Xavier Carpentier

ESSAYS ON THE LAW AND ECONOMICS OF INTELLECTUAL PROPERTY

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To Emma

Abstract

This dissertation consists of four essays on the law and economics of intellectual property (IP). The first essay deals with trade secret law. The second and third essays consider specific patent doctrines in models of sequential innovation. The fourth essay is a comparative analysis of the incentive properties of different IP regimes when innovation is cumulative.

The first essay investigates how the combination of damages and criminal fines, which sanctions the misappropriation of a trade secret through bribery, affects the incentives to innovate and imitate. Counterintuitively, the trade secret owner's payoff can decrease when the criminal fine increases. It is always possible to design a socially optimal trade secret law which sets the criminal fine equal to zero. Bribery can be socially optimal. In that case, trade secret protection is ensured by a strictly positive level of damages which differs depending whether the imitator can or cannot reverse-engineer the innovation.

The second essay analyzes the role of the doctrine of estoppel in a model of sequential innovations. The first innovation is patented and the following one infringes the patent (for example, it is an application to another industry of the patented innovation). The doctrine punishes a patentholder who threatened to sue an infringer and then remained silent for a while before enforcing her patent: the patent may be unenforceable. In the model, the patentholder can enforce her patent before or after the infringer has developed his innovation. Counterintuitively, the doctrine can make the infringer worse off, though it is designed to protect him. Also, the doctrine can induce more delay in litigation, though it punishes delays. Under specific circumstances, the doctrine of estoppel can be treated as a new instrument of patent policy aimed at reducing the hold-up problem. The effect of patent validity on players' welfare and on the equilibrium outcome is also analyzed.

The third essay considers the doctrine of laches, again in a context of sequential innovations. Like the doctrine of estoppel, the doctrine of laches punishes a patentholder who delayed enforcing her patent. But this doctrine does not require an initial threat of litigation followed by a period of silence, and the patent remains enforceable. However, the patentholder cannot collect damages to compensate her for infringement that occured during the period of delay. The analysis incorporates uncertainty about the profitability of the

follow-on innovation. Hence, *both* the timing of investment in the follow-on innovation and the timing of litigation are endogenized. The doctrine can spur or deter investment in the follow-on innovation. Also, it can speed-up investment or delay it. It can hurt the infringer. The effect of the patentholder's compensation via damages is also analyzed. An increase in this compensation can speed-up or delay investment in the follow-on innovation and can paradoxically make the patentholder worse off.

The fourth essay, a joint work with Klaus Kultti, is a comment of a widely discussed article by James Bessen and Eric Maskin (B&M) (2002). The authors argue than patents can *reduce* aggregate R&D investment when innovation is cumulative. We extend their model in two directions: we endogenize the level of R&D investment and, beside "patents" and "no protection", we introduce a third IP regime called "copyright". We find that when innovation is cumulative, "patents" always yield more aggregate R&D than "no protection" (in contrast to B&M). Also, a copyright regime may implement the socially optimal investment by reducing R&D incentives compared to a patent regime.

Keywords: trade secret, patent, copyright, damages, injunctions, doctrines of laches and estoppel, sequential innovation, incentives.

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Helsinki, September 2006

Xavier Carpentier

Contents

Introduction

1 Questions and motivation

- What is the socially optimal combination of criminal and civil penalties to punish misappropriation of trade secrets through bribery?
- When and how is the "doctrine of estoppel", which renders a patent unenforceable under specific circumstances, an instrument of patent policy?
- The "doctrine of laches" penalizes a patentholder who delayed enforcing her patent. How does this doctrine and the level of compensatory damages affect the incentives to infringe the patent and to litigate the infringer?
- Bessen and Maskin (2002) argue that patents can *hinder* innovation when it is sequential and firms' R&D investments are exogenous. Does this conclusion survive when R&D investments are endogenous? And what is the optimal legal protection to offer sequential innovators?

*

Intellectual property rights (IPRs) have long been acknowledged to be crucial mechanisms for the support of innovation and economic growth.[1] In line with Jeremy Bentham's utilitarian view on IPR, most economists, policy makers and lawyers argue that the absence of property rights over the knowledge embodied in an innovation would spur imitation and competition, thereby reducing the innovator's profit. Anticipating this outcome, the potential innovator would be deterred from investing in R&D in the first place. As a result, the benefits of IPRs for growth are usually considered to outweigh their costs in terms of monopolistic distortions and technological diffusion. Nevertheless, in recent years, this concept has been either challenged, or adapted, on the basis of several observations.

- First, innovation is sequential. Although this is not a new phenomenon, it has gained momentum and become a real concern amongst academics and practitioners. An innovation either improves upon a previous one, applies it in another sector, or is obtained through the use of the previous innovation as a basis for R&D. In all cases, a dilemma arises. The IPR system must ensure that the holder of a patent over the first innovation is properly rewarded for opening "research avenues". Yet, since IP law typically fulfills this objective by allowing the patentholder to collect revenues from the follow-on innovation,[2] this can create disincentives for the follow-on innovators themselves. In many industries such as the software or the semiconductor industry, where an innovation is often an incremental improvement over a previous one, patents have been criticized for creating excessive rights over future innovations (Bessen and Maskin, 2002).

- Second, innovations are more and more complementary. Many innovations are "composite" in the sense that they rely themselves on the combination of several other complementary innovations. The DVD standard or the standards for mobile telecommunication such as the GSM, the CDMA or the WCDMA standards combine a myriad of complementary technologies, often owned by different firms. In the biomedical sector, R&D usually

[1] The economic literature on endogenous growth formalizes the role of R&D investment in promoting technological progress and growth. IPRs have been introduced in this literature by O'Donoghue and Zweimüller (2004).

[2] I call a "follow-on" innovation any innovation that builds upon a previous one. As mentioned, it can be an improvement, an application (like the use of the "simplex algorithm" is AT&T patent 4,744,028) or it results from the use of the previous innovation as a research tool (such as the Cohen-Boyer patent on the technology enabling insertion of foreign genetic material into a bacteria).

requires the use of patented technologies known as "research tools". The potential downsides of fragmentation of ownership are the increasing transaction costs associated with obtaining a license for all patented technologies, the multiplication of the mark-ups imposed by licensors on licensees and the overall higher risk of patent infringement. In some sectors (telecommunications, consumer electronics,...) industry participants have developed institutional arrangements to alleviate the problem: "patent pools" license a pool of patented technologies, at one single price, to downstream users. But other sectors, like the biomedical sector, have not been as successful. Acknowledging this problem, Heller and Eisenberg (1998) argue: "the tragedy of the commons's metaphor helps explain why people overuse shared resources. However, the recent proliferation of intellectual property rights in biomedical research suggests a different tragedy, an "anticommons", in which people underuse scarce resources because too many people can block each other".

- Third, there is a race towards more patent protection in some industries and a wider use of alternative means of protection in others. The widespread expression "knowledge-based economy" highlights the importance of knowledge in economies where information and communication technologies play a central role. This is reflected by a massive increase in patent applications and patent grants in the last decade in this and related sectors. According to the OECD's "compedium of patent statistics" published in 2004: "two technology fields contributed substantially to the overall surge in patenting: biotechnology and ICT. Between 1991 and 2000, biotechnology and ICT patent applications to the EPO increased by 10.9% and 9.5% respectively compared to 6.9% for all EP patent applications". Some authors such as Jaffe and Lerner (2004) argue that this trend is coupled with a decrease in patent quality (due to an overburdened Patent Office or to regulatory capture). As a result, low quality patents are granted[3] which can nevertheless exert a non-deserved anticompetitive pressure. Also, the overall increase in patent applications should not hide the fact that many firms do not believe in patents as the best mechanism to protect their innovations. In an important survey from 2000, Cohen, Nelson and Walsh highlight the importance of trade secrets, typically preferred by a majority of the manufacturers interviewed.

[3]patents for innovations that do not meet the patentability requirements of novelty, non-obviousness and usefulness.

These phenomena: sequentiality, complementarity, race towards protection, low quality of the patents granted and diversification of the protection strategies, combine and form a more complex innovation and IPR landscape, where the ownership over IP is fragmented and the likelihood of IPR infringement is higher.[4] Maybe reflecting this growing complexity and the higher risk of infringement, statistics show an unprecedented increase in intellectual property (IP) litigation. A recent report by LexisNexis reveals that the continued upward trend in patent litigation in the United States resulted in a 130% increase in patent case filings between 1988 and 2003. This fact reminds us that an IPR is effective only to the extent that the right holder is willing to enforce it against a challenger. A patent, a copyright, a trademark, are all rights to exclude others from using the innovation for commercial purpose without the consent of the owner. A trade secret is a right not to disclose the details of an innovation and to exclude anyone who violates this right (i.e obtains disclosure illegally). Importantly, this points out to the central role played by legal determinants in the actual value of IPR. By legal determinants, I mean the various rules that govern IPR litigation.[5] This dissertation is dedicated to improve our understanding of the impact of these rules on the incentives to innovate.

The four questions displayed above are, broadly, the four issues investigated in the essays of this dissertation. Particularly in the three first essays, I focus on specific legal rules affecting intellectual property disputes.[6] These legal rules include both the "remedies" available to intellectual property holders whose right has been infringed, and the "defenses" available to alleged infringers. A "remedy" is a mechanism by which the right holder is compensated. A typical remedy is the award of damages which compensate the intellectual property holder for a loss of profit due to infringement. A "defense" is a mechanism by which the accused infringer may try to avoid compensating the right holder. This "Law and Economics" approach enables me to inquire notions and economic situations which have been either neglected or only preliminary investigated in the literature so far. Ultimately, my research can contribute

[4]This explains the recent surge of interest for IPR litigation insurance. See Lanjouw and Schankerman (2002) and Görtz and Konnerup (2001).

[5]IPR legislation is broader than the mere Law concerning IPR enforcement. For instance, patent legislation also concerns patent applications. Given that there is a possibility of nearly simultaneous innovations, two rules are currently in force. In the US, the "first-to-invent" rule means that the patent issues to the first inventor provided the date of the first invention is documented. In all other countries, the "first-to-file" means that the patent issues to the first *applicant*. See Scotchmer and Green (1990).

[6]Also in the fourth essay, though in a more remote manner.

to technology policy debates, when the instruments of this policy include the legal remedies and defenses available in intellectual property disputes. Given that these mechanisms are widely used in practice, analyzing their effects on the incentives for innovation and litigation is important.

2 The economics of intellectual property rights

According to the Constitution of the United States[7] (Art. 1, section 8, clause 8):

"The Congress shall have Power To promote the Progress of Science and useful Arts, by securing for limited Times to Authors and Inventors the exclusive Right to their respective Writings and Discoveries".

The fact that IPRs are constitutional rights in the United States alerts us of their importance in the eyes of the Founding Fathers. Despite their early recognition by modern States as instruments of innovation policy, IPRs did not attract much attention from economists before William Nordhaus' pathbreaking contribution in 1969. Takalo (1999) offers a clear review of this scarce economic literature from the 18th century until the 1960's. He discusses in particular Jeremy Bentham's initial insights. Since Nordhaus (1969), the literature has flourished. To find an order in this "forest" it is worthwhile to classify the contributions using a framework represented in Figure 1. By comparison, my own contributions in this dissertation are reported in Figure 2. Contributions are ordered according to two dimensions: the type of literature they belong to (horizontal axis) and the main policy issue they address (vertical axis). This classification does not pretend to be exhaustive and some original contributions do not perfectly fit in. Yet, most of the relevant literature can be thought of through this model.

- *The literature dimension* (horizontal axis). Although many consider the economics of IPR as part of the "Law and Economics" literature, a closer look suggests a more nuanced statement. In the so-called "patent race" literature, where many firms compete in R&D under a winner-take-all rule, patents are essentially prizes. This literature belongs more to the "Industrial Organization" field in the sense that it investigates firms' competitive behaviour and its impact on social welfare. It does not focus on how the Law

[7] Capital letters are as in the original text.

determines the value of the "prize", namely the patent. By contrast, some papers are mainly interested in that issue. They model patent litigation and investigate how legal rules determine infringement and litigation strategy. These papers belong more to the "Law and Economics" tradition.

- *The policy dimension* (vertical axis). The basic tension between static and dynamic efficiency is at the root of most articles: securing a right to exclude others from using the innovation provides incentives to invest in R&D and promotes dynamic efficiency, but it reduces market competition and thereby static efficiency. Nordhaus (1969, 1972) formalizes this issue and design a socially optimal patent life which accounts for this trade-off. Recently, new contributions have highlighted the potential dynamic inefficiency created by IPRs (Bessen and Maskin, 2002; Boldrin and Levine, 2003).

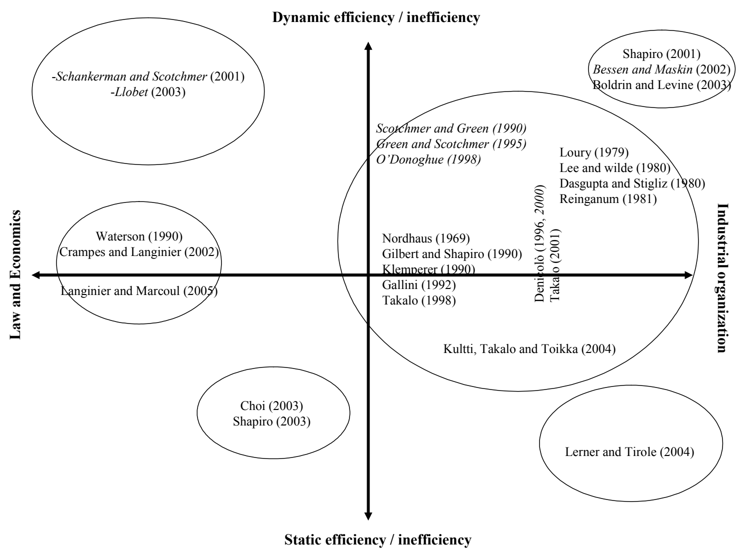


Figure 1. The economics of IPRs: a classification

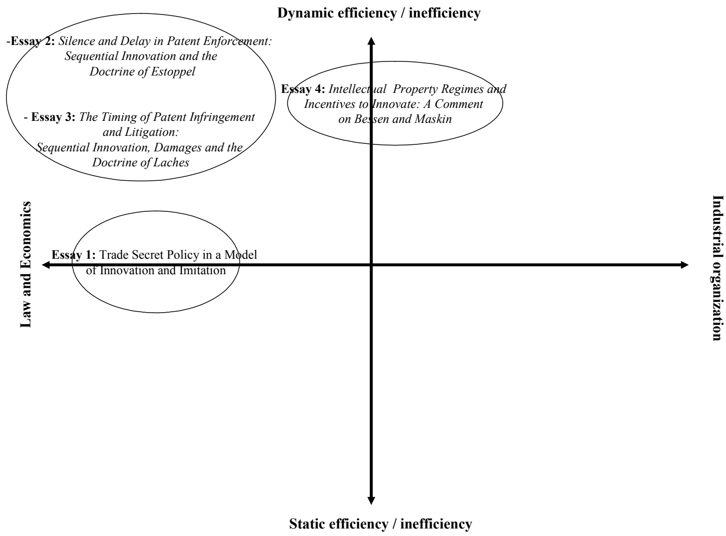


Figure 2: The contribution of the dissertation

The different essays of this dissertation build on gaps or issues identified in the literature. The economic literature on trade secret laws is still underdeveloped. Therefore, the first essay attempts to improve our understanding of the economic consequences of trade secret laws. Regarding patents and sequential innovation, the literature has largely overlooked the questions of patent litigation and litigation timing. The second and the third essays look at these aspects. Finally, in the fourth essay, a joint work with Klaus Kultti, we address the robustness of the model proposed by Bessen and Maskin (2002) for sequential innovations. But before I turn to the content of the essays, I present a review of this literature. Based on the framework[8] in Figure 1, I identify groups of papers that highlight particular problems and discuss these contributions in more depth.

One-shot innovation and the static vs. dynamic efficiency dilemma. After Nordhaus (1969, 1972), it is possible to distinguish two research programmes[9]. The first programme is represented by the so-called "patent race" literature, pioneered by Loury (1979) and Lee and Wilde (1980). It focuses on the effects of patents on R&D competition. A key insight from this literature, besides its fundamental breakthrough in modelling R&D competition, is the emphasis on the over-investment induced by a patent system. In the models proposed, the patent is assumed

[8]The literature on cumulative innovation is indicated in italics.

[9]This is developed in Takalo (1999).

to be "perfect" i.e to fully prevent imitation. Thus, the payoff gap between the winner of the R&D race (the first to patent) and the loser(s) is large. As a result, firms tend to overinvest in R&D (from society's point of view) and patents generate a waste of resources. The second research programme develops Nordhaus' initial contribution on patent design. The recognition of the static/dynamic efficiency dilemma is the cornerstone of all papers in this tradition. The major contribution of this literature is the introduction of a second instrument of patent policy, called "patent scope" (or "breadth") which determines the value of the flow of profit accruing to the innovator over the life of her patent. With two instruments (life and scope), the problem of designing a socially optimal patent becomes one of optimal mixing: which combination of scope and life maximizes social welfare? Gilbert and Shapiro (1990), Klemperer (1990), Gallini (1992), Takalo (1998), Kanniainen and Stenbacka (2000) all belong to this tradition. Papers differ in the way "scope" is modeled. In Gilbert and Shapiro (1990), the scope is just captured through the flow profit earned by the patentholder, while in Gallini (1992) or Takalo (1998), the scope depends on the rivals' cost of inventing around the patent. Contributors disagree about the socially optimal length/breadth mix. Denicolò (1996) proposes a theorem that reconciles these different results, showing that the optimal mix depends on the concavity in patent scope of the social welfare and the incentive to innovate functions. Takalo (2001) refines his findings. Denicolò (1996) also builds a model which combines a patent race stage with a market competition stage where patent protection is imperfect: he thus bridges the gap between the two research programmes mentioned.

Other papers analyze the optimality of a patent renewal system, through the payment of renewal fees. Scotchmer (1998) shows that, when firms have private information about R&D cost and innovation value, any direct incentive mechanism can be implemented by a renewal mechanism. Cornelli and Schankerman (1999) introduce moral hazard (on the R&D effort undertaken by the firms) and show that a menu of patent lives can do better than a uniform life.

Cumulative innovation and the issue of dynamic inefficiency. Scotchmer and Green (1990) and Green and Scotchmer (1995) are pathbreaking papers because they acknowledge the cumulative nature of innovation and investigate how the patent system can affect innovation incentives in this context[10]. Scotchmer and Green (1990) propose a model with two sequential innovations. There are two firms competing in each stage to obtain the innovation. The first

[10] Recognition of the cumulativeness nature of innovation, and its implication for patent policy, can be traced

innovation can be patented (disclosed) or kept secret. Various trade-offs are analyzed. For instance, patenting helps the rival to achieve faster the second innovation due to disclosure but it also protects the patentholder from independent discovery. Green and Scotchmer (1995) introduce the notion of "patent breadth" in the context of cumulative innovation[11]. The "breadth" of the patent determines whether a follow-on innovation infringes this patent or not. Essentially, the authors acknowledged a previously mentioned dilemma for sequential innovations: the first innovator must be rewarded for opening new research paths and so she should collect profits from innovations that build on her own. But this can reduce the incentives of follow-on innovators. They discuss mechanims that alleviate this tension (such as ex-ante agreements). Several papers have complemented this research. Chang (1995) shows that inventions having a small stand-alone value relative to subsequent improvements should be offered broad protection. O'Donoghue (1998) and O'Donoghue, Scotchmer and Thisse (1998) extend the analysis to an infinite sequence of innovations. O'Donoghue introduces the notions of "lagging breadth" (which determines whether a product of inferior quality infringes the patent) and "leading breadth (which determines whether a product of superior quality infringes the patent). These papers also incorporate an important fact: an innovation can be patentable even if it infringes a previous patent[12] (Merges and Nelson, 1990). By distinguishing "leading breadth" -which determines infringement of a previous patent- from "novelty" (or "patentability"), these papers advanced our understanding of patent law and its economic implications. Matutes, Regibeau and Rockett (1996) compare two protection regimes ("length" and "scope") when a patented innovation has applications for other markets. Denicolò (2000) extends his 1996's contribution to the case of cumulative innovations. He combines a two-stage patent races framework with a discussion of "forward patent policy". "Forward patent policy" is a policy determining how a patent can allow the innovator to benefit from subsequent (related) innovations by others. Denicolò considers two instruments of forward patent policy: "leading breadth" of the original patent and "patentability" of the follow-on innovation. All these papers show that a proper balance between rewarding an initial innovator and encouraging future ones can be achieved through a proper design of forward patent policy instruments.

In recent years, a more "radical" literature has emerged which argues that the sum of static

back to two non formal papers by Merges and Nelson (1990) and Scotchmer (1991).

[11] In another paper, O'Donoghue will call it "leading breadth".

[12] If the innovation improves sufficiently the previously patented innovation, it can be patented. Then, there are two "overlapping patents". The oldest patent is called "dominant" and the newest one is called "subservient".

and dynamic inefficiencies created by patents in key industries call for their abandonment. Bessen and Maskin (2002) argue that in industries where innovation is sequential and complementary, patents can reduce aggregate R&D and be socially detrimental. This is because a patent on an initial innovation confers its holder a property right over subsequent improvements.[13] In the fourth essay of this dissertation the robustness of Bessen and Maskin's result is challenged. Boldrin and Levine (2003) unveil conditions under which innovations can occur in a perfectly competitive environment, making patents a pure social cost.

Incorporating litigation and legal determinants in the economics of IPR. To the best of my knowledge, the first articles dealing with patent litigation (each in a different manner) are Meurer (1989) and Waterson (1990). Meurer (1989) builds a model of settlement in the shadow of litigation when the patentholder has private information regarding the validity of the patent. "Patent validity" is an important notion in this literature. The Patent Office must check the "patentability" of the innovation i.e check that it is novel, non-obvious and useful. Imperfect screening by the Patent Office encourages alleged infringers to challenge the validity of the patent once they are sued. Meurer compares the effect of two litigation cost allocation rules on the probability of settlement and litigation.[14] Aoki and Hu (1999) develop this line of investigation and model the settlement of patent litigation as a Nash bargaining game. Waterson (1990) proposes a three-stage game where the innovator decides whether to patent, the entrant decides where to locate in the product space and the (possible) patentholder decides whether to litigate the entrant. Settlement is ruled out. Waterson analyzes, *inter alia*, how the legal parameters of his model affect the decision to litigate and the entrant's decision to locate in the product space. This line of inquiry is pursued in Crampes and Langinier (2002). The authors allow for endogenous patent monitoring (the patentholder supervises the market to verify whether or not infringement occured). They also allow for settlement, as an alternative to trial and renunciation. They derive some counter-intuitive results: the likelihood of entry can increase with the penalty and with the cost of settlement. Choi (1998) notices that a trial transfers information about patent validity to future (potential) infringers. When

[13] In practice, the leading breadth of a patent is usually significant as explained by Merges and Nelson (1990) through the use of several examples.

[14] The "American rule" says that each party (the "plaintiff", i.e the patentholder and the "defendant" i.e the infringer) pay its own litigation costs, regardless who wins. The "British rule" says that the party which loses pays its own litigation costs and those of the winner.

entry (i.e infringement) is costly, this can have two effects. It can delay entry (infringers enter a war of attrition whereby each expects the other one to be litigated first so that if the patent is deemed valid, they avoid sinking the entry cost). But depending on the degree of patent protection, the information transeferred can also accelerate entry. Also, there is a growing literature focusing on specific legal doctrines. Lanjouw and Lerner (2001) show that patentholders can ask for a preliminary injunction[15] to create financial difficulties for the infringer. Schankerman and Scotchmer (2001) analyze damage doctrines ("lost profit" versus "unjust enrichment") and defense doctrines (the doctrine of laches). Aoki and Small (2004) look at the doctrine of "essential facilities". Llobet (2003) models the "doctrine of equivalents". Langinier and Marcoul (2005) analyze the role of the "doctrine of contributory infringement" in network industries. This doctrine states that anyone who materially helps another party to infringe a patent can be sanctioned as well. Anton and Yao (2005) analyze the "lost profit" doctrine of damages.

Of the last contributions mentioned, Schankerman and Scotchmer (2001) and Llobet (2003) are the most notable ones. Indeed, they recognize that, when innovation is cumulative and the follow-on innovation infringes a previous patent, a dispute may arise between the patentholder and the infringer. Since Green and Scotchmer (1995), most papers dealing with sequential innovation have assumed a division of profit between the two parties. But these papers abstract from the issue of how legal doctrines affect this division. By focusing on specific damage rules and other doctrines, Schankerman and Scotchmer (2001) and Llobet (2003) undoubtedly improved our understanding of the role of IPR when innovation is sequential.

The issue of complementarity. In a widely cited article published in the review *Science* in 1998, Heller and Eisenberg argue that patents can hinder innovation due to a complementarity issue. In the biomedical sector, most innovations can be developed only by combining a variety of "upstream" patented technologies used as complementary R&D inputs (or "research tools"). These upstream patents tend to form a complex net of rights: the risk of infringing one or several of the patents is high while the cost of securing a license from all upstream patentholders becomes very high. According to the authors, this situation of "blocking patents" reduces R&D incentives. Shapiro (2001) discusses how firms "navigate" this "patent thicket". He ex-

[15] The doctrine of preliminary injunctions is a motion which forces the alleged infringer to stop producing before the Court has reached its final conclusions on the case. Given that trials can last for years, this is a powerful instrument for patentholders. Stopping production can put the infringer in a difficult financial position.

plains the social benefit of "patent pools" and "cross-licensing agreements". A patent pool is an institutional arrangement whereby patentholders agree to license a pool of complementary technologies at a single price. A well-known result due to Cournot (1838) is that the selling of a bundle of complementary products by a single seller increases welfare compared to the situation where each seller independently prices one of the complementary goods. This result can be adapted to understand the benefit of patent pools. In an important contribution, Lerner and Tirole (2004) discuss various antitrust rules that govern the formation of these pools. Choi (2003) shows that the lower the validity of the patents, the more a patent pool should be encouraged.

Alternative forms of IP protection. Although most papers in the field of IPRs are interested by patents, there is a literature looking at alternative forms of IP protection. The literature on copyright emerged in the 1980's with the growing concern that technologies facilitating copying of copyrighted content could reduce artists' and publishers' incentives. Novos and Waldman (1984), Johnson (1985), Liebowitz (1985) pioneered this research. Takeyama (1994), Shy and Thisse (1999) show that in industries with network effects, such as the software industry, the absence of copyright protection can benefit creators. Recent contributions focus on modern issues such as Peer-to-Peer networks (Takeyama, Gordon and Towse (eds.), 2005), contributory infringement in network industries (Langinier and Marcoul, 2005) and Digital Right Management (Scotchmer and Park, 2005). Takalo (1999) compares copyright with patents in a general equilibrium search model. Copyright allows for multiple independent discoveries, contrary to patents. There is also a growing literature on trade secrets. Trade secrets are either considered as an alternative to patenting (Takalo, 1998; Anton and Yao, 2004) or as the only mean of protecting non-patentable innovations[16]. In the latter category, Rönde (2001) and Motta and Rönde (2003) look at firms' strategies (such as "covenants not to compete" in employment contracts) to reduce the risk of knowledge leakage due to rivals poaching employees.

The question of ex-ante versus ex-post licensing. Essays 2 and 3 in this dissertation deal with patent infringement when innovation is sequential and assume that ex-ante licensing (i.e. the patentholder offering a royalty contract to the potential infringer before he invests in the follow-

[16] In Europe, financial innovations and many computer-implemented innovations are still unpatentable subject matters, making secrecy a fundamental form of IP protection.

on innovation) cannot take place. Many of the inefficiencies discussed, such as hold-up, come from this assumption. Although in line with observations (see below), this modeling assumption is at odds with a strand of the literature - in particular Suzanne Scotchmer's contributions - which analyzes patent policy under the assumption that ex-ante licensing is always possible and always happens. Therefore, beyond refering to observations and statistics about the scarcity of ex-ante licensing, it is important to have a theory for why ex-post licensing and hold-up occur. This is proposed by Bessen (2004) and I find it important to report his argument. Consider a firm wishing to develop an innovation of value v at cost c. If this innovation does not infringe a patent, it is developed if $v \geq c$. Suppose this innovation infringes a patent. If the firm develops it without the consent of the patentholder, i.e. without an ex-ante license, it will have to secure a license ex-post. v will be shared, say $s_1 v$ for the patentholder ($s_1 \in [0,1]$). Ex-ante, the infringer will invest if and only if $(1-s_1)v \geq c$: with ex-post licensing, the hold-up issue implies that infringers with cost $c \in ((1-s_1)v, v]$ will not invest. This is a social cost. Suppose now the patentholder observes $c \in ((1-s_1)v, v]$. He will be willing to offer ex-ante a license with a royalty rate $s_0 \leq s_1$. The infringer invests as long as $(1-s_0)v \geq c$ and $(1-s_0)v \geq (1-s_1)v$ i.e. hold-up is mitigated. Because the ex-ante license solves the hold-up issue, we should never observe ex-post licenses. Bessen argues that this is at odds with facts: Anand and Khanna (2000) found that only 5% or 6% of licensing deals occurred ex-ante in most industries. Grindley and Teece (1997) show that major licensors such as Texas Instrument or Hewlett-Packard do not conclude ex-ante agreements. I add to Bessen's references the contribution of Arora, Cohen and Walsh (2003) who show that in a survey most firms acknowledged that they do not try to secure ex-ante licenses. Bessen (2004) show that ex-post licensing can occur in equilibrium and hold-up is not solved. His idea is that there is typically asymmetric information between the patentholder and the infringer concerning the cost of developing the innovation. Suppose c is distributed according to F conditional on $0 \leq c \leq v$ (with $F(0) = 0$, $F(v) = 1$, F twice continuously differentiable and log-concave). The patentholder offers s_0 that maximizes $s_0 v F((1 - s_0)v)$. There exists a unique interior solution s_0^*. If $s_1 \leq s_0^*$ the infringer will refuse the ex-ante offer and there will be ex-post licensing in equilibrium. If in addition $(1 - s_0^*)v < (1 - s_1)v < c$ the hold-up issue remains and there will be neither ex-ante licensing nor investment.

I now turn to the content of the dissertation. As I noticed above, the essays build on gaps or issues identified in the literature previously reviewed. The first essay attempts to improve our understanding of the economic consequences of trade secret laws. The literature has largely overlooked the questions of patent litigation and litigation timing when innovation is sequential. The second and the third essays look at these aspects. Finally, in the fourth essay, a joint work with Klaus Kultti, we address the robustness of the model proposed by Bessen and Maskin (2002).

3 Essays and results

In this section, I review each of the four essays. For each of them, I explain the question they seek to answer, I summarize the main features of the model and the main conclusions I reach.

3.1 Essay 1: Trade Secret Policy in a Model of Innovation and Imitation

Secrecy appears to be a crucial mechanism used by firms to protect their intellectual property. Nelson, Cohen and Walsh (2000) report that, in a survey administered to 1478 R&D labs in the United States, "patents tend to be the least emphasized [mechanism] in the majority of manufacturing industries and secrecy and lead time the most". Trade secrets are protected by a well-defined body of laws: criminal laws sanction anyone who attempts to acquire a trade secret by improper means (such as bribery) and civil laws allow for compensation of the trade secret owner when misappropriation is detected. In addition, it has been recognized by the Supreme Court of the United States in 1974[17] that the purpose of trade secret laws is to allow "the individual inventor to reap the rewards of his labor". In other words, trade secret law is considered in the U.S. as an instrument of innovation policy. A trade secret is not only an alternative available to innovators who decide not to patent. It is a positive form of IP protection, recognized by the Courts, and protected against misappropriation. Despite these elements, the economic literature has mainly focused on patent and the optimal design of patent law. Even if statistics do not exist, casual observations suggest that there is a disproportionate number of papers dealing with patents. In the literature, secrecy usually appears as a "default option" when the patent is foregone: the details of the innovation are not disclosed and if the information leaks out, the concealed knowledge is lost and no compensation is availaible for

[17] *Kewanee Oil Co. v. Bicron Corp.*

the trade secret owner. In recent years, important contributions have begun to fill the gap and analyze trade secrets in more depth (Rönde (2001), Motta and Rönde (2003)). Yet, very few papers attempt to formally tackle the issue of trade secret law design. To the best of my knowledge, exceptions are Friedman, Landes and Posner (FL&P) (1991) and Fosfuri and Rönde (2004). FL&P (1991) is mainly a verbal discussion of how trade secret law influences trade secret owners' incentives to invest in the protection of their trade secret. They do not introduce the two remedies for trade secret misappropriation (damages and criminal fines) in a game-theoretical model and thus they cannot design a socially optimal trade secret policy. This is what I do in this essay. Fosfuri and Rönde (2004) look at damages but not criminal fines, since they are not concerned with bribery. Also, they look at a model that substantially differs from the one proposed in this essay: in theirs, innovation is cumulative and trade secret leakage can occur through employees' poaching by rival firm. The novelty of the first essay is that it considers the design of a socially optimal trade secret policy when this policy consists of *two* legal remedies commonly used by Courts to protect trade secret owners: criminal fines and damages.

The model is simple. First, an innovator invests in R&D and, if successful, discloses the details of the innovation to an employee who is hired for production. Then, an imitator has two options: invest in reverse-engineering to duplicate the innovation (this is legal) or bribe the employee (which is illegal). Bribery is sanctioned with probability p (which reflects inter alia the probability of detection). The bribed employee has to pay the criminal fine F while the imitator (a firm) has to pay damages D in addition to the criminal fine. Only the damages D compensate the trade secret owner for the loss of profit due to illegal imitation. The criminal fine has only a deterrence effect and does not compensate the trade secret owner.[18] I seek to answer two questions: *i)* How do changes in the legal parameters (damages and criminal fine) affect incentives to imitate and innovate? *ii)* What is the socially optimal combination of criminal fines and damages?

[18]This is even clearer if the criminal penalty considered is imprisonment instead of a monetary fine. I consider this possibility in the essay.

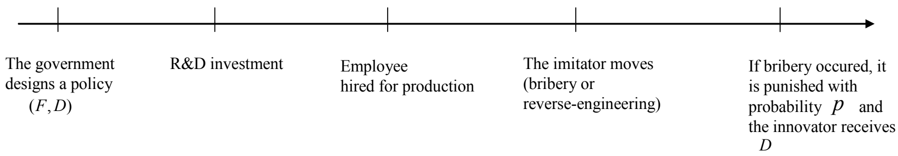


Figure 3: timeline

The positive analysis of damages and criminal fines. Concerning the first question, the most notable result is that the innovator's payoff may *decrease* when the criminal fine increases. This is counter-intuitive since the criminal fine aims at protecting the trade secret owner from misappropriation. I show that an increase in F enhances the imitator's incentive to reverse-engineer the innovation instead of bribing the employee. For any given level of damages D, a higher probability of reverse-engineering implies a lower probability that the innovator gets the compensation D. Indeed, this compensation is earned only when bribery takes place and is detected.

A normative inquiry: trade secret law design. The socially optimal trade secret policy is designed to take into account a classic trade-off in the economic analysis of IPR. Setting F and D so high that imitation is deterred may guarantee a monopoly to the trade secret owner. This provides the highest incentives for innovation. But at the same time, monopoly distortions lower static efficiency. In contrast, a moderate level of damages can allow the imitator to stay in the market and the resulting duopoly enhances static efficiency. The cost of this solution is that the damages may not be high enough to provide as much innovation incentive as when imitation is deterred. I show that the acquisition of the trade secret through bribery is socially optimal if the imitator cannot reverse-engineer. If he can, reverse-engineering may be the socially optimal acquisition option. If it is socially optimal that bribery occurs, the optimal level of damages is lower when the imitator can reverse-engineer than when he cannot. This is because the level of reverse-engineering increases with the level of damages: the higher the damages, the more the imitator wants to invest in reverse-engineering to avoid bribery. The main contribution of this policy analysis is to show that, regardless whether the imitator can or cannot reverse-engineer, it is always possible to implement the socially optimal policy by a strictly positive level of damages and a criminal fine equal to zero.

3.2 Essays 2 and 3: Sequential Innovation and Hold-Up

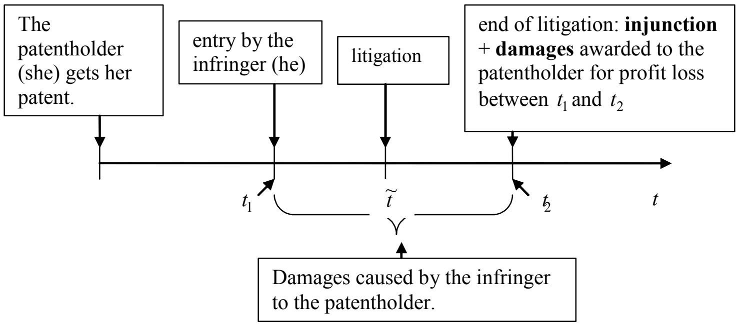
In the next two essays, I turn to patents. These essays deal with sequential innovation and more precisely the "hold-up" issue that may arise in this context. Firms often infringe previous patents when they develop their own. Consider for instance the early years of the aviation industry. The Wright brothers held a very broad patent on a pioneering method that enabled to pilot an airplane sustaining controlled flight. Glenn Curtiss came up with another innovation improving on the Wright's technology: he introduced the use of a steering on a stick, the control device still used today. The Wright brothers sued arguing that Curtiss' improvements fell into the boundaries of their patent[19]. When an innovator develops an application of a previously patented innovation, or when he uses this patented innovation as an input in his R&D process ("research tools" are often patented), he exposes himself to litigation by the patentholder. I call the "infringing" innovation a "follow-on" innovation. The patentholder has an incentive to sue for infringement in order to obtain compensation. I focus on this situation that appears to be widespread. In the literature on patent policy for sequential innovation, I follow Chang (1995) or Denicolò (2000) who assume that licensing agreements are not possible before the infringer engage in R&D. A strong argument in favor of this assumption is that the follow-on innovator may be reluctant to disclose his idea to the patentholder, by fear that the latter could steal it. I presented previously Bessen's theory for why ex-post licensing might occur in equilibrium, creating hold-up. What distinguishes my approach from most of the previous literature is that I provide a formal model of patent litigation over sequential innovations. To do so, I ask a general question: What defenses are available to a company which developed a follow-on innovation infringing a previous patent? I concentrate on two related defenses called the "doctrine of estoppel" (essay 2) and the "doctrine of laches" (essay 3). In essence, both doctrines "punish" a patentholder who delayed enforcing her patent against the alleged infringer. Hence, analyzing these defenses allows me to focus on another largely overlooked question: the timing of patent litigation. The requirements of the doctrines and their consequences differ. These differences are reported in Table 1 below.

[19]See Shulman (2002).

	the doctrine of estoppel	**the doctrine of laches**
requirements	The patentholder threatens to litigate and then delays litigation	The patentholder delays litigation
effects	The patent is unenforceable (the patentholder cannot obtain an injunction or damages and the infringer is free)	The patent remains enforceable. The patentholder cannot collect damages for infringement that occured *during the delay period*. But she can obtain compensation if the infringer wants to continue infringing the patent
comparison		The effect is less stringent than under the doctrine of estoppel. This is because the patentholder did not *threaten* to litigate at the outset.
rationale	The law considers that the infringer may be hurt by delayed litigation: he may interpret a delay as a sign that litigation will not take place. Thus, he may invest in the infringing activity or destroy evidentiary documents that would be useful if litigation took place.	

Table 1: Differences between the doctrine of estoppel (Essay 2) and the doctrine of laches (Essay 3).

In Figure 4, I propose a simplified representation of how patent Law compensates a patentholder for infringement, in the case of a dispute over sequential innovations. Typically, damages can be awarded to the patentholder to cover the loss that occured prior to the judgment (this loss represents the licensing revenues that the patentholder should have earned, had a licensing agreement taken place). In addition, an injunction can be granted. It forces the infringer to stop production and negotiate a license with the patentholder in order to continue to use the patented innovation. In general, this license allows the patent holder to get revenues equivalent to the damages awarded by the Court.



Injunction: the infringer is forced to stop infringement and settle with the patentholder (if he wants to continue using the patented innovation after t_2).

Damages: a transfer from the infringer to the patentholder to compensate the patentholder for infringement that occurred between t_1 and t_2.

Figure 4: Definitions of damages and injunctions when innovation is sequential.

3.3 Essay 2: Efficient Delay in Patent Enforcement: Sequential Innovation and the Doctrine of Estoppel

One defense available to a company which developed a follow-on innovation infringing a previous patent is the "doctrine of estoppel". According to this rule, if the patentholder threatened to sue and then remained silent for an "unreasonably long time", the patent may be simply unenforceable.[20] I construct a game-theoretical model to assess the effect of this doctrine on the incentives to invest in the follow-on innovation and litigate the follower. This model enriches our understanding of patent disputes and unveils new determinants of patent policy for sequential innovations.

I build on an idea from Schankerman and Scotchmer (2001): patentholders can be informed of infringement and litigate infringement before the infringing product is fully developed and brought into the market. For example, biotechnology companies often learn that a pharmaceutical firm is infringing one of their patents while the pharmaceutical firm is still developing its

[20] See Table 1.

new drug. My model is as follows. A patentholder realizes that her patent is being infringed. The infringer still needs to invest resources in developing a commercializable product based on a prototype that has already been obtained. Investment is endogenized and determines the probability that development is successful. The patentholder can enforce her patent before or after development succeeds. If she decides not to enforce immediately, she can decide to initially "threaten to sue" the infringer.[21] If the infringer does not respond to this threat, she can remain silent until after development has succeeded. This strategy exposes herself to the application of the doctrine of estoppel[22]. Building on the remarks in Lemley and Shapiro (2005), I assume uncertainty in the application of the Law so that the doctrine of estoppel applies probabilistically, even when its basic requirements are fulfilled. Given this basic set-up, I ask: *i)* How do the doctrine of estoppel and patent validity affect the patentholder's and the infringer's payoffs?[23] *ii)* When and how does the doctrine of estoppel constitute a new instrument of "forward patent protection"?[24]

Players' payoffs. My main contributions here are to show that the infringer can be better off if the probability that the doctrine of estoppel applies decreases and if patent validity increases. Both results are counter-intuitive. They arise from the fact that an increase in these parameters can induce an equilibrium switch that hurts the infringer. More accurately, an increase in the estoppel probability can induce the patentholder to enforce her patent before development of the follow-on innovation while the infringer may prefer enforcement to happen after. A decrease in patent validity can have the same effect on the patentholder's enforcement timing. More importantly, I show that both parties, the patentholder and the infringer, are better off when the doctrine's design is such that the patentholder first threatens to sue the infringer and then remains silent until after development has succeeded.

The doctrine of estoppel can be designed to alleviate the hold-up problem. I show that, when the infringer is credit-constrained, the doctrine of estoppel can work as a new instrument of "forward patent protection". The probability that the doctrine of estoppel applies can be

[21]In practice, patentholders send "notice of infringement" to alleged infringers, arguing that they will vigorously enforce their patents. There is no standard form for the notice of infringement: it is basically a letter informing the infringer of the patentholder's intentions.

[22]See Table 1 again.

[23]It is important to look at this dimension as final payoffs are the ultimate sources of innovation incentives.

[24]I defined "forward patent protection" in section 2.

designed so as to minimize the hold-up problem arising from the fact that the patentholder can collect revenues from the follow-on innovation. It turns out that it is socially optimal to design the doctrine so as to generate a threat of litigation followed by a period of delay. However, when the infringer is not credit-constrained, the doctrine of estoppel does not alleviate the hold-up problem.

3.4 Essay 3: The Timing of Patent Infringement and Litigation: Sequential Innovation, Damages and the Doctrine of Laches

The spirit of this third essay is close to that of the previous one. I still focus on patent litigation when innovation is sequential. A follow-on innovation infringes a previous patent and the patentholder has to enforce her patent if she wants to obtain compensation. However, there are three main differences with the second essay.

- First and foremost, I endogenize the timing of investment in the infringing innovation. There are two periods and, at the outset, the demand for the infringing product is uncertain. Uncertainty is resolved eventually. Given that investment involves a sunk cost K, the infringer, who is the leader of the game, has to decide whether to invest before or after uncertainty is resolved. In contrast, the timing of investment is irrelevant in the model of the second essay because there is no exogenous uncertainty regarding the profitablity of the follow-on innovation. Exogenous uncertainty is introduced to reflect a reality affecting all innovative industries. Prominent examples include the pharmaceutical industry or the aviation industry.
- Second, I introduce litigation costs c.[25] If litigation takes place, both the patentholder and the infringer have to bear these costs. The patentholder is the follower in the sense that she reacts to infringement. The patentholder's compensation consists in a fraction of the infringer's profits[26]. If the infringer invested before uncertainty was resolved, the patentholder herself faces a "real option" problem. With costly litigation, she can litigate in period 1 or delay until uncertainty is resolved.

[25] In the second essay, I abstract from these costs for tractability reasons: they would not alter the key insights but they might make the model more cumbersome.

[26] This compensation rules captures the essence of the "reasonable royalty" damages doctrine, which is discussed in the essay.

- Third, a delay in litigation can be sanctioned. I focus here on the "doctrine of laches".[27] According to this doctrine, a delay does not make the patent unenforceable (as the doctrine of estoppel does). It simply prevents recovery of damages that occured during the delay period. In the model, if the patentholder does not litigate in period 1 but delays until period 2, she cannot recover period 1 damages but is entitled to compensation if the infringer wants to continue using the patent in period 2.

Arguably, this model is stylized. Yet, it allows me to investigate issues that have not been looked at before. I seek to answer two questions: *i)* First, how do the patentholder's compensation and the doctrine of laches affect players' payoff? *ii)* Second, how do they affect the timing of investment in the follow-on innovation?

Players' payoff. I show that counter-intuitively an increase in the patentholder's compensation can make her worse off. The explanation differs substantially from a similar result in the second essay[28]. Here, an increase in the patentholder's expected compensation reduces the share of the profits obtained by the infringer. Ceteris paribus, this can encourage the infringer to delay investment and forego period 1 profits. A consequence of this is that litigation becomes unprofitable for the patentholder: he would only obtain period 2 damages and this is not enough to cover the cost of litigation. Also counter-intuitively, the doctrine of laches can hurt the infringer, although it is designed to protect him.

The timing of investment in the follow-on innovation. I find that an increase in the patentholder's compensation can delay or speed-up investment. Also, the doctrine of laches can have the same two opposite effects on investment timing. The occurence of one of these two outcomes depends on the parameters of the model. The doctrine of laches can encourage the patentholder to litigate in period 1, i.e. before uncertainty is resolved, instead of delaying. This increases the cost of infringement in period 1 which is now equal to the sunk investment cost K and the litigation cost c. *Ceteris paribus*, this can encourage the infringer to delay investment. The doctrine of laches can also deter the patentholder from litigating in any period. In that case, infringement is not penalized and the infringer obtains the full profit from his innovation. Everything else equal, this increased expected payoff can induce him to invest "earlier", i.e in period 1.

[27] See table 1.

[28] There, I show that an increase in patent validity can make the patentholder worse-off too.

My contribution offers a new perspective on understanding the drivers of innovation is some key industries. In the aviation industry, the construction of new airplanes is usually decided on the basis of partial information about the potential demand. Demand uncertainty is typically resolved over time. At the same time, constructors like Airbus or Boeing permanently innovate and are likely to face patented technologies that need to be incorporated in the aircraft design. Ex-ante licensing agreements with competitors who own these patents is often excluded because of the risk that this competitor would steal the development idea. The present analysis offers some insights on how to handle patent disputes in this industry. Also, my contribution sheds light on the heated debate about patent "trolls". These are patent licensing companies who often delay aggressive enforcement againts manufacturers to extract the highest surplus. Provided it is well designed, the doctrine of laches could be an instrument against "trolls".

3.5 Essay 4: Intellectual Property Regimes and Incentives to Innovate: A Comment on Bessen and Maskin (joint with Klaus Kultti)

A shorter version of this essay has been published as a chapter in Bruun (ed.) (2005). In this essay, we revisit a widely discussed paper by Bessen and Maskin (B&M) (2002) entitled "Sequential Innovation, Patents and Imitation". The authors argue that in industries where innovation is sequential, like the software and the semiconductor industries, patents can *hinder* innovation instead of encouraging it. This paper has played an important role in policy debates in Europe, when the European Commission launched the discussions on the project of "patents for computer-implemented innovations" (quickly assimilated to "software patents"). Recently, on July 7, 2005, the European Parliament rejected the legislation that would have allowed patents for softwares. Our intent in this essay is twofold. First, and mainly, we assess the robustness of B&M's major result. Second, we investigate which of three alternative intellectual property regimes (patent, copyright and no protection) is socially preferable. B&M compare two IP regimes (a "patent" regime and a regime with no protection) first in the context of a one-shot innovation, then when innovation is sequential. In the latter case, a patent on the first innovation confers a property right over subsequent innovations. This happens in many industries where innovation is "incremental" because small improvements often end up within the scope of the initial patent. Because of transaction costs, it is possible that the patentholder cannot license its technology to her rival.[29] The latter is excluded from future R&D and aggregate R&D is reduced. In a regime with no IP protection, no firm is ever excluded: the dynamic incentives associated with the prospect of being always in the R&D race can outweigh the static disincentives associated with the absence of property right over each innovation (and the resulting imitation).

Our main objective being to test the robustness of B&M's analysis, we seek to remain as close as possible to the original framework, allowing only for two modifications. It turns out that these modifications yield contrasting and refined results.

Robustness. B&M consider a model where one firm conducts R&D and a second firm has to decide whether or not to engage in R&D too, i.e pay the *exogenously* given R&D cost c. Instead, we assume that the two firms simultaneously (and non-cooperatively) decide their level of R&D effort: we *endogenize* the level of R&D investment. We obtain that patents *always* yield more aggregate R&D than a regime with no IP protection. This is in contrast with B&M. When the R&D effort is endogenized, in a patent regime, firms have a strong incentive to be the winner of the first patent. Indeed, because a patent excludes the loser of the first R&D race from future R&D, firms try to win the first contest and invest much for that. In contrast, in a regime with no IP protection, the loser of the first R&D contest is not excluded from future R&D, i.e. she can use the idea of the first innovation to conduct future research: given that R&D is always possible, obtaining the first patent is not essential: R&D incentive measured as the endogenously determined R&D investment is lower.

The socially preferrable IP regime. We also show that patents usually yield overinvestment (from society's view point), a point already made by the "patent race" literature. However, we obtain this outcome in a model of cumulative innovation. The explanation follows from the previous remarks. A patent regime creates excessive incentives to be the winner of the pioneer (first) patent. We introduce a third IP regime, which is "moderate" in the sense that it imperfectly protects against imitation and does not exclude the loser of the first race from participating in the subsequent ones. We call it a "copyright regime" because we believe its features are consistent with Case Law over copyrights. Contrary to B&M who informally argue that a copyright regime is socially preferable because it allows for *more* aggregate R&D,

[29] The transaction cost considered by Bessen and Maskin is an asymmetry of information regarding the rival's R&D cost: that can prevent licensing from occuring.

we propose the opposite justification: copyrights can solve the problem of overinvestment by *moderating* R&D incentives.

4 Implications and new challenges

Research opens up more questions than it provides answers. This dissertation seems to follow this rule. Here, I would like to briefly develop some implications of my results for technology policy and mention challenges for future research.

4.1 Implications for technology policy

The very rationale behind technology policy is the belief in the existence of market failures for the supply and the diffusion of new technologies. Indeed, Mowery (1995) defines technology policy as a group of "policies that are intended to influence the decisions of firms to develop, commercialize or adopt new technologies". In that sense, IPR legislation exemplifies technology policy. IPRs intend to remedy the suspected[30] lack of innovation incentives that would occur in an environment where innovations are not protected. Fine-tuning the instruments of IPR policy guarantees innovation incentives while allowing for technology diffusion that can spur both imitation, which enhances static efficiency, and future innovation. One of the objectives of this dissertation is to unveil new potential instruments of technology policy, integrate them in game-theoretical models and derive quantitative and qualitative conclusions regarding their effects on innovation. The instruments considered are widely used legal mechanisms such as "criminal fines" for trade secret misappropriation or the "doctrine of estoppel" for delays in patent litigation.

The virtue of a criminal fine equal to zero. When trade secrets are protected against misappropriation by two mechanisms, damages and criminal fines, it is always possible to design a socially optimal policy by setting the criminal fine equal to zero. Only damages are used as instruments of innovation policy. In the literature on crime and punishment, a seminal result due to Becker (1968) is that criminal fines should be *maximized* so as to guarantee deterrence

[30] This common wisdom, already mentioned above, tends to ignore the role of secrecy and lead time as drivers of (temporary) monopoly situations.

while minimizing the cost of detection and enforcement. But in the context of trade secret theft, when damages are awarded, criminal fines should be *minimized* because increasing them provides incentives for reverse-engineering which is a pure social cost[31].

Patent breadth does not guarantee forward patent protection. This is actually implied by the very *existence* of the doctrine of estoppel: even if there is actual patent infringement by a follow-on innovation (because the "leading breadth" of the patent is wide enough), the patentholder may not collect revenues from the infringer due to the interposition of the doctrine of estoppel. In other words, infringement does not guarantee compensation. In the economic context analyzed in essay 2, provided the doctrine of estoppel is well-designed (provided it is enforced probabilistically), it turns out that the patentholder can benefit from the doctrine of estoppel. This analysis suggests that the doctrine of estoppel, like the doctrine of laches for that matter, should be considered as new instruments of "forward patent protection" (O'Donoghue (1998), Denicolò (2000)).

Patent Law can affect the timing of innovation The literature on "real options" tells that technology policy can affect not only the supply and the diffusion of innovations, but also the timing of new innovations. Kanniainen and Takalo (2000) show how the existence of patents can slow down technological progress. In the third essay, I show how specific legal rules (the doctrine of laches and the doctrine of "reasonable royalty" damages) can have contrasted effects on the timing of innovation. Depending on parameters values, the dcotrine of laches can speed-up or delay the follow-on innovation. The level of damages accruing to the patentholder also affects this timing.

When innovation is sequential, a "moderate" IP regime can improve social welfare by reducing R&D incentives. The fourth essay shows how policy recommendations can be sensitive to modeling choices. We challenge Bessen and Maskin's idea that a patent system reduces aggregate R&D when innovation is cumulative. We also stress that a moderate IP regime, akin to a "copyright regime", may be useful, not in encouraging R&D as Bessen and Maskin argue, but in decreasing R&D intensity.

[31] I explain in section 7 of the essay that in a more dynamic perspective, reverse-engineering may have benefits since it enables engineers to better understand how an innovation "works".

4.2 Challenges for future research on the economics of IPRs

I believe economics should continue to dig deeper into IP laws and elaborate models that enable us to assess the efficiency properties of specific legal rules, in specific economic contexts. In that sense, both the mushrooming literature mentioned in section 2 and the essays in this dissertation go in this direction. Undoubtedly, this line of research suggests several possible inquiries. My research on trade secret laws could be extended to account for sequential innovation. As mentioned in section 7 of the essay, the optimal combination of criminal fines and damages could be affected by this alternative environment. The doctrines of estoppel and laches could be analyzed in a more static framework where the patented innovation would be imitated for commercial purpose (and not only used as a basis for a follow-on innovation). Even though I believe the most interesting results are obtained in the context of sequential innovation, imitation would still deserve to be looked at. The model of cumulative innovation in the last essay captures the "strength" of IP protection against imitation as the probability that the innovator obtains an injunction forcing the imitator to exit the market. This is a very stylized representation of IPR litigation, and it would be interesting to embed a more elaborate litigation structure in a model of cumulative innovation with an infinite horizon. Also, combining both "lagging breadth" (protection against imitation) and "leading breadth" (protection against future innovations) in the same model should be feasible[32]. So far, our model treats "leading breadth" in a discrete manner (either first innovator has an infinite right over future innovations, or she has no right): this could be relaxed and a more realistic model of leading breadth could be investigated.

Future research should also try to connect the IPR field with other branches of the economic literature. Even if the taxonomy illustrated by Figure 1 is not exhaustive, it is fair to say that papers seldom inquire IPR from a "political economy" point of view or a "financial economics" point of view. The majority of the literature proposes an "Industrial Organization" approach or a "Law and Economics" approach. Below, I mention some notable exceptions and I briefly develop ideas for future research in two domains: the political economy of IPR and the relationships between IPR and finance.

The political economy of IPR. Scotchmer (2004) proposes a political economy model of IPR.

[32] O'Donoghue (1998) eventually abstracts from "lagging breadth" and focuses only on "leading breadth" and the "patentability requirement".

She chooses to focus on IP "treaties" i.e. on global issues such as the harmonization of domestic IPR legislations and the "national treatment" of foreign inventors. She uses simple partial equilibrium welfare measures and discusses countries' incentives to agree on these issues. Other papers analyze IPR in North-South trade models (Helpman (1993), Grossman and Lai (2001)). But none of these contributions incorporate the "state of the art" from the political economy literature. It would be worthwhile to introduce lobbying models in the spirit of Grossman and Helpman (2001) in the research on IPR policy. Lobbying governments seems to be a common practice and understanding the welfare effects of this practice constitutes an important agenda.

Finance and IPR. There is a need to combine the economic literature on corporate finance with the literature on IPR. There are two reasons for that. First, the literature on enterprise financing and financial constraints often ignores the possible role of IPR as a collateral in debt financing contracts. The reason for this shortcoming is that banks have long been reluctant to use intangibles as a collateral due to the difficulty to assess their actual value. Progress in patent valuation tends to modify this. According to Kramer and Patel (2003), "the value of intangible assets as a percentage of market capitalization of US companies increased from 20% in 1978 to 73% in 1998". As a result, using intangibles in general and IPR in particular as collateral is an important objective that could decrease the "financing gap" for small innovative firms. This calls for further theoretical and empirical research. Second, IPR management in itself can be a financial burden: a patent must be monitored and enforced if necessary. This is has substantial costs. Llobet's and Suarez's (2005) work is a first attempt to tackle the issue of patent litigation financing. They propose a model that compares the resorting to a financier after infringement has occured with alternative arrangements such as a "patent litigation insurance" scheme. Their simple model could be extended in several directions. For instance, they only consider insurance schemes for potential plaintiffs (patentholders) but not for defendants (alleged infringers). Investigating the latter type of scheme (called "liability insurance") would reward future research.

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Trade Secret Policy in a Model of Innovation and Imitation

Abstract

The paper proposes a leader-follower model of innovation and imitation when the innovation is protected by a trade secret. The imitator has two non-exclusive options to acquire this innovation: to bribe an employee of the innovating firm or to engage in costly, uncertain but legal reverse-engineering. Misappropriation through a bribe is sanctioned by criminal fines (for the imitator and the employee) and civil damages (for the imitator). The paper investigates how changes in the legal environment affect players' incentives, and analyzes trade secret law design. It shows that counterintuitively the innovator's payoff can decrease when the criminal fine increases. Also, it is socially optimal to set the criminal fine to zero for both the imitator and the employee. Bribery can be socially optimal (it avoids monopoly distorsions and the waste of reverse-engineering costs) provided intellectual property protection is ensured by a strictly positive level of damages (to compensate the innovator and thereby provide R&D incentives) which differs depending whether the imitator can or cannot engage in reverse-engineering.

JEL classification codes: *K42* (Illegal behavior and the enforcement of the law), *O31* (Innovation and Incentives), *O32* (Management of technological innovation and R&D), *O34* (Intellectual property rights).

Keywords: trade-secret, reverse-engineering, bribery, damages, criminal fines, deterrence, innovation incentives.

1 Introduction

Secrecy appears to be a major mechanism used by firms to protect their intellectual property (see below Cohen, Nelson and Walsh, 2000). In this paper, I address the largely overlooked question of a socially optimal trade secret policy. Following current legal practices, two sources of law are considered: criminal law (with criminal fines) and civil law (with damages), which both protect from trade secret misappropriation such as bribery. I have three main results: First, increasing the criminal fine may counter-intuitively hurt the trade secret owner. Then, it is possible to implement the socially optimal trade secret policy by a criminal fine set to zero and a strictly positive level of damages, which varies depending on the ability of the imitator to conduct reverse-engineering. Finally, bribery may be socially preferred to either reverse-engineering or to the absence of entry by the imitator. These conclusions offer new insights on intellectual property design and innovation policy.

According to the Uniform Trade Secret Act (UTSA)[1], a trade secret is any piece of "information including a formula, pattern, compilation, program, device, method, technique, process, that: *i)* derives independent economic value, actual or potential, from not being generally known to the public or to other persons who can obtain economic value from its disclosure; and *ii)* is the subject of efforts that are reasonable under the circumstances to maintain its secrecy". A trade secret owner is protected by the law from a rival who acquires the innovation by improper means (such as bribing an employee) but not from a rival who independently duplicates the innovation. A trade secret owner can sue a suspected rival and obtain civil remedies in the form of damages and/or injunctions. In addition, several countries have criminalized the misappropriation of trade secrets subjecting anyone who knowingly acquires or reveals a trade secret by improper means, without the consent of the owner, to fines or emprisonment. An economic analysis of trade secret law is relevant for, at least, three reasons. First, since 1974 and the Supreme Court decision in *Kewanee Oil Co. v. Bicron Corp.*, it has been recognized that the purpose of trade secret law is to allow "the individual inventor to reap the rewards of his labor". In other words, the protection of trade secrets against acquisition by improper means aims at providing incentives both against misappropriation and for innovation in the first place. Despite this decision, Courts continue to diverge in the way civil remedies are computed. Second, in an empirical analysis based on a questionnaire administered to 1478 R&D labs in the United States, Cohen, Nelson and Walsh (2000) show that, of all the mechanisms used by firms to protect their innovations, "patents tend to be the least emphasized by firms in the majority of manufacturing industries and secrecy and lead time tend to be emphasized most heavily". Finally, Lerner (1994) shows that in a sample of 530 manufacturing firms from Massachusetts, 43% of intellectual property litigation involved trade secrets.

[1] One of the main texts regarding trade secrets law in the United States.

