is auction, 30 percent of buyers and sellers stay, and 70 percent of them move. Using $\theta = 1$ in (25) we have $\beta = \sigma = 1/2$: if the number of buyers equals that of the sellers, and bargaining is used, one half of buyers and sellers move and the other half of them stays. If the two pools of traders are equally large, there is too much moving and too little staying if auction is used. Auction gives the agents too strong an incentive to move, which leads to an excessive amount of empty locations. Bargaining treats the agents more evenly when sharing the surplus in a trade, and this in turn makes the populations of movers and stayers more balanced. A numerical analysis showed that for all values of θ , the proportions of staying sellers and staying buyers is larger in case of bargaining than in case of auction.

5.2 Relative Utility of a Buyer and a Seller in Two-Sided Search

Increasing the relative number of buyers to sellers will make the sellers better off and the buyers worse off. The value of being a seller relative to the value of being a buyer depends on the trading mechanism. In case of auction, $\frac{V_s^m}{V_b^w} = \frac{\beta e^{-\varphi}}{1 - e^{-\varphi} - \varphi e^{-\varphi}}$, and $\frac{V_s^w}{V_b^m} = \frac{1 - e^{-\gamma} - \gamma e^{-\gamma}}{\sigma e^{-\gamma}}$. In equilibrium $V_s^m = V_s^w$ and $V_b^m = V_b^w$. Using (11) and (12) and

denoting the utility of a seller as V_s and the utility of a buyer as V_b we have $\frac{V_s}{V_b} = \frac{\beta e^{-\varphi}}{\sigma e^{-\gamma}}$,

and using the equilibrium values for β and σ we get $\frac{V_s}{V_b} = \frac{(1 + \theta) \left[1 - \frac{1}{\theta} \ln(1 + \theta) \right]}{\left(1 + \frac{1}{\theta} \right) \left[1 - \theta \ln \left(1 + \frac{1}{\theta} \right) \right]}$.

In case of bargaining, by a similar argument we have $\frac{W_s}{W_b} = \theta$. Figure 2 shows that as θ increases, a seller's relative utility increases faster if auction is used than if bargaining is used.

5.3 Efficiency of Two-Sided versus One-Sided Search

Here we compare the number of matches resulting from one-sided search versus two-sided search. If all the sellers wait and all the buyers move, the number of matches is $S(1 - e^{-\theta})$, whereas if all the buyers wait and all the sellers move, the number of matches is $B(1 - e^{-1/\theta})$. These hold whether the trading mechanism is auction or bargaining.

Proposition 4 *One-sided search is more efficient than two-sided search. This holds for auction as well as for bargaining.*

Proof. i) Auction: in two-sided search, the number of matches is $\frac{\theta S}{1+\theta} \left(2 - \theta \ln \left(1 + \frac{1}{\theta} \right) - \frac{1}{\theta} \ln(1+\theta) \right)$ (see Remark 1), and in one-sided search it is $S(1 - e^{-\theta})$ if all the sellers wait and all the buyers search, and $B(1 - e^{-1/\theta})$ if all the buyers wait and all the sellers search. Using fact $\ln(1+x) \geq \frac{x}{1+x} \forall x > 0$ gives that $S(1 - e^{-\theta}) - \frac{\theta S}{1+\theta} \left(2 - \theta \ln \left(1 + \frac{1}{\theta} \right) - \frac{1}{\theta} \ln(1+\theta) \right) \geq S \left\{ 1 - e^{-\theta} - \frac{\theta}{1+\theta} \left(2 - \frac{\theta}{1+\theta} - \frac{1}{1+\theta} \right) \right\} = S \left(1 - e^{-\theta} - \frac{\theta}{1+\theta} \right)$, which has the same sign as $1 - e^{-\theta} - \theta e^{-\theta}$, which is positive. In the latter case $B(1 - e^{-1/\theta}) - \frac{\theta S}{1+\theta} \left(2 - \theta \ln \left(1 + \frac{1}{\theta} \right) - \frac{1}{\theta} \ln(1+\theta) \right)$ has the same sign as $\theta(1 - e^{-1/\theta}) - \frac{\theta}{1+\theta} \left(2 - \theta \ln \left(1 + \frac{1}{\theta} \right) - \frac{1}{\theta} \ln(1+\theta) \right)$, which is at least as large as $\theta(1 - e^{-1/\theta}) - \frac{\theta}{1+\theta}$, which has the same sign as $1 - e^{-1/\theta} - \frac{1}{\theta} e^{-1/\theta}$, which is positive. That is, one-sided search produces more matches than two-sided search.

ii) Bargaining: in two-sided search, the number of matches is $S(1 - e^{-\sigma\theta})$, where $\sigma \leq 1$, and where σ and θ satisfy equation (25) (see Remark 3). It is easily noted that $S(1 - e^{-\sigma\theta}) \leq S(1 - e^{-\theta})$, that is, one-sided search when all sellers wait produces at least as many matches as two-sided search. If in one-sided search all the buyers wait, the difference in the number of matches is $S(1 - e^{-\sigma\theta}) - B(1 - e^{-1/\theta})$, which has the same sign as $1 - \theta - e^{-\sigma\theta} + \theta e^{-1/\theta}$. In two-sided search, σ and θ satisfy equilibrium condition $\frac{\sigma-1}{\theta} = 0$, and plugging the latter into $1 - \theta - e^{-\sigma\theta} + \theta e^{-1/\theta}$ implies that $S(1 - e^{-\sigma\theta}) - B(1 - e^{-1/\theta})$ has the same sign as $1 - e^{\sigma/\theta}$, which is negative. One-sided search when all buyers wait produces at least as many matches as two-sided search. ■

Figures 3 and 4 (in the Appendix) illustrate the different cases presented in part (i) of the proof, and figures 5 and 6 (in the Appendix) illustrate cases of part (ii). In Figure 3,

$$E1 = \frac{S(1 - e^{-\theta})}{\frac{\theta S}{1 + \theta} \left(2 - \theta \ln \left(1 + \frac{1}{\theta} \right) - \frac{1}{\theta} \ln(1 + \theta) \right)}, \text{ and in Figure 4,}$$

$$E2 = \frac{B(1 - e^{-1/\theta})}{\frac{\theta S}{1 + \theta} \left(2 - \theta \ln \left(1 + \frac{1}{\theta} \right) - \frac{1}{\theta} \ln(1 + \theta) \right)}. \text{ In Figure 5, } E3 = \frac{S(1 - e^{-\theta})}{S(1 - e^{-\sigma\theta})}, \text{ and}$$

in Figure 6, $E4 = \frac{B(1 - e^{-1/\theta})}{S(1 - e^{-\sigma\theta})}$, where in $S(1 - e^{-\sigma\theta})$ the values of σ and θ satisfy equation (25). The relative efficiency gain from one-sided search compared to two-sided search is at its largest when θ is roughly between 0.7 and 1.5, and the largest gain is approximately between 1.63 and 2.08.

5.4 Efficiency in One-Sided Search

One-sided search has two alternative configurations: i) all sellers wait and all buyers search, thus the number of matches is $S(1 - e^{-\theta})$, or ii) all buyers wait and all sellers search, which produces $B(1 - e^{-1/\theta})$ matches. Their relative efficiency depends only on θ . Note that the trading rule has no effect on the number of matches if search is one-sided, because all searchers just choose each location with equal probability.

Proposition 5 *If $B > (<)S$, a market where all buyers search and all sellers wait produces more (less) matches than a market where all sellers search and all buyers wait.*

Proof. See the Appendix. ■

In Figure 7 (in the Appendix), $E5 = \frac{S(1 - e^{-\theta})}{B(1 - e^{-1/\theta})}$. We see that the maximum efficiency loss from ‘wrong’ choices of search and wait is about ten percent, and it happens when the ratio of buyers to sellers is about 3 or 1/3. As θ approaches zero or infinity, the efficiency loss approaches zero.

6 Stability of Two-Sided Search

Let us study replicator dynamics in order to gain insight to the nature of equilibria. There are two variables that determine the equilibria: the proportions of buyers and sellers, β and σ , who stay at their locations. This means that the analysis can be conducted graphically in a (β, σ) -plane (see Lu and McAfee, 1996). The idea behind the replicator dynamics presupposes myopically behaving agents. Those who trade are replaced by identical agents and these agents go to the ‘market’ where their type did best in the previous period. The stable points under replicator dynamics are a subset of Nash-equilibria, and we can think of the dynamics as a kind of criterion to choose between equilibria.

To conduct the analysis we determine two curves called the buyers’ equilibrium curve BE, and the sellers’ equilibrium curve SE. The BE is got by equating the buyers’ expected utility in the market where they wait and in the market where they move. The SE is determined analogously, and the equilibrium we study is determined by the intersection of the BE and the SE.

6.1 Stability of Two-Sided Search in Case of Auction

Equating V_b^w and V_b^m from (7) and (9) yields the BE which after some manipulation is given by

$$(1 - e^{-\varphi} - \varphi e^{-\varphi}) (1 - \delta e^{-\gamma} - \delta \gamma e^{-\gamma}) - \sigma e^{-\gamma} (1 - \delta + \delta \beta e^{-\varphi}) = 0. \quad (28)$$

Equating V_s^w and V_s^m from (8) and (10) yields SE:

$$(1 - e^{-\gamma} - \gamma e^{-\gamma}) (1 - \delta e^{-\varphi} - \delta \varphi e^{-\varphi}) - \beta e^{-\varphi} (1 - \delta + \delta \sigma e^{-\gamma}) = 0. \quad (29)$$

On the (β, σ) -plane the point $(0, 1)$ corresponds to the market where all the buyers move and all the sellers wait, and point $(1, 0)$ corresponds to the market where all the sellers move and all the buyers wait.

The positions of the equilibrium curves

First we show that the equilibrium curves slope downwards. Totally differentiating (28) and (29) yield

$$\frac{d\sigma}{d\beta} = \frac{\delta\gamma\theta e^{-\gamma}(1 - e^{-\varphi} - \varphi e^{-\varphi}) + \sigma\theta e^{-\gamma}(1 - \delta + \delta\beta e^{-\varphi}) + \delta\sigma e^{-\gamma}e^{-\varphi}}{-\theta^{-1}\varphi e^{-\varphi}(1 - \delta e^{-\gamma} - \delta\gamma e^{-\gamma}) - e^{-\gamma}(1 - \delta + \delta\beta e^{-\varphi}) - \delta\theta^{-1}\beta\sigma e^{-\gamma}e^{-\varphi}} < 0, \quad (30)$$

$$\frac{d\sigma}{d\beta} = \frac{\gamma\theta e^{-\gamma}(1 - \delta e^{-\varphi} - \delta\varphi e^{-\varphi}) + e^{-\varphi}(1 - \delta + \delta\sigma e^{-\gamma}) + \delta\beta\sigma\theta e^{-\gamma}e^{-\varphi}}{-\delta\theta^{-1}\varphi e^{-\varphi}(1 - e^{-\gamma} - \gamma e^{-\gamma}) - \beta\theta^{-1}e^{-\varphi}(1 - \delta + \delta\sigma e^{-\gamma}) - \delta\beta e^{-\gamma}e^{-\varphi}} < 0. \quad (31)$$

That is, BE and SE are downward-sloping. Next we determine how the curves behave close to points $(0, 1)$ and $(1, 0)$. Let us first consider BE. There are four cases.

1. $(\beta, \sigma) \rightarrow (0, \sigma)$ in which case $\gamma = \theta$ and BE becomes $(1 - e^{-\varphi} - \varphi e^{-\varphi}) \times (1 - \delta e^{-\theta} - \delta\theta e^{-\theta}) - \sigma e^{-\theta}(1 - \delta) = 0$, which is equivalent to $\frac{\sigma(1 - \delta)e^{\varphi}}{e^{\varphi} - 1 - \varphi} = e^{\theta} - \delta(1 + \theta)$. The right-hand side of the latter equation is fixed; the left-hand side is zero when $\sigma = 0$, and it grows without limit when σ approaches unity. The left-hand side is also increasing in σ . Thus, we know that BE always goes through point $(0, \sigma)$ where $\sigma < 1$.

2. $(\beta, \sigma) \rightarrow (\beta, 1)$ in which case $\varphi = 0$, and BE becomes $-\sigma e^{-\gamma}(1 - \delta + \delta\beta) = 0$ which is never true. Thus, BE does not go through $(\beta, 1)$ where $\beta > 0$.

3. $(\beta, \sigma) \rightarrow (\beta, 0)$ in which case $\varphi = \theta^{-1}$, and the BE becomes $\left(1 - e^{-1/\theta} - \frac{1}{\theta}e^{-1/\theta}\right) \times (1 - \delta e^{-\gamma} - \delta\gamma e^{-\gamma}) = 0$ which never holds. Thus, we know that BE never goes through $(\beta, 0)$ where $\beta < 1$.

4. $(\beta, \sigma) \rightarrow (1, \sigma)$ in which case $\gamma = 0$, and BE becomes $(1 - e^{-\varphi} - \varphi e^{-\varphi})(1 - \delta) - \sigma(1 - \delta + \delta e^{-\varphi}) = 0$ which is equivalent to $\frac{\sigma(e^{\varphi} - \delta e^{\varphi} + \delta)}{e^{\varphi} - 1 - \varphi} = 1 - \delta$. Here the right-hand side is fixed, and the left-hand side is zero when $\sigma = 0$ and it grows without limit when σ approaches unity. The left-hand side is also increasing in σ . Thus, we know that BE always goes through point $(1, \sigma)$ where $\sigma > 0$.

In sum, for all θ , BE goes through points $(0, \sigma)$ where $\sigma < 1$, and $(1, \sigma)$ where $\sigma > 0$. Completely analogous reasoning shows that for all θ , SE goes through $(\beta, 1)$ where $\beta > 0$, and $(\beta, 0)$ where $\beta < 1$. From (13) and (14) we know that BE and SE intersect exactly

once. The BE and SE-curves are presented in Figure 8. The arrows depicting the behavior out of steady state result from the evolutionary stability analysis presented below.

Evolutionary stability

The entering agents' action set consists of two actions corresponding to the two markets. Let us denote the buyers' strategy by β and the sellers' strategy by σ . The first is the probability of buyers going to the market where buyers wait and the second is the probability of sellers going to the market where sellers wait. The probabilities of going to the other market are naturally $1 - \beta$ for the buyers and $1 - \sigma$ for the sellers. One can also think of these as the population shares of agents going to the two markets. To define replicator dynamics let us first establish notation for the buyers' and sellers' average expected utilities A_b and A_s , given population shares β and σ in the markets where they wait. Now $A_b = (1 - \beta)V_b^m + \beta V_b^w$ and $A_s = (1 - \sigma)V_s^m + \sigma V_s^w$. In the replicator dynamics the population shares are determined by the following differential equations: $\frac{d\beta}{dt} = \beta (V_b^w - A_b)$ and $\frac{d\sigma}{dt} = \sigma (V_s^w - A_s)$.

Definition 1 *An equilibrium (β, σ) is evolutionarily stable if there exists a neighborhood of (β, σ) , where the replicator dynamics converges to the equilibrium.*

Since there are only two possible choices for the agents, the analysis of the replicator dynamics can be easily performed graphically (see e.g. Lu and McAfee, 1996). We get one of the the main results of this paper:

Theorem 1 *If the trading mechanism is auction, then i) the market where both buyers and sellers use mixed strategies over the wait and move decisions is not evolutionarily stable, and ii) a market with one-sided coordination problem is evolutionarily stable.*

Proof. It is enough to locate the buyers' and sellers' equilibrium curves and determine the entering agents' behavior off the equilibrium curves, where the shares adjust according to the replicator dynamics. When the ratios of agents in the two markets are such that (β, σ) is above the BE-curve, the entering buyers go to, or prefer, the market where sellers wait. This is almost self-evident since on the BE-curve the buyers are indifferent between the markets. Above the BE-curve there are relatively lot of sellers in the market

where they wait, and this makes the market preferable to the buyers. By a similar logic, when the ratios of agents in the two markets are such that (β, σ) is above the SE curve, the entering sellers go to, or prefer, the market where the buyers wait. Of course, below the curves the agents' preferences are just opposite. With this knowledge we can draw the arrows that depict the direction of the dynamics in Figure 8. ■

Let us finally see how the intersection of the BE and the SE depends on θ :

Theorem 2 *In case of auction, the basin of attraction for the market where sellers wait grows when the ratio of buyers to sellers grows.*

Proof. We see from the proof of Proposition 1 that $\frac{d\beta}{d\theta} > 0$ and $\frac{d\sigma}{d\theta} < 0$. This means that as θ grows, the intersection of BE and SE goes towards the point $(1, 0)$, and the basin of attraction for the market where the sellers wait and the buyers move grows. In this sense the larger θ is, the more likely it is that the equilibrium market structure is at point $(0, 1)$. ■

This result is in accordance with the results where buyers and sellers may choose to move or wait but where there is still only one-sided coordination problem in any market, i.e. there are no empty locations but those who move always meet someone. See for example Kultti, Miettunen, Takalo, and Virrankoski (2004).

6.2 Stability of Two-Sided Search in Case of Bargaining

The analysis goes analogously to the case of auction. Equating W_b^w and W_b^m from (19) and (22) gives the buyers' equilibrium, BE, where

$$\frac{\beta}{\varphi} - \frac{\gamma}{\sigma} + \frac{2(1-\delta)}{\delta(1-e^{-\varphi})} - \frac{2(1-\delta)\gamma}{\delta\sigma(1-e^{-\gamma})} = 0, \quad (32)$$

and setting $W_s^m = W_s^w$ from (20) and (21) gives the sellers' equilibrium SE:

$$\frac{\sigma}{\gamma} - \frac{\varphi}{\beta} + \frac{2(1-\delta)}{\delta(1-e^{-\gamma})} - \frac{2(1-\delta)\varphi}{\delta\beta(1-e^{-\varphi})} = 0. \quad (33)$$

It can be shown that BE and SE are downward sloping, like in the case of auction. The behavior of BE and SE near points $(0, 1)$ and $(1, 0)$ is analyzed next.

