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Abstract

In the strategic level decision-making of road network maintenance, the condition of the network plays a central role, both as an input to the evaluation process and an outcome of the proposed maintenance works. The condition is perceived by the road users and affects the costs of the traffic flow on the network. Without maintenance the condition of the network deteriorates. The current or future condition and the deterioration rate of the road network can be estimated using measurements or statistical models. The resulting estimates of both methods exhibit variation to some degree. But how accurate is the condition information when based on current measurements as opposed to predicting the condition based on previous years' measurements? How does the time interval between the measurements (1 to 3 years) affect the accuracy of the condition information?

A data set of International roughness index (IRI) measurements over three years (2000 – 2002) was drawn from the road condition data base of the Finnish Road Administration (Finnra). The data was randomly partitioned into two sets of equal size and one of these sets was used for developing regression models that predict the deterioration of roughness over one and two years. The models were validated using the other half of the data set not used for fitting the models.

Since the IRI-values were found to be approximately log-normally distributed, the models were developed for the natural logarithms of IRI. The comparison of the residual distribution in the logarithmic model and measuring accuracy in logarithmic terms facilitates direct consideration of the relative accuracies. The results indicated an increase of 2.5 – 4.6 %-units in the standard deviation involved in the prediction of the logarithmic IRI of a road section compared to measuring it. The benefits from measuring instead of predicting the current condition from previous measurement have to be compared with the cost of data collection. The results of this study emphasise the importance of the accuracy of the measured data.

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1 Introduction and objectives

The purpose of strategic level decision-making is to justify the funding needed for road network maintenance and to allocate these funds effectively between different sub-networks and maintenance actions. The condition of the road network is an essential input in the decision-making process, and it enables the quantification of benefits from maintenance, both to the road user and the agency.

Road condition, or rather its decay, is manifested in the change of the surface profile and cracking of the surface layer. The surface profile, usually considered as the longitudinal and transversal unevenness of the road, is measured for example using a road surface monitoring vehicle based on laser technology. The cracking of the surface layer can be monitored visually or using a high-speed data collection of digital images connected with automated image processing.

Longitudinal unevenness has the greatest impact on the driving comfort and the road user costs (Sayers et al. 1986). The most widely used measure of longitudinal unevenness is the International Roughness Index, IRI (see e.g. UMTRI 2004). Several variables are calculated from the transverse profile of the road, the most obvious of them being the rut depth. The extent and severity of cracking of the surface layer can be expressed for individual types of cracking, or a combined damage index can be used. However, in this paper, we use data for longitudinal unevenness in terms of IRI [mm/m] to demonstrate our methodology that is applicable to any condition variable measured and modelled for a road network.

Often it is not reasonable to measure the entire road network in consideration each year. Various performance models have been proposed, that can be used for predicting either future or current road condition based on previous measurements (European Commission 1999, Jämsä 2000, Odoki & Kerali 2000). From this ground, the question arises how accurate the condition information is in such a case. It is known that certain variation in the measured condition variables exists and that this variation affects the estimated deterioration of the road and the accuracy of the predicted condition information.

The focus of this paper is on how to collect the road network condition data for strategic level economic analysis. The objective of the research was to provide answers to the following questions:
1. How does predicting road network condition based on previous years' measurements, instead of measuring the condition, affect the accuracy of the condition distribution?
2. How does the time interval between the measurements (1 to 3 years) affect the accuracy of the condition information?

In section 2 of this paper, the data set used in the analysis, and the procedures of choosing it are described. The methodology used in modelling and in the estimation of the accuracy of both measured and modelled values is described in section 3. In section 4 the results are shown and discussed, and in section 5, the summary and conclusions of this study are presented.

2 Data

2.1 Description of the data

A data set was drawn from the road condition data base (Kurre) of the Finnish Road Administration (Finnra). For the purposes of this study, only data for roughness measurements (IRI, mm/m) was considered (Sayers et al. 1986, Sayers 1995). A similar analysis can, and indeed should, be made for any of the various road condition variables included in the condition data base.