These elements suggest that trade secrets are a crucial tool used by firms to protect their intellectual property and that trade secret laws should deserve a careful investigation. Yet, so far, economic research has mainly focused on patents and patent law. In particular, the "patent design" literature has examined various issues such as the optimal length/breadth mix, both in a static setting and in models with sequential innovations (Gilbert and Shapiro (1990), Klemperer (1990), Gallini (1992), Green and Scotchmer (1995), Denicolò (1996) or Takalo (1998)). This paper aims at filling a gap in the previous literature by introducing an analysis of "trade secret law design". To do so, a simple innovation-imitation model is developed where imitation can take two forms: by reverse-engineering, the imitator can duplicate the innovation. This is legal. He can also bribe an employee of the innovating firm to obtain the details of the concealed innovation and avoid having to conduct reverse-engineering. Such misappropriation is illegal. These two options are not mutually exclusive: investment in reverse-engineering is endogenized, in the spirit of Takalo (1998) or Kanniainen and Stenbacka (2000), and, in case of failure, the imitator can still engage in bribery. Alternatively, he can propose a bribe without sinking the cost of reverse-engineering. This set-up distinguishes imitation of an innovation protected by a patent from imitation of an innovation protected by a trade secret: the disclosure requirement in patent law makes bribery essentially unnecessary. Which imitation strategy is followed is determined both by the cost of reverse-engineering and by the legal environment. The legal environment consists of two instruments: damages (a civil remedy) aiming at compensating the trade secret owner for misappropriation and at deterring misappropriation, and a criminal fine (paid by both the imitator and the employee) only aiming at deterring misappropriation, as the fine is not a transfer to the trade secret owner but to the government. The objective of the paper is to analyze the optimal combination of damages and criminal fines.

Results. First I find that although increasing damages benefits the trade secret owner, increasing the criminal fine may hurt her. This is because a higher criminal fine increases the incentives for reverse-engineering whereby the innovator receives no compensation. Also, the trade secret owner can be better-off when the imitator is more efficient in financing reverse-engineering or when the imitation becomes technologically easier to obtain than the innovation. Second, I show that the acquisition of the trade secret through bribery is socially optimal if the imitator cannot engage in reverse-engineering but if he can, reverse-engineering may be the socially optimal acquisition option. Indeed, when reverse-engineering is not possible, bribery guarantees a duopoly and a proper level of damage compensation can implement satisfactory innovation incentives. But when reverse-engineering is possible, the innovator is better off when the imitator prefers it to bribery. Thus when innovation incentives are more important than monopoly distorsions, society should design a trade secret policy that implements reverse-engineering. Third, if it is socially optimal that bribery occurs, the optimal level of damages is lower when the imitator can reverse-engineer than when he cannot. This is because damages increase investment in reverse-engineering which is a waste of resources for society. Damages should thus be reduced to mitigate this cost. Finally, regardless whether the imitator can or cannot conduct reverse-engineering, it is always possible to implement the socially optimal policy by a strictly positive level of damages and a criminal fine equal to zero. This is because the criminal punishment does not compensates the innovator and thus fails to provide innovation incentives.

Previous literature. This paper is related to three strands of the literature. First, it relates to the literature on intellectual property rights with reverse-engineering. Gallini (1992), Takalo (1998) and Kanniainen and Stenbacka (2000) show that allowing for costly imitation has implications for patent policy. More recently, Maurer and Scotchmer (2002) analyze the independent inventor defense (forbidden by patent law) in a model where the cost of duplicating the invention is explicitly introduced. They show that if the independent inventor defense is allowed, the original inventor might have an incentive to license its technology to preempt duplication by competitors. Samuelson and Scotchmer (2000) informally discuss the law and economics of reverse-engineering emphasizing inter alia the benefits of reverse-engineering which allows follow-on innovators to better "de-construct" technically challenging inventions, develop know-how and thereby socially valuable improvements. Finally Scotchmer and Park (2005) propose a model where technical protection against piracy of digital goods can be circumvented at some cost. They show that the threat of circumvention lowers the price of digital content. Second, my paper relates to the literature on trade secrets. Friedman, Landes and Posner (1991) informally discuss the economics of trade secret law and more formally investigate the innovator's incentive to invest in measures intended to conceal her innovation, in the absence of strategically behaving rivals. I abstract from this issue in my model[2] and focus more on the trade-offs faced by the imitator in acquiring the trade secret. Fosfuri and Ronde (2004)

[2] I nevertheless discuss the question in section 7.

propose a two-stage cumulative innovation model where the loser of the first-stage innovation (protected by a trade secret) can hire the employee of the winner. They do look at damages, but not at criminal fines as they do not consider bribery as the vehicle of misappropriation. They focus on how firms' incentive to "cluster" (i.e to locate in the same geographical area) is influenced by the parameters of the model, including damages.[3] Motta and Ronde (2004) analyze the effects of covenant-not-to-compete clauses in employment contracts. They show that, counter-intuitively, the absence of such covenants can benefit the trade secret owner as the threat of the employee leaving for another firm forces this trade secret owner to pay a bonus to her employee, thereby raising his incentives to work. Third, although patenting is not an option in my paper, it does relate to the literature on patenting versus secrecy. Denicolò and Franzoni (2004) compare patent and secrecy in a model where reverse-engineering is endogenized and show that the disclosure motive alone is sufficient to justify the grant of patents. Kultti, Takalo and Toikka (2006) propose a general equilibrium model of innovation and argue that patenting might be preferred to secrecy even when the protection offered is low: indeed, the trade-off is not between patenting or concealing but between patenting or letting another firm patent.

A roadmap. In section 2, I briefly review criminal and civil protection against trade secret misappropriation in Europe and in the United States. In section 3, I present the assumptions of the model. This model is a simple game of innovation and imitation and it is solved by backward induction. Hence, in section 4, I start by deriving the equilibrium outcomes of the imitation subgame. Moving one step backwards, I investigate in section 5 the incentive to innovate. More accurately, I analyze how changes in the parameters of the model affect innovation incentives. In section 6, I design a socially optimal trade secret law. In section 7, I summarize my main findings and discuss some of my modeling choices and unaddressed issues.

[3] As they acknowledge in their last section, they cannot propose a full welfare analysis as they do not have a specific model of market competition and most of their analysis does not endogenize R&D investment. In my model, the level of reverse-engineering effort is endogenized.

2 The law of trade secret misappropriation

As emphasized by the IPR Helpdesk[4], "there was a noticeable movement towards increased trade secret protection in many countries of the world during the 1990's and a surprising uniformity in the treatment of trade secrets. Trade secret theft now constitutes a crime in many countries". The French criminal code has had provisions relating to the theft of trade secrets since 1844 (Art. 411 Code Pénal). In addition, damages are available to the private litigant (Art. L. 621-1 Code de la Propriété Intellectuelle). In Germany, unfair competition law created criminal penalties in 1909 and trade secret owners can obtain damages. In Italy also trade secret theft is a crime (Art. 513, 623, Codice Penali) and civil remedies are available (Art. 2598 (3), 2600 Codice Civile). In Spain, the enactment of a new criminal code effective as from 24 May 1996 imposes fines and imprisonment for a number of trade secrets crimes such as the divulgation of stolen trade secrets by a third party. In the United States, although trade secret laws are state granted rights, two main federal texts deal with trade secrets: the Uniform Trade Secret Act (UTSA) and the Economic Espionage Act (EEA). The UTSA states that trade secret owners can be granted civil remedies to compensate for the lost profit due to misappropriation. Under the EEA of 1996, the theft of trade secrets is now a federal crime[5].

The so-called "Avery Dennison Case" illustrates an investigation under the EEA. Avery Dennison is a manufacturer of adhesive products (postage stamps, diaper tape). It employs 16,000 employees worldwide. In 1996, the company started suspecting that one of its employee, Tehong Lee, was being paid in exchange for trade secret information by Four Pillars, a major competitor. After internal inquiry, the case was referred to the FBI. The investigation was successful in finding clear and convincing evidence of Lee being bribed. He was charged under section 1832 of the EEA. Four Pillars was sentenced to pay 80 millions dollars in damages to Avery Dennison while the chairman of the company was sentenced to pay 250 000 dollars as criminal fine[6]. Suspicions of corruption and trade secret theft are regularly reported in the media. More examples of such investigations under the EEA can be found of the web site of the United States Department of Justice.

[4] The IPR Helpdesk is a project of the European Commission DG Enterprise. It assists potential and current contractors taking part in Community funded research on intellectual property rights issues. www.ipr-helpdesk.org/index.htm

[5] The EEA concerns both theft by foreign companies in the context of economic espionage, and theft by national companies.

[6] As I finalize this paper, I do not yet know the Court's decision for the corrupt employee.

3 The assumptions of the model

Players. I consider a game between three risk-neutral players: two firms I and E and an employee labelled "e". I assume that firm I is an innovator ("she") whereas firm E is an entrant ("he") also referred to as the imitator. As in Takalo (1998) or Kanniainen and Stenbacka (2000) the role of the innovator and of the imitator is exogenously given. I assume I and E are big corporations.

Actions. Initially firm I invests in R&D to obtain a non patentable innovation. Secrecy is then the only alternative to protect her innovation.[7] R&D is uncertain and costly. The probability to succeed in R&D is given by:

$$q(x) = 1 - e^{-x}, \tag{1}$$

where x is R&D investment. This specification allows me to derive closed-form solutions and satisfies the following properties: $\frac{\partial q(x)}{\partial x} \geq 0$, $\frac{\partial^2 q(x)}{\partial x^2} \leq 0$. The effective cost of R&D is given by:

$$c(x) = x. \tag{2}$$

If R&D is successful, player I hires an employee to produce a marketable good based on the innovation. I assume that there is a large number of potential employees on the job market whose reservation wage is normalized to 0. A contract between the innovator and the employee is simply a wage $w = 0$ paid by the innovator.

Then the imitator moves. His possible actions are depicted in Figure 1 which represents the imitation subgame. Imitation can take two forms: *duplication through reverse-engineering* and *offering a bribe* to the innovator's employee.

[7] Ignoring patents is a strong assumption.One argument is that "the subject matter that can be protected by trade secrets is broader than that which can be protected by patents. Trade secret protection is available for *both* technical information and information that does not relate to technical innovations. Non-technical information for which trade secret protection can exist includes: business and marketing plans, and customer lists. Patent protection is generally available for technical innovations ...". This is taken from Chilling Effects Clearinghouse website at http://www.chillingeffects.org/index.cgi. It means that there are innovations which are valuable and yet non patentable. Nevertheless my model does not intend to focus only on these non patentable innovations and so ignoring patents is a simplifying assumption intended to make it more tractable. Also, a (somehow unsatisfactory) argument could be that focusing only on trade secret policy is a similar imperfection as it is for the bunch of papers which discuss patent policy to abstract from trade secret policy...

- *Duplication through reverse-engineering.* This is legal, but uncertain. The probability to succeed is

$$q_R(y) = 1 - e^{-\gamma y}, \tag{3}$$

where y is the investment in reverse-engineering. I assume that duplication is more straightforward than innovation: $\gamma \in [1, +\infty)$.The intuition is that for the same amount invested in innovation and duplication, the probability to succeed is weakly higher with the duplication technology. With a perfectly protected process innovation, γ is likely to be close to 1 i.e. "duplication" is essentially a "rediscovery" and can be assimilated to "independent research". But if the innovation cannot be perfectly concealed, for example because the product commercialized constitutes the innovation *itself,* γ may be large.

The cost of reverse-engineering is:

$$c_R(y) = \beta y, \tag{4}$$

where $\beta \geq 1$. Contrary to γ which captures a *technological* property of reverse-engineering compared to innovation, β captures a *financial* asymmetry between I and E. It denotes the relative cost of capital between firm E and firm I, with the restriction that the cost of capital is at least as high for E as it is for I. Two remarks are in order here. First, although the parameter β may seem to complicate unnecessarily the model, it turns out that it simplifies the analysis[8]. Second, given that β is introduced, the restriction that firm I's cost of capital is at least as high as firm E ($\beta \geq 1$) is a useful simplication.[9] Introducing another parameter $\alpha \geq 1$ to capture firm I's cost of capital may be more general but would make the model less clear and would add an unnecessary layer of complexity.

- *Bribing the employee* (which is also referred to as "misappropriation"[10]). Bribing is modeled as an ultimatum game whereby the imitator makes a take-it-or-leave-it offer b to

[8] If I were to set $\beta = 1$, I would have to assume $\pi^d \geq 0$, instead of $\pi^d \geq 1$ as I do later in the analysis, in order to generate all the equilibria that I currently characterize. Although $\pi^d \geq 0$ is more natural, it turns out that these alternative assumptions make the analysis more cumbersome: the number of scenarios to consider (depending on the parameters) becomes much larger than it is now, and no substantially new insights are obtained. Hence I believe the current modeling choices, though redundant and debatable, actually improves the analysis.

[9] Financiers who have invested in the innovating firm might be unwilling to finance a second one.

[10] Clearly, misappropriation can take other forms, some of which are not considered crimes. See Fosfuri and Ronde (2004).

the employee in exchange for the details of the innovation.[11] Bribing is illegal and exposes both the imitator and the employee to a criminal fine $F \in [0, +\infty)$. For simplicity, I assume that the fine is the same for both players. In addition, the imitator may be forced to pay damages $D \in [0, +\infty)$ to the trade secret owner. However, a bribe allows the imitator to avoid research. Damages have a deterrence and a compensation effect since they make misappropriation more costly and conditional on misappropriation occuring and being proved, they compensate the trade secret owner for her profit loss. Criminal fines only have a deterrence effect: they do not compensate the trade secret owner ex-post, but they punish the wrongdoers, thereby reducing their incentives for bribery. F should be interpreted as the disutility imposed on the imitator and the employee if bribery is found to have occurred, *regardless of the nature of this disutility.* F can be a monetary fine. I assume the imitator is a corporation with enough ressources to pay such a monetary fine in addition to damages. However, the employee may not have the ressources to pay F upfront. In that case, he may be sentenced to jail and I assume that the disutility associated with prison can be expressed in monetary terms by F. The distinction between a monetary fine and prison for the employee impacts the analysis only in section 6 where I investigate the design of a socially optimal F. In section 6, I assume that the employee pays a monetary fine transferred to the government who can use it for socially valuable activities. In Appendix D, in contrast to section 6, I assume the employee is imprisoned (a net cost to society). In either case, I show that the socially optimal F is the same: zero. Hence, in equilibrium, the *nature* of the criminal punishment for the employee is irrelevant.[12]

These two strategies are not exclusive from each other: the imitator may try to reverse-

[11] The *relative occurence* of the different equilibria derived in the paper is most likely sensitive to this bargaining assumption. If the imitator were to have less bargaining power, he would be less inclined to bribe the employee and would rather engage in reverse-enginering. I aknowledge that the bargaining assumption may bias the policy conclusions concerning the optimal magnitude of the criminal fine and damages. However I believe the *nature* of the equilibria would be robust to alternative bargaining assumptions.

[12] Bribing differs from hiring the employee: my understanding is that if the newly hired employee were to reveal the innovation to his new employer, it would not be considered a crime. But the new employer might be liable for damages. In practice, hiring the employee is often impossible due to "covenant-not-to-compete" (although the Courts of California refuse to enforce these covenants).

engineer first and approach the employee conditional on a failure in reverse-engineering. Although I investigate the effects of a combination of a criminal fine (F) and a civil remedy (D) on the incentives to innovate and imitate, I do not explicitly model the enforcement process.[13] This is a simplifying assumption. Indeed, it implies that the owner of the trade secret, conditional on finding evidence of a bribe, is committed to file both a civil suit (to obtain damages) and a criminal suit (to punish the employee and the competitor). Regarding the criminal suit, we can imagine that the innovator aims at building a reputation to deter future employees and competitors from misappropriation, but I do not model this feature. Here, the only way for the trade secret owner to obtain damages is to prove misappropriation i.e. to find evidence of a bribe. I assume that the obtention of such evidence occurs with exogenous probability, only if bribery occured. If it did not occur, this probability is zero.The probability that a judgment is made in favor of the trade secret owner is denoted $p > 0$ and captures different dimensions. It reflects the quality of the monitoring technology of the trade secret owner. It also reflects the quality of the police and the Attorney General in investigation and prosecution: because a bribe is considered a crime, the trade secret owner must involve government representatives in the enforcement of her right, as in the "Avery Dennison case" discussed previously. For simplicity, I assume that the obtention of this evidence is costless. I emphasize that p is the probability that a judgement is made in favor of the trade secret owner. Even if players know that, given the level of the legal parameters D and F, bribery must have occured, I assume that the Court does not necessarily rule in favor of the innovator: in practice, the trade secret owner will be required to prove that she actually had a secret i.e. that she took all necessary actions to conceal her innovation. Finally, I do not assume that the employee is fired if bribery is proved. In reality, this would most likely be the case: the cost for the innovator of hiring a new employee might be smaller than the benefit from "building a reputation" (i.e. firing the employee to show determination in protecting trade secrets).[14] By focusing on two methods of acquiring

[13]Hence I abstract from litigation costs which, in practice, can be substantial. Such costs might deter bribery (the employee and the imitator would have to pay these costs if they are caught) but they can also deter the innovator from litigating, thereby encouraging bribery...The overall effect is unclear without properly modeling the costs and it would be worthwhile in future research to do so, maybe by simplifying other aspects of the model for tractability reasons.

[14]Worker turnover would increase the cost of being detected for the employee (he would have to find a new job which might be complicated due to "bad reputation") and therefore for the imitator (he would have to offer an higher bribe). I abstract from this aspect here and leave it for future research.

the trade secret (bribery and reverse-engineering), I intend to keep the model tractable while highlighting important trade-offs. Of course, one can think of other acquisition mechanisms. One such mechanism is licensing. An implicit assumption of the current model is that licensing of a trade secret is impossible or very difficult: in the bargaining process, the trade secret owner would reveal the secret to the rival who then could start using it without finalizing the deal[15]. Even if there may be legal protections against that, evidenciary issues remain. This problem constitutes one of the main argument in favor of patents (see Arora, Fosfuri and Gambardella, 2001): a patent guarantees verifiable ownership despite disclosure and therefore it facilitates technology licensing compared to secrecy.

Payoffs. The final payoffs depend on whether or not the innovator is successful in R&D and on the actions of the imitator. If the innovator is successful in R&D, there are five possible states of the world. First, both misappropriation and reverse engineering are unprofitable so that the innovator faces no competition and earns the monopoly profit π^m. Second, misappropriation is unprofitable but reverse engineering is profitable. If the imitator succeeds, there is competition between the two firms and both earn the duopoly profit π^d.[16] But, and this is the third case, if the imitator fails, the innovator faces no competition: he obtains the monopoly profit π^m. Fourth, reverse engineering is unprofitable but misappropriation is profitable. Competition follows and each firm obtains π^d (minus an expected punishment for the imitator, plus expected damages for the trade secret owner). Finally, both reverse engineering and misappropriation are profitable. In that case, competition will necessarily follow. Of course, the occurence of these different states will depend on the parameters of the model. In most of the analysis, I do not specify the nature of market competition and I simply assume that $2\pi^d \leq \pi^m$. In the end of the paper, I develop a simple model of Cournot competition to offer more precise conclusions.

The timing of the game is as follows. First, society designs a policy (F, D). Second, the innovator invests in R&D, succeeds or fails. If she succeeds, she hires an employee to produce the innovation and informs this employee about the concealed details of the innovation. Finally, the competitor chooses an imitation strategy as described by the extensive form in Figure 1.

[15] Anton and Yao (1994) propose a model where the party with the idea can earn rents despite the inexistence of property right on the idea (by threatening to give the idea for free to other firms, should the other party steal the idea). Yet, it is not guaranteed that such other firms exist.

[16] I assume $\pi^d \geq 1$. This restriction allows me to limit the number of parameter configurations to analyze and therefore makes the exposition of the analysis much less cumbersome.

The game is solved by backward induction and the solution concept is the subgame perfect equilibrium.

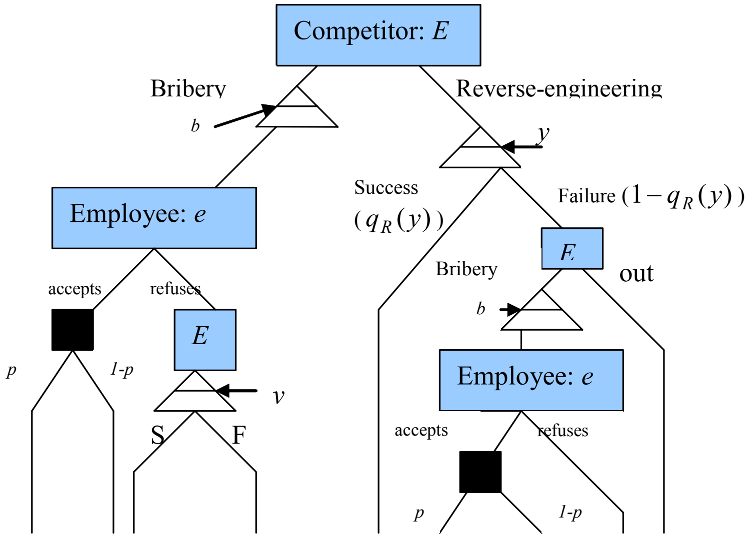


Figure 1: Extensive form for the imitation subgame. (The black nodes represent "Nature").

4 Equilibrium outcomes of the imitation subgame

Assume first that the imitator started with reverse-engineering (right branch of the imitation subgame tree, starting from the initial decision node).

■ *The bribing subgame.* Suppose the imitator failed in reverse engineering. He still has the opportunity to bribe the employee, as long as this is a profitable option. He offers a bribe b which satisfies the individual rationality constraint of the employee. Given the probability p of being detected and the criminal fine F, this constraint is given by:

$$b - pF \geq 0.$$

In equilibrium, the bribe is:

$$b^* = pF. \tag{5}$$

The employee's expected payoff is $U_e = 0$ and that of the imitator is thus:

$$U_E = \pi^d - p(D + F) - b^* = \pi^d - p(D + 2F).$$

This payoff satisfies the individual rationality constraint of the imitator for bribery as long as $U_E \geq 0$. This constraint can be expressed by two equivalent boundary conditions:

$$F \leq \frac{\pi^d}{2p} - \frac{1}{2}D = F_1(D), \tag{6}$$

or[17]:

$$\pi^d \geq p(D + 2F) = \widetilde{\pi}^d. \tag{7}$$

This analysis yields an intuitive result:

Lemma 1 ***misappropriation deterrence.*** *When the punishments for bribery are high enough ($F > F_1(D)$), bribery does not occur and the imitator gets $U_E^{NO} = 0$. When the punishments for bribery are low enough ($F \leq F_1(D)$), bribery occurs and the imitator gets $U_E^B = \pi^d - p(D+2F)$. In both cases, the employee gets a zero payoff.*

Moving one step backward, I now analyze the imitator's incentives to invest in reverse-engineering. This depends on whether he can resort to bribery if reverse-engineering fails.

■ *Reverse engineering when bribery is not deterred.* By lemma 1, if the imitator fails in reverse-engineering, his payoff is $U_E = \pi^d - p(D + 2F)$. His objective is to choose the level of reverse engineering effort, y, that maximizes:

$$q_R(y)\pi^d + (1 - q_R(y)) \left[U_E^B\right] - c_R(y).$$

Substituting for $q_R(y) = 1 - e^{-\gamma y}$, $c_R(y) = \beta y$ and U_E^B, this objective function rewrites as:

$$\pi^d - p(D + 2F) + (1 - e^{-\gamma y}) \left[p(D + 2F)\right] - \beta y. \tag{8}$$

The first order condition yields the optimal effort in reverse engineering when misappropriation is not deterred[18]:

$$y^* = \frac{1}{\gamma} \ln \left(\frac{\gamma p(D + 2F)}{\beta} \right). \tag{9}$$

Define $F_2(D)$ as:

$$F_2(D) = \frac{\beta}{2\gamma p} - \frac{1}{2}D, \tag{10}$$

[17] The condition on π^d will enable to draw Figure 2, while the condition on F is used to draw Figure 5, later in the analysis.

[18] Denoting the objective function V_E , the second-order condition holds since $\frac{\partial^2 V_E}{\partial y^2} = -\gamma^2 e^{-\gamma y} p(D+2F) \leq 0$ for all y, y^* is a global maximum.

and define $\widetilde{\gamma}$ as:

$$\widetilde{\gamma} = \frac{\beta}{p(D+2F)}. \tag{11}$$

Lemma 2 *The optimal investment in reverse engineering is positive as long as* $\gamma \geq \widetilde{\gamma}$ *and is equal to zero if* $\gamma < \widetilde{\gamma}$.

Proof. $\frac{1}{\gamma} \ln \left(\frac{\gamma p(D+2F)}{\beta} \right) \geq 0$ if and only if $\frac{\gamma p(D+2F)}{\beta} \geq 1$ or $\gamma \geq \frac{\beta}{p(D+2F)} = \widetilde{\gamma}$ by (11). When $\gamma < \widetilde{\gamma}$, there is the corner solution $y^* = 0$ as the marginal cost of reverse-engineering exceeds its marginal benefit. ∎

It is straightforward to show that if bribery is profitable and $\gamma \geq \widetilde{\gamma}$, the investment in reverse engineering is increasing in the probability of misappropriation being established (p), in the level of the damages (D) and in the level of the criminal fine (F). It is decreasing in the relative cost of capital parameter (β).[19] Also, the investment in reverse engineering $y^*$and the punishments for misappropriation F and D are "complements" in the sense that the *higher* the level of the criminal fine or the level of damages, the *less* the imitator obtains if he fails in reverse-engineering: *ceteris paribus*, this encourages him to invest *mor*e in reverse-engineering.

■ *Reverse-engineering when bribery is deterred.* By lemma 1, the imitator's payoff conditional on failing in reverse-engineering is zero and his objective is to choose the level of reverse-engineering effort y which maximizes:

$$q_R(y)\pi^d - c_R(y).$$

Substituting for $q_R(y) = 1 - e^{-\gamma y}$ and $c_R(y) = \beta y$ gives:

$$(1 - e^{-\gamma y})\pi^d - \beta y. \tag{12}$$

The first-order condition yields the optimal reverse-engineering effort[20] when misappropri-

[19]Partially differentiating y^* with respect to p, D, F and β, we have: $\frac{\partial y^*}{\partial p} = \frac{1}{\gamma p} \geq 0$; $\frac{\partial y^*}{\partial D} = \frac{1}{\gamma(D+2F)} \geq 0$; $\frac{\partial y^*}{\partial F} = \frac{2}{\gamma(D+2F)} \geq 0$; $\frac{\partial y^*}{\partial \beta} = -\frac{1}{\gamma\beta} < 0$.

[20]I use the same notation y^* as in the previous case. Although it might be misleading at first sight, it has no incidence on the rest of the paper and it allows me not to increase the already significant number of symbols.

ation is deterred[21]:

$$y^* = \frac{1}{\gamma} \ln \left(\frac{\gamma \pi^d}{\beta} \right). \tag{13}$$

Define:

$$\widehat{\gamma}(\pi^d) = \frac{\beta}{\pi^d}. \tag{14}$$

Lemma 3 *The optimal investment in reverse-engineering is positive as long as* $\gamma \geq \frac{\beta}{\pi^d} = \widehat{\gamma}(\pi^d)$. *Otherwise, the optimal investment is equal to zero.*

Proof. As for lemma 2, the proof is immediate upon inspection of (13). ■

Simple comparative statics show that if bribery is deterred and $\gamma \geq \widehat{\gamma}(\pi^d)$, the optimal investment in reverse-engineering is decreasing in the level of market competition. Also, the higher the relative cost of capital (the higher is β), the lower his effort in reverse-engineering.[22]

So far I have assumed that the imitator started with reverse-engineering. I now analyze the situation where the competitor chooses to bribe the employee before committing to reverse-engineering (left branch starting from the initial decision node of the imitation game tree in Figure 1). Notice that in equilibrium, the employee will always accept the bribe b and his participation constraint will bind so that the level of the bribe is again given by (5). In case reverse-engineering is not possible, bribery occurs (first) if and only if the damages and criminal fine are not too high i.e. condition $F \leq F_1(D)$ is satisfied. Suppose then that reverse-engineering is possible. Two conditions are required for the competitor to start with bribing the employee. First, bribery must not be deterred. This implies that condition $F \leq F_1(D)$ holds. Second, his payoff must be higher than what he obtains from engaging first in reverse-engineering and bribing the employee only conditional on reverse-engineering failure. If the first condition does not hold, then bribery cannot be an equilibrium strategy and if the second condition does not hold, then bribing *before* reverse-engineering cannot be an equilibrium strategy either. Thus, to analyze whether bribery occurs before reverse-engineering requires computing the imitator's payoff under each strategy. These payoffs obviously depend on the imitation options available and chosen by the imitator. I use the equilibrium investment in reverse-engineering calculated

[21] Denoting again the objective function V_E , the second-order condition holds since $\frac{\partial^2 V_E}{\partial y^2} = -\gamma^2 e^{-\gamma y} \pi^d \leq 0$ for all y. Hence y^* is a global maximum.

[22] Computing the relevant partial derivatives: $\frac{\partial y^*}{\partial \pi^d} = \frac{1}{\gamma \pi^d} \geq 0$ (the lower π^d, i.e the higher market competition, the lower is y^*). Also $\frac{\partial y^*}{\partial \beta} = -\frac{1}{\gamma \beta} \leq 0$.

before, as well as lemma 1. The proof is in Appendix A.2. If bribery is deterred and reverse-engineering is profitable, the imitator's payoff is:

$$U_E = \pi^d - \frac{\beta}{\gamma}\left[1 + \ln\left(\frac{\gamma\pi^d}{\beta}\right)\right]. \tag{15}$$

If bribery is deterred and reverse-engineering is not profitable, he cannot imitate and his payoff is:

$$U_E = 0. \tag{16}$$

If bribery is not deterred but he starts with reverse-engineering, his payoff is:

$$U_E = \pi^d - \frac{\beta}{\gamma}\left[1 + \ln\left(\frac{\gamma p(D+2F)}{\beta}\right)\right]. \tag{17}$$

Finally, if bribery is not deterred and reverse-engineering is not profitable, the payoff is:

$$U_E = \pi^d - p(D+2F). \tag{18}$$

I can now derive the equilibria of the imitation subgame. Depending on the values of the parameters, the imitation subgame has four possible equilibrium outcomes summarized in the following proposition and illustrated by Figure 2 in the (γ, π^d) space.[23]. In this proposition, R stands for "reverse-engineering" (only), RB stands for "reverse-engineering first and bribery conditional on reverse-engineering failure", B stands for "bribery" (only) and NO stands for "neither bribery nor reverse-engineering".

Proposition 1 ***Equilibria of the imitation subgame.***

- *If bribery is deterred ($\pi^d < \widetilde{\pi}^d$), the imitation subgame has two equilibrium outcomes: When $\gamma \geq \widehat{\gamma}(\pi^d)$ the competitor tries to reverse-engineer the innovation (outcome R) and when $\gamma < \widehat{\gamma}(\pi^d)$ he does not (outcome NO).*
- *If bribery is not deterred ($\pi^d \geq \widetilde{\pi}^d$), the subgame again has two equilibrium outcomes: When $\gamma \geq \widetilde{\gamma}$, the competitor engages in reverse-engineering first and, if he fails, bribes the*

[23] $\widetilde{\gamma}$ is independent of π^d and $\widetilde{\pi}^d$ is independent of γ. Then, $\widehat{\gamma}(\pi^d)$ is a straightforward decreasing function of π^d which is analyzed in Appendix A. Of course, this representation holds for a particular configuration of parameters such that $\frac{\beta}{p(D+2F)} > 1$, $p(D+2F) > 1$ and $\beta > 1$.

employee to acquire the innovation. When bribed, the employee accepts the offer (outcome RB). When $\gamma < \widetilde{\gamma}$ he bribes the employee immediately. When bribed, the employee always accept the offer (outcome B).

Proof. See Appendix A.3. ■

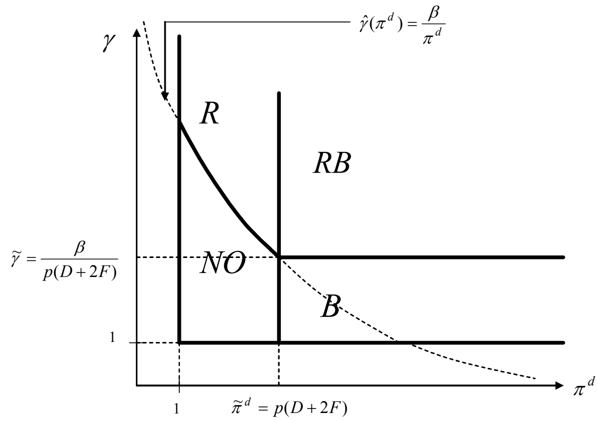


Figure 2: The four equilibrium outcomes of the imitation game

5 Incentives to innovate

I now turn to the previous stage of the game where the innovator chooses her R&D investment. I investigate how the innovator's payoff is affected by changes in the criminal fine or the damages. The main insight is that an increase in the criminal fine can reduce her payoff. I start by computing the innovator's payoff corresponding to each equilibrium outcome of the imitation subgame. I denote $\Delta\pi = \pi^m - \pi^d$. If bribery is deterred and reverse-engineering is profitable (outcome R), the innovator's payoff before R&D investment is:

$$U_I^R = \pi^m - \left(1 - \frac{\beta}{\gamma}\frac{1}{\pi^d}\right)\Delta\pi. \tag{19}$$

If bribery is deterred and reverse-engineering is not profitable (outcome NO), her payoff is:

$$U_I^{NO} = \pi^m. \tag{20}$$

Then, when reverse-engineering occurs first and bribery only if reverse-engineering fails (outcome RB), her payoff is:

$$U_I^{RB} = \pi^d + \left(\frac{\beta}{\gamma} \frac{1}{(D + 2F)} \right) D. \tag{21}$$

Finally, when bribery is not deterred and reverse-engineering is not profitable (outcome B), it is:

$$U_I^B = \pi^d + pD. \tag{22}$$

The proof is reported in Appendix B.1. The innovator's expected payoff is affected by changes in the criminal fine F or the damages D through two different channels. First, it is affected by the effect of a marginal change in F or D on the boundaries $\widetilde{\gamma}$ and $\widetilde{\pi}^d$ between the four equilibrium outcomes of the imitation game (see Figure 2). This may induce a switch from one equilibrium to another. Second, it is affected by a marginal change in F or D within the parameter regions where bribery is not deterred, namely RB and B. To facilitate the analysis, it is useful to define two scenarios and to illustrate them by two separate graphics (in Figure 3, where the right-hand side graphic illustrates scenario 1 and the left-hand side graphic illustrates scenario 2):[24]

- Under the first scenario, reverse-engineering cannot happen. This correspond to observing $\gamma < \widehat{\gamma}(\pi^d)$ as can be seen in Figure 2. However, bribery is possible.
- Under the second scenario, reverse-engineering can happen. This correponds to observing $\gamma \geq \widehat{\gamma}(\pi^d)$. In addition, bribery is possible as well.

[24] Appendices A.1 and B.2 expose the straightforward computations that enabled me to draw these two graphics. In these graphics, I report the values $\underline{D}$ (intersection between $F_2(D)$ and the $D-$axis) and $\overline{D}$ (intersection between $F_1(D)$ and the $D-$axis). These values play a crucial role in the analysis of the socially optimal trade secret policy (section 6).

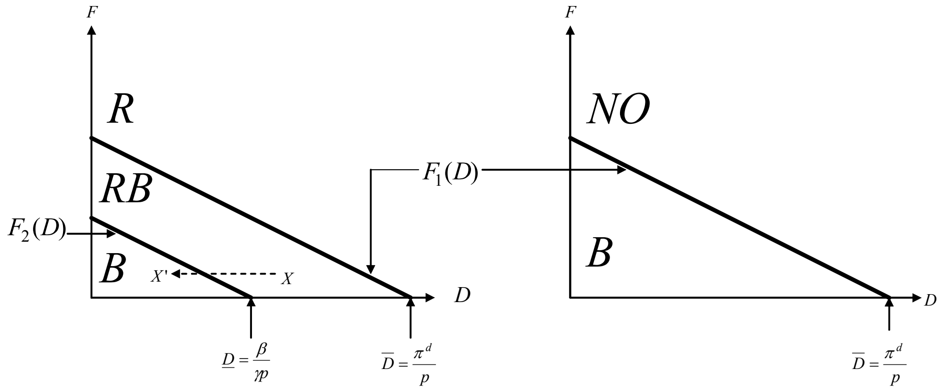


Figure 3: Equilibrium outcomes of the imitation game in the (F, D) space

Consider the first scenario. When reverse-engineering cannot happen, the innovator faces either bribery or no imitation (right-hand side graphics in Figure 3). It is straightforward that she prefers no imitation since in this case she receives the monopoly profit. This can be proved formally by comparing U_I^{NO} in (20) with U_I^B in (22). Therefore, an increase in the criminal fine F or the damages D that yield a switch from equilibrium B to NO makes the innovator better off (as does an increase in the damages, compatible with outcome B since $\frac{\partial U_I^B}{\partial D} = p \geq 0$.

Proposition 2 *If reverse-engineering cannot happen, the innovator is always better-off when the level of the criminal fine and of the damages increases.*

This intuitive result is not robust to the case where reverse-engineering can happen. There are three possible outcomes: only reverse-engineering occurs (R), reverse-engineering occurs first and, in case of failure, bribery takes place (RB), or only bribery occurs (B). Figure 4 represents the innovator's payoff as a function of F, holding D constant (left-hand-side graphic) and as a function of D holding F constant (right-hand-side graphic).[25]

[25] In this figure, the boundary D_1 is the equivalent of F_1 and D_2 is the equivalent of F_2. Appendix B.3 details the computations that enable me to draw these two graphics.

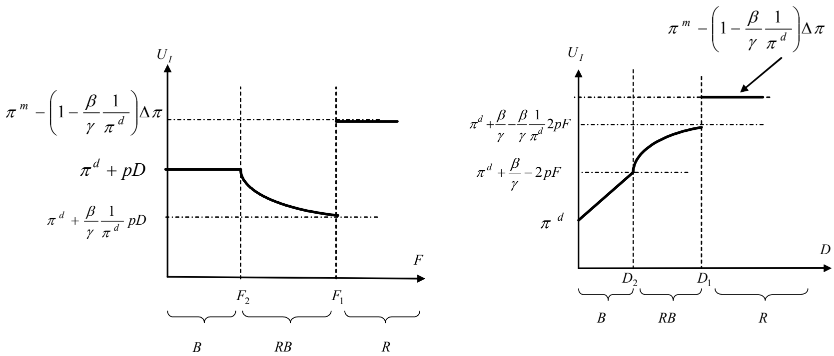


Figure 4: Innovator's payoff when reverse-engineering can happen.

Proposition 3 *Suppose reverse-engineering can happen. Everything else equal, an increase in the level of damages unambiguously increases the innovator's expected payoff. However, everything else equal, an increase in the criminal fine may decrease her payoff.*

Proof. See Appendix B.3. ■

The counter-intuitive second part of this proposition is illustrated by the left-hand-side graphic in Figure 4. The explanation comes from the fact that the criminal fine F is not a monetary compensation. Increasing F so as to switch from equilibrium B to RB while keeping D constant means that the probability for the innovator to be uncompensated increases: indeed, in RB, the imitator may succeed in reverse-engineering which results in a duopoly without any monetary compensation. Formally, in outcome RB, the innovator is compensated by damages D with probability $[1 - q_R(y^*)]\, p$ where y^* is the investment in reverse-engineering given by (9), $1 - q_R(y^*)$ is the probability that reverse-engineering fails and p is the probability that the Court rules in favor of the innovator. By contrast in outcome B, the innovator is compensated with probability p. Now it is obvious that $[1 - q_R(y^*)]\, p \le p$ so that the probability to be uncompensated is higher in outcome RB. This holds for *any* level of D. Consider then an increase in D holding F constant. An increase in D so as to switch from equilibrium B to equilibrium RB while holding F constant always improves the innovator's expected payoff as can be seen in the right-hand side graphic in Figure 4. Counter-intuitively, this means that the innovator is ex-ante better-off if the probability to be uncompensated increases (as a switch from B to RB implies reverse-engineering before bribery). To understand that, we must keep in mind that the increase in D offsets the higher probability to be uncompensated. An implication of these results is given by the following corollary:

Corollary 1 *Suppose reverse-engineering cannot happen. Everything else equal, the innovator can be better off when the imitator becomes more efficient in financing reverse-engineering (i.e when β decreases) and when imitation becomes technologically easier compared to innovation (γ increases).*

To see that, consider the left-hand side graphic in Figure 3. Suppose that the legal environment is given by a configuration (F, D) corresponding to X. Now, an increase in β shifts the $F_2(D)$ curve upward as $F_2(D) = \frac{\beta}{2\gamma p} - \frac{D}{2}$. As a result, X may end up below (and not above) the line $F_2(D)$, i.e in region B. This effect would correspond to a switch from configuration X to configuration X'. But we saw previously that this switch reduces the innovator's payoff. The same kind of argument explains the result about imitation easiness (i.e the parameter γ). The only case where the innovator is indifferent between the imitator trying to reverse-engineer first and him bribing the employee immediately is when $U_I^B = U_I^{RB}$. This yields a condition on F that coincides with $F_2(D)$. Re-expressing it as a condition on D yields:

$$D = D_2(F) = \frac{\beta}{\gamma p} - 2F \tag{23}$$

Finally, I show in Appendix B.5 that an increase in the criminal fine or the damages that yields a switch from equilibrium B (bribery only) to R (reverse-engineering only) or from RB (reverse-engineering and bribery if it fails) to R (reverse-engineering only) unambiguously increases the innovator's payoff. The intuition for the former result is simple: with reverse-engineering, there is a chance that the innovator remains a monopolist; with bribery, the expected compensation offered by the level of damages compatible with this outcome does not offer a benefit as high as the benefit of being maybe a monopoly (if reverse-engineering fails). The latter result is intuitive as well: when the competitor has only one option (reverse-engineering but not bribery), the probability that the innovator remains a monopolist is higher. Hence she is better off. I conclude this section by presenting the equilibrium investment in innovation.

Lemma 4 ***Incentives to innovate.*** *If bribery is deterred and reverse-engineering is profitable, the investment in innovation is given by:*

$$x^{R,*} = \ln\left(\frac{\gamma\pi^{d2} + \beta\Delta\pi}{\gamma\pi^d}\right). \tag{24}$$

If bribery is deterred and reverse-engineering is unprofitable, it is:

$$x^{NO,*} = \ln \pi^m. \tag{25}$$

If bribery is not deterred and reverse-engineering is profitable:

$$x^{RB,*} = \ln \left(\frac{\pi^d \gamma(D + 2F) + \beta D}{\gamma(D + 2F)} \right). \tag{26}$$

If bribery is not deterred and reverse-engineering is unprofitable:

$$x^{B,*} = \ln \left(\pi^d + pD \right). \tag{27}$$

Proof. The innovator chooses the level of investment x which maximizes

$$(1 - e^{-x})U_I^k - x,$$

for $k = R, NO, RB$ and B. This objective function is well-behaved. Substituting for the values of U_I^k and solving for each outcome successively yields the above results. ■

6 Trade secret law design

At the outset, society wants to design the criminal fine and the damages (i.e. a policy (F, D)) that maximizes social welfare. I assume the social planner considers p as an exogenous parameter.[26] In designing this policy, the social planner observe β, π^d, γ and p. Similar to the analysis

[26]This is a simplifying assumption. I propose three justifications. First, in practice, p may not only be a function of the police's effort on which the social planner may have an influence, but it may also be a function of other parties' efforts: the innovator could determine *positively* p while the corrupt parties (the imitator and the employee) can influence p *negatively.* A second reason is that, even assuming that the main determinant of p is the police investigation, a government may find it much more convenient to implement a trade secret policy consisting only in specific values for F and D, than to implement a policy that tries to influence police's effort as well. In practice, this latter variable is likely to be much more difficult to influence by a governmental decision than the mere levels of F and D. Indeed, p would depend itself on a wide range of parameters (policemen's incentives, budget allocation, political considerations, bureaucratic inertia...) that would tend to complicate the implementation of the socially optimal p. Finally, and most importantly, I have stressed that p is not only the probability to find clear and convincing evidence of bribery: it is the probability that a judgment is made in favor of the innovator. This is affected by several other determinants (lawyers' effort, effort to protect the trade secret...). As a result, and as a first step, I find it relevant to focus only on F and D as the instruments of technology policy.

of innovation incentives, two scenarios can be distinguished (see Figure 3). Under the first scenario, reverse-engineering cannot happen. Under the second scenario, reverse-engineering can happen. I call these two scenarios scenario 1 and scenario 2. The relative occurence of these two scenarios is independent of the policy (F, D). Since the social planner is assumed to observe which scenario actually occurs, an optimal trade secret policy can be designed for each scenario separately. A standard assumption in the following analysis is that the sum of consumer surplus and profit(s) is higher under duopoly than under monopoly.

6.1 Reverse-engineering cannot happen

I allow F and D to take any value between 0 and $+\infty$. Intuitively, with F and D close to 0, it will be difficult to prevent bribery to occur, while setting F and D close to infinity would most likely deter bribery. This intuition is formally captured by lemma 1. This lemma states that for any F such that $F \leq F_1(D)$, bribery occurs (outcome B) whereas for any $F > F_1(D)$, it is deterred (outcome NO). The right-hand side graphic on Figure 3 represents these two outcomes. The social planner faces the following alternative: design (F, D) such that $F \leq F_1(D)$ and bribery will occur or design (F, D) such that $F > F_1(D)$ and bribery does not occur. I compare social welfare for both outcomes.

■ *Consider first social welfare when bribery occurs (outcome B).* Under the assumption that the criminal fine is a transfer between the government and the wrongdoers, what is the benefit of an innovation to society? Denoting w^d this benefit:

$$w^d = \underbrace{\pi^d + pD}_{\text{innovator}} + \underbrace{\pi^d - pF - p(D+F)}_{\text{imitator}} + \underbrace{pF - pF}_{\text{employee}} + \underbrace{2pF}_{\text{government}} + \underbrace{cs^d.}_{\text{consumers}} \tag{28}$$

This expression simplifies to:

$$w^d = 2\pi^d + cs^d. \tag{29}$$

According to lemma 4, anticipating she would get $\pi^d + pD$ from being successful in R&D, the innovator invests $x^{B,*} = \ln\left(\pi^d + pD\right)$. Thus equilibrium social welfare is given by:

$$W = \left(1 - \frac{1}{\pi^d + pD}\right) w^d - \ln\left(\pi^d + pD\right). \tag{30}$$

Indeed, the innovator succeeds in R&D with probability $\left(1 - \frac{1}{\pi^d + pD}\right)$ and society gets w^d given by (29). The equilibrium cost of R&D is $\ln\left(\pi^d + pD\right)$. I now investigate what combination

(F, D) maximizes social welfare W conditional on $(F, D) \in B$ i.e. conditional on bribery not being deterred. Notice first that W is independent of F. And clearly,

$$\frac{\partial W}{\partial D} = \frac{p}{(\pi^d + pD)} \left(\frac{w^d}{\pi^d + pD} - 1 \right) > 0.$$

Indeed, $\frac{w^d}{\pi^d + pD} > 1$ if and only if $cs^d + \pi^d > pD$. The highest D compatible with outcome B is $D = \overline{D} \triangleq \frac{\pi^d}{p}$. Since $cs^d + \pi^d > \pi^d$, $cs^d + \pi^d > pD$ holds for lower values of D. Consequently, the solution is a corner solution: the highest D such that $(F, D) \in B$ is:

$$\overline{D} \triangleq \frac{\pi^d}{p}. \tag{31}$$

At this point, $F = 0$. Plugging in $F = 0$ and $D = \frac{\pi^d}{p}$ into W gives the best that society can get conditional on not deterring bribery, under scenario 1:

$$W^{B,1} = \left(1 - \frac{1}{2\pi^d}\right) w^d - \ln(2\pi^d), \tag{32}$$

where B stands for "bribery" and 1 for "scenario 1".

Lemma 5 *When bribery occurs and reverse-engineering cannot happen, the optimal policy consists in:*

$$\begin{cases} F = 0 \\ D = \frac{\pi^d}{p}. \end{cases}$$

■ *Consider then social welfare if bribery is deterred (outcome NO).* By definition, the resulting outcome is *independent* of the policy parameters F and D: they are so high that the potential imitator is deterred from bribing the innovator's employee. In addition, under scenario 1, reverse-engineering does not happen. Hence, conditional on a success by the innovator, the market structure is a monopoly so that society gets $w^m = \pi^m + cs^m$. Lemma 4 states that the innovator would invest $x^{NO,*} = \ln \pi^m$. Thus, social welfare is given by:

$$W^{NO} = \left(1 - \frac{1}{\pi^m}\right) w^m - \ln(\pi^m) \tag{33}$$

The next step consists in comparing $W^{B,1}$ and W^{NO}. I cannot provide a clear-cut result regarding whether society would be better-off under bribery or not. The trade-off is a "classic" trade-off in the economic analysis of intellectual property rights: strengthening intellectual property protection, by setting (F, D) that deters bribery increases incentive to innovate at the cost of a monopolistic distortions. Alternatively, relaxing intellectual property protection by designing a policy (F, D) such that bribery occurs yields lower incentives for innovation, but guarantees a duopoly. It is possible to go further in the analysis of this trade-off by specifiying the nature of market competition. I do this by assuming Cournot competition with linear demand.[27] The inverse-demand for the innovation is given by $p = a - q$ where a is the highest willingness to pay for the good and the marginal cost of production is normalized to zero for both firms. Standard computations yield the monopoly profit $\pi^m = \frac{1}{4}a^2$, the consumer surplus under monopoly $cs^m = \frac{1}{8}a^2$ and social welfare $w^m = \frac{3}{8}a^2$. Also, the duopoly profit is $\pi^d = \frac{1}{9}a^2$, the consumer's surplus under duopoly is $cs^d = \frac{2}{9}a^2$ so that social welfare is $w^d = \frac{4}{9}a^2$. To guarantee that $\pi^d > 1$, I impose the restriction $a > 3$.

Proposition 4 *Suppose the imitator cannot reverse-engineer. Under Cournot competition with linear demand and a marginal cost of production of zero, the optimal trade secret policy is to set the criminal final equal to zero and the damages equal to the "adjusted unjust enrichment" level*[28] *$(D = \frac{\pi^d}{p})$. Then, bribery occurs.*

Proof. Plugging in the values for π^m, π^d, w^m, w^d into $W^{B,1}$ and W^N yields:

$$\begin{cases} W^{B,1} \simeq 0.44a^2 - 2 - \ln\left(0.22a^2\right) \\ W^{NO} \simeq 0.375a^2 - 1.5 - \ln\left(0.25a^2\right) \end{cases}$$

It follows that $W^{B,1} \geq W^{NO}$ if and only if:

$$0.065a^2 - 0.5 \geq \ln(0.888)$$

which is equivalent to:

$$a \geq 2.42.$$

But since $a > 3$, $W^{B,1} \geq W^{NO}$ always holds. ■

[27] Linear demand is a strong assumption and I aknowledge that the following results might be amended under alternative functional forms. However in this context, the main advantage of linearity is that it is tractable enough, though it probably limits the generalization of the result.

[28] "Unjust enrichment" is a damage doctrine which means that the wrongdoer (infringer, thief...) must "disgorge his ill-gotten profit". That is: he must pay as damages the profit he obtained by illegally exploiting the intellectual property. Here, the wrongdoer is the imitator who earned π^d. This profit is adjusted to account for the probability p that the ruling is in favor of the trade secret owner (detection of bribery and other determinants affect this probability). This measure of damages implies that the imitator has deep pockets.

Under these competition assumptions, Proposition 4 shows that implementing bribery is socially optimal[29]. This means that the decrease in innovation incentives (they are lower when bribery occurs than when it is deterred) is compensated, in welfare terms, by an increase in consumer surplus (bribery guaranteeing a duopoly). The contribution of proposition 4 is that the criminal fine should be set equal to zero. The reason is that, if bribery is implemented, it is best for society to increase the damages to their maximum level compatible with bribery: this provides the maximum incentives to the innovator who is compensated by damages but not by criminal fines (or jail sentences). But maximizing the damages D while remaining in the bribery outcome implies minimizing the criminal fine F. Indeed, D and F play the *same* role: they both decrease the imitator's incentive for bribery. In the next section, I investigate trade secret policy when reverse-engineering can happen and I obtain different results.

6.2 Reverse-engineering can happen

Again, I allow F and D to take any value between 0 and $+\infty$. Three outcomes can occur, depending on the values of F and D. These three outcomes are depicted by the left-hand side graphic in Figure 3. For $F \leq F_2(D)$, reverse-engineering does not happen and the imitator bribes the employee (outcome B). For $F \in [F_2(D), F_1(D)]$, reverse-engineering is tried first and, conditional on failure, the employee is bribed (outcome RB). Finally, for $F > F_1(D)$, only reverse-engineering happens (outcome R). Thus, the social planner has now *three* options.

■ *Consider first social welfare if the social planner sets F and D compatible with outcome B.* The analysis is similar to the analysis proposed under scenario 1 for outcome B. The only difference is that the set of parameters (F, D) compatible with B is smaller. Indeed, $F_2(D) < F_1(D)$. An innovation would yield a benefit w^d to society, as given by (29). The innovator would invest $x^{B,*} = \ln\left(\pi^d + pD\right)$. Hence, social welfare is again given by (30). I investigate which combination (F, D) maximizes W conditional on $(F, D) \in B$. Clearly W is

[29] If F and D are such as in proposition 3, we know that bribery occurs. Hence, the probability p must be interpreted as more than just the probability of detecting bribery. But I have stressed that point throughout the paper. In practice, even if bribery is acknowledged, Courts typically rule in favor of the trade secret owner if and only if she can prove that the trade secret was indeed a secret, i.e. if she can prove that she *actively concealed her intellectual property*. p could reflect the quality of this protection against leakage.

independent of F, and we know from section 6.1 that $\frac{\partial W}{\partial D} > 0$. The solution is a corner solution: the largest D compatible with outcome B is:

$$\underline{D} \triangleq \frac{\beta}{\gamma p}. \tag{34}$$

For that value of D, we have $F = 0$. Plugging in these values of F and D into W yields society's highest welfare conditional on implementing outcome B, under scenario 2:

$$W^{B,2} = \left(1 - \frac{\gamma}{\gamma \pi^d + \beta}\right) w^d - \ln\left(\pi^d + \frac{\beta}{\gamma}\right). \tag{35}$$

■ *Then, consider social welfare if the social planner sets F and D compatible with outcome RB.* In that case, the rival first invests in reverse-engineering and then bribes the employee if reverse-engineering failed. Denoting y^* this investment, we can compute society's welfare conditional on a innovation occuring. With probability $q_R(\ y^*)$ the imitator succeeds and there is a duopoly so that society gets w^d. With probability $1 - q_R(\ y^*)$ the imitator fails and bribes the employee: society's welfare is given by (29), that is w^d. Regardless of whether the imitator succeeds in reverse-engineering, society gets w^d. However, the cost of reverse-engineering must be substracted. This cost is $c_R(y^*)$. From (9): $y^* = \frac{1}{\gamma} \ln\left(\frac{\gamma p(D+2F)}{\beta}\right)$. Hence, $c_R(y^*) = \frac{\beta}{\gamma} \ln\left[\frac{\gamma}{\beta} p\left(D + 2F\right)\right]$. Conditional on an innovation occuring, society's welfare is $w^d - \frac{\beta}{\gamma} \ln\left[\frac{\gamma}{\beta} p\left(D + 2F\right)\right]$. Lemma 4 states that the innovator, anticipating this outcome, invests $x^{RB,*} = \ln\left(\frac{\pi^d \gamma(D+2F)+\beta D}{\gamma(D+2F)}\right)$ so that the probability to have an innovation is:

$$q(x^{RB,*}) = \left(1 - \frac{\gamma(D + 2F)}{\pi^d \gamma(D + 2F) + \beta D}\right).$$

The equilibrium cost of innovation is:

$$c(x^{RB,*}) = \ln\left(\pi^d + \frac{\beta}{\gamma}\frac{D}{D + 2F}\right).$$

The objective function of the social planner is:

$$\begin{aligned} W &= \left(1 - \frac{\gamma(D + 2F)}{\pi^d \gamma(D + 2F) + \beta D}\right)\left\{w^d - \frac{\beta}{\gamma} \ln\left[\frac{\gamma}{\beta} p\left(D + 2F\right)\right]\right\} \\ &\quad - \ln\left(\pi^d + \frac{\beta}{\gamma}\frac{D}{D + 2F}\right). \end{aligned} \tag{36}$$

The social planner sets F and D compatible with outcome RB, such that (36) is maximized.

Lemma 6 *For society, the best policy* (F, D) *compatible with the imitator engaging first in reverse-engineering and bribing the employee in case of failure is:*

$$\begin{cases} F = 0 \\ D = \frac{\beta}{\gamma p} \end{cases} \tag{37}$$

Proof. See Appendix C. ■

Notice that this policy $\left\{F = 0\ ;\ D = \frac{\beta}{\gamma p}\right\}$ is identical to the policy maximizing welfare under outcome B. In fact, if $F = 0$ and $D = \frac{\beta}{\gamma p}$, the imitator is indifferent between bribery only (outcome B) and reverse-engineering first followed by bribery if it fails (outcome RB). For these values of F and D the investment in reverse-engineering is zero which means that bribery occurs for sure. Plugging in $F = 0$ and $D = \frac{\beta}{\gamma p}$ into (36) yields society's highest welfare conditional on implementing outcome RB :

$$W^{RB} = \left(1 - \frac{\gamma}{\gamma \pi^d + \beta}\right) w^d - \ln\left(\pi^d + \frac{\beta}{\gamma}\right). \tag{38}$$

Comparing (35) and (38), it is clear that social welfare is *the same*. Let us denote this welfare value by $W^{B,2}$. The economic intuition is the following. On the interval $D \in [0, \frac{\beta}{\gamma p}]$ which is compatible with outcome B, social welfare is increasing in D. But on the interval $D \in [\frac{\beta}{\gamma p}, \frac{\pi^d}{p}]$ compatible with outcome RB, social welfare is decreasing in D. It turns out that social welfare is maximized for $D = \frac{\beta}{\gamma p}$ and takes the value $W^{B,2}$. This is represented in Figure 5 below. This non-monotonicity comes from the combination of two effects: the "innovation incentives effect" and the "reverse-engineering cost effect": For low values of D (compatible with outcome B), reverse-engineering does not occur. Increasing D increases innovation incentives and it is good for society. But for higher values of D (compatible with outcome RB), increasing D increases investment in reverse-engineering. This is a waste of resources since even if reverse-engineering fails, the imitator will bribe the employee and society would benefit from having a duopoly. Decreasing D enables to decrease this waste of resources. It turns out that on this interval, this second effect dominates the innovation incentives effect. As a result, the optimal level of damages is lower when the imitator can reverse-engineer than when he cannot. Indeed, when reverse-engineering cannot happen, the "reverse-engineering cost effect" does not matter. Only the "innovation incentives effect" matters and socially optimal damages are higher. Proposition 5 summarizes this finding.

Proposition 5 *Under bribery, when the imitator can reverse-engineer, social welfare is maximized by setting:*

$$\begin{cases} F = 0 \\ D = \frac{\beta}{\gamma p} \end{cases}$$

such that there is no investment in reverse-engineering. The level of damages is lower than when the imitator cannot reverse-engineer the innovation.

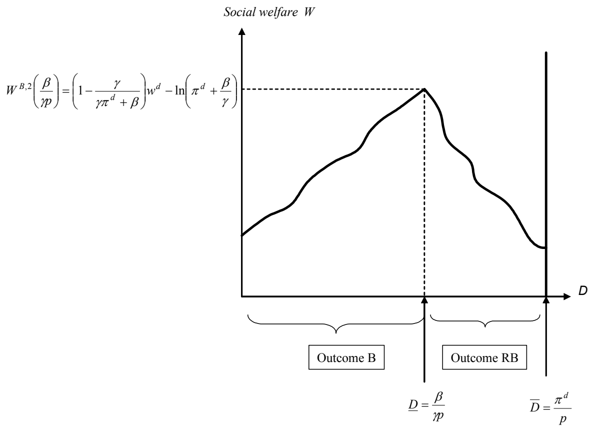


Figure 5: the welfare-maximizing level of damages under scenario 2 when bribery is possible.

A counterintuitive implication of proposition 5 is that:

Proposition 6 *The easier it is to reverse-engineer the innovation, the lower the socially optimal compensation D for the innovator.*

In other words, the socially optimal level of damages $D = \frac{\beta}{\gamma p}$ decreases with γ. This comes again from the "reverse-engineering cost effect": the easier it is to reverse-engineer, the more it takes to prevent the imitator from doing so: by decreasing the damages, the punishment for bribery decreases and reverse-engineering is less tempting. Proposition 5 does not state what the socially optimal policy is when reverse-engineering can happen. To that end, $W^{B,2}$ must be compared with the social welfare when only reverse-engineering occurs.