1. $(\beta, \sigma) \rightarrow (0, \sigma)$ in which case $\gamma = \theta$ and BE becomes, after some manipulation, as

$$\frac{\sigma}{1 - e^{-\varphi}} = \frac{\delta\theta}{2(1 - \delta)} + \frac{\theta}{1 - e^{-\theta}}. \quad (34)$$

The right-hand side is constant, and the left-hand side approaches infinity when σ approaches unity, and the left-hand side approaches zero when σ approaches zero. The left-hand side is increasing in σ . Then we know that BE goes through point $(0, \sigma)$, where $\sigma \in (0, 1)$.

2. $(\beta, \sigma) \rightarrow (\beta, 1)$ in which case $\varphi = 0$, and BE becomes

$$-\beta = \frac{2(1 - \delta)}{\delta} \left[\frac{\varphi}{(1 - e^{-\varphi})} \right] \quad (35)$$

The limit of right-hand side of (34) when σ approaches one is, using L'Hospital's rule, equal to $\frac{2(1 - \delta)\theta}{\delta}$, and we conclude that BE does not go through $(\beta, 1)$ where $\beta \geq 0$.

3. $(\beta, \sigma) \rightarrow (\beta, 0)$ in which case $\varphi = \theta^{-1}$, and BE becomes

$$\gamma = \frac{-2(1 - \delta)\gamma}{\delta(1 - e^{-\gamma})} \quad (36)$$

which never holds. Thus, BE does not go through $(\beta, 0)$ where $\beta < 1$.

4. $(\beta, \sigma) \rightarrow (1, \sigma)$ in which case $\gamma = 0$, and BE becomes

$$-\frac{1}{\varphi} = \frac{2(1 - \delta)}{\delta} \left[\frac{1}{1 - e^{-\varphi}} - \frac{\gamma}{\sigma(1 - e^{-\gamma})} \right], \quad (37)$$

where $\lim_{\beta \rightarrow 1} \frac{\gamma}{1 - e^{-\gamma}} = 1$. When β approaches one, BE becomes

$$1 = \frac{2(1 - \delta)\varphi}{\delta} \left[\frac{1}{\sigma} - \frac{1}{1 - e^{-\varphi}} \right]. \quad (38)$$

The term in brackets is decreasing in σ , and it approaches infinity if σ approaches zero. We can conclude that BE goes through point $(1, \sigma)$ where $\sigma > 0$. The analysis for SE-curve goes analogously and it is not presented here. In sum, BE and SE are situated like in the case of auction, presented in Figure 8. Also, the arrows depicting out of steady state behavior are similar to the case of auction. We can state the following (the proofs are similar to proofs of Theorems 1 and 2, and they are omitted):

Theorem 3 *If the trading mechanism is bargaining, then i) the market where both buyers and sellers use mixed strategies over the wait and move decisions is not evolutionarily stable, and ii) a market with one-sided coordination problem is evolutionarily stable.*

Theorem 4 *In case of bargaining, the basin of attraction for the market where sellers wait grows when the ratio of buyers to sellers grows.*

Like in case of auction, the intersection of BE and SE moves toward point $(1, 0)$ when θ grows. We see that from Proposition 2 and its proof.

7 Conclusion

Traditionally, the directed search literature has simply *assumed* that either all sellers wait and all buyers search, or vice versa. This paper makes a step towards *explaining* who search and who wait. We show that one-sided search is an evolutionarily stable equilibrium in a model where both buyers and sellers have the opportunity to search or wait. We begin with two-sided search equilibrium which is shown to be unstable, and show that the market converges to either of the two stable one-sided search equilibria: i) all sellers wait and all buyers search, or ii) all buyers wait and all sellers search. The likelihood of these one-sided equilibria depends on the relative number of buyers and sellers: Increasing (decreasing) the number of buyers relative to sellers increases the probability that the market converges to an equilibrium where all sellers wait (search) and all buyers search (wait). This result holds whether we assume that the trading mechanism is auction or bargaining.

In addition to the above main result, we derive several results about efficiency: i) In two-sided search equilibrium, bargaining is more efficient than auction. The result originates from the assumed physical nature of search: if an agent leaves his location in order to search, the location becomes empty, and an agent of the opposite side who happens to choose this location will not trade. Bargaining internalizes this externality better than auction. ii) One-sided search is more efficient than two-sided search. This is a very intuitive result, because in two-sided search, searching agents can end up in an empty location. iii) In one-sided search it is more efficient if the members of the larger pool search and the members of the smaller pool wait, than vice versa.

Of course, auction and equal-split bargaining are not the only trading rules one can apply in this kind of model. One could use a mixture of auction and bargaining in the

following way: If the stayer meets just one mover, they split the resulting surplus in half. If more than one mover comes to the stayer, there is an auction, and each of the movers get their reservation values, and the stayer gets one minus a mover's reservation value. However, the results regarding stability are not likely to change, and the efficiency of mixture of auction and equal-split bargaining is likely to lie somewhere between the two.

It seems that in principle one could find a surplus-sharing rule that maximizes the number of trades, given the matching process. Such rules are presented by Hosios (1990) and Mortensen (1982). The 'Hosios rule' states that efficiency is obtained if and only if the surplus going to an agent equals his marginal contribution to matches. The Hosios rule is applicable only if the matching function has constant returns to scale. Mortensen's rule is applicable also in case of non-constant returns, but it is required that the initiator of a match (which, in the present model, would be a mover rather than the stayer) can be identified. The rule - in case of constant returns to scale matching function - says that efficiency is achieved if the initiator is allocated the whole surplus of the match. However, these rules are designed for pairwise meetings, and applying them in the present model is not straightforward since a stayer may meet many movers in one period. Recently, Julien, Kennes, and King (2002) have presented a similar kind of rule in a search model where there may be multiple meetings. However, the coordination problem in their model is one-sided, and they assume that it is the seller side (sellers of labor) that waits.

One could give up the assumption that search is physical in nature. Then agents could take and receive contacts at the same time. This seems to lead to a very difficult analysis, since one would have to deal with a vast net of contacts. While maintaining the physical search, one could introduce heterogeneity into the model: sellers could have different kinds of goods, or different amounts of them, or different buyers could value the same good differently. It may then well happen that a two-sided search equilibrium is stable.

8 Appendix

1. Derivation of equilibrium β and σ in case of auction (expressions (13) and (14)).

Solving $e^{-\varphi}$ from (12) and using the result in (11) gives $\sigma e^{-\gamma} = 1 - \frac{1}{\beta} (1 + \varphi) (1 - e^{-\gamma} - \gamma e^{-\gamma})$, and we obtain

$$e^{-\gamma} = \frac{1 - \frac{1}{\beta} - \frac{\varphi}{\beta}}{\sigma - \frac{1}{\beta} (1 + \gamma + \varphi + \gamma\varphi)}. \quad (\text{A1})$$

By a similar procedure we get

$$e^{-\varphi} = \frac{1 - \frac{1}{\sigma} - \frac{\gamma}{\sigma}}{\beta - \frac{1}{\sigma} (1 + \gamma + \varphi + \gamma\varphi)}, \quad (\text{A2})$$

and $\frac{e^\gamma}{e^\varphi} = \frac{\sigma - 1 - \gamma}{\beta - 1 - \varphi} = \theta$. Using $\theta e^\varphi = e^\gamma$ in (11) yields $\sigma = \theta (e^\varphi - 1 - \varphi) = \theta \left(e^\varphi - 1 - \frac{1 - \sigma}{\theta} \right) \Leftrightarrow e^\varphi = \frac{1 + \theta}{\theta} \Leftrightarrow \varphi = \ln \left(1 + \frac{1}{\theta} \right) \Leftrightarrow \sigma = 1 - \theta \ln \left(1 + \frac{1}{\theta} \right)$. Using $\theta e^\varphi = e^\gamma$ we have $e^\gamma = 1 + \theta \Leftrightarrow \beta = 1 - \frac{1}{\theta} \ln (1 + \theta)$.

2. Derivation of moving buyer's trading probability in case of bargaining

In equation (16), the arrival of sellers in a buyer's location is determined by Poisson parameter φ . There are k other buyers with probability $\frac{\varphi^k e^{-\varphi}}{k!}$, and the buyer trades with probability $\sum_{k=0}^{\infty} e^{-\varphi} \frac{\varphi^k}{k!} \frac{1}{k+1} = \frac{1 - e^{-\varphi}}{\varphi}$.

3. Proof of Proposition 5

Expression $S(1 - e^{-\theta}) - B(1 - e^{-1/\theta})$ has the same sign as expression $1 - e^{-\theta} - \theta + \theta e^{-1/\theta}$ which equals zero if $\theta = 0$ or if $\theta = 1$. Differentiating yields

$$\partial (1 - e^{-\theta} - \theta + \theta e^{-1/\theta}) / \partial \theta = e^{-\theta} + (1 + 1/\theta) e^{-1/\theta} - 1 \text{ which is positive if } \theta = 1.$$

Next we show that equation

$$1 - e^{-\theta} - \theta + \theta e^{-1/\theta} = 0 \quad (\text{A3})$$

has exactly one strictly positive solution, $\theta = 1$. Let us study function $g(h) = e^h - 1 - he^h + he^{h-1/h}$, which has the same zeroes as the left-hand side of (A3).

i) Here it is shown that equation $1 - e^{-\theta} - \theta + \theta e^{-1/\theta} = 0$ has no solution in interval $\theta \in (0, 1)$. We immediately see that $g(0) = g(1) = 0$. The derivative of g is

$$g'(h) = -he^h + e^{h-1/h} \left(h + 1 + \frac{1}{h} \right). \quad (\text{A4})$$

We can see that $g'(1) = 3 - e > 0$. In the right-hand side of (A4) we have $\lim_{h \rightarrow 0} \left(\frac{e^{h-1/h}}{h} \right) = \lim_{h \rightarrow 0} \left(\frac{e^{\frac{h^2-1}{h}}}{h} \right)$, and using L'Hôpital's rule, it is equal to $\lim_{h \rightarrow 0} \left(e^{\frac{h^2-1}{h}} \left(\frac{h^2+1}{h^2} \right) \right) = \lim_{h \rightarrow 0} \left(e^{h-\frac{1}{h}} \right) = 0$. We see that $\lim_{h \rightarrow 0} g'(h) = 0$. The second derivative of g is

$$g''(h) = -e^h - he^h + he^{h-1/h} \left(2 + h + \frac{2}{h} + \frac{1}{h^3} \right). \quad (\text{A5})$$

Using L'Hôpital's rule we get $\lim_{h \rightarrow 0} g''(h) = -1$. Thus, at first $g(h)$ is decreasing. Next we show that $g(h) \neq 0$ in interval $h \in (0, 1)$. If $g(h) = 0$ in interval $h \in (0, 1)$ and if g attained strictly positive values, there should be at least two zeroes. Before the last zero g would reach a maximum and its derivative would be zero. Let us denote the last maximum of g (where it is positive) by k . Thus we know that $g'(k) = -ke^k + e^{k-1/k} \left(k + 1 + \frac{1}{k} \right) = 0$ and $g(k) = e^k - 1 - ke^k + ke^{k-1/k} > 0$. From these conditions we get

$$g(k) = e^k - 1 - ke^k + \frac{k^3 e^k}{1 + k + k^2} > 0, \quad (\text{A6})$$

which holds if and only if $e^k - 1 - k - k^2 > 0$. We have $e^h - 1 - h - h^2 = 0$ at $h = 0$ and $e^h - 1 - h - h^2 < 0$ at $h = 1$. It is easy to see that $e^h - 1 - h - h^2 < 0$ in interval $h \in (0, 1]$. Thus, the assumption that g is positive at $h \in (0, 1)$ leads to a contradiction. The result is that equation $1 - e^{-\theta} - \theta + \theta e^{-1/\theta} = 0$ has no solution in interval $\theta \in (0, 1)$.

ii) To show that equation $1 - e^{-\theta} - \theta + \theta e^{-1/\theta} = 0$ has no solution at $\theta > 1$ it is enough to show that $g'(h) = -he^h + e^{h-1/h} \left(h + 1 + \frac{1}{h} \right) > 0$ when $h > 1$, because $g(1) = 0$ and $g'(h) > 0$ at $h = 1$. The sign of $g'(h)$ is positive if and only if $v(h) \equiv 1 - e^{-1/h} - \frac{1}{h}e^{-1/h} - \frac{1}{h^2}e^{-1/h} < 0$. We see that $v(1) < 0$ and $\lim_{h \rightarrow \infty} v(h) = 0$. Further, $v'(h) = 1 + e^{-1/h} \frac{1}{x^3} \left(1 - \frac{1}{x} \right)$, which is positive if $h > 1$. That is, $v(h) < 0$ if $h > 1$, thus $g'(h) = -he^h + e^{h-1/h} \left(h + 1 + \frac{1}{h} \right) > 0$ when $h > 1$.

We have shown that equation $1 - e^{-\theta} - \theta + \theta e^{-1/\theta} = 0$ has exactly two solutions, $\theta = 0$ and $\theta = 1$. If $\theta \in (0, 1)$, then $1 - e^{-\theta} - \theta + \theta e^{-1/\theta} < 0$, and if $\theta > 1$, then $1 - e^{-\theta} - \theta + \theta e^{-1/\theta} > 0$, resulting in $S(1 - e^{-\theta}) < B(1 - e^{-1/\theta})$ if $\theta \in (0, 1)$, and $S(1 - e^{-\theta}) > B(1 - e^{-1/\theta})$ if $\theta > 1$. ■

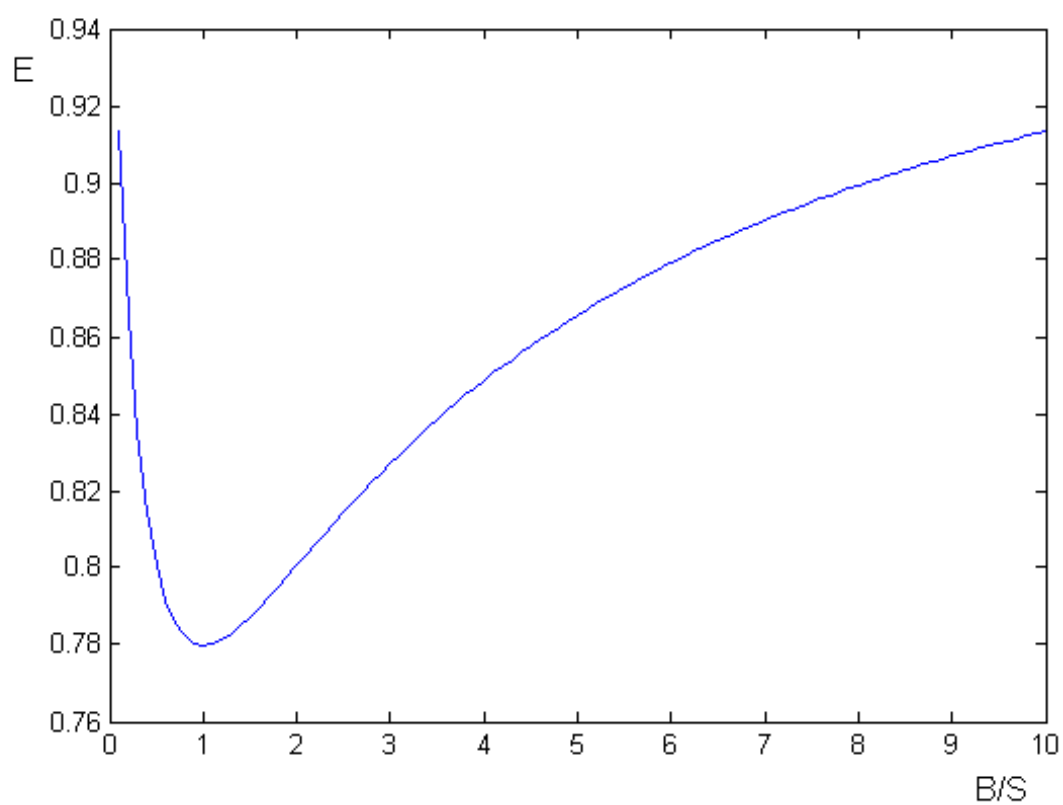


Figure 1: Number of matches in two-sided search with auction / Number of matches in two-sided search with bargaining