The condition data in the data base is stored in 100-meter sections. For the data set used in this paper, three consecutive years 2000-2002 were considered. All 100-meter sections with a roughness measurement on all three years 2000, 2001 and 2002 were selected for the analysis. Sections with recorded maintenance activities between the two measurements were excluded. Further reduction of data is described in section 2.2. The data base covers the whole network of paved public roads in Finland, which is approximately 50 000 km. With these selection criteria a data set of 65 592 observations (6 559 km) was considered.

The distribution of IRI-values and the values of the natural logarithm of IRI for the data from year 2002 are shown in Figure 1 and the statistics that describe the entire data set are shown in Table 1. The mean value increases slightly in time which indicates deterioration of the road network. It can be noted that also the standard deviation slightly increases in time. The minimum and maximum values together with several percentile points are shown in Table 1 as well.
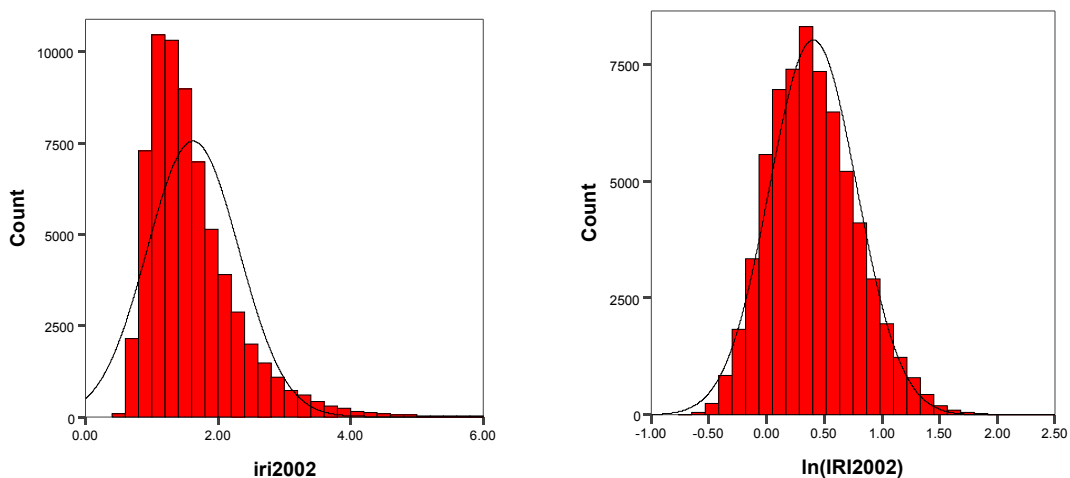
Figure 1   Distribution of IRI (left diagram) and the natural logarithm of IRI (right diagram) in 2002. Fitted normal curves are superimposed on the histograms in order to facilitate comparison of the distributions.

It is clearly seen, that the distribution of IRI is by no means normal, whereas the distribution of the logarithmic IRI is fairly close to normal. Our experience is that many road condition variables exhibit similar behaviour. They are, by definition, assigned positive values only, and extremely high values are not restricted. Even if some of the high values are caused by errors, some of them depict actual road condition. As a consequence, the resulting distributions are right skewed like the one for IRI shown in the left diagram in Figure 1. The skewness of the distribution is also illustrated by the skewness coefficient of IRI far greater

than 0 (see Table 1). The skewness values for the logarithmic IRI range from 0.404 to 0.428. The excess kurtosis is close to 0 for logarithmic values which also indicates normality of the distribution. We conclude that the logarithmic values of IRI can be described as being approximately normally distributed, i.e. IRI values can be considered log-normally distributed. As will be shown later, the use of logarithmic values also allows direct consideration of the relative accuracies of the measured and modelled values.