■ *Consider social welfare if the social planner sets F and D compatible with outcome R.* This outcome occurs when F and D are so high that bribery is deterred. Yet, the imitator still has the possibility to engage in reverse-engineering. Conditional on an innovation occuring, society's welfare is independent of F and D: if the imitator succeeds in reverse-engineering, there is a duopoly and society gets w^d, if the imitator fails, there is a monopoly and society gets w^m. According to (13), the imitator invests $y^* = \frac{1}{\gamma}\ln\left(\frac{\gamma\pi^d}{\beta}\right)$. The probability of success in reverse-engineering is thus $q_R(y^*) = \left(1 - \frac{\beta}{\gamma\pi^d}\right)$ and the cost of reverse-engineering is $c_R(y^*) = \frac{\beta}{\gamma}\ln\left(\frac{\gamma\pi^d}{\beta}\right)$. Lemma 4 states that the innovator, anticipating this outcome, invests $x^{R,*} = \ln\left(\frac{\gamma(\pi^d)^2+\beta\Delta\pi}{\gamma\pi^d}\right)$. The probability of success is $q(x^{R,*}) = \left(1 - \frac{\gamma\pi^d}{\gamma(\pi^d)^2+\beta\Delta\pi}\right)$ and the cost: $c(x^{R,*}) = \ln\left(\pi^d + \frac{\beta\Delta\pi}{\gamma\pi^d}\right)$. Social welfare is a constant given by:

$$
\begin{aligned}
W^R &= \left(1 - \frac{\gamma\pi^d}{\gamma(\pi^d)^2 + \beta\Delta\pi}\right)\left[w^m + \left(1 - \frac{\beta}{\gamma\pi^d}\right)\Delta w - \frac{\beta}{\gamma}\ln\left(\frac{\gamma\pi^d}{\beta}\right)\right] \\
&\quad - \ln\left(\pi^d + \frac{\beta\Delta\pi}{\gamma\pi^d}\right).
\end{aligned} \tag{39}
$$

The next step consists in comparing $W^{B,2}$ and W^R to know whether society is better-off under bribery or reverse-engineering. As for scenario 1, the model needs to be specified to make the comparison possible. However, even by specifying Cournot competition, the nature of the expressions for $W^{B,2}$ and W^R does not allow me to propose an analytical comparison. Hence, I resort to a numerical analysis whereby specific values are attached to the parameters. The condition for society to prefer bribery to reverse-engineering is that:

$$
\begin{aligned}
\left(1 - \frac{\gamma}{\gamma\pi^d + \beta}\right) w^d - \ln\left(\pi^d + \frac{\beta}{\gamma}\right) \quad &\geq \\
\left(1 - \frac{\gamma\pi^d}{\gamma(\pi^d)^2 + \beta\Delta\pi}\right) \quad &\times \\
&\left[\begin{array}{c} w^m + \left(1 - \frac{\beta}{\gamma\pi^d}\right)\Delta w \\ -\frac{\beta}{\gamma}\ln\left(\frac{\gamma\pi^d}{\beta}\right) \end{array}\right] \\
&- \ln\left(\pi^d + \frac{\beta\Delta\pi}{\gamma\pi^d}\right).
\end{aligned} \tag{40}
$$

Assuming Cournot competition with linear demand, the numerical analysis is conducted in Appendix E. It shows that, depending on the values of the parameters, W^R is above or below $W^{B,2}$. Therefore, without loss of generality, I can conclude:

Proposition 7 *Suppose that reverse-engineering can happen ($\gamma \geq \widehat{\gamma}(\pi^d)$). Then,*

- *If (40) holds, the socially optimal trade secret policy is to set the criminal fine to zero and the damages to $D = \frac{\beta}{\gamma p}$. Bribery occurs.*

- *If (40) does not hold, the socially optimal trade secret policy can be implemented by setting $F = 0$ and $D > \frac{\pi^d}{p}$. Bribery is deterred but reverse-engineering occurs.*

Deterring bribery is not socially optimal when reverse-engineering is impossible, but it can be socially optimal when reverse-engineering is possible. This is shown by the numerical analysis in Appendix E. Deterring bribery means that reverse-engineering takes place. In section 5, I showed that the innovator prefers reverse-engineering to bribery. This suggests that when society implements reverse-engineering instead of bribery under scenario 2, providing incentives for innovation is more important than avoiding monopoly distortions (since reverse-engineering can fail, a monopoly could happen). In fact, the numerical analysis shows that reverse-engineering is implemented precisely when the parameter β is very low and γ very high (see table 5). But that means that reverse-engineering is very likely to succeed. Therefore the probability of monopoly distortions is small.

7 Discussion

The main message of this paper is that trade secret law, so far largely neglected in the economic literature, should be considered as an instrument of innovation policy. In this last section, I summarize my major findings and I discuss some unaddressed questions. My model yields four conclusions. First, the trade secret owner is always better-off when the damages increase, but an increase in the criminal fine may hurt her (proposition 3). Also, she can be better-off when the relative cost of capital to finance reverse-engineering is lower and when imitation is easier compared to innovation (corollary 1). Second, if the imitator cannot conduct reverse-engineering, it is socially optimal to let bribery occur under Cournot competition. But if the imitator can conduct reverse-engineering, it can be socially optimal to deter bribery and implement reverse-engineering (propositions 4 and 7). Third, regardless whether the imitator can or cannot conduct reverse-engineering, it is always possible to implement the socially optimal trade secret policy by a strictly positive level of damages and a criminal fine equal to zero (propositions 4 and 7). Fourth, when it is socially optimal that bribery occurs, the level of damages which maximizes social welfare is lower when the imitator is able to conduct reverse-engineering than when he is not able to do so. The optimal criminal fine is zero (propositions 5 and 7). I now discuss some additional issues.

Investment in protecting the trade secret. I have deliberately abstracted from the trade secret owner's incentives to protect its trade secret. The main reason is that the literature has already investigated this issue. Friedman, Landes and Posner (1991) analyze the problem. Motta and Rønde (2002) look at a contractual possibility: the "covenant not to compete" which is a clause whereby the employee accepts not to work for a rival firm for a period of time after he has left his current employer. My model could be extended to account for the innovator's investment to make reverse-engineering of his product more difficult (by endogenizing γ). But its tractability would be altered. In particular, doing so would complicate the already cumbersome derivation of an optimal policy. The equilibrium effect of introducing protection measures would depend whether technical protection and legal protection are substitutes or complements. If they are substitutes, legal protection would save investments in technical protection.

Trade secret policy in a more dynamic perspective. I show that even when the imitator can conduct reverse-engineering, it can be better for society to deter him from doing so and implement bribery (proposition 7). From a more dynamic perspective, reverse-engineering has benefits which are not accounted for in the present model. In particular, by conducting reverse-engineering firms often learn methods and techniques which can prove valuable for future R&D (see Samuelson and Scotchmer, 2004). Hence, in a more dynamic setting where the imitator would also be able to innovate in later periods, my conclusions could be amended.

Appendix

Appendix A: The imitation subgame

Appendix A.1: Analysis of the functions $F_1(D)$, $F_2(D)$, and $\widehat{\gamma}(\pi^d)$. The expressions for $F_1(D)$, $F_2(D)$, and $\widehat{\gamma}(\pi^d)$ are given by (6), (10) and (14) respectively.

- The function $F_1(D)$ is decreasing in D. $F_1(0) = \frac{\pi^d}{2p}$ and $F_1(D) = 0$ for $D = \frac{\pi^d}{p} = \overline{D}$. Indeed, $\frac{\partial F_1(D)}{\partial D} = -\frac{1}{2} < 0$. Then, $F_1(0)$ is obtained by substituting $D = 0$. Finally $\frac{\pi^d}{2p} - \frac{1}{2}D = 0 \Longleftrightarrow D = \frac{\pi^d}{p}$.

- In the (F, D) space, the function $F_2(D)$ is decreasing in D. $F_2(0) = \frac{\beta}{2\gamma p}$ and $F_2(D) = 0$ for $D = \frac{\beta}{\gamma p} = \underline{D}$. Indeed, $\frac{\partial F_2(D)}{\partial D} = -\frac{1}{2} < 0$. Then, $F_2(0)$ is obtained by setting $D = 0$. Finally, $\frac{\beta}{2\gamma p} - \frac{D}{2} = 0 \Longleftrightarrow D = \frac{\beta}{\gamma p}$.

- In the (γ, π^d) space, the function $\widehat{\gamma}(\pi^d)$ is decreasing in π^d and $\lim_{\pi^d \mapsto 0} \widehat{\gamma}(\pi^d) = +\infty$ while $\lim_{\pi^d \mapsto +\infty} \widehat{\gamma}(\pi^d) = 0$. This is obvious given the form of $\widehat{\gamma}(\pi^d)$.

Appendix A.2: The imitator's payoff.

- *If bribery is deterred*, the imitator invests a non-zero amount in reverse-engineering given by (13), as long as $\gamma \geq \widehat{\gamma}(\pi^d)$. Substituting $y^* = \frac{1}{\gamma} \ln \left(\frac{\gamma \pi^d}{\beta} \right)$ into the objective function (12) gives (15). If $\gamma < \widehat{\gamma}(\pi^d)$, the optimal investment in reverse-engineering is the corner solution 0. Substituting 0 into (12) yields $U_E = 0$ i.e (16).

- *If bribery is not deterred*, the imitator invests a non-zero amount in reverse-engineering given by (9) as long as $\gamma \geq \widetilde{\gamma}$. Substituting $y^* = \frac{1}{\gamma} \ln \left(\frac{\gamma p(D+2F)}{\beta} \right)$ into the objective function (8) gives (17). If $\gamma < \widetilde{\gamma}$, the optimal investment in reverse-engineering is the corner solution 0. However, since misappropriation is not deterred, the imitator will bribe the employee and its payoff is given by lemma 1. (18) follows.

Appendix A.3: Proof of proposition 1. I use lemmas 1, 2 and 3.

- *Bribery is deterred.* By definition, this means that $\pi^d \leq \widetilde{\pi}^d$. The imitator can only engage in reverse-engineering. He will invest a positive amount in reverse-engineering if and only if $\gamma \geq \widehat{\gamma}(\pi^d)$. If this does not hold, he does not invest in reverse-engineering.

- *Bribery is not deterred.* This means that $\pi^d > \widetilde{\pi}^d$. The imitator engages in reverse-engineering if and only if $\gamma \geq \widetilde{\gamma}$. I now show that, if this condition holds, the imitator always invests in reverse-engineering *first* and bribes the employee if reverse-engineering fails. This amounts at showing that:

$$\pi^d - \frac{\beta}{\gamma} \left[1 + \ln \left(\frac{\gamma}{\beta} p(D + 2F) \right) \right] \geq \pi^d - p(D + 2F),$$

  which is equivalent to:

$$1 + \ln \left(\frac{\gamma}{\beta} p(D + 2F) \right) \leq \frac{\gamma}{\beta} p(D + 2F).$$

Denoting $k = \frac{\gamma}{\beta} p(D + 2F)$, this inequality can be writen:

$$1 + \ln k \leq k$$

which holds for all $k \in [0, +\infty)$. If $\gamma < \widetilde{\gamma}$ he offers a bribe immediately to the employee since reverse-engineering is not an option.

Appendix B. Innovation

Appendix B.1: The innovator's payoff

- *Suppose bribery is deterred* but reverse-engineering is profitable. The innovator obtains π^m if reverse-engineering fails (which occurs with probability $q_R(y^*) = 1 - \frac{\beta}{\gamma} \frac{1}{\pi^d}$) and she obtains π^d if reverse-engineering succeeds. This occurs with probability $1 - q_R(y^*) = \frac{\beta}{\gamma} \frac{1}{\pi^d}$. It follows that her expected payoff his given by (19). If reverse-engineering is not profitable, obviously the innovator faces no competition. This explains (20).

- *Suppose now that bribery is not deterred.* If in addition reverse-engineering is profitable and succeeds (which occurs with probability $q_R(y^*) = 1 - \frac{\beta}{\gamma} \frac{1}{p(D+2F)}$) the innovator obtains π^d. If reverse-engineering fails (with probability $1 - q_R(y^*) = \frac{\beta}{\gamma} \frac{1}{p(D+2F)}$), the imitator bribes the employee and the payoff for the innovator is $\pi^d + pD$. It follows that her ex-ante expected payoff is given by (21). Finally, if reverse-engineering is unprofitable, her payoff is obviously $\pi^d + pD$.

Appendix B.2: Drawing Figure 3. This figure takes into account the fact that the boundary $\widehat{\gamma}(\pi^d) = \frac{\beta}{\pi^d}$ is independent of the policy parameters F and D.

- The right-hand side graphic represents the case where $\gamma < \widehat{\gamma}(\pi^d)$. We know from Figure 2 that for these values of γ, two outcomes can occur: NO (the imitator is out) and B (only bribery occurs). The boundary between these two outcomes is given by $\pi^d = p(D + 2F)$ or, equivalently, $F = \frac{\pi^d}{2p} - \frac{1}{2} D = F_1(D)$. The analysis of this function in Appendix A enables to draw $F_1(D)$ in the (F, D) space.

- The left-hand side graphic represents the case where $\gamma \geq \widehat{\gamma}(\pi^d)$. We know from Figure 2 that for these values of γ, three outcomes can occur: R (reverse-engineering only), RB (reverse-engineering first and bribe is failure), B (bribery only). The boundary between B and RB is $\gamma = \frac{\beta}{p(D+2F)}$, equivalent to $F = \frac{\beta}{2\gamma p} - \frac{1}{2}D = F_2(D)$. And the boundary between RB and R is $\pi^d = p(D+2F)$, equivalent to $F = F_1(D)$. Now, notice that $F_1(D) \geq F_2(D)$ if and only if $\frac{\pi^d}{2p} - \frac{1}{2}D \geq \frac{\beta}{2\gamma p} - \frac{1}{2}D$ or $\gamma \geq \frac{\beta}{\pi^d} = \widehat{\gamma}(\pi^d)$ which holds by assumption. Again, using Appendix A, the two boundaries $F_1(D)$ and $F_2(D)$ can be represented in the (F, D) space.

Appendix B.3: Drawing Figure 4 and proof of proposition 3. The focus is on scenario 2 defined by $\gamma \geq \widehat{\gamma}(\pi^d)$.

- Consider first U_I as a function of F, holding D constant. Suppose $F \leq F_2(D)$. Observing Figure 3, we know that the corresponding outcome is B. It follows that $U_I^B = \pi^d + pD$ which is independent of F. Then, suppose that $F \in [F_2(D), F_1(D)]$. Observing Figure 3 again, the corresponding outcome is RB and: $U_I^{RB} = \pi^d + \left(\frac{\beta}{\gamma}\frac{1}{D+2F}\right) D$. Clearly, $\frac{\partial U_I^{RB}}{\partial F} = -\frac{\beta}{\gamma}\frac{2D}{(D+2F)^2} \leq 0$. This proves the second part of proposition 2. In addition, $U_I^{RB}|_{F=F_2} = \pi^d + \left[\frac{\beta}{\gamma}\frac{1}{D+2\left(\frac{\beta}{2\gamma p}-\frac{D}{2}\right)}\right] D = \pi^d + pD$ and $U_I^{RB}|_{F=F_1} = \pi^d + \left[\frac{\beta}{\gamma}\frac{1}{D+2\left(\frac{\pi^d}{2p}-\frac{D}{2}\right)}\right] D = \pi^d + \frac{\beta}{\gamma}\frac{pD}{\pi^d}$. Finally, suppose that $F > F_1(D)$. This corresponds to outcome R and $U_I^R = \pi^m - \left(1 - \frac{\beta}{\gamma}\frac{1}{\pi^d}\right)\Delta\pi$ is independent of F. $U_I^R \geq U_I^{RB}|_{F=F_1}$ if and only if $\Delta\pi \geq \pi^d$ or $\pi^m \geq 2\pi^d$ which holds by assumption.

- Then, consider U_I as a function of D holding F constant. Suppose $D \leq D_2(F)$ where D_2 is such that $F = \frac{\beta}{2\gamma p} - \frac{1}{2}D_2$ or $D_2 = \frac{\beta}{\gamma p} - 2F$. The corresponding outcome is B and: $U_I^B = \pi^d + pD$ which is linearly increasing in D. $U_I^B|_{D=0} = \pi^d$ and $U_I^B|_{D=D_2} = \pi^d + \frac{\beta}{\gamma} - 2pF$. Suppose then that $D \in [D_2(F), D_1(F)]$ where D_1 is such that $F = \frac{\pi^d}{2p} - \frac{1}{2}D_1$ or $D_1 = \frac{\pi^d}{p} - 2F$. On this interval, the corresponding outcome is RB and $U_I^{RB} = \pi^d + \left(\frac{\beta}{\gamma}\frac{1}{D+2F}\right) D$. Notice that $\frac{\partial U_I^{RB}}{\partial D} = \frac{\beta}{\gamma}\frac{2F}{(D+2F)^2} \geq 0$. And $U_I^{RB}|_{D=D_2} = \pi^d + \left(\frac{\beta}{\gamma}\frac{1}{\frac{\beta}{\gamma p}-2F+2F}\right)\left(\frac{\beta}{\gamma p} - 2F\right) = \pi^d + \frac{\beta}{\gamma} - 2pF$. Also, $U_I^{RB}|_{D=D_1} = \pi^d + \left(\frac{\beta}{\gamma}\frac{1}{\frac{\pi^d}{p}-2F+2F}\right)\left(\frac{\pi^d}{p} - 2F\right) = \pi^d + \frac{\beta}{\gamma} - \frac{\beta}{\gamma}\frac{2pF}{\pi^d}$. Finally, suppose that $D \geq D_1(F)$. This corresponds to outcome R and $U_I^R = \pi^m - \left(1 - \frac{\beta}{\gamma}\frac{1}{\pi^d}\right)\Delta\pi$ is independent of D. $U_I^R \geq U_I^{RB}|_{D=D_1}$ if and only if $\Delta\pi \geq \pi^d - 2pF$ which holds.

Appendix B.4: I propose here an additional result concerning the effect on the innovator's payoff of a *joint* increase in the criminal fine and the damages.

Lemma 7 *Suppose $\gamma \geq \widehat{\gamma}(\pi^d)$. An increase in both F and D that yields a switch from equilibrium B to equilibrium RB is beneficial to the innovator if and only if the value of the damages D^{RB} compatible with equilibrium RB is high enough:*

$$D^{RB} \geq \underline{D}^{RB} = \frac{2\gamma p F^{RB} D^B}{\beta - \gamma p D^B}. \tag{41}$$

Hence, an increase in the compensatory damages can make the trade secret owner worse-off if it is coupled with an increase in the criminal fine. I consider the following numerical example: I set $\pi^d = 40$, $\beta = 16$, $\gamma = 2$, $p = .3$. A configuration compatible with outcome B is $(F^B, D^B) = (5, 10)$. This yields $U_I^B = 43$. Then, consider two configurations compatible with outcome RB. First, $(F^{RB}, D^{RB}) = (15, 19)$. In this example, $D^{RB} = 19 > \underline{D}^{RB} = 18$. Hence the condition of the above lemma is satisfied. As a result, the innovator is better-off when both legal parameters increase ($U_I^{RB} = 43.01 > U_I^B$). Then consider $(F^{RB}, D^{RB}) = (15, 13)$. Now D^{RB} does not satisfy condition (41): $13 < 18$. As a result, the innovator is worse off after an increase in both F and D (indeed, $U_I^{RB} = 42.41 < U_I^B$).

Appendix B.5: Proof that the innovator prefers R to B and R to RB.

- *Consider first how U_I is affected by a change in D, holding F constant.* In outcome B, the highest payoff the innovator can get is obtained when D reaches its maximal value compatible with outcome B, that is, when $D = \frac{\beta}{\gamma p}$. Indeed, we know from Appendix B.2 that $\frac{\partial U_I^B}{\partial D} > 0$ and the highest D is given by $D_2 \left|_{F=0}\right. = \frac{\beta}{\gamma p}$. At this point, $U_I^B = \pi^d + \frac{\beta}{\gamma}$. We know that $U_I^R = \pi^m - \left(1 - \frac{\beta}{\gamma} \frac{1}{\pi^d}\right) \Delta\pi = \pi^d + \frac{\beta}{\gamma} \frac{\Delta\pi}{\pi^d}$. It follows that $U_I^R \geq U_I^B$ if and only if $\Delta\pi \geq \pi^d$ or $\pi^m \geq 2\pi^d$ which holds by assumption. Then, the highest payoff the innovator can obtain in outcome RB is reached for $D = D_1 \left|_{F=0}\right. = \frac{\pi^d}{p}$. This comes from the fact that $\frac{\partial U_I^B}{\partial D} > 0$ and the maximal D compatible with outcome RB is $D = D_1 \left|_{F=0}\right.$. At this point, $U_I^{RB} = U_I^{RB,\max} = \pi^d + \frac{\beta}{\gamma}$ while, again, $U_I^R = \pi^d + \frac{\beta}{\gamma} \frac{\Delta\pi}{\pi^d}$. Since $\Delta\pi \geq \pi^d$, it follows that $U_I^R \geq U_I^{RB,\max}$.

- *Consider now how U_I is affected by a change in F holding D constant.* In outcome B, the innovator's payoff is independent of F and the maximal payoff she can get is obtained for $D = D_2|_{F=0}$. At this point, we have $U_I^B = \pi^d + \frac{\beta}{\gamma}$ and so the previous analysis applies. In outcome RB, the innovator's payoff is a decreasing function of F so that the highest payoff is obtained when F reaches its *minimum* value compatible with this outcome: $F = F_2$. At this point, $U_I^{RB} = \pi^d + pD$.

Appendix C. Trade secret law design, proof of lemma 6.

The proof is in three steps. In *Step 1,* I derive the expression for $\frac{\partial W}{\partial F}$. In *Step 2,* I show that $\frac{\partial W}{\partial F} < 0$. From that, it can be concluded that the socially optimal F is the minimal F compatible with outcome RB. Given that, *step 3* investigates the socially optimal D.

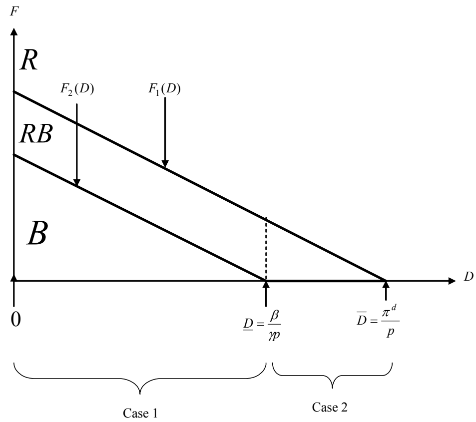
- *Step 1*: I compute $\frac{\partial W}{\partial F}$. The expression for W is given by (36). Hence, we have:

$$\begin{aligned}\frac{\partial W}{\partial F} = & \left[-\frac{2\gamma\left(\gamma\pi^d(D+2F)+\beta D\right)-2\gamma^2(D+2F)\pi^d}{\left(\pi^d\gamma(D+2F)+\beta D\right)^2}\right]\left[\begin{array}{c} w^d- \\ \frac{\beta}{\gamma}\ln\left(\frac{\gamma}{\beta}p(D+2F)\right)\end{array}\right] \\ & +\left[\frac{\pi^d\gamma(D+2F)+\beta D-\gamma(D+2F)}{\pi^d\gamma(D+2F)+\beta D}\right]\left(-\frac{\beta}{\gamma}\frac{2}{(D+2F)}\right) \\ & -\frac{\frac{\beta}{\gamma}\left(-\frac{2D}{(D+2F)^2}\right)}{\pi^d+\frac{\beta D}{\gamma(D+2F)}}. \end{aligned} \tag{42}$$

After rearranging terms, this expression simplifies to:

$$\begin{aligned}\frac{\partial W}{\partial F} = & -\frac{2\gamma\beta D}{\gamma\pi^d(D+2F)}\left[\frac{w^d-\frac{\beta}{\gamma}\ln\left(\frac{\gamma}{\beta}p(D+2F)\right)}{\pi^d\gamma(D+2F)+\beta D}-\frac{1}{\gamma(D+2F)}\right]- \\ & \frac{\beta}{\gamma}\frac{2}{(D+2F)}\frac{\beta D+\gamma(D+2F)(\pi^d-1)}{\pi^d\gamma(D+2F)+\beta D}. \end{aligned} \tag{43}$$

- *Step 2*: I denote T the term in square brackets in (43). Notice that $T \geq 0$ is a sufficient condition for $\frac{\partial W}{\partial F} < 0$. Hence, I investigate the sign of T in the relevant ranges of F and D values such that outcome RB occurs. The following figure illustrates the situation.



☐ `Case1`. Suppose that $D \in [0, \underline{D}]$. The condition that $(F, D) \in RB$ implies that $F \in [F_2(D), F_1(D)]$, for all $D \in [0, \underline{D}]$ where $F_2(D) \geq 0$.

☐ `Case 2.` Suppose that $D \in [\underline{D}; \overline{D}]$. The condition that $(F, D) \in RB$ implies that $F \in [0, F_1(D)]$, for all $D \in [\underline{D}, \overline{D}]$.

I show that in both cases, $T \geq 0$ so that $\frac{\partial W}{\partial F} < 0$.

☐ `Case 1.` After re-arranging the terms, the condition $T \geq 0$ is equivalent to:

$$\underbrace{\gamma(D + 2F)\left(\pi^d + cs^d\right)}_{f(F)} \geq \underbrace{\beta(D + 2F)\ln\left[\frac{\gamma}{\beta}p(D + 2F)\right] + \beta D}_{g(F)} \tag{44}$$

$T \geq 0$ implies that (44) holds for all $F \in [F_2(D), F_1(D)]$. Define the left-hand side term in (44) as $f(F)$ and the right-hand side term as $g(F)$. Plugging in the values of $F_2(D)$ and $F_1(D)$ yields $f(F_2(D)) = \frac{\beta}{p}\left(\pi^d + cs^d\right)$, $f(F_1(D)) = \frac{\gamma\pi^d}{p}\left(\pi^d + cs^d\right)$, $g(F_2(D)) = \beta D$ and $g(F_1(D)) = \frac{\beta\pi^d}{p}\ln\left(\frac{\gamma\pi^d}{\beta}\right) + \beta D$. In addition:

$$\frac{\partial f(F)}{\partial F} = 2\gamma\left(\pi^d + cs^d\right) > 0, \tag{45}$$

and:

$$\frac{\partial g(F)}{\partial F} = 2\beta\left\{\ln\left[\frac{\gamma}{\beta}p(D + 2F)\right] + 1\right\} > 0. \tag{46}$$

Lemma 8 $\frac{\partial f(F)}{\partial F} > \frac{\partial g(F)}{\partial F}$ *for* $F \in [0, F_1(D)]$.

Proof. A necessary and sufficient condition for $\frac{\partial f(F)}{\partial F} > \frac{\partial g(F)}{\partial F}$ is that

$$\gamma\left(\pi^d + cs^d\right) > \beta\left\{\ln\left[\frac{\gamma}{\beta}p(D+2F)\right] + 1\right\},$$

which is equivalent to:

$$\frac{\gamma}{\beta}\pi^d + \frac{\gamma}{\beta}cs^d > \ln\left[\frac{\gamma}{\beta}p(D+2F)\right] + 1. \tag{47}$$

Denoting $k = \frac{\gamma}{\beta}$, $y = \pi^d$ and $z = p(D+2F)$, we have $y \geq z$ since $F \leq F_1(D) = \frac{\pi^d}{2p} - \frac{D}{2}$ is equivalent to $\pi^d \geq p(D+2F)$. Rewriting (51):

$$ky + kcs^d > \ln(kz) + 1$$

Now we know $ky \geq \ln(ky) + 1$ as a general property. In addition, we have $ky \geq kz$. It follows that:

$$ky + kcs^d > ky \geq \ln(ky) + 1 \geq \ln(kz) + 1,$$

so that indeed (47) is satisfied. ■

Then, I compare $f(F_2(D))$ and $g(F_2(D))$.

Lemma 9 $g(F_2(D)) < f(F_2(D))$.

Proof. A necessary and sufficient condition for $g(F_2(D)) < f(F_2(D))$ is that

$$pD < \pi^d + cs^d \tag{48}$$

where $D \in [0, \frac{\beta}{\gamma p}]$. Plugging in the largest value of D into (51) yields:

$$\pi^d + cs^d > \frac{\beta}{\gamma}. \tag{49}$$

By assumption (scenario 2), we have $\pi^d \geq \frac{\beta}{\gamma}$. And obviously, $\pi^d + cs^d > \pi^d$. That implies that (49) holds. ■

Combining lemmas 8 and 9, I conclude that $f(F) > g(F)$ or $T > 0$ on $[F_2(D), F_1(D)]$.

☐ `Case 2.` The condition $T \geq 0$ is still equivalent to (44). Both f and g are defined on $F \in [0, F_1(D)]$. I compute $f(0) = \gamma D(\pi^d + cs^d)$, $g(0) = \beta D \ln\left(\frac{\gamma}{\beta}pD\right) + \beta D$. The values of $\frac{\partial f(F)}{\partial F}$ and $\frac{\partial g(F)}{\partial F}$ are still given by (45) and (46) respectively. Hence, lemma 8 still holds.

Lemma 10 $f(0) > g(0)$.

Proof. A necessary and sufficient condition for $f(0) > g(0)$ is that:

$$\gamma D(\pi^d + cs^d) > \beta D \ln\left(\frac{\gamma}{\beta} pD\right) + \beta D,$$

which is equivalent to:

$$\frac{\gamma}{\beta}\pi^d + \frac{\gamma}{\beta} cs^d > \ln\left(\frac{\gamma}{\beta} pD\right) + 1 \tag{50}$$

Denoting $k = \frac{\gamma}{\beta}$, $y = \pi^d$ and $z = pD$ we have $y \geq z$ since $D \leq \frac{\pi^d}{p}$. Hence, $ky \geq kz$. In addition, $ky \geq \ln(ky) + 1$ is a general property. This implies:

$$ky + kcs^d > ky \geq \ln(ky) + 1 \geq \ln(kz) + 1. \tag{51}$$

Hence, (50) holds. ■

Consequently, combining lemmas 8 and 10, I conclude that $f(F) > g(F)$ or $T > 0$ on $F \in [0, F_1(D)]$.

- *Step 3*: Given that it is optimal to set $F = F_2(D)$ for all $D \in \left[0, \frac{\beta}{\gamma p}\right]$ and $F = 0$ for all $D \in \left[\frac{\beta}{\gamma p}, \frac{\pi^d}{p}\right]$, I now investigate the socially optimal damage level.

☐ `Case 1`. The focus is on $D \in [0, \frac{\beta}{\gamma p}]$. In that case, the minimal F compatible with outcome RB is $F = F_2(D) = \frac{\beta}{2\gamma p} - \frac{D}{2}$. Plugging this value of F into society's objective function (36) yields:

$$\begin{aligned} W &= \left[1 - \frac{\gamma\left(D + \frac{\beta}{\gamma p} - D\right)}{\gamma\pi^d\left(D + \frac{\beta}{\gamma p} - D\right) + \beta D}\right]\left\{w^d - \frac{\beta}{\gamma}\ln\left[\frac{\gamma}{\beta}p(D + \frac{\beta}{\gamma p} - D)\right]\right\} \\ &\quad - \ln\left(\pi^d + \frac{\beta}{\gamma}\frac{D}{D + \frac{\beta}{\gamma p} - D}\right). \end{aligned}$$

After simplification, this expression reduces to:

$$W = \left(1 - \frac{1}{\pi^d + pD}\right) w^d - \ln\left(\pi^d + pD\right).$$

Then,

$$\frac{\partial W}{\partial D} = \frac{p}{\pi^d + pD}\left(\frac{w^d}{\pi^d + pD} - 1\right).$$

This expression is strictly positive if and only if:

$$w^d > \pi^d + pD.$$

The highest D compatible with outcome RB and the case where $D \in [0, \frac{\beta}{\gamma p}]$ is clearly $D = \underline{D} = \frac{\beta}{\gamma p}$. Plugging this value of D into the previous inequality gives:

$$2\pi^d + cs^d > \pi^d + \frac{\beta}{\gamma}. \tag{52}$$

Now, since $\gamma \geq \frac{\beta}{\pi^d}$ under scenario 2, it follows that $\pi^d \geq \frac{\beta}{\gamma}$. Clearly, the following two inequalities hold:

$$\begin{cases} 2\pi^d + cs^d > \pi^d + \pi^d \\ \pi^d + \pi^d \geq \pi^d + \frac{\beta}{\gamma}. \end{cases}$$

But this implies that (52) holds. So $\frac{\partial W}{\partial D} > 0$ and society sets the maximal D which is $D = \underline{D} = \frac{\beta}{\gamma p}$. For this value of the damages, society's welfare is given by:

$$W = \left(1 - \frac{\gamma}{\gamma\pi^d + \beta}\right) w^d - \ln\left(\pi^d + \frac{\beta}{\gamma}\right) \tag{53}$$

☐ Case 2. The focus is now on $D \in \left[\frac{\beta}{\gamma p}, \frac{\pi^d}{p}\right]$. In that case, the minimal F compatible with outcome RB is $F = 0$. Plugging this value of F into society's objective function (36) and rearranging the terms yields:

$$W = \left(1 - \frac{\gamma}{\pi^d\gamma + \beta}\right)\left[w^d - \frac{\beta}{\gamma}\ln\left(\frac{\gamma}{\beta}pD\right)\right] - \ln\left(\pi^d + \frac{\beta}{\gamma}\right). \tag{54}$$

Differentiating with respect to D:

$$\frac{\partial W}{\partial D} = -\frac{\beta}{\gamma}\left(1 - \frac{\gamma}{\pi^d\gamma + \beta}\right)\frac{1}{D} < 0 \tag{55}$$

As a result, society sets the smallest D compatible with $D \in \left[\frac{\beta}{\gamma p}, \frac{\pi^d}{p}\right] : D = \underline{D} = \frac{\beta}{\gamma p}$. Plugging in this value of D into 36) yields society's welfare:

$$W = \left(1 - \frac{\gamma}{\gamma\pi^d + \beta}\right) w^d - \ln\left(\pi^d + \frac{\beta}{\gamma}\right) \tag{56}$$

Notice that in both cases (case 1 or case 2), the socially optimal damages is $D = \underline{D} = \frac{\beta}{\gamma p}$ and social welfare for this level of the damages is the same. This situation is depicted on the figure below.

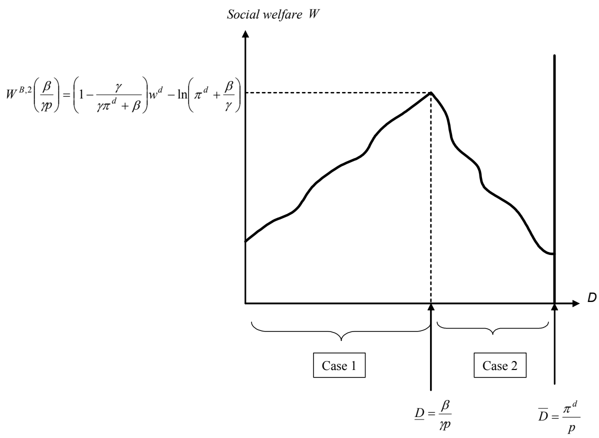


Figure 6. Social welfare as a function of D (for the minimal value of F), compatible with outcome RB.

This concludes the proof of lemma 12.

Appendix D. Trade secret law design when the employee is imprisoned.

As noticed in section 3, the employee may not have the possibility to pay a monetary fine F. An alternative interpretation of F for the employee is that it represents a monetary equivalent for the disutility of being imprisoned (with the restriction that this disutility is equal to the imitator's disutility from paying the fine).[30] But, ex-post, prison is a net social cost for society. While a monetary fine can be seen as a transfer from the criminal to the government which can use it for financing socially valuable activities, a jail sentence imposes two types of cost on society ex-post: the criminal's disutility and the cost of keeping him in jail. Assuming for simplicity that this latter cost is zero, the (static) social benefit of an innovation which is imitated illegally is now:

$$w=\underbrace{\pi^d+pD}_{\text{innovator}}+\underbrace{\pi^d-pF-p(D+F)}_{\text{imitator}}+\underbrace{pF-pF}_{\text{employee}}+\underbrace{pF}_{\text{government}}+\underbrace{cs^d}_{\text{consumers}}=w^d-pF, \qquad (57)$$

from which reverse-engieering costs must be substracted if reverse-engineering happens. Comparing with (28), the employee's disutility from being imprisoned is not transferable to the government, hence the cost pF.

[30] Assuming that sentence to jail and monetary fines are perfect substitutes is strong. It would be interesting in future research to introduce these two alternatives in the same model and see how it affects trade secret protection.

The analysis conducted in section 6 can be applied. Here, I denote society's objective function by W'. The overall message of Appendix D is that the optimal trade secret policy derived in section 6 is still valid when the employee faces a risk of imprisonment instead of a monetary fine. In particular, in equilibrium, the socially optimal F is set equal to zero (so that imprisonment is not an equilibrium outcome). Then, trade secret protection is ensured by a strictly positive level of damages identical to that derived in section 6.

☐ Scenario 1 ($\gamma < \frac{\beta}{\pi^d} = \widehat{\gamma}(\pi^d)$). Two outcomes must be considered.

- If the social planner does not deter bribery (outcome B), social welfare is given by:

$$W' = \left(1 - \frac{1}{\pi^d + pD}\right)\left(w^d - pF\right) - \ln\left(\pi^d + pD\right). \tag{58}$$

This expression is analogous to the welfare expression in section 6.1 for outcome B. The only difference is the cost pF, as explained above. Clearly, $\frac{\partial W'}{\partial F} \leq 0$ so that the socially optimal F is $F = 0$. Given that, $W' = W$ where W is defined by the welfare expression in section 6.1 for outcome B. Then, it has been shown in section 6 that $\frac{\partial W}{\partial D} > 0$ so that the socially optimal D is the highest D compatible with outcome B. Following the analysis in section 6.1, this value of the damages is $D = \overline{D} = \frac{\pi^d}{p}$. Plugging in $F = 0$ and $D = \overline{D}$ into W' yields the same value $W^{B,1}$ given by (32).

- If the social planner sets F and D so high that bribery is deterred (outcome NO), social welfare W'^{NO} is a constant given by (33): $W'^{NO} = W^{NO}$.

- Consequently, the comparison is between W^B and W^{NO}, so that the analysis conducted in section 6.1 is unchanged.

☐ Scenario 2 ($\gamma \geq \frac{\beta}{\pi^d} = \widehat{\gamma}(\pi^d)$). Three outcomes must be considered.

- If society sets F and D such that outcome B occurs, its objective function is the same as in section 6.2 for outcome B. The same analysis applies as well. In particular, the socially optimal F is $F=0$ and the socially optimal D is the highest D compatibe with this outcome. Under scenario 2, this value of D is $\underline{D}=\frac{\beta}{\gamma p}$. And social welfare is given by (39). There is no change compared to the analysis in section 6.2.

- If society sets F and D so high that bribery is deterred, only reverse-engineering occurs (outcome R) and social welfare is given by the constant given in (43). Here again, there is no change compared to section 6.2.

- Now, suppose society sets F and D compatible with outcome RB.

This case is less straightforward. Indeed, if the imitator succeeds in reverse-engineering, bribery does not occur and society gets w^d. If he fails in reverse-engineering, which occurs with probability $1-q_R(y^*)$, society obtains w^d-pF : indeed, bribery occurs and the employee in jail implies a net social cost as discussed previously. We have $y^*=\frac{1}{\gamma}\ln\left[\frac{\gamma p(D+2F)}{\beta}\right]$. Hence, the cost of reverse-engineering beeing $c_R(y^*)=\frac{\beta}{\gamma}\ln\left[\frac{\gamma p(D+2F)}{\beta}\right]$ and the probabililty of success being $1-\frac{\beta}{\gamma p(D+2F)}$, it follows that, conditionnal on an innovation occuring, society's benefit is:

$$w^d-\underbrace{\frac{\beta}{\gamma p(D+2F)}}_{\substack{\text{probability of}\\ \text{reverse-engineering}\\ \text{failure}}}\times\underbrace{pF}_{\substack{\text{net social}\\ \text{cost}}}-\underbrace{\frac{\beta}{\gamma}\ln\left[\frac{\gamma p(D+2F)}{\beta}\right]}_{\substack{\text{cost of}\\ \text{reverse-engineering}}}. \tag{59}$$

As a result, society's objective function is given by:

$$\begin{aligned} W' &= \left(1-\frac{\gamma(D+2F)}{\pi^d\gamma(D+2F)+\beta D}\right)\left(w^d-\frac{\beta F}{\gamma(D+2F)}-\frac{\beta}{\gamma}\ln\left[\frac{\gamma p(D+2F)}{\beta}\right]\right) \\ &\quad -\ln\left(\pi^d+\frac{\beta}{\gamma}\frac{D}{D+2F}\right). \end{aligned} \tag{60}$$

Society sets F and D compatible with outcome RB such that the above function is maximized. Like in Appendix C, I proceed in three steps. First, I compute $\frac{\partial W'}{\partial F}$ (*step 1*). Then I show it is negative (*step 2*). Finally, I derive the optimal D given that F is set at its minimum value (*step 3*).

Step 1. First, I show that $\frac{\partial W'}{\partial F} \leq 0$. The expression for $\frac{\partial W'}{\partial F}$ is given by:

$$\begin{aligned}
\frac{\partial W'}{\partial F} &= \left[-\frac{2\gamma\beta D}{\left(\gamma\pi^d(D+2F)+\beta D\right)^2}\right]\left\{w^d - \frac{\beta F}{\gamma(D+2F)} - \frac{\beta}{\gamma}\ln\left[\frac{\gamma p(D+2F)}{\beta}\right]\right\} \\
&\quad - \left[\frac{(\pi^d-1)\gamma(D+2F)+\beta D}{\pi^d\gamma(D+2F)+\beta D}\right]\left[\frac{2\beta}{\gamma(D+2F)} + \frac{\beta D}{\gamma(D+2F)^2}\right] \\
&\quad + \frac{2\beta\gamma D}{\gamma(D+2F)\left(\gamma\pi^d(D+2F)+\beta D\right)}.
\end{aligned}$$

This involved expression can be rewritten as follows:

$$\begin{aligned}
\frac{\partial W'}{\partial F} &= \left[-\frac{2\gamma\beta D}{\left(\gamma\pi^d(D+2F)+\beta D\right)^2}\right]\left\{w^d - \frac{\beta}{\gamma}\ln\left[\frac{\gamma p(D+2F)}{\beta}\right]\right\} \\
&\quad + \frac{2\beta\gamma D}{\gamma(D+2F)\left(\gamma\pi^d(D+2F)+\beta D\right)} \\
&\quad + \frac{2\gamma\beta D}{\left(\gamma\pi^d(D+2F)+\beta D\right)^2} \times \frac{\beta F}{\gamma(D+2F)} \\
&\quad - \left[\frac{(\pi^d-1)\gamma(D+2F)+\beta D}{\pi^d\gamma(D+2F)+\beta D}\right]\left[\frac{2\beta}{\gamma(D+2F)} + \frac{\beta D}{\gamma(D+2F)^2}\right].
\end{aligned}$$

The terms of this expression can again be rearranged to

$$\begin{aligned}
\frac{\partial W'}{\partial F} &= -\frac{2\gamma\beta D}{\gamma\pi^d(D+2F)}\left[\frac{w^d - \frac{\beta}{\gamma}\ln\left(\frac{\gamma}{\beta}p(D+2F)\right)}{\pi^d\gamma(D+2F)+\beta D} - \frac{1}{\gamma(D+2F)}\right] \\
&\quad - \frac{\beta}{\gamma}\frac{2}{(D+2F)}\frac{\beta D+\gamma(D+2F)(\pi^d-1)}{\pi^d\gamma(D+2F)+\beta D} \\
&\quad + \frac{2\gamma\beta D}{\left(\gamma\pi^d(D+2F)+\beta D\right)^2} \times \frac{\beta F}{\gamma(D+2F)} \\
&\quad - \beta\frac{(\pi^d-1)\gamma(D+2F)+\beta D}{\left[\pi^d\gamma(D+2F)+\beta D\right]\gamma(D+2F)} \times \left(2 + \frac{D}{D+2F}\right).
\end{aligned}$$

Now, I denote the *two first terms* of this expression by L. Then:

$$\begin{aligned}
\frac{\partial W'}{\partial F} &= L \\
&\quad + \frac{2\gamma\beta D}{\left(\gamma\pi^d(D+2F)+\beta D\right)^2} \times \frac{\beta F}{\gamma(D+2F)} \\
&\quad - \beta\frac{(\pi^d-1)\gamma(D+2F)+\beta D}{\left[\pi^d\gamma(D+2F)+\beta D\right]\gamma(D+2F)} \times \left(2 + \frac{D}{D+2F}\right).
\end{aligned} \tag{61}$$

Step 2. Notice that L is exactly the expression for $\frac{\partial W}{\partial F}$ derived in Appendix C. That is to say, L expresses how social welfare is affected by a marginal change in F, when the employee pays a monetary fine instead of being imprisoned. And Appendix C (steps 1 and 2) proved that:

$$L = \frac{\partial W}{\partial F} < 0.$$

Hence, for $\frac{\partial W'}{\partial F} < 0$, it must be that the sum of the two last terms in (61) is negative:

$$\begin{aligned} 0 \geq\ & \frac{2\gamma\beta D}{(\gamma\pi^d(D+2F)+\beta D)^2} \times \frac{\beta F}{\gamma(D+2F)} - \\ & -\beta\frac{(\pi^d-1)\gamma(D+2F)+\beta D}{[\pi^d\gamma(D+2F)+\beta D]\,\gamma(D+2F)} \times \left(2+\frac{D}{D+2F}\right). \end{aligned}$$

This inequality can be simplified to:

$$0 \geq \frac{2\gamma\beta DF}{\gamma\pi^d(D+2F)+\beta D} - \frac{\left[(\pi^d-1)\gamma(D+2F)+\beta D\right](3D+4F)}{D+2F},$$

or

$$\left[(\pi^d-1)\gamma(D+2F)+\beta D\right](3D+4F)\left[\gamma\pi^d(D+2F)+\beta D\right] \geq 2\gamma\beta DF\,(D+2F)\,. \tag{62}$$

Suppose π^d is set at its minimal value: $\pi^d = 1$. Then, the previous inequality reads as:

$$\beta D\,(3D+4F)\,[\gamma(D+2F)+\beta D] \geq 2\gamma\beta DF\,(D+2F)\,,$$

or

$$(3D+4F)\,[\gamma(D+2F)+\beta D] \geq 2\gamma F\,(D+2F)\,.$$

Developing this expression yields:

$$3D\gamma(D+2F)+3D^2\beta+4\gamma F(D+2F)+4F\beta D \geq 2\gamma F\,(D+2F)\,. \tag{63}$$

Clearly: $4\gamma F(D+2F) \geq 2\gamma F\,(D+2F)$. Since all the terms on the left-hand side are positive, we can conclude that (63) always holds. So, (62) holds for $\pi^d = 1$. But notice that increasing π^d would only increase the value of the left-hand side term in expression (62). As a result, if (62) holds for the minimal value of π^d, it holds necessarily for larger values.

It follows from this analysis that $\frac{\partial W'}{\partial F} < 0$.

Given that $\frac{\partial W'}{\partial F} < 0$, the socially optimal F is the minimal F compatible with outcome RB. But this conclusion is identical to the conclusion reached in section 6 when the employee payed a monetary criminal fine.

Step 3. After setting F equal to its minimal value, it is possible to derive the socially optimal damages D.

$\square$ `Case 1.` $D \in \left[0, \frac{\beta}{\gamma p}\right]$. I substitute for $F = F_2(D) = \frac{\beta}{2\gamma p} - \frac{D}{2}$ into society's objective function (60). This yields, after simplification:

$$W' = \left(1 - \frac{1}{\pi^d + pD}\right)\left(w^d - \frac{\beta}{2\gamma} + p\frac{D}{2}\right) - \ln\left(\pi^d + pD\right).$$

Maximizing this function with respect to D yields, after rearranging the terms:

$$\frac{\partial W'}{\partial D} = \frac{p}{\pi^d + pD}\left(\frac{2w^d - \frac{\beta}{\gamma} + pD + (\pi^d + pD - 1)(\pi^d + pD) - 2(\pi^d + pD)}{2(\pi^d + pD)}\right).$$

This expression is positive if and only if:

$$2w^d - \frac{\beta}{\gamma} + pD + (\pi^d + pD - 1)(\pi^d + pD) - 2(\pi^d + pD) \geq 0,$$

or

$$(\pi^d + pD)^2 + \pi^d + 2cs^d - 2pD - \frac{\beta}{\gamma} \geq 0.$$

It is enough to show that $(\pi^d + pD)^2 + \pi^d - 2pD - \frac{\beta}{\gamma} \geq 0$ holds. Developing this expression and re-arranging the terms, it is equivalent to:

$$-\frac{\beta}{\gamma} + \pi^{d2} + 2pD(\pi^d - 1) + (pD)^2 \geq 0. \tag{64}$$

Now, notice that $\pi^{d2} \geq \frac{\beta}{\gamma}$ holds since $\gamma \geq \frac{\beta}{\pi^d}$ (this is the definition of scenario 2) and in addition $\pi^d \geq 1$ by assumption. Also, since $\pi^d \geq 1$, $2pD(\pi^d - 1) \geq 0$. As a result, inequality (64) holds.

We can conclude $\frac{\partial W'}{\partial D} \geq 0$ so that the socially optimal D is the highest value of D compatible with $D \in \left[0, \frac{\beta}{\gamma p}\right]$: it is $\underline{D} = \frac{\beta}{\gamma p}$. For this level of the damages, social welfare is:

$$W' = \left(1 - \frac{\gamma}{\pi^d\gamma + \beta}\right) w^d - \ln\left(\pi^d + \frac{\beta}{\gamma}\right). \tag{65}$$

Clearly, this expression is the same as the expression derived in Appendix C (where the employee faces monetary fines instead on prison).

$\square$ `Case 2.` $D \in \left[\frac{\beta}{\gamma p}, \frac{\pi^d}{p}\right]$. The minimal value of F in that case is $F = 0$. Plugging in $F = 0$ into W' given by (60) yields:

$$W' = \left(1 - \frac{\gamma}{\pi^d\gamma + \beta}\right)\left[w^d - \frac{\beta}{\gamma}\ln\left(\frac{\gamma}{\beta}pD\right)\right] - \ln\left(\pi^d + \frac{\beta}{\gamma}\right).$$

Notice that this expression is the same as the expression (54) for W when the employee faces monetary fine instead of jail. This makes sense: $F = 0$ means that the employee actually faces no criminal punishment. Hence, the distinction between monetary fine and prison disutillity is irrelevant and the social welfare function is the same as in Appendix C. The result in Appendix C holds here as well: the socially optimal damages are set equal to $D = \underline{D} = \frac{\beta}{\gamma p}$ and for this value of the damages, social welfare is:

$$W' = \left(1 - \frac{\gamma}{\pi^d \gamma + \beta}\right) w^d - \ln\left(\pi^d + \frac{\beta}{\gamma}\right). \tag{66}$$

Clearly, (65) and (66) are the same. Figure 5, which combines and illustrates the analysis conducted for cases 1 and 2, is still relevant in this appendix. The socially optimal trade secret policy compatible with outcome RB consists in setting a criminal punishment equal to zero and the damages equal to $\frac{\beta}{\gamma p}$. Lemma 6 and proposition 4 still holds under the assumptions of this appendix.

Appendix E. Numerical analysis for the socially optimal trade secret policy under scenario 2 ($\gamma \geq \frac{\beta}{\pi^d} = \widehat{\gamma}(\pi^d)$) and under Cournot competition

Society is better-off implementing a policy that incites to bribery if and only if $W^{B,2} \geq W^R$ or:

$$\begin{aligned} \left(1 - \frac{\gamma}{\gamma\pi^d + \beta}\right) w^d - \ln\left(\pi^d + \frac{\beta}{\gamma}\right) \quad \geq \quad & \left(1 - \frac{\gamma\pi^d}{\gamma(\pi^d)^2 + \beta\Delta\pi}\right) \\ & \times \left[w^m + \left(1 - \frac{\beta}{\gamma\pi^d}\right)\Delta w - \frac{\beta}{\gamma}\ln\left(\frac{\gamma\pi^d}{\beta}\right)\right] \\ & - \ln\left(\pi^d + \frac{\beta\Delta\pi}{\gamma\pi^d}\right). \end{aligned} \tag{67}$$

I assume the same model of Cournot competition as in section 6.1. The values of π^d, π^m, cs^d, cs^m, w^d, w^m are the same. Plugging in these values into the above expression and re-arranging yields the condition:

$$\begin{aligned} 0 \quad \leq \quad & \frac{4\gamma a^4 + 36\beta a^2 - 36\gamma a^2}{9a^2\gamma + 81\beta} - \left(\frac{4\gamma a^2 + 45\beta - 36\gamma}{4\gamma a^2 + 45\beta}\right)\left[\frac{4}{9}a^2 - \frac{5}{8}\frac{\beta}{\gamma} - \frac{\beta}{\gamma}\ln\left(\frac{\gamma}{\beta}\frac{a^2}{9}\right)\right] \\ & - \ln\left(\frac{\gamma\frac{a^2}{9} + \beta}{\gamma\frac{a^2}{9} + \frac{5}{4}\beta}\right). \end{aligned} \tag{68}$$

I denote the difference between the two first terms by S. In the logarithm function, I denote $\gamma\frac{a^2}{9}+\beta$ by Q and $\gamma\frac{a^2}{9}+\frac{5}{4}\beta$ by G. Then, (68) can be rewritten as:

$$0 \leq S - \ln\left(\frac{Q}{G}\right). \tag{69}$$

In Excel, I can specify numerical values for the parameters a, β and γ. I can also write a formula for S, Q and G. Notice that the formula for S is quite involved. To check that this formula is correct, I also divided S into several simpler formulas and I obtained exactly the same values as with a single formula. The results reported below are those derived with the single Excel formula for S.

Of course, a numerical analysis is potentially infinite and its purpose cannot be to generate a proposition. Instead, my analysis illustrates the fact that the condition (69) does or does not hold depending on the parameters' value. In the tables below, what matters is the sign of the number in the last column. A positive number indicates that *(69)* holds which means that society is better-off not deterring bribery. On the contrary, a negative number means that society is better-off deterring bribery.

Table 1: *Increasing parameter a for arbitrary values of β and γ such that $\gamma \geq \frac{\beta}{\pi^d} = \frac{9\beta}{a^2}$ (since $\pi^d = \frac{a^2}{9}$).*

$a2$	$a4$	β	γ	Q	G	$ln(Q/G)$	S	$S-ln(Q/G)$
9	81	2	4	6	6,5	-0,08004	0,048362	0,128404
16	256	2	4	9,111111	9,611111	-0,05343	0,390317	0,443742
25	625	2	4	13,11111	13,61111	-0,03743	0,701568	0,738994
36	1296	2	4	18	18,5	-0,0274	0,963753	0,991152
64	4096	2	4	30,44444	30,94444	-0,01629	1,367537	1,383826
81	6561	2	4	38	38,5	-0,01307	1,525855	1,538927
100	10000	2	4	46,44444	46,94444	-0,01071	1,663533	1,674241
121	14641	2	4	55,77778	56,27778	-0,00892	1,784901	1,793825
144	20736	2	4	66	66,5	-0,00755	1,893174	1,900722

Tables 2 to 4: *Increasing β for γ and a constant such that $\gamma \geq \frac{9\beta}{a^2}$.*

a^2	a^4	β	γ	Q	G	$\ln(Q/G)$	S	$S - \ln(Q/G)$
9	81	1	10	11	11,25	-0,02247	-0,04828	-0,02581
9	81	2	10	12	12,5	-0,04082	-0,04396	-0,00313
9	81	3	10	13	13,75	-0,05609	-0,01819	0,037901
9	81	4	10	14	15	-0,06899	0,015029	0,084022
9	81	5	10	15	16,25	-0,08004	0,048362	0,128404
9	81	6	10	16	17,5	-0,08961	0,077784	0,167396
9	81	7	10	17	18,75	-0,09798	0,101073	0,199053
9	81	8	10	18	20	-0,10536	0,117035	0,222396
9	81	9	10	19	21,25	-0,11192	0,125085	0,237003
9	81	10	10	20	22,5	-0,11778	0,125	0,242783

a^2	a^4	β	γ	Q	G	$\ln(Q/G)$	S	$S - \ln(Q/G)$
16	256	1	6	11,66667	11,91667	-0,0212	0,170877	0,192079
16	256	2	6	12,66667	13,16667	-0,03871	0,289199	0,327914
16	256	3	6	13,66667	14,41667	-0,05343	0,390317	0,443742
16	256	4	6	14,66667	15,66667	-0,06596	0,474867	0,540825
16	256	5	6	15,66667	16,91667	-0,07676	0,542326	0,61909
16	256	6	6	16,66667	18,16667	-0,08618	0,592537	0,678715
16	256	7	6	17,66667	19,41667	-0,09445	0,625737	0,720189
16	256	8	6	18,66667	20,66667	-0,10178	0,642415	0,744198
16	256	9	6	19,66667	21,91667	-0,10832	0,643203	0,751525
16	256	10	6	20,66667	23,16667	-0,11419	0,628799	0,742991

a^2	a^4	β	γ	Q	G	$\ln(Q/G)$	S	$S - \ln(Q/G)$
25	625	1	4	12,11111	12,36111	-0,02043	0,438655	0,459087
25	625	2	4	13,11111	13,61111	-0,03743	0,701568	0,738994
25	625	3	4	14,11111	14,86111	-0,05179	0,901316	0,953101
25	625	4	4	15,11111	16,11111	-0,06408	1,055272	1,119351
25	625	5	4	16,11111	17,36111	-0,07472	1,170794	1,245517
25	625	6	4	17,11111	18,61111	-0,08403	1,252293	1,336323
25	625	7	4	18,11111	19,86111	-0,09224	1,302964	1,395202
25	625	8	4	19,11111	21,11111	-0,09953	1,325362	1,424892
25	625	9	4	20,11111	22,36111	-0,10605	1,32164	1,427691
25	625	10	4	21,11111	23,61111	-0,11192	1,293662	1,40558

a^2	a^4	β	γ	Q	G	$\ln(Q/G)$	S	$S - \ln(Q/G)$
36	1296	1	3	13	13,25	-0,01905	0,732259	0,751308
36	1296	2	3	14	14,5	-0,03509	1,1596	1,194692
36	1296	3	3	15	15,75	-0,04879	1,47581	1,5246
36	1296	4	3	16	17	-0,06062	1,716123	1,776748
36	1296	5	3	17	18,25	-0,07095	1,896301	1,967253
36	1296	6	3	18	19,5	-0,08004	2,025582	2,105625
36	1296	7	3	19	20,75	-0,08811	2,110256	2,198363
36	1296	8	3	20	22	-0,09531	2,155011	2,250321
36	1296	9	3	21	23,25	-0,10178	2,163552	2,265334
36	1296	10	3	22	24,5	-0,10763	2,138918	2,246549

Tables 5 and 6: *Increasing* γ *with* β *and* a *given such that* $\gamma \geq \frac{9\beta}{a^2}$.

a^2	a^4	β	γ	Q	G	$\ln(Q/G)$	S	$S - ln(Q/G)$
9	81	3	4	7	7,75	-0,10178	0,110017	0,2118
9	81	3	5	8	8,75	-0,08961	0,077784	0,167396
9	81	3	6	9	9,75	-0,08004	0,048362	0,128404
9	81	3	7	10	10,75	-0,07232	0,024762	0,097083
9	81	3	8	11	11,75	-0,06596	0,0065	0,072458
9	81	3	9	12	12,75	-0,06062	-0,00749	0,053136
9	81	3	10	13	13,75	-0,05609	-0,01819	0,037901
9	81	3	11	14	14,75	-0,05219	-0,02638	0,025804
9	81	3	12	15	15,75	-0,04879	-0,03266	0,016129
9	81	3	13	16	16,75	-0,04581	-0,03747	0,008336
9	81	3	100	103	103,75	-0,00726	-0,02359	-0,01634
9	81	3	700	703	703,75	-0,00107	-0,00411	-0,00304
9	81	3	10000	10003	10003,75	-7,5E-05	-0,0003	-0,00022

a^2	a^4	β	γ	Q	G	$ln(Q/G)$	S	$S - ln(Q/G)$
25	625	3	2	8,555556	9,305556	-0,08403	1,252293	1,336323
25	625	3	3	11,33333	12,08333	-0,06408	1,055272	1,119351
25	625	3	4	14,11111	14,86111	-0,05179	0,901316	0,953101
25	625	3	5	16,88889	17,63889	-0,04345	0,787678	0,831128
25	625	3	6	19,66667	20,41667	-0,03743	0,701568	0,738994
25	625	3	7	22,44444	23,19444	-0,03287	0,634212	0,667081
25	625	3	8	25,22222	25,97222	-0,0293	0,580041	0,609343
25	625	3	9	28	28,75	-0,02643	0,535458	0,561891
25	625	3	10	30,77778	31,52778	-0,02408	0,498059	0,522135
25	625	3	11	33,55556	34,30556	-0,0221	0,466185	0,48829
25	625	3	12	36,33333	37,08333	-0,02043	0,438655	0,459087
25	625	3	40	114,1111	114,8611	-0,00655	0,181675	0,188226
25	625	3	700	1947,444	1948,194	-0,00039	0,017955	0,01834
25	625	3	10000	27780,78	27781,53	-2,7E-05	0,001766	0,001793

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Efficient Delay in Patent Enforcement: Sequential Innovation and the Doctrine of Estoppel

Abstract

The doctrine of estoppel punishes a patentholder who threatened to sue an alledged infringer and then remained silent for a while before enforcing her patent: the patent may become unenforceable. I analyze the implications of this doctrine in a model of patent litigation when innovation is sequential and the follow-on innovation infringes a previous patent. Once informed, the patentholder can enforce her patent before or after a commercial follow-on product is developed. I show that, under some circumstances, the doctrine of estoppel can be designed so as to yield delayed enforcement. This benefits both the patentholder and the infringer. Also, judicial uncertainty in the application of the doctrine is socially optimal.

JEL classification codes: *O31* (Innovation and incentives), *O32* (Intellectual property rights), *K42* (Illegal behavior and the enforcement of the law).

Keywords: patent, infringement, doctrine of estoppel.

1 Introduction

Firms often infringe patents when they develop their own innovations. For instance, in the early years of the aviation industry, in order to develop new pathbreaking technologies, many aircraft companies infringed the broad pioneer patent held by the Wright brothers on a system for "airplane stabilization and steering". Merges and Nelson (1990) propose an exhaustive survey of the effects of broad patents on the development of improved technologies or application technologies[1]. The problem of patent scope when innovation is sequential has generated much

[1]Regarding the aviation industry, they explain that "the problems caused by the initial pioneer patent were compounded as improvements and complementary patents owned by different companies came into existence."

attention in the last decade. However, patent disputes -and litigation- over sequential innovations has been largely overlooked. Furthermore, the issue of litigation timing has been almost ignored, even if casual observations suggest that patentholders delay patent enforcement. This gap is regrettable given the widespread occurence of patent disputes and the existence of legal rules that affect the timing of litigation. In this paper, I look at these issues. I investigate the role of the "doctrine of estoppel" in patent infringement cases when innovation is sequential. This doctrine punishes[2] a patentholder who first threatened to sue an alleged infringer and then delayed enforcement of her patent for an unreasonably long time (see section 2 for a more extensive presentation of the doctrine). When the application of the doctrine of estoppel is probabilistic, the analysis reveals that both the patentholder and the infringer may benefit from a threat of a suit followed by a period of silence: the "stringency" of the doctrine of estoppel, defined as the probability that it applies given that a set of basic requirements are fulfilled, serves to determine the occurence of this equilibrium outcome. Most papers dealing with "forward patent protection" assume that "leading breadth"- which determines whether a follow-on innovation infringes a previous patent- is the *only* instrument of patent policy. I derive some conditions for the doctrine of estoppel to be another instrument to take into consideration. Hence, my analysis has implications for patent policy.