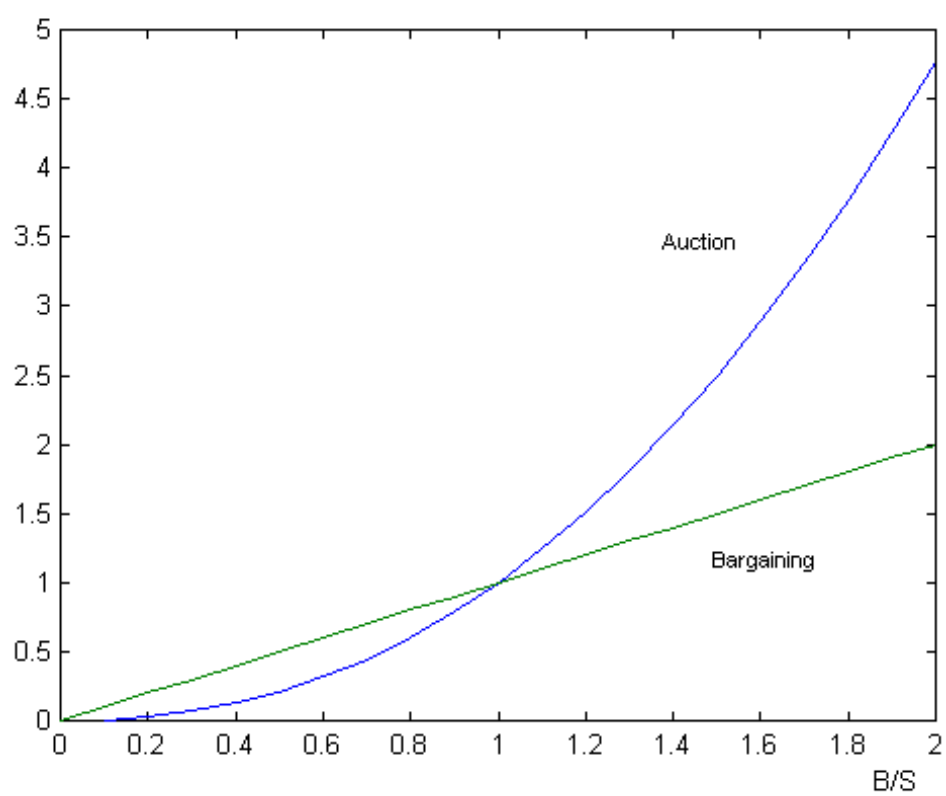


Figure 2: Utility of seller / utility of buyer in two-sided search, in case of auction and bargaining

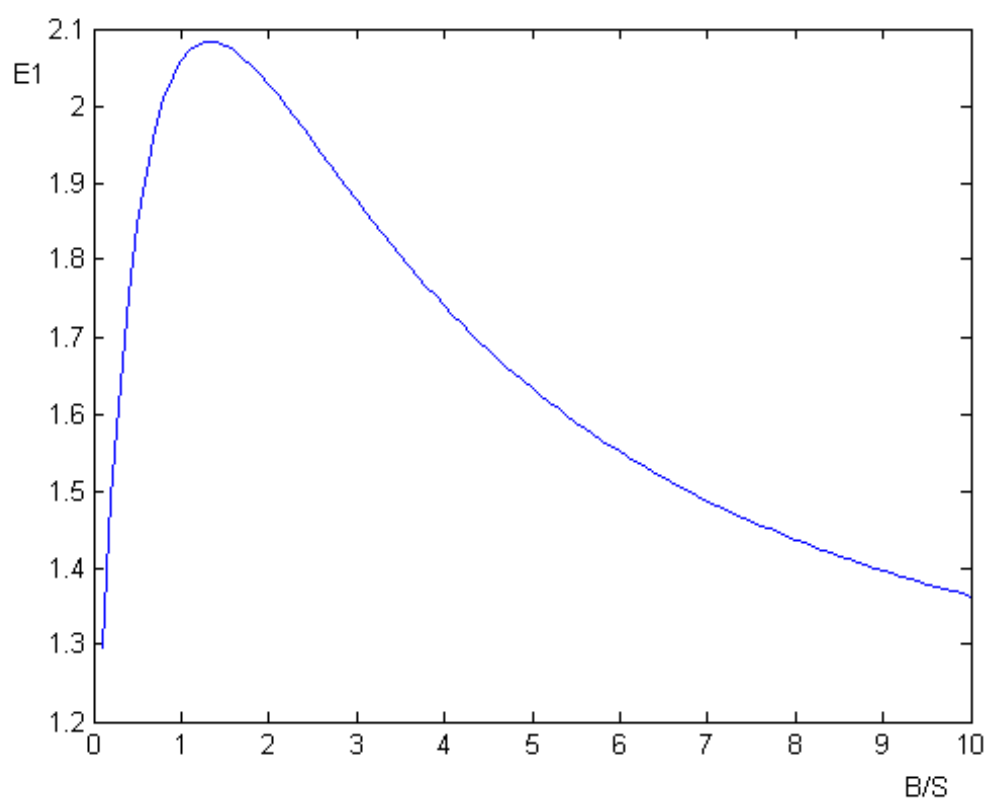


Figure 3: Number of matches when one-sided search and all sellers wait / Number of matches when two-sided search and auction

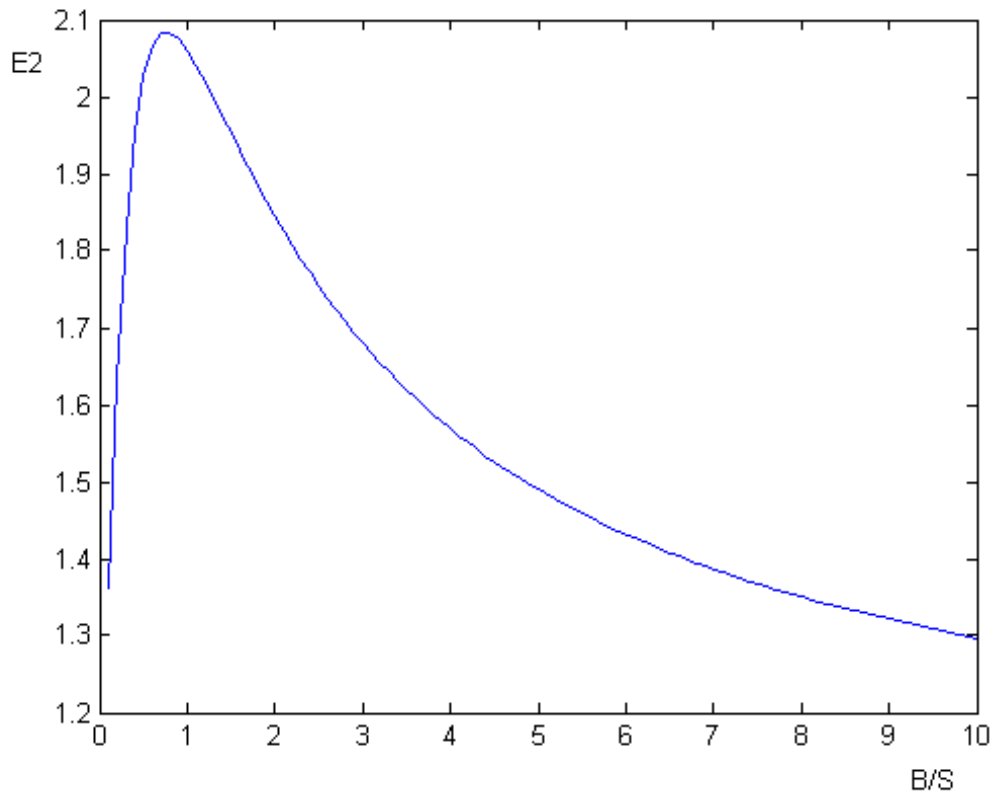


Figure 4: Number of matches when one-sided search and all buyers wait / Number of matches when two-sided search and auction

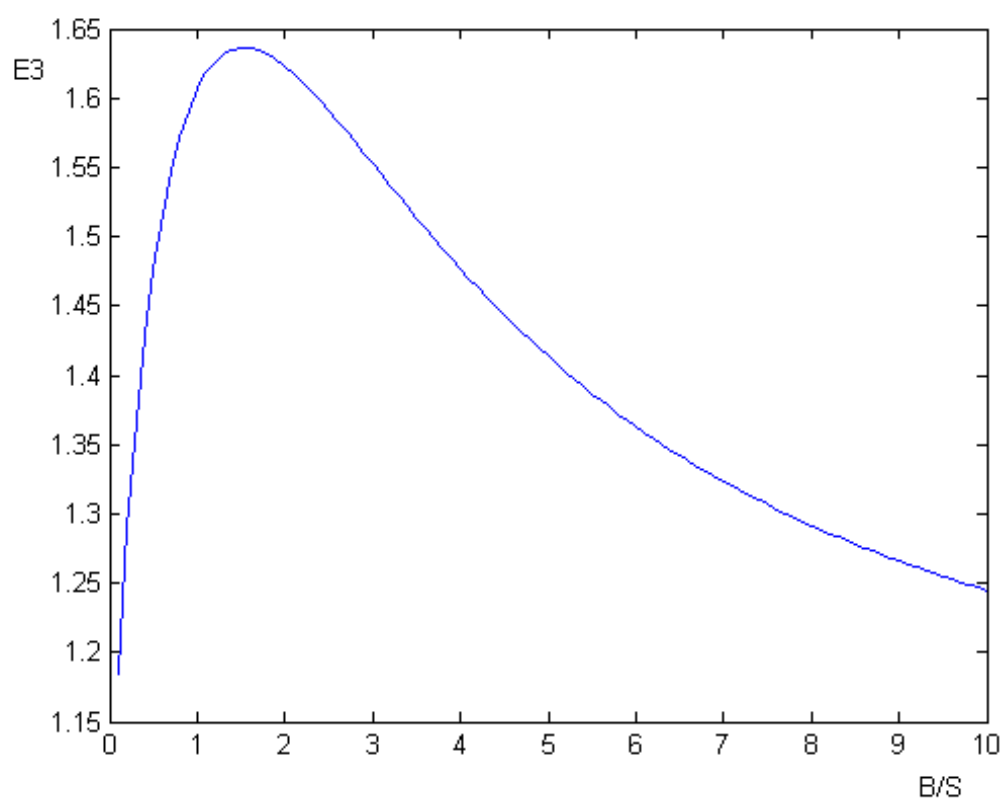


Figure 5: Number of matches when one-sided search and all sellers wait / Number of matches when two-sided search and bargaining

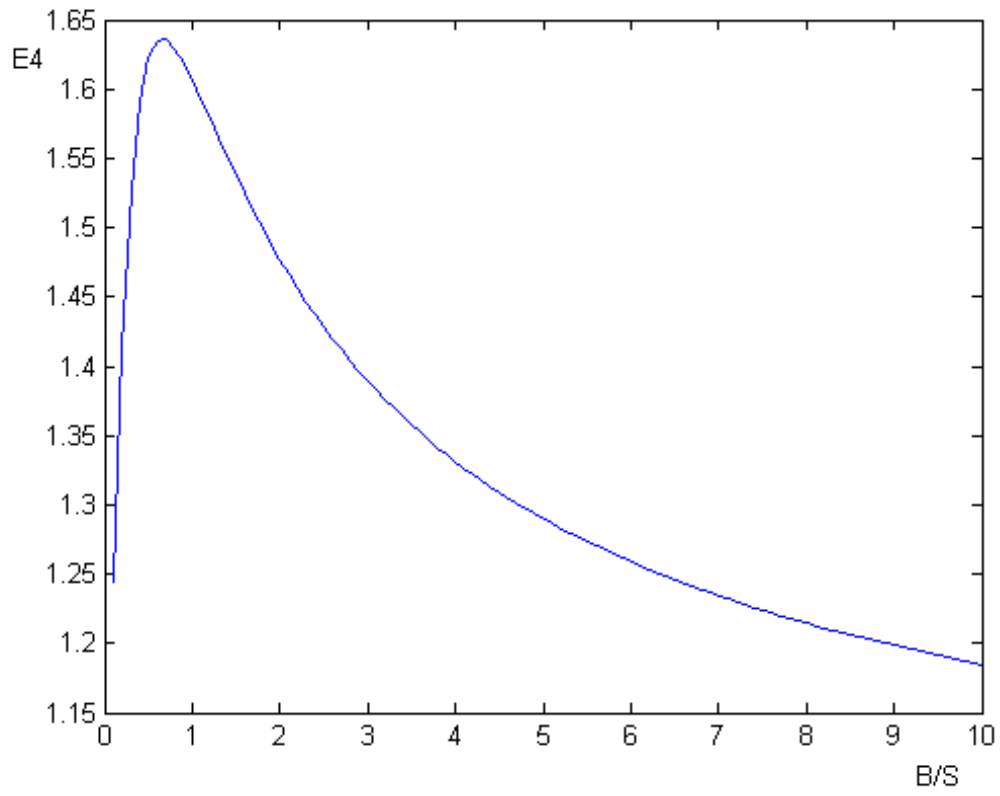


Figure 6: Number of matches when one-sided search and all buyers wait / Number of matches when two-sided search and bargaining

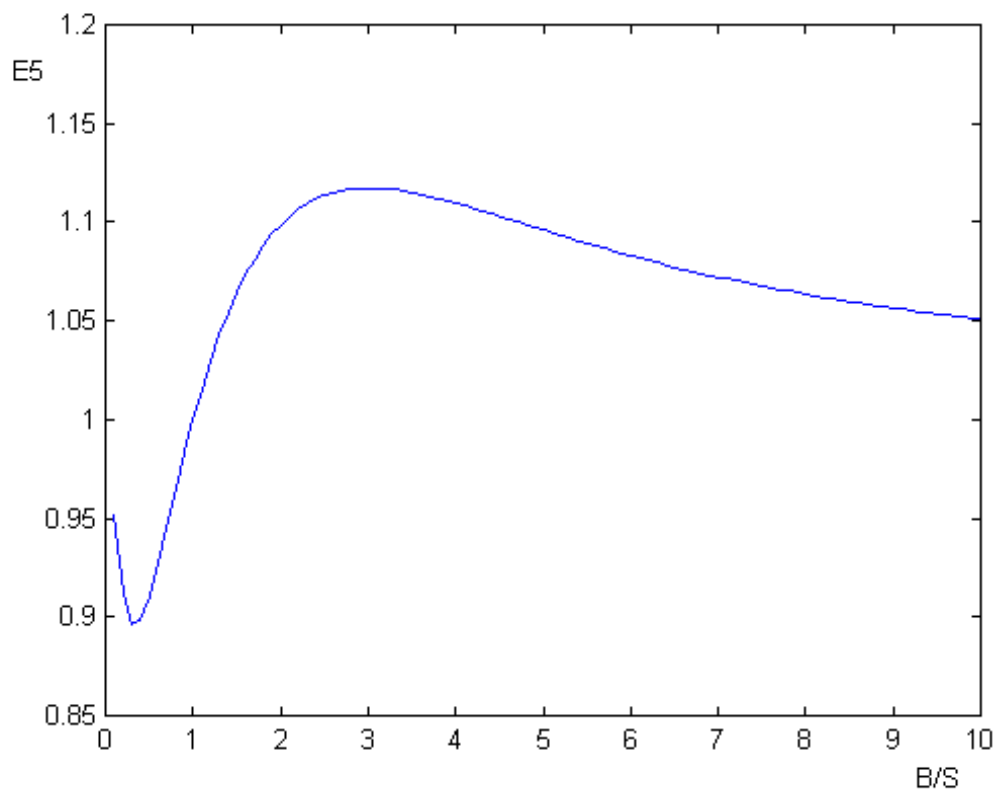


Figure 7: Number of matches when one-sided search and all buyers search / Number of matches when one-sided search and all sellers search

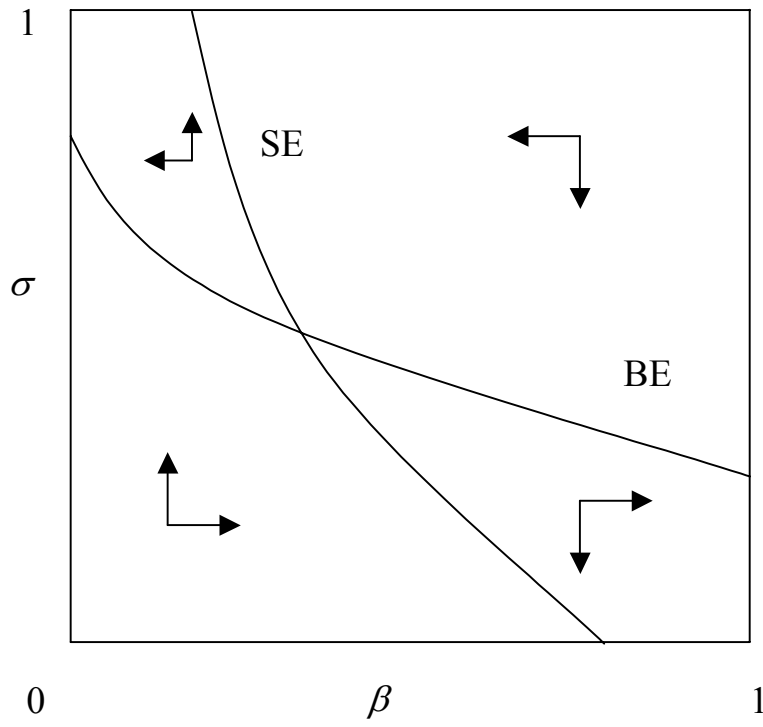


Figure 8: Two-sided search is evolutionarily unstable

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Price Distribution in a Symmetric Economy

Klaus Kultti*

Juha Virrankoski†

*University of Helsinki, kkultti@valt.helsinki.fi

†Helsinki School of Economics, virranko@hkkk.fi

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Price Distribution in a Symmetric Economy

Klaus Kultti and Juha Virrankoski

Abstract

We consider an economy with symmetric buyers and symmetric sellers. The sellers are in locations and post prices simultaneously. The buyers observe the prices, and each buyer visits one location. The buyers act independently and employ symmetric mixed strategies. We show that when there are several sellers in a location, the Nash equilibrium features price dispersion, i.e. the sellers post different prices. The equilibrium strategy of the sellers is a non-atomic distribution.

KEYWORDS: price distribution, search

1 Introduction

The problem of price distribution has a long history in economic theory. The basic problem stems from the observation that seemingly similar goods are sold for seemingly different prices, which violates the law of one price. This finding is noted for example in Pratt, Wise and Zeckhauser (1979) and in Froot, Kim and Rogoff (2001). In a perfect market this cannot be an equilibrium, and consequently the profession has proceeded to introduce models with less than perfect markets to address the phenomenon. In sum, “To generate a price distribution, there must be some heterogeneity among sellers and/or buyers”, (McMillan and Rothschild, 1994, 912).

We present a model with ex ante totally symmetric agents and publicly and costlessly known informative prices, that still features a price distribution in equilibrium. Actually, there are no equilibria in which a price distribution does not exist. The idea is, not too surprisingly, simple and it is based on a model by Kultti (2003). In that model the equilibrium market structure is determined; the sellers can group themselves as they want, and the two main results of the article are the following: First, the Walrasian market is not an equilibrium. Secondly, depending on the ratio of buyers to sellers there might be any number of sellers in a location. The results hint that it is worthwhile to study situations where there are two or more sellers in the same locality or very close to each other, so that a buyer can visit several sellers within a period.

The driving forces that generate a price distribution in the present model are (i) sellers’ capacity constraint, and (ii) assumption of more than one seller in a location (we consider a special case of two sellers in a location). Observed prices direct the buyers to the locations. Buyers are assumed to employ independent mixed strategies regarding the location they visit. The lower is the average price in a location, the more buyers choose that location. The sellers can thus affect the arrival of buyers, but the realisation of the number of buyers in a location is still random. The seller with the lowest price in the location sells whenever at least one buyer shows up, the seller with the second-lowest price sells whenever at least two buyers come, and so on. The capacity constraint creates monopoly power for the sellers if at least two buyers come to a location. On the other

hand, only one buyer appearing leads to Bertrand competition between the sellers. These two forces pull the price in opposite directions. We show that a uniform price is not an equilibrium, and that a pure-strategy equilibrium does not exist.

Up to date, the models resulting in price distribution have assumed that information on prices is imperfect, or that the agents are otherwise heterogeneous *ex ante*. In models of the former type, buyers, for example, get to know the price charged by any individual seller only by visiting the sellers sequentially (Diamond 1971, Salop and Stiglitz 1982). In these models *ex ante* identical buyers know the price distribution, but they do not know *ex ante* the price charged by any individual seller. Alternatively, buyers can sample price information before visiting a seller. In the model of Butters (1977), sellers choose the number of costly advertisements to be sent to randomly selected buyers. In Burdett and Judd (1983), buyers choose how much to invest in getting price quotations. A similar type of model is presented in Kandel and Simhon (2002). In these three models, the buyers know the price quotations of some sellers, thus the buyers are heterogeneous in their information prior to meeting a seller.

A price distribution can be generated also by assuming *ex ante* heterogeneous agents. Consumers may differ in their willingness to pay (Diamond 1987), or firms differ in their costs of production, and consumers differ in their search costs (Carlson and McAfee 1983). In these two models, buyers select the sellers randomly as they do in the models of Diamond, and Salop and Stiglitz, mentioned above. This imperfection is absent in the wage-posting model of Montgomery (1991). Firms who want to hire labor post wage offers, without cost, and job seekers observe all the offers, without cost. Job seekers then choose which firm to visit on the basis of observed wage offers. Wage distribution results if firms value the vacancies differently. Montgomery's model has a typical property of "directed search", namely, that buyers are directed to sellers by the latter's price quotations.

In discrete-time random matching models it is easy to get an *ex post* price distribution if one thinks that trades are consummated in auctions; the price will differ depending on how many buyers come to a seller (Lu and McAfee 1996). But here it is a little misleading to call the outcomes prices. They are rather terms of trade. To talk about

price distribution one should, in our opinion, have meaningful prices, i.e. terms of trade that the buyers can observe in advance and base their decision to visit any particular seller on this information.

The rest of the present paper is organised as follows. In section 2 we present the model. In section 3 we show that a pure-strategy equilibrium does not exist. In section 4 we prove the existence of a symmetric mixed-strategy equilibrium when the number of agents is finite, and we characterise the equilibrium. Section 5 concludes.

2 The Model

The basic set-up is a static model of directed search with buyers and sellers. The sellers are in so called locations which can be interpreted as physical sites, like stores, or booths in a market place. There can be several sellers in a location, and we study a model where there are two sellers in each location. There are B risk-neutral buyers with a unit demand and S risk-neutral sellers with a unit supply of an identical and indivisible good. A seller values the object at zero, and a buyer values it at one.

The agents play a three-stage game where in the first stage sellers post prices that are publicly observed. In the second stage the buyers choose a symmetric mixed strategy that tells the probability of visiting any particular location, and the buyers go to locations. In the last stage trade occurs. If in a particular location there is one buyer, he purchases from the low-price seller, and if both sellers charge the same price, one seller is chosen randomly. If there are two or more buyers, one of them is chosen randomly to buy the low-price good, and one buyer is chosen randomly to buy the high-price good.

If all the locations are similar, the equilibrium behaviour of buyers looks like they were randomly allocated on the locations. Consequently, the number of buyers arriving in a particular location would be a binomial random variable with parameters $(B, 2/S)$. Binomials are cumbersome to deal with, so we assume that the number of buyers and sellers is large, in which case the binomial distribution can be approximated with a Poisson-distribution with rate $2B/S$.¹

¹We focus on the case of two sellers in a location, but most of the results and definitions extend in a

3 No Pure-Strategy Equilibrium Exists

The heuristics of the non-existence of a pure-strategy equilibrium is as follows: The sellers in a certain location are in a Bertrand competition situation if only one buyer comes to that location, and thus they do not post equal prices in equilibrium. This is shown in lemma 1 and its proof. Instead of choosing a price, a seller can be thought to choose the expected number of buyers arriving to the location, given the other seller's price and the buyers' expected utility. The objective function of a low-price seller is different from that of a high-price seller, leading to different choices. Thus, an asymmetric pure-strategy equilibrium does exist. This is shown in proposition 1 and its proof.

Let us first present two definitions:

Definition 1 *An equilibrium is a pair of density functions $(p_{i1}, p_{i2})_{i=1}^{S/2}$ for each location where p_{ij} is seller j 's density function in location i , and a mixed strategy $(\pi_h)_{h=1}^{S/2}$ for each buyer such that each seller's strategy is an optimal response to other sellers' strategies and the buyers' strategies, and each buyer's mixed strategy is the optimal response to the sellers' prices and other buyers' strategies.²*

Definition 2 *A symmetric pure-strategy equilibrium is such that each seller's density function is an identical mass point, and all the buyers go to each location with the same probability.*

The following result applies to any number of sellers in a location:

Lemma 1 *In a pure-strategy equilibrium no two sellers in the same location quote equal prices*

Proof. Assume that exactly two prices are equal, say $p_1 = p_2 > 0$, and assume that this is the k th lowest price in the location. If $k - 1$ or fewer buyers appear, neither of the sellers quoting the k th lowest price will trade. Assume that the probability that exactly k buyers appear is χ . In this case either seller trades with probability $\chi/2$. Lowering

straightforward way to the n -seller case.

²This definition extends to a case of n sellers per location.

the price by any amount means that the seller with the lower price trades in this case with probability χ , which is clearly a profitable deviation. The case where more than two prices are equal goes analogously. To complete the proof we must show that in equilibrium the sellers do not quote zero prices. But if this were the case a seller could increase his price a little and still trade with positive probability since there would be buyers coming to the location. This is because the buyers' mixed strategy is continuous in prices³. ■

With two sellers per location the Poisson parameter is $\theta = \frac{B}{S/2} = \frac{2B}{S}$ when all the low-price sellers in all locations post the same price and all the high-price sellers in all locations post the same price, i.e. all the locations are similar. Let us assume that there exists a pure-strategy equilibrium where one seller in each location posts price p_1 and the other seller posts p_2 , where $p_1 < p_2$. The sellers' expected utilities are determined by

$$V_s(p_1) = (1 - e^{-\theta})p_1, \quad (1)$$

$$V_s(p_2) = (1 - e^{-\theta} - \theta e^{-\theta})p_2, \quad (2)$$

where $1 - e^{-\theta}$ is the probability that at least one buyer comes to the location, guaranteeing that the low-price seller sells his object. The high-price seller trades if and only if at least two buyers appear; this happens with probability $1 - e^{-\theta} - \theta e^{-\theta}$.

Lemma 2 *In any pure-strategy equilibrium where sellers in some location quote prices p_1 and p_2 ($p_1 < p_2$), we have $V_s(p_2) > V_s(p_1)$.*

Proof. First, denote $\theta \equiv \theta(p_1, p_2)$ and $\theta_\epsilon \equiv \theta(p_1 - \epsilon, p_1)$, where $\epsilon > 0$. Note that $\theta_\epsilon > \theta$, because each buyer chooses a location where prices are $p_1 - \epsilon$ and p_1 with a higher probability than a location where prices are p_1 and p_2 . Suppose that $V_s(p_2) \leq V_s(p_1)$.

³Consider two locations such that in one location the prices are p_1 and p_2 , $p_1 < p_2$, and in the other location the prices are q_1 and q_2 , $q_1 < q_2$. The corresponding Poisson parameters are γ and β . The buyers' mixed strategy which determines γ and β is given by condition $\frac{1-e^{-\gamma}}{\gamma}(1-p_1) + \frac{1-e^{-\gamma}-\gamma e^{-\gamma}}{\gamma}(1-p_2) = \frac{1-e^{-\beta}}{\beta}(1-q_1) + \frac{1-e^{-\beta}-\beta e^{-\beta}}{\beta}(1-q_2)$. That is, a buyer's utility from choosing a (p_1, p_2) -location equals his utility from choosing a (q_1, q_2) -location (see expression 5). The values of γ and β are continuous functions of prices.

The seller who charges p_2 can now make a profitable deviation by charging $p_1 - \epsilon$ instead of p_2 . His utility would be equal to

$$V_s(p_1 - \epsilon) = (1 - e^{-\theta\epsilon}) (p_1 - \epsilon). \quad (3)$$

We want to show that $V_s(p_1 - \epsilon) > V_s(p_1)$ which is equivalent to

$$(1 - e^{-\theta\epsilon}) (p_1 - \epsilon) > (1 - e^{-\theta})p_1. \quad (4)$$

Notice that the Poisson-rate $\theta(p_1, p_1) \equiv \theta + \nu$, $\nu > 0$, where both sellers posting the same price p_1 satisfies $\theta_\epsilon > \theta(p_1, p_1) > \theta$. Thus, the left-hand side of (4) can be approximated down by $(1 - e^{-\theta-\nu})(p_1 - \epsilon)$, and it is enough to show that $(1 - e^{-\theta-\nu})(p_1 - \epsilon) > (1 - e^{-\theta})p_1$. But it is immediate that this holds for small enough ϵ because $1 - e^{-\theta-\nu} > 1 - e^{-\theta}$. Therefore $V_s(p_2) \leq V_s(p_1)$ cannot hold in equilibrium. ■

The buyers' utilities are determined by

$$\begin{aligned} V_b &= \left[e^{-\theta} + \theta e^{-\theta} \frac{1}{2} + \frac{\theta^2}{2!} e^{-\theta} \frac{1}{3} + \dots + \frac{\theta^k}{k!} e^{-\theta} \frac{1}{k+1} + \dots \right] (1 - p_1) \\ &\quad + \left[\theta e^{-\theta} \frac{1}{2} + \frac{\theta^2}{2!} e^{-\theta} \frac{1}{3} + \dots + \frac{\theta^k}{k!} e^{-\theta} \frac{1}{k+1} + \dots \right] (1 - p_2) \\ &= \frac{1 - e^{-\theta}}{\theta} (1 - p_1) + \frac{1 - e^{-\theta} - \theta e^{-\theta}}{\theta} (1 - p_2). \end{aligned} \quad (5)$$

A buyer pays p_1 for sure if no other buyers appear in the location; this happens with probability $e^{-\theta}$. Each remaining term in the series captures the probability that $k - 1$ other buyers appear, times the probability that the given buyer trades with the low-price seller. A buyer pays p_2 only if at least one other buyer comes to the location.

Proposition 1 *A pure-strategy equilibrium does not exist.*

Proof. ⁴Consider the decision of a seller, such that the seller takes the prices of all the other sellers and each buyer's utility V_b as given. Assuming an infinite number of

⁴The authors thank Robert Shimer for suggesting this proof. The result can also be proved by considering deviations by a small group (of non-zero measure) of agents, and then letting the size of the group approach zero. This method has the flavour of a single agent deviating as in the standard test for a Nash equilibrium. The proof is available from the authors upon request.

sellers means that a single seller's decision about the price he charges has no effect on V_b . Let seller 1 be the low-price seller charging p_1 , and let seller 2 be the high-price seller charging p_2 , where $p_1 < p_2$. Seller 1 chooses p_1 to maximise $V_s(p_1) = (1 - e^{-\theta})p_1$, taking p_2 as given. We proceed by using buyer's utility V_b as a constraint. We have $V_b = \frac{1-e^{-\theta}}{\theta}(1 - p_1) + \frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta}(1 - p_2)$, and solving p_1 from it and substituting into $V_s(p_1)$ gives that seller 1 maximises

$$V_s(p_1) = 1 - e^{-\theta} + (1 - e^{-\theta} - \theta e^{-\theta})(1 - p_2) - \theta V_b. \quad (6)$$

Seller 1 effectively chooses θ of his location by choosing p_1 . Maximising $V_s(p_1)$ with respect to θ , taking V_b as given, gives first-order condition

$$e^{-\theta} + \theta e^{-\theta}(1 - p_2) - V_b = 0. \quad (7)$$

Seller 2 maximises $V_s(p_2) = (1 - e^{-\theta} - \theta e^{-\theta})p_2$ by choosing p_2 , taking p_1 and V_b as given. Solving p_2 from the equation for V_b and substituting into $V_s(p_2)$ results that seller 2, by choosing p_2 , effectively chooses θ to maximise

$$V_s(p_2) = 1 - e^{-\theta} - \theta e^{-\theta} + (1 - e^{-\theta})(1 - p_1) - \theta V_b. \quad (8)$$

The first-order condition for seller 2 is

$$\theta e^{-\theta} + e^{-\theta}(1 - p_1) - V_b = 0. \quad (9)$$

Together, the first-order conditions result in $p_1 = \theta p_2$. Utilising this result in lemma 2 gives $V_s(p_2) > V_s(p_1) \Leftrightarrow (1 - e^{-\theta} - \theta e^{-\theta})p_2 > (1 - e^{-\theta})p_1 \Leftrightarrow (1 - e^{-\theta} - \theta e^{-\theta})p_2 > (1 - e^{-\theta})\theta p_2 \Leftrightarrow 1 - e^{-\theta} > \theta$. The last inequality does not hold for any value of θ . We conclude that a pure-strategy equilibrium does not exist. ■

The non-existence of a pure-strategy equilibrium holds also when the number of buyers and sellers is finite; just replace the Poisson distribution with a binomial distribution.

4 Existence of Equilibrium

We still have to show that a symmetric mixed-strategy equilibrium exists⁵. Since the number of buyers a seller meets is Poisson-distributed, there are actually an infinite number of buyers and sellers in the economy. Most of the time it does not matter whether one thinks that there are just very many, but a finite number, or infinitely many agents. However, we can prove the existence of a symmetric mixed-strategy equilibrium only in the finite case. That all the results above hold for the finite case is straightforward, since the only difference is that instead of a Poisson-distribution one has to consider a binomial distribution. The second part of this section shows that in the infinite case the support of a symmetric mixed strategy does not have a gap.

4.1 Existence of a Symmetric Mixed-Strategy Equilibrium when the Number of Buyers and Sellers is Finite

Consider a modification to the present model such that the number of buyers and sellers is finite. Assuming two sellers in a location requires that S is even. If the buyers chose locations randomly (not directed by price quotations), the number of buyers who come to a location would be a binomial random variable with parameters $(B, 2/S)$. Denoting μ_{ki} the probability that k buyers come to a location i , we have $\mu_{ki} = \binom{B}{k} \alpha_i^k (1 - \alpha_i)^{B-k}$, where $\alpha_i \equiv 2/S$ is the probability that a given buyer chooses location i . In case of price posting, buyers' choices between locations are affected by prices announced, and α_i becomes a function of prices, too: $\alpha_i = \alpha[2/S, (p_i^1, p_i^2), p_{-i}]$, where p_i^1 and p_i^2 are the prices in location i , and p_{-i} captures the prices in all the other locations. However, the functional form of $\alpha[2/S, (p_i^1, p_i^2), p_{-i}]$ is not needed for our purposes. It suffices to know that because buyers use a (symmetric) mixed strategy, $\alpha[2/S, (p_i^1, p_i^2), p_{-i}]$ is continuous in prices, and therefore μ_{ki} is also continuous in prices. The only possible cause of non-existence of a mixed-strategy equilibrium is that the sellers' utilities are discontinuous if $p_i^1 = p_i^2$: If either seller increases or lowers his price a little, the seller with the lower price

⁵It is not obvious to us that asymmetric mixed strategy equilibria do not exist. We can show that mixed strategy equilibria with different but overlapping supports cannot exist.

trades whenever one or more buyers appear, whereas the higher-price seller trades if and only if at least two buyers appear. This causes a discrete jump in both agent's utilities. If $p_i^1 \neq p_i^2$, the sellers' utilities are continuous in prices, because a change in price does not affect the sellers' trading probabilities once the number of buyers in the location is realised. A price change affects only μ_{ki} which is a continuous function of prices, because the buyers have a mixed strategy.