When innovation is sequential in the sense that a second innovation builds on the knowledge embodied in a previous innovation, and the first innovation is patented, a conflict may arise regarding the division of profit from the second innovation. Indeed, if the second innovation is perceived as infringing the patent over the first innovation (the first patent has enough "leading breadth"), then the first and the second innovators have to share the profit from the follow-on innovation. This situation has been extensively analyzed in the literature since the pioneering works of Scotchmer and Green (1990) and Green and Scotchmer (1995). Matutes, Regibeau and Rockett (1996) investigate the case where the follow-on innovation is an application of the first patented innovation (for another market). They give several examples such as lasers or algorithm technologies (as first patented innovations) which have applications in "medical technology, aerospace, telecommunications and electronics" (as follow-on innovations)[3]. Scotchmer (1996) shows that patents on second-generation products are not necessary to encourage their development. Schankerman and Scotchmer (2001) focus on the case where the first patented

[2] by making the patent unenforceable against the alleged infringer.

[3] This literature abstracts from patent litigation.

innovation is a "research tool". A proper design of leading breadth is important for two reasons: if the first innovator has no rights over the follow-on innovation, she may lack the incentive to invest in the first innovation (especially when all revenues are collected from the application). But if the second innovator does not capture enough from the second innovation's profit, he himself may lack incentives to invest in it. In addition to the aforementioned papers, two important contributions are Chang (1995) and Denicolo (2000). An attempt to formalize the notion of leading breadth by using legal categories is Llobet (2003). He argues- following in that sense the verbal discussion in Scotchmer (1991)- that the "doctrine of equivalents" can be used to capture leading breadth[4]. A gap in the literature is the consideration of leading breadth as the only instrument of "forward patent protection". In practice, even if the second innovation infringes the first patent and no ex-ante agreement took place, a legal dispute may arise. The patentholder would be in the role of the "plaintiff" arguing that her patent has been infringed. The infringer would be in the role of the "defendant". Typically, the infringer would try to invalidate the patent. More generally, he would use the "defenses" available against the patentholder, while the patentholder would call for "remedies" to compensate for the infringement of her patent. The doctrine of estoppel is one of the few defenses available to the infringer: if the conditions of its application are fulfilled, then, even if there is patent infringement, the patentholder may not be able to collect any revenue from the second innovation.

Results. I propose a model which accounts for the role of the doctrine of estoppel in patent litigation when innovation is sequential. My results differ depending whether the infringer is credit-constrained at the time litigation takes place. Suppose the patentholder is informed of ongoing infringement in innovation development and litigates before the infringer exerts the final effort in developing this innovation. If the infringer is credit-constrained, he may not have the money to pay upfront a licensing fee. As a result, the patentholder offers to share the future proceeds from the innovation and the infringer will not exert the first-best effort level in development. I show that the doctrine of estoppel can make the patentholder better-off and the infringer worse-off: anticipating that the doctrine might make the patent unenforceable if enforcement is delayed after an initial threat of litigation, the infringer exerts

[4]The "doctrine of equivalents" necessitates to determine the level of the technological contribution embodied in the follow-on innovation. The more innovative it is, the less likely the Court will recognize infringement of the first patent (i.e. the smaller the leading breadth of this patent).

more effort thereby increasing the innovation success probability and the players' payoffs. But if the probability that the doctrine of estoppel applies is too high, this positive incentive effect will cease since the patentholder will refrain from delaying enforcement: less effort translates into a lower success probability: a high probability of estoppel counter-intuitively reduces the infringer payoff. Hence, the doctrine works as an incentive mechanism counterbalancing the negative effect of the profit-sharing contract. I characterize the socially optimal policy and show that it is designed to induce an (efficient) litigation delay. However when the infringer is not credit-constrained (i.e he can pay upfront a fixed licensing fee to the patentholder), he does not have to share the proceeds from the innovation. He exerts the first-best effort level. The patentholder may still want to delay enforcement but without threatening to sue the infringer at the outset i.e. she does not *need to* expose herself to estoppel as the absence of profit-sharing does not reduce the infringer's incentives. Delayed enforcement in that case is inefficient but the doctrine of estoppel cannot mitigate this inefficiency since the patentholder never exposes herself to its application.

Previous literature. To the best of my knowledge, this paper is the first to analyze the economics of the doctrine of estoppel. It is related to Schankerman and Scotchmer (2001)'s analysis of the "doctrine of laches". They assume that the doctrine of laches prevents a patentholder from obtaining an injunction if she delays enforcing her patent. This assumption is *at odds with the current implications of the doctrine of laches.* I analyze the doctrine of laches in-depth in a companion paper (Carpentier, 2005). There I explain, by referring to Case Law, that the doctrine of laches allows the patentholder to get an injunction even if she delayed litigation[5]. The only doctrine which prevents the patentholder from getting an injunction is the *doctrine of estoppel.* This is because this doctrine makes the patent completely unenforceable. And this doctrine has more requirements than a mere delay in litigation. That being said, *both doctrines are about delayed litigation.* As a result, and despite several differences between their model and mine, I find it important to compare my results with those in Schankerman and Scotchmer (2001). In their discussion of the doctrine of laches, they show that if the patentholder has full bargaining power, the doctrine of laches should prevent delay in patent enforcement. This contrasts with my results where the doctrine of estoppel should be designed to induce a delay. Also, I show that the infringer may be hurt by an increase in the probability that the doctrine of

[5] It only prevent her from getting damages to compensate for infringement that occured in the delay period.

estoppel applies and that the infringer may benefit from delayed litigation. In their model the infringer is always better-off when litigation is less delayed. Finally, I discuss the relationship between patent validity and the occurence of delayed litigation. More generally, my analysis belongs to a new literature that tries to analyze how specific legal doctrines affect patent litigation and innovation incentives: Lanjouw and Lerner (2001) (the doctrine of "preliminary injunctions"), Schankerman and Scotchmer (2001) (the doctrines of "unjust enrichment", "lost profit" and "laches"), Llobet (2003) (the "doctrine of equivalents"), Anton and Yao (2004) (the doctrine of "lost profit"), Aoki and Small (2004) (the doctrine of "essential facilities"), Langinier and Marcoul (2005) (the doctrine of "contributory infringement"). Finally, my paper relates to Llobet and Suarez (2005) who consider the impact of financial constraints on the financing of patent litigation. I also deal with the issue of financial constraint but the financial constraint in my model does not concern the financing of litigation.

A roadmap. In section 2, I briefly review the legal concepts that I formalize in the model. Then, in section 3, I turn to presenting the assumptions of the model. In section 4, I conduct the equilibrium analysis (summarized in proposition 1) and derive a first set of results. In this section, I assume that the infringer has no wealth at the time of litigation and is credit-constrained. In Section 5, I analyze the doctrine of estoppel from the point of view of patent policy. In Section 6, I discuss the case where the infringer has some wealth or is not credit-constrained at the time of litigation. Section 7 concludes.

2 Legal notions

The model proposed in this paper formalizes a number of legal concepts. In this section, I briefly review these concepts. The first one is the "doctrine of estoppel". Then, I discuss two other notions: "the notice of infringement" and "the declaratory judgment".

The doctrine of estoppel. The model proposed in this paper relies on the decision in *Meyers v. Asics Corp., 974 F.2d 1309, 1308-1309 (Fed.Circ. 1992).* In this case, the patentholder threatened vigorous enforcement of its patent, but then did nothing for an unreasonably long time[6]. In that circumstance, the Court held that the doctrine of estoppel applied: the patent was unenforceable against the alledged infringer. Of course, this implies the infringer convinced the

[6]See "estoppel" on the IPwatchdog website, at www.ipwatchdog.com

Court that he interpreted the prolonged silence of the patentholder as an intent not to enforce the patent. The possibility of this ruling is confirmed in Lerner and Poltorak (2002): "Although mere silence does not give rise to equitable estoppel, *extended inaction after issuing a notice of infringement may well do so*[7]" (page 134). Even if this basic requirement for application of the doctrine is fulfilled (a threat to sue followed by a period of silence), it is important to notice that the final decision remains subject to the examination of various other "facts". Hence, in the model, even if the patentholder decides to remain silent after she issued a notice of infringement, I assume that the doctrine of estoppel will apply with a given probability. The idea is simple: the higher this probability, the less stringent are the requirements of the doctrine. In other words, the higher this probability, the more the Court considers that extented silence after a notice of infringement is a sufficient requirement for applying the doctrine of estoppel[8]. From a "descriptive" point of view, this formulation captures the fact that many exogenous elements may influence the application of the doctrine, so that uncertainty is always present. From a "prescriptive" point of view, this formulation allows to discuss the effect of the "stringency" of the requirement on the players' equilibrium strategies and payoffs: in section 5, I analyze the probability that the doctrine of estoppel applies as an instrument of patent policy.[9]

The notice of infringement. When a patentholder suspects infringement of her patent, she must notify the alledged infringer about her concerns. One possibility is to send a letter called a "notice of infringement", before filing a suit. The other possibility is to file a suit immediately (thereby informing the alledged infringer that a concern exists). The "notice of infringement" letter (called "notice of infringement" for simplicity) can be of different types. Some patentholders threaten to sue vigorously the infringer in the hope that he will stop the infringing activity. The danger with sending a notice of infringement is that the infringer is entitled to ask a "declaratory judgment" to the Court. I now discuss this notion.

The declaratory judgment (DJ). If the infringer receives a notice of infringement, whereby the patentholder threatens to sue him, she can ask the Court for a "declaratory judgment".

[7]Italics added.

[8]In the model, this probability is exogenous.

[9]My model does not incorporate time as such. More simply, I assume that not litigating immediately after the "threat", but delaying until he innovation is developed, is considered by the Court as sufficient to possibly apply the doctrine of estoppel.A more realistic model could incorporate time.

This is a ruling about the validity of the patent: if the patent is judged invalid by the Court, the patentholder is not entitled to enforce her patent anymore. Two remarks are in order. First, one may ask why it is beneficial for the infringer to learn about patent validity through a DJ: he would learn about validity through a normal trial as well. The answer is that in practice, asking for a DJ means that the infringer chooses the Court where he wants the patent to be scrutinized. Given that some Courts are more patent-friendly, this can make a big difference. This feature does not appear in the model where I assume there is one Court only. But this remark implies a new question: why is it that the patentholder ever sends a notice of infringement, given that it seems to provide advantages to the infringer? This paper proposes a new explanation.

To summarize: first, the patentholder suspects infringement. She can send a notice of infringement or enforce immediately the patent by filing a suit. If she sends a notice of infringement, the alledged infringer is entitled to ask for a "declaratory judgment" whereby the Court rules about patent validity. If the infringer does not ask for this judgment, the patentholder can enforce immediately her patent or delay, in which case she exposes herself to the doctrine of estoppel. In the next section, I develop the assumptions of the model. This model formalizes the three legal notions discussed above. Then, in section 4, I conduct the equilibrium analysis.

3 The assumptions of the model

The situation at the outset. I consider a game between two players. A patentholder (she) also called player A, and an infringer (he) also called player B. At the outset, after research, player B has obtained a "prototype" which still needs to be developed in order to obtain a marketable invention. Development requires investment (effort) by player B. This investment is non verifiable by a third party. I assume in most of the paper that player B has no wealth of its own and is credit-constraint. This assumption, relaxed in section 6, would make sense for instance if player B was a start-up with no cash-flow, which has to resort to external sources of finance. The literature on R&D financing has stressed problems associated with the financing of these firms (higher risk of default, lack of previous achievements to assess the quality of the company, lack of collateral and more generally, asymmetric information). The supply of credit may be limited due to capacity constraints (lack of venture capitalists with expertise in the field of R&D). If it is marketed, player B's invention is assumed to infringe player A's patent. However, it does not reduce player A's current revenue. This situation corresponds for example to the situation where player B's invention is an application of player A's patent for another market: player A's market share is unaffected by infringement, however, she is entitled to collect (royalty) revenues from player B. Like in Schankerman and Scotchmer (2001), one can think that player A has a patent on a "research tool" that player B used in research. Like in Chang (1995) or Denicolò (2000), I assume that player B refrains at the outset from contacting the patentholder to sign an ex-ante agreement. One of Chang's argument is that player B will be concerned by the possibility that the patentholder can steal his idea and bring a product on the market before him.[10] In the important survey by Arora, Cohen and Walsh (2003), many firms acknowledged that they do not always try to secure an ex-ante license but simply infringe the patents (despite the expected consequences). However if the patentholder becomes informed of an ongoing act of infringement by player B, she has the power to enforce her patent and a licensing agreement before B has concluded the developement of his innovation is possible.[11] I assume that player A can become informed of player B's infringement after B conducted research but before B engaged into development. In the model, player A becomes informed exogenously. In practice, there are many channels through which a patentholder can be informed that another firm is engaged into an infringing activity before the final product is commercialized. For example, in the context of patented research tools, Schankerman and Scotchmer (2001) write: "According to interviews that we conducted with patent counsel in biotechnology firms, the owner of a research tool typically learns about infringement when the infringer conducts field trials, which usually begin about halfway in the development process (...)". Another channel to learn that infringement may have occurred is when the infringer

[10] Chang writes (the terminology refers to his paper): "Firm 2 may find that it cannot induce firm 1 to agree to an R&D joint-venture without disclosing its idea. Such disclosure, however, would undermine the bargaining power of firm 2". Contrary to my model, Chang does not deal with research tools or application of a technology in other markets, but with improvements of a first-generation invention. Yet I do believe the danger of information disclosure is valid in the cases of research tools and applications. Many biotechnology firms have patented research tools and in addition develop follow-on inventions using these tools. Finally, if player B has an idea to develop a radically new product using player A's patented technology currently applied in another industry, he might be reluctant to disclose this idea to player A in the fear that she might develop this product (i.e. create a new market) before him. This is especially true if firm A has resources while firm B is more constrained.

[11] At that stage, I would argue that player B does not fear to reveal his idea to the patentholder since he is advanced in the development of his innovation.

himself applies for a patent on his prototype[12]. A patent is a public document and so the original patentholder may learn that a new patented innovation infringes her own patent. In that case, there is a context of overlapping patents where the new patent is called "subservient" and the original (oldest) patent is called "dominant" as discussed in Merges and Nelson (1990).

The game: actions and payoffs. In what follows, I describe players' actions and final payoffs. Figure 1 below helps to better grasp the order of moves. The game begins when player A is informed that player B infringes her patent. Player A faces *a first alternative*: to send a notice of infringement, or not. I assume that sending this notice is interpreted by the Court as a "threat of litigation".

If she does not send this notice (right-hand side branch of the game tree, starting from the initial node), she faces a second alternative. She can enforce her patent immediately (file a suit and ask an injunction to the Court), or she can delay enforcement. Delaying means enforcing the patent after player B has invested effort in development, and succeeded. Enforcing immediately implies that player B has not yet invested in development. In either case (delayed or immediate enforcement), player A obtains an injunction with probability θ (the validity of the patent) and, in that case, makes a take-it-or-leave-it offer to player B. This offer consists in offering to player B a share $(1-\gamma)$ of the return v from player B's innovation. Player A keeps the remaining share. When player B has no wealth at this contract stage, he cannot pay a fixed licensing fee. In Appendix C, I show that player B could ask a financier to pay the fee in exchange for a future share of v. If the financier has the bargaining power in proposing the financing contract to player B, it turns out that the patentholder can do as well by offering only a sharing contract so that the infringer does not have to contact the financier.

If she sends a notice of infringement before filing a suit and asking an injunction (left-hand side branch of the game tree, starting from the initial node), player A takes a first risk. Player B has the possibility to ask a declaratory judgment. If it occurs, player A will

[12] According to Judd et al. (2003): "conventional discussions of patent policy focus on the optimal duration and breadth of patent protection. These discussions typically assume that a firm does not receive that patent until the R&D process is complete. This is not true of actual innovation processes where a firm often bears significant expenditures after it receives a patent. For example, drug firms can patent a drug before they have proven its efficacy and its safety". They also refer to Jewkes et al. (1969): "the first patents for xerography were granted many years before the first copy machine and far more money was spent on development of the transistor after the patent was granted than before":

immediately ask an injunction for infringement. If player B asks a declaratory judgment, then with probability θ the patent is valid, the declaratory judgment is refused and player A gets an injunction.[13] With probability $1-\theta$ the patent is invalid and player B is "free". But if player B does not ask an declaratory judgement, player A faces the same alternative described above: she can enforce her patent immediately or she can delay until she observes a success by the infringer. Here comes the main difference with delaying enforcement without a notice of infringement. Indeed, If player A delays after she has sent a notice of infringement and the infringer did not ask a declaratory judgment, she exposes herself to the doctrine of estoppel (see section 2). This is the second risk faced by player A. The doctrine of estoppel means that if the patentholder remained silent after having threatened to sue an infringer, her patent may be unenforceable. I assume that this occurs probabilistically. With probability ϕ the doctrine of estoppel applies and the patent is unenforceable: the patentholder gets 0 and the infringer gets v. With probability $1-\phi$ the doctrine of estoppel does not apply and the patent is enforceable. If it is valid, an injunction is granted to the patentholder and she makes a take-it-or-leave-it offer to the infringer. Modeling probabilistically the application of the doctrine of estoppel obeys two objectives. First, it captures a central feature of law enforcement: uncertainty. Shapiro and Lemley (2005) emphasize that patents are only "probabilistic" property rights. My formulation includes this aspect. Second, the parameter ϕ fulfills a prescriptive objective: it enables me to characterize the optimal stringency of the doctrine of estoppel.

Regardless of player A's action and the timing of litigation, player B has to invest in developing his prototype into a commercial product. Development is successful with probability $p(x)$ where x is player B's effort. The development cost is given by $c(x) = \frac{1}{2}\alpha x^2$. I assume $\alpha \geq v$. These assumptions guarantee that all the considered maximization problems are well-behaved. To simplify, I assume no litigation cost.

[13]The possibility for the patentholder to obtain an injunction before the innovation is actually on the market might be challenged. I borrow this assumption from Schankermann and Scotchmer (2001).

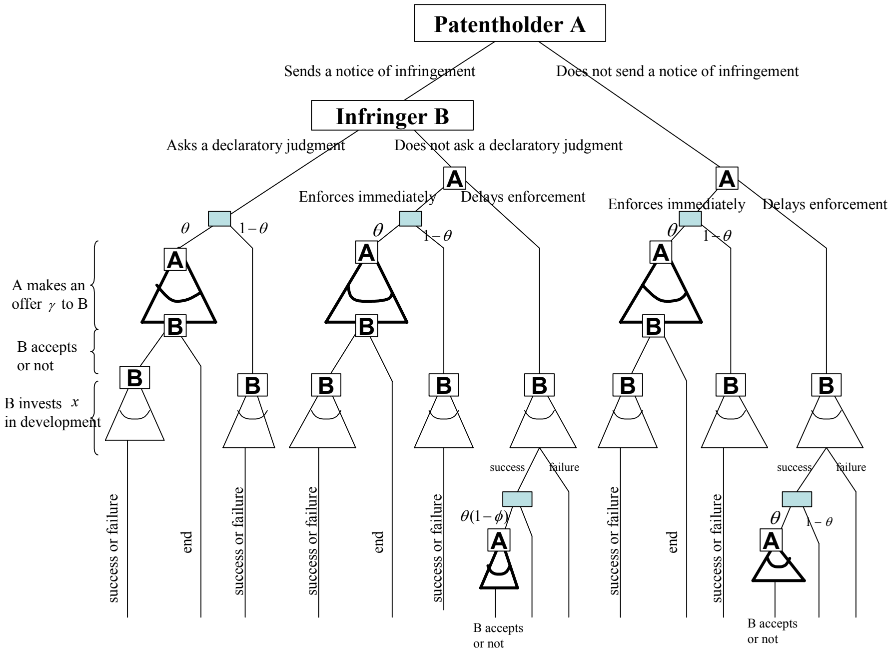

Figure 1: Game tree. The grey nodes represent moves by nature.

I now turn to the equilibrium analysis. The possible equilibrium outcomes are summarized by proposition 1 and represented in Figure 4 below.

4 Equilibrium analysis when the infringer is credit-constrained

This section is organized as follows. In subsection 4.1, I analyze the case where player A did not send a notice of infringement to player B (right branch of the game tree, starting from the initial node). In that case, she has the choice between enforcing her patent immediately or delaying: I derive the condition for the patentholder to prefer one option to the other. In subsection 4.2, I analyze the case where she sent such a notice (left branch of the game tree, starting from the initial node). In subsection 4.3, I present the equilibrium outcomes of the game. Finally subsection 4.4 discusses the effects of estoppel and patent validity on the players'payoffs and on the occurence of the different equilibria.

4.1 The patentholder did not send a notice of infringement

I focus on the right-hand side branch of the game tree (starting from the initial node). Proceeding by backward induction, I distinguish between two occurencies: the patentholder decided to enforce immediately her patent, or she decided to delay.

■ *Immediate enforcement.* Suppose player A enforced immediately her patent and obtained an injunction. Then, she proposes a contract $(1-\gamma^i)$ to player B. Superscript "i" stands for "immediate enforcement". Given this contract, player B decides how much to invest in developing her invention. Her objective is to choose x that maximizes:

$$x(1-\gamma^i)v - \frac{1}{2}\alpha x^2$$

The first-order condition yields player B's optimal investment:

$$x^i = \frac{v(1-\gamma^i)}{\alpha}. \tag{1}$$

Anticipating this investment, player A makes an offer that would maximize her own expected payoff given by:

$$\frac{v(1-\gamma^i)}{\alpha}\gamma^i v$$

The first-order condition yields player A's optimal offer:

$$\gamma^{i*} = \frac{1}{2} \tag{2}$$

Plugging in (2) into (1) yields the optimal investment if player A obtained an injunction after an immediate enforcement:

$$x^{i*} = \frac{v}{2\alpha}. \tag{3}$$

Now, suppose player A enforced immediately, but did not obtain the injunction. Then, player B knows she can appropriate the whole value from her innovation and she chooses the first-best level of investment:

$$x^{FB} = \frac{v}{\alpha} \tag{4}$$

Ex-ante, an injunction is granted to player A only if her patent is valid (i.e. with probability θ). Thus, I can now compute players' expected payoffs in case of an immediate enforcement without an initial notice of infringement:[14]

[14] The patentholder gets a share of the profit if and only if her patent is valid (with probability θ): $\pi_A^i = \theta x^{i*}\frac{v}{2} = \theta\frac{v^2}{4\alpha}$. For the infringer, if the patent is valid, he has to share the expected profit $x^{i*}v$ with the patentholder, but if the patent is invalid, he can keep the expected profit $x^{FB}v$ for himself. His payoff is $\pi_B^i = \theta\left[x^{i*}\frac{v}{2} - \frac{1}{2}\alpha\left(x^{i*}\right)^2\right] + (1-\theta)\left[x^{FB}v - \frac{1}{2}\alpha\left(x^{FB}\right)^2\right] = \left(1-\frac{3}{4}\theta\right)\frac{v^2}{2\alpha}$

$$\begin{cases} \pi_A^i = \theta \frac{v^2}{4\alpha} \\ \pi_B^i = (1 - \frac{3}{4}\theta)\frac{v^2}{2\alpha} \end{cases} \tag{5}$$

■ *Delayed enforcement.* Suppose player A has delayed her reaction. Ex-post, she will react provided player B has succeeded in developing and marketing her innovation. If she reacts ex-post and obtains the injunction, then she proposes a contract γ^d to player B such that player B will accept it. Superscript "*d*" stands for "delay". Formally, this condition is given by:

$$(1 - \gamma^d)v \geq 0.$$

Optimally, player A proposes:

$$\gamma^{d*} = 1 \tag{6}$$

At the previous stage, player B chooses her investment, anticipating the possibility of an injunction. An injunction is granted to player A with probability θ. Player B's objective is now to choose x which maximizes:

$$x(1-\theta)v - \frac{1}{2}\alpha x^2.$$

The first order condition yields the optimal investment in development conditional on the patentholder not sending a notification of infringement and delaying her reaction:

$$x^{d*} = \frac{(1-\theta)v}{\alpha}. \tag{7}$$

When the patentholder plays this strategy, players'payoffs are:

$$\begin{cases} \pi_A^d = \frac{\theta(1-\theta)v^2}{\alpha} \\ \pi_B^d = \frac{(1-\theta)^2 v^2}{2\alpha}. \end{cases} \tag{8}$$

■ *The patentholder's choice between delayed and immediate enforcement.* Given the preceding analysis, the following lemma gives the condition on θ (the probability that the patent is valid) such that the patentholder delays her enforcement.

Lemma 1 ***(Delayed enforcement without initial notice of infringement).*** *If the patentholder did not send a notice of infringement to the infringer at the outset, she delays enforcement if and only if:*

$$\theta \leq \frac{3}{4} = \theta'.$$

Proof. The proof is immediate upon comparison of π_A^d in (8) and π_A^i in (5):$\frac{\theta(1-\theta)v^2}{\alpha} \geq \theta\frac{v^2}{4\alpha}$ holds if and only if $\theta \leq \frac{3}{4}$. ■

The patentholder delays enforcement when the probability that the patent is valid is low enough. The intuition is that the infringer anticipates that the patent is likely to be invalidated ex-post so that he will not have to share the returns v from the innovation. As a result, he exerts more effort in development. This increases the success probability and thereby the patentholder's *expected* payoff. This is why the patentholder has an incentive to delay enforcement. When the likelihood that the patent is valid is too high (here: $\theta > \frac{3}{4}$), this positive effect on effort diminishes and the patentholder prefers enforcing her patent immediately.

4.2 The patentholder sent a notice of infringement

Now, I turn to the case where the patentholder sent a notice of infringement. Remember that this notice is interpreted by the Court as a "threat to litigate". Hence, if the patentholder delays litigation after he sent the notice, *the doctrine of estoppel may apply.* I focus on the left-hand side branch of the game tree (starting from the initial node).

■ *The infringer asks a declaratory judgment.* With probability θ, the Court recognizes that the patent is valid and the patentholder is granted an injunction. With probability $1-\theta$ the patent is invalid and player B is free to infringe it. Notice that the analysis is now equivalent to that conducted in the previous subsection when the patentholder decided to enforce her patent immediately. Indeed, if she obtains an injunction, the patentholder makes an offer $\gamma = \gamma^{i*}$ to the infringer who then invests $x = x^{i*}$. Players' expected payoffs if the infringer asked a declaratory judgment are given by (5). Adapting notations so that DJ means "declaratory judgment":

$$\begin{cases} \pi_A^{DJ} = \theta\frac{v^2}{4\alpha} \\ \pi_B^{DJ} = (1-\frac{3}{4}\theta)\frac{v^2}{2\alpha} \end{cases} \tag{9}$$

■ *The infringer did not ask a declaratory judgment.* In that circumstance, the patentholder faces an enforcement timing problem alike the one analyzed in the previous subsection. She can enforce her patent immediately or she can remain silent until the infringer succeeds in developing a marketable invention. "Remaining silent" is equivalent to delaying enforcement. The essential difference with the previous subsection is that if she remains silent, the patentholder may be "punished" by the doctrine of estoppel. Indeed, she will be in a situation where she threatened to sue the infringer, but remained silent for a while afterward. As a result, her patent may be unenforceable. This occurs with probability ϕ.

☐ If the patentholder enforces immediately her patent, the outcome is again identical to that analyzed in subsection 4.1 when she enforces immediately: she obtains an injunction with probability θ and makes an offer $\gamma = \gamma^{i*}$ to the infringer who then invests $x = x^{i*}$. And with probability $1-\theta$ the infringer is free: he invests $x = x^{FB}$. Players' expected payoffs are again given by (5). Adapting notations and denoting "no declaratory judgment" by NDJ :

$$\begin{cases} \pi_A^{NDJ,i} = \theta \frac{v^2}{4\alpha} \\ \pi_B^{NDJ,i} = (1 - \frac{3}{4}\theta)\frac{v^2}{2\alpha} \end{cases} \tag{10}$$

☐ However, if the patentholder decides to remain silent ("delay"), the analysis is now different. Indeed, facing the possibility of litigation after he has invested in development, the infringer chooses an investment in development which maximizes:

$$xv\left[1 - \theta\left(1 - \phi\right)\right] - \frac{1}{2}\alpha x^2. \tag{11}$$

With probability $(1-\theta)$, the patent is invalid and the infringer obtains v. With probability θ it is valid, but with probability ϕ the doctrine of estoppel applies making the patent unenforceable: the infringer obtains v. Finally, with probability $\theta(1-\phi)$ the patent is valid *and* the doctrine of estoppel does not apply: under this scenario, the patentholder obtains the injunction and makes a take-it-or-leave offer to player B. Optimally, given this bargaining assumption, she can extract all the surplus by offering $\gamma = \gamma^{d*} = 1$ as in (6) and leaving the infringer with nothing. This explains the form of player B's objective function (11). The first-order condition associated with the maximization of (11) with respect to x yields the optimal investment in development, conditional on the patentholder having sent a notice of infringement and then remaining silent:

$$x^{N,d} = \frac{\left[1 - \theta\left(1 - \phi\right)\right]v}{\alpha}. \tag{12}$$

Intuitively, this investment is increasing in the probability that the patent is not valid (θ) and in the probability that the doctrine of estoppel applies (ϕ). I can now compute players' payoffs if the patentholder delays enforcement after having sent a notice of infringement and conditional on the infringer not asking a declaratory judgment:

$$\begin{cases} \pi_A^{NDJ,d} = \frac{[1-\theta(1-\phi)]\theta(1-\phi)v^2}{\alpha} \\ \pi_B^{NDJ,d} = \frac{[1-\theta(1-\phi)]^2 v^2}{2\alpha}. \end{cases} \tag{13}$$

□ The following lemma gives the condition on θ such that the patentholder prefers to delay enforcement after having threatened to sue, if the infringer did not ask a declaratory judgment.

Lemma 2 ***(Delayed enforcement after an initial notice of infringement).*** *If the patentholder threatened to litigate the infringer at the outset, and the infringer did not ask a declaratory judgement, then the patentholder delays her enforcement if and only if:*

$$\theta \leq \frac{3-4\phi}{4(1-\phi)^2} = \widetilde{\theta}(\phi). \tag{14}$$

Proof. Upon inspection of (10) and (13), $\pi_A^{NDJ,d} \geq \pi_A^{NDJ,i}$ if and only if $\theta \leq \frac{3-4\phi}{4(1-\phi)^2}$ which is the result in (14). ■

There is a similarity between lemma 1 and lemma 2: the patentholder delays enforcement if the probability that the patent is valid is low enough. In lemma 2 however, the cutoff value $\widetilde{\theta}$ below which the patentholder delays incorporates the estoppel parameter ϕ. In Appendix A, I analyze the main properties of $\widetilde{\theta}(\phi)$.Comparative statics show that this value is increasing in ϕ when $\phi < \frac{1}{2}$ i.e when the estoppel probability is low. This means that increasing the probability that the doctrine of estoppel applies increases the range of θ values for which the patentholder delays enforcement and exposes herself to estoppel. This is counterintuitive and I discuss the reason for this phenomenon in section 4.4. Moving one step backward, I now analyze the decision of the infringer to ask (or not to ask) a declaratory judgment.

■ *The infringer's choice between asking a declaratory judgement or not.* As discussed previously, this motion is available to alledged infringers threatened to be sued by patentholders (via a notice of infringement). The advantage of a declaratory judgement is that the validity of the patent can be assessed immediately. If it is invalid, then the infringer knows he will not be "held-up" ex-post by the patentholder and he exerts the first-best level of effort. When deciding whether to ask for a declaratory judgment or not, the infringer anticipates the response of the patentholder. In particular, if he does not ask such a judgment, he knows, from lemma 2, that the patentholder will delay enforcement if and only if $\theta \leq \widetilde{\theta}$. I now analyze the cases.

- $\theta > \widetilde{\theta}$. If the infringer does not ask a declaratory judgment, the patentholder enforces her patent immediately (by lemma 2) and, from (10) the infringer obtains $\pi_B^{NDJ,i} = \left(1 - \frac{3}{4}\theta\right) \frac{v^2}{2\alpha}$. From (9), his expected payoff if he asks a declaratory judgment is $\pi_B^{DJ} = \left(1 - \frac{3}{4}\theta\right) \frac{v^2}{2\alpha}$ as well. Hence, he is *i*ndifferent between asking and not asking a declaratory judgment but it is reasonable to assume that *he does not ask it* (if the declaratory judgment were slightly costly, as it is in practice, this would indeed be the equilibrium[15]).

- $\theta \leq \widetilde{\theta}$. If the infringer does not ask a declaratory judgment, the patentholder delays the enforcement of her patent and, from (13), the infringer obtains $\pi_B^{NDJ,d} = \frac{[1-\theta(1-\phi)]^2 v^2}{2\alpha}$. From (9), his expexted payoff if he asks a declaratory judgment is again $\pi_B^{DJ} = \left(1 - \frac{3}{4}\theta\right) \frac{v^2}{2\alpha}$. The following lemma gives the condition on θ such that the infringer prefers to ask a declaratory judgment when $\theta \leq \widetilde{\theta}$.

Lemma 3 ***(Declaratory Judgment action)****. If the infringer is threatened to be sued by the patentholder, he asks a declaratory judgment if and only if:*

$$\theta \leq \frac{5 - 8\phi}{4(1-\phi)^2} = \widehat{\theta}(\phi). \tag{15}$$

Proof. Upon inspection of $\pi_B^{NDJ,d}$ and π_B^{DJ}. It follows that $\pi_B^{DJ} \geq \pi_B^{NDJ,d}$ if and only if $\left(1 - \frac{3}{4}\theta\right) \frac{v^2}{2\alpha} \geq \frac{[1-\theta(1-\phi)]^2 v^2}{2\alpha}$ which simplifies to $\theta \leq \frac{5-8\phi}{4(1-\phi)^2}$. ■

The condition in lemma 3 says that the infringer asks a declaratory judgment if the probability that the patent is valid is low enough.[16] This makes sense: if the patent is unlikely to be valid (here: $\theta \leq \widehat{\theta}$), the infringer prefers to test patent validity before he invests in innovation development (i.e. he asks a declaratory judgment). The chances that the patent is invalidated are high and therefore the chances that the infringer will not have to share the returns from

[15]If the patentholder does not ask a declaratory judgment, the infringer has to pay the cost of the infringement suit launched by the patentholder. When infringers ask a declaratory judgment, my understanding is that patentholders retaliate anyway with an infringement suit so that filing for a declaratory judgment represents an incremental cost over the cost of defending oneself against infringement. In practice, an important benefit of such declaratory judgement suits is that they give the infringer the possibility to choose the jurisdiction. The present model does not incorporate all these aspects.

[16]Notice that $\widehat{\theta} < \widetilde{\theta}$ if $\phi \geq \frac{1}{2}$. And if $\phi < \frac{1}{2}$, $\widehat{\theta}$ is larger than 1 i.e. it is not defined.

the innovation are high. In such a case, he will exert the first-best effort which maximizes the success probability and his expected payoff. But when the patent is likely to be valid (here: $\theta > \widehat{\theta}$), the chances to exert the first-best effort are low when the infringer asks a declaratory judgment. As a result, this becomes a less interesting option. The function $\widehat{\theta}(\phi)$ plays an important role in representing the equilibrium outcomes in the (θ, ϕ) space. In Appendix A, I analyze the main properties of $\widehat{\theta}(\phi)$.

4.3 Equilibrium

To determine the different equilibrium outcomes of the game, I need, moving one step backward, to unveil the conditions for the patentholder to send a notice of infringement. I include this analysis in the proof of the existence of the three equilibrium outcomes given in proposition 1 and illustrated by Figure 2. The proof is reported in Appendix A.

Proposition 1 ***(Equilibrium outcomes of the game.)*** *Depending on the values of θ and ϕ, the game has three possible equilibrium outcomes:*

- *The patentholder does not send an initial notice of infringement and enforces her patent immediately ("immediate enforcement" I).*
- *The patentholder sends a notice of infringement, the infringer does not ask a declaratory judgment and the patentholder remains silent until the infringer succeeds ("estoppel exposure" E).*
- *The patentholder does not send a notice of infringement and delays the enforcement of her patent ("delayed enforcement" D).*

Proof. See Appendix A. ■

The three possible equilibrium outcomes can be represented in the (θ, ϕ) space. For that matter, I use the properties of the functions $\widetilde{\theta}$, $\widehat{\theta}$ and $\widetilde{\widetilde{\theta}}$ (see Appendix A). The value $\overset{\bullet}{\theta}$ (computed in Appendix A) corresponds to the intersection between $\widetilde{\widetilde{\theta}}$ and $\widehat{\theta}$. $\overset{\bullet}{\theta}$ is a straight line in the (θ, ϕ) space. In the next subsection, I investigate the *reasons* for the "estoppel exposure" equilibrium to occur.

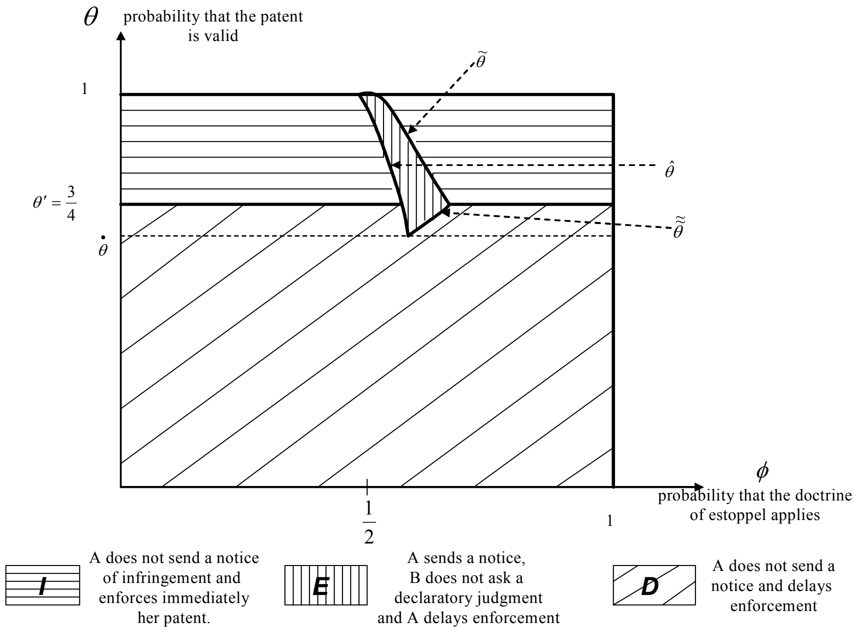

Figure 2: Equilibrium outcomes

4.4 Effects of estoppel and patent validity

The main result of this section is that an increase in the probability that the doctrine of estoppel applies can have counterintuitive effects on players' payoffs: it can make the infringer worse off and the patentholder better off. As a result, both players prefer the outcome where the patentholder exposes herself to estoppel. The intuition is that when the estoppel probability enters the infringer's effort optimization, an increase in this probability increases the infringer's optimal effort which in turn increases the likelihood of success. This increases both players' *expected* payoffs. But if the estoppel probability is too high, it also reduces the probability that the patent is enforceable and thus the patentholder will refrain from exposing her, making the infringer worse off compared to the previous situation.

Proposition 2 *An increase in the probability ϕ that the doctrine of estoppel applies can:*

- *make both the patentholder and the infringer better off, or*

- *make both the patentholder and the infringer worse off.*

Below I provide a proof, and develop the economic intuition for the two counter-intuitive results of this proposition:[17]

The infringer is worse off. Suppose that initially θ and ϕ are compatible with equilibrium E. Now, holding patent validity θ constant, an increase in the estoppel probability ϕ induces a switch from E to D. This can be seen of Figure 2. I compare the infringer's payoff in the two equilibrium outcomes. In equilibrium E it is $\pi_B^{NDJ,d} = \frac{(1-\theta(1-\phi))^2 v^2}{2\alpha}$ and in equilibrium D it is $\pi_B^d = \frac{(1-\theta)^2 v^2}{2\alpha}$. It follows that $\pi_B^{NDJ,d} \geq \pi_B^d$ if and only if $(1-\theta) \leq 1$ which always holds. Hence, an increase in the estoppel probability which induces a switch from E to D makes the infringer worse off. The intuition is as follows: a higher estoppel probability encourages the patentholder not to take the risk of sending the notice of infringement. As a result, there is a switch from equilibrium E (where the patentholder exposes herself to estoppel) to equilibrium D (where she does not). And in equilibrium D the infringer is not protected by the doctrine of estoppel anymore: the probability that the patentholder obtains an injunction is thus higher and so the infringer's payoff is lower. The same analysis can be conducted for the case where an increase in the estoppel probability ϕ induces a switch from equilibrium E to equilibrium I (the infringer's payoff is reduced).

The patentholder is better off. Suppose that initially θ and ϕ are compatible with equilibrium D. Now, holding patent validity θ constant, an increase in the estoppel probability ϕ induces a switch from D to E. Initially, the patentholder's payoff in equilibrium D is $\pi_A^d = \frac{\theta(1-\theta)v^2}{\alpha}$. After the increase in ϕ, her payoff in the new equilibrium E is $\pi_A^{NDJ,d} = \frac{[1-\theta(1-\phi)]\theta(1-\phi)v^2}{\alpha}$. And $\pi_A^{NDJ,d} \geq \pi_A^d$ if and only if $\theta \geq \frac{1}{2-\phi} = \tilde{\tilde{\theta}}(\phi)$. Clearly, this holds since equilibrium E is defined *inter alia* by $\theta \geq \tilde{\tilde{\theta}}(\phi)$. Hence, an increase in the estoppel probability makes the patentholder better off. The intuition is as follows: an increase in the estoppel probability ϕ has two effects. First, it increases the likelihood that the patent will be unenforceable if the patentholder delays enforcement after she sent a notice of infringement. This decreases the patentholder's expected payoff. Second, an increase it ϕ provides incentives to the infringer: anticipating that the patent is more likely to be unenforceable, he invests more in development. And this increases the patentholder's expected payoff: because the infringer invests more in

[17] I consider the two other results as intuitive.

development, the probability that an innovation occurs increases. For intermediate values of the estoppel probability, it turns out that the second effect dominates. The same analysis can be conducted for the case where an increase in the estoppel probability ϕ induces a switch from equilibrium I to equilibrium E (the patentholder's payoff increases).

Proposition 3 *An increase in the probability ϕ that the doctrine of estoppel applies can induce a switch from the "immediate enforcement" or "delayed enforcement" equilibrium to the "estoppel exposure" equilibrium.*

This result is a consequence of proposition 2: because an increase in the probability ϕ that the doctrine of estoppel applies can *benefit* the patentholder, this encourages her to expose herself to the doctrine, namely to send a notice of infringement and to delay enforcement if the infringer does not ask a declaratory judgment. Notice, by considering Figure 2, that if the doctrine of estoppel does not exist ($\phi = 0$), then the patentholder never takes a risk: he does not send a notice of infringement (he merely delays enforcement if $\theta \leq \frac{3}{4}$ and enforces immediately otherwise). This is the very existence of the doctrine of estoppel which, provided it is applied probabilistically,[18] induces an equilibrium where the patentholder exposes herself. A corollary of these results is:

Corollary 1 *An increase in the probability ϕ that the doctrine of estoppel applies can induce delayed litigation more often or less often.*

I now turn to analyzing the influence of the probability of patent validity on players'payoff and on the equilibrium outcomes.

Proposition 4 *Depending on the paramters, An increase in the probability that the patent is valid:*

- *can make the patentholder and the infringer better off,*

[18] Of course, if the doctrine was enforced for sure ($\phi = 1$), Figure 2 also reveals that the patentholder would not take a risk and would behave exactly as if the doctrine did not exist: delay enforcement for $\theta \leq \frac{3}{4}$ and enforce immediately otherwise.

- *can make the patentholder and the infringer worse off.*

These results mirror those of proposition 2. There are two counter-intuitive insights. First, the patentholder can be worse off if the validity of her patent increases. Second, the infringer can be better off if the validity of the patent increases. The two other insights are intuitive. I provide an explanation for the two counter-intuitive results.

The infringer is better off. Suppose that θ and ϕ are given so that equilibrium D prevails. Now, looking at Figure 2, an increase in θ leaving ϕ unchanged may induce a switch from D to E. The infringer's payoff switches from $\pi_B^d = \frac{(1-\theta)^2 v^2}{2\alpha}$ in equilibrium D to $\pi_B^{NDJ,d} = \frac{[1-\theta(1-\phi)]^2 v^2}{2\alpha}$ in equilibrium E. Now consider the following numerical example: Initially, $\phi = 0.64$ and $\theta = 0.73$. Then in equilibrium D, $\pi_B^d = 0.0729\frac{v^2}{2\alpha}$. If θ increases to 0.736 and ϕ is unchanged, the new equilibrium is E and $\pi_B^{NDJ,d} = 0.54\frac{v^2}{2\alpha}$. Clearly, the infringer is better-off after the increase in patent validity induced a change from equilibrium D to equilibrium E[19]. The intuition is that an increase in patent validity encourages the patentholder to send a notice of infringement (i.e expose herself to estoppel), hence the equilibrium switch. As I explained for proposition 2, the infringer is better off if the patentholder expose herself to estoppel because the probability that the patent will be unenforceable increases (from $(1-\theta)$ to $\phi(1-\theta)$).

The patentholder is worse off. In Figure 3, I represent the patentholder's payoff as a function of patent validity θ, holding the estoppel probability constant at $\phi = 0$ (the same argument holds for other values of the estoppel probability). It is clear on this figure that locally, an increase in patent validity reduces the patentholder's payoff. The intuition is as follows. In equilibrium D the patentholder's payoff $\pi_A^d = \theta(1-\theta)\frac{v^2}{\alpha}$ is non-monotonic in θ. There are *two effects*: a higher θ means that the probability to win the injunction increases, but at the same time this effect decreases the incentives of the infringer to invest in development, thereby reducing the expected payoff of the patentholder. For high enough values of the patent validity θ, namely $\theta \in \left[\frac{1}{2}, \frac{3}{4}\right]$, the second effect dominates and the patentholder's payoff is reduced.[20] Such inverse-U shape

[19]Denoting θ^E a patent validity probability compatible with outcome E and θ^D a patent validity compatible with D, it can be shown that the infringer is better-off in equilibrium E than in equilibriumD if and only if $\theta^E \leq \frac{\theta^D}{1-\phi}$. This comes from comparing π_B^d and $\pi_B^{NDJ,d}$.

[20]This result echoes a result in a companion paper on the "doctrine of laches" (Carpentier, 2005). There, I show that an increase in patent validity can make the patentholder worse-off as well. However, the rationale is very different from this paper: it involves the timing of infringement and the cost of litigation, in the context

is well established in the analysis of innovation and market structure (Kamien and Schwartz, 1975, 1982; Aghion et al., 2002). My analyis provides a new explanation for the observation.

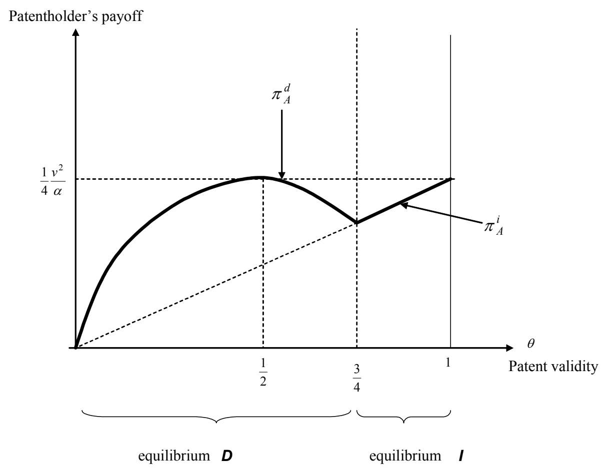

Figure 3: The patentholder's payoff as a function of patent validity θ holding the estoppel probability constant at $\phi = 0$

A consequence of these remarks is:

Corollary 2 *An increase in the probability θ that the patent is valid can induce a switch from a safe to the risky equilibrium and induce more often or less often litigation delay.*

An increase in patent validity allows the patentholder to expose herself more to estoppel (i.e to send a notice of infringement and to delay if the infringer does not ask a declaratory judgement). This is because a higher probability that the patent is valid compensates the probability that the doctrine of estoppel applies. But if validity is too high, the incentives of the infringer decrease. Thereby, if litigation is delayed, the probability that the product is developed decreases and this affects negatively the patentholder's expected payoff. Ceteris paribus, this encourages her not to delay. This explains why an increase in patent validity can have two opposite effects: induce more often or less often delay in litigation.

of a simple "real option" model. The timing of infringement is irrelevant in the present paper, and I ignored litigation costs for tractability reasons. Of course, both the "doctrine of estoppel" and the "doctrine of laches" being about punishing a patentholder who delays patent enforcement, it is natural to expect some similar results. What distinguishes the two papers is mainly the *economic set-up* in which the two doctrines are analyzed. The result also echoes Llobet (2003).

5 The doctrine of estoppel and the hold-up problem

When innovation is sequential, patents may create a hold-up problem. As in this paper, the holder of the first patent is entitled to collect revenues from the follow-on innovation and this decreases the infringer's effort in developing the innovation. Patent policy may help alleviating this problem. I now discuss the results of the previous analysis from a patent policy perspective, by considering the doctrine of estoppel ϕ as an instrument of patent policy (taking other parameters as given) which can affect the hold-up issue. Despite its previously discussed advantages, my probabilistic formulation for the doctrine of estoppel has a shortcoming: advising a Court to randomize the application of the doctrine does not seem easily implementable. Yet, it appears that keeping a certain degree of uncertainty is precisely what "Case Law" is about. Once a case is decided, Courts write reports in which they seek to justify their decision and clarify some requirements for applying a specific judgment, keeping at the same time a number of potential questions unanswered. This uncertainty aims at letting future judges decide their case according to its own specificity. Here, society is assumed to choose the level of ϕ, i.e the "stringency" of the requirements to apply the doctrine of estoppel. The lower is ϕ, the the more stringent are these requirements. Society chooses the estoppel policy that maximizes the the sum of the players' payoffs. Since the joint payoff is maximized when the development effort is maximized[21], the estoppel policy ϕ^* that maximizes the joint payoff also minimizes the hold-up issue (i.e. implements a level of effort closest to the first-best level).

Before going further into this investigation, notice that the stringency of the doctrine of estoppel (captured by ϕ) is irrelevant for a wide range of θ values. This is illustrated in Figure 2: For all $\theta < \overset{\bullet}{\theta}$, a change in ϕ has no effect on the equilibrium outcome D: the patentholder does not send a notice of infringement and she delays enforcement. As a result, the following results about the design of an optimal ϕ are relevant only when the validity of the patent is high enough: $\theta \geq \overset{\bullet}{\theta}$. In the discussion below, I use ϕ as the policy instrument. The following figure is identical to Figure 3, except that it represents ϕ as a function of θ.[22]

[21] By "maximized effort", I mean a second-best effort closest to the first-best effort.

[22] $\widetilde{\widetilde{\phi}}$ is the inverse of $\widetilde{\widetilde{\theta}}$, $\widetilde{\phi}$ is the inverse of $\widetilde{\theta}$ and $\widehat{\phi}$ is the inverse of $\widehat{\theta}$: the detail of the computations that yield to $\widetilde{\widetilde{\phi}}$, $\widetilde{\phi}$ and $\widehat{\phi}$ are provided in Appendix B.

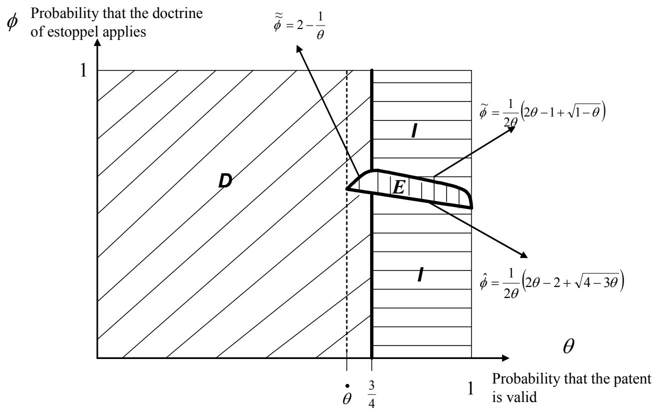

Figure 4: Equilibrium outcomes when the estoppel probability ϕ is a function of patent validity θ.

I compute the joint payoff W for all three equilibrium outcomes (E, I, D) separately.[23]

- *Equilibrium I:* the patentholder does not send a notice of infringement and enforces immediately her patent. The payoffs are given by (5). Adding them yields:

$$W^I = \frac{v^2}{2\alpha}\left(1 - \frac{\theta}{4}\right). \tag{16}$$

- *Equilibrium D*: the patentholder does not send a notice of infringement and delays patent enforcement. The players'payoffs are given by (8). Adding them yields:

$$W^D = \frac{v^2}{2\alpha}\left(1 - \theta^2\right). \tag{17}$$

Notice that both $W^I$and W^D are independent of ϕ.

- *Equilibrium E*: the patentholder sends a notice of infringement, the infringer does not ask a declaratory judgment and the patentholder delays enforcement. The payoffs are given by (13), yielding:

$$W^E(\phi) = \frac{v^2}{2\alpha}\left\{1 - \left[\theta\left(1-\phi\right)\right]^2\right\}. \tag{18}$$

[23]Maximizing the joint payoff could be the objective of a social planner in a small open economy (like Finland) where domestic consumer surplus is marginal (most revenues being collected abroad).

Computing $\frac{\partial W^E(\phi)}{\partial \phi}$ yields:

$$\frac{\partial W^E(\phi)}{\partial \phi} = \frac{v^2}{\alpha}\theta^2(1-\phi) \geq 0.$$

Hence, society prefers to increase ϕ to its maximum value compatible with equilibrium outcome E. In Figure 4, this maximum value is $\tilde{\tilde{\phi}}(\theta) = 2 - \frac{1}{\theta}$ if $\theta \in \left[\overset{\bullet}{\theta}, \frac{3}{4}\right]$ and $\tilde{\phi}(\theta) = \frac{1}{2\theta}\left(2\theta - 1 + \sqrt{1-\theta}\right)$ if $\theta > \frac{3}{4}$. This implies the following lemma:

Lemma 4 *The best estoppel policy in equilibrium E is the maximum ϕ compatible with this equilibrium:*

$$\phi = \begin{cases} \tilde{\phi}(\theta) = \frac{1}{2\theta}\left(2\theta - 1 + \sqrt{1-\theta}\right) & \text{if } \theta > \frac{3}{4} \\ \tilde{\tilde{\phi}}(\theta) = 2 - \frac{1}{\theta} & \text{if } \theta \in \left[\overset{\bullet}{\theta}, \frac{3}{4}\right] \end{cases}. \tag{19}$$

Plugging in the values of ϕ defined in (20) back into (19) yields the highest value for W^E :

$$W^{E,*} = \begin{cases} \frac{v^2}{2\alpha}\frac{3}{4}\left(1 + \sqrt{1-\theta}\right) & \text{if } \theta > \frac{3}{4} \\ \frac{v^2}{2\alpha}\theta\left(2-\theta\right) & \text{if } \theta \in \left[\overset{\bullet}{\theta}, \frac{3}{4}\right] \end{cases}. \tag{20}$$

The detailed calculations are in the Appendix B. Now, I can compare W^I, W^D and $W^{E,*}$. It turns out that $W^{E,*}$ is higher than either W^I or W^D (this is also proved in Appendix B). Hence:

Proposition 5 *Suppose θ is high enough ($\theta \geq \overset{\bullet}{\theta}$). If society's objective is to maximize the sum of the patentholder's and the infringer's payoff, the optimal estoppel policy is :*

$$\phi^* = \begin{cases} \tilde{\phi}(\theta) = \frac{1}{2\theta}\left(2\theta - 1 + \sqrt{1-\theta}\right) & \text{if } \theta > \frac{3}{4} \\ \tilde{\tilde{\phi}}(\theta) = 2 - \frac{1}{\theta} & \text{if } \theta \in \left[\overset{\bullet}{\theta}, \frac{3}{4}\right] \end{cases}. \tag{21}$$

This implements the estoppel exposure equilibrium E.

Proof. See Appendix B. ■

Looking at Figure 4, it is clear that the optimal estoppel corresponds to the boundaries $\tilde{\phi}$ and $\tilde{\tilde{\phi}}$ of the "estoppel exposure" region: the highest ϕ compatible with this region is indeed given by these two boundaries. In section 4, I showed that the patentholder and the infringer prefer to be in equilibrium E. Here, I made this result more precise by establishing that the sum of their payoffs increases with the probability ϕ that the doctrine of estoppel applies in equilibrium E. The main insight of this analysis is that the "estoppel exposure" outcome is the optimal outcome. For that to be the case, the doctrine of estoppel should be "lenient enough" (in the sense that the probability that it applies is quite high). Indeed, $\phi^* \geq \frac{1}{2}$ in (22). [24]

6 The absence of credit constraint

If the infringer were not credit-constrained, he could pay *upfront* to the patentholder a fixed licensing fee equal to the expected net payoff from developing the innovation. He would become the "residual claimant" and therefore he would exert the first-best effort in developing his innovation. This changes the possible equilibrium outcomes of the game:

Proposition 6 *If the infringer is not credit-constrained, there are two equilibrium outcomes:*

- *If $\theta \leq \frac{1}{2}$, the patentholder does not send a notice of infringement and delays the enforcement of her patent. The infringer invests $x^{d*} = \frac{(1-\theta)v}{\alpha}$.*
- *If $\theta > \frac{1}{2}$, the patentholder does not send a notice of infringement and enforces immediately her patent. The infringer exerts the first-best level of effort $x^{FB} = \frac{v}{\alpha}$.*

Proof. See Appendix D. ■

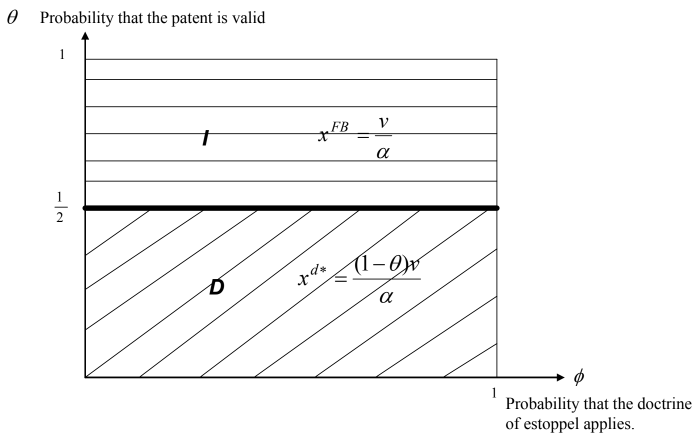

Figure 5: Equilibrium outcomes when the infringer is not credit-constrained.

[24]This result on the optimality of judicial uncertainty echoes the notion of "constructive ambiguity" in the analysis of central banks' monetary policy and lender-of-last resort function

I now provide the economic intuition behind the difference between presence and absence of a credit constraint.

- In the model with credit constraint, the optimal contract in case of immediate enforcement was a profit-sharing contract where the infringer would receive $\frac{1}{2}v$ (half the return from the innovation) instead of the full return v. This created disincentives to invest in development (since obviously $\frac{1}{2}v < v$), thereby reducing the probability of success. This decreased the patentholder's *expected* payoff. This is why the patentholder found it useful to expose herself to the application of the doctrine of estoppel (as long as the probability that it applied would remain lower than 1) and to delay enforcement: the possibility of estoppel, and thus of the patent being unenforced, encouraged the infringer to invest more in development, increasing the success probability and thereby the patentholder's expected payoff.

- Here, the credit constraint is absent. In case of immediate enforcement, the infringer can pay upfront a lump-sum to the patentholder (he has the money for that) and then he decides how much effort to put in development. Because he is the residual claimant, he gets v in case of success. The optimal effort is thus the first-best effort. Anticipating this, in the contract stage, the patentholder, who has the bargaining power, makes an offer which extracts all the surplus from the infringer.[25] What she gets is an amount corresponding to the first-best effort by the infringer. As a result, even if the patentholder can still find it beneficial to delay enforcement[26], she does not need to expose herself to estoppel anymore: she does not need to use estoppel to counterbalance the decreased incentives of the infringer caused by a profit-sharing contract.

To summarize, when the infringer is credit-constrained, estoppel works as an incentive mechanism. A first consequence is that:

[25] This suggests that the infringer may want to pretend that he is credit-constrained just to raise its bargaining power with the patentholder. I thank Vesa Kanniainen for this remark.

[26] When $\theta \leq \frac{1}{2}$ the patentholder delays enforcement. If she enforces immediately, she gets, via a lump sum, the *net* expected payoff from the innovation (return v minus the development cost $c(x^{FB}) = \frac{1}{2}\frac{v^2}{\alpha}$). If she delays, *the infringer* bears the cost. To avoid bearing the development cost, the patentholder may thus prefer to delay enforcement. However she does so only if patent validity is unlikely ($\theta \leq \frac{1}{2}$). Indeed, a high patent validity reduces the infringer's effort in case patent enforcement is delayed and this adversely affects the patentholder's *expected* payoff.

Corollary 3 *If the infringer is not credit-constrained, the estoppel policy has no effect on the equilibrium outcome and on the hold-up issue.*

Notice that the hold-up issue remains valid when $\theta \leq \frac{1}{2}$ as the infringer does not exert the first-best level of effort. This corollary means that the doctrine of estoppel is an effective patent policy instrument *only when the infringer cannot make an upfront payment to the patentholder.* Another observation is that a high patent validity probability is an insurance against hold-up:

Corollary 4 *An increase in patent validity θ can suppress the hold-up problem.*

Proof. Consider an increase in θ which induces a switch from equilibrium D to equilibrium I. This implies that the investment in the follow-on innovation increases from x^{d*} to x^{FB}, the first-best level of effort. But by definition of the first-best effort, there is no hold-up anymore. ■

When litigation is delayed, low patent validity gives incentives to the infringer to exert more effort in development, which increases the patentholder's expected payoff. Coupled with the fact that the patentholder does not bear the development cost (see footnote 26), this encourages her to delay. This effect is reduced when patent validity is high and the patentholder will prefer to enforce early and get a lump sum transfer corresponding to the infringer exerting the first-best effort. Corollary 4 is an additional argument in favor of improving patent quality. There is a vivid debate in the United States on this issue, some arguing that an overloaded Patent Office is unable to conduct a thorough prior-art search and grants patents of dubious quality.[27] I show that one benefit of patent "quality" is that it might alleviate hold-up when innovation is sequential. [28]

7 Conclusion

When developing innovations, firms often infringe previous patents. This is because the "leading breadth" of most pioneer patents is often substantial: the patent claims to cover many possible

[27] It is estimated for example that in electronics, the probability that a randomly considered patent is valid is around 43%. See the website of OceanTomo.