Dasgupta and Maskin (1986) present conditions for the existence of a symmetric mixed-strategy equilibrium when utilities are discontinuous. Applied to the present model, a symmetric, atomless mixed-strategy equilibrium of this price-posting game exists if all the agents' utilities are bounded and three other conditions are satisfied: First, the sum of payoffs of all buyers and sellers must be upper semi-continuous. Second, all agents' utilities must be weakly lower semi-continuous in prices. Third, the definition of weak lower semi-continuity must have a strict inequality at points $p_i^1 = p_i^2 = p_i$. Obviously, the individual utilities are bounded from below by zero and from above by one. Call the sellers in a location i as 1 and 2. Their utilities are $V_i^1(B, S, p_i^1, p_i^2; p_{-i})$ and $V_i^2(B, S, p_i^1, p_i^2; p_{-i})$. Let $V_j(B, S, p_1^1, p_1^2, \dots, p_{S/2}^1, p_{S/2}^2)$ be a buyer's utility. For short, we use $V_i^1(p_i^1, p_i^2)$, $V_i^2(p_i^1, p_i^2)$, and V_j . Denote by $\mu_{ki}(p_i^1, p_i^2)$ the probability that k buyers come to location i . Suppose that $p_i^1 = p_i^2$, such that $p_i^1 \in (0, 1)$. Let $\mu_k \equiv \mu_k(p_i^1, p_i^2)$, $\mu_{k,\epsilon-} \equiv \mu_k(p_i^1 - \epsilon, p_i^2)$, and $\mu_{k,\epsilon+} \equiv \mu_k(p_i^1 + \epsilon, p_i^2)$, where ϵ is a small positive number. Let $p_i^1 \rightarrow p_i-$ and $p_i^1 \rightarrow p_i+$ denote p_i^1 approaching p_i from below or above.

Lemma 3 *The sum of the sellers' and buyers' utilities, $\sum_{i=1}^{S/2} [V_i^1(p_i^1, p_i^2) + V_i^2(p_i^1, p_i^2)] + \sum_{j=1}^B V_j$ is upper semi-continuous for $\forall p_i \in (0, 1)$.*

Proof. See Appendix. ■

Lemma 4 *All utilities $V_i^1(p_i^1, p_i^2)$, $V_i^2(p_i^1, p_i^2)$, and V_j are weakly lower semi-continuous in p if $p \in (0, 1)$.*

Proof. See Appendix. ■

Lemma 5 *The definition of weak lower semi-continuity has a strict inequality at points $p_i^1 = p_i^2 = p_i$.*

Proof. See Appendix. ■

Proposition 2 *There exists a symmetric, atomless mixed-strategy equilibrium where $(p_i^1, p_i^2) \in (0, 1)$, if the number of buyers and sellers is finite.*

Proof. By boundedness of utilities and by lemmas 3, 4, and 5. ■

4.2 Properties of a Symmetric Mixed-Strategy Equilibrium when the Number of Buyers and Sellers is Infinite

Let us assume that there are infinitely many buyers and sellers, and that there exists a symmetric equilibrium where both sellers in each location use the same mixed strategy. We show that the sellers' strategy is a distribution function F with support $[a, A]$, where $a > 0$. When a seller quotes price $p_1 \in [a, A]$, his expected utility is equal to $\int_a^A p_1 P(p_1, p_2) dF(p_2)$ where $P(p_1, p_2)$ is the probability that the seller trades when his price is p_1 and the price of the other seller in the same location is p_2 . This probability is determined by the buyers' mixed strategy. If a buyer decides to visit a location where the quoted prices are p_1 and p_2 , $p_1 < p_2$, he expects to get utility $V_b = \frac{1-e^{-\theta}}{\theta}(1-p_1) + \frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta}(1-p_2)$, where θ is the Poisson-parameter governing the arrival of buyers. The value of θ depends on F , p_1 , p_2 , prices in all the other locations, B , and S , but these are suppressed in the notation. The lowest price that a seller quotes results in a trade with a probability less than unity: the probability of trading would be equal to one only if θ were infinitely large. This, however, is not consistent with buyers' equilibrium behaviour. Since quoting the lowest price must yield the seller expected utility V_s , we have $a > V_s$. A seller's utility V_s is equal to zero if he charges a price equal to zero or larger than one. Obviously, a price larger than one means no trading. A price of zero does not belong to the support of F either, because a seller can increase his price above zero while still having a positive trading probability, because the buyers' strategy is continuous in prices. Thus, we have $V_s > 0$, and we conclude that $a > 0$.

Proposition 3 *The sellers' equilibrium strategy has no mass points, i.e. the equilibrium strategy is non-atomic.*

Proof. If there were a mass point, the situation would be like in the pure strategy case, and one of the sellers could profit by decreasing his price a little. ■

Proposition 4 *The support of the equilibrium strategy has no gaps.*

Proof. See Appendix. ■

5 Conclusion

The empirical observation that seemingly similar goods are sold for seemingly different prices has been a theoretical puzzle to economists. We have shown that in equilibrium there is a non-degenerate price distribution even when all the buyers are identical, all the sellers are identical, all the goods are identical, and when posting and observing prices is costless. All previous studies where price distribution results in equilibrium assume some kind of ex ante heterogeneity either among the sellers or among the buyers, or imperfect information about prices quoted by individual sellers. We consider a discrete-time urn-ball -type search model where the sellers are in locations and post prices. Each buyer can visit one location, and the buyers base their decision to visit the locations on the prices observed. The buyers are assumed to act independently and to employ symmetric mixed strategies. We show that when there are several sellers in a location, the Nash equilibrium features price dispersion, i.e. the sellers post different prices. Assuming many sellers in a location is not just a trick to yield price distribution. The assumption can be defended by a result that when trades are consummated in auctions, the equilibrium market structure features many sellers per location (Kultti 2003). In the present model, the seller with the lowest price makes a transaction even if only one buyer arrives. The seller with the second-lowest price makes a transaction only if at least two buyers arrive, and so on. The non-degenerate price distribution results because the sellers have a capacity constraint, and because there are more than one seller in a location. The sellers compete both within a location and between locations by posting prices, but this does not lead to Bertrand competition, because the capacity constraint endows the sellers with some monopoly power. We show that a pure-strategy equilibrium does not exist, and that the

symmetric mixed-strategy equilibrium has no mass points nor gaps. The existence of a symmetric mixed-strategy equilibrium is proved when the number of buyers and sellers is finite. Unfortunately, we cannot solve the equilibrium mixed strategy explicitly nor prove its existence when there are infinitely many buyers and sellers. Also, the uniqueness of the equilibrium strategy remains an open question.

6 Colophon

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Klaus Kultti: Department of Economics, University of Helsinki, P.O. Box 54, 00014 University of Helsinki, Finland. Tel +358-9-191 24892, email kkultti@valt.helsinki.fi

Juha Virrankoski: Department of Economics, Helsinki School of Economics, P.O. Box 1210, 00101 Helsinki, Finland. Tel. +358-9-43138582, fax +358-9-43138738, email virranko@hkkk.fi.

7 Appendix

7.1 Proofs Related to Subsection 4.1

Proof of Lemma 3

First, V_j is continuous in prices, because the buyers use mixed strategies such that μ_{ki} is continuous in prices. Therefore $\sum_{j=1}^B V_j$ is upper semi-continuous in prices. Sellers 1 and 2 in location i post prices in interval $[p_i - \epsilon, p_i + \epsilon]$. If $V_i^1(p_i^1, p_i^2) + V_i^2(p_i^1, p_i^2)$ is upper semi-continuous when $p_i^1 = p_i^2$, then (i) $V_i^1(p_i^1, p_i^2) + V_i^2(p_i^1, p_i^2)$ is upper semi-continuous for all (p_i^1, p_i^2) , because the sellers' utilities are discontinuous if and only if $p_i^1 = p_i^2$, and (ii) $\sum_{i=1}^{S/2} [V_i^1(p_i^1, p_i^2) + V_i^2(p_i^1, p_i^2)]$ is upper semi-continuous. Let both sellers choose the

same price p_i , where $p_i \in (0, 1)$. Their utilities are

$$V_i^1(p_i^1, p_i^2) = V_i^2(p_i^1, p_i^2) = \mu_1 p_i / 2 + (1 - \mu_0 - \mu_1) p_i = (1 - \mu_0 - \mu_1 / 2) p_i. \quad (10)$$

If only one buyer arrives, a seller trades with probability one half; if two or more buyers appear, a seller trades for certain. The sum of the above utilities is

$$V_i^1(p_i^1, p_i^2) + V_i^2(p_i^1, p_i^2) = [2(1 - \mu_0) - \mu_1] p_i. \quad (11)$$

Let now seller 1 choose $p_i - \epsilon$, and let seller 2 choose p_i . Denote their utilities as $V_{\epsilon-}^1(p_i - \epsilon, p_i)$ and $V_{\epsilon-}^2(p_i - \epsilon, p_i)$, they are equal to

$$V_{\epsilon-}^1(p_i - \epsilon, p_i) = (1 - \mu_{0,\epsilon-})(p_i - \epsilon), \quad (12)$$

$$V_{\epsilon-}^2(p_i - \epsilon, p_i) = (1 - \mu_{0,\epsilon-} - \mu_{1,\epsilon-}) p_i \quad (13)$$

The difference between the sum of the payoffs is denoted by $\Delta \Sigma V_{\epsilon-}$, and

$$\begin{aligned} \Delta \Sigma V_{\epsilon-} &\equiv V_{\epsilon-}^1(p_i - \epsilon, p_i) + V_{\epsilon-}^2(p_i - \epsilon, p_i) - V_i^1(p_i^1, p_i^2) - V_i^2(p_i^1, p_i^2) \\ &= [2(\mu_0 - \mu_{0,\epsilon-}) + \mu_1 - \mu_{1,\epsilon-}] p_i - (1 - \mu_{0,\epsilon-}) \epsilon. \end{aligned} \quad (14)$$

When $\epsilon \rightarrow 0$, then $\mu_{0,\epsilon-} \rightarrow \mu_0$ and $\mu_{1,\epsilon-} \rightarrow \mu_1$ (because of buyers' mixed strategies), yielding $\Delta \Sigma V_{\epsilon-} \rightarrow 0$.

Suppose that seller 1 chooses $p_i + \epsilon$ and seller 2 chooses p_i . Their utilities are

$$V_{\epsilon+}^1(p_i + \epsilon, p_i) = (1 - \mu_{0,\epsilon+} - \mu_{1,\epsilon+})(p_i + \epsilon), \quad (15)$$

$$V_{\epsilon+}^2(p_i + \epsilon, p_i) = (1 - \mu_{0,\epsilon+}) p_i, \quad (16)$$

and

$$\begin{aligned} \Delta \Sigma V_{\epsilon+} &\equiv V_{\epsilon+}^1(p_i + \epsilon, p_i) + V_{\epsilon+}^2(p_i + \epsilon, p_i) - V_i^1(p_i^1, p_i^2) - V_i^2(p_i^1, p_i^2) \\ &= [2(1 - \mu_{0,\epsilon+}) - \mu_{1,\epsilon+} + 2(1 - \mu_0) + \mu_1] p_i + (1 - \mu_{0,\epsilon+} - \mu_{1,\epsilon+}) \epsilon \end{aligned} \quad (17)$$

which approaches zero as $\epsilon \rightarrow 0$ because $\mu_{k,\epsilon+} \rightarrow \mu_k$ as $\epsilon \rightarrow 0$, for $k = 0, 1$. Because $\Delta \Sigma V_{\epsilon-} \rightarrow 0$ and $\Delta \Sigma V_{\epsilon+} \rightarrow 0$, $V_i^1(p_i^1, p_i^2) + V_i^2(p_i^1, p_i^2)$ is continuous and therefore also upper semi-continuous for all $p \in (0, 1)$. Therefore, $\sum_{i=1}^{S/2} [V_i^1(p_i^1, p_i^2) + V_i^2(p_i^1, p_i^2)] + \sum_{j=1}^B V_j$ is upper semi-continuous in prices. ■

Proof of Lemma 4

A buyer's utility $V_j \left(B, S, p_1^1, p_1^2, \dots, p_{S/2}^1, p_{S/2}^2 \right)$ is continuous in prices (see lemma 3 and its proof). Assume that sellers 1 and 2 in location i post prices in interval $[p_i - \epsilon, p_i + \epsilon] \in (0, 1)$. As in lemma 3, we consider the case $p_i^1 = p_i^2 = p_i$, which is a discontinuity point in sellers' utilities. The limits of utilities of seller 1 when he chooses $p_i - \epsilon$ or $p_i + \epsilon$ are (look at (12) and (15) in the proof of lemma 3)

$$\lim_{p_i^1 \rightarrow p_i -} \inf V_i^1(p_i^1, p_i^2) = (1 - \mu_{0, \epsilon -}) (p_i - \epsilon), \quad (18)$$

$$\lim_{p_i^1 \rightarrow p_i +} \inf V_i^1(p_i^1, p_i^2) = (1 - \mu_{0, \epsilon +} - \mu_{1, \epsilon +}) (p_i + \epsilon). \quad (19)$$

Seller 1's utility $V_i^1(p_i^1, p_i^2)$ is weakly lower semi-continuous in prices, if for all p_i^1 , there exists $\lambda \in [0, 1]$ such that for all p_i^2

$$\lambda \lim_{p_i^1 \rightarrow p_i -} \inf V_i^1(p_i^1, p_i^2) + (1 - \lambda) \lim_{p_i^1 \rightarrow p_i +} \inf V_i^1(p_i^1, p_i^2) \geq V_i^1(p_i^1, p_i^2). \quad (20)$$

Obviously, this holds for all $p \in (0, 1)$ if $p_i^1 \neq p_i^2$. For $p_i^1 = p_i^2 = p_i$ we have

$$\begin{aligned} & \lambda \lim_{p_i^1 \rightarrow p_i -} \inf V_i^1(p_i^1, p_i^2) + (1 - \lambda) \lim_{p_i^1 \rightarrow p_i +} \inf V_i^1(p_i^1, p_i^2) \\ &= [(\lambda - 1) \mu_{1, \epsilon +} + 1 - \mu_{0, \epsilon +}] p_i + [(1 - \lambda) (1 - \mu_{0, \epsilon +} - \mu_{1, \epsilon +}) - \lambda (1 - \mu_{0, \epsilon -})] \epsilon \\ &= [1 - \mu_0 + (\lambda - 1) \mu_1] p_i \end{aligned} \quad (21)$$

because $\mu_{k, \epsilon -} \rightarrow \mu_k$ and $\mu_{k, \epsilon +} \rightarrow \mu_k$ for all k when $\epsilon \rightarrow 0$ (because of the buyers' mixed strategies). Remembering that $V_i^1(p_i^1, p_i^2) = (1 - \mu_0 - \mu_1/2) p_i$, we have that $V_i^1(p_i^1, p_i^2)$ is weakly lower semi-continuous in prices at p_i if there exists $\lambda \in [0, 1]$ such that $[1 - \mu_0 + (\lambda - 1) \mu_1] p_i \geq (1 - \mu_0 - \mu_1/2) p_i$. The inequality holds if $\lambda \in [1/2, 1]$. That is, $V_i^1(p_i^1, p_i^2)$ is weakly lower semi-continuous in prices. Similarly, $V_i^2(p_i^1, p_i^2)$ is weakly lower semi-continuous in prices. ■

Proof of Lemma 5

For the definition of weak lower semi-continuity to have a strict inequality at points $p_i^1 = p_i^2 = p_i$ requires that there exists $\lambda \in [0, 1]$ such that

$$\lambda \lim_{p_i^1 \rightarrow p_i -} \inf V_i^1(p_i^1, p_i^2) + (1 - \lambda) \lim_{p_i^1 \rightarrow p_i +} \inf V_i^1(p_i^1, p_i^2) > V_i^1(p_i^1, p_i^2), \quad (22)$$

$$\lambda \lim_{p_i^2 \rightarrow p_i -} \inf V_i^2(p_i^1, p_i^2) + (1 - \lambda) \lim_{p_i^2 \rightarrow p_i +} \inf V_i^2(p_i^1, p_i^2) > V_i^2(p_i^1, p_i^2), \quad (23)$$

for all $p_i \in (0, 1)$. This requires that there exists $\lambda \in [0, 1]$ such that $[1 - \mu_0 + (\lambda - 1)\mu_1]p_i > (1 - \mu_0 - \mu_1/2)p_i$, for all $p_i \in (0, 1)$. One easily sees that this condition is satisfied if $\lambda \in (1/2, 1]$. ■

7.2 Proof of Proposition 4

The idea of the proof is that if the support of the strategy has a gap, a seller can move some probability mass from a part of the support immediately above the gap to the upper bound of the gap, without fear of the other seller in the location undercutting his price. This new strategy leads to an improvement, and the original strategy cannot constitute an equilibrium.