[28] This holds in the credit constraint case as well.

technological improvements or to be useful for a wide range of applications. Most of the previous literature focuses only on leading breadth as an instrument of "forward patent protection" (to reward pioneers while alleviating the hold-up problem). It also largely abstracts from the issue of litigation and litigation timing. In this paper, I introduce a model of patent litigation over sequential innovations and I analyze another instrument of "forward patent protection": the doctrine of estoppel. The relevance of the doctrine of estoppel as a patent policy instrument hinges upon the financial situation of the infringer.

When the infringer is not credit-constrained (or is wealthy), the doctrine of estoppel cannot minimize the hold-up problem which occurs when patent validity is low (proposition 6 and corollary 3). But an increase in patent validity can suppress the hold-up problem (corollary 4). When the infringer has no wealth at the time of litigation, and in addition he is credit-constrained, the doctrine of estoppel can be designed to minimize the hold-up issue (propositions 5). The optimal estoppel policy is a probability strictly larger than 0 and strictly less than 1 meaning that *society benefits from judicial uncertainty.* I also show that an increase in the probability that the doctrine applies can make both the patentholder and the infringer better or worse off, depending on parameters' values (proposition 2). It can induce more often or less often delayed litigation (proposition 3 and corollary 1). Finally, an increase in patent validity can also make both players better or worse off (proposition 4) and induce more often or less often delayed litigation (corollary 2). The main contribution of this analysis is to show that delayed litigations can be welfare enhancing and that there are instruments which can be designed to generate these delays.

Appendix

Appendix A: Equilibrium outcomes of the litigation game

In this Appendix, I first state a series of three lemmas which summarize the essential properties of $\widetilde{\theta}(\phi)$ and $\widehat{\theta}(\phi)$. Then I prove proposition 1.

Lemma 5 ***Properties of*** $\widetilde{\theta}(\phi)$: *$\widetilde{\theta}(\phi)$ is strictly increasing in ϕ for $\phi \in [0, \frac{1}{2})$ and weakly decreasing in ϕ for $\phi \in [\frac{1}{2}, 1]$. Moreover, $\widetilde{\theta}(0) = \frac{3}{4}$, $\widetilde{\theta}(\phi) = 0$ if and only if $\phi = \frac{3}{4}$ and $\lim_{\phi \mapsto 1^-} \widetilde{\theta}(\phi) = -\infty$. Finally, $\frac{\partial^2 \widetilde{\theta}(\phi)}{\partial \phi^2} \leq 0$ if and only if $\phi \leq \frac{1}{4}$.*

Proof. Computing $\widetilde{\theta}(0)$ and $\lim_{\phi \mapsto 1^-} \widetilde{\theta}(\phi)$, as well as solving $\widetilde{\theta}(\phi) = 0$ is straightforward. The derivative of $\widetilde{\theta}(\phi)$ with respect to ϕ is given by:

$$\frac{\partial \widetilde{\theta}(\phi)}{\partial \phi} = \frac{2(1-\phi)-1}{2(1-\phi)^3}.$$

It follows that $\frac{\partial \widetilde{\theta}(\phi)}{\partial \phi} > 0$ if and only if $\phi < \frac{1}{2}$. The second derivative is:

$$\frac{\partial^2 \widetilde{\theta}(\phi)}{\partial \phi^2} = \frac{4(1-\phi)-3}{2(1-\phi)^4}.$$

It follows that $\frac{\partial^2 \widetilde{\theta}(\phi)}{\partial \phi} \geq 0$ if and only if $\phi \leq \frac{1}{4}$. ■

Lemma 6 ***Properties of*** $\widehat{\theta}(\phi)$*:* $\widehat{\theta}(\phi)$ *is increasing in* ϕ *on* $[0, \frac{1}{4})$ *and weakly decreasing in* ϕ *on* $[\frac{1}{4}, 1]$*. Moreover,* $\widehat{\theta}(0) = \frac{5}{4} > 1$*;* $\widehat{\theta}(\phi) = 0$ *if and only if* $\phi = \frac{5}{4}$ *and* $\widehat{\theta}(\phi) = 1$ *if and only if* $\phi = \frac{1}{2}$*. Finally,* $\lim_{\phi \longmapsto 1^-} \widehat{\theta}(\phi) = -\infty$.

Proof. Computing $\widehat{\theta}(0)$ and $\lim_{\phi \mapsto 1^-} \widehat{\theta}(\phi)$, as well as solving $\widehat{\theta}(\phi) = 0$ is straigthforward. The derivative of $\widehat{\theta}(\phi)$ with respect to ϕ is:

$$\frac{\partial \widehat{\theta}(\phi)}{\partial \phi} = \frac{1-4\phi}{2(1-\phi)^2}.$$

It follows that $\frac{\partial \widehat{\theta}(\phi)}{\partial \phi} > 0$ if and only if $1 - 4\phi > 0$ or $\phi < \frac{1}{4}$.

$\widehat{\theta}(\phi) = 1$ if and only if $5 - 8\phi = 4(1-\phi)^2$ which simplifies to $\phi^2 = \frac{1}{4}$ or $\phi = \frac{1}{2}$. ■

Proof of Proposition 1.

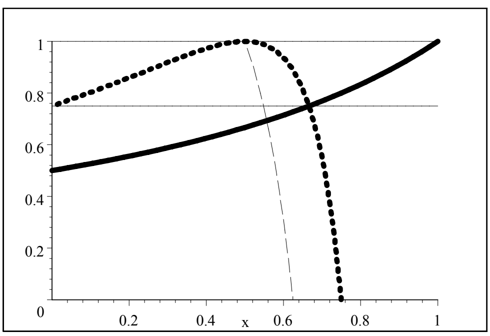

Figure 6: Plotting the functions θ' (thin solid line), $\widetilde{\theta}(\phi)$ (thick dots) and $\widehat{\theta}(\phi)$ (dash line) and $\widetilde{\widetilde{\theta}}(\phi)$ (thick solid line). Parameter ϕ is on the x-axis.

I successively establish the existence of the three equilibrium outcomes presented in proposition 1. To do so, I analyze the conditions for the patentholder to send a notice of infringement. The infringer's best-response to the patentholder's moves have been established in 4.2. I use these results to establish the equilibrium outcomes. I call (θ, ϕ) a configuration of parameters θ and ϕ.

1. Equilibrium outcome I: The patentholder does not send a notice of infringement and enforces immediately her patent. I show that this equilibrium holds when the parameters (θ, ϕ) are compatible with the region defined by $\theta \geq \theta' = \frac{3}{4}$ and $\theta < \widehat{\theta}(\phi)$ and the region defined by $\theta \geq \theta' = \frac{3}{4}$ and $\theta > \widetilde{\theta}(\phi)$.

When $\theta \geq \theta' = \frac{3}{4}$, by lemma 1, if the patentholder does not send a notice of infringement, she enforces her patent immediately. Her payoff is then given by π_A^i in (5). If (θ, ϕ) is such that $\theta < \widehat{\theta}(\phi)$, lemma 3 states that if the the infringer asks a declaratory judgment if the patentholder sends him a notice of infringement and the patentholder's payoff is given by π_A^{DJ} in (9). Clearly, comparing π_A^i in (5) and π_A^{DJ} in (9), the patentholder is indifferent between sending a notice of infringement or not, and I assume that she does not send such a notice (if this action were slightly costly, this would indeed be the equilibrium strategy). **Suppose** (θ, ϕ) **is such that** $\theta > \widetilde{\theta}(\phi)$. Since this implies that (θ, ϕ) is such that $\theta \geq \widehat{\theta}(\phi)$, as above, the infringer does not ask a declaratory judgment. And since $\theta > \widetilde{\theta}(\phi)$, the patentholder does not delay enforcement. Her payoff is thus $\pi_A^{NDJ,i}$ given by (10). Again, this is exactly the same payoff she would obtain by not sending a notice of infringement and so it is reasonable to assume that she would not send such a notice.

Hence, when (θ, ϕ) is such that $\theta \geq \theta' = \frac{3}{4}$ and $\theta < \widehat{\theta}(\phi)$ or such that $\theta \geq \theta' = \frac{3}{4}$ and $\theta > \widetilde{\theta}(\phi)$ the patentholder does not send a notice of infringement and enforces immediately her patent.

2. Equilibrium outcome D: the patentholder does not send a notice of infringement and delays enforcement. I show that this equilibrium exist for all parameters (θ, ϕ) such that $\theta < \theta' = \frac{3}{4}$ and $\theta < \widetilde{\widetilde{\theta}}(\phi)$ where $\widetilde{\widetilde{\theta}}(\phi) = \frac{1}{2-\phi}$.

Suppose (θ, ϕ) is such that $\theta < \theta' = \frac{3}{4}$. Lemma 1 says that if the patentholder does not send a notice of infringement, she delays the enforcement of her patent. Her payoff is given by π_A^d in (8). **Suppose in addition that** (θ,ϕ) **is such that** $\theta \leq \widehat{\theta}(\phi)$. If player A sends a notice of infringement, by lemma 3, the infringer would ask a declaratory judgment and player A would get π_A^{DJ} given in (9). Comparing π_A^d in (8) with π_A^{DJ} given in (9), the patentholder prefers not to send a notice of infringement (and delay enforcement) if and only if $\pi_A^d \geq \pi_A^{DJ}$ or $\theta(1-\theta)\frac{v^2}{\alpha} \geq \theta\frac{v^2}{4\alpha}$. This is equivalent to $\theta \leq \frac{3}{4}$ which always holds in that case. Hence, the patentholder does not send any notice of infringement and delays enforcement. **Suppose then that** (θ,ϕ) **is such that** $\theta > \widehat{\theta}(\phi)$ **but** $\theta < \widetilde{\theta}(\phi)$. Because $\theta \geq \widehat{\theta}(\phi)$, by lemma 3, if the patentholder does not send a notice of infringement, the infringer does not ask a declaratory judgment. By lemma 2, since $\theta \leq \widetilde{\theta}(\phi)$, the patentholder delays enforcement. Her payoff is $\pi_A^{NDJ,d}$ given in (13). Comparing with π_A^d in (8), she prefers to send a notice of infringement if and only if $\pi_A^{NDJ,d} \geq \pi_A^d$ or $\frac{[1-\theta(1-\phi)]\theta(1-\phi)v^2}{\alpha} \geq \frac{\theta(1-\theta)v^2}{\alpha}$. This is equivalent to $\theta \geq \frac{1}{2-\phi}$. I denote $\widetilde{\widetilde{\theta}}(\phi) = \frac{1}{2-\phi}$. If (θ,ϕ) is such that $\theta \in [\widehat{\theta}(\phi), \widetilde{\theta}(\phi)]$ and $\theta \geq \widetilde{\widetilde{\theta}}(\phi)$, the patentholder sends a notice of infringement. If (θ,ϕ) is such that $\theta \in [\widehat{\theta}(\phi), \widetilde{\theta}(\phi)]$ but $\theta < \widetilde{\widetilde{\theta}}(\phi)$,[29] she does not send this notice and delays enforcement. **Finally suppose that** (θ,ϕ) **is such that** $\theta \geq \widetilde{\theta}(\phi)$. In this case, $\theta \geq \widehat{\theta}(\phi)$ again so that the infringer would not ask a declaratory judgment if the patentholder were to send a notice of infringement. The patentholder would obtain $\pi_A^{NDJ,i}$ in (10). Because $\theta \geq \widetilde{\theta}(\phi)$, she would enforce immediately her patent. If she does not send such a notice, she obtains π_A^d in (8). As I found previously, $\pi_A^d \geq \pi_A^{NDJ,i}$ if and only if $\theta \leq \frac{3}{4} = \theta'$ which holds in this case. The patentholder does not send a notice of infringement and delays enforcement.

3. Equilibrium outcome E: The patentholder sends a notice of infringement, the infringer does not ask a declaratory judgment and the patentholder delays enforcement. I show that this equilibrium outcome exists for parameters (θ,ϕ) within the region circumscribed by the boundaries $\widetilde{\theta}(\phi)$, $\widehat{\theta}(\phi)$ and $\widetilde{\widetilde{\theta}}(\phi)$.

Suppose first that $\theta \geq \theta' = \frac{3}{4}$ **and** (θ,ϕ) **is such that** $\theta \in [\widehat{\theta}(\phi), \widetilde{\theta}(\phi)]$. Since $\theta \geq \widehat{\theta}(\phi)$, by lemma 3, the infringer does not ask a declaratory judgment and since $\theta \leq \widetilde{\theta}(\phi)$, by lemma 2, the patentholder delays enforcement. As a result, the patentholder's payoff is given by $\pi_A^{NDJ,d}$ in (13). Comparing π_A^i in (5) and $\pi_A^{NDJ,d}$ in (13), *the patentholder prefers to send a notice of infringement.* Indeed, $\pi_A^{NDJ,d} \geq \pi_A^i$ if and only if $\frac{[1-\theta(1-\phi)]\theta(1-\phi)v^2}{\alpha} \geq \theta\frac{v^2}{4\alpha}$ which holds if and only if $\theta \leq \frac{3-4\phi}{4(1-\phi)^2} = \widetilde{\theta}(\phi)$. This clearly holds in the current case. Suppose then that

[29]By this I mean that the configuration (θ,ϕ) belong to the region defined by the boundaries $\widetilde{\theta}$, $\widehat{\theta}$, $\widetilde{\widetilde{\theta}}$.

$\theta < \theta' = \frac{3}{4}$. I established in the proof of the existence of equilibrium D that if (θ, ϕ) is such that $\theta \in [\widehat{\theta}(\phi), \widetilde{\theta}(\phi)]$ and $\theta \geq \widetilde{\widetilde{\theta}}(\phi)$, the patentholder sends a notice of infringement. *QED.*

Determination of the value $\overset{\bullet}{\theta} \simeq 0.69$. To determine this value, I proceed in two steps. Define $\widetilde{\widetilde{\theta}}(\phi) = \frac{1}{2-\phi}$.

- *Step 1*: I first re-express $\theta = \widetilde{\widetilde{\theta}}(\phi)$ so that ϕ is a function of θ. Then I re-express $\theta = \widehat{\theta}(\phi)$ so that ϕ is a function of θ.

I have $\widetilde{\widetilde{\theta}}(\phi) = \frac{1}{2-\phi}$ from which I obtain: $\widetilde{\widetilde{\phi}}(\theta) = 2 - \frac{1}{\theta}$.

Then I have $\widehat{\theta}(\phi) = \frac{5-8\phi}{4(1-\phi)^2}$. Rewriting this expression so as to isolate the estoppel parameter ϕ:

$$4\theta\phi^2 + 8\phi(1-\theta) + 4\theta - 5 = 0.$$

This quadractic form has two roots

$$\begin{cases} \phi_1(\theta) = \frac{1}{2\theta}\left(2\theta - 2 - \sqrt{4-3\theta}\right) \\ \phi_2(\theta) = \frac{1}{2\theta}\left(2\theta - 2 + \sqrt{4-3\theta}\right). \end{cases}$$

Clearly only $\phi_2(\theta)$ is relevant here since $\phi_1(\theta) \leq 0$ (while ϕ must be non-negative). Let us denote $\phi_2(\theta) = \widehat{\phi}(\theta)$

- *Step 2:* The value $\overset{\bullet}{\theta}$ can be found by equating $\widetilde{\widetilde{\phi}}(\theta)$ with $\widehat{\phi}(\theta)$:

$$2 - \frac{1}{\theta} = \frac{1}{2\theta}\left(2\theta - 2 + \sqrt{4-3\theta}\right),$$

which is equivalent to:

$$4\theta^2 + 3\theta - 4 = 0.$$

This equation has two roots but only one is positive and thus relevant:

$$\theta = \overset{\bullet}{\theta} = \frac{-3+\sqrt{73}}{8} \simeq 0.69.$$

QED.

Appendix B: The doctrine of estoppel and the hold-up problem

Drawing of Figure 5: Deriving the expressions for $\widetilde{\widetilde{\phi}}(\theta)$, $\widetilde{\phi}(\theta)$ and $\widehat{\phi}(\theta)$.

- I established in Appendix A that $\widetilde{\widetilde{\phi}}(\theta) = 2 - \frac{1}{\theta}$.

- I also established that $\widehat{\phi}(\theta) = \frac{1}{2\theta}\left(2\theta - 2 + \sqrt{4 - 3\theta}\right)$. Notice that $\frac{\partial\widetilde{\widetilde{\phi}}(\theta)}{\partial\theta} = \frac{1}{\theta^2} \geq 0$ so that $\widetilde{\widetilde{\phi}}(\theta)$ is increasing in θ. Furthermore:

$$\frac{\partial\widehat{\phi}(\theta)}{\partial\theta} = \frac{2\theta\left(2 - \frac{3}{2\sqrt{4-3\theta}}\right) - 2\left(2\theta - 2 + \sqrt{4 - .3\theta}\right)}{4\theta^2}.$$

After simple computations, it can be shown that this is negative provided that $9\theta + 24 \geq 0$, which holds. Hence, $\widehat{\phi}(\theta)$ is decreasing. Now, I show that $\widehat{\phi}(\theta) \geq \frac{1}{2}$. To that end, I define $f(\theta) = 2\theta - 2 + \sqrt{4 - 3\theta}$. Then, $\lim\limits_{\theta \longrightarrow 0} \widehat{\phi}(\theta) = \frac{1}{2}\lim\limits_{\theta \longrightarrow 0} \frac{f(\theta) - f(0)}{\theta - 0} = \frac{1}{2}f'(0) = \frac{1}{2}\frac{5}{4} = \frac{5}{8}$. In addition, $\widehat{\phi}(1) = \frac{1}{2}$. I can conclude that $\widehat{\phi}(\theta) \in [\frac{1}{2}, \frac{5}{8}]$ and so $\widehat{\phi}(\theta) \geq \frac{1}{2}$.

- $\widetilde{\theta}(\phi) = \frac{3 - 4\phi}{4(1-\phi)^2}$. Rewriting this expression so as to isolate the estoppel parameter as a function of patent validity θ:

$$4\theta\phi^2 + 4(1 - 2\theta) + 4\theta - 3 = 0.$$

This equation has two roots:

$$\begin{cases} \phi_1(\theta) = \frac{1}{2\theta}\left(2\theta - 1 - \sqrt{1-\theta}\right) \\ \phi_2(\theta) = \frac{1}{2\theta}\left(2\theta - 1 + \sqrt{1-\theta}\right). \end{cases}$$

The only relevant root is the one which belongs to the interval $[\frac{1}{2}, 1]$.[30] I show next that $\phi_1(\theta) = \frac{1}{2\theta}\left(2\theta - 1 - \sqrt{1-\theta}\right)$ does *not* belong to $[\frac{1}{2}, 1]$ (so that, by deduction, the only relevant root is $\phi_2(\theta)$).

Notice that $\phi_1(\theta)$ is non-negative if and only $\theta \geq \frac{3}{4}$. Then, $\frac{\partial\phi_1(\theta)}{\partial\theta} = \frac{\frac{1}{2}(1-\theta)^{-\frac{1}{2}}\theta + 1 + (1-\theta)^{\frac{1}{2}}}{2\theta^2} \geq 0$. Hence, $\phi_1(\theta)$ is increasing. Its maximum value is reached for $\theta = 1$ and is given by $\frac{1}{2}$. I can conclude that $\phi_1(\theta) \in [0, \frac{1}{2}]$. And so, the only relevant root is $\phi_2(\theta)$. I denote $\phi_2(\theta) = \widetilde{\phi}(\theta)$.

I can conclude:

$$\begin{cases} \widetilde{\widetilde{\phi}}(\theta) = 2 - \frac{1}{\theta} \\ \widehat{\phi}(\theta) = \frac{1}{2\theta}\left(2\theta - 2 + \sqrt{4 - 3\theta}\right) \\ \widetilde{\phi}(\theta) = \frac{1}{2\theta}\left(2\theta - 1 + \sqrt{1-\theta}\right), \end{cases} \tag{22}$$

[30] Indeed, as it is obvious in Figure 2, the decreasing line that represents $\widetilde{\theta}$ is drawn from values of ϕ such that $\phi \geq \frac{1}{2}$.

and draw these functions in Figure 4. *QED.*

Calculation of $W^{R,*}$. I distinguish between two cases.

- Suppose first $\theta \in \left[\overset{\bullet}{\theta}, \frac{3}{4}\right]$. The maximum value of ϕ compatible with equilibrium E is then given by $\widetilde{\widetilde{\phi}}(\theta) = 2 - \frac{1}{\theta}$. Plugging this value of ϕ into W^E yields:

$$W^{E,*} = \frac{v^2}{2\alpha}\left\{2\theta\left[1-\left(2-\frac{1}{\theta}\right)\right]\left[1-\theta\left(1-\left(2-\frac{1}{\theta}\right)\right)\right]+\left[1-\theta\left(1-\left(2-\frac{1}{\theta}\right)\right)\right]^2\right\}.$$

This expression simplifies to:

$$W^{R,*} = \frac{v^2}{2\alpha}\theta\left(2-\theta\right).$$

- Suppose then that $\theta \geq \frac{3}{4}$. The maximum value of ϕ compatible with equilibrium E is now given by $\widetilde{\phi}(\theta) = \frac{1}{2\theta}\left(2\theta - 1 + \sqrt{1-\theta}\right)$. Plugging in this expression into W^E yields:

$$W^{E,*} = \frac{v^2}{2\alpha}\left\{\begin{array}{c} 2\theta\left(1-\left(1-\frac{1}{2\theta}+\frac{\sqrt{1-\theta}}{2\theta}\right)\right)\left[1-\theta+\frac{1}{2}\left(2\theta-1+\sqrt{1-\theta}\right)\right] \\ +\left[1-\theta+\frac{1}{2}\left(2\theta-1+\sqrt{1-\theta}\right)\right]^2 \end{array}\right\}.$$

After calculation, this expression simplifies to:

$$W^{E,*} = \frac{v^2}{2\alpha}\frac{3}{4}\left(1+\sqrt{1-\theta}\right).$$

QED.

Proof of Proposition 6: I simply compare $W^{E,*}$ with W^D when $\theta \in \left[\overset{\bullet}{\theta}, \frac{3}{4}\right]$ and I compare W^E with W^D when $\theta \geq \frac{3}{4}$.

- Suppose first $\theta \in \left[\overset{\bullet}{\theta}, \frac{3}{4}\right]$. $W^{E,*} \geq W^D$ if and only if:

$$\frac{v^2}{2\alpha}\theta\left(2-\theta\right) \geq \frac{v^2}{2\alpha}\left(1-\theta^2\right),$$

which is equivalent to:

$$\theta \geq \frac{1}{2}.$$

But this is the case on $\left[\overset{\bullet}{\theta}, \frac{3}{4}\right]$ since $\overset{\bullet}{\theta} \simeq 0,69$. It follows that $W^{E,*} \geq W^D$.

- Suppose then that $\theta \geq \frac{3}{4}$. $W^{E,*} \geq W^I$ if and only if:

$$\frac{v^2}{2\alpha}\frac{3}{4}\left(1+\sqrt{1-\theta}\right) \geq \frac{v^2}{2\alpha}\left(1-\frac{\theta}{4}\right),$$

which is equivalent to:

$$\theta^2 + 7\theta - 8 \leq 0.$$

The equation $\theta^2 + 7\theta - 8 = 0$ has two roots: -8 and 1. It follows that $\theta^2 + 7\theta - 8 \leq 0$ for all $\theta \in [-8, 1] \supseteq [0, 1]$. Hence, I can conclude that for $\theta \geq \frac{3}{4}$, $\theta^2 + 7\theta - 8 \leq 0$. It follows that $W^{E,*} \geq W^I$. *QED.*

Appendix C: Optimal licensing with a financially constrained infringer

I propose a simple model to show that if the infringer has no cash when the patentholder offers him a contract, and in addition he is credit-constrained, then the patentholder cannot do better than offering him a "sharing" contract $\gamma = \frac{1}{2}$. To do so, I assume that if the patentholder asks an upfront licensing fee T, then the infringer can ask a financier to pay this license. Yet, there is "credit-constraint" because of a moral hazard issue. The financier accepts to pay T in exchange for a return on the revenues v generated by the innovation. I assume that the financier has the bargaining power (maybe because there are several alternative projects to finance). He offers the infringer to pay T in exchange of a share ρ of future revenues. In that case, I show that the best contract the patentholder can propose to the infringer is an upfront payment $T = \frac{v^2}{4\alpha}$ and a share $\gamma = 0$. The patentholder's payoff is thus $\frac{v^2}{4\alpha}$. But if she were to offer only a sharing contract $\gamma = \frac{1}{2}$ so that the infringer does not need to contact a financier, then the patentholder gets $\frac{v^2}{4\alpha}$ as well, according to (5) in section 4.

The timing of the game is as follows:

1) First, player A (the patentholder: she) offers a licensing contract to player B (the infringer of the patent: he) that stipulates: *i)* a fixed licensing fee T (an upfront payment) and *ii)* a royalty rate which consists in a share γ of the revenues collected from the development of the infringing innovation.

2) Then, upon observation of the offer, player B accepts or refuses the contract. If he accepts, he contacts a financier who offers him to pay T but in exchange for a payment from player B. This payment can occur only if player B succeeds in developing his innovation since

he is protected by limited liability. With no loss of generality, I assume that the financier, in case of success, obtain a share ρ from the profit v.

3) Finally, player B exerts effort x in development. The probability of success is $p(x) = x$ and the disutility $c(x) = \frac{1}{2}\alpha x^2$.

This game is solved by backward induction.

Third stage. Conditional on the offer γ by the patentholder and ρ by the financier, player B's objective is to choose x that maximizes:

$$x(1-\gamma)(1-\rho)v - \frac{1}{2}\alpha x^2.$$

The first-order condition of this well-behaved problem is:

$$x^* = \frac{(1-\gamma)(1-\rho)v}{\alpha}.$$

Second stage. At the previous stage, the financier offers the contract ρ. The problem of the financier is thus to choose ρ that maximizes:

$$\underbrace{-T}_{\text{initial outlay}} + \underbrace{x^*\rho v(1-\gamma)}_{\text{expected revenue}}.$$

Given that $x^* = \frac{x(1-\gamma)(1-\rho)}{\alpha}$, the first-order condition associated with this well-behaved problem is:

$$\rho^* = \frac{1}{2}.$$

In equilibrium, the probability of successful development is given by:

$$x^* = \frac{(1-\gamma)v}{2\alpha},$$

and the financier's net expected payoff is:

$$-T + \underbrace{\frac{(1-\gamma)^2 v^2}{4\alpha}}_{\widetilde{T}}.$$

The individual rationality constraint of the financier is thus:

$$T \leq \widetilde{T}.$$

First stage. At the outset, player A designs an optimal two-part contract. Her problem is:

$$\left\{ \begin{array}{l} \underset{T,\gamma}{Max} \underbrace{T}_{\text{fixed fee}} + \overbrace{\underbrace{\frac{(1-\gamma)v}{2\alpha}}_{\text{probability of success}} \times \underbrace{\gamma v.}_{\text{royalties revenue}}}^{\text{expected royalties: } E} \\ \text{such that: } T \leq \widetilde{T} \text{ and } T \geq 0 \text{ ; } \gamma \geq 0. \end{array} \right. \tag{23}$$

Notice that:

$$\left\{ \begin{array}{c} \frac{\partial \widetilde{T}(\gamma)}{\partial \gamma} = -2(1-\gamma)\frac{v^2}{4\alpha} \leq 0 \\ \frac{\partial^2 \widetilde{T}}{\partial \gamma^2} = \frac{v^2}{2\alpha} > 0 \end{array} \right.$$

And

$$\left\{ \frac{\partial E(\gamma)}{\partial \gamma} \geq 0 \Longleftrightarrow \gamma \leq \frac{1}{2} \right.$$

In addition, $\widetilde{T}(1) = 0$ and $\widetilde{T}(0) = \frac{v^2}{4\alpha} = \widetilde{T}^{\max}$. Also, $E(0) = E(1) = 0$. And $\widetilde{T}(\frac{1}{2}) = \frac{v^2}{16\alpha}$ while $E(\frac{1}{2}) = \frac{v^2}{8\alpha}$. I can represent $\widetilde{T}$ and E as functions of γ :

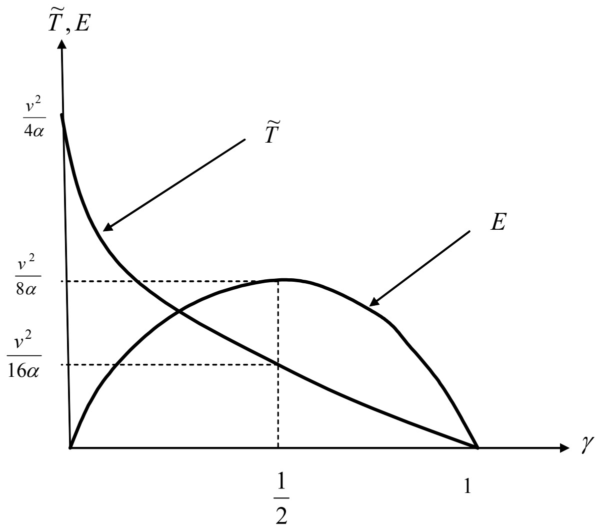

Figure 7: Patentholder's revenues from the fixed licensing fee and the royalties, when a financier pays the licensing fee.

Player A (the patentholder) can fix T so that the individual rationality constraint of the financier is binding. This implies that $T = \widetilde{T}$. In that case, the financier still accepts to pay the fixed fee, but the patentholder extracts all the surplus. Then, (23) becomes $\underset{\gamma}{Max}$ $\frac{(1-\gamma)^2v^2}{4\alpha}+\frac{(1-\gamma)\gamma v^2}{2\alpha}$. The first-order condition is $-\gamma v^2=0$. So the optimal γ is $\gamma^*=0$. For this value of γ, the optimal licensing fee is $T^*=\frac{v^2}{4\alpha}$. The patentholder's payoff is $\frac{v^2}{4\alpha}$, which is exactly what she obtains by offering $\gamma^*=\frac{1}{2}$ and no fixed licensing fee so that the infringer does not contact a financier. What drives this result is that everytime the infringer has to contact the financier, the financier has the bargaining power and offers to share the revenues v in exchange for T. This share is deciced by the financier as $\rho^*=\frac{1}{2}$, regardless of the magnitude of T. Therefore, the infringer has to pay ρ^*v in addition to γv $(\forall\gamma)$ and this reduces his incentives to invest in developing the follow-on innovation. To mitigate this effect, the patentholder has to set the smallest possible γ $(\gamma^*=0)$ and the highest possible T. But another solution is to avoid totally the presence of the financier by offering a sharing contract $\gamma^*=\frac{1}{2}$ only.

Appendix D. Discussion: solving the model when the infringer is not credit-constrained.

I repeat here the different steps followed in section 4, but I assume that the infringer can make an upfront payment to the patentholder if she litigates before the innovation is developed.

1. The patentholder does not send a notice of infringement.

- If she enforces immediately, she obtains the injunction with probability θ and makes an offer T to the infringer where T is an upfront payment. The infringer would be the residual claimant on his innovation. He would thus invest x that maximizes $xv-\frac{1}{2}\alpha x^2-T$. The optimal x is given by the first-best level: $x^{FB}=\frac{v}{\alpha}$ and the infringer's expected payoff is thus $\frac{v^2}{2\alpha}-T$. By making a take-it-or-leave-it offer $T=\frac{v^2}{2\alpha}$, the patentholder extracts all the surplus from the infringer. Before litigation, players' expected payoffs are:

$$\begin{cases} \pi_A^i=\theta\frac{v^2}{2\alpha} \\ \pi_B^i=(1-\theta)\frac{v^2}{2\alpha}. \end{cases} \tag{24}$$

- If she delays enforcement, the patentholder obtains the same payoff as the one derived in section 4:

$$\begin{cases} \pi_A^d=\frac{\theta(1-\theta)v^2}{\alpha} \\ \pi_B^d=\frac{(1-\theta)^2v^2}{2\alpha}. \end{cases} \tag{25}$$

- Now the patentholder prefers to delay if and only if $\pi_A^d \geq \pi_A^i$ or $\theta \leq \frac{1}{2} = \theta''$.

2. The patentholder sends a notice of infringement.

- If the infringer asked a declaratory judgment, the payoffs are identical to π_A^i and π_B^i in (26).

- If the infringer did not ask a declaratory judgement, the patentholder can enforce immediately (her payoff is $\pi_A^i = \theta\frac{v^2}{2\alpha}$) or delay. If she delays, she exposes herself to the application of the doctrine of estoppel. Following the results in section 4, players'payoffs would be:

$$\begin{cases} \pi_A^{NDJ,d} = \frac{[1-\theta(1-\phi)]\theta(1-\phi)v^2}{\alpha} \\ \pi_B^{NDJ,d} = \frac{[1-\theta(1-\phi)]^2 v^2}{2\alpha}. \end{cases} \tag{26}$$

Clearly, the patentholder delays if and only if $\pi_A^{NDJ,d} \geq \pi_A^i$ or:

$$\theta \leq \frac{1-2\phi}{2(1-\phi)^2} = \widetilde{\theta}(\phi) \tag{27}$$

Notice that $\widetilde{\theta}(\phi) \leq \frac{1}{2} = \theta''$ (see Figure 8 below).

- When does the infringer ask for a declaratory judgment?

☐ If $\theta \leq \widetilde{\theta}(\phi)$, he gets $\pi_B^{NDJ,d} = \frac{(1-\theta)^2 v^2}{\alpha}$ if he does not ask a declaratory judgement (as the patentholder delays litigation) and he gets $\pi_B^i = \pi_B^{DJ} = (1-\theta)\frac{v^2}{2\alpha}$ if he asks a declaratory judgment. $\pi_B^{DJ} \geq \pi_B^{NDJ,d}$ if and only if:

$$\theta \leq \frac{1-2\phi}{(1-\phi)^2} = \widehat{\theta}(\phi) \tag{28}$$

It turns out that $\widehat{\theta}(\phi) > \widetilde{\theta}(\phi)$, hence for all $\theta \leq \widetilde{\theta}(\phi)$ the infringer asks a declaratory judgment and the players get: $\pi_A^{DJ} = \pi_A^i = \theta\frac{v^2}{2\alpha}$ and $\pi_B^{DJ} = \pi_B^i = (1-\theta)\frac{v^2}{2\alpha}$.

☐ If $\theta > \widetilde{\theta}(\phi)$, the infringer gets $\pi_B^{NDJ.i} = (1-\theta)\frac{v^2}{2\alpha}$ if he does not ask a declaratory judgment (as the patentholder enforces immediately). If he asks a declaratory judgment, he gets $\pi_B^i = \pi_B^{DJ} = (1-\theta)\frac{v^2}{2\alpha}$ as well. It is reasonable to assume that he does not ask a declaratory judgment (as it would be the case if this procedure were slightly costly). Hence, players's payoffs are $\pi_A^{NDJ,i} = \pi_A^i = \theta\frac{v^2}{2\alpha}$ and $\pi_B^{NDJ,i} = \pi_B^i = (1-\theta)\frac{v^2}{2\alpha}$.

To conclude, the infringer asks for a declaratory judgment if and only if $\theta \leq \widetilde{\theta}(\phi)$.

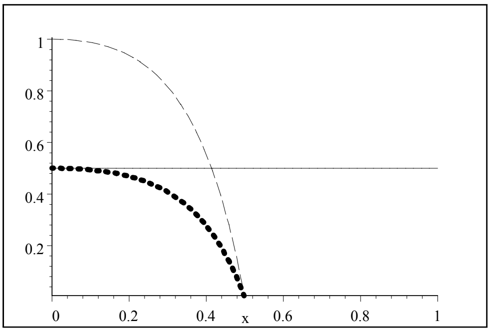

Figure 8: Plotting the functions θ'', $\widetilde{\theta}(\phi)$ (thick dots) and $\widehat{\theta}(\phi)$ (dash line).

3. When does the patentholder send a notice of infringement? I distinguish between two cases.

- $\theta \geq \frac{1}{2} = \theta''$. If the patentholder does not send a notice of infringement, she enforces her patent immediately since $\theta \geq \frac{1}{2}$. She obtains $\pi_A^i = \theta\frac{v^2}{2\alpha}$. If she sends a notice of infringement, there are two possibilities. If $\theta \leq \widehat{\theta}(\phi)$ the infringer asks a declaratory judgment and the patentholder gets $\pi_A^{DJ} = \theta\frac{v^2}{2\alpha}$. If $\theta > \widehat{\theta}(\phi)$, the infringer does not ask a declaratory judgment and the patentholder enforces immediately (since $\theta \geq \frac{1}{2} \geq \widetilde{\theta}(\phi)$). Again, $\pi_A^{NDJ,i} = \theta\frac{v^2}{2\alpha}$. Hence, regardless what the patentholder does, she obtains $\theta\frac{v^2}{2\alpha}$. It is reasonable to assume that she does not send a notice of infringement and enforces immediately the patent. Hence, if $\theta \geq \frac{1}{2}$, the patentholder does not send a notice of infringement and enforces immediately the patent. The equilibrium is I.

- $\theta < \frac{1}{2} = \theta''$. If the patentholder does not send a notice of infringement, she delays and get $\pi_A^d = \frac{\theta(1-\theta)v^2}{\alpha}$. If she sends a notice of infringement, there are two possibilities. If $\theta \leq \widetilde{\theta}(\phi)$ the infringer asks a declaratory judgment and $\pi_A^{DJ} = \theta\frac{v^2}{2\alpha}$. If $\theta > \widetilde{\theta}(\phi)$ the infringer does not ask a declaratory judgment and the patentholder enforces immediately the patent. She gets $\pi_A^{NDJ,i} = \theta\frac{v^2}{2\alpha} = \pi_A^{DJ}$. Hence, she refuses to send a notice of infringement if and only if $\pi_A^d > \pi_A^{DJ}$ or $\theta < \frac{1}{2}$ which holds here. As a result, for all $\theta < \frac{1}{2} = \theta''$, the patentholder does not send a notice of infringement and delay patent enforcement. The equilibrium is D.

This concludes the proof of proposition 7.

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The Timing of Patent Infringement and Litigation: Sequential Innovation, Damages and the Doctrine of Laches

Abstract

Often, firms infringe patents when developing their own innovations. I analyze the implications of the doctrine of laches in a model where a follow-on innovation infringes a previous patent. The doctrine of laches penalizes a patentholder who delayed enforcing her patent once infringement has been detected: she does not obtain damages for infringement that occurred in the delay period. However the patent remains enforceable. There are two periods, there is exogenous uncertainty regarding the profitability of the follow-on innovation and litigation is costly. As a result, the infringer can invest before or after uncertainty is resolved and the patentholder can litigate before or after as well. I show that the doctrine can spur or deter investment. It can also speed-up investment or delay it. It can hurt the infringer though it is meant to protect him. The effect of the patentholder's compensation via damages is also analyzed. An increase in this compensation can speed-up or delay investment, and it can paradoxically make the patentholder worse-off.

JEL classification codes: *O31* (Innovation and incentives), *O32* (Intellectual property rights), *K42* (Illegal behavior and the enforcement of the law).

Keywords: patent, litigation, reasonable royalty damages, doctrine of laches, investment under uncertainty.

1 Introduction

This paper analyzes the incentive effects of the *level of damages* and the *doctrine of laches* in a model of patent dispute over sequential innovations. When innovation is sequential, the owner of a patent over the first innovation is often entitled to collect revenues from the second (follow-on) innovation. This occurs for example when the second innovation is an application of the first one. The patentholder can litigate and collect damages to be compensated for infringement, and then negotiate with the infringer to obtain royalties if the infringer wants to continue exploiting the patent. The doctrine of laches punishes the patentholder if she delayed litigation after infringement has begun: the patentholder is not entitled to get damages for infringement that occured during the delay period. However, the patentholder can still enforce her patent and thus collect licensing revenues if the infringer wishes to continue exploiting the patent[1]. I propose a model which incorporates these features. I investigate the effects of the damages and the doctrine of laches on the timing of investment by the infringer and the timing of litigation by the patentholder. I show that the doctrine not only affects the timing of litigation, but also and perhaps most importantly, the timing of investment in the follow-on innovation. I also derive, *inter alia*, two counterintuitive results: first, an increase in the level of compensatory damages can *hurt* the patentholder and second, the doctrine of laches, meant to protect the infringer, can *hurt* him[2]. Overall, my analysis suggests that it is worthwhile to deepen our understanding of legal mechanisms that play a role in patent disputes when innovation is sequential: *in fine*, these mechanisms impact innovation incentives.

It has long been acknowledged that innovations build on previous ones. Consider for example the biotechnology and pharmaceutical industries. Medicines are often developed by using previously patented innovations, such as the PCR technology for replicating DNA in test tubes (see Schankerman and Scotchmer (2001) for an extensive list of such "research tools"). The software industry also illustrates this phenomenon. Bessen and Maskin (2002) argue that previously patented technologies required to develop a follow-on technology *hinder* innovation in industries where innovation is complementary and sequential. The reason is that the follow-on innovator typically needs to obtain the right to use the previously patented innovation. When such a right is not secured by a licensing agreement prior to engaging in research and

[1] The "doctrine of laches" differs from the "doctrine of estoppel" analyzed in a companion paper in two ways. First, the application of the doctrine of estoppel has more requirements than a mere delay. Second, if these requirements of the "doctrine of estoppel"are fulfilled, the patent is *completely unenforceable*: the patentholder cannot collect any revenue from the infringer. Under the requirements of the "doctrine of laches", the patent remains enforceable: the patentholder does not collect revenues for infringement that occured during the delay period (damages) but she collects revenues for future act of infringement (licensing revenues if the infringer wants to continue using the patented invention).

[2] The rationale for these results differ from similar results obtained in a companion paper about the doctrine of estoppel.

development, the infringer may find himself involved in a legal dispute ex-post. Indeed, the patentholder is entitled to litigate and collect damages to be compensated for infringement. The patent literature dealing with sequential innovation often abstracts from specific legal factors affecting patent disputes[3]. By focusing on some of these determinants (the doctrine of laches, the level of compensatory damages and litigation costs), this paper aims at filling a gap. In this paper, a firm has an idea which can be developed into a commercializable product at a given (sunk) cost. Development requires using a previously patented technology and ex-ante agreements with the patentholder are ruled out. If this firm (called the infringer) invests and infringes the patent, the patentholder can litigate and collect damages ex-post. Notice that infringement does not reduce the patentholder's profit. However, its patent allows the patentholder to collect part of the revenues earned by the infringer. Litigation is costly as well, for both the infringer and the patentholder. Given this basic set-up, I introduce *uncertainty regarding the demand for the innovation that the infringer wishes to develop.* There are two periods and *uncertainty is revealed at the end of the first period*: with a given probability, a demand exists for the innovation and revenues are generated from which the patentholder can collect damages. With the complementary probability there is no demand and no revenues: the patentholder do not collect any damages (I rule out punitive damages). The infringer is the leader and decides when to invest: before uncertainty is resolved (at the beginning of period 1) or after (at the beginning of period 2). The patentholder is a follower and litigates only if infringement occured. If the infringer invested at the beginning of period 1, the patentholder decides whether she litigates immediately (i.e before uncertainty is resolved) or she delays until period 2 (when uncertainty is resolved). This delayed litigation is punished if the doctrine of laches applies: the patentholder cannot get damages for infringement that occured in the first period. However, she still can get licensing revenues from the second period profit if the infringer continues to produce his infringing product.

Results. I have five main results. I present them and provide a brief economic intuition for some of them.

- First, I show that the doctrine of laches triggers earlier litigation and can decrease the likelihood of litigation. The former effect is intuitive: because it punishes a patentholder who delays enforcement, the doctrine encourages prompt litigation. The latter effect is

[3]Exceptions are Schankerman and Scotchmer (2001) or Llobet (2003).

interesting and I discuss later how it can be used in the debate over patent trolls. By forcing the patentholder to litigate early (i.e. before commercial success of the infringement is known) the doctrine essentially makes enforcement more costly so that litigation may not be profitable. When the patentholder's compensation is low enough, it can deter enforcement. This effect is ususally neglected in legal discussions of the doctrine.

- Second, I show that an increase in the patentholder's compensation can *delay* investment in the follow-on innovation (when the compensation is in an intermediate value range). This is because this increased compensation reduces the infringer's payoff, making investment under uncertainty more costly: waiting becomes the preferred strategy. However, for higher values of the patentholder's compensation, an increase in this compensation can *speed-up* investment. This is because the increased compensation encourages the patentholder to litigate regardless of the timing of investment (without the increase, litigation would not occur if the infringer were to delay). This lowers the infringer's payoff from delaying. This effect can dominate the payoff-reducing effect caused by an increased compensation and encourage the infringer to invest early.

- Then I show that counterintuitively an increase in the patentholder's compensation can make her worse off. The increase in the patentholder's compensation encourages the infringer to delay investment (see the second result) which reduces the total income to be shared (from an income over two periods to an income over one period). Increasing the compensation (the "share of the pie") does not compensate for this smaller income to be shared (the "size of the pie"), as long as the compensation remains in a low enough value range.

- The doctrine of laches can have opposite effects on the timing of the infringing investment. Depending on the model's parameters, it can speed-up or delay investment. It can also deter or spur investment. Section 5 analyses in detail the economic forces behind these different conclusions.

- Finally, I analyze when and why the doctrine of laches can hurt the infringer, though it is designed to protect him. The reason is that, by encouraging early litigation (see the first result) and forcing the infringer to pay the litigation cost before uncertainty is resolved, the doctrine makes investment more costly. When the sunk investment cost and the patentholder's compensation are high enough, it can be shown that the infringer is deterred from investing, leaving him worse-off compared to a situation where the doctrine does not apply. My contribution is to show that the doctrine fulfills its original intention (being a defense argument benefiting the accused infringer) only when the patentholder's compensation is low enough.

The doctrine of laches in brief: legal requirements. The doctrine of laches is a "defense" available to the infringer. That means that the infringer can invoke the doctrine to defend himself if the patentholder litigates him. To be successful with this defense, the infringer needs to show that the patentholder delayed litigation and that this delay caused a prejudice. If the Court is convinced, the punishment for the patentholder is simple: she cannot obtain damages for infringement that occured during the delay period. However, the patent is still enforceable. Thus, the patentholder can collect damages for infringement occuring *after* litigation started, and she can collect licensing revenues if the infringer wants to continue producing his infringing product. Legal information about the doctrine of laches can be found from various sources. A particularly clear and well illustrated paper is Szendro (2002). As emphasized by Szendro (2002), "patentees against whom the laches defense has been successfully invoked are barred from collecting only those damages that accrued prior to filing suit. Patentees may recover damages flowing from infringing activity conduct that takes place after commencement of an infringment action, even where the accused infringer successfully invokes the laches defense. Accordingly, interposition of laches does not permit the alledged infringer to lawfully continue the infringing conduct. Continued infringement remains the subject of litigation that may require settlement, entering into licensing agreements that require the payment of royalties to the patentee (...)".

Related literature. To the best of my knowledge, my paper is the first to investigate the joint effect of the *level of damages* and the *doctrine of laches* on the incentives to infringe and litigate. Schankerman and Scotchmer (2001) analyze the doctrine of laches but assume that the doctrine prevents a patentholder from obtaining an injunction (they do not investigate the doctrine in the case where the patentholder is compensated by damages). This is at odds with the facts: the doctrine of laches *allows the patentholder to get an injunction to prevent future infringement.* Its role is only to prevent the patentholder from collecting damages for infringement that occured during the delay period. My model is also related to Choi (1998). As in Choi, both the timing of infringement and the timing of litigation are endogenized. Otherwise, my approach is substantially different in the issues investigated and the results obtained. In Choi (1998), there is an incumbent patentholder and two entrants. Entry reduces the profit of the patentholder. The first litigation reveals whether the patent is valid or not. As a result, a waiting game can arise where the two entrants expect the other one to pay the cost of entry first (the other one entering only if the patent is invalid). But a "preemption game" can arise as well, because for some parameter values, the patentholder has an incentive not to litigate the first entrant in order not to reveal validity information to the second entrant. By contrast, in my model, infringement creates new revenues to be shared between the patentholder and the infringer (there is no profit erosion). The timing of litigation is driven by litigation costs and uncertainty regarding the profitability of the infringing innovation. The timing of infringement is affected by the sunk investment cost and uncertainty regarding the profitability of the innovation. The revelation of patent validity plays no role. Hence, the dynamics of my model do not rely on the same economic forces as in Choi (1998). Most importantly, my inquiry focuses on the doctrine of laches. I solve the model under two regimes, one where the doctrine applies and one where it does not, and I analyze the effect of the doctrine on players' welfare, on litigation timing and infringement timing. This is not the focus of Choi who assumes away the application of the doctrine of laches. More broadly, my paper is related to a mushrooming literature which attempts to deepen our understanding of patent disputes by *analyzing the economic impacts of various doctrines*: Lanjouw and Lerner (2001) (the doctrine of "preliminary injunctions"), Schankerman and Scotchmer (2001) (the doctrines of "unjust enrichment", "lost profit" and "laches"), Llobet (2003) (the "doctrine of equivalents"), Anton and Yao (2004) (the doctrine of "lost profit"), Aoki and Small (2004) (the doctrine of "essential facilities"), Langinier and Marcoul (2005) (the doctrine of "contributory infringement").

A roadmap. In section 2, I present the main assumptions of the model (players, actions, payoffs and timing). In section 3, I conduct the equilibrium analysis. I solve the game under two legal regimes: a regime where the doctrine of laches does not apply and a regime where the doctrine applies. This enables me to propose a comparison of the two regimes in a later section. Despite the conceptual simplicity of the model, the equilibrium analysis is long and sometimes cumbersome. This is because there are several "scenarios" to analyze, depending on the parameters of the model. In order to streamline the display of the investigation, I relegate many steps of this analysis to the Appendix. From this analysis, I am able to characterize the equilibrium outcomes of the game under both regimes. I use graphics that illustrate the different equilibrium outcomes. In the last two sections, I use these figures to derive economic insights: In section 4, I analyze the effect of strengthening the patentholder's compensation. In section 5, I analyze the effects of the doctrine of laches compared to a regime where it does not apply. Section 6 concludes.

2 Model setting

Players, actions, payoffs. I consider two players, a patentholder (she) labelled H and a potential infringer (he) labelled I. At the outset, the patentholder has a patent on an innovation A and the potential infringer is able to develop an innovative product B. I do not consider investment in obtaining innovation A and simply assume that a patent exists[4]. The previously patented innovation is required as an input in the development of the new product B. This product, if developed by the infringer, does not compete away the patentholder's profit. However, because of her patent, the patentholder can collect damages. Such a situation of "sequential innovations" is common in practice and has been extensively scrutinized in the economic literature (Matutes, Regibeau and Rockett (1996); Schankermann and Scotchmer (2001)). For example, think of the patentholder as a biotechnology firm owning a patent on a research tool like a gene sequence (A), and the infringer as a pharmaceutical company contemplating developing a new drug (B) against a specific disease. If the development of this drug requires the use of the gene sequence, there is a risk of infringement. Like Chang (1995) or Denicolo (2000), I assume that ex-ante licensing is impossible. It is not surprising that a follow-on innovator refrains from engaging in ex-ante licensing agreements with patentholders. One reason is that there are several patents that may be infringed and it is too costly (both in terms of time and money) to secure a license for each patent before any success in research and development (patent pools try to alleviate this problem). Another reason is that the follower is unwilling to engage in a costly ex-ante bargaining process because he expects to be able to

[4] This is a common feature of many models of patent litigation. For some exceptions, see Llobet (2003) or Aoki and Hu (2003).

"invent around" the patent when conducting R&D[5]. Finally, by not signing an ex-ante licensing agreement, the follower avoids to disclose his idea to the patentholder who may otherwise be able to steal it and bring a product to the market first[6].

There are two periods. To simplify the problem, I assume no discounting between periods. At the beginning of period 1, there is uncertainty regarding whether innovation B will generate any profit. Specifically, with probability α the profit from B will be π (in both periods), whereas with probability $1-\alpha$, the profit will be zero (in both periods). Uncertainty is resolved at the end of the first period.

	probability	period 1	period 2
profit from product B	α	π	π
profit from product B	$(1-\alpha)$	0	0

- The infringer is the leader in the game. He has to decide whether to invest before uncertainty is resolved (i.e. at the beginning of period 1) or after (i.e. at the beginning of period 2). Investment is a sunk cost $K \in [0, +\infty)$. Delaying has a cost: if the infringer prefers to delay investment until uncertainty is resolved, and the venture turns out to be profitable, first period profit is foregone. But delaying also has a benefit: if the venture is unprofitable (which occurs with probability $1-\alpha$), K is "saved". Hence, the infringer faces a simple problem of investment under uncertainty.

- The patentholder is a follower. Conditional on observing infringement of her patent, she can litigate to obtain damages that will compensate her for the loss of licensing revenues she would have obtained, had an ex-ante licensing agreement been signed. The Court decides whether the patent is valid. It is valid with exogenous probability θ. Then, damages can be awarded. I assume the calculation of damages goes as follows. The Court allows the patentholder to collect a share ρ of the profit π earned by the infringer during the period of infringement. Thus the patentholder gets $\rho\pi$ as damages for infringement in a given period. This way of modeling damages can be found in Langinier and Marcoul (2005) (who investigates the doctrine of contributory infringement). In practice,

[5] As such, this does not explain that a licensing agreement is not signed: the easier it is for the infringer (follow-on innovator) to invent around the patent, the higher his share of the surplus in the bargaining agreement. However, if this bargaining process is costly, the infringer may prefer to avoid it.

[6] Chang (1995) also discusses other reasons why ex-ante agreements may not be signed.

Court and interested parties often rely on "rules of thumb" to calculate damages and/or royalty payments. One popular rule is that the patentholder should be entitled 25% of the infringer's operating profit, known as the "25% rule" (Parr and Smith, 2000).[7]Once the Court has calculated damages for infringement that occured prior to litigation, the patentholder and the infringer are free to bargain to share *future* revenues. Indeed, it is in the best interest of both parties that the "infringer" continues to produce his innovation, since it generates a profit that can be shared. I assume that if the infringer invested in period 1 while the patentholder litigated in period 2, then the patentholder gets $\rho\pi$ as damages for period 1 infringement and $\rho\pi$ as licensing revenues for the second period as well.[8] I define $\theta\rho \triangleq \delta$ and call δ the "compensatory rule". More accurately, it is the "expected compensatory rule", since θ is the probability that the patent is valid (and so the probability that the patentholder gets compensated).[9]

Given this compensatory rule, the patentholder has to decide whether he litigates. Litigation costs c for both players.[10] The allocation of litigation costs follows the American rule whereby each party pays its own expenditures for litigation. If the infringer invested in period 1, the patentholder herself faces a "real option problem". She can litigate immediately (i.e in period 1) before uncertainty is resolved, or she can delay litigation until period 2. If she delays litigation until period 2, she obtains damages for infringement that occured in period 1 *only if the doctrine of laches does not apply.* I assume that the profit from the innovation is high enough compared to the cost of litigation: $\pi \geq 6c$. This is a simplifying assumption aiming at reducing the number of scenarios to analyze. Indeed, there is already an important number of scenarios. Increasing

[7]This method of compensation resembles, but is not accurately equivalent to, the compensation by a "reasonable royalty" (see *Georgia-Pacific Corp. v. United States Plywood Corp.*, 38 F. Supp. 1116, 1970). The idea is to give the patentholder a level of royalty that she would have gotten, had an ex-ante licensing agreement been signed. The difficulty here is that such an agreement, if it were possible, would yield a sharing of the net profit i.e. it would incorporate the sunk investment cost K. Schankermann and Scotchmer (2001) show that calculating damages along these lines involves a continuum of equilibria.

[8]If the doctrine of laches does not apply.

[9]I sometimes refer to δ as the level of patent protection.

[10]I abstract from out-of-Court settlement, which might be seen as a strong assumption. However, my model can also be interpreted as a model of settlement where c would be the settlement cost. Many prominent economic papers emphasize the importance of settlement costs (e.g. Crampes and Langinier, 2002; Daughety and Reinganum, 2005), either direct costs (lawyers'fees) or opportunity costs (re-allocation of time and effort to solving the dispute). Of course, this approach of settlement would still be a reduced-form approach. A proper model of settlement would allow for bargaining in the shadow of litigation.

this number would hardly yield additional economic insights but it would considerably increase the length of the analysis.

Legal regimes. I solve the game under two alternative regimes. In the first regime (the "no laches regime", labelled N), the doctrine of laches does not apply. In the second regime (the "laches regime", labelled L), the doctrine of laches applies[11]. The difference between these two regimes matters only when the infringer invested in period 1 and the patentholder delayed litigation until period 2:

- In the "no laches regime": the infringer cannot invoke the doctrine of laches and so the patentholder gets damages for period 1 infringement (she is not punished for having delayed litigation).

- In the "laches regime": the infringer can invoke the doctrine of laches. If he does so, the patentholder is punished for having delayed litigation and cannot obtain damages for period 1 infringement. However, *she can get licensing revenues for future exploitation of her patent.* Under the assumptions of the model, she does not get damages $\rho\pi$ for infringement in period 1, but she gets $\rho\pi$ as licensing revenues to compensate for exploitation of her patent in period 2.[12]

Timing of the game. 1) The potential infringer decides whether to invest in period 1 or to delay his decision until uncertainty is resolved. 2) If the infringer invested in period 1, the patentholder decides whether to litigate early or to wait until demand uncertainty is resolved. 3) Uncertainty is resolved.[13] In period 2, the timing of the game depends on period 1's actions: If the infringer delayed investment until date 2, he will invest whenever the demand turns out to be high enough. Conditional on infringement, the patentholder litigates in period 2 or

[11] To investigate the effects of the doctrine of laches, I need a "benchmark" where the doctrine does not apply.

[12] Notice that this way of modeling the doctrine of laches is consistent with Szendro's definition of the doctrine. In particular, the doctrine of laches does not make the patent unenforceable. It punishes the patentholder who delayed simply by preventing her to recover damages for the delay period.

[13] Here investment is not needed for uncertainty to be resolved. This is in line with the literature on investment under uncertainty (Dixit and Pyndick, 1994; Kanniainen and Takalo, 2000) where the investor first observes the evolution of the demand -which follows a stochastic process- and can invest at any point in time. Here the stochastic process is simplified. On the contrary, the literature on experimentation (see Bergemann and Välimäki, 2000) considers situations where learning occurs only through (possibly costly) experimentation.

renunciates. If the infringer invested in period 1 but the patentholder delayed litigation, she can litigate at the beginning of period 2. This game is represented in extensive form in Figure 1.

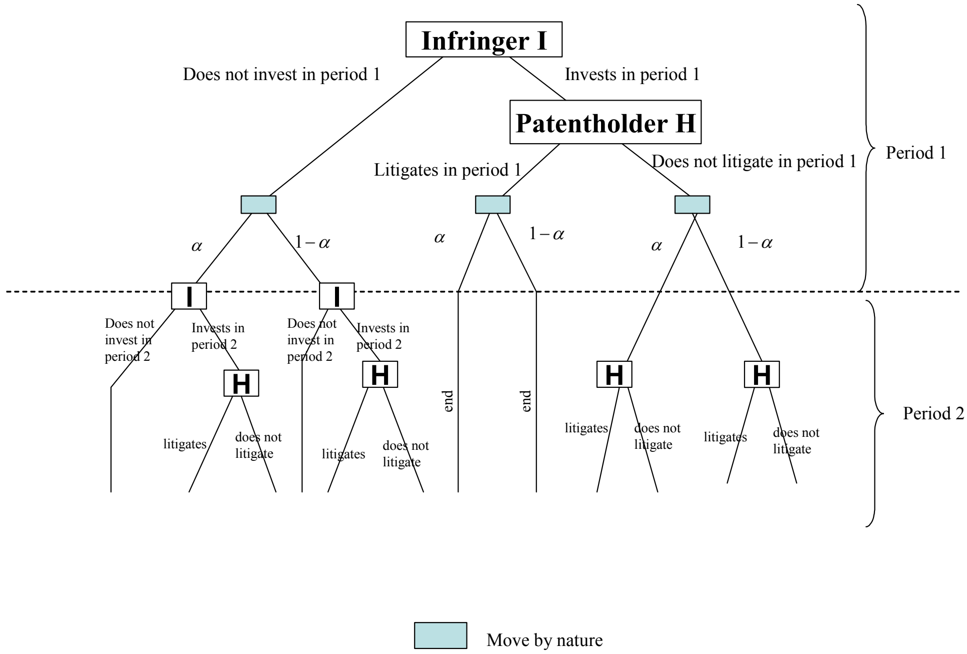


Figure 1: Extensive form

Notation. I denote by $U^r_{i,t}(a)$ player i's payoff (for $i = H, I$) at time $t \in \{1, 2\}$ in regime $r \in \{N, L\}$ when action $a \in A_i$ is chosen. The infringer's action set is given by $A_I = \{i, n\}$ and the patentholder's action set is $A_H = \{l, nl\}$. i means "investment", n means "no investment", l means "litigation" nl means "no litigation".

3 Equilibrium analysis

This two-period game is solved by backward induction and the solution concept is the subgame perfect equilibrium. First, in section 3.1 I analyze the patentholder's litigation decision. This decision depends, inter alia, on the period in which the infringer invested. Facing infringement, the patentholder has to decide whether and when to litigate. In section 3.2, I analyze the infringer's investment decision. He himself has to decide whether and when to invest.[14]

3.1 Litigation timing

Before I analyze the litigation decision, notice that, since invoking the doctrine of laches entails no cost, it is a dominant strategy for the infringer to do so (in the "laches regime"): the doctrine would allow him not to pay the damages $\rho\pi$ for the first period infringement. The patentholder observes that infringement has occured and decides whether she litigates. There are two possibilities. First, the infringer delayed investment until period 2. In that case, the patentholder de facto decides in period 2 whether she litigates or not and the regime (laches or not) is irrelevant. The other possibility is that the infringer invested in period 1. In that case, the patentholder decides whether to litigate immediately, i.e. in period 1, or to delay litigation until period 2. The main benefit of delaying litigation is that litigation costs are saved if the infringing venture does not generate any demand. I analyze in section 3.1.1 the patentholder's decision if the infringer invested in period 2. Assuming then that the infringer invested in period 1, I analyze in section 3.1.2 the patentholder's decision in the "no laches regime". Finally, in section 3.1.3, I analyze her decision in the "laches regime".

3.1.1 Litigation when the infringer invested in period 2

Suppose the infringer delayed investment. The regime is irrelevant: the patentholder can only litigate in period 2 and uncertainty is resolved at that time. She litigates provided this is profitable i.e. provided $\delta\pi \geq c$ or $\delta \geq \frac{c}{\pi} \triangleq \overline{\delta}_L$.

Lemma 1 ***(Litigation timing when the infringer invested in period 2).*** *When the infringer delayed investment until period 2, the patentholder litigates if and only if the compensatory rule is high enough i.e.* $\delta \geq \frac{c}{\pi} = \overline{\delta}_L$.

[14]Section 3 is mainly concerned by the technical analysis of the model. Because this analysis turns out to be cumbersome, many analytical steps are proposed in the Appendix.

3.1.2 The patentholder's decision in the "no laches regime"

Suppose the infringer invested in period 1. By litigating in period 1, the patentholder gets $-c+\alpha\left(\rho\pi+\rho\pi\right)$: she pays the litigation cost and gets compensation for period 1 and period 2 ($2\rho\pi$) if her patent is valid (probability θ) and if the follow-on innovation is a commercial success (probability α). By waiting and litigating in period 2 only if the innovation is a success, her expected payoff is $\alpha\left[-c+\theta\left(\rho\pi+\rho\pi\right)\right]$. Clearly the second option is more profitable as the patentholder saves the litigation cost in case the follow-on innovation does not generate any profit. Hence delaying is a dominant strategy and litigation is delayed and occurs if and only if $-c+\theta\left(\rho\pi+\rho\pi\right)\geq 0$ or $\delta\geq\frac{c}{2\pi}=\overline{\delta}_N$.

Lemma 2 ***(litigation timing in the "no laches regime" when the infringer invested in period 1).*** *When the compensatory rule δ is higher than $\overline{\delta}_N$, the patentholder delays litigation. When $\delta<\overline{\delta}_N=\frac{c}{2\pi}$, litigation is unprofitable, either in period 1 or in period 2.*

Still assuming that the infringer invested in period 1, I now turn to the case where the doctrine of laches applies.

3.1.3 The patentholder's decision in the "laches regime"

Suppose first the patenholder delays litigation until uncertainty is resolved. She litigates in period 2 provided this is profitable. Since the doctrine of laches applies, she obtains royalties for period 2, but no damages for period 1. Her net litigation payoff in period 2 is thus:

$$U_{H,2}^{L}(l)=\theta\rho\pi-c\triangleq\delta\pi-c. \tag{1}$$

Indeed, with probability θ the patent is valid and the Court allows the patentholder to get a share ρ of the forthcoming second period profit. Notice that this net payoff differs from the net payoff when the doctrine of laches did not apply. When the doctrine of laches applies, the net payoff from delayed litigation is lower since the first-period damages are forgone. $U_{H,2}^{L}(l)$ is increasing in δ and decreasing in c. Hence, there exists a value δ *above* which litigation is profitable. Denoting $\overline{\delta}_L$ this value (the subscript L referring to the "laches regime"):

$$U_{H,2}^{L}(l)\begin{cases} <0 \ \text{ if } \ \delta<\overline{\delta}_L\triangleq\frac{c}{\pi} \\ =\delta\pi-c\geq 0 \ \text{ if } \ \delta\geq\overline{\delta}_L\triangleq\frac{c}{\pi}. \end{cases} \tag{2}$$

In period 1, the patentholder computes her payoff if she does not litigate immediately, anticipating her period 2 net payoff:

$$U^L_{H,1}(nl) = \begin{cases} 0 \ \text{ if } \ \delta < \overline{\delta}_L \triangleq \frac{c}{\pi} \\ \alpha(\delta\pi - c) \ \text{ if } \ \delta \geq \overline{\delta}_L \triangleq \frac{c}{\pi}. \end{cases} \tag{3}$$

The expressions in (3) come from (2). Indeed, if $\delta < \overline{\delta}_L$, she would not litigate in period 2 (since her net payoff would be negative according to (2)). Hence, if she does not litigate in period 1 either, she obtains 0. And if $\delta \geq \overline{\delta}_L$, she would litigate in period 2, but only if the infringing venture generates revenues, which occurs with probability α. In that case, she gets $\delta\pi - c$. The patentholder also computes her net payoff if she litigates immediately (i.e. in period 1):

$$U^L_{H,1}(l) = -c + \theta\left[\alpha(2\rho\pi)\right] \triangleq -c + \alpha 2\delta\pi. \tag{4}$$

She has to pay the litigation cost c and if her patent is valid (with probability θ) she gets a share ρ from both period 1 and period 2 profits. The next step consists in determining a condition on δ (the compensatory rule) such that $U^L_{H,1}(l) \geq U^L_{H,1}(nl)$, where these net payoffs are given by (3) and (4). Because the net payoff $U^L_{H,1}(nl)$ differs depending whether $\delta < \overline{\delta}_L$ or $\delta \geq \overline{\delta}_L$, I distinguish between these two cases. "Case 1" means that $\delta \in [0, \overline{\delta}_L]$ and "case 2" means that $\delta \in [\overline{\delta}_L, 1]$.

□ Case 1: $\delta \in [0, \overline{\delta}_L]$. On this interval, given (3), $U^L_{H,1}(nl) = 0$. It follows that the condition $U^L_{H,1}(l) \geq U^L_{H,1}(nl)$ is equivalent to $U^L_{H,1}(l) \geq 0$. Using (4), this condition holds if and only if $-c + \alpha 2\delta\pi \geq 0$ or:

$$\delta \geq \frac{c}{2\alpha\pi} \triangleq \overline{\overline{\delta}}_L. \tag{5}$$

Notice that $\overline{\overline{\delta}}_L \leq \overline{\delta}_L$ if and only if $\alpha \geq \frac{1}{2}$. From that remark, I can conclude:

- If $\alpha < \frac{1}{2}$, then $\overline{\overline{\delta}}_L > \overline{\delta}_L$. This implies that for any $\delta \in [0, \overline{\delta}_L]$, $\delta < \overline{\overline{\delta}}_L$. So, (5) is violated and $U^L_{H,1}(l) \geq 0$ does not hold: the patentholder does not litigate.
- If $\alpha \geq \frac{1}{2}$, then $\overline{\overline{\delta}}_L \leq \overline{\delta}_L$. This implies that (5) holds for some $\delta \in [0, \overline{\delta}_L]$. More accurately, for $\delta < \overline{\overline{\delta}}_L$, (9) does not hold and the patentholder does not litigate. But for $\delta \in [\overline{\overline{\delta}}_L, \overline{\delta}_L]$, (5) holds and so the patentholder litigates in period 1.

□ Case 2: $\delta \in [\overline{\delta}_L, 1]$. On this interval, using (3) and (4), $U^L_{H,1}(l) \geq U^L_{H,1}(nl)$ if and only if $-c + \alpha 2\delta\pi \geq \alpha(\delta\pi - c)$ or:

$$\delta \geq \frac{(1-\alpha)c}{\alpha\pi} \triangleq \overline{\overline{\overline{\delta}}}_L. \tag{6}$$

Proceeding as I did above, notice that $\overline{\overline{\overline{\delta}}}_L \geq \overline{\delta}_L$ if and only if $\alpha \leq \frac{1}{2}$. From that remark, I can conclude:

- If $\alpha \leq \frac{1}{2}$, then $\overline{\overline{\overline{\delta}}}_L \geq \overline{\delta}_L$. This implies that for all $\delta \in [\overline{\delta}_L, 1]$, we have the following partition. If $\delta \in [\overline{\delta}_L, \overline{\overline{\overline{\delta}}}_L]$, condition (6) is violated and so the patentholder delays litigation. And if $\delta \in [\overline{\overline{\overline{\delta}}}_L, 1]$, condition (6) holds and the patentholder litigates in period 1.
- If $\alpha > \frac{1}{2}$, then $\overline{\overline{\overline{\delta}}}_L < \overline{\delta}_L$. This implies that for all $\delta \in [\overline{\delta}_L, 1]$, $\delta \geq \overline{\overline{\overline{\delta}}}_L$. So, condition (6) holds and the patentholder litigates in period 1.

Lemma 3 ***(Litigation timing in the "laches regime" when the infringer invested in period 1).***

- *If the probability of commercial success is high ($\alpha \geq \frac{1}{2}$), the patentholder does not litigate when the compensatory rule δ is lower than $\overline{\overline{\overline{\delta}}}_L$ and litigates early for $\delta \in [\overline{\overline{\overline{\delta}}}_L, 1]$.*
- *If the probabililty of commercial success is intermediate ($\alpha \in [\frac{c}{c+\pi}, \frac{1}{2})$), the patentholder does not litigate when $\delta \in [0, \overline{\delta}_L]$, delays litigation for $\delta \in [\overline{\delta}_L, \overline{\overline{\overline{\delta}}}_L]$ and litigates early for $\delta \in [\overline{\overline{\overline{\delta}}}_L, 1]$.*
- *If the probability of success is low ($\alpha \in [0, \frac{c}{c+\pi})$), the patentholder does not litigate when $\delta \in [0, \overline{\delta}_L]$ and delays litigation when $\delta \in [\overline{\delta}_L, 1]$.*

The various equilibrium actions of the patentholder are represented in Figure 2. The thick solid lines represent boundaries between different regions where a particular litigation strategy occurs in equilibrium. The case where the infringer invested in period 2 is represented by the right-hand-side graphic. Following lemma 3, in the (α, δ) space, the boundary value $\overline{\delta}_L$ separates a region where litigation occurs from a region where it does not occur. Intuitively, an increase in the compensatory rule δ yields a switch from "no litigation" to "litigation", for any given level of the litigation cost. The case where the infringer invested in period 1 is represented by the two left-hand-side graphics. Notice that I distinguish between the "no laches regime" (bottom graphic) and the "laches regime" (top graphic). The boundary $\overline{\overline{\alpha}}_L$ is the inverse of $\overline{\overline{\delta}}_L$ and $\overline{\overline{\overline{\alpha}}}_L$ is the inverse of $\overline{\overline{\overline{\delta}}}_L$[15]. Comparing the bottom graphic with the top graphic shows the main effects of the doctrine of laches on the patentholder's litigation strategy. These effects are stated in proposition 1 below.

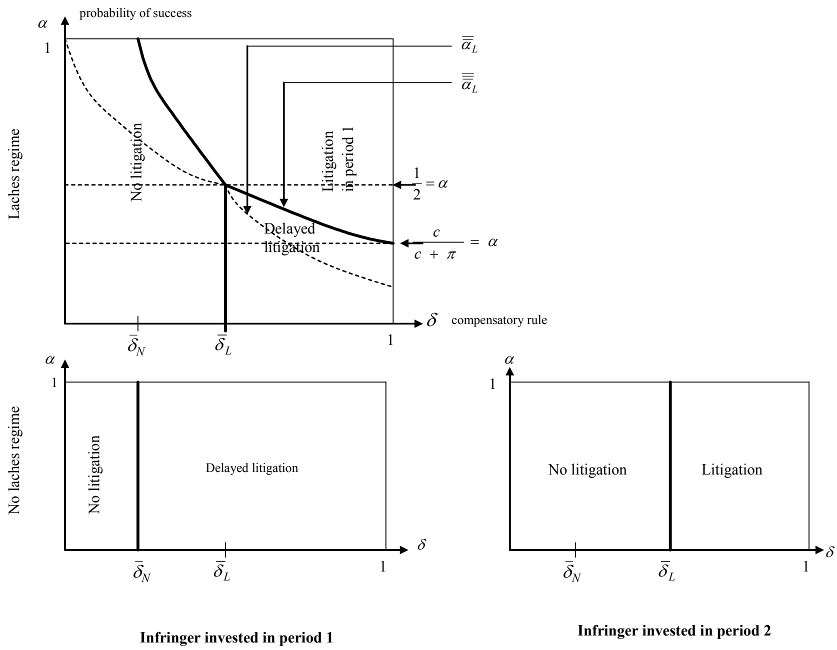


Figure 2: Litigation in either regime

Proposition 1 ***(the doctrine of laches and litigation).*** *The doctrine of laches has two possible effects on litigation compared to a regime where it does not apply:*

- *first, it increases the likelihood of early litigation,*
- *second, it decreases the likelihood of litigation.*

[15]Since $\overline{\overline{\delta}}_L = \frac{c}{2\alpha\pi}$ it follows that $\overline{\overline{\alpha}}_L = \frac{c}{2\delta\pi}$. And since $\overline{\overline{\overline{\delta}}}_L = \frac{(1-\alpha)c}{2\pi}$ it follows that $\overline{\overline{\overline{\alpha}}}_L = \frac{c}{\delta\pi+c}$. Both $\overline{\overline{\alpha}}_L$ and $\overline{\overline{\overline{\alpha}}}_L$ are decreasing in δ and they intersect at $\delta = \frac{c}{\pi} = \overline{\delta}_L$. The boundary value $\alpha = \frac{c}{c+\pi}$ in the above lemma represents the intersection between $\overline{\overline{\overline{\alpha}}}_L$ and $\delta = 1$.

Proof. The proof is straigthforward upon inspection of the cutoff values. ■

Both results in proposition 1 are intuitive. The first result is the most expected: because the doctrine of laches punishes the patentholder who delays by reducing the amount of damages she can recover, it forces some patentholders to react in a timely manner (i.e. in period 1). The second result comes from the fact that litigation is costly and there is uncertainty about whether the infringing innovation will be profitable. The patentholder herself faces a real option problem: if she delays litigation, and infringement turns out to be unprofitable, she saves litigation costs. But the interposition of the doctrine of laches forces her to litigate earlier, that is, before uncertainty is resolved. For any given c and α, and a low enough compensatory rule δ, litigating early will be non profitable and so litigation will be deterred.

Now, I move one step backward and I investigate the infringer's decision. He faces a "real option" problem as well in the sense that he can invest in period 1 or in period 2 (or not at all). In making his decision, the infringer anticipates how the patentholder will react. It means that he anticipates whether the patentholder will litigate and if she does, in which period it happens.

3.2 Investment timing by the infringer

As for litigation, I distinguish between the two regimes. First, in section 3.2.1, I analyze the investment decision in the "no laches regime". Then, in section 3.2.2, I analyze the investment decision in the "laches regime". In both cases, different scenarios must be analyzed depending on the values of the parameters. Displaying the analysis for every scenarios in this section would be cumbersome and would only slow down the progression towards deriving economic insights. As a result, part of the necessary analytical steps of this section are given in Appendix A. Also I shall assume that the probability that the innovation is profitable is high enough, namely $\alpha \geq \frac{1}{2}$. This is clearly a simplifying assumption. Like the previous simplifying assumption ($\pi \geq 6c$), it aims at reducing the number of scenarios to investigate. Notice that when $\alpha \geq \frac{1}{2}$, the two effects of the doctrine of laches on litigation are still captured: the doctrine induces earlier litigation or it deters litigation. This can be seen by comparing the two left-hand side graphics in Figure 2. Hence, the essential economic insights regarding the influence of these two effects on the timing of investment can be derived when $\alpha \geq \frac{1}{2}$.

3.2.1 The infringer's decision in the "no laches regime"

At the beginning of period 1, the infringer must decide whether and when he invests (and thereby infringes the patent). He knows that the doctrine of laches does not apply. He anticipates the patentholder's litigation strategy if he invests in period 1 (represented by the left-hand-side "bottom graphic" in Figure 2). He also anticipates the patentholder's litigation strategy when he invests in period 2 (represented by the right-hand side graphic in Figure 2). Based on these two graphics, there are three scenarios to consider depending on the value of the compensatory rule δ.

- *Scenario 1:* $\delta \in [\overline{\delta}_L, 1]$. The infringer faces litigation in period 2 regardless of the timing of investment.
- *Scenario 2:* $\delta \in [\overline{\delta}_N, \overline{\delta}_L]$. The infringer will not face litigation if he invests in period 2. However, he will face litigation in period 2 if he invests in period 1.
- *Scenario 3:* $\delta \in [0, \overline{\delta}_N]$. The infringer will never face litigation.

Here, I only report the detailed analysis for scenario 1.[16] For each scenario, I conclude by a lemma where I summarize the infringer's investment decision (lemmas 4, 5 and 6) Also, it is useful to define here two values that play a role in the forthcoming analysis: $\widehat{\alpha} = \frac{\pi}{2(\pi - c)}$ and $\widehat{\widehat{\alpha}} = \frac{\pi}{2\pi - 3c}$.

■ *Scenario 1.* $\delta \in [\overline{\delta}_L, 1]$. Suppose the infringer delays investment until uncertainty is resolved. He invests in period 2 provided that there is a demand for his product. This occurs with probability α. If he invests in period 2, his net payoff is:

$$U_{I,2}^{N}(i) = -K - c + \theta(1-\rho)\pi + (1-\theta)\pi \triangleq -K - c + \pi(1-\delta). \tag{7}$$

Because the infringer knows that the patentholder will litigate in period 2 he will face litigation cost c in addition to the sunk investment cost K. With probability θ the patent is

[16] The methodology used to solve this problem is similar to that used for analyzing the patentholder's litigation decision. Because I repeat the same analytical steps for all three scenarios, the details of the reasoning for scenarios 2 and 3 is reported in Appendix A.1.

valid and the patentholder collects a share ρ of second period profit π. With probability $1-\theta$ the patent is invalid. Since $U_{I,2}^{N}(i)$ is decreasing in K, there exists a value of K below which investment is profitable. Denoting $\overline{K}_{N,1}$ this value (N is for the "no laches regime" and 1 refers to scenario 1) it follows that:

$$U_{I,2}^{N}(i) = -K - c + \pi(1-\delta) \begin{cases} < 0 \ \text{ if } \ K > \overline{K}_{N,1} \triangleq \pi(1-\delta) - c \\ \geq 0 \ \text{ if } K \leq \overline{K}_{N,1} \triangleq \pi(1-\delta) - c. \end{cases} \tag{8}$$

In period 1, the infringer can compute his payoff if he does not invest in period 1:

$$U_{I,1}^{N}(n) = \begin{cases} 0 \quad \text{if } \ K > \overline{K}_{N,1} \triangleq \pi(1-\delta) - c \\ \alpha\left[\pi(1-\delta) - c - K\right] \quad \text{if } \ K \leq \overline{K}_{N,1} \triangleq \pi(1-\delta) - c. \end{cases} \tag{9}$$

If $K > \overline{K}_{N,1}$, the infringer would not invest in period 2. So, if he does not invest in period 1, he gets 0. If $K \leq \overline{K}_{N,1}$, the infringer would invest in period 2 if he does not invest in period 1, provided the demand for his product exists. This occurs with probability α.The payoff from investing in period 1 is:

$$U_{I,1}^{N}(i) = -K + \alpha\left[\theta 2\pi(1-\rho) + (1-\theta)2\pi - c\right] \triangleq -K + \left\{\alpha\left[2\pi(1-\delta) - c\right]\right\}. \tag{10}$$

Indeed, if he invests in period 1, the infringer faces litigation in period 2, provided the demand for the infringing products exists. This occurs with probability α. Then, with probability θ the patent is valid and the patentholder gets a share ρ of both period 1 and period 2 profits (the sum being 2π). With probability $1-\theta$ the infringer keeps the sum of the profits for himself. In any case, he has to pay the litigation cost c. The next step consists in determining a condition on K such that $U_{I,1}^{N}(i) \geq U_{I,1}^{N}(n)$ i.e such that the infringer invests in period 1. These two net payoffs are given by (9) and (10). Because $U_{I,1}^{N}(n)$ differs depending whether $K > \overline{K}_{N,1}$ or $K \leq \overline{K}_{N,1}$, I distinguish between these two cases. "Case 1" means that $K > \overline{K}_{N,1}$ and "case 2" means that $K \leq \overline{K}_{N,1}$.

□ Case 1. If $K > \overline{K}_{N,1}$, delaying investment is never profitable. Investing today is profitable as long as $U_{I,1}^{N} \geq 0$ which is equivalent to

$$K \leq 2\alpha\pi(1-\delta) - \alpha c \triangleq \overline{\overline{K}}_{N,1}. \tag{11}$$

□ Case 2. If $K \leq \overline{K}_{N,1}$, delaying yields a non-negative profit. As a result, the infringer will invest today if and only if $U_{I,1}^{N}(i) \geq U_{I,1}^{N}(n)$ or

$$K \leq \frac{\alpha}{1-\alpha}\pi(1-\delta) \triangleq \overline{\overline{\overline{K}}}_{N,1}. \tag{12}$$

From this analysis, I derive the timing of investment by the infringer when the compensatory rule δ belongs to the interval $[\overline{\delta}_L, 1]$ (scenario 1). To do so, I analyze in more depth the respective positions of $\overline{K}_{N,1}$, $\overline{\overline{K}}_{N,1}$ and $\overline{\overline{\overline{K}}}_{N,1}$. This is done in Appendix A.1 and I obtain the following result:

Lemma 4 *Under scenario 1, in the "no laches" regime, the infringer invests in period 1 when the sunk cost of investment K is lower than $\overline{\overline{\overline{K}}}_{N,1}$ and he does not invest for all $K > \overline{\overline{\overline{K}}}_{N,1}$.*

■ Scenario 2. $\delta \in [\overline{\delta}_N, \overline{\delta}_L]$. In Appendix A.1, I detail the analysis of this scenario. The methodology is identical to that used for scenario 1 above, but the payoffs, and thus the "boundary" values $\overline{K}_{N,2}$, $\overline{\overline{K}}_{N,2}$ and $\overline{\overline{\overline{K}}}_{N,2}$, are different:

$$\begin{cases} \overline{K}_{N,2} \triangleq \pi \\ \overline{\overline{K}}_{N,2} \triangleq 2\alpha\pi(1-\delta) - \alpha c \\ \overline{\overline{\overline{K}}}_{N,2} \triangleq \frac{\alpha}{1-\alpha}\left[\pi(1-2\delta) - c\right]. \end{cases} \tag{13}$$

As shown in Appendix A.1, it is necessary to define two values. First, the function $\overset{\bullet}{\delta} = \frac{2\alpha\pi-\pi-\alpha c}{2\alpha\pi}$ such that $\overset{\bullet}{\delta} \in [\overline{\delta}_N, \overline{\delta}_L]$. Then the kinked curve $\overset{\bullet}{K} = \overline{\overline{K}}_{N,1}$ if $\delta \in [\overline{\delta}_N, \overset{\bullet}{\delta}]$ and $\overset{\bullet}{K} = \overline{\overline{\overline{K}}}_{N,2}$ if $\delta \in [\overset{\bullet}{\delta}, \overline{\delta}_L]$. I show in Appendix A.1 that the following lemma holds:

Lemma 5 *Under scenario 2, in the "no laches" regime, there are three possibilities depending on the value of the probability α that the innovation is profitable.*

- *If the probability that the innovation is profitable is relatively high i.e. $\alpha \in (\frac{1}{2}, \widehat{\alpha}]$, the infringer invests in period 1 provided the sunk investment cost is lower than $\overline{\overline{\overline{K}}}_{N,2}$. He delays investment if $K \in (\overline{\overline{\overline{K}}}_{N,2}, \overline{K}_{N,2}]$ and he does not invest if $K \geq \overline{K}_{N,2}$.*
- *If $\alpha \in (\widehat{\alpha}, \widehat{\widehat{\alpha}}]$, the infringer invests in period 1 if $K \leq \overset{\bullet}{K}$, delays investment if $K \in [\overset{\bullet}{K}, \overline{K}_{N,2}]$ and does not invest if $K \geq \overset{\bullet}{K}$ and $K \geq \overline{K}_{N,2}$.*
- *If $\alpha \in (\widehat{\widehat{\alpha}}, 1]$, the infringer invests in period 1 if $K \leq \overline{\overline{K}}_{N,1}$. Otherwise, he does not invest.*

■ *Scenario 3.* $\delta \in [0, \overline{\delta}_N]$. The analysis of this scenario is detailed in Appendix A.1. For the same reason as in scenario 2, I report here the three boundaries:

$$\begin{cases} \overline{K}_{N,3} \triangleq \pi \\ \overline{\overline{K}}_{N,3} \triangleq 2\alpha\pi \\ \overline{\overline{\overline{K}}}_{N,3} \triangleq \frac{\alpha}{1-\alpha}\pi. \end{cases} \tag{14}$$

Lemma 6 *Under scenario 3, in the "no laches" regime, the infringer invests if* $K \leq \overline{\overline{K}}_{N,3}$. *Otherwise he does not invest.*

Combining the results concerning the timing of investment (lemmas 4 to 6) with those concerning litigation (lemmas 1 and 2), I obtain four different equilibrium outcomes when the doctrine does not apply and the probability of commercial succees of the innovation is high ($\alpha \geq \frac{1}{2}$)[17]:

- The infringer invests in period 1 and the patentholder does not litigate (EN).
- The infringer invests in period 1 and the patentholder litigates in period 2 (ED).
- The infringer invests in period 2 and the patentholder does not litigate (DN).
- The infringer does not invest (NO).

The rationale behind the names given to each outcome is as follows. The first block letter refers to the infringer's action: E means early investment (period 1) and D means delayed investment (period 2). The second block letter refers to the patentholder's action: E means early litigation (period 1), D means delayed litigation (period 2), and N means no litigation. Finally, NO means no investment (and so no litigation). To help figuring out the different equilibrium outcomes in the "no laches regime", I present three figures N1 to N3.[18] The label N refers to the "no laches" regime. These figures represent the equilibrium outcomes of the game in the (K, δ) space. K is the sunk investment cost born by the infringer and δ is the compensation rule which governs the share of the profit obtained by the patentholder. There are three figures because, when $\delta \in [\overline{\delta}_N, \overline{\delta}_L]$, the equilibrium outcomes are affected by the value of α. There are three intervals to consider for $\alpha \geq \frac{1}{2}$: $\alpha \in \left[\frac{1}{2}, \widehat{\alpha}\right]$, $\alpha \in (\widehat{\alpha}, \widehat{\widehat{\alpha}}]$, $\alpha \in (\widehat{\widehat{\alpha}}, 1]$. This comes from lemma 5. These figures will be analyzed more in-depth in sections 4 and 5. However,

[17] I do not detail the exact parameters values for which a particular equilibrium outcome occurs. The exposition would be tedious otherwise. Again, this is done in Appendix A.1

[18] With 3 parameters α, δ and K, another possibility would have been to consider a 3-dimensional representation of the parameter space. I do not follow this path but it should be possible to go that way. Also, it should be possible to provide computer-generated graphics to improve accuracy in the slopes and respective positioning of the lines. Yet, the proposed figures are based upon an accurate analytical analysis of each function (see Appendix A).

notice that the higher the sunk cost K and the higher the patentholder's compensation (i.e. the higher is δ), the less often investment occurs. This is intuitive: a higher K renders investment more costly and a higher δ reduces the share obtained by the infringer (for all K), thereby making investment less attractive.

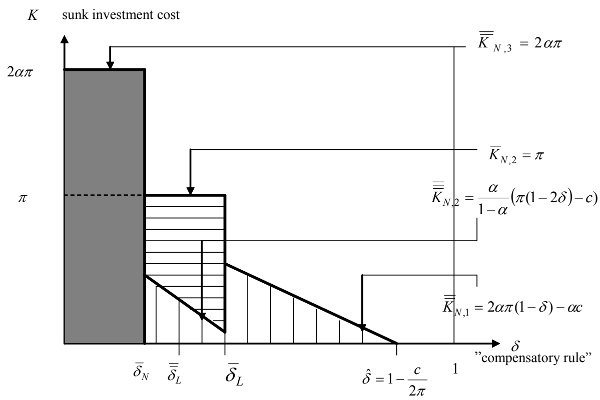


Figure N1: $\alpha \in \left[\frac{1}{2}, \hat{\alpha}\right]$.

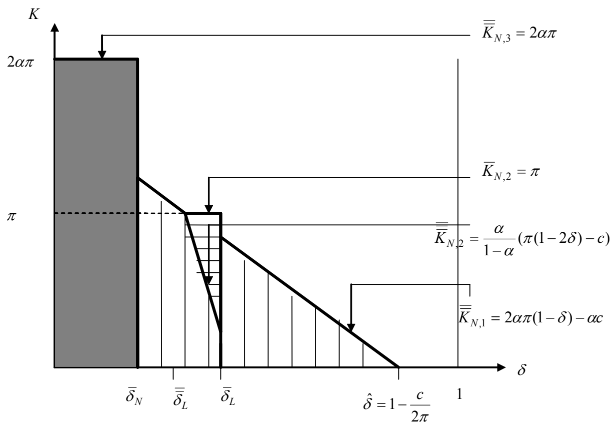


Figure N2: $\alpha \in \left[\hat{\alpha}, \hat{\hat{\alpha}}\right]$.

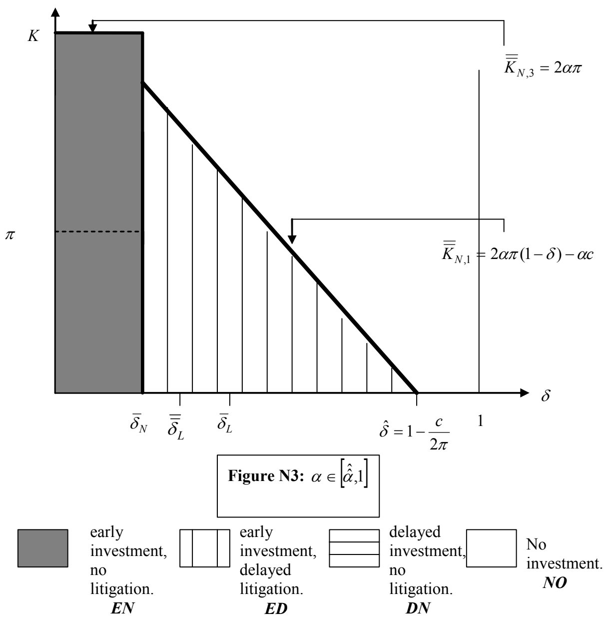


Figure N3: $\alpha \in \left[\hat{\hat{\alpha}}, 1\right]$

3.2.2 The infringer's decision in the "laches regime"

In making his decision, the infringer anticipates the patentholder's reaction if he invests in period 1 (represented by the left-hand-side top graphic in Figure 2). He also anticipates the patentholder's reaction if he invests in period 2 (represented by the right-hand side graphic in Figure 2). Based on the observation of these two graphics, there are three scenarios to consider in the "laches regime" depending on the value of the compensatory rule δ:

- *Scenario 1*: $\delta \in [\overline{\delta}_L, 1]$. The infringer faces litigation in the period of investment.
- *Scenario 2:* $\delta \in [\overline{\overline{\delta}}_L, \overline{\delta}_L]$. The infringer will not face litigation if he delays investment. However, he will face litigation in period 1 if he invests in period 1.
- *Scenario 3:* $\delta \in [0, \overline{\overline{\delta}}_L]$.The infringer will never face litigation.

For each scenario, the infringer decides whether and when to invest. Again, the methodology used to solve this timing problem is identical to that used in the previous sections. Here, I detail only the first scenario. This enables me to stress the difference with the "no laches regime".

The detailed analysis of scenarios 2 and 3 is reported in Appendix A.2. Also it is useful to define here two values that play a role in the analysis below: $\widetilde{\alpha} = \frac{\pi+2c}{2\pi}$ and $\widetilde{\widetilde{\alpha}} = \frac{\pi+c}{2(\pi-c)}$.

■ *Scenario 1.* $\delta \in [\overline{\delta}_L, 1]$. Suppose the infringer delays investment until period 2. He invests in period 2 provided that there is a demand for his product. This occurs with probability α. If he invests in period 2, his net payoff is:

$$U^L_{I,2}(i) = -K - c + \theta(1-\rho)\pi + (1-\theta)\pi \triangleq -K - c + \pi(1-\delta) \tag{15}$$

This is unchanged compared to the "no laches regime". Indeed, the regime does not matter when the infringer delays investment until period 2. As noticed for the "no laches regime", $U^L_{I,2}(i)$ is increasing in π and $1-\delta$ but it is decreasing in K. Hence, there is a value K below which the infringer would invest in period 2. Denoting $\overline{K}_{L,1}$ this value (L referring to the "laches regime" and 1 to scenario 1), it follows that:

$$U^L_{I,2}(i) = -K - c + \pi(1-\delta) \begin{cases} < 0 \text{ if } K > \overline{K}_{L,1} \triangleq \pi(1-\delta) - c \\ \geq 0 \text{ if } K \leq \overline{K}_{L,1} \triangleq \pi(1-\delta) - c. \end{cases} \tag{16}$$

In period 1, the infringer can compute his payoff if he does not invest in period 1:

$$U^L_{I,1}(n) = \begin{cases} 0 \text{ if } K > \overline{K}_{L,1} \\ \alpha\left[-K - c + \pi(1-\delta)\right] \text{ if } K \leq \overline{K}_{L,1}. \end{cases} \tag{17}$$

This is still identical to the "no laches regime". Also, in period 1, the infringer computes his net payoff if he invests immediately. This payoff differs from the "no laches regime":

$$U^L_{I,1}(i) = -K - c + 2\alpha\pi(1-\delta). \tag{18}$$

Here, the doctrine of laches encourages the patentholder to litigate in period 1 if the infringer invests in period 1. As a result, the infringer faces litigation costs c in period 1, before uncertainty is resolved. In the "no laches regime", the situation was different: the patentholder preferred to delay litigation until period 2 and, as a result, the infringer faced litigation costs only if a demand for the infringing product turned out to exist, i.e only with probability α. This difference plays a crucial role in the analysis of the doctrine of laches in section 5. The next step consists in determining a condition on K such that $U^L_{I,1}(i) \geq U^L_{I,1}(n)$, that is: such that the infringer prefers to invest in period 1. These two net payoffs are given by (17) and (18).

Because $U_{I,1}^{L}(n)$ differs depending whether $K > \overline{K}_{L,1}$ or $K \leq \overline{K}_{L,1}$, I distinguish between these two cases. "Case 1" means that $K > \overline{K}_{L,1}$ while "case 2" means that $K \leq \overline{K}_{L,1}$.

□ Case 1: $K > \overline{K}_{L,1}$. Delaying investment is not profitable. The infringer invests today if and only if $U_{I,1}^{L}(i) \geq 0$ or

$$K \leq 2\alpha\pi(1-\delta) - c \triangleq \overline{\overline{K}}_{L,1}. \tag{19}$$

□ Case 2: $K \leq \overline{K}_{L,1}$. Delaying investment yields a non-negative payoff. The infringer invests today if and only if $U_{I,1}^{L}(i) \geq U_{I,1}^{L}(n)$ or if:

$$K \leq \frac{\alpha\pi(1-\delta) + \alpha c - c}{1-\alpha} \triangleq \overline{\overline{\overline{K}}}_{L,1}. \tag{20}$$

Lemma 7 *Under scenario 1, in the laches regime, the infringer invests in period 1 if the sunk investment cost is such that $K \leq \overline{\overline{K}}_{L,1}$. If $K > \overline{\overline{K}}_{L,1}$ he does not invest.*

■ *Scenario 2.* $\delta \in [\overline{\overline{\delta}}_L, \overline{\delta}_L]$. In Appendix A.2, I detail the analysis corresponding to this scenario. The methodology is identical to that used for scenario 1 above, but the payoffs, and thus the "boundaries" functions $\overline{K}_{L,2}$, $\overline{\overline{K}}_{L,2}$ and $\overline{\overline{\overline{K}}}_{L,2}$, are different:

$$\begin{cases} \overline{K}_{L,2} \triangleq \pi \\ \overline{\overline{K}}_{L,2} \triangleq 2\alpha\pi(1-\delta) - c \\ \overline{\overline{\overline{K}}}_{L,2} \triangleq \frac{\alpha\pi(1-2\delta)-c}{1-\alpha}. \end{cases} \tag{21}$$

As shown in Appendix A.2, it is necessary to define two values. First, the function $\overset{\bullet}{\overset{\bullet}{\delta}} = \frac{2\alpha\pi - c - \pi}{2\alpha\pi}$ such that $\overset{\bullet}{\overset{\bullet}{\delta}} \in [\overline{\overline{\delta}}_L, \overline{\delta}_L]$. Then the kinked curve $\overset{\bullet}{\overset{\bullet}{K}} = \overline{\overline{K}}_{L,1}$ if $\delta \in [\overline{\overline{\delta}}_L, \overset{\bullet}{\overset{\bullet}{\delta}}]$ and $\overset{\bullet}{\overset{\bullet}{K}} = \overline{\overline{\overline{K}}}_{L,2}$ if $\delta \in [\overset{\bullet}{\overset{\bullet}{\delta}}, \overline{\delta}_L]$. I show in Appendix A.2 that:

Lemma 8 *Under scenario 2, in the laches regime, there are three possibilities depending on the probability α that the innovation is profitable:*

- *If the probability that the innovation is profitable is relatively high i.e. $\alpha \in \left[\frac{1}{2}, \widetilde{\alpha}\right]$, the infringer invests in period 1 if the sunk investment cost is low enough i.e. $K \leq \overline{\overline{\overline{K}}}_{L,2}$, delays investment for $K \in [\overline{\overline{\overline{K}}}_{L,2}, \overline{K}_{L,2}]$ and does not invest if $K \geq \overline{K}_{L,2}$.*

- *If* $\alpha \in \left[\widetilde{\alpha}, \widetilde{\widetilde{\alpha}}\right]$, *the infringer invests in period 1 if* $K \leq \overset{:}{K}$. *If* $K \in [\overset{:}{K}, \overline{K}_{L,2}]$, *the infringer delays investment. If* $K > \overset{:}{K}$ *and* $K > \overline{K}_{L,2}$, *the infringer does not invest.*
- *If* $\alpha \in \left[\widetilde{\widetilde{\alpha}}, 1\right]$, *the infringer invests in period 1 if* $K \leq \overline{\overline{K}}_{L,1}$. *He does not invest if* $K > \overline{\overline{K}}_{L,1}$.

■ *Scenario 3.* $\delta \in [0, \overline{\overline{\delta}}_L]$. For these values of the compensatory rule δ, the patentholder does not litigate. Hence, the analysis is formally equivalent to the no laches case. The three boundaries on K are given in (14). Lemma 6 applies here.

Now, I can combine the results concerning the timing of investment (lemmas 6,7 and 8) with those concerning litigation (lemmas 1 and 3). I obtain four different equilibrium outcomes when the doctrine applies and the probability of commercial succees is high ($\alpha \geq \frac{1}{2}$). [19]

- The infringer invests in period 1 and the patentholder does not litigate (EN).
- The infringer invests in period 1 and the patentholder litigates in period 1 (EE).
- The infringer invests in period 2 and the patentholder does not litigate (DN).
- The infringer does not invest (NO).

The logic behind the names of the outcomes is similar to that in the "no laches" regime. I present figures L1 to L3 to illustrate the different equilibrium outcomes of the game in a livelier manner. There are three figures because, when the compensatory rule δ belongs to $[\overline{\overline{\delta}}_L, \overline{\delta}_L]$, the timing of investment depends on α: three intervals must be considered separately on $\alpha \in [\frac{1}{2}, 1]$. At first sight, these figures are quite similar to the ones representing the equilibrium regions in the "no laches" regime. In fact, the main differences come from the respective position of the boundaries between the regions. This will be analyzed in section 5.

[19] As for the "no laches" regime, I do not detail the exact parameters' values for which a particular outcome occurs: the exposition would be tedious. This is done in Appendix A.2.

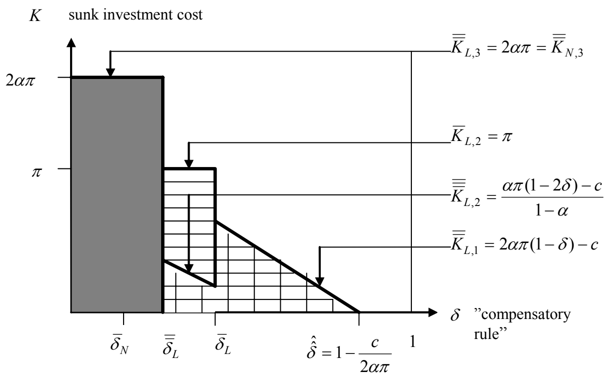


Figure L1: $\alpha \in \left[\frac{1}{2}, \tilde{\alpha}\right]$.

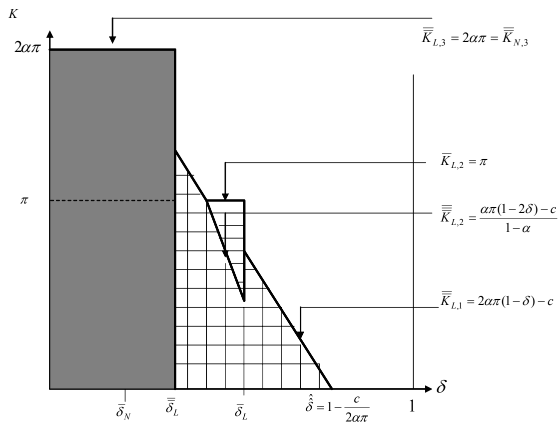


Figure L2: $\alpha \in \left[\tilde{\alpha}, \tilde{\tilde{\alpha}}\right]$

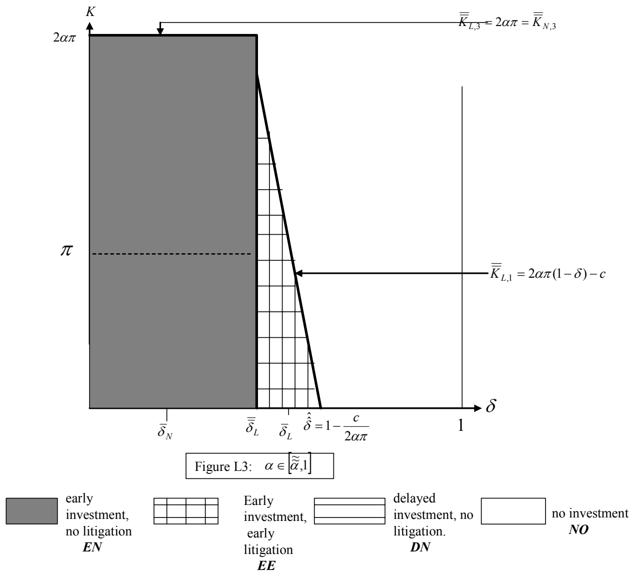


Figure L3: $\alpha \in [\tilde{\tilde{\alpha}}, 1]$

Now that I have derived the equilibrium outcomes in both regimes, I turn to analyzing the economic insights of the model. In section 4, I investigate the effects of the compensatory rule δ, in both the "no laches" and the laches regimes. In section 5, I analyze the effect of the doctrine of laches, compared to the situation where it does not apply.

4 Effect of the compensatory rule on investment and players' payoffs

Lemmas 1, 2 and 3 constituted a first step into analyzing the effect of the compensatory rule δ on litigation. Now, I analyze how this rule affects the timing of investment and players' welfare. The main results are captured by propositions 2 and 3 below.

Proposition 2 *In either regime, an increase in the compensatory rule δ can*

- *delay investment or*
- *speed-up investment.*

Delayed investment is intuitive. An increase in the compensatory rule δ reduces the infringer's gross payoff (since it increases the patentholder's gross payoff). As a result, for any c (cost of litigation), α (probability of success) and K (sunk cost) given, this payoff reduction encourages the infringer to delay investment: this is consistent with the basic insight from a real option setting. Assume that in each period the infringer gets a gross payoff $\Pi(\delta)$ which decreases with δ. If he invests in period 1, he gets $-K + \alpha \left[\underbrace{\Pi(\delta)}_{\text{period 1}} + \underbrace{\Pi(\delta)}_{\text{period 2}} \right]$. If he delays investment, he gets $\alpha \left[-K + \underbrace{\Pi(\delta)}_{\text{period 2}} \right]$. Delaying is preferrable if and only if $K \geq \frac{\alpha}{1-\alpha}\Pi(\delta)$. The threshold value is decreasing in δ meaning that delaying becomes the preferred option for a wider range of K values.

An accelerated investment is less intuitive. The reason is that, in a real option setting, one expects a decrease in the investor's payoff to delay investment, as explained above. But in the present setting, one needs to consider the effect of an increase in the compensatory rule δ on the patentholder's behavior. The basic reason behind the second insight of proposition 4 is the possibility of an equilibrium switch due to an increase in δ. To see this, consider the increase from δ_2 to δ_3 in Figure 3 below. This figure concerns the "no laches" regime but the same rationale applies to the laches regime. When $\delta = \delta_2$ the infringer delays and the patentholder does *not* litigate. Suppose now that $\delta = \delta_3$ and $\delta_3 > \delta_2$. If the infringer were to delay (i.e stick to the same strategy), he would now face litigation. This is because the increase in the compensatory rule from δ_2 to δ_3 provides incentives for the patentholder to litigate. As a result, the infringer's gross payoff from delaying is lower when $\delta = \delta_3$ than when $\delta = \delta_2$ (since he faces litigation for $\delta = \delta_3$ but not for $\delta = \delta_2$) . Ceteris paribus, this implies that investing early (in period 1), becomes more attractive. So, an increase in the compensatory rule can indeed speed-up investment.

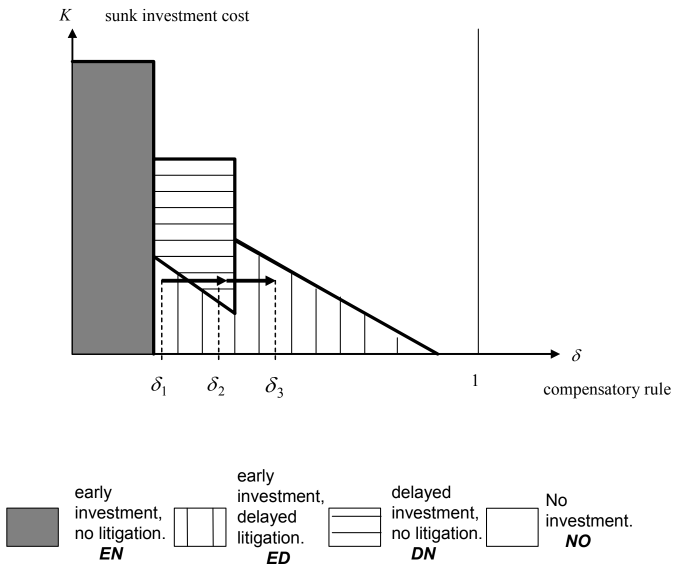


Figure 3: Effect of an increase of the compensatory rule δ in the "no laches" regime.

Proposition 3 *An increase in the compensatory rule δ can make the patentholder worse-off in both regimes.*

To see this[20], consider Figure 3 above. Again, it concerns the "no laches" regime but the same rationale applies in the laches regime. Focus now on an increase from δ_1 to δ_2. This induces a switch from an equilibrium with early investment and delayed litigation (ED) to an equilibrium with delayed investment and no litigation (DN). Clearly the patentholder is worse-off as she does not litigate anymore under DN and so she is not compensated. To understand this insight, one needs to remember the first effect derived in proposition 4: an increase in the compensatory rule δ incites the infringer to delay investment. It implies that for a given cost of litigation c, litigation becomes less attractive for the patentholder: when investment is delayed $(\delta = \delta_2)$, she can obtain compensation only from the second-period profit $(\delta_2\pi)$ whereas when

[20]Anticipating that the increase in the compensatory rule will make her worse-off, the patentholder could take actions to prevent such an outcome. For example, she could commit to a policy of cap damages. I thank Tuomas Takalo for this remark which points out that an increase in δ may not actually have such a negative outcome.

investment is not delayed ($\delta = \delta_1$) she can obtain compensation from both period 1- and period 2- profits ($\delta_1\pi + \delta_1\pi$). For low enough values of δ_2 the increase in δ does not compensate the decrease in the "pie" that the two players must share (this "pie" decreases from $\pi + \pi$ to π). And so the patentholder is worse off.[21]

Summarizing, there are two main results. First, a decrease in the infringer's gross payoff (through an increase in the patentholder's compensation) can speed-up investment. This differs from the standard implication of the real option set-up. Second, an increase in the patentholder's compensation can make her worse off.[22] I now turn to the analysis of the doctrine of laches.

5 Regime comparison

In section 4, I focused on the effect of the compensatory rule, in either regime. In this section, I compare the laches and the "no laches" regimes, for any given level of the compensatory rule. I ask: what are the qualitative effects induced by the doctrine of laches, compared to a regime where it does not apply? Proposition 1 constituted a first step into this comparative analysis. Now, the idea is to investigate, for any given level of the parameters of the model, how the implementation of a laches defense affects: the timing of investment into the infringing activity, the equilibrium outcomes of the infringement-litigation game and players' payoff. Comparing the regimes implies to consider separately five different cases, depending on the magnitude of α. To see that, consider figures N1 to N3 and figures L1 to L3: the timing of investment changes depending on cutoff values for α, which are not the same in the laches and in the "no laches" regimes. As a result, five intervals must be considered: $\alpha \in \left[\frac{1}{2}, \widehat{\alpha}\right]$, $\alpha \in \left[\widehat{\alpha}, \widehat{\widehat{\alpha}}\right]$, $\alpha \in \left[\widehat{\widehat{\alpha}}, \widetilde{\alpha}\right]$,

[21] In a companion paper on the doctrine of estoppel, I show that a higher probability of a valid patent can hurt the patentholder. However, the argument in the present paper is totally different: the patentholder's payoff is reduced when δ increases due to a change in the *timing* of investment by the infringer.

[22] Two of Choi (1998)'s results echo these findings. However the underlying economic explanations are totally different. He shows that an increase in patent validity can "accelerate" entry. This is because for some parameters values, there is no "room" for two infringers: being the second entrant is unprofitable and there is a race to be the first one. Choi also shows that an increase in patent validity can reduce the patentholder's payoff. This is because the first entry is accomodated and occurs immediately due to the "preemption" race: the patentholder's profit is reduced because entry is "accelerated". In my model, an increase in the compensation rule reduces the patentholder's profit, not because it generates earlier infringement, but because it *delays* infringement (see the interpretation of proposition 3). Hence, the explanation is the opposite of Choi's.

$\alpha \in \left[\widetilde{\alpha}, \widetilde{\widetilde{\alpha}}\right]$ and $\alpha \in \left[\widetilde{\widetilde{\alpha}}, 1\right]$. Appendix B.1 shows that the cutoff values are indeed ranked in this way. To be as accurate as possible, the comparative analysis is conducted for each interval separately. Yet, it turns out that no additional insight is obtained by considering intervals others than $\alpha \in \left[\frac{1}{2}, \widehat{\alpha}\right]$. Hence, in what follows, I concentrate on the interval $\alpha \in \left[\frac{1}{2}, \widehat{\alpha}\right]$ which exhibits all the possible effects induced by introducing a defense of laches. The four other cases are treated in Appendix B.2. In section 5.1 I analyze the effect of the doctrine on the occurence and the timing of investment in the follow-on innovation. In section 5.2, I investigate the effect of the doctrine of players' welfare.

5.1 Investment and equilibrium outcomes

Figure 4 below illustrates the effect of the doctrine of laches on the equilibrium outcomes. The figure is obtained by superposing Figure N1 and Figure L1. A dotted line represents a boundary under the doctrine of laches while a solid line represents the same boundary in the no laches regime. As shown in the figure, the doctrine of laches induces a change of the equilibrium outcome for six parameters configurations denoted I, I', J, M, O and P. It does so essentially, but not only, by modifying some boundaries between the equilibrium regions. This is why it is important to compare analytically the position of these boundaries[23].

[23]It can be seen that the doctrine of laches affects the boundaries $\overline{\overline{K}}_{N,1}$ and $\overline{\overline{K}}_{N,2}$. Appendix B.3 shows the relative position of $\overline{\overline{K}}_{N,1}$ and $\overline{\overline{K}}_{L,1}$ as well as the relative position of $\overline{\overline{K}}_{N,2}$ and $\overline{\overline{K}}_{L,2}$.

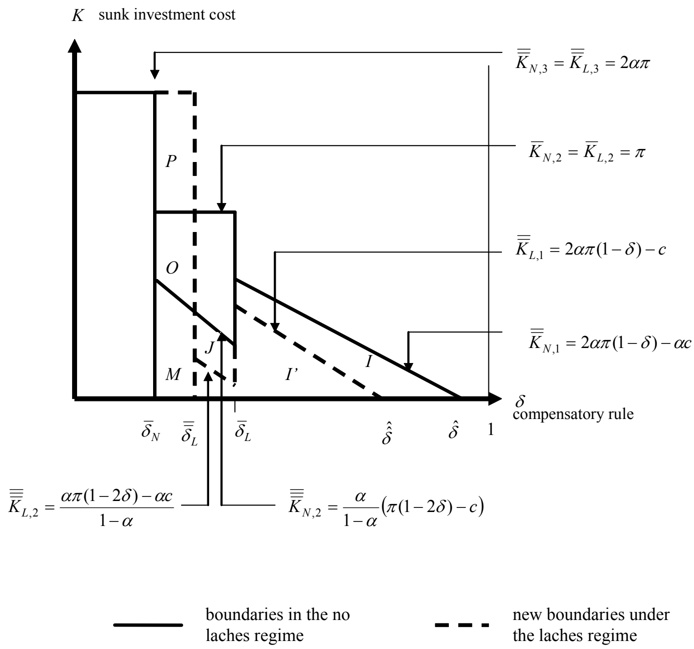


Figure 4: Effect of a switch from the "no laches" regime to the laches regime, when $\alpha \in \left[\frac{1}{2}, \widehat{\alpha}\right]$.

The main insight from this section is that the doctrine of laches can have *opposite effects* on the occurence and the timing of investment, depending on the parameters of the model[24].

Proposition 4 ***(the doctrine of laches and the investment into the infringing activity).*** *The doctrine of laches can have four different effects:*

- *it can deter investment (configuration I),*
- *it can delay investment (configuration J),*
- *it can speed-up investment (configuration O),*
- *it can spur investment (configuration P).*

[24]The proposition does not exactly say that "anything can happen". It says that *within specific parameter regions*, only one effect is *determined* to occur. However, between different parameter regions, the effects can be opposite.

I prove this proposition, and give the economic intuitions, by analyzing successively the four configurations I, J, O, P.[25]

- Configuration I : The doctrine of laches may induce a switch from an equilibrium where investment occurs in period 1 and litigation is delayed (ED), to an equilibrium where investment does not occur at all (NO). For these values of the compensatory rule ($\delta \geq \overline{\delta}_L$), the infringer does not benefit from delaying investment: the patentholder's compensation is too high for the infringer to give up period-1 profit. Hence, the infringer's trade-off is between investing in period 1 and not investing at all. The doctrine of laches has a time-inconsistency effect: by lemma 1, we know that if the doctrine of laches is available, the infringer will always invoke it as a defense argument when the patentholder litigates. Anticipating that, as shown in proposition 1, the patentholder may litigate earlier, that is: before uncertainty is fully resolved. As a result, the infringer would face litigation costs with probability 1 if he were to invest in period 1. On the contrary, in the absence of the doctrine, the infringer would face litigation only with probability α, as the patentholder would delay litigation until uncertainty is resolved and litigate only when demand is high (which occurs with probability α). The prospect of being involved in patent litigation at an early stage can discourage the infringer to invest, although he would have invested in the absence of the doctrine. This effect is illustrated in Figure 4 : for $\delta \geq \overline{\delta}_L$, the boundary $\overline{\overline{K}}_{N,1}$ that separates investment and no investment in a "no laches" regime switches to $\overline{\overline{K}}_{L,1}$ under the doctrine of laches. Because $\overline{\overline{K}}_{L,1} \leq \overline{\overline{K}}_{N,1}$, it follows that for all $K \in [\overline{\overline{K}}_{L,1}, \overline{\overline{K}}_{N,1}]$, investment does not occur anymore. The first effect of the doctrine of laches is identified: it may deter investment in the follow-on innovation.

- Configuration J : The doctrine may induce a switch from an equilibrium where investment occurs in period 1 and litigation is delayed (ED) to an equilibrium where investment is delayed and litigation does not occur (DN). Notice first that for $\delta \in \left[\overline{\overline{\delta}}_L, \overline{\delta}_L\right]$, delaying investment *can be* profitable. *If the doctrine of laches applies* and the infringer invests in period 1, he faces litigation in period 1 (see Figure 2). *If the doctrine of laches does not apply* and the infringer invests in period 1, he faces litigation in period 2 only if the

[25] Configurations I' and M are analyzed in Appendix B.4 as they do not yield any new insights.

investment turns out to be profitable, i.e with probability α (see Figure 2 as well). Hence, the "real" cost of investing in period 1 is higher under the doctrine of laches $(K + c)$ than in the "no laches" regime $(K + \alpha c)$. Consequently, everything else equal, this higher cost implies that delaying investment becomes more attractive, for some values of K. Configuration J illustrates this effect. This is the second effect of the doctrine: it may delay investment.

- Configuration O : The doctrine may induce a switch from an equilibrium where investment is delayed and litigation is deterred (DN) to an equilibrium where investment occurs in period 1 and litigation is still deterred (EN). As for configuration M, the doctrine of laches deters litigation for these parameters values, if the infringer invests in period 1. But if he invests in period 1 in the absence of the doctrine, he faces litigation in period 2. The prospect of not being litigated under the doctrine of laches increases the expected reward from investment, for all values of α and K (due to the absence of litigation costs and damages). This incites the infringer to invest in period 1 instead of delaying. In the absence of the doctrine, if he were to invest in period 1, he would face litigation in period 2 while if he were to delay, he would not face litigation. This latter effect dominates in the absence of the doctrine, and the infringer has an incentive to delay investment. This is the third effect of the doctrine: it may "speed-up" investment.

- Configuration P : The doctrine may induce a switch from an equilibrium where investment is deterred (NO) to an equilibrium where it occurs in period 1 and litigation is deterred (EN). As for configurations M and O, the doctrine of laches deters litigation. Here, and for the same reasons advanced to explain the qualitative changes for configuration O, the prospect of not being litigated encourage the infringer to invest (and to invest early). On the contrary, in the "no laches" regime, anticipating litigation, the infringer was deterred from investing for these high values of K. This is the fourth effect of the doctrine: it spurs investment.

Having analyzed how the doctrine of laches affects the incentives to invest in the follow-on innovation, I turn to investigating how it affects players' payoff.

5.2 Players' payoff

There are two results. First, the doctrine of laches can make both players worse off (proposition 5). This is a straightfoward implication of the effect isolated in proposition 4 for configuration I: the fact that the doctrine deters investment. Second, the doctrine can leave the players indifferent or make the patentholder worse off and the infringer better off.

Proposition 5 *When the probability of success is high ($\alpha \geq \frac{1}{2}$) and patent protection is strong ($\delta \geq \overline{\delta}_L$) or intermediate ($\delta \in \left[\overline{\overline{\delta}}_L, \overline{\delta}_L\right]$), a regime where the doctrine of laches applies can make both the patentholder and the infringer worse off compared to a regime where it does not apply.*

Proof.

- Consider first strong patent protection ($\delta \geq \overline{\delta}_L$). From proposition 4, we know that for configuration I investment is deterred in a laches regime, while it would occur in a "no laches" regime, implying that both players are worse-off with the doctrine of laches. Consider then configuration I'. We can compute players' payoffs in both regimes. First in the "no laches" regime N (litigation is delayed):

$$\begin{cases} U_H^N = -\alpha c + 2\alpha\pi(1-\delta) \\ U_I^N = -K - \alpha c + 2\alpha\pi(1-\delta). \end{cases}$$

Then, in the laches regime L (litigation is not delayed):

$$\begin{cases} U_H^L = -c + 2\alpha\pi(1-\delta) \\ U_I^L = -K - c + 2\alpha\pi(1-\delta). \end{cases}$$

It follows that $U_H^N \geq U_H^L$ and $U_I^N \geq U_I^L$.

- Consider then intermediate patent protection $\left(\delta \in \left[\overline{\overline{\delta}}_L, \overline{\delta}_L\right]\right)$. Here the two relevant configurations are J and I'. I compute players' payoffs under either regime, for each configuration. Consider configuration J. In the "no laches" regime N:

$$\begin{cases} U_H^N = -\alpha c + 2\alpha\pi(1-\delta) \\ U_I^N = -K - \alpha c + 2\alpha\pi(1-\delta) \end{cases}$$

and in the laches regime L :

$$\begin{cases} U_H^N = 0 \\ U_I^L = \alpha(\pi - K). \end{cases}$$

Clearly, the patentholder is worse-off in regime L. The infringer is worse-off if and only if:

$$K \leq \frac{\alpha\pi(1-2\delta) - \alpha c}{1-\alpha} = \overline{\overline{\overline{K}}}_{N,2},$$

which holds for configuration J. Then, consider configuration I'. It has been proved above that for this configuration, players are better-off in a no laches regime. ■

This proposition states a counterintuitive result: the defense available to the defendant (infringer) can make him worse off. To understand this point, consider simply the explanation for the equilibrium switch characterizing configuration I : the doctrine of laches deters investment compared to a regime where it does not apply. This implies that no profit can be generated, which leaves both the patentholder and the infringer worse off. It can be established however (see Appendix C) that when the compensatory rule is low ($\delta < \overline{\overline{\delta}}_L$), a regime where the doctrine of laches applies can leave the patentholder indifferent or make her worse off compared to a regime where it does not apply and it can leave the infringer indifferent or make him better off. This is more conform to the explicit objective of the doctrine of laches than the result derived in proposition 5: As a "defense" argument, the doctrine is supposed to benefit the infringer (the "defendant" in the trial) and sanction the patentholder (if she adopts the prohibited behavior). My contribution is to show that, if this is the desired effect of the doctrine, it can be achieved only by taking into account the patentholder's compensation through designing a "low enough" compensatory rule ($\delta < \overline{\overline{\delta}}_L$).

6 Conclusion

In this paper, I analyzed the joint effects of the doctrine of laches and compensatory damages on the incentives to infringe a patent and to enforce this patent. "Infringement" is equivalent to an investment in a follow-on innovation which requires the patented technology. Both the infringer and the patentholder have a "real option" problem. The profitability of the infringing product is initially uncertain. The infringer is the leader and can invest before or after uncertainty is resolved. The patentholder is the follower and, if the infringer invested before uncertainty was resolved, she herself can litigate before or after profitability becomes known. Litigation is costly for both players. Delayed litigation can be punished by the doctrine of laches which prevents the patentholder from getting damages for infringement that occured during the delay period.