Assume that the support of the equilibrium mixed strategy has a gap. Let us choose a gap (g, G) such that $[G, A]$ belongs to the support of the mixed strategy F . The ex-ante expected utility of the seller with distribution $F(p_1)$ is

$$\begin{aligned}
& \int_a^G \int_a^{p_1} p_1 (1 - e^{-\theta(p_1, p_2)} - \theta(p_1, p_2) e^{-\theta(p_1, p_2)}) dF(p_2) dF(p_1) + \\
& \int_a^G \int_{p_1}^A p_1 (1 - e^{-\theta(p_1, p_2)}) dF(p_2) dF(p_1) + \\
& \int_G^{G+\epsilon} \int_a^{p_1} p_1 (1 - e^{-\theta(p_1, p_2)} - \theta(p_1, p_2) e^{-\theta(p_1, p_2)}) dF(p_2) dF(p_1) + \\
& \int_G^{G+\epsilon} \int_{p_1}^A p_1 (1 - e^{-\theta(p_1, p_2)}) dF(p_2) dF(p_1) + \\
& \int_{G+\epsilon}^A \int_a^{p_1} p_1 (1 - e^{-\theta(p_1, p_2)} - \theta(p_1, p_2) e^{-\theta(p_1, p_2)}) dF(p_2) dF(p_1) + \\
& \int_{G+\epsilon}^A \int_{p_1}^A p_1 (1 - e^{-\theta(p_1, p_2)}) dF(p_2) dF(p_1),
\end{aligned} \tag{24}$$

which we denote by $I_1 + I_2 + I_3 + I_4 + I_5 + I_6$ with obvious correspondences. Consider another strategy which is otherwise similar to F but the seller moves mass $F(G+\epsilon) - F(G)$ to point G . Now the seller's expected utility is

$$\begin{aligned}
& I_1 + I_2 + [F(G+\epsilon) - F(G)] \int_a^G G (1 - e^{-\theta(G, p_2)} - \theta(G, p_2) e^{-\theta(G, p_2)}) dF(p_2) \\
& + [F(G+\epsilon) - F(G)] \int_G^A G (1 - e^{-\theta(G, p_2)}) dF(p_2) + I_5 + I_6.
\end{aligned} \tag{25}$$

We show that ϵ can be chosen so that (25) > (24) which is equivalent to

$$\begin{aligned}
 & \int_G^{G+\epsilon} \int_a^{p_1} p_1 (1 - e^{-\theta(p_1, p_2)} - \theta(p_1, p_2) e^{-\theta(p_1, p_2)}) dF(p_2) dF(p_1) + \\
 & \int_G^{G+\epsilon} \int_{p_1}^A p_1 (1 - e^{-\theta(p_1, p_2)}) dF(p_2) dF(p_1) \\
 & < [F(G + \epsilon) - F(G)] \int_a^G G (1 - e^{-\theta(G, p_2)} - \theta(G, p_2) e^{-\theta(G, p_2)}) dF(p_2) + \\
 & [F(G + \epsilon) - F(G)] \int_G^A G (1 - e^{-\theta(G, p_2)}) dF(p_2),
 \end{aligned} \tag{26}$$

which in turn is equivalent to

$$\begin{aligned}
 & \int_G^{G+\epsilon} \int_a^G p_1 (1 - e^{-\theta(p_1, p_2)} - \theta(p_1, p_2) e^{-\theta(p_1, p_2)}) dF(p_2) dF(p_1) + \\
 & \int_G^{G+\epsilon} \int_G^{p_1} p_1 (1 - e^{-\theta(p_1, p_2)} - \theta(p_1, p_2) e^{-\theta(p_1, p_2)}) dF(p_2) dF(p_1) + \\
 & \int_G^{G+\epsilon} \int_{p_1}^{G+\epsilon} p_1 (1 - e^{-\theta(p_1, p_2)}) dF(p_2) dF(p_1) + \\
 & \int_G^{G+\epsilon} \int_{G+\epsilon}^A p_1 (1 - e^{-\theta(p_1, p_2)}) dF(p_2) dF(p_1) \\
 & < [F(G + \epsilon) - F(G)] \int_a^G G (1 - e^{-\theta(G, p_2)} - \theta(G, p_2) e^{-\theta(G, p_2)}) dF(p_2) + \\
 & [F(G + \epsilon) - F(G)] \int_G^{G+\epsilon} G (1 - e^{-\theta(G, p_2)}) dF(p_2) + \\
 & [F(G + \epsilon) - F(G)] \int_{G+\epsilon}^A G (1 - e^{-\theta(G, p_2)}) dF(p_2),
 \end{aligned} \tag{27}$$

which we denote by $H_1 + H_2 + H_3 + H_4 < K_1 + K_2 + K_3$ with obvious correspondences. We immediately notice that by letting ϵ approach zero, we can make $|H_1 - K_1|$ and $|H_4 - K_3|$ as small as we please. We note that $\theta(p, q)$ is decreasing in both of its arguments and that both $1 - e^{-\theta}$ and $1 - e^{-\theta} - \theta e^{-\theta}$ are increasing in θ .

Let us approximate H_2 and H_3 upwards. Changing the order of integration in H_2

and replacing the first argument of θ by G we get

$$H_2 < \int_G^{G+\epsilon} \int_{p_2}^{G+\epsilon} p_1 \begin{pmatrix} 1 - e^{-\theta(G, p_2)} \\ -\theta(G, p_2) e^{-\theta(G, p_2)} \end{pmatrix} dF(p_1) dF(p_2) \quad (28)$$

$$< \int_G^{G+\epsilon} (G + \epsilon) \begin{pmatrix} 1 - e^{-\theta(G, p_2)} \\ -\theta(G, p_2) e^{-\theta(G, p_2)} \end{pmatrix} \int_{p_2}^{G+\epsilon} dF(p_1) dF(p_2) \quad (29)$$

$$= \int_G^{G+\epsilon} (G + \epsilon) (1 - e^{-\theta(G, p_2)} - \theta(G, p_2) e^{-\theta(G, p_2)}) \times [F(G + \epsilon) - F(p_2)] dF(p_2) \quad (30)$$

$$< (G + \epsilon) (1 - e^{-\theta(G, G)} - \theta(G, G) e^{-\theta(G, G)}) \times \left[\int_G^{G+\epsilon} F(G + \epsilon) F(p_2) - \int_G^{G+\epsilon} \frac{1}{2} F^2(p_2) \right] \quad (31)$$

$$= (G + \epsilon) (1 - e^{-\theta(G, G)} - \theta(G, G) e^{-\theta(G, G)}) \times \left[F^2(G + \epsilon) - F(G + \epsilon) F(G) - \frac{1}{2} F^2(G + \epsilon) + \frac{1}{2} F^2(G) \right]. \quad (32)$$

In a completely analogous way we can approximate H_3 upwards by

$$H_3 < (G + \epsilon) (1 - e^{-\theta(G, G)}) \times \left[\frac{1}{2} F^2(G + \epsilon) - \frac{1}{2} F^2(G) - F(G) F(G + \epsilon) + F^2(G) \right]. \quad (33)$$

Summing up (32) and the right-hand side of (33) yields the following upper bound for $H_2 + H_3$:

$$H_2 + H_3 < (G + \epsilon) \left(1 - e^{-\theta(G, G)} - \frac{1}{2} \theta(G, G) e^{-\theta(G, G)} \right) \times [F^2(G + \epsilon) - 2F(G) F(G + \epsilon) + F^2(G)]. \quad (34)$$

We show that this can be made less than K_2 . Formally,

$$\begin{aligned} & (G + \epsilon) \left(1 - e^{-\theta(G, G)} - \frac{1}{2} \theta(G, G) e^{-\theta(G, G)} \right) \times [F^2(G + \epsilon) - 2F(G) F(G + \epsilon) + F^2(G)] \\ & < [F(G + \epsilon) - F(G)] \int_G^{G+\epsilon} G (1 - e^{-\theta(G, p_2)}) dF(p_2) \end{aligned} \quad (35)$$

$\Longleftrightarrow$

$$\begin{aligned} & (G + \epsilon) [F(G + \epsilon) - F(G)] \left(1 - e^{-\theta(G, G)} - \frac{1}{2} \theta(G, G) e^{-\theta(G, G)} \right) \\ & < \int_G^{G+\epsilon} G (1 - e^{-\theta(G, p_2)}) dF(p_2). \end{aligned} \quad (36)$$

In (36) the right-hand side is greater than $G(1 - e^{-\theta(G, G+\epsilon)})[F(G+\epsilon) - F(G)]$. But

$$(G+\epsilon)[F(G+\epsilon) - F(G)] \left(1 - e^{-\theta(G, G)} - \frac{1}{2}\theta(G, G)e^{-\theta(G, G)}\right) \quad (37)$$

$$< G(1 - e^{-\theta(G, G+\epsilon)})[F(G+\epsilon) - F(G)]$$

$$\Longleftrightarrow$$

$$\epsilon(e^{-\theta(G, G)} - e^{-\theta(G, G+\epsilon)}) - (G+\epsilon)\frac{1}{2}\theta(G, G)e^{-\theta(G, G)} < 0. \quad (38)$$

Let us choose ϵ so small that both $|H_1 - K_1|$ and $|H_4 - K_3|$ are less than

$\frac{1}{2}(\frac{1}{2}\theta(G, G)e^{-\theta(G, G)})$. In the left-hand side of (38) we have approximated $H_2 + H_3 - K_2$ upwards, and since $\epsilon(e^{-\theta(G, G)} - e^{-\theta(G, G+\epsilon)}) < 0$, we have shown that the new strategy leads to an improvement. Thus, the support of the mixed-equilibrium strategy F cannot contain gaps. ■

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Frictions in Project-Based Supply of Permits*

MATTI LISKI¹ and JUHA VIRRANKOSKI²

¹*Department of Economics, Helsinki School of Economics, P.O. Box 1210, 00101 Helsinki, Finland (E-mail: matti.liski@hkkk.fi);* ²*Helsinki School of Economics, P.O. Box 1210, 00101 Helsinki, Finland (E-mail: virranko@hkkk.fi)*

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Abstract. Emissions trading in climate change can entail large overall cost savings and transfers between developed and developing countries. However, the search for acceptable JI or CDM projects implies a deviation from the perfect market framework used in previous estimations. Our model combines the search market for projects with a frictionless permit market to quantify the supply-side frictions in the CO₂ market. We also decompose the effects of frictions into the effects of search friction, bargaining, and bilateralism. A calibration using previous cost estimates of CO₂ reductions illustrate changes in cost savings and allocative implications.

Key words: climate change, matching, pollution permits

JEL classification: C61, H23, Q49

1. Introduction

The reconciliation of interests of developed (DC) and developing (LDC) countries is close to the heart of the climate change negotiations. LDC country representatives often find climate change policies unfair because these countries are not responsible for the existing stock of carbon dioxide (CO₂) and other greenhouse gases,¹ whereas DC countries often prefer to avoid emissions reductions at home because such reductions are considerably less costly in LDC countries. An obvious solution to the conflict is a treaty which allows DC-country polluters to finance abatement projects in LDC countries and use the created reductions against their higher emissions at home. A large number of studies have estimated the overall cost-saving potential and allocative implications of the above trading using a perfect market framework.² However, if the LDC-country project hosts are unable to undertake some or even majority of the profitable projects, e.g., due to credit constraints, there will be a supply-side distortion in the permit market, implying a deviation from the frictionless ideal and thereby potentially a large change in the estimated implications of trading. While many experts have noted the deviation (e.g. Barrett 1999; Hahn and Stavins 1999; Scmalensee 1998), it has not been framed in any systematic way. To this end, we use a simple search model of CO₂-abatement projects to decompose the effects of the deviation on losses and

allocations into the effects of (i) search friction, (ii) bargaining and (iii) bilateralism. A calibration based on previous cost estimates of reducing CO₂ emissions in the European Union (EU) and Eastern European (EE) countries is used to gauge the decomposition and the quantitative magnitudes of the resulting losses.

The market for CO₂-abatement projects can be viewed as a search market for three main reasons. First, CO₂ reductions are difficult to verify, and they cannot be approved without project-by-project estimation of baseline and post-investment emissions. The ease of verification and approval depends on a number of project-specific factors which are learned in negotiations. Second, financing partners are typically energy producers and traders experienced with the technologies they are already using at home, whereas a hosting partner typically offers abatement opportunities which are best materialized using a particular technology.³ Thus, a provider with a given technology needs to search for a project that best matches the technology.⁴ Third, some hosts in LDC countries are likely to have low creditworthiness with respect to their projects which may necessitate contracting with DC country partners. A contract with a solid DC country firm may restore the creditworthiness of the project because the firm is experienced with the technology, or because reductions become more easily acceptable to third-party verifiers or to regulators.⁵

The above reasons for search apply to projects generating reductions in LDC countries but not to trading with reductions once the reductions have been approved. Thus, frictions potentially characterize the project market but not the permit market where, for example, “Emissions Reduction Units” (ERUs) for Joint Implementation (JI) projects and “Certified Emissions Reductions” (CERs) for Clean Development Mechanism (CDM) Projects are traded.^{6,7} Our objective is to link the search market for these projects to the overall permit market and analyze what determines the supply-side friction, and how it affects the cost-savings and allocative implications of emissions trading.⁸

We provide the first systematic approach to frictions in the project market by decomposing the effects of frictions on both sides of the market into the effects of search friction, bargaining, and bilateralism. Search friction indicates that real resources are lost when time is spent in search. Bargaining effect arises in costly search since the transfers between project hosts and financiers are determined in private negotiations, even though there will be a uniform price for the permits created by the project. Bilateralism due to the hosts’ need for DC-country partners will affect the gains from trade, for example, because the identity of the financier changes.

We use, first, the decomposition to identify circumstances where the frictionless ideal is achieved. Second, we use the decomposition to address allocative implications: while search friction damages both sides of the market, bargaining and bilateralism can lead to large welfare gains on one side of the market. Finally, we calibrate the decomposition to compare the implications of various degrees of frictions to those produced by a typical computable general equilibrium model.

We set up the model in section 2 where traded quantities and transfers in individual trades as well as the total number of transactions are solved. In the limit when frictions disappear, this model approaches the frictionless ideal. In section 3, we decompose the effects of frictions on gains on both sides of the market. Having identified the forces at work, we calibrate the model in section 4. Section 5 concludes and summarizes the implications of the results to the implementation of emissions trading in climate change.

2. The Model

We postulate a two-region model where the populations of polluters (firms) in both regions are large and where an emissions cap is implemented by issuing in each period a given fixed number of emissions rights (permits) that are usable only in the period of issuance.⁹ Usable permits can be transferred by making an investment which reduces the emission rate for a host of an emissions reduction project below the host's permit endowment. We assume that each host (region s firms) has one emissions-reducing project. By assumption, hosts cannot undertake their projects alone, but they need a bilateral contract with a firm from region b . We call the region b firm who either transfers the technology or finances the investment as the "provider".¹⁰ Providers of the projects need not be regulated polluters but for clarity we assume that providers are region b polluters.¹¹ Individual providers can in principle undertake many projects. Throughout this paper we will assume that once reductions are verified, they can be sold in the permit market.

This framework, which is sketched above and made more precise below, is consistent both with cap-and-trade and credit-based interpretations of emissions trading, if bilateral contracting solves verification problems of reductions.¹² Under the cap-and-trade interpretation, the host's endowment of permits cannot be moved without an emissions-reducing investment which requires bilateral contracting due to problems discussed above. This cap-and-trade interpretation applies to JI projects. Also, we can think that the sellers have already unilaterally undertaken and sold all reductions which do not violate their technical and financial constraints. This paper is about these remaining projects. Under the credit-based interpretations of emissions trading, the project is credited with transferable permits when the investment is made. This interpretation applies to CDM projects. In both cases, transfers imply no changes in the aggregate quantity of emissions,¹³ although the sellers are not explicitly under the cap in latter case.

2.1. MATCHING: MARKET FOR PROJECTS

That project hosts cannot undertake their projects alone implies a deviation from the perfect market framework. We explain first how the bilateral contracting between hosts and providers is coordinated.¹⁴ We call the market for projects as the project market, and emphasize that this market is separate from the permit

market which will be a frictionless market by assumption. We assume the following sequence of moves in the project market:

- (1) Search. A provider seeks for an unmatched project host and is sought by them. The resulting meetings are outcomes of time-taking matching process.¹⁵
- (2) Bargaining. When a provider and a host have met, they bargain over the size of the project (quantities to be exchanged) and over the division of the total value of the project.
- (3) Trading. Having agreed on the terms of trade, the provider finances the project, and the host leaves the pool of unmatched agents. However, the provider keeps on searching since a project undertaken is assumed to have no effect on the provider's ability to undertake another project.¹⁶ The expected length of individual project equals that of the investment. When the match breaks down due to breakdown of structures and equipment, the host again enters the pool of unmatched traders (stage 1).¹⁷

We assume that the aggregate number of matches per unit of time between the providers and hosts is given by matching function $m(B, S)$ where B is the number of providers and S is the number of unmatched sellers.¹⁸ Number B is a given fixed constant whereas S is determined by a steady-state condition. The reason for this asymmetry is that individual providers keep on financing projects irrespective of the number of projects already financed, but an individual project host must leave the project market when the project is undertaken. Function $m(B, S)$ is assumed to be increasing in S and satisfies $m(B, 0) = 0$.¹⁹ The number of all projects (matched and unmatched) is a given fixed constant and denoted by N . Throughout the paper we assume that time is continuous and extends to infinity; and the inflow of new projects just equals the outflow of matches that break down. The last assumption implies

$$m(B, S) = \lambda(N - S), \quad (1)$$

where $\lambda > 0$ is the constant separation rate of traders that characterizes the expected length of projects.^{20,21} For given λ , equation (1) determines a unique-sized pool of projects not yet financed, S . From now on, we consider only a steady state pool S in this sense. Note that the rate with which potential project partners arrive is $q_b \equiv m(B, S)/B$ for the provider and $q_s \equiv m(B, S)/S$ for the host.

We will use q_b and q_s to characterize traders' *relative degree of search friction*. We can say that the market is tight for the providers if $q_b < q_s$, and tight for the hosts if $q_b > q_s$.²² Parameter λ characterizes the *market-level search friction*: the larger is the value of λ , the shorter is the expected duration of a project, which, at the market level, makes the pool of matched hosts smaller. Similarly, if λ is close to zero, bilateral contracts are almost everlasting, implying that almost all potential matches are realized.

Next we proceed towards the decisions made in an individual project. To this end, we first define the technologies that determine the relationship between pollution and revenues. Then, we explain how the market for reductions (permits) is

assumed to work. The permit market will give the expected market price for each ton of reductions produced in a project, so it will be a critical determinant of the overall surplus created by a project.

2.2. TECHNOLOGIES AND ALLOCATIONS

The rate of revenue for a polluter of type i ($= b, s$) is $R_i(x_i)$ which is increasing and strictly concave in the emission rate x_i . The revenue function arises from a relationship between the firm's output and a vector of CO₂-intensive inputs. We denote the per-period endowment of permits for a polluter in region i by e_i and assume that the pre-trade productivity of type b is higher, $R'_b(e_b) > R'_s(e_s)$. If e_s satisfies $R'_s(e_s) = 0$, then the seller's endowment equals business-as-usual emissions and thus the allocation is not binding (the CDM case). If the hosts are explicitly under the overall emissions cap (the JI case), it is natural to assume that the endowment is binding, so that $R'_s(e_s) > 0$. Throughout the paper, we assume that the technologies and allocations are such that there exists an interior frictionless allocation satisfying

$$(x_b^* - e_b) = \frac{N}{B}(e_s - x_s^*) \text{ and } R'_b(x_b^*) = R'_s(x_s^*) > 0. \quad (2)$$

Thus, the total emission cap per period is $Be_b + Ne_s$.

2.3. PERMIT MARKET

Suppose that a project leads to a reduction flow $e_s - x_s$. We assume that this quantity can be sold in a spot market for permits in each period during the lifetime of the project.²³ The spot market for usable permits is continuously open and frictionless, given the supply from ongoing projects – the number of ongoing projects is determined by the matching process between buyers willing to finance a project and sellers willing to host a project. Since the number of ongoing projects is $N - S$, the number of ongoing projects per region b polluter is

$$K = \frac{N - S}{B}.$$

Users of the permits can obtain permits from the spot market at any time, so that the permits generated by a financed project have no other value than that given by the market price. Let p denote the spot price and note that in equilibrium we must have

$$R'_b(x_b) = p, \quad (3)$$

$$x_b = e_b + (e_s - x_s)K, \quad (4)$$

where (3) defines the demand curve for permits, and (4) is the accounting relationship between the demanded quantity and supply from ongoing projects. To

determine the equilibrium “size”, $e_s - x_s$, for each project undertaken we must next consider the decisions made in an individual project.

2.4. PROJECTS

Let

$$\Sigma(p) = \max_{x_s} \{p(e_s - x_s) - C(x_s)\},$$

where $C(x_s) \equiv R_s(e_s) - R_s(x_s)$, denote the maximized surplus flow for a project, given the price p for permits. Thus, $e_s - x_s$ is the quantity that an individual seller would supply if the seller is a price taker and if $C(x_s)$ is the only cost of reducing pollution. However, we assume that there is also a one-time set-up cost for the project. We denote this cost by c and assume that c does not vary across projects. To exclude degenerate equilibria, we assume throughout the paper that

$$\Sigma(p) - (r + \lambda)c \geq 0, \quad (5)$$

where r is the interest rate, and thus $(r + \lambda)c$ is the effective cash outflow from an investment of size c and of expected length $1/\lambda$.

Under these assumptions, the expected net overall surplus is the same across projects and defined by (5). By this and the fact that under the assumed search technology each provider searches for one project at a time, we can treat each individual project between the provider and host separately, without any need to keep track of the number of projects already financed by an individual provider.²⁴ Let W_b denote the expected capital value of the project for a provider, gross of the investment cost c . The value W_b satisfies

$$rW_b = p(e_s - x_s) - T - \lambda W_b, \quad (6)$$

where T is the provider-to-host transfer (flow) and $p(e_s - x_s)$ is the market valuation of the created permits (flow). Since individual projects are small, project partners take the market price as given, but with rational expectations they expect a price satisfying (3) and (4) (the steady state price will be stationary, so we have dropped the dependence on time). The expected capital loss from the event that the project breaks down is characterized by λW_b . Thus, the provider's net capital gain from a project is

$$W_b - c = \frac{p(e_s - x_s) - T}{r + \lambda} - c,$$

where the explicit values for T is yet to be found. For the project host, the value of the relationship is affected by the option value of continued search, since once the project is sold to a provider it cannot be sold to another provider before the current project breaks down. Let V_s denote the expected value of search for a host

(the option value of continued search). Let W_s be the expected value of an ongoing relationship. These values satisfy

$$rV_s = R_s(e_s) + q_s(W_s - V_s), \quad (7)$$

where $R_s(e_s)$ is the revenue flow without reductions, and $q_s(W_s - V_s)$ characterizes the expected gain from selling the project. Once the project is sold, the host's valuation for the project satisfies

$$rW_s = R_s(x_s) + T - \lambda(W_s - V_s). \quad (8)$$

Note that the set-up cost for the project is absent in equations (7)–(8) because it is the buyer who finances the project.²⁵ Thus, the host's capital gain from the project satisfies

$$\begin{aligned} W_s - V_s &= \frac{1}{\rho} \{R_s(x_s) + T - R_s(e_s)\} \\ &= \int_0^\infty \{T - C(x_s)\} e^{-\rho t} dt, \end{aligned}$$

where $\rho \equiv r + \lambda + q_s$ is the effective discount rate for a host.

We assume that, given a market price p , the transfer as well as the reductions are determined by dividing the total capital gain $(W_b - c) + (W_s - V_s)$ according to Nash bargaining solution which is characterized by

$$(T, x_s) \in \arg \max_{T, x_s} \{(W_b - c)^\beta (W_s - V_s)^{1-\beta}\}, \quad (9)$$

where $\beta \in (0, 1)$ is the buyer's (exogenous) bargaining power. For simplicity, we adopt the usual practice of assuming the symmetric Nash bargaining solution where $\beta = 1/2$.

PROPOSITION 1. *Given (5), the Nash bargaining solution implies the following equilibrium allocations and transfers: (i) (x_b, x_s) satisfies $R'_b(x_b) = R'_s(x_s)$ and (4), and (ii) for $\beta = \frac{1}{2}$, $T = C(x_s) + \frac{\rho}{r+\lambda+\rho}[\Sigma(p) - (r+\lambda)c]$ where $C(x_s) \equiv R_s(e_s) - R_s(x_s)$.*

Proof. See Appendix. ■

COROLLARY 1. *A host gets at least one half of the net surplus flow.*

Note first that polluters prefer to choose quantities that maximize their joint trading surplus, i.e., they prefer to achieve $\Sigma(p)$. The result holds not only for $\beta = 1/2$ but for any $\beta \in (0, 1)$. Note next that the seller reduces its emissions when entering a match, which results in the loss $C(x_s)$ in the host's revenue. The transfer flow T compensates the seller for this loss and, in addition, allocates to it the fraction $\frac{\rho}{r+\lambda+\rho}$ of the net surplus flow $\Sigma(p) - (r+\lambda)c$. The host's share depends on the

parameter ρ that is the host's effective discount rate. It characterizes the host's relative degree of search friction. A low q_s indicates difficulties in finding a partner, which reduces the effective discount rate, thereby increasing host's capital gain from the match. Similarly, if q_s and thus ρ is high, the host will heavily discount the gains from the relationship, which increases the host's capital gain.

Recall that the frictionless allocation and the frictionless price are denoted by (x_b^*, x_s^*) and p^* , respectively.

PROPOSITION 2. *Given (5) and $\lambda > 0$, (i) $p > p^*$ and (ii) $x_i < x_i^*$, and (iii) (x_b, x_s) approaches (x_b^*, x_s^*) as λ approaches zero.*

Proof. See Appendix. ■

Note that bargaining does not distort the size of the project, given the market price. This together with the fact that the number of ongoing projects per buyer, K , approaches total number of projects per provider, N/B , as projects become sufficiently long lasting (λ small but positive), implies that the market-level supply approaches the first-best supply (see (2) and (3)–(4)). There are more reductions per project than there would be under frictionless allocation. The reason is the “unsatisfied demand” for permits due to the fact that search friction reduces the number of ongoing projects. This distortion as well as the deviation from the frictionless price vanishes if projects are almost everlasting.

3. Decomposition of Losses from Frictions

Given that bilateralism and search friction distort the traded quantities, they obviously imply market-level reduction in gains from trade. We proceed toward these losses by first decomposing the effects of frictions on project hosts' gains from trade. The decomposition shows that frictions can benefit one side of the market.

3.1. DECOMPOSITION OF LOSSES FOR PROJECT HOSTS

Recall first that we have deviated from the perfect market framework because the hosts are unable to finance their projects alone. Conversely, going to a frictionless market implies that hosts do not need buyers to finance projects – the identity of the financier is changed. In view of this, the first-best capital value of a representative host is

$$Q_s^* \equiv \{R_s(x_s^*) + p^*(e_s - x_s^*) - \lambda c\}/r.$$

In an attempt to keep later expressions short, we use $T^* \equiv p^*(e_s - x_s^*)$. We assume that the cost flow λc is not too large so that $Q_s^* \geq 0$. In project-based trading, the capital value of a representative host is

$$Q_s \equiv \left\{ R_s(e_s) \frac{\lambda}{q_s + \lambda} + (R_s(x_s) + T) \frac{q_s}{q_s + \lambda} \right\} / r,$$

where $\frac{\lambda}{q_s + \lambda}$ and $\frac{q_s}{q_s + \lambda}$ are the shares of unmatched and matched hosts in equilibrium, respectively. This expression is obtained by summing up the capital values for searching and matched project hosts and dividing by the number of hosts, i.e., $Q_s = \{V_s S + W_s(N - S)\}/N$. Using the definitions Q_s^* and Q_s , the loss due to frictions (or the gain if negative) in the rate of return for a host is seen to be

$$\begin{aligned} r(Q_s^* - Q_s) = & [R_s(x_s) + T - R_s(e_s)] \frac{\lambda}{q_s + \lambda} \\ & + [T^* - T] \\ & + [R_s(x_s^*) - R_s(x_s) - \lambda c]. \end{aligned} \quad (10)$$

Expression (10) decomposes the effects of friction into the effects of (i) search friction, (ii) bargaining, and (iii) bilateralism. The effect due to search friction is captured by the first line which is the share of unmatched hosts times the lost net trading surplus flow per project. The bargaining effect is $T^* - T$ due to the fact that the negotiated transfer generally deviates from the frictionless transfer per host.²⁶ The effect from bilateralism is the discrepancy in raw revenue $R_s(x_s^*) - R_s(x_s) - \lambda c$ which is caused by distortions in traded quantities and the change in the identity of the financier.

We consider next factors that determine the significance of the three effects. Parameter λ , which characterizes the length of bilateral contracts (the expected length is $1/\lambda$), alters search friction directly. To avoid violating (5), assume $c = 0$ for a while and suppose that λ is very large. Then ongoing trading becomes impossible, implying that the loss $r(Q_s^* - Q_s)$ is close to $R_s(x_s^*) + T^* - R_s(e_s)$. This is the surplus flow that a host would achieve if frictionless trading opportunities existed. Thus, all of the potential surplus is lost.

Assume then that λ is close to zero. Projects are now almost everlasting, implying that the loss due to search friction almost vanishes. Only the bargaining effect $T^* - T$, which must be positive, remains. For if λ is close to zero, then host's share of surplus flow $\frac{\rho}{r + \lambda + \rho}$ is close to $\frac{r + q_s}{2r + q_s}$, implying that $T^* - T$ is close to

$$p^*(e_s - x_s^*) - C(x_s^*) - \frac{\rho}{r + \lambda + \rho} \Sigma(p^*) = \frac{r}{2r + q_s} \{p^*(e_s - x_s^*) - C(x_s^*)\} > 0.$$

So, the host gains from bargaining when projects are sufficiently long lasting.

Outside the above limiting case, an increase in λ decreases the host's share of the surplus towards $1/2$. This happens because an increase in λ increases steady-state S , making q_s smaller. Thus, a sufficient increase in the market-level search friction makes the project market tighter for the hosts and thereby the division of the surplus flow more symmetric.

3.2. DECOMPOSITION OF LOSSES FOR PERMIT BUYERS

In order to construct a similar type of decomposition for permit users we need to assume that the providers come from the pool of regulated firms in region b .

That is, firms which enter the negotiations with project hosts are the firms buying permits from the permit market. Without this assumption, part of the gains from trade would be realized by nonregulated participants, i.e., by anyone who has the ability (technological or financial) to undertake projects. The analysis so far does not require that the identity of the financier is restricted in this way, but for the sake of the decomposition of losses we now make this restriction.²⁷

With the above restriction a representative region b firm has a double role: he is a permit buyer in the permit market and a provider in the project market. Denote the rate of return for a representative buyer by rQ_b . This rate has two components. First, the revenue flow from producing and buying permits from the permit market is $R_b(x_b) - p(x_b - e_b)$ per buyer. Second, as the provider the representative region b firm expects a revenue from the project market. The overall rate of return from the project market for the provider side is

$$\begin{aligned} & q_b B(W_b - c) \\ & - \lambda(N - S)W_b \\ & + (N - S)\{p(e_s - x_s) - T\} \\ = & (N - S)\{p(e_s - x_s) - T - \lambda c\}. \end{aligned}$$

The first line gives the market-level rate with which projects create capital gain. The second line gives the corresponding rate for the loss from the breaking projects. The third line is the surplus flow from ongoing projects. Summing up gives the last line since in equilibrium the inflow of new projects equals the outflow of breaking projects, $q_b B = \lambda(N - S)$. Dividing the last line by the number of providers gives the rate of return from the project market for the representative region b firm:

$$K\{p(e_s - x_s) - T - \lambda c\}.$$

Summing up the rate of return from the permit market and project market gives

$$\begin{aligned} rQ_b &= R_b(x_b) - p(x_b - e_b) + K\{p(e_s - x_s) - T - \lambda c\} \\ &= R_b(x_b) - K\{T + \lambda c\}. \end{aligned}$$

Thus, the representative region b firms pays $K\{T + \lambda c\}$ rather than $p(x_b - e_b)$ for the permit transfer. The payment includes the negotiated transfer T for K projects and the cost flow from refinancing the structures and equipment that break down; an average provider has K projects which break down with rate λ . The first-best rate of return is simply $rQ_b^* \equiv R_b(x_b^*) - p^*(e_b - x_b^*) = R_b(x_b^*) - T^*$, so the loss due to frictions (or the gain if negative) in the rate of return for a host is seen to be

$$\begin{aligned} & R_b(x_b^*) - R_b(x_b) \\ & + KT - T^* \\ & + K\lambda c \\ = & r(Q_b^* - Q_b). \end{aligned}$$

The first line is again the loss due to search friction which reduces the overall supply and thereby leads to a loss in the raw revenue from production. The second line is the loss due to bargaining: the average transfer from the buyer to the seller side is KT rather than T^* . The third line gives the loss from bilateralism which comes from the change in the identity of the financier.

3.3. MARKET-LEVEL LOSS

Given the decomposition of the effects on private welfare, the step to the market-level loss is straightforward. The polluters' first-best total capital value is $Q^* \equiv Q_b^* B + Q_s^* N$. The polluters' total capital value in project-based trading is $Q \equiv Q_b B + Q_s N$. Using the hosts' and providers' decompositions yields the difference in the flow surplus:

$$r(Q^* - Q) = B [R_b(x_b^*) - R_b(x_b)] + N [R_s(x_s^*) - R_s(x_s)] - S [C(x_s^*) + \lambda c].$$

The first and second terms give the distortions in after-trade revenues for all active traders (positive by Proposition 2). The last term gives the reduction in abatement cost due to fact that some hosts are not reducing their emissions when there is search friction. By the first fundamental welfare theorem, $Q^* - Q$ is nonnegative. By Proposition 2, the loss $Q^* - Q$ becomes vanishingly small as frictions disappear: when λ approaches zero, $x_i^* \rightarrow x_i$ and $S \rightarrow 0$.

4. Application

To illustrate quantitative magnitudes, we next calibrate the model using previous cost estimates of CO₂ reductions for the European Union (EU) and East Europe (EE). There are numerous estimates for various regions of the world (see Weyant and Hill 1999) for an overview of recent cost estimates produced by thirteen research teams). We focus on the above two regions because the estimated demand for permits is high in the EU region and the abatement costs are low in the EE region. We adopt estimates from the Global Trade and Environment Model (GTEM) (see Tulpulé et al. 1999).²⁸ All monetary values are measured in 1992 U.S. dollars, but in the sequel we express results as percentage deviations from the GTEM equilibrium.

The steps of the calibration are the following. First, we fit the Nordhaus' (Nordhaus 1991) marginal abatement-cost function to the pre- and after-trade marginal costs given by the GTEM results. The marginal cost of emissions reduction for a polluter of type i ($= b, s$) is

$$c_i(x_i) = \alpha_i + \frac{1}{\gamma_i} \ln(x_i/x_i^{bau}) \geq 0, \quad (11)$$

where x_i is the emission flow, x_i^{bau} is the business-as-usual emission flow, i.e., the emission flow without any emissions reductions, and α_i and γ_i are parameters.

Table I. Losses from friction

	Host's loss	Provider's loss	Spot price
$\lambda = 0.001$	0.5	-0.2	176
$\lambda = 1$	0.4	6.7	209
$\lambda = 10$	-340.1	71.3	522

Losses are percentages of the representative host's frictionless surplus flow.