Interestingly, my research has implications in the heated debate over patent trolls. Patent trolls are patentholders who do not themselves manufacture any good and rely only on a licensing business model: they assert patents- often bought from third party assignees, and considered of dubious validity- against manufacturers in order to obtain royalties and damages. Some have argued that the doctrine of laches could be a useful instrument against trolls (Barker, 2005). My analysis suggests than this can be the case (proposition 1 establishes that the doctrine can deter litigation), but the doctrine can also have an adverse effect. It can make the infringer worse off, when it forces the patentholder to litigate before commercial uncertainty is resolved, thereby increasing the cost of investment by an amount equal to the the litigation cost, and eventually detering investment. My model suggests that the doctrine of laches is more likely to have a positive effect against trolls (i.e. deter litigation) when the compensatory rule is low.

I also show that the doctrine has different effects on the timing of investment in the follow-on innovation depending on parameters values: it can deter or spur investment. It can speed-up or delay investment (proposition 4). I establish that an increase of the patentholder's compensation can delay or speed-up investment (proposition 2). It can make the patentholder worse-off (proposition 3).

My main message is that the doctrine of laches should be taken into account as a meaningful instrument of patent policy. But its design should be carefully considered in relation to other patent policy instruments such as the patentholder monetary compensation.

Appendix

Appendix A: Investment timing when $\alpha \geq \frac{1}{2}$.

In the main text, many analytical steps have been omitted in order to simplify the progression towards the economic results gathered in sections 4 and 5. In Appendix A, I report these omited steps.

Appendix A.1: The infringer's decision in the "no laches regime".

In Appendix A.1, I report the omitted analytical steps for scenarios 1, 2 and 3.

■ *Scenario 1:* $\delta \in [\overline{\delta}_L, 1]$.

The analysis of this scenario is conducted in detail in section 3.2.1. Here, I analyze the respective positions of the three boundaries $\overline{K}_{N,1}(\delta)$, $\overline{\overline{K}}_{N,1}(\delta)$ and $\overline{\overline{\overline{K}}}_{N,1}(\delta)$ defined by (8), (11) and (12).

It can be shown that $\overline{\overline{K}}_{N,1}(\delta) \geq \overline{K}_{N,1}(\delta)$. Indeed, this inequality holds if and only if $(2\alpha - 1)\pi(1-\delta) + (1-\alpha)c \geq 0$ which holds for all $\alpha \geq \frac{1}{2}$ and $\delta \in [\overline{\delta}_L, 1]$. Also, it can be shown that $\overline{\overline{\overline{K}}}_{N,1}(\delta) \geq \overline{K}_{N,1}(\delta)$. This inequality holds if and only if $\frac{2\alpha-1}{1-\alpha}\pi(1-\delta) + c \geq 0$ which holds for all $\alpha \geq \frac{1}{2}$ and $\delta \in [\overline{\delta}_L, 1]$. Hence:

$$\begin{cases} \overline{\overline{K}}_{N,1}(\delta) \geq \overline{K}_{N,1}(\delta) \\ \overline{\overline{\overline{K}}}_{N,1}(\delta) \geq \overline{K}_{N,1}(\delta) \end{cases} \tag{22}$$

It follows that for all $K \geq \overline{K}_{N,1}(\delta)$ the infringer invests in period 1 if $K \leq \overline{\overline{K}}_{N,1}(\delta)$ but does not invest otherwise. And for all $K \leq \overline{K}_{N,1}(\delta)$ he invests in period 1. So, for all $K \leq \overline{\overline{K}}_{N,1}(\delta)$, the infringer invests in period 1. This is stated in lemma 5.

Analysis of $\overline{\overline{K}}_{N,1}(\delta) = 2\alpha\pi(1-\delta) - \alpha c$. This function is obviously downward sloping with $\overline{\overline{K}}_{N,1}(0) = 2\alpha\pi - \alpha c$ and $\overline{\overline{K}}_{N,1}(\delta) = 0 \Longleftrightarrow \delta = 1 - \frac{c}{2\pi} = \widehat{\delta} > \frac{c}{\pi} = \overline{\delta}_L$ since $\pi \geq 3c$. In addition, $\overline{\overline{K}}_{N,1}(\frac{c}{\pi}) = 2\alpha\pi - 3\alpha c$ and $\overline{\overline{K}}_{N,1}(\frac{c}{2\pi}) = 2\alpha\pi - 2\alpha c$.

■ *Scenario 2:* $\delta \in [\overline{\delta}_N, \overline{\delta}_L]$.

Suppose the infringer delayed investment. His payoff if he invests is $U^N_{I,2}(i) = \pi - K$. Indeed, the patentholder does not litigate. Hence:

$$U^N_{I,2}(i) = \pi - K \begin{cases} < 0 & \text{if} \quad K > \overline{K}_{N,2} \triangleq \pi \\ \geq 0 & \text{if} \quad K \leq \overline{K}_{N,2} \triangleq \pi. \end{cases} \tag{23}$$

In period 1, the infringer's expected payoff if he does not invest is:

$$U^N_{I,1}(n) = \begin{cases} 0 & \text{if} \quad K > \overline{K}_{N,2} \triangleq \pi \\ \alpha(\pi - K) & \text{if} \quad K \leq \overline{K}_{N,2} \triangleq \pi. \end{cases} \tag{24}$$

By contrast, his payoff if he invests in period 1 is:

$$U^N_{I,1}(i) = -K + 2\pi\alpha(1-\delta) - \alpha c, \tag{25}$$

since the patentholder will litigate in period 2 and obtain a share of both period 1 and period 2 profits (the doctrine of laches does not apply).

Again, I can distinguish between two cases.

$\diamondsuit$ If $K > \overline{K}_{N,2}$, delaying investment is never profitable. Is it profitable to invest today? The condition for profitability is $U_{I,1}^{N}(i) \geq 0$ which is equivalent to:

$$K \leq 2\alpha\pi(1-\delta) - \alpha c \triangleq \overline{\overline{K}}_{N,2}. \tag{26}$$

$\diamondsuit$ If $K \leq \overline{K}_{N,2}$, delaying investment yields a non-negative profit. The infringer invests today if and only if $U_{I,1}^{N}(i) \geq U_{I,1}^{N}(n)$ or:

$$K \leq \frac{\alpha}{1-\alpha}\left[\pi(1-2\delta) - c\right] \triangleq \overline{\overline{\overline{K}}}_{N,2}. \tag{27}$$

As for scenario 1, it remains to analyze in the (K,δ) space the respective position of $\overline{K}_{N,2}$, $\overline{\overline{K}}_{N,2}$ and $\overline{\overline{\overline{K}}}_{N,2}$ defined by (23), (26) and (27). The difficulty in that case is that these positions depend on the value of α. As a result, I must distinguish again between cases depending on the value of α on the interval $\alpha \in [\frac{1}{2}, 1]$ (remember that I assumed $\alpha \geq \frac{1}{2}$).

Notice first that $\overline{\overline{K}}_{N,2}(\delta) = \overline{\overline{K}}_{N,1}(\delta)$. Hence, from the above analysis I know that $\overline{\overline{K}}_{N,2}(\delta)$ is downward sloping with $\overline{\overline{K}}_{N,2}(\delta) = 0$ at $\delta = \widehat{\delta}$. I can compute $\overline{\overline{K}}_{N,2}(\overline{\delta}_N) = 2\alpha(\pi - c)$. Then, $\overline{K}_{N,2}(\delta) = \pi$ is a constant. Finally, $\overline{\overline{\overline{K}}}_{N,2}(\delta)$ is linear and decreasing in δ and $\overline{\overline{\overline{K}}}_{N,2}(\overline{\delta}_N) = \frac{\alpha\pi - 2\alpha c}{1-\alpha}$. Notice that the line representing $\overline{\overline{\overline{K}}}_{N,2}(\delta)$ is steeper than that of $\overline{\overline{K}}_{N,2}(\overline{\delta}_N)$. Indeed, $\frac{2\alpha\pi}{1-\alpha} \geq 2\alpha\pi$ always holds. Finally, $\overline{\overline{\overline{K}}}_{N,2}(\frac{c}{\pi}) = \frac{\alpha}{1-\alpha}(\pi - 3c) \geq 0$ since $\pi \geq 3c$.

In order to analyze the respective positions of $\overline{K}_{N,2}(\delta)$, $\overline{\overline{K}}_{N,2}(\delta)$ and $\overline{\overline{\overline{K}}}_{N,2}(\delta)$, I define the following values: $\widehat{\alpha} = \frac{\pi}{2(\pi - c)}$ and $\widehat{\widehat{\alpha}} = \frac{\pi}{2\pi - 3c}$. First, notice that $\widehat{\alpha} \geq \frac{1}{2}$ if and only if $c \geq 0$ which holds. Also, notice that $\widehat{\widehat{\alpha}} \geq \widehat{\alpha}$ if and only if $2\pi(\pi - c) \geq \pi(2\pi - 3c)$ which holds. In addition, $\widehat{\widehat{\alpha}} \leq 1$ if and only if $\pi \geq 3c$ which holds by assumption.

$\diamondsuit$*Consider first the interval* $\alpha \in [\frac{1}{2}, \widehat{\alpha}]$.

For these values of α, we have $\overline{\overline{K}}_{N,2}(\overline{\delta}_N) \leq \overline{K}_{N,2}(\overline{\delta}_N)$ if and only if $\alpha \leq \frac{\pi}{2(\pi - c)} = \widehat{\alpha}$ which holds by assumption. And since $\overline{K}_{N,2}(.)$ is constant while $\overline{\overline{K}}_{N,2}(.)$ is downward sloping, it follows that $\overline{K}_{N,2}(\delta) \geq \overline{\overline{K}}_{N,2}(\delta)$.

Also, I have $\overline{\overline{\overline{K}}}_{N,2}(\overline{\delta}_N) \leq \overline{\overline{K}}_{N,2}(\overline{\delta}_N)$ if and only if $\alpha \leq \widehat{\alpha}$ which holds by assumption. Since $\overline{\overline{\overline{K}}}_{N,2}(\overline{\delta}_N)$ is steeper than $\overline{\overline{K}}_{N,2}(\overline{\delta}_N)$ it follows that $\overline{\overline{K}}_{N,2}(\delta) \geq \overline{\overline{\overline{K}}}_{N,2}(\delta)$. Hence,

$$\overline{K}_{N,2}(\delta) \geq \overline{\overline{K}}_{N,2}(\delta) \geq \overline{\overline{\overline{K}}}_{N,2}(\delta). \tag{28}$$

Hence, for all $K > \overline{K}_{N,2}(\delta)$ the infringer does not invest. For $K \leq \overline{K}_{N,2}(\delta)$ he invests in period 1 if $K \leq \overline{\overline{\overline{K}}}_{N,2}(\delta)$ and he delays investment if $K > \overline{\overline{\overline{K}}}_{N,2}(\delta)$. This is stated in lemma 5.

$\diamondsuit$ *Consider then the interval* $\alpha \in (\widehat{\alpha}, \widehat{\widehat{\alpha}}]$.

I know from the preceding analysis that $\overline{K}_{N,2}(\delta) < \overline{\overline{K}}_{N,2}(\delta)$ and $\overline{\overline{K}}_{N,2}(\delta) < \overline{\overline{\overline{K}}}_{N,2}(\delta)$. I investigate the condition for the lines representing $\overline{\overline{K}}_{N,2}(\delta)$ and $\overline{\overline{\overline{K}}}_{N,2}(\delta)$ to intersect $\overline{K}_{N,2}(\delta) = \pi$ at a point $\delta \in [\overline{\delta}_N, \overline{\delta}_L]$. To that end, I solve $\overline{\overline{K}}_{N,2}(\delta) = \pi$ for δ. This yields $\delta = \frac{2\alpha\pi - \pi - \alpha c}{2\alpha\pi} = \overset{\bullet}{\delta}$. Then I solve $\overline{\overline{\overline{K}}}_{N,2}(\delta) = \pi$ for δ. This yields $\delta = \overset{\bullet}{\delta}$ as well. The conditions for $\overset{\bullet}{\delta} \in [\overline{\delta}_N, \overline{\delta}_L]$ are $\overset{\bullet}{\delta} \geq \overline{\delta}_N$ and $\overset{\bullet}{\delta} \leq \overline{\delta}_L$. The first condition amounts at showing that $\alpha \geq \frac{\pi}{2(\pi - c)} = \widehat{\alpha}$, which holds. The second condition implies that $\alpha \leq \frac{\pi}{2\pi - 3c} = \widehat{\widehat{\alpha}}$, which holds. These two conditions are clearly satisfied on the interval $(\widehat{\alpha}, \widehat{\widehat{\alpha}}]$.

Hence we have:

$$\begin{cases} \overline{\overline{\overline{K}}}_{N,2}(\delta) \geq \overline{\overline{K}}_{N,2}(\delta) \geq \overline{K}_{N,2}(\delta) & \text{if } \delta \in [\overline{\delta}_N, \overset{\bullet}{\delta}] \\ \overline{\overline{\overline{K}}}_{N,2}(\delta) \leq \overline{\overline{K}}_{N,2}(\delta) \leq \overline{K}_{N,2}(\delta) & \text{if } \delta \in [\overset{\bullet}{\delta}, \overline{\delta}_L] \end{cases} \tag{29}$$

Define the kinked curved $\overset{\bullet}{K}(\delta)$ as:

$$\overset{\bullet}{K}(\delta) = \begin{cases} \overline{\overline{K}}_{N,2}(\delta) & \text{if } \delta \in [\overline{\delta}_N, \overset{\bullet}{\delta}] \\ \overline{\overline{\overline{K}}}_{N,2}(\delta) & \text{if } \delta \in [\overset{\bullet}{\delta}, \overline{\delta}_L] \end{cases} \tag{30}$$

It follows that for all $K < \overset{\bullet}{K}(\delta)$ the infringer invests in period 1. For $K \geq \overset{\bullet}{K}(\delta)$ but $K \leq \overline{K}_{N,2}(\delta)$, he delays investment until period 2. Finally, for all $K \geq \overset{\bullet}{K}(\delta)$ and $K > \overline{K}_{N,2}(\delta)$, he does not invest. This is stated in lemma 5.

$\diamondsuit$ *Finally, consider* $\alpha \in [\widehat{\widehat{\alpha}}, 1]$.

From the preceding analysis, we know that $\overset{\bullet}{\delta} \geq \overline{\delta}_L$ for every α in this interval. Hence, for all $\delta \in [\overline{\delta}_N, \overline{\delta}_L]$:

$$\overline{\overline{\overline{K}}}_{N,2}(\delta) \geq \overline{\overline{K}}_{N,2}(\delta) \geq \overline{K}_{N,2}(\delta). \tag{31}$$

Consequently, for all $K \geq \overline{K}_{N,2}(\delta)$ and $K \leq \overline{\overline{K}}_{N,2}(\delta)$, the infringer invests in period 1. For all $K > \overline{\overline{K}}_{N,2}(\delta)$ he does not invest. And for all $K \leq \overline{K}_{N,2}(\delta)$, he invests in period 1. It follows that for all $K \leq \overline{\overline{K}}_{N,2}(\delta) = \overline{\overline{K}}_{N,1}(\delta)$ the infringer invests in period 1 and does not invest for larger values of K. This is stated in lemma 5.

■ *Scenario 3:* $\delta \in [0, \overline{\delta}_N]$.

Suppose the infringer delayed investment. In period 2, his net payoff from investing is $U_{I,2}^N(i) = \pi - K$ since the patentholder does not litigate. Hence:

$$U_{I,2}^N(i) = \pi - K \begin{cases} < 0 & \text{if } K > \overline{K}_{N,3} \triangleq \pi \\ \geq 0 & \text{if } K \leq \overline{K}_{N,3} \triangleq \pi. \end{cases} \tag{32}$$

In period 1, the infringer's expected payoff if he does not invest is:

$$U_{I,1}^F(n) = \begin{cases} 0 & \text{if } K > \overline{K}_{N,3} \\ \alpha(\pi - K) & \text{if } K \leq \overline{K}_{N,3}. \end{cases} \tag{33}$$

If the infringer invests in period 1, he obtains:

$$U_{I,1}^N(i) = -K + \alpha 2\pi, \tag{34}$$

as the patentholder will never litigate.

◇ If $K \geq \overline{K}_{N,3}$, delaying investment is never profitable. Investing today is profitable if and only if $U_{I,1}^N(i) \geq 0$ or:

$$K \leq 2\alpha\pi \triangleq \overline{\overline{K}}_{N,3}. \tag{35}$$

◇ If $K < \overline{K}_{N,3}$, delaying yields a non-negative profit. The infringer invests today if and only if $U_{I,1}^N(i) \geq U_{I,1}^N(n)$ or:

$$K \leq \frac{\alpha}{1-\alpha}\pi \triangleq \overline{\overline{\overline{K}}}_{N,3}. \tag{36}$$

Again, I have to analyze in the (K, δ) space the respective position of $\overline{K}_{N,3}(\delta)$, $\overline{\overline{K}}_{N,3}(\delta)$ and $\overline{\overline{\overline{K}}}_{N,3}(\delta)$, respectively defined by (32), (35) and (36).

Obviously, $2\alpha\pi \geq \pi$ if and only if $\alpha \geq \frac{1}{2}$ which holds, so that $\overline{\overline{K}}_{N,3}(\delta) \geq \overline{K}_{N,3}(\delta)$. And $\frac{\alpha}{1-\alpha}\pi \geq \pi$ if and only if $\alpha \geq \frac{1}{2}$ which holds as well, so that $\overline{\overline{\overline{K}}}_{N,3}(\delta) \geq \overline{K}_{N,3}(\delta)$. Hence,

$$\begin{cases} \overline{\overline{K}}_{N,3}(\delta) \geq \overline{K}_{N,3}(\delta) \\ \overline{\overline{\overline{K}}}_{N,3}(\delta) \geq \overline{K}_{N,3}(\delta). \end{cases} \tag{37}$$

It follows that for all $K \geq \overline{K}_{N,3}(\delta)$ the infringer invests in period 1 provided that $K \leq \overline{\overline{K}}_{N,3}(\delta)$, otherwise he does not invest. And for all $K \leq \overline{K}_{N,3}(\delta)$ the infringer invests in period 1. So, for all $K \leq \overline{\overline{K}}_{N,3}(\delta)$ the infringer invests in period 1. This is stated in lemma 6.

Analysis of $\overline{\overline{K}}_{N,3}(\delta) = 2\alpha\pi$. Notice only that $\overline{\overline{K}}_{N,3}(\delta) = 2\alpha\pi$ is a constant.

The following lemma combines the above analysis with the analysis of litigation conducted in section 3. It gives the exact values of the parameters for which a specific equilibrium outcome occurs. Figures **N1**, **N2** and **N3** capture these features in a livelier manner.

Lemma 9 ***(Equilibrium outcomes when the doctrine of laches does not apply)***. *When the probability of commercial success is high ($\alpha \geq \frac{1}{2}$),*

- *If patent protection is strong ($\delta \in [\overline{\delta}_L, 1]$), the infringer invests in period 1 if the sunk cost is low enough ($K \leq \overline{\overline{K}}_{N,1}$) and does not invest otherwise. If he invests, the patentholder delays litigation.*

- *If patent protection is weak ($\delta \in [0, \overline{\delta}_N)$), the infringer invests in period 1 if the sunk cost is low enough ($K \leq \overline{\overline{K}}_{N,3}$) and does not invest otherwise. If he invests, the patentholder does not litigate.*

- *If patent protection is intermediate ($\delta \in [\overline{\delta}_N, \overline{\delta}_L]$), the timing of investment depends on the probability of commercial success:*

  - *When success is moderately likely ($\alpha \in \left[\frac{1}{2}, \widehat{\alpha}\right]$), the infringer invests in period 1 if $K \leq \overline{\overline{\overline{K}}}_{N,2}$ and the patentholder delays litigation. The infringer delays investment until period 2 if $K \in [\overline{\overline{\overline{K}}}_{N,2}, \overline{K}_{N,2}]$ and the patentholder does not litigate. And the infringer does not invest if $K \geq \overline{K}_{N,2}$.*
  - *When success is likely ($\alpha \in (\widehat{\alpha}, \widehat{\widehat{\alpha}}]$), the infringer invests in period 1 if $K \leq \dot{K}$ and the patentholder delays litigation. The infringer delays investment if $K \in [\dot{K}, \overline{K}_{N,2}]$ and the patentholder does not litigate. And the infringer does not invest if $K \geq \dot{K}$ and $K \geq \overline{K}_{N,2}$.*
  - *When success is very likely ($\alpha \in (\widehat{\widehat{\alpha}}, 1]$), the infringer invests in period 1 if $K \leq \overline{\overline{K}}_{N,1}$ and the patentholder delays litigation. Otherwise, the infringer does not invests.*

Appendix A.2. The infringer's decision in the "laches regime"

The methodology here is similar to that in Appendix A.1.

■ *Scenario 1:* $\delta \in [\overline{\delta}_L, 0]$.

I want to analyze the respective of the three functions $\overline{K}_{L,1}(\delta)$, $\overline{\overline{K}}_{L,1}(\delta)$ and $\overline{\overline{\overline{K}}}_{L,1}(\delta)$, respectively defined by (16), (19) and (20). I can show that $\overline{\overline{K}}_{L,1}(\delta) \geq \overline{K}_{L,1}(\delta)$. Indeed, this inequality holds if and only if $\pi(1-\delta)(2\alpha-1) \geq 0$ which holds for all $\alpha \geq \frac{1}{2}$ and $\delta \in [\overline{\delta}_L, 1]$. Also, we can show that $\overline{\overline{\overline{K}}}_{L,1}(\delta) \geq \overline{K}_{L,1}(\delta)$. This inequality holds if and only if $\frac{\alpha\pi(1-\delta)+\alpha c-c}{1-\alpha} \geq \pi(1-\delta)-c$ which holds for all $\alpha \geq \frac{1}{2}$. Hence:

$$\begin{cases} \overline{\overline{K}}_{L,1}(\delta) \geq \overline{K}_{L,1}(\delta) \\ \overline{\overline{\overline{K}}}_{L,1}(\delta) \geq \overline{K}_{L,1}(\delta) \end{cases} \tag{38}$$

From that, we can conclude that if $K \geq \overline{K}_{L,1}(\delta)$, the infringer invests in period 1 if and only if $K \leq \overline{\overline{K}}_{L,1}(\delta)$ and if $K \leq \overline{K}_{L,1}(\delta)$, the infringer always invest in period 1. So, for all $K \leq \overline{\overline{K}}_{L,1}(\delta)$, the infringer invests in period 1. Otherwise he does not invest. This is stated in lemma 7.

Analysis of $\overline{\overline{K}}_{L,1}(\delta)$. The function $\overline{\overline{K}}_{L,1}(\delta) = 2\alpha\pi(1-\delta) - c$ is linear and decreasing in δ. In addition, $\overline{\overline{K}}_{L,1}(\overline{\delta}_L) = 2\alpha\pi - c(2\alpha+1)$ and $\overline{\overline{K}}_{L,1}(\delta) = 0$ for $\delta = 1 - \frac{c}{2\alpha\pi} = \widehat{\widehat{\delta}}$.

■ *Scenario 2:* $\delta \in [\overline{\overline{\delta}}_L, \overline{\delta}_L]$.

If the infringer did not invest in period 1, his net payoff from investing in period 2 is $U^L_{I,2}(i) = -K + \pi$. Indeed, he faces no litigation in period 2. It follows that:

$$U^L_{I,2}(i) = -K + \pi \begin{cases} < 0 \ \text{ if } \ K > \overline{K}_{L,2} \triangleq \pi \\ \geq 0 \ \text{ if } K \leq \overline{K}_{L,2} \triangleq \pi. \end{cases} \tag{39}$$

In period 1, the infringer's payoff if he does not invest is:

$$U^L_{I,1}(n) = \begin{cases} 0 \ \text{ if } \ K > \overline{K}_{L,2} \triangleq \pi \\ \alpha(\pi - K) \ \text{ if } \ K \leq \overline{K}_{L,2} \triangleq \pi. \end{cases} \tag{40}$$

His payoff if he invests in period 1 is:

$$U^L_{I,1}(i) = -K - c + 2\alpha\pi(1-\delta), \tag{41}$$

since the patentholder litigates in period 1 (before uncertainty is resolved).

$\diamondsuit$Suppose $K \geq \overline{K}_{N,2}$. Then $U^L_{I,1}(i) \geq 0$ if and only if:

$$K \leq 2\alpha\pi(1-\delta) - c \triangleq \overline{\overline{K}}_{L,2}. \tag{42}$$

$\diamondsuit$Suppose $K < \overline{K}_{N,2}$. Then $U^L_{I,1}(i) \geq U^L_{I,1}(n)$ if and only if:

$$K \leq \frac{\alpha\pi(1-2\delta)-c}{1-\alpha} \triangleq \overline{\overline{\overline{K}}}_{L,2}. \tag{43}$$

$\square$ On this interval, I analyze the functions $\overline{K}_{L,2}(\delta)$, $\overline{\overline{K}}_{L,2}(\delta)$ and $\overline{\overline{\overline{K}}}_{L,2}(\delta)$ respectively defined by (39), (42) and (43). Notice first that $\overline{\overline{K}}_{L,2}(\delta) = \overline{\overline{K}}_{L,1}(\delta)$. Hence, from the above analysis, I know that $\overline{\overline{K}}_{L,2}(\delta)$ is downward sloping with $\overline{\overline{K}}_{L,2}(\delta) = 0$ at $\delta = \hat{\hat{\delta}}$. I can compute $\overline{\overline{K}}_{L,2}(\overline{\overline{\delta}}_L) = 2\alpha\pi - 2c$. Then, $\overline{K}_{L,2}(\delta) = \pi$ is a constant. Finally, $\overline{\overline{\overline{K}}}_{L,2}(\delta)$ is linear and decreasing in δ and $\overline{\overline{\overline{K}}}_{L,2}(\overline{\overline{\delta}}_L) = \frac{\alpha\pi-2c}{1-\alpha}$. Notice that the line representing $\overline{\overline{\overline{K}}}_{L,2}(\delta)$ is steeper than that of $\overline{\overline{K}}_{L,2}(\delta)$. Indeed, $\frac{2\alpha\pi}{1-\alpha} \geq 2\alpha\pi$ always holds.

$\square$ In order to analyze the respective positions of $\overline{K}_{L,2}(\delta)$, $\overline{\overline{K}}_{L,2}(\delta)$ and $\overline{\overline{\overline{K}}}_{L,2}(\delta)$, I define the following values: $\widetilde{\alpha} = \frac{\pi+2c}{2\pi}$ and $\widetilde{\widetilde{\alpha}} = \frac{\pi+c}{2(\pi-c)}$. Notice first that $\widetilde{\alpha} \geq \frac{1}{2}$ if and only if $2c \geq 0$ which holds. Also, notice that $\widetilde{\widetilde{\alpha}} \geq \widetilde{\alpha}$ if and only if $(\pi+2c)(\pi-c) \leq (\pi+c)\pi$ or $-2c^2 \leq 0$ which holds. In addition, $\widetilde{\widetilde{\alpha}} \leq 1$ if and only if $\pi - 3c \geq 0$ which holds by assumption.

$\diamondsuit$ *Consider first the interval* $\alpha \in \left[\frac{1}{2}, \widetilde{\alpha}\right]$.

For these values of α, $\overline{\overline{K}}_{L,2}(\delta) \leq \overline{K}_{L,2}(\delta) = \pi$. To establish this result, it is sufficient to show that $\overline{\overline{K}}_{L,2}(\overline{\overline{\delta}}_L) \leq \overline{K}_{L,2}(\overline{\overline{\delta}}_L) = \pi$ because $\overline{\overline{K}}_{L,2}(\delta)$ is decreasing in δ while $\overline{K}_{L,2}(\delta) = \pi$ is constant. And $\overline{\overline{K}}_{L,2}(\overline{\overline{\delta}}_L) \leq \overline{K}_{L,2}(\overline{\overline{\delta}}_L)$ if and only if $2\alpha\pi - 2c \leq \pi$ or $\alpha \leq \frac{\pi+2c}{2\pi} = \widetilde{\alpha}$ which holds. Then, I can show that $\overline{\overline{\overline{K}}}_{L,2}(\delta) \leq \overline{\overline{K}}_{L,2}(\delta)$. Again, to establish this result, it is sufficient to show that $\overline{\overline{\overline{K}}}_{L,2}(\overline{\overline{\delta}}_L) \leq \overline{\overline{K}}_{L,2}(\overline{\overline{\delta}}_L)$ since $\overline{\overline{\overline{K}}}_{L,2}(\delta)$ is steeper than $\overline{\overline{K}}_{L,2}(\delta)$. But this inequality amounts at $\frac{\alpha\pi-2c}{1-\alpha} \leq 2\alpha\pi - 2c$ which holds for all $\alpha \leq \widetilde{\alpha}$. Hence, I have:

$$\overline{\overline{\overline{K}}}_{L,2}(\delta) \leq \overline{\overline{K}}_{L,2}(\delta) \leq \overline{K}_{L,2}(\delta). \tag{44}$$

It follows that for all $K > \overline{K}_{L,2}(\delta)$, the infringer does not invest. For $K \leq \overline{K}_{L,2}(\delta)$, he invests in period 1 provided $K \leq \overline{\overline{\overline{K}}}_{L,2}(\delta)$ and he delays if $K \in (\overline{\overline{\overline{K}}}_{L,2}(\delta), \overline{K}_{L,2}(\delta)]$. This is stated in lemma 8.

$\diamondsuit$ *Consider then* $\alpha \in \left[\widetilde{\alpha}, \widetilde{\widetilde{\alpha}}\right]$.

I know from the preceding analysis that $\overline{\overline{K}}_{L,2}(\overline{\overline{\delta}}_L) \geq \overline{K}_{L,2}(\overline{\delta}_L) = \pi$ and in addition $\overline{\overline{\overline{K}}}_{L,2}(\overline{\overline{\delta}}_L) \geq \overline{\overline{K}}_{L,2}(\overline{\overline{\delta}}_L)$. I investigate the condition for both $\overline{\overline{K}}_{L,2}(\delta)$ and $\overline{\overline{\overline{K}}}_{L,2}(\delta)$ to intersect $\overline{K}_{L,2}(\delta) = \pi$ at a point $\delta \in [\overline{\overline{\delta}}_L, \overline{\delta}_L]$. To that end, I solve $\overline{\overline{K}}_{L,2}(\delta) = \pi$ for δ. This gives $\delta = \frac{2\alpha\pi - c - \pi}{2\alpha\pi} = \overset{:}{\delta}$ and I solve $\overline{\overline{\overline{K}}}_{L,2}(\delta) = \pi$ for δ. This *also* yields $\delta = \frac{2\alpha\pi - c - \pi}{2\alpha\pi} = \overset{:}{\delta}$. The conditions for $\overset{:}{\delta} \in [\overline{\overline{\delta}}_L, \overline{\delta}_L]$ are: $\overset{:}{\delta} \geq \overline{\overline{\delta}}_L$ and $\overset{:}{\delta} \leq \overline{\delta}_L$. The first condition amounts at $\frac{2\alpha\pi - c - \pi}{2\alpha\pi} \geq \frac{c}{2\alpha\pi}$ or $\alpha \geq \widetilde{\alpha}$ while the second condition amounts at $\frac{2\alpha\pi - c - \pi}{2\alpha\pi} \leq \frac{c}{\pi}$ or $\alpha \leq \frac{\pi + c}{2(\pi - c)} = \widetilde{\widetilde{\alpha}}$. These two conditions are clearly satisfied. Hence, I have:

$$\begin{cases} \overline{\overline{\overline{K}}}_{L,2}(\delta) \geq \overline{\overline{K}}_{L,2}(\delta) \geq \overline{K}_{L,2}(\delta) & \text{if } \delta \in [\overline{\overline{\delta}}_L, \overset{:}{\delta}] \\ \overline{\overline{\overline{K}}}_{L,2}(\delta) \leq \overline{\overline{K}}_{L,2}(\delta) \leq \overline{K}_{L,2}(\delta) & \text{if } \delta \in [\overset{:}{\delta}, \overline{\delta}_L]. \end{cases} \tag{45}$$

Defines the kinked curved $\overset{:}{K}(\delta)$ by:

$$\overset{:}{K}(\delta) = \begin{cases} \overline{\overline{K}}_{L,2}(\delta) & \text{if } \ \delta \leq \overset{:}{\delta} \\ \overline{\overline{\overline{K}}}_{L,2}(\delta) & \text{if } \ \delta > \overset{:}{\delta} \end{cases} \tag{46}$$

It follows that for all $K \leq \overset{:}{K}(\delta)$ the infringer invests in period 1, and for all K such that $K \leq \overline{K}_{L,2}(\delta)$ and $K \geq \overset{:}{K}(\delta)$ he delays investment until period 2. For $K > \overset{:}{K}(\delta)$ and $K > \overline{K}_{L,2}(\delta)$ he does not invest. This is stated in lemma 8.

$\diamondsuit$ *Consider finally* $\alpha \in \left[\widetilde{\widetilde{\alpha}}, 1\right]$.

From the preceding analysis, I know that $\overset{:}{\delta} \geq \overline{\delta}_L$. Hence, for all $\delta \in [\overline{\overline{\delta}}_L, \overline{\delta}_L]$, I have:

$$\overline{\overline{\overline{K}}}_{L,2}(\delta) \geq \overline{\overline{K}}_{L,2}(\delta) \geq \overline{K}_{L,2}(\delta). \tag{47}$$

It follows that for all $K \leq \overline{\overline{\overline{K}}}_{L,2}(\delta)$ the infringer invests in period 1, otherwise he does not invest. This is again stated in lemma 8.

■ *Scenario 3:* $\delta \in [0, \overline{\overline{\delta}}_L]$.

The analysis is identical to the "no laches regime". See Appendix A.1.

On this interval, three functions must be considered: $\overline{K}_{L,3}(\delta)$, $\overline{\overline{K}}_{L,3}(\delta)$ and $\overline{\overline{\overline{K}}}_{L,3}(\delta)$. The analysis is equivalent to the analysis of the "no laches" regime since $\overline{K}_{L,3}(\delta) = \overline{K}_{N,3}(\delta)$, $\overline{\overline{K}}_{L,3}(\delta) = \overline{\overline{K}}_{N,3}(\delta)$ and $\overline{\overline{\overline{K}}}_{L,3}(\delta) = \overline{\overline{\overline{K}}}_{N,3}(\delta)$. Therefore, it is not necessary to detail the analysis and, following the result for the no laches regime, I can state that for all $K \leq \overline{\overline{K}}_{L,3} = 2\alpha\pi$, the infringer invests in period 1. Otherwise, he does not invest. This is stated in lemma 6.

The following lemma combines the above analysis with the analysis of litigation conducted in section 3. It gives the exact values of the parameters for which a specific equilibrium outcome occurs. Figures **L1**, **L2** and **L3** capture these features in a livelier manner.

Lemma 10 ***Equilibrium outcomes under the doctrine of laches).*** *When the probability of commercial success is high ($\alpha \geq \frac{1}{2}$),*

- *If patent protection is strong ($\delta \in [\overline{\delta}_L, 1]$), the infringer invests in period 1 if the sunk cost is low enough ($K \leq \overline{\overline{K}}_{L,1}$) and the patentholder litigates in period 1. If $K > \overline{\overline{K}}_{L,1}$ he does not invest.*
- *If patent protection is weak ($\delta \in [0, \overline{\overline{\delta}}_L]$, the infringer invests in period 1 if the sunk cost is low enough ($K \leq \overline{\overline{K}}_{L,3}$) and the patentholder does not litigate. He does not invest if $K > \overline{\overline{K}}_{L,3}$.*
- *If patent protection is intermediate ($\delta \in [\overline{\overline{\delta}}_L, \overline{\delta}_L]$), the timing of investment depends on the probability that the innovation is profitable:*
  - *When $\alpha \in \left[\frac{1}{2}, \widetilde{\alpha}\right]$, the infringer invests in period 1 if $K \leq \overline{\overline{\overline{K}}}_{L,2}$ and the patentholder litigates in period 1. He delays investment until period 2 for $K \in [\overline{\overline{\overline{K}}}_{L,2}, \overline{K}_{L,2}]$ and the patentholder does not litigate. He does not invest if $K \geq \overline{K}_{L,2}$.*
  - *When $\alpha \in \left[\widetilde{\alpha}, \widetilde{\widetilde{\alpha}}\right]$, the infringer invests in period 1 if $K \leq \ddot{K}$ and the patentholder litigates in period 1. If $K \in [\ddot{K}, \overline{K}_{L,2}]$, the infringer delays investment and the patentholder does not litigate. If $K > \ddot{K}$ and $K > \overline{K}_{L,2}$, the infringer does not invest.*
  - *When $\alpha \in \left[\widetilde{\widetilde{\alpha}}, 1\right]$, the infringer invests in period 1 if $K \leq \overline{\overline{K}}_{L,1}$ and the patentholder litigates in period 1. He does not invest if $K > \overline{\overline{K}}_{L,1}$.*

Appendix B

Appendix B.1: I show that $\frac{1}{2} \leq \widehat{\alpha} \leq \widehat{\widehat{\alpha}} \leq \widetilde{\alpha} \leq \widetilde{\widetilde{\alpha}}$.

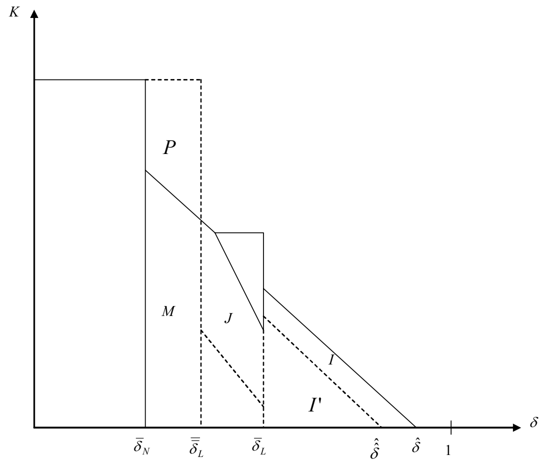
I have: $\widehat{\alpha} = \frac{\pi}{2(\pi - c)}$; $\widehat{\widehat{\alpha}} = \frac{\pi}{2\pi - 3c}$; $\widetilde{\alpha} = \frac{\pi + 2c}{2\pi}$ and $\widetilde{\widetilde{\alpha}} = \frac{\pi + c}{2(\pi - c)}$.

Notice that $\frac{1}{2} \leq \widehat{\alpha}$ if and only if $\pi \geq \pi - c$ which holds. Then, $\widehat{\alpha} \leq \widehat{\widehat{\alpha}}$ if and only if $2(\pi - c) \geq 2\pi - 3c$ which holds as well. And $\widehat{\widehat{\alpha}} \leq \widetilde{\alpha}$ if and only if $2\pi^2 \leq (\pi + 2c)(2\pi - 3c)$ which is equivalent to $\pi \geq 6c$ which holds by assumption. Finally, $\widetilde{\alpha} \leq \widetilde{\widetilde{\alpha}}$ if and only if $(\pi + 2c)2(\pi - c) \leq (\pi + c)2\pi$ which is equivalent to $-2c^2 \leq 0$. This clearly holds.

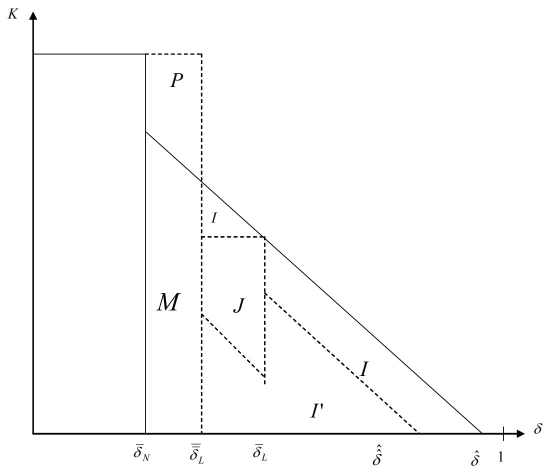
I can conclude that $\frac{1}{2} \leq \widehat{\alpha} \leq \widehat{\widehat{\alpha}} \leq \widetilde{\alpha} \leq \widetilde{\widetilde{\alpha}}$. *QED.*

Appendix B.2. I conduct the comparative analysis between the laches and the no laches regime, for each of the following intervals: $\alpha \in \left[\widehat{\alpha}, \widehat{\widehat{\alpha}}\right]$, $\alpha \in \left[\widehat{\widehat{\alpha}}, \widetilde{\alpha}\right]$, $\alpha \in \left[\widetilde{\alpha}, \widetilde{\widetilde{\alpha}}\right]$ and $\alpha \in \left[\widetilde{\widetilde{\alpha}}, 1\right]$. I proceed with a graphical comparison, as in section 5.1.

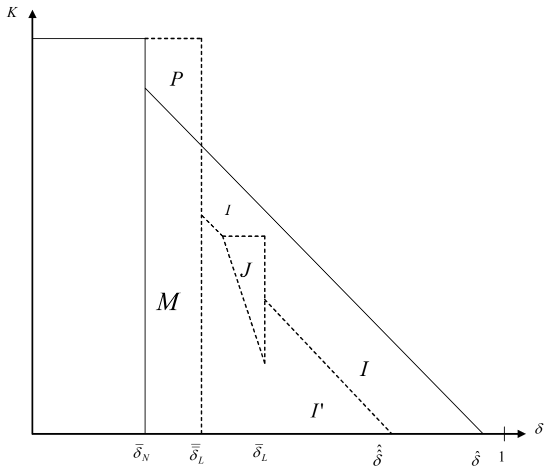
When $\alpha \in \left[\widehat{\alpha}, \widehat{\widehat{\alpha}}\right]$, the relevant graphics to compare are N2 (for the "no laches regime) and L1 (for the laches regime). I superpose these two graphics to obtain the following figure. As in Figure 4, the solid lines represent boundaries between the different equilibria in the no laches regime. The dotted lines represent the boundaries in the (new) laches regime. The capital letters represent parameters configurations that are affected by a switch to the laches regime. Notice that all these configurations (I, I', M, P, J) have been encountered when analyzing the case $\alpha \in \left[\frac{1}{2}, \widehat{\alpha}\right]$. Hence, there is no additional insight when $\alpha \in \left[\widehat{\alpha}, \widehat{\widehat{\alpha}}\right]$.



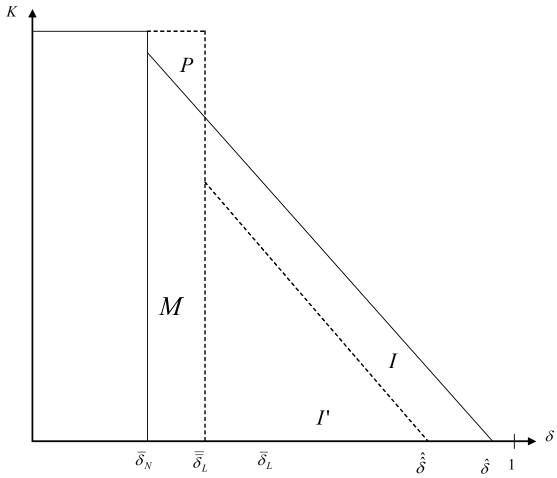
I repeat this analysis for the case where $\alpha \in \left[\widehat{\widehat{\alpha}}, \widetilde{\alpha}\right]$. The relevant graphics to compare are now N3 and L1. As shown in the following figure, the parameters configurations affected by the change of regime (I, I', M, P, J) have been encountered in the case $\alpha \in \left[\frac{1}{2}, \widehat{\alpha}\right]$:



When $\alpha \in \left[\widetilde{\alpha}, \widetilde{\widetilde{\alpha}}\right]$ I compare N3 and L2. All the parameters configurations affected by the change of regime (I, I', M, P, J) have been analyzed before:



When $\alpha \in \left[\tilde{\tilde{\alpha}}, 1\right]$ I compare N3 and L3. Again, all parameters configurations (I, I', M, P) have been encountered before:



Hence, focusing only on $\alpha \in \left[\frac{1}{2}, \widehat{\alpha}\right]$ entails no loss of generality.

Appendix B.3. I show here that $\overline{\overline{K}}_{N,1}(D) \geq \overline{\overline{K}}_{L,1}(D)$ and $\overline{\overline{\overline{K}}}_{N,2}(D) \geq \overline{\overline{\overline{K}}}_{L,2}(D)$. This is straightforward: we have $\overline{\overline{K}}_{N,1}(D) = 2\alpha\pi(1-D) - \alpha c \geq \overline{\overline{K}}_{L,1}(D) = 2\alpha\pi(1-D) - c$ since $\alpha \leq 1$ and by the same token, $\overline{\overline{\overline{K}}}_{N,2}(D) = \frac{\alpha\pi(1-2D)-\alpha c}{1-\alpha} \geq \overline{\overline{\overline{K}}}_{L,2}(D) = \frac{\alpha\pi(1-2D)-c}{1-\alpha}$.

Appendix B.4. I analyze here configurations I' and M and establish that the timing or occurence of investment in the follow-on innovation is not affected.

- Configuration I' : The doctrine of laches may induce a switch from an equilibrium where investment occurs in period 1 and litigation is delayed (ED) to an equilibrium where both occur in period 1 (EE). Section 3 analyzed and explained the intuition for this change in litigation behavior: the doctrine of laches encourages the patentholder *not to delay* precisely because delay is punished by a reduction of the damages collected. Notice that for the values of K such as in configuration I', the infringer still invests (in period 1), despite early litigation.

- Configuration M : The doctrine induces a switch from an equilibrium where investment occurs in period 1 and litigation is delayed (ED) to an equilibrium where investment occurs in period 1 and litigation is deterred (EN). Here, the doctrine of laches does not affect the timing of investment. But, as stated in lemma 3, it deters litigation. More precisely, for all $\delta \in \left[\overline{\delta}_N, \overline{\overline{\delta}}_L\right]$, (delayed) litigation occurs in a "no laches" regime but does not occur in a laches regime. I provided an explanation for this effect in section 3.

Appendix C.

I show that when $\alpha \geq \frac{1}{2}$ and $\delta < \overline{\overline{\delta}}_L$ the doctrine of laches can leave the patentholder indifferent or make her worse off and leave the infringer indifferent or make him better off.

For all $\delta \leq \overline{\delta}_N$, litigation does not occur under either regime so that a regime change leaves the players indifferent. For $\delta \in \left[\overline{\delta}_N, \overline{\overline{\delta}}_L\right]$, consider three configurations: M, O and P. For configuration M; introducing a laches defense deters litigation and so makes the patentholder worse-off and the infringer better-off (he still invests in period 1 but does not pay damages). For configuration O, the patentholder would not litigate in either regime. So introducing a laches defense leaves her indifferent. But the infringer would invest earlier so that his expected payoff is $-K + 2\alpha\pi$ in a laches regime, and $\alpha(\pi - K)$ in the "no laches" regime. He is better-off in a laches regime since $-K + 2\alpha\pi \geq \alpha(\pi - K)$ if and only if $K \leq \frac{\alpha}{1-\alpha}\pi = \overline{\overline{\overline{K}}}_{L,3}$, which holds for configuration O. Finally, for configuration P, the patentholder is not affected by the introduction of a laches defense (in the laches regime, she does not litigate and in the "no laches" regime, there is no investment in the first place). But the infringer is clearly better-off as the defense of laches makes investment profitable.

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Intellectual Property Regimes and Incentive to Innovate: A Comment on Bessen and Maskin

Xavier Carpentier Klaus Kultti

Abstract

When innovation is cumulative, James Bessen and Eric Maskin (2002) (denoted B&M) argue that patents may generate less aggregate investment than a regime with no intellectual property rights and that society may be better-off without such rights. We extend their model in two directions: we endogenize the level of R&D investment and we introduce a third form of intellectual property right: copyright. We obtain refined and contrasted results when innovation is cumulative: patents always yield *more* aggregate investment than no protection (in contrast to B&M), and a copyright regime can implement the socially optimal investment by *reducing* R&D incentives compared to a patent regime (again in contrast to B&M).

JEL classification codes: *O31* (Innovation and Incentives), *O32* (Management of Technological Innovation), *O34* (Intellectual Property Rights).

Keywords: patent, copyright, sequential innovation.

1 Introduction

In the last twenty years, there has been a trend in the United States toward a strengthening of the patent system. Recent papers by Jaffe (1999), Gallini (2002) or Lerner (2003) acknowledge this evolution. Reflecting this reinforcement, the "expansion of the realm of patentability"[1] has been emphasized by many: there are now patents for gene sequences, financial formulas and computer sofwares, for example. Economists warned against the possible side-effects of this development. James Bessen and Eric Maskin (2002) (denoted B&M) contribute to the criticisms by arguing that in industries where innovations are cumulative and complementary, as in the software or the semi-conductor industries, patents might be an impediment to innovation rather than an "engine" as they are traditionnally perceived. Innovations are cumulative when

[1] Jaffe (1999).

each innovation builds on the previous one. They are complementary in the sense of B&M if each firm takes a possibly different research path, which increases the overall probability that at least one firm will come up with the innovation. A patent on one innovation confers its holder a "hold-up" right over subsequent innovations, if performing the latter requires the right to use the former. Assuming ex-ante licensing is impossible or imperfect as in B&M, only the patentholder will have the incentive (the right) to engage in R&D for further innovations. This can restrict the number of firms performing research and the aggregate R&D investment is reduced. Were a patent absent, each innovation could be imitated legally and used freely for next researches. The (static) disincentives associated with the loss of the patent for a successful firm could be more than compensated by the (dynamic) gains associated with the prospects of being always in the R&D race, being allowed to imitate a winner, and being able to become an innovator. B&M first propose a theoretical model that supports this view, in which they stress the social merit of an intellectual property (IP) regime with no legal protection as compared to a regime with patents. Then, they conduct an empirical investigation of the transition from an IP regime with copyrights towards a regime with more patents in the US software industry during the late 1980's. They show, in particular, that this trend has generated a decrease of R&D investment at the firm level. B&M's paper has been very influential in European policy debates over the appropriate IP regime for software. In particular, after the European Commission launched the discussions about this issue, opponents of software "patents" repeatedly cited the paper as an argument against such patents. Given the influence of this paper and the importance of the issues it discusses, we elaborate a closely related model (so that comparisons are possible) which extends B&M's model in two directions:

- First, we endogenize the *level* of R&D investment[2]. Two firms compete in R&D by choosing simultaneously and non-cooperatively an investment level which determines the probability of R&D success.

- Second, we refine the instruments of IP protection policy. Where B&M consider two possible regimes: patent and no protection, we consider three possible regimes which differ according to the extent of the property right offered to an innovator. Our "patent regime" and our "no protection regime" are identical to B&M's patent and "no protection" regimes. However, we introduce a moderate IP protection regime, that we call "copyright

[2] BM consider an *endogenous* decision to do or not to do R&D, with an *exogenously* given R&D cost.

regime", with the following features: first, independent identical discoveries are allowed ; second, protection against imitation is imperfect and third, an innovator has no property right over future innovations. We justify these assumptions in section 2.

Except for these two modifications, we remain as close as possible to B&M. This is because we want to easily contrast our results with theirs. In doing so, we inherit many of their assumptions. In this context, we ask two questions: Do BM's main conclusions still hold? What IP regime, if any, can implement the socially optimal R&D investment?

Results. First, and in contrast to B&M, we find that when innovation is cumulative, a patent regime always yields more R&D investment than a "no protection" regime. Second, a regime with no IP protection never yields the socially optimal R&D investment when innovation is cumulative. Third, still for cumulative innovations, we show that for some values of the parameters, a moderate IP protection akin to copyright implements the socially optimal investment. But we show that the role of this moderate regime can be to mitigate R&D incentives. This also contrasts with B&M who argue than under cumulative innovation a copyright regime can provide more R&D incentives than patents.[3]

A roadmap. We follow the organization of B&M's paper. In section 2, we present the main assumptions of our model. In section 3, we analyze the static case (a one-shot innovation). In section 4, we analyze the cumulative innovation case. In section 3 and 4, we work with a general functional form for the probability of R&D success. Focusing on a general function allows us to derive more general intermediate results. However, in order to obtain additional results, we need to work with a specific functional form. We do so in section 5. We then obtain our core results and emphasize how they differ from B&M. Section 6 concludes.

2 The assumptions of the model

Our general framework follows closely B&M. Two firms compete in R&D. We consider two cases: a one-shot innovation of value v (static case), and cumulative innovations. In this second

[3] Although we analyze the "one-shot innovation" case (section 3), our results are not fundamentally different from those in B&M, and thus are not reported here.

case, we assume that in each period, innovation is incremental: it yields additional value v to the previous innovation. Like BM, we assume that firms capture the whole social value v of an innovation and that, if competition occurs, it does not reduce firms' rent[4]. Also, each innovation builds on the previous one, so that access to an innovation in any given period is required to generate the next ones.

Contrary to B&M, we endogenize the *level* of R&D invesment: we assume that firms are symmetric and that the probability of success in R&D is given by $p(x_i)$ where x_i denotes the level of R&D investment by firm i. The cost of R&D is assumed to be linear: $c(x_i) = \alpha x_i$ (with $\alpha \geq 1$). In the first part of the paper we work with a general functional form for $p(x_i)$. In the last part, we turn to a specific functional form $p(x) = 1 - e^{-x}$ to obtain additional results. To guarantee the existence of an interior solution for all maximization problems considered, we assume that $p'(x_i) > 0$, $p''(x_i) \leq 0$ and $2\left[p'(x)\right]^2 \leq -(1-p(x))p''(x) = p(x)p''(x) - p''(x)$ (this assumption is referred to as assumption 1). Assumption 1 is a sufficient but not a necessary condition that guarantees that all the objective functions we consider have a maximum, as shown in the technical appendix of the paper (Appendix I). It is satisfied by various probability of success functions such as $p(x) = \frac{x}{1+x}$ or $p(x) = 1 - x^{-a}$ (for this last function, a restriction on x must be imposed to guaratee a positive sign). An implication of assumption 1, namely $\left[p'(x)\right]^2 \leq -(1-p(x))p''(x)$, enables us to derive unambiguous results in section 3.

In addition to endogenizing R&D investment, we extend B&M by introducing a third regime. Like them, we consider a patent regime and a "no protection" regime. But we also consider an intermediate regime with moderate protection that we call the copyright regime. The differences between these regimes is as follows.

- *The patent regime.* It has maximal protection against imitation: the winner of the patent gets v and the loser gets 0. In addition, a patent has maximal "forward protection"[5]. This means that the patentholder has full property rights over subsequent innovations. As a result, we assume that when innovation is cumulative, only the owner of the patent

[4]This means that the monopoly profit is v whereas if imitation occurs and is "legal", the duopoly profit is $\frac{v}{2}$. We are aware that this "collusive" assumption is not fully satisfactory. Our paper being a comment of *other aspects* of Bessen and Maskin, we borrow this assumption from their analysis.

[5]A patent confers property right not only over the currently patented innovation *but also over future innovations*, especially when these subsequent innovations are not too different from the current one. This is discussed in Merges and Nelson (1990).

on the first innovation will conduct subsequent research[6]. Finally, in a patent regime, independent discoveries are not allowed: if both firms innovate, only one gets a patent[7].

- *The "no protection regime".* In this regime, there is no protection against imitation so that when one firm innovates, the other one imitates legally and both earn $\frac{v}{2}$. In addition, because there is no property right, the innovative firm has no right over subsequent innovations: there are always two firms conducting R&D in every period. The absence of property rights also implies that there is no "independent discovery" issues.

- *The copyright regime.* The protection against imitation is imperfect. In any period, an imitation is allowed with probability θ[8]. Accordingly, payoffs are $(1-\theta)\frac{v}{2}$ for the imitator and $\theta v + (1-\theta)\frac{v}{2} = (1+\theta)\frac{v}{2}$ for the innovator, where $\frac{v}{2}$ is the profit when imitation is allowed. Under copyright protection, we assume that the copyright owner has no property right over subsequent innovations. Finally, we assume that copyright protection allows for independent discoveries. Justifications for these assumptions are offered below. Notice that by setting $\theta = 1$ we obtain protection against imitation in the patent regime. By setting $\theta = 0$ we obtain the "no protection" regime.

	protection against imitation (backward protection)	property right over future innovation (forward protection)	right for independent discoveries
patent	*full: θ=1*	*full*	*no*
copyright	*imperfect: 0<θ<1*	*no*	*yes*
no protection	*no: θ=0*	*no*	*yes*

Table 1: Intellectual property regimes

[6]This equivalent to a regime where the follow-on innovation is unpatentable and infringes the first innovation. Denicolò (2000) shows that only the first patentholder has an incentive to engage in R&D for the follow-on innovation.

[7]We acknowledge that the assumptions for the patent regime are strong (patents can be invented around, new inventions can be patentable...). Besides inheriting the assumptions from B&M, we stress that some of these assumptions appear in the early patent race literature (full backward protection creates a winner-take-all effect like in our paper). Also, what we really want to capture, following B&M, is that the patent regime is believed to be the *strongest* IP regime available, in particular stronger than copyright. Hence the regime characterizations should be understood in relation to each other.

[8]In practice, this means that the innovator can sue the imitator for infringement (at no cost) and obtain an injunction with probability θ. The injunction would force the imitator to stop producing the imitation.

We emphasize that the regimes' names are just labels. It would be possible to call the patent regime a "high protection" regime. But we want to stick to Bessen and Maskin's terminology so as to facilitate comparison. Our understanding of a copyright is that it is a moderate form of protection compared to a patent. In particular, because it protects an expression and not an idea, we expect that it is easier to come with a non-infringing imitation than in a patent regime (by simply changing the expression but using the same idea). Thus, we believe that a crucial distinctive feature of the copyright is that it prevents from "hold-up" by its owner. This is supported by the recent "jurisprudence" in the United States. In the famous case between Apple Computer versus Microsoft and Hewlett-Packard, a ruling was made in favor of the latters against Apple who argued that Microsoft Window´s program and HP´s New Wave software had "copied the "look and feel" of Macintosh's graphic-based operating system. The Court clearly favored a strict interpretation of the copyright whereby the "idea" of a particular expression can be used for developing a different expression (another software in this case). Notice however that this example contrasts with the trend observed in the 80's that strengthened copyright protection for softwares[9]. A similar judgment by the US Supreme Court was pronounced in the case *Lotus Development Corp. v. Borland International Inc.*, 96 Daily Journal D.A.R. 495 (Jan.16, 1996): The Court let stand a judgment by the Court of Appeals for the First Circuit that denied infringement by Borland's spreadsheet program of the Lotus 1-2-3 program. Lotus claimed that the (acknowledged) introduction of Lotus´ menus command hierarchy in Borland's program was illegal. However, the Court of Appeals for the First Circuit stated against Lotus by refering to the rule governing copyright protection under title 17 of the U.S.Code, Section 102(b):"[I]n no case does copyright protection for an original work of authorship extend to any idea, procedure, process, system, method of operation, concept, principle or discovery, regardless of the form in which it is described, explained, illustrated or embodied in such work". This is remarkably explained by Bunker (2002)[10], who concludes: "(...) it seems clear that broad protection for software under the laws of copyright is dead. (...) copyright protects against copying (...) [but] provides no protection against independent creation. (...)[on the contrary],

[9]It is important to distinguish two phenomena: first, the extent of the *patent* protection for softwares (while initiated by *Diamond v. Diehr (1981)*, it has not been extented to all types of softwares) and the evolution of the *copyright* protection for softwares which shows two trends: In the 1980's, an extension of this protection (strong protection against imitation) and in the 1990's a comeback to a strict application of copyright law (weak protection against imitation). We base our definition of the copyright regime on this last trend.

[10]Lawyer at Knobbe Martens Olson & Bear LLP.

since a patent can protect an idea or a concept, the patent claims can be written in broad terms to cover the novel combination of elements". These remarks enable us to state that, under a copyright regime, the idea of an innovation can be used to develop further innovations so that the two firms remain R&D competitors in every period.

3 Analysis of the static case

In this brief section, we analyze firms' and society's R&D investments in each regime, in the case where there is a single innovation.

3.1 Firms

Consider first firm i's objective function in the copyright regime. Firm i's objective is to choose x_i (given x_j) that maximizes:

$$U_i = -c(x_i) + p(x_i)\left\{p(x_j)\frac{v}{2} + [1-p(x_j)]\,\frac{v}{2}(1+\theta)\right\} + [1-p(x_i)]\,p(x_j)\frac{v}{2}(1-\theta). \qquad (1)$$

Firm i faces an R&D cost $c(x_i)$. Then, with probability $p(x_i)$ it succeeds. If the rival firm j succeeds as well, under a copyright regime, independent discoveries are allowed: each firm gets $\frac{v}{2}$. If firm j does not succeed, it can imitate: in that case, firm i gets $\frac{v}{2}(1+\theta)$ where θ captures the protection against imitation in the copyright regime. With the complementatry probability $1-p(x_i)$ firm i does not innovate, but with probability $p(x_j)$ firm j does. In that case, firm i can imitate and get $\frac{v}{2}(1-\theta)$.

It can be rewritten as:

$$U_i = -c(x_i) + \frac{v}{2}\left\{p(x_i)\left[1+\theta\left(1-p(x_j)\right)\right] + (1-p(x_i))p(x_j)(1-\theta)\right\}.$$

The first-order conditions characterize a symmetric Nash equilibrium in R&D investment, x^*, implicitely defined by[11]:

$$p'(x^*) = \frac{2c'(x^*)}{v\left[1+\theta-p(x^*)\right]}. \qquad (2)$$

[11]It is straightforward to check that it is a maximum. $\frac{\partial^2 U_i}{\partial x_i^2} = -c''(x_i) + \underbrace{p''(x_i)}_{\leq 0}\frac{v}{2}\left(\underbrace{1+\theta-p(x_j)}_{\geq 0}\right) \leq 0.$

Investment in the "no protection" regime is obtained by setting $\theta = 0$ in (2) and investment in the patent regime is obtained by setting $\theta = 1$[12].