Parameters: $r = 0.06$, $m(B, S) = B^{0.5} S^{0.5}$, $B = 1500$, $N = 500$, $c = 3$.

We obtain the cost schedules for the representative EU buyer and EE seller.²⁹ These can be used to reproduce the first-best equilibrium (the GTEM equilibrium), and to calculate the equilibrium both under barter transfers and freely transferable permits.

Second, we specify the matching function. We assume that the aggregate meeting rate is given by the Cobb-Douglas function $m(B, S) = B^a S^{1-a}$ where $a = 1/2$. This imposes no *a priori* search advantage to either side. Moreover, if we let $\theta = B/S$ denote the “market tightness”, we can express the matching rates for individual traders in the project market as $q_s = q(\theta) = \theta^{1/2}$ and $q_b = \frac{q(\theta)}{\theta} = \theta^{-1/2}$. Recall that B and N are given fixed constants, and S is endogenously defined by (1). We assume that

$$B = 1500, N = 500.$$

That is, there are 1500 potential providers in the EU region and 500 potential project hosts in the EE region.³⁰

Table I shows the calibrated losses for the representative providers and hosts and the spot price when projects are almost everlasting ($\lambda = 0.001$), short-lived ($\lambda = 1$), and extremely short-lived ($\lambda = 10$).³¹ Recall first that if the supply-side friction vanishes (λ small), the after-trade allocation per trader and the spot price become close to their first-best levels (proposition 2). This is illustrated by the first row: the loss per trader is insignificant and the frictionless price is almost achieved ($p = 176$ in 1992 U.S. dollars). The host loses and the provider gains slightly, when compared to the frictionless surplus flow from trading. Note next that a sufficient increase in friction leads to a considerable gain on the seller side – the representative seller's surplus flow exceeds the frictionless one more than by a factor of three. This is explained by the soaring spot price for permits generated by the seller's project. We consider next the decomposition of these losses.

Table II shows the decomposition of the above losses for the representative host. As explained in the previous section the loss from search friction vanishes as the separation rate approaches zero. While the search-friction loss increases with market-level increase in friction (larger λ), the loss is more than compensated by the gain coming from bargaining, which explains the overall gain from frictions in the last row.

Table II. Decomposition of losses for a representative host

	Search friction	Bargaining	Bilateralism	Total
$\lambda = 0.001$	0	0.5	0	0.5
$\lambda = 1$	26.3	-67.5	41.5	0.4
$\lambda = 10$	2330	-5150.9	2480.3	-340.1

Losses are percentages of the representative host's frictionless surplus flow.

Parameters: $r = 0.06$, $m(B, S) = B^{0.5}S^{0.5}$, $B = 1500$, $N = 500$, $c = 3$.

Table III. Decomposition of losses for a representative provider

	Search friction	Bargaining	Bilateralism	Total
$\lambda = 0.001$	0	-0.2	0	-0.2
$\lambda = 1$	5.0	-0.2	1.9	6.7
$\lambda = 10$	88.1	-20.6	3.8	71.3

Losses are percentages of the representative host's frictionless surplus flow.

Parameters: $r = 0.06$, $m(B, S) = B^{0.5}S^{0.5}$, $B = 1500$, $N = 500$, $c = 3$.

For the buyer side the decomposition is presented in Table III. The search-friction effect dominates for the buyer because it reduces the effective permit flow from projects and thereby implies a reduction in revenues from production. It may seem surprising that also the buyer gains from bargaining when frictions are increased. The bargaining effect becomes negative for the buyer when the number of projects undertaken approaches zero (larger λ) because the effective transfer flow from the buyer side to the seller side vanishes; the high-price effect that benefits the host side cancels out for the buyers because of the double role of region b firms as permit users and providers of projects.

5. Conclusion

“International transfers, in one form or another, are likely to serve as both the building blocks of globally optimal action and the cement of global cooperation” (IPCC 1995: 71, section 2.4.2).

Emissions trading is a potential way of linking transfers from rich to poor countries and achieving the efficient allocations of emissions reductions. The majority of all previous emissions trading models assume efficiency in trading (see Weyant and Hill 1999). Given the earlier experience from DC-LDC trade (Marin and Schnitzer 1995) and the built-in friction in the suggested trading mechanisms (Barrett 1998; Hahn and Stavins 1999), we see no reason to consider only efficient trading institutions in climate change. We deviated from the perfect market framework by assuming that the project-based supply for permits requires costly search. We still assumed that the permit market is frictionless. We decomposed

the resulting effects on losses and allocations into the effects of search friction, bargaining, and bilateralism.

Our decomposition can be used to study circumstances which seem reasonable in climate change. Our calibration suggested that project hosts can considerably gain relative to the frictionless benchmark if the degree of friction in the project market is high. The calibration, as well as our theory, showed that full cost saving potential is achieved in the limit where the friction in the project market vanishes. These results suggest a potential conflict between efficiency and the hosts' gain from trading: When the market is "tight" for the buyer side, hosts may prefer bilateral negotiations which entail total welfare losses. Thus, friction *per se* need not damage all, and because more friction can even benefit one side of the market, there is a potential incentive to build excessive friction into the trading rules in climate change negotiations.

Notes

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1. We will interpret all greenhouse gases in CO₂ equivalent units because the distinction between gases is not central to this paper. The full text and discussion of the 1997 Kyoto Protocol can be found at the website of the Climate Change Secretariat, <http://www.unfccc.int>. See also Grubb et al. (1999).
2. According to an overview of recent estimates produced by thirteen research teams, emissions trading have the potential of reducing the total cost of the 1997 Kyoto Protocol by about half in DC countries (Weyant and Hill 1999). See Barrett (1998) for a discussion about trading mechanisms and the political economy of the Protocol.
3. Projects in the pilot trading program include investments in (i) renewable energy (hydro, wind, solar, wood), (ii) energy efficiency (thermal protection, heat exchanges, heat pipelines), (iii) fugitive gas capture (methane utilization), (iv) fuel switching (boilers), (v) agriculture, (vi) afforestation, (vii) reforestation. For the distribution of abatement among the above categories, see UNFCCC (2001). Detailed project descriptions can be found in various issues of the "Joint Implementation Quarterly" (<http://www.northsea.nl/jiq>).
4. These first two problems are well-reported in Lile et al. (1999) who provide lessons from case studies. For example, the case of Aquatech Services, Inc. illustrates both of the above reasons for search. First, each installation of the same methane capture technology implies a different cost because site-specific differences require testing and calibration of the technology. Second, the firm ultimately terminated the described project because of the difficulties in the approval process.
5. The early experience from the pilot program "Activities Implemented Jointly" under the Framework Convention on Climate Change indicates that contracts are bilateral arrangements where DC-country partners finance abatement projects in EE/LDC countries. As no climate change treaty is yet ratified and implemented, this is only an experimental program where investments are real, however. Project partners participate in the hope that permits generated can be transferred when/if the climate treaty enters into force (this can be seen from a typical contract,

see Jepma (2000: 9–10). As of May 2003, contracts for 156 projects were available online at <http://unfccc.int/>. For a succinct description of the pilot program, see Stavins (2002).

6. JI projects are in countries whose emissions are under the Kyoto cap. CDM projects are in nonparticipating countries. The reasons for search apply to both types of projects irrespective of whether the credits created by projects can be traded in a cap-and-trade program.
7. To our knowledge, climate-change negotiations suggest that a project can freely sell ERUs or CERs created, once the crediting procedure is over. This implies that ERUs and CERs are not different from tradable emission permits, so we call ERUs and CERs permits.
8. The literature on Joint Implementation projects emphasizes the important role of asymmetric information (e.g. Hagem 1996; Wirl et al. 1998), but because it does not extend project-based trading to a market level, it cannot link the project market to the overall permit market and address the total cost saving potential. Previous approaches to frictions in emissions trading (e.g. Liski 2001, 2002; Stavins 1995) do not model bargaining nor the source of friction explicitly and, hence, they cannot address the allocative effects properly.
9. Intertemporal trading can be a source of cost savings due to uncertainty (Schnach 1998), scale economies (Liski 2002), or smoothing of abatement activities when the cap on emissions becomes tighter over time (Ellerman 2000). We do not have these reasons for banking in our model.
10. Recall that in climate change most profitable pollution abatement projects are in less developed, highly indebted countries where project hosts are lacking creditworthiness to finance their projects. A bilateral contract with a partner from a developed country is assumed to restore the creditworthiness of the project or facilitate a creditor-debtor relationship.
11. It might be helpful to think providers as region b polluters who are already using a better technology (less polluting) at home than that used by region s polluters. Then, through projects, region s polluters become clones of region b polluters in technological sense. Formally, our setup does not require this type of restriction on the identity of the provider until in later sections where we want to consider the overall gains from emissions trading in both regions s and b .
12. Carbon reductions are difficult to verify and only high quality tons will be accepted by all interested parties. In an anonymous market, it may be difficult to ascertain whether the permits are backed by real reductions, and the incentives to buy permits of unknown quality will depend on liability rules and inspection probabilities. Bilateral contracting can potentially facilitate private monitoring by the parties who are interested in building up a reputation as a provider of high quality tons.
13. This property is important because we want to consider the cost of achieving a given cap on total pollution. See Stavins (2002) for a discussion about cap-and-trade and credit-based programs. The idea of a given cap and the fact that in climate change trades are projects led to this modelling framework where trades are long-lasting relationships. An alternative is to assume one-time transactions, and that transacting agents leave the market for good and are replaced by identical agents (see e.g. De Fraja and Sákovics 2001; Mortensen and Wright 2002). This latter approach also ensures a stationary market environment but implies an unbounded population for agents having transacted, which is not consistent with the aggregate cap on endowments (pollution licenses). Yet another alternative approach would be to assume search from an exhaustible pool of projects (Spulber 1996). Our approach comes close to this if projects are assumed to be almost everlasting.
14. Our approach will be closest to labor market search models (see Pissarides 2000). Other topics addressed by search models include price and wage dispersion (Burdett and Judd 1983; Diamond 1971) and money (Kiyotaki and Wright 1991; Trejos and Wright 1995).
15. If trades are subject to regulatory approval or verification of reductions, the process is completed at the time traders are matched. Potential delays from this source can be thought of as being incorporated into the matching process defined below.

16. Projects are technically independent. Also, we assume that the provider's creditworthiness is not affected by the number of projects financed.
17. This last sentence implies that we are circulating a given set of agents through stages 1–3.
18. See Pissarides (2000: 6), for a discussion about the usefulness of the concept.
19. The first assumption excludes multiple equilibria, the first and second together guarantee the existence of equilibrium. At this stage we do not make any assumption of returns to scale: all our theoretical results are independent of returns to scale in matching function. In the numerical application we specify $m(B, S) = B^a S^{1-a}$ where $a = 1/2$.
20. The project breaks down when the structures and equipment break down. Of course, the length of the project could also be a matter of contracting. However, when the technical length of the project is made equal to the length of the substitution, the project is credited for the exact amount of abatement produced. Finally, while λ is data for traders, it will alter the buyer-seller contract which will be explained below.
21. The assumption about stationarity is not central to the result that losses due to frictions can be decomposed as we do below. However, along the transition towards stationary trading environment, losses and the decomposition are changing. The characterization provided here is thus a long-run equilibrium.
22. Note that Pissarides uses $B/S (= q_s/q_b)$ as the measure of market tightness (Pissarides 2000).
23. The other alternative is that the project is immediately credited with a quantity that equals the expected overall reductions during the lifetime of the project. This approach would create a discrepancy between the credited and actual reductions because the lifetime of the project is uncertain. Also, we believe that the approach adopted in the text is closer to the crediting procedures agreed in climate change negotiations.
24. One can assume that region b polluters use consultants (middlemen) in finding suitable projects (one middleman per polluter). Let V_b denote the expected gain from asking the middleman to find one project. The expected waiting time for a successful project is $1/q_b$, and once the project has been financed, another middleman can be hired with the expected gain V_b . The overall expected capital gain for a provider from operating in the project market depends on the expected number of V_b 's realized per provider along the equilibrium path.
25. Note that the seller's lack of creditworthiness *with respect to projects* (e.g., due to technical inability) does not prevent one from using equations (7) and (8): these equations give the values of firms' assets, given the need for bilateral contracts.
26. To clarify potential sources of confusion, recall that the frictionless transfer is the market price times supplied quantity per project.
27. This restriction is not unreasonable since the financing partners are likely to be regulated energy producers and traders who are experienced with the technologies they are already using at home.
28. We use GTEM estimates rather than a combination of estimates for three reasons. First, estimates are to a large extent driven by scenarios about changes, for example, in energy efficiency and output which vary between studies. To keep the scenarios and the estimates consistent, we use the results of a single study. Second, all data needed for the calibration is reported in Tulpulé et al. (1999: 269, Table 6). Third, according to a review of different simulation approaches (Weyant and Hill 1999), the GTEM estimates are not outliers among the various estimates.
29. The calibration involves entirely straightforward calculations which are available upon request. According to the GTEM data, the pre-trade emissions are $x_b/x_b^{bau} = 0.747$ and $x_s/x_s^{bau} = 0.764$, and the after-trade emissions are $x_b/x_b^{bau} = 0.908$ and $x_s/x_s^{bau} = 0.464$. Solving (11) with these gives $(\alpha_b, \alpha_s, \gamma_b, \gamma_s) = (-90.027, -33.412, -0.00036279, -0.00366679)$.
30. We have no data about the relative sizes of these pools. In an earlier version we experimented with different sizes of trader pools. We found that this experiment was not central to this illustration, so this final version reports only the case $B = 1500, N = 500$.
31. Obviously, there exists no data about actual technical life of the JI and CDM projects, so the purpose of these number is only to illustrate the effect of hypothetical lifetimes.

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Appendix: Proofs

PROOF OF PROPOSITION 1

Step 1. Maximizing $(W_b - c)^\beta (W_s - V_s)^{1-\beta}$ with respect to T gives the first-order condition $(1 - \beta)(W_b - c) \frac{\partial W_s}{\partial T} + \beta(W_s - V_s) \frac{\partial W_b}{\partial T} = 0$. Using $W_b = \frac{p(e_s - x_s) - T}{r + \lambda}$, $W_s = \frac{R_s(x_s) + T + \lambda V_s}{r + \lambda}$, and $W_s - V_s = \frac{R_s(x_s) - R_s(e_s) + T}{r + \lambda + q_s}$ where $R_s(x_s)$ is associated with arbitrary (x_b, x_s) , gives $(1 - \beta)(W_b - c) = \beta(W_s - V_s)$.

Step 2. Maximization of $(W_b - c)^\beta (W_s - V_s)^{1-\beta}$ with respect to (x_b, x_s) is uniquely characterized by $(1 - \beta)(W_b - c)R'_s(x_s) - \beta(W_s - V_s)p = 0$ where x_b and x_s satisfy (4). Using the result $(1 - \beta)(W_b - c) = \beta(W_s - V_s)$ of step 1 gives $R'_s(x_s) = p = R'_b(x_b)$.

Step 3. Plugging the allocation (x_b, x_s) given in step 2 into the sharing rule given in step 1 and using $\beta = 1/2$, yields the transfer given in the proposition. ■

PROOF OF PROPOSITION 2

We first prove item (ii). Equation (2) gives $Be_b + Ne_s = Bx_b^* + Nx_s^*$, and equation (4) gives $Be_b + Ne_s = Bx_b + Nx_s + S(e_s - x_s)$. Combining these yields

$$Bx_b + Nx_s + S(e_s - x_s) = Be_b + Ne_s = Bx_b^* + Nx_s^*, \quad (12)$$

and because $e_s > x_s$, we can write

$$Bx_b + Nx_s < Be_b + Ne_s = Bx_b^* + Nx_s^*. \quad (13)$$

If $x_b^* < x_b$, then $R'_b(x_b^*) > R'_b(x_b)$, and by $R'_b(x_b^*) = R'_s(x_s^*)$ and $R'_b(x_b) = R'_s(x_s)$ we have $R'_s(x_s^*) > R'_s(x_s) \Leftrightarrow x_s^* < x_s$, a contradiction to (13). If $x_b^* = x_b$, we analogously argue that $x_b^* = x_b$, again contradicting (13). Thus, we are left with the case $x_b^* > x_b$,

which implies $R'_b(x_b^*) < R'_b(x_b)$ and $R'_s(x_s^*) < R'_s(x_s) \Leftrightarrow x_s^* > x_s$. That is, only $x_i^* > x_i$, $i = b, s$, is possible. ■

Item (i) Proof of $p > p^*$ is easy: $R'_b(x_b^*) = R'_s(x_s^*) = p^*$, and $R'_b(x_b) = R'_s(x_s) = p$. Since $x_b^* > x_b$, we have $R'_b(x_b^*) < R'_b(x_b) \Leftrightarrow p > p^*$. ■

Item (iii) When λ approaches zero, S approaches zero, and $Bx_b + Nx_s = Be_b + Ne_s = Bx_b^* + Nx_s^*$. Applying the reasoning in proof of item (ii) shows that $(x_b, x_s) \rightarrow (x_b^*, x_s^*)$ if $\lambda \rightarrow 0$. ■

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Runeberginkatu 14-16
00100 Helsinki
Puh. (09) 4313 8310, fax (09) 495 617
Sähköposti: kirjak@kyypari.hkkk.fi

Helsingin kauppakorkeakoulu
Julkaisutoimittaja
PL 1210
00101 Helsinki
Puh. (09) 4313 8579, fax (09) 4313 8305
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