Lemma 1 *A firm's R&D investment is strictly increasing in the level of backward protection θ, in the value v of the innovation and in the firm's R&D efficiency.*

Proof. See Appendix A. ■

3.2 Society

Society's objective is to choose x_1 and x_2 that maximize:

$$U_s = -c(x_1) - c(x_2) + \{1 - (1 - p(x_1))(1 - p(x_2))\} v. \tag{3}$$

Society faces both the cost for firm i and the cost for firm j. But what matters is that at least one firm innovates. The probability that at least one firm innovates is $1-(1-p(x_1))(1-p(x_2))$. Given firms' symmetry, society would like each firm to invest the same amount $x^{*,s}$ implicitely defined by:

$$p'(x^{*,s}) = \frac{c'(x^{*,s})}{v\left[1 - p(x^{*,s})\right]}. \tag{4}$$

Lemma 2 *The socially optimal R&D investment is higher than a firm's investment in the "no protection" regime, but lower than a firm's investment in the patent regime.*

Proof. See Appendix A. ■

This result is in line with BM. The patent regime creates overinvestment due to the "winner-take-all" effect. Our model enables us to refine B&M's findings: in particular, we now characterize the optimal IP policy which is moderate (proposition 1) and we derive comparative statics over the socially optimal level of protection against imitation (corollary 1).

[12] Indeed, in the patent regime, firm i's objective is given by (1) with $\theta = 1$. If both firms succeed, they get the patent with probability $\frac{1}{2}$ so that their expected payoff from a simultaneous independent discovery is $\frac{v}{2}$ (as in the copyright regime).

Proposition 1 *The optimal intellectual property policy is equal to the probability of failure at the socially optimal R&D investment:*

$$\theta^* = 1 - p(x^{*,s}). \tag{5}$$

Proof. See Appendix A. ■

To derive this result, we have set $x_s^* = x^*(\theta)$ and solved for θ. Alternatively, we could have haved substituted for $x^*(\theta)$ into U_i and assumed that society maximizes $2U_i(x^*(\theta))$. This would yield the same optimal θ.

Corollary 1 *The optimal intellectual property policy is decreasing in the social value v of the innovation and in the R&D efficiency of the firms.*

Proof. See Appendix A. ■

This is also explained by the winner-take-all effect: the higher the prize v, the less important becomes the protection θ to encourage innovation. This suggests that an increase in v increases firms' incentives *more* than society's incentives. Hence, the optimal IP policy should be adjusted to *reduce* firms' incentives. This is obtained by setting a lower θ^*. The same rationale explains the result about R&D efficiency. A symmetric increase in R&D efficiency increases firms' investment more than society's optimal investment. The optimal IP policy is set so as to reduce firms' incentives. Like B&M, we find that in the static model patents yield overinvestment and "no protection" yields underinvestment. We have refined their results by defining the optimal policy (under the assumptions of this clearly stylized model) and deriving simple comparative statics. We now turn to the cumulative innovation case. We begin this analysis in section 4 with a general functional form for $p(x)$. We show in particular than the absence of protection always fails to implement the socially optimal investment (proposition 2). Our main results are obtained in section 5 where we specify a functional form for $p(x)$.

4 Cumulative Innovation

We now turn to analyzing the case where innovation is cumulative. To that end, it its worthwhile to define the following functions:

$$\begin{cases} h(x) = \frac{1-p(x)}{p'(x)} \\ q(x) = 1-(1-p(x))(1-p(x)), \end{cases}$$

where $p(x)$ is the probability of R&D success. The ratio $h(x)$ is introduced to simplify notations. Under assumption 1, $h(x)$ is weakly increasing in x. We also define $q(x)$ as the probability that at least one firm innovates in any period, when both firms' investment in R&D is given by x.

4.1 Patent regime

We must now distinguish the first period from subsequent periods. If only one firm innovates in the first period, it has full property right over all subsequent innovations, which deters the rival from engaging in R&D[13] (see our definition of the patent regime in section 2). $\widehat{x}_i$ denotes firm's i R&D investment in the first period and $\widetilde{x}$ the investment of the patentholder in all subsequent periods. Firm i's objective in the first period is to choose $\widehat{x}_i$ (given $\widehat{x}_j$) that maximizes:

$$U_i^2 = -c(\widehat{x}_i) + p(\widehat{x}_i)\left\{p(\widehat{x}_j)\frac{1}{2}\left(v+U_i^1\right) + (1-p(\widehat{x}_j))\left[v+U_i^1\right]\right\}. \tag{6}$$

Superscripts 2 and 1 indicate the number of firms able to conduct R&D. In the first period, there are two firms. Firm i pays the cost of R&D $c(\widehat{x}_i)$ and innovates with probability $p(\widehat{x}_i)$. In that case, if firm j innovates as well (with probability $p(\widehat{x}_j)$), firm i gets the patent with probability $\frac{1}{2}$: it receives v as the first period profit and, in the following periods, it is the only firm conducting R&D. Hence, the value function is defined as U_i^1 and it is defined precisely in expression (7) below. If firm j does not innovate (with probability $1-p(\widehat{x}_j)$), firm i gets the

[13] B&M allow for ex-ante licensing with the restriction that it is imperfect due to assymetric information. Hence, sometimes, ex-ante licensing does not occur in their model and aggregate R&D is reduced (i.e. one firm does not conduct R&D). In our model, introducing ex-ante licensing might have two effects. It can reinforce our main conclusion that the patent regime provides more R&D incentives or it can change it (anticipating that they might be licensed if they do not succeed in R&D, firms might free-ride). It is not clear which effect would prevail but, in any case, it would contrast with B&M (for example if ex-ante licensing creates free-riding thereby reducing aggregate R&D).

patent for sure.[14] And in all subsequent periods, the holder of the first patent chooses $\widetilde{x}$ that maximizes[15]:

$$U_i^1 = -c(\widetilde{x}) + p(\widetilde{x}) \left[v + U_i^1 \right], \tag{7}$$

or:

$$U_i^1 = \frac{1}{1 - p(\widetilde{x})} \left[-c(\widetilde{x}) + vp(\widetilde{x}) \right].$$

Solving first for the optimal $\widetilde{x}$, we obtain that $\widetilde{x}^*$ is implicitly defined by:

$$c(\widetilde{x}^*) = v - c'(\widetilde{x}^*) h(\widetilde{x}^*). \tag{8}$$

Then, we can substitute for $U_i^1(\widetilde{x}^*)$ in U_i^2. This yields:

$$U_i^2 = -c(\widehat{x}_i) + p(\widehat{x}_i) \left[1 - \frac{1}{2} p(\widehat{x}_j) \right] \frac{v - c(\widetilde{x}^*)}{1 - p(\widetilde{x}^*)}.$$

Maximizing U_i^2 with respect to $\widehat{x}_i$ yields a first-order condition which, given symmetry, implicitly defines the equilibrium R&D investment in the first race, $\widehat{x}^{*,pat}$ (the notation *pat* means that the regime considered is the patent regime):

$$c(\widetilde{x}^*) = v - c'(\widehat{x}^{*,pat}) \frac{1 - p(\widetilde{x}^*)}{p'(\widehat{x}^{*,pat}) \left[1 - \frac{1}{2} p(\widehat{x}^{*,pat}) \right]}. \tag{9}$$

Finally we obtain:

$$p(\widehat{x}^{*,pat}) = 2 \left[1 - \frac{c'(\widehat{x}^{*,pat}) p'(\widetilde{x}^*)}{p'(\widehat{x}^{*,pat}) c'(\widetilde{x}^*)} \right]. \tag{10}$$

[14]Some of our assumptions make the model close to a discrete version of a patent race. In particular, R&D is memoryless. The main difference with this literature is that if both firms fail to innovate in one period, the R&D race stops. We inherit this strong assumption from B&M.

[15]Notice that we need to solve first the optimal investment in all periods but the first one and then the optimal investment in the first period. Hence, we proceed by backward induction here. However, for all periods except the first one (as well as in the copyright and no protection regimes or for society) the model is not properly dynamic in the sense of intertemporal optimization. Indeed, in every period, the environment is unchanged (there is no law of motion affecting the parameters values) and the decision making problem is identical in nature and outcome to that of all other periods since i) R&D is memoryless (firms do not accumulate knowledge) ii) the prize v is the same. Here for example, in every period, player i would have to choose $\widetilde{x}$ which maximizes $V_i^1(\widetilde{x}) = -c(\widetilde{x}) + p(\widetilde{x})(v + V_i^1(\widetilde{y}))$ where $\widetilde{y}$ is the optimum in the next period. It is clear that if $\widetilde{x}$ is optimal, then $\widetilde{x} = \widetilde{y}$.In every period the optimum is the same.

4.2 Copyright and "no protection" regimes

We start with the copyright regime. Firm i's objective is to choose $\widehat{x}_i$ that maximizes:

$$\begin{aligned} U_i^2 &= -c(\widehat{x}_i) + p(\widehat{x}_i)\left\{p(\widehat{x}_j)\left[\frac{v}{2}+U_i^2\right] + (1-p(\widehat{x}_j)\left[\frac{v}{2}(1+\theta)+U_i^2\right]\right\} \\ &\quad +(1-p(\widehat{x}_i))p(\widehat{x}_j)\left[\frac{v}{2}(1-\theta)+U_i^2\right]. \end{aligned} \tag{11}$$

Superscript 2 indicates there are two firms conducting R&D. Firm i faces an R&D cost $c(\widehat{x}_i)$ and innovates with probability $p(\widehat{x}_i)$. If firm j innovates as well (with probability $p(\widehat{x}_j)$), both firms are allowed to produce the innovation (simultaneous independent discoveries are allowed), and, under the assumptions of the copyright regime, both firms can use the current innovation for next ones: hence the value function U_i^2 indicates that in the next period, the same R&D competition occurs with the same expected payoffs. If firm j does not innovate, it can imitate and in the current period firm i obtains $\frac{v}{2}(1+\theta)$. But again, under the assumptions of the copyright regime, the idea embodied in the current innovation can be freely used by firm j for next R&D contests. If firm i does not innovate in the current period but firm j does, firm i imitates and earns $\frac{v}{2}(1-\theta)$. In addition, it can use the idea of firm j's innovation to conduct R&D in future periods. Expression (12) can be rewritten as:

$$U_i^2 = \frac{1}{1-p(\widehat{x}_i)-p(\widehat{x}_j)+p(\widehat{x}_i)p(\widehat{x}_j)}\left\{-c(\widehat{x}_i)+\frac{v}{2}\left\{\begin{array}{c} p(\widehat{x}_i)\left[1+\theta\left(1-p(\widehat{x}_j)\right)\right]+ \\ (1-p(\widehat{x}_i))p(\widehat{x}_j)(1-\theta) \end{array}\right\}\right\}. \tag{12}$$

The first-order conditions characterize a symmetric Nash equilibrium in R&D investment, $\widehat{x}^{*,cop}$, implicitely defined by:

$$c(\widehat{x}^{*,cop}) = \frac{v}{2}\left[1+\theta(1-p(\widehat{x}^{*,cop}))\right] - c'(\widehat{x}^{*,cop})h(\widehat{x}^{*,cop}). \tag{13}$$

Clearly, by setting $\theta = 0$, we obtain the equilibrium investment under no protection:

$$c(\widehat{x}^{*,n}) = \frac{v}{2} - c'(\widehat{x}^{*,n})h(\widehat{x}^{*,n}). \tag{14}$$

Lemma 3 *Firm's R&D invesment is increasing in the level of backward protection θ, in the value of the innovation v, and in the efficiency of R&D.*

Proof. See Appendix A. ■

We now turn to deriving the optimal R&D investment from society's point of view.

4.3 Society

Society's objective is to choose x_1 and x_2 that maximize[16]:

$$U_s = \frac{1}{1 - p(\widehat{x}_1) - p(\widehat{x}_2) + p(\widehat{x}_1)p(\widehat{x}_2)} \left\{ -c(\widehat{x}_1) - c(\widehat{x}_2) + v\left[p(\widehat{x}_1) + p(\widehat{x}_2) - p(\widehat{x}_1)p(\widehat{x}_2)\right] \right\}. \quad (15)$$

Given firms' symmetry, society would like each firm to invest the same amount $\widehat{x}_s^*$ implicitly defined by:

$$c(\widehat{x}^{*,s}) = \frac{1}{2}\left[v - c'(\widehat{x}^{*,s})h(\widehat{x}^{*,s})\right]. \quad (16)$$

Before comparing the socially optimal investment with actual investments in the different regimes, we define the "expected aggregate investment". B&M argue that patents reduce the aggregate investment in the economy (by restricting, in their model, the number of firms engaged in R&D). Hence, for the comparison to be meaningful, we need a measure of the aggregate investment in our model as well.

Definition 1: *Expected aggregate investments are given by:*

$$\begin{cases} X^{pat} = 2\widehat{x}^{*,pat} + q(\widehat{x}^{*,pat})\left\{\widetilde{x}^* + p(\widetilde{x}^*)\left[\widetilde{x}^* + ...\right]\right\} = 2\widehat{x}^{*,pat} + \frac{q(\widehat{x}^{*,pat})\widetilde{x}^*}{1-p(\widetilde{x}^*)} \textit{ in the patent regime} \\ X^k = 2\widehat{x}^{*,k} + q(\widehat{x}^{*,k})\left\{2\widehat{x}^{*,k} + q(\widehat{x}^{*,k})\left[2\widehat{x}^{*,k} + q(\widehat{x}^{*,k})(...)\right]\right\} = \frac{2\widehat{x}^{*,k}}{1-q(\widehat{x}^{*,k})} \textit{ for } k = cop, n \\ X^s = \frac{2\widehat{x}^{*,s}}{1-q(\widehat{x}^{*,s})} \quad \textit{for the social optimum.} \end{cases} \quad (17)$$

From this definition, we derive the following lemma:

Lemma 4 *The expected aggregate investment is always larger under copyright protection than under no protection: $X^{cop} > X^n$*

Proof. This comes from the fact that the no protection regime is equivalent to $\theta = 0$ in the copyright regime, and we know from lemma 3 that individual per period investment increases with θ, while it is obvious that the expected aggregate investment increases with the individual per period investment. ■

The intuition for this lemma is straightforward: because it provides *some* protection against imitation, the copyright regime implies less free-riding in R&D investment than the no protection regime.

[16]We do not verbally detail the rationale for this objective function: it obeys the same logic as in the previous sections. For society, it is enough that at least one firm innovates.

Proposition 2 *The socially optimal expected aggregate investment is always larger than the expected aggregate investment in the no protection regime ($X^s > X^n$).*

Proof. The proof is in two steps.

First, we prove that $\widehat{x}^{*,s} > \widehat{x}^{*,n}$. This proof is by contradiction.

Assume that $\widehat{x}^{*,s} \leq \widehat{x}^{*,n}$. This implies $c(\widehat{x}^{*,s}) \leq c(\widehat{x}^{*,n})$. Given (14) and (16), this yields:

$$c'(\widehat{x}^{*,s})h(\widehat{x}^{*,s}) \geq 2c'(\widehat{x}^{*,n})h(\widehat{x}^{*,n}).$$

Clearly, $2c'(\widehat{x}^{*,n}) > c'(\widehat{x}^{*,s})$ and $c'(x) = \alpha$. Hence, we have:

$$h(\widehat{x}^{*,s}) > h(\widehat{x}^{*,n}).$$

And since $h(x)$ is increasing in x, it implies:

$$\widehat{x}^{*,s} > \widehat{x}^{*,n},$$

which contradicts our initial assumption.

Then, we show that $\widehat{x}^{*,s} \geq \widehat{x}^{*,n}$ implies $X^s > X^n$. But this is straightforward given (17). ■

From proposition 2, we know that a regime with no IP protection will *never* implement the socially optimal R&D investment: in every period, society would want each firm to invest more than they actually do when IP protection is absent. This result holds for all functional forms $p(x_i)$ satisfying assumptions 1.

Proposition 3 *For society, the "best copyright regime" is designed by setting a level of backward protection θ^* such that:*

$$\theta^* = \min\left\{\frac{1}{v}\frac{c'(\widehat{x}^{*,s})}{p'(\widehat{x}^{*,s})}, 1\right\}. \tag{18}$$

Proof. Society designs θ such that $\widehat{x}^{*,s} = \widehat{x}^{*,cop}$. This implies $c(\widehat{x}^{*,cop}) = c(\widehat{x}^{*,cop})$ and so:

$$\frac{v}{2}\left[1 + \theta(1 - p(\widehat{x}^{*,s}))\right] - c'(\widehat{x}^{*,s})h(\widehat{x}^{*,s}) = \frac{1}{2}\left[v - c'(\widehat{x}^{*,s})h(\widehat{x}^{*,s})\right].$$

Solving for θ, this yields $\theta = \frac{1}{v}\frac{c'(\widehat{x}^{*,s})}{p'(\widehat{x}^{*,s})}$. ■

To understand this result, consider Figure 1. $\widehat{x}^{*,s}$ is independent of θ while we know from lemma 3 that $\widehat{x}^{*,cop}$ is increasing in θ. The left-hand side figure represents a situation where $\widehat{x}^{*,s}$ and $\widehat{x}^{*,cop}$ intersect at $\theta < 1$. The right-hand side figure represents a situation where they intersect at $\theta > 1$. It follows that in this case, the optimal θ is the corner solution 1.

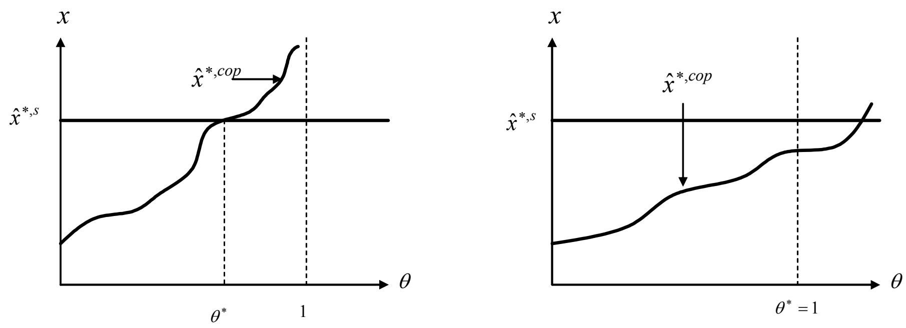

Figure 1: Designing the best copyright regime.

This suggests that a copyright regime (characterized by a right for independent discovery, no forward protection and some backward protection) can implement the socially optimal level of R&D investment. A condition for that is that $\widehat{x}^{*,s}$ and $\widehat{x}^{*,cop}$ intersect at $\theta \leq 1$. This depends of course on the form of $p(.)$ as well as the magnitude of v. Proposition 3 implies that:

- If $\frac{1}{v}\frac{c'(\widehat{x}^{*,s})}{p'(\widehat{x}^{*,s})} \leq 1$, the socially optimal IP policy is implemented by a copyright regime with a protection against imitation given by $\theta^* = \frac{1}{v}\frac{c'(\widehat{x}^{*,s})}{p'(\widehat{x}^{*,s})}$, no forward protection and a right for independent discovery.

- If $\frac{1}{v}\frac{c'(\widehat{x}^{*,s})}{p'(\widehat{x}^{*,s})} > 1$, we only know that the "best copyright regime" is obtained by setting $\theta^* = 1$. But this does *not necessarily* implement the socially optimal investment and thus we cannot conclude that this regime is the best one for society.

We now know that the socially optimal investment can sometimes be implemented by a properly defined copyright regime. But we have not yet compared the patent regime and the no protection regime (which is a crucial point in B&M's paper). Also, we have not yet compared the socially optimal investment with the investment in the patent regime. To do so, we have to specify a functional form for the probability of R&D success.

5 Application: $p(x) = 1 - e^{-x}$ and $c(x)=x$

As shown by Kultti (2003), the probability function $1-e^{-x}$ is particularly attractive for analyzing R&D[17]. Two remarks are in order with this specification. First, the sufficient condition for a maximum given by assumption 1 is not satisfied. However, we show in the technical appendix that a maximum is reached by the objective functions in every IP regime. Otherwise, all the results of the previous section still apply. In this model, applying the general formulas derived in the previous section, we have:

$$\begin{cases} \widetilde{x}^{*} = v - 1 \\ \widehat{x}^{*,pat} = v - 1 - \ln 2 + \ln(1 + e^{-\widehat{x}^{*,pat}}) \\ \widehat{x}^{*,cop} = \frac{v}{2} - 1 + v\theta e^{-\widehat{x}^{*,cop}} \\ \widehat{x}^{*,n} = \frac{v}{2} - 1 \\ \widehat{x}^{*,s} = \frac{1}{2}(v-1), \end{cases} \tag{19}$$

As previously we cannot derive closed-form solutions for $\widehat{x}^{*,pat}$ and $\widehat{x}^{*,cop}$, but for $\widetilde{x}^{*}$, $\widehat{x}^{*,n}$, $\widehat{x}^{*,s}$ we can. Also, we have assumed that society wants each firm to invest the same amount.

Result 1: *In every period, the aggregate investment is always larger in the patent regime than in the no protection regime. Hence, the expected aggregate investment is always larger in the patent regime than in the regime without protection.*

Proof. By definition 1, we have the expected aggregate investment in the patent regime given by:

$$X^{pat} = 2\widehat{x}^{*,pat} + \frac{q(\widehat{x}^{*,pat})\widetilde{x}^{*}}{1 - p(\widetilde{x}^{*})}$$

And we have the expected aggregate investment under no protection which can be re-expressed as:

$$X^{n} = 2\widehat{x}^{*,n} + \frac{q(\widehat{x}^{*,n})2\widehat{x}^{*,n}}{1 - q(\widehat{x}^{*,n})}$$

Under the assumption $p(x) = 1 - e^{-x}$, we have $q(\widehat{x}^{*,pat}) = 1 - e^{-2\widehat{x}^{*,pat}}$ and $q(\widehat{x}^{*,n}) =$

[17] Indeed, R&D can be viewed as a process by which investment is made "step-by-step": first a limited amount x is invested and, conditional on failure to generate anything valuable, a second amount y is invested and so on...Kultti (2003) shows that the only function p which solves the functional equation $p(x+y) = p(x) + (1 - p(x))p(y)$ is precisely $p(x+y) = 1 - e^{-(x+y)}$ so that it is *equivalent* - in terms of the probability to succeed- to invest $x + y$ today or x first and y conditional on failure.

$1 - e^{-2\widehat{x}^{*,n}}$. We can rewrite X^{pat} and X^n as:

$$\begin{cases} X^{pat} = 2\widehat{x}^{*,pat} + (1 - e^{-2\widehat{x}^{*,pat}})\frac{\widetilde{x}^*}{e^{-\widetilde{x}^*}} \\ X^n = 2\widehat{x}^{*,n} + (1 - e^{-2\widehat{x}^{*,n}})\frac{2\widehat{x}^{*,n}}{e^{-2\widehat{x}^{*,n}}} \end{cases}$$

Suppose: $i)$ $\widehat{x}^{*,pat} > \widehat{x}^{*,n}$ and $ii)$ $\widetilde{x}^* > 2\widehat{x}^{*,n}$. Then, $i)$ implies $2\widehat{x}^{*,pat} > 2\widehat{x}^{*,n}$ and $1 - e^{-2\widehat{x}^{*,pat}} > 1 - e^{-2\widehat{x}^{*,n}}$ and $ii)$ implies $e^{-\widetilde{x}^*} < e^{-2\widehat{x}^{*,n}}$. So if $i)$ and $ii)$ hold, it suffices to conclude that $X^{pat} > X^n$. Next, we prove that $i)$ and $ii)$ indeed hold.

$i)$ $\widehat{x}^{*,n} = \frac{v}{2} - 1$ and $\widehat{x}^{*,pat} = v - 1 - \ln 2 + \ln(1 + e^{-\widehat{x}_h^*})$. $\widehat{x}^{*,pat} > \widehat{x}^{*,n}$ implies $\frac{v}{2} - \ln 2 + \ln(1 + e^{-\widehat{x}^{*,pat}}) > 0$ which holds (as $v \geq 2$ and $\ln(1 + e^{-\widehat{x}^{*,pat}}) > 2$, $\forall \widehat{x}^{*,pat}$).

$ii)$ $2\widehat{x}^{*,n} = v - 2$ and $\widetilde{x}^* = v - 1$. Clearly, $\widetilde{x}^* > 2\widehat{x}^{*,n}$.

Hence, conditions $i)$ and $ii)$ hold. ■

This result contrasts with B&M who claim that, when innovation is cumulative, patents are likely to reduce aggregate investment in the economy compared to a regime with no IP rights. In our model the patent regime always yields a higher aggregate investment in the economy than a no protection regime. The difference between ours and B&M's result lies in the assumptions about R&D investment. We have endogenized the level of R&D investment. With patents, firms have a strong incentive to win the first patent because the loser is excluded from future R&D. This effect is absent in the no protection regime since no firm is ever excluded. In addition, with no protection, firms tend to "free ride" on each other in every period. These incentives translate into a higher level of aggregate R&D investment in the patent regime. In contrast, B&M assume that R&D is a fixed cost and measure aggregate R&D as the number of firms participating in research. We claim that their main conclusion (patents hinder innovation) hinges upon the assumption of an exogenous R&D cost.

Result 2: *If* v is large enough ($v \geq 2.386$), *the per-period aggregate investment in the patent regime is larger than the socially optimal per-period aggregate investment.*

Proof. See Appendix B. ■

We are not able to prove this result for small values of v ($v \in [2, 2.386)$). However, our result is quite general since 2.386 is a fairly small value. It shows that even in the cumulative innovation case, a patent regime tends to generate *excessive* R&D investment from society's point of view.

Result 3: *Provided the value of the innovation satisfies* $v \geq e^{\frac{1}{2}(v-1)}$*, the socially optimal R&D investment can be implemented by a copyright regime with protection against imitation given by:*

$$\theta^* = \frac{e^{\frac{1}{2}(v-1)}}{v}. \tag{20}$$

Proof. Applying proposition 3 to this model, we have $c'(\widehat{x}_s^*) = 1$ and $p'(\widehat{x}_s^*) = e^{\frac{1}{2}(v-1)}$. Hence the result. ∎

Corollary 2 $\frac{\partial \theta^*}{\partial v} \geq 0$.

Proof. $\frac{\partial \theta^*}{\partial v} = \frac{e^{\frac{1}{2}(v-1)}}{v^2}\left(\frac{1}{2}v - 1\right) \geq 0$ since $v \geq 2$. ∎

Notice that the role of the optimal IP policy is to *mitigate* incentives to innovate. Indeed, in the patent regime, there can be *too much* aggregate investment in the first period, compared to the social optimum (result 2). In that case, our conclusions for cumulative innovations are qualitatively similar to that for a "one-shot" innovation: from society's view point, aggregate per-period investment is excessive in the patent regime, not sufficient in the absence of protection, and optimal in a copyright regime *for some values of the parameters (*see result 3*).* The current model does not allow us to be more accurate. But notice that this conclusion stands in contrast to B&M who emphasize that patents would *reduce* aggregate R&D and that a copyright regime could alleviate this problem.

6 Conclusion

Bessen and Maskin argue that when innovation is cumulative, the absence of intellectual property rights can generate more R&D investment than a patent regime and that the absence of such rights can be socially preferred to patents. Building on their model, we show that their conclusions are dependent, in particular, on their assumption of an exogenous R&D cost.

- By endogenizing the amount invested in R&D, we find that the absence of IP protection *never* yields the socially optimal investment in R&D, for a wide class of R&D models (that include a probability of R&D success and a cost of R&D).

- Specifying the functional form for the probability of success, we find, in contrast to BM, that a regime with no IP protection *always* yields *less* R&D investment than a patent regime.

- Also the patent regime can yield overinvestment even when innovation is cumulative.

- Finally, our analysis enables us to show that a moderate IP protection regime (with a right for independent simultaneous discoveries, no forward protection and imperfect backward protection), like a copyright regime, *can* implement the socially optimal per-period investment.

We have characterized the socially optimal level of protection against imitation ("backward protection") and this protection is typically stronger in the cumulative innovation case than in the static case. Clearly, that a moderate regime implements the socially optimal investment echoes B&M's point in favor of a mild IP regime (such as the copyright regime). But we stress that, in our model, this serves at *mitigating firms' incentives* and not at providing more R&D incentives, i.e. at solving an overinvestment problem. We believe this comment of B&M's paper is important given the role played by their paper in the debates around the software patent directive in Europe. We claim that their analysis should be considered cautiously in future discussions amongst practitioners.

Appendix

Appendix A: Proofs of the results derived in sections 3 and 4.

Proof of lemma 1. We want to derive comparative statics on x^*. We define:

$$F(x^*, \theta, v) = v\left[1 + \theta - p(x^*)\right] p'(x^*) - 2c'(x^*) = 0.$$

1) Effect of the protection against imitation θ : The implicit function theorems states that $\frac{dx^*}{d\theta} = -\frac{\frac{\partial F(x^*,\theta,v)}{\partial\theta}}{\frac{\partial F(x^*,\theta,v)}{\partial x^*}}$, provided that $\frac{\partial F(x^*,\theta,v)}{\partial x^*} \neq 0$. We have $\frac{\partial F(x^*,\theta,v)}{\partial\theta} = vp'(x^*)$. And: $\frac{\partial F(x^*,\theta,v)}{\partial x^*} = v\left[1 + \theta - p(x^*)\right] p''(x^*) - vp'(x^*)^2 - 2c''(x)$.

First we show that $\frac{\partial F(x^*,\theta,v)}{\partial x^*} < 0$. Given our assumptions, $p''(x^*) \leq 0$, $p'(x^*) > 0$ and $c''(x) = 0$, we have: $\frac{\partial F(x^*,\theta,v)}{\partial x^*} < 0$.

Then we consider $\frac{\partial F(x^*,\theta,v)}{\partial \theta} = vp'(x^*)$. It follows that $\frac{\partial F(x^*,\theta,v)}{\partial \theta} > 0$ since $p'(x^*) > 0$.

We can conclude that $\frac{dx^*}{d\theta} > 0$.

2) Effect of the value of the innovation: The implicit function theorem states that $\frac{dx^*}{dv} = -\frac{\frac{\partial F(x^*,\theta,v)}{\partial v}}{\frac{\partial F(x^*,\theta,v)}{\partial x^*}}$, provided that $\frac{\partial F(x^*,\theta,v)}{\partial x^*} \neq 0$. We have $\frac{\partial F(x^*,\theta,v)}{\partial v} = [1+\theta - p(x^*)]\, p'(x^*)$. And $\frac{\partial F(x^*,\theta,v)}{\partial x^*} \leq 0$ as above. Then $\frac{\partial F(x^*,\theta,v)}{\partial v} > 0$ since $p'(x^*) > 0$. Hence, $\frac{dx^*}{dv} > 0$.

3) Effect of the R&D efficiency: Define: $F(x^*, \alpha) = v\,[1+\theta - p(x^*)]\, p'(x^*) - 2\alpha = 0$. Applying the implicit functions theorem and following the same rationale as above, $\frac{\partial F(x^*,\alpha)}{\partial x^*} = -vp'(x^*)^2 + v\,[1+\theta-p(x^*)]\, p''(x^*) < 0$ and $\frac{\partial F(x^*,\alpha)}{\partial \alpha} = -2 < 0$. It follows that $\frac{dx^*}{d\alpha} \leq 0$. *QED.*

Proof of lemma 2. The proof is by contradiction. "*pat*" denotes the patent regime and "*n*" the "no protection regime". Assume that $p'(x) > 0$, $p''(x) \leq 0$ and $c'(x)$ is constant.

1) Suppose that $x_s^* < x_n^*$. Then it must be that $p'(x^{*,s}) \geq p'(x^{*,n})$ since $p''(x) \leq 0$. Substituting for the values of these derivatives given by (2) and (4), we have $\frac{1}{v[1-p(x_s^*)]} \geq \frac{2}{v[1-p(x_n^*)]}$ which implies:

$$p(x^{*,n}) \leq 2p(x^{*,s}) - 1. \tag{21}$$

But clearly, $2p(x^{*,s}) - 1 < p(x^{*,s})$. Those two inequalities combine into:

$$p(x^{*,n}) \leq 2p(x^{*,s}) - 1 < p(x^{*,s}), \tag{22}$$

which implies, since $p'(x) \geq 0$:

$$x^{*,n} < x^{*,s}. \tag{23}$$

This clearly contradicts our initial assumption.

2) Suppose $x^{*,s} > x^{*,pat}$. This implies that $p'(x^{*,s}) \leq p'(x^{*,pat})$ since $p''(x) \leq 0$. Given (2) and (4), this equivalent to $\frac{1}{v[1-p(x^{*,s})]} \leq \frac{2}{v[2-p(x^{*,pat})]}$ or

$$p(x^{*,pat}) \geq 2p(x^{*,s}). \tag{24}$$

And clearly

$$2p(x^{*,s}) > p(x^{*,s}). \tag{25}$$

Combining (26) and (27) yields

$$p(x^{*,pat}) \geq 2p(x^{*,s}) > p(x^{*,s}), \tag{26}$$

which implies, since $p'(x) \geq 0$

$$x^{*,pat} > x^{*,s}. \tag{27}$$

This obviously contradicts our initial assumption. *QED*

Proof of proposition 1: Society can optimally choose θ by constraining the individual R&D's investment $x^*(\theta)$ to be equal to the socially optimal R&D investment $x^{*,s}$ which is independent of θ. Formally $x^*(\theta) = x^{*,s}$ implies $p'(x^*(\theta)) = p'(x^{*,s})$. Using (2) and (4):

$$\frac{2c'(x^{*,s})}{v\left[1+\theta-p(x^{*,s})\right]} = \frac{c'(x^{*,s})}{v\left[1-p(x^{*,s})\right]},$$

which is equivalent to

$$\theta = 1 - p(x^{*,s}) = \theta^*.$$

QED.

Proof of corollary 1: We analyze how θ^* is affected by a change in v and in R&D efficiency.

1) value of the innovation: By the chain rule, $\frac{d\theta^*}{dv} = -\frac{dp(x^{*,s})}{dx^{*,s}}\frac{dx^{*,s}}{dv}$. We have $\frac{\partial p(x^{*,s})}{\partial x^{*,s}} > 0$ by assumption, so $sign\left(\frac{d\theta^*}{dv}\right) = -sign\left(\frac{dx^{*,s}}{dv}\right)$. Define $G(x^{*,s}; v) = v\left[1-p(x^{*,s})\right]p'(x^{*,s}) - c'(x^{*,s}) = 0$. By the implicit functions theorem, assuming that $\frac{\partial G(x^{*,s},v)}{\partial x^{*,s}} \neq 0$, we have $\frac{dx^{*,s}}{dv} = -\frac{\frac{\partial G(x^{*,s},v)}{\partial v}}{\frac{\partial G(x^{*,s},v)}{\partial x^{*,s}}}$. Computing this derivative, we get $\frac{\partial G(x^{*,s},v)}{\partial v} = p'(x^{*,s})(1-p(x^{*,s})) \geq 0$ and $\frac{\partial G(x^{*,s},v)}{\partial x^{*,s}} = p''(x^{*,s})v(1-p(x^{*,s})) - vp'(x^{*,s})^2 - c''(x^{*,s}) < 0$. Hence $\frac{dx^{*,s}}{dv} \geq 0$ and it follows that $\frac{d\theta^*}{dv} \leq 0$.

2) R&D efficiency:

By the chain rule, $\frac{d\theta^*}{d\alpha} = -\frac{dp(x^{*,s})}{dx^{*,s}}\frac{dx^{*,s}}{d\alpha}$. We have $\frac{\partial p(x^{*,s})}{\partial x^{*,s}} > 0$ so $sign\left(\frac{d\theta^*}{d\alpha}\right) = -sign\left(\frac{dx^{*,s}}{d\alpha}\right)$. Define $H(x^{*,s},\alpha) = v\left[1-p(x^{*,s})\right]p'(x^{*,s}) - \alpha = 0$. By the implicit functions theorem, assuming that $\frac{\partial H(x^{*,s},v)}{\partial x^{*,s}} \neq 0$, we have $\frac{dx^{*,s}}{d\alpha} = -\frac{\frac{\partial H(x^{*,s},\alpha)}{\partial \alpha}}{\frac{\partial H(x^{*,s},\alpha)}{\partial x^{*,s}}}$. Computing this derivative, we get $\frac{\partial H(x^{*,s},\alpha)}{\partial \alpha} = -1 < 0$ and $\frac{\partial H(x^{*,s},\alpha)}{\partial x^{*,s}} = p''(x^{*,s})v(1-p(x^{*,s})) - vp'(x^{*,s})^2 < 0$. Hence, $\frac{dx^{*,s}}{d\alpha} < 0$ and it follows that $\frac{d\theta^*}{d\alpha} > 0$. *QED.*

Proof of lemma 3. The optimal investment in the copyright regime is implicitly defined by:

$$c(\widehat{x}^{*,cop}) = \frac{v}{2}\left[1+\theta(1-p(\widehat{x}^{*,cop}))\right] - c'(\widehat{x}^{*,cop})h(\widehat{x}^{*,cop}).$$

To analyze comparative statics on $\widehat{x}^{*,cop}$ we use the implicit functions theorem. We define:

$$K(\widehat{x}^{*,cop};\theta;v)=c(\widehat{x}^{*,cop})-\frac{v}{2}\left[1+\theta(1-p(\widehat{x}^{*,cop}))\right]+c'(\widehat{x}^{*,cop})h(\widehat{x}^{*,cop})=0.$$

1) backward protection θ : The implicit functions theorem states that, provided $\frac{\partial K}{\partial \widehat{x}^{*,cop}}\neq 0$, we have: $\frac{\partial \widehat{x}^{*,cop}}{\partial \theta}=-\frac{\frac{\partial K}{\partial \theta}}{\frac{\partial K}{\partial \widehat{x}^{*,cop}}}$. Computing this ratio yields: $\frac{\partial K}{\partial \theta}=-\frac{v}{2}(1-p(\widehat{x}^{*,cop}))\leq 0$ and $\frac{\partial K}{\partial \widehat{x}^{*,cop}}=c'(\widehat{x}^{*,cop})+\frac{v}{2}\theta p'(\widehat{x}^{*,cop})+c''(\widehat{x}^{*,cop})h(\widehat{x}^{*,cop})+c'(\widehat{x}^{*,cop})h'(\widehat{x}^{*,cop})$. We have $h'(\widehat{x}^{*,cop})=\frac{-p'(\widehat{x}^{*,cop})^2-[1-p(\widehat{x}^{*,cop})]p''(\widehat{x}^{*,cop})}{p'(\widehat{x}^{*,cop})^2}$. Given assumption 1, $h'(\widehat{x}^{*,cop})\geq 0$. In addition, $c''(\widehat{x}^{*,cop})=0$, $c'(\widehat{x}^{*,cop})=\alpha>0$ and $p'(\widehat{x}^{*,cop})\geq 0$. It follows that $\frac{\partial K}{\partial \widehat{x}^{*,cop}}>0$.

We can conclude that for both models the conditions of application of the implicit function theorem are satisfied and that $\frac{\partial \widehat{x}^{*,cop}}{\partial \theta}\geq 0$.

2) value of the innovation: The implicit functions theorem states that, provided $\frac{\partial K}{\partial \widehat{x}^{*,cop}}\neq 0$, we have: $\frac{\partial \widehat{x}^{*,cop}}{\partial v}=-\frac{\frac{\partial K}{\partial v}}{\frac{\partial K}{\partial \widehat{x}^{*,cop}}}$. We already know the sign of $\frac{\partial K}{\partial \widehat{x}^{*,cop}}$ from above. And $\frac{\partial K}{\partial v}=-\frac{1}{2}\left[1+\theta(1-p(\widehat{x}^{*,cop}))\right]\leq 0$. Hence we can conclude that $\frac{\partial \widehat{x}^{*,cop}}{\partial v}\geq 0$.

3) R&D efficiency: Define: $K(\widehat{x}^{*,cop};\alpha)=\alpha\widehat{x}^{*,cop}-\frac{v}{2}\left[1+\theta(1-p(\widehat{x}^{*,cop}))\right]+\alpha h(\widehat{x}^{*,cop})=0$. Applying the implicit functions theorem and following the same rationale as above, we have $\frac{\partial K}{\partial \alpha}=\widehat{x}^{*,cop}+h(\widehat{x}^{*,cop})\geq 0$. We already know that $\frac{\partial K}{\partial \widehat{x}^{*,cop}}>0$. Hence, $\frac{\partial \widehat{x}^{*,cop}}{\partial \alpha}\leq 0$. *QED.*

Appendix B: Proofs of the results in section 5 (the application section).

Proof of result 2:

Consider first investment in every period but the first one. In the patent regime: $\widetilde{x}^*=v-1$ is directly the aggregate per-period investment (since there is only one firm conducting R&D in all subsequent periods). And for society, the per-period aggregate investment should be $2\times\widehat{x}^{*,s}=v-1$. It follows that in all period except the first one, the socially optimal aggregate investment is equal to the aggregate investment in a patent regime.

Consider then the first period. In the patent regime, we have $\widehat{x}^{*,pat}=v-1-\ln 2+\ln\left(1+e^{-\widehat{x}^{*,pat}}\right)$. It follows that the aggregate investment is $2\times$ $\widehat{x}^{*,pat}=2v-2-2\ln 2+$

$2\ln\left(1+e^{-\widehat{x}^{*,pat}}\right)$. And the socially optimal aggregate investment in the first period is $2\times\widehat{x}^{*,s} = v-1$ again. Now, $2\times\ \widehat{x}^{*,pat} \geq 2\times\widehat{x}^{*,s}$ if and only if $2\ln\left(1+e^{-\widehat{x}^{*,pat}}\right) \geq -v+2.386$. For all $v \geq 2.386$, the right-hand side is negative so that the inequality holds (the left-hand side being positive). *QED.*

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Technical Appendix

Appendix I

We show that assumption 1 ($2[p'(x)]^2 \leq -(1-p(x))p''(x)$) is a sufficient condition for the objective functions to reach a maximum in all intellectual property regimes, when innovation is cumulative.

1 Patent regime

■ *Every period but the first period.* The objective function is:

$$U_i^1 = \overbrace{\frac{1}{1-p(\widetilde{x})}}^{T} \left[\underbrace{-c(\widetilde{x}) + vp(\widetilde{x})}_{U} \right].$$

$$\frac{\partial^2 U_i^1}{\partial \widetilde{x}^2} = \frac{\partial^2 T}{\partial \widetilde{x}^2} U + T \frac{\partial^2 U}{\partial \widetilde{x}^2} + 2 \frac{\partial T}{\partial \widetilde{x}} \frac{\partial U}{\partial \widetilde{x}}.$$

And

$$\frac{\partial T}{\partial \widetilde{x}} = \frac{p'(\widetilde{x})}{[1-p(\widetilde{x})]^2}; \; \frac{\partial^2 T}{\partial \widetilde{x}^2} = \frac{p''(\widetilde{x})(1-p(\widetilde{x})) + 2(p'(\widetilde{x}))^2}{[1-p(\widetilde{x})]^3}$$

$$\frac{\partial U}{\partial \widetilde{x}} = -c'(\widetilde{x}) + vp'(\widetilde{x}); \; \frac{\partial^2 U}{\partial \widetilde{x}^2} = -c''(\widetilde{x}) + vp''(\widetilde{x}).$$

Hence, after substitution:

$$\frac{\partial^2 U_i^1}{\partial \widetilde{x}^2} = \frac{[p''(\widetilde{x})(1-p(\widetilde{x})) + 2p'(\widetilde{x})^2][-c(\widetilde{x}) + vp(\widetilde{x})]}{[1-p(\widetilde{x})]^3} + \frac{-c''(\widetilde{x}) + vp''(\widetilde{x})}{1-p(\widetilde{x})} + \frac{2p'(\widetilde{x})(-c'(\widetilde{x}) + vp'(\widetilde{x}))}{[1-p(\widetilde{x})]^2}.$$

This must be less or equal to zero for $\widetilde{x}^*$ to be a maximum, which yields:

$$[p''(\widetilde{x})(1-p(\widetilde{x})) + 2p'(\widetilde{x})^2][-c(\widetilde{x}) + vp(\widetilde{x})] + \left\{ \begin{array}{c} (1-p(\widetilde{x}))^2[-c''(\widetilde{x}) + vp''(\widetilde{x})] \\ +2p'(\widetilde{x})(1-p(\widetilde{x}))[-c'(\widetilde{x}) + vp'(\widetilde{x})] \end{array} \right\} \leq 0. \quad (1)$$

Given our assumptions on $p(.)$ and $c(.)$, condition (1) is given by:

$$[p''(\widetilde{x})(1-p(\widetilde{x})) + 2p'(\widetilde{x})^2][-\alpha\widetilde{x} + vp(\widetilde{x})] + \left\{ \begin{array}{c} (1-p(\widetilde{x}))^2 vp''(\widetilde{x}) \\ +2p'(\widetilde{x})(1-p(\widetilde{x}))[-\alpha + vp'(\widetilde{x})] \end{array} \right\} \leq 0.$$

After developing and re-arranging terms, we have:

$$\underbrace{-2\alpha p'(\widetilde{x})(1-p(\widetilde{x}))}_{-} + (v-\alpha\widetilde{x})\left[\underbrace{p''(\widetilde{x})(1-p(\widetilde{x})) + 2p'(\widetilde{x})^2}_{H(\widetilde{x})}\right] \leq 0.$$

Now consider $H(\widetilde{x})$. By assumption 1, we know that it is less or equal to zero. Hence, the above inequality holds and we can conclude that $\frac{\partial^2 U_i^1}{\partial \widetilde{x}^2} \leq 0$.

■ *First period:*

$$U_i^2 = -c(\widehat{x}_i) + p(\widehat{x}_i)\left[1 - \frac{1}{2}p(\widehat{x}_j)\right]R,$$

where R is a constant given by $R = \frac{v-c(\widetilde{x}^*)}{1-p(\widetilde{x}^*)}$.

In that case,

$$\frac{\partial^2 U_i^2}{\partial \widehat{x}_i^2} = -c''(\widehat{x}_i) + p''(\widehat{x}_i)R\left[1 - \frac{1}{2}p(\widehat{x}_j)\right],$$

Given our assumptions on $p(.)$ and $c(.)$,

$$\frac{\partial^2 U_i^2}{\partial \widehat{x}_i^2} = p''(\widehat{x}_i)R\left[1 - \frac{1}{2}p(\widehat{x}_j)\right] \leq 0 \quad \text{since} \quad p'' \leq 0,$$

2 Copyright and "no protection" regimes

We have:

$$U_i^2 = \overbrace{\frac{1}{1-p(\widehat{x}_i)-p(\widehat{x}_j)+p(\widehat{x}_i)p(\widehat{x}_j)}}^{T}\left\{\underbrace{-c(\widehat{x}_i) + \frac{v}{2}\left[p(\widehat{x}_i)(1+\theta-p(\widehat{x}_j)) + p(\widehat{x}_j)(1-\theta)\right]}_{U_i}\right\}.$$

And

$$\frac{\partial^2 U_i^2}{\partial \widehat{x}_i^2} = \frac{\partial^2 T}{\partial x_i^2}U_i + T\frac{\partial^2 U_i}{\partial x_i^2} + 2\frac{\partial T}{\partial x_i}\frac{\partial U_i}{\partial x_i},$$

with

$$\frac{\partial T}{\partial \widehat{x}_i} = \frac{p'(\widehat{x}_i)(1-p(\widehat{x}_j))}{\underbrace{(1-p(\widehat{x}_i)-p(\widehat{x}_j)+p(\widehat{x}_i)p(\widehat{x}_j))^2}_{S}} \quad ;$$

$$\frac{\partial^2 T}{\partial \widehat{x}_i^2} = \frac{(1-p(\widehat{x}_j))\left[p''(\widehat{x}_i)\overbrace{(1-p(\widehat{x}_i)-p(\widehat{x}_j)+p(\widehat{x}_i)p(\widehat{x}_j))}^{S} + 2p'(\widehat{x}_i)^2(1-p(\widehat{x}_j))\right]}{\underbrace{(1-p(\widehat{x}_i)-p(\widehat{x}_j)+p(\widehat{x}_i)p(\widehat{x}_j))^3}_{S}},$$

and

$$\frac{\partial U_i}{\partial \widehat{x}_i} = -c'(\widehat{x}_i) + \frac{v}{2}p'(\widehat{x}_i)(1+\theta-p(\widehat{x}_j)) \quad ; \quad \frac{\partial^2 U_i}{\partial \widehat{x}_i^2} = -c''(\widehat{x}_i) + \frac{v}{2}p''(\widehat{x}_i)(1+\theta-p(\widehat{x}_j)),$$

Substituting back into the expression for $\frac{\partial^2 U_i^2}{\partial \widehat{x}_i^2}$:

$$\begin{aligned}\frac{\partial^2 U_i^2}{\partial \widehat{x}_i^2} &= \frac{(1-p(\widehat{x}_j))\left[p''(\widehat{x}_i)S + 2p'(\widehat{x}_i)^2(1-p(\widehat{x}_j))\right]}{S^3}[U_i] + \frac{1}{S}\left[-c''(\widehat{x}_i) + \frac{v}{2}p''(\widehat{x}_i)(1+\theta-p(\widehat{x}_j))\right] \\ &+2\frac{p'(\widehat{x}_i)(1-p(\widehat{x}_j))}{S^2}\left[-c'(\widehat{x}_i) + \frac{v}{2}p'(\widehat{x}_i)(1+\theta-p(\widehat{x}_j))\right].\end{aligned}$$

This expression must be less or equal to zero, which yields:

$$\left\{\begin{array}{c}(1-p(\widehat{x}_j)) \\ \left[p''(\widehat{x}_i)S + 2p'(\widehat{x}_i)^2(1-p(\widehat{x}_j))\right][U_i]\end{array}\right\} + \left\{\begin{array}{c}S^2\left[-c''(\widehat{x}_i) + \frac{v}{2}p''(\widehat{x}_i)(1+\theta-p(\widehat{x}_j))\right] + \\ 2Sp'(\widehat{x}_i)(1-p(\widehat{x}_j)) \\ \left[-c'(\widehat{x}_i) + \frac{v}{2}p'(\widehat{x}_i)(1+\theta-p(\widehat{x}_j))\right]\end{array}\right\} \leq 0 \tag{2}$$

Given our assumptions on $p(.)$ and $c(.)$, condition (2) is given by:

$$(1-p(\widehat{x}_j))\left[p''(\widehat{x}_i)S + 2p'(\widehat{x}_i)^2(1-p(\widehat{x}_j))\right][U_i] + \left\{\begin{array}{c}S^2\frac{v}{2}p''(\widehat{x}_i)(1+\theta-p(\widehat{x}_j)) + \\ 2Sp'(\widehat{x}_i)(1-p(\widehat{x}_j)) \\ \left[-\alpha + \frac{v}{2}p'(\widehat{x}_i)(1+\theta-p(\widehat{x}_j)\right]\end{array}\right\} \leq 0.$$

Re-arranging terms, this condition is equivalent to:

$$-2\alpha Sp(\widehat{x}_i)(1-p(\widehat{x}_j)) + \left\{\begin{array}{c}(1-p(\widehat{x}_j))p''(\widehat{x}_i)SU_i + \\ 2p'(\widehat{x}_i)^2(1-p(\widehat{x}_j))U_i + \\ S^2\frac{v}{2}p''(\widehat{x}_i)(1+\theta-p(\widehat{x}_j)) + \\ 2Sp'(\widehat{x}_i)(1-p(\widehat{x}_j))\frac{v}{2}p'(\widehat{x}_i)(1+\theta-p(\widehat{x}_j))\end{array}\right\} \leq 0.$$

Developing, re-arranging the expression into brackets and simplifying yields:

$$\underbrace{-2\alpha Sp(\widehat{x}_i)}_{-} + \left[\underbrace{U_i + S\frac{v}{2}(1+\theta-p(\widehat{x}_j))}_{+}\right]\left[\underbrace{p''(\widehat{x}_i)S + 2p'(\widehat{x}_i)^2(1-p(\widehat{x}_j))}_{H(\widehat{x}_i,\widehat{x}_j)}\right] \leq 0.$$

Now consider $H(\widehat{x}_i, \widehat{x}_j)$. Given that $S = 1-p(\widehat{x}_i)-p(\widehat{x}_j)+p(\widehat{x}_i)p(\widehat{x}_j) = (1-p(\widehat{x}_i))(1-p(\widehat{x}_j))$, we have:

$$H(\widehat{x}_i, \widehat{x}_j) = p''(\widehat{x}_i)(1-p(\widehat{x}_i))(1-p(\widehat{x}_j)) + 2p'(\widehat{x}_i)^2(1-p(\widehat{x}_j).$$

This is less or equal to zero if and only if:

$$p''(\widehat{x}_i)(1-p(\widehat{x}_i))+2p'(\widehat{x}_i)^2 \leq 0.$$

But this is exactly our assumption 1.

Hence, $H(\widehat{x}_i,\widehat{x}_j) \leq 0$ and we can conclude that $\frac{\partial^2 \widehat{U}_i^2}{\partial \widehat{x}_i^2} \leq 0$.

3 Society

We have

$$U_s = \overbrace{\frac{1}{1-p(\widehat{x}_1)-p(\widehat{x}_2)+p(\widehat{x}_1)p(\widehat{x}_2)}}^{T}\left\{\underbrace{-c(\widehat{x}_1)-c(\widehat{x}_2)+v\left[p(\widehat{x}_1)+p(\widehat{x}_2)-p(\widehat{x}_1)p(\widehat{x}_2)\right]}_{U_s}\right\},$$

and:

$$\frac{\partial^2 U_s}{\partial \widehat{x}_1^2} = \frac{\partial^2 T}{\partial \widehat{x}_1^2}U_s + T\frac{\partial^2 U_s}{\partial \widehat{x}_1^2} + 2\frac{\partial T}{\partial \widehat{x}_1}\frac{\partial U_s}{\partial \widehat{x}_1},$$

with:

$$\frac{\partial T}{\partial x_1} = \frac{p'(\widehat{x}_1)(1-p(\widehat{x}_2))}{\underbrace{(1-p(\widehat{x}_1)-p(\widehat{x}_2)+p(\widehat{x}_1)p(\widehat{x}_2))}_{S}{}^2};$$

$$\frac{\partial^2 T}{\partial x_1^2} = \frac{(1-p(\widehat{x}_2))\left[p''(\widehat{x}_1)\overbrace{(1-p(\widehat{x}_1)-p(\widehat{x}_2)+p(\widehat{x}_1)p(\widehat{x}_2))}^{S}+2p'(\widehat{x}_1)^2(1-p(\widehat{x}_2))\right]}{\underbrace{(1-p(\widehat{x}_1)-p(\widehat{x}_2)+p(\widehat{x}_1)p(\widehat{x}_2))}_{S}{}^3},$$

and

$$\frac{\partial U_s}{\partial x_1} = -c'(\widehat{x}_1)+vp'(\widehat{x}_1)(1-p(\widehat{x}_2)) \quad ; \quad \frac{\partial^2 U_s}{\partial x_1^2} = -c''(\widehat{x}_1)+vp''(\widehat{x}_1)(1-p(\widehat{x}_2)).$$

Substituting back into the expression for $\frac{\partial^2 U_s}{\partial \widehat{x}_1^2}$:

$$\begin{aligned}\frac{\partial^2 U_s}{\partial \widehat{x}_1^2} &= \frac{(1-p(\widehat{x}_2))\left[p''(\widehat{x}_1)S+2p'(\widehat{x}_1)^2(1-p(\widehat{x}_2))\right]}{S^3}U_s + -c''(\widehat{x}_1)+\frac{1}{S}[vp''(\widehat{x}_1)(1-p(\widehat{x}_2))]+ \\ &\quad 2\frac{p'(\widehat{x}_1)(1-p(\widehat{x}_2))}{S^2}[-c'(\widehat{x}_1)+vp'(\widehat{x}_1)(1-p(\widehat{x}_2))].\end{aligned}$$

This must be less or equal to zero. This condition yields:

$$(1-p(\widehat{x}_2))\left[p''(\widehat{x}_1)S+2p'(\widehat{x}_1)^2(1-p(\widehat{x}_2))\right]U_s+\left\{\begin{array}{c} S^2[vp''(\widehat{x}_1)(1-p(\widehat{x}_2))]+ \\ 2Sp'(\widehat{x}_1)(1-p(\widehat{x}_2)) \\ [-c'(\widehat{x}_1)+vp'(\widehat{x}_1)(1-p(\widehat{x}_2))] \end{array}\right\}\leq 0. \quad (3)$$

Given our assumptions on $p(.)$ and $c(.)$, condition (3) is given by:

$$(1-p(\widehat{x}_2))\left[p''(\widehat{x}_1)S+2p'(\widehat{x}_1)^2(1-p(\widehat{x}_2))\right]U_s+\left\{\begin{array}{c} S^2[vp''(\widehat{x}_1)(1-p(\widehat{x}_2))]+ \\ 2Sp'(\widehat{x}_1)(1-p(\widehat{x}_2)) \\ [-\alpha+vp'(\widehat{x}_1)(1-p(\widehat{x}_2))] \end{array}\right\}\leq 0.$$

After developing and re-arranging the terms:

$$\underbrace{-2\alpha Sp'(\widehat{x}_1)(1-p(\widehat{x}_2))}_{-}+(1-p(\widehat{x}_2))(\underbrace{U_s+Sv}_{+})\left[\underbrace{Sp''(\widehat{x}_1)+2p'(\widehat{x}_1)^2(1-p(\widehat{x}_2))}_{H(\widehat{x}_1,\widehat{x}_2)}\right]\leq 0.$$

Consider $H(\widehat{x}_1,\widehat{x}_2)$. Given that $S=1-p(\widehat{x}_1)-p(\widehat{x}_2)+p(\widehat{x}_1)p(\widehat{x}_2)=(1-p(\widehat{x}_1))(1-p(\widehat{x}_2))$,we have:

$$H(\widehat{x}_1,\widehat{x}_2)=(1-p(\widehat{x}_1))(1-p(\widehat{x}_2))p''(\widehat{x}_1)+2p'(\widehat{x}_1)^2(1-p(\widehat{x}_2)).$$

This is less or equal to zero if and only if:

$$(1-p(\widehat{x}_1))p''(\widehat{x}_1)+2p'(\widehat{x}_1)^2\leq 0$$

which is our assumption 1.

Hence, we can conclude that $\frac{\partial^2 U_s}{\partial \widehat{x}_1^2}\leq 0$.

To conclude, notice that by symmetry, the same demonstration holds for the second derivative of the objective function with respect to $\widehat{x}_2$.

Appendix II: Application: $p(x)=1-e^{-x}$ and $c(x)=x$.

In this case, assumption 1 is not sufficient to guarantee that the objective function reaches a maximum. Nevertheless, we can show that this specification guarantees a maximum in all IP regimes.

Notice that $p'(x)=e^{-x}$; $p'(x)=-e^{-x}$; $1-p(x)=e^{-x}$.

- **Patent regime**. In all period but the first one, the objective function writes as $U_i^1 = e^{\widetilde{x}}\left[-\widetilde{x}+v\left(1-e^{-\widetilde{x}}\right)\right]$. It follows that $\frac{\partial U_i^1}{\partial \widetilde{x}} = e^{\widetilde{x}}\left(-\widetilde{x}+v-1\right)$ and $\frac{\partial^2 U_i^1}{\partial \widetilde{x}^2} = e^{\widetilde{x}}(-\widetilde{x}+v-2)$.

  This is negative for all $\widetilde{x} \geq v-2$. Given that $\widetilde{x}^* = v-1$, $\widetilde{x}^*$ is a maximum.

In the first period, using the result in Appendix I, we have: $\frac{\partial^2 U_i^2}{\partial \widehat{x}_i^2} = -e^{-\widehat{x}_i} R\left[1-\frac{1}{2}(1-e^{-\widehat{x}_j})\right]$. The term into brackets is positive. Now, $R = \frac{v-c(\widetilde{x}^*)}{1-p(\widetilde{x}^*)} = \frac{1}{e^{-\widetilde{x}^*}} \geq 0$. Hence, $\frac{\partial^2 U_i^2}{\partial \widehat{x}_i^2} \leq 0$.

- **Copyright regime (and no protection)**. Replacing $p(x)$ by $1-e^{-x}$ and $c(x)$ by x in expression (13), the objective function writes as:

$$U_i^2 = e^{\widehat{x}_i+\widehat{x}_j}\left\{-\widehat{x}_i+\frac{v}{2}\left[(1-e^{-\widehat{x}_i})(1+\theta e^{-\widehat{x}_j})+e^{-\widehat{x}_i}(1-e^{-\widehat{x}_j})(1-\theta)\right]\right\}.$$

After computation, it follows that:

$$\frac{\partial U_i^2}{\partial \widehat{x}_i} = e^{\widehat{x}_i+\widehat{x}_j}\left[-\widehat{x}_i+e^{-\widehat{x}_i}(\frac{v}{2}-1)+\theta e^{-\widehat{x}_i}e^{-\widehat{x}_j}(\frac{v}{2}-1)+\theta e^{-\widehat{x}_j}\right].$$

And then:

$$\frac{\partial^2 U_i^2}{\partial \widehat{x}_i^2} = e^{\widehat{x}_i+\widehat{x}_j}\left(-1-\widehat{x}_i+\theta e^{-\widehat{x}_j}\right).$$

This is less or equal to zero if and only if $-\widehat{x}_i+\theta e^{-\widehat{x}_j} \leq 1$ which always holds since $\theta e^{-\widehat{x}_j} \in [0,1]$.

- **Society.** Replacing $p(x)$ by $1-e^{-x}$ and $c(x)$ by x in expression (17), the objective function writes as:

$$U_s = e^{\widehat{x}_1+\widehat{x}_2}\left[-(\widehat{x}_1+\widehat{x}_2)+v\left(1-e^{-\widehat{x}_1-\widehat{x}_2}\right)\right]. \tag{4}$$

Notice that for society, what matters is the sum $\widehat{x}_1+\widehat{x}_2=\widehat{x}$. Indeed, how investment is divided between firm 1 and firm 2 is irrelevant. Hence, we can assume that society optimizes directly with respect to $\widehat{x}$ (the sum of each firm's investment), and then decide how to split the result between the firms. It simplifies the calculations. Replacing $\widehat{x}_1+\widehat{x}_2$ by one variable $\widehat{x}$ in (6) yields:

$$\widehat{x} = e^{\widehat{x}}\left[-\widehat{x}+v(1-e^{-\widehat{x}})\right]. \tag{5}$$

It follows that $\frac{\partial U_s}{\partial \widehat{x}} = e^{\widehat{x}}(v - 1 - \widehat{x})$ and $\frac{\partial^2 U_s}{\partial \widehat{x}^2} = e^{\widehat{x}}(v - 2 - \widehat{x})$. This is less than or equal to zero if and only if $\widehat{x} \geq v - 2$. Given that the optimal $\widehat{x}$ was found to be $\widehat{x}^{*,s} = v - 1$, and $v - 1 > v - 2$, $\widehat{x}^{*,s}$ a maximum.

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