Table 1   **Statistics of the data set used in the analysis**

		IRI2000	IRI2001	IRI2002	ln(IRI2000)	ln(IRI2001)	ln(IRI2002)
N		65592	65592	65592	65592	65592	65592
Mean		1.50	1.55	1.62	0.335	0.365	0.404
Std. Deviation		0.615	0.656	0.693	0.366	0.373	0.380
Skewness		1.892	2.052	1.932	0.404	0.428	0.415
Excess kurtosis		7.47	9.22	7.43	0.18	0.22	0.14
Minimum		0.47	0.44	0.47	-0.755	-0.821	-0.755
Percentiles	1	0.67	0.69	0.70	-0.400	-0.371	-0.357
	5	0.80	0.82	0.85	-0.223	-0.198	-0.163
	10	0.89	0.91	0.94	-0.117	-0.094	-0.062
	20	1.02	1.04	1.08	0.020	0.039	0.077
	30	1.13	1.16	1.20	0.122	0.148	0.182
	40	1.24	1.28	1.32	0.215	0.247	0.278
	50	1.36	1.40	1.46	0.307	0.336	0.378
	60	1.49	1.54	1.61	0.399	0.432	0.476
	70	1.66	1.72	1.79	0.507	0.542	0.582
	80	1.89	1.96	2.05	0.637	0.673	0.718
	90	2.27	2.36	2.49	0.820	0.859	0.912
	95	2.66	2.78	2.93	0.978	1.022	1.075
	99	3.61	3.80	3.98	1.284	1.335	1.381
Maximum		9.99	11.77	11.02	2.302	2.466	2.400

2.2 Variation in the data and exclusion of odd observations

The roughness is measured using vehicles that are part of the normal traffic flow. A number of factors are known to affect the variation in the obtained data from any single year:
- The accuracy of the measurement equipment.
- The variability between different equipment of the same kind.
- The variation in the lateral position of the measurement vehicle with the same operator.
- The variation in the lateral position between the operators.
- The conditions prevailing at the time of measurement.
- The seasonal variation of the profile.

In addition to that, variation of the data in consecutive years is caused by:
- The variation of the profile from year to year (deterioration).
- Maintenance works that are not recorded.

Each vehicle has certain measurement accuracy, which is the variation of measured values between repeated runs along the same profile. This variation can be considered random and

is due to the mechanics of the measuring apparatus. A random error also exists between vehicles driving along exactly the same longitudinal profile. In practice, comparison of results from repeated runs using either the same or different vehicles in practically the same conditions (weather, etc.) is hampered by variation in the actually measured profile due to unavoidable lateral wander of the vehicle. In this study, the effect of variation in the lateral position both due to each operator and among several operators is assumed to be random.

The seasonal variation of the profile causes additional variation in the IRI values. The measurements for this data set were carried out between April and November. The effect of the seasonal variation has not been quantified. The effects of different sources of variation have been studied in detail by Karamihas et al. (1999).

Deterioration is the trend we are trying to capture using the logarithmic regression model. The other factors of variation mentioned above cause random variation around this trend. In our data set, we cannot explicitly determine the contribution of each factor to the total variation. Every year, quality control measurements covering some 1-4 % of the measuring program have been done. In the quality control measurements done in 2001 and 2002 (Hätälä and Ruotoistenmäki 2002), it was required that in 90 % of the cases, the absolute value of the difference in the measured IRI-value between any two vehicles (and operators) within the same year is less than 0.5 mm/m. The deterioration of IRI between two years' measurements should in principle be non-negative, but in the presence of measurement errors an improvement of 0.5 mm/m in the measured IRI due to random variation could be allowed.

The roughness of a road section can also be improved because maintenance works are actually carried out between any two measurements, but not recorded in the data base. The length of the works can be very short (<100 meters), or they can extend to several hundred meters. The works can be of any kind, but typically they are either light maintenance (correcting ruts) or localised reconstruction work. In a large data set, such as the one used in this study, sections with improved IRI with no recorded maintenance work do exist, even though the distribution of IRI shows that deterioration generally takes place (Figure 1 and Table 1).

We wanted to eliminate the most obvious errors due to unrecorded maintenance works from the data set. However, exclusion of single outliers, i.e. any *single* observations that show an improvement of IRI greater than 0.5 mm/m would cause a serious truncation bias in the data, see Figure 2a. This is undesirable and results in data that does not conform to basic requirements of regression analysis. We have therefore chosen to exclude only those measurements for which any *two or more* consecutive 100-meter sections have a change in the measured IRI less than -0.5 mm/m between any two years: IRI2002 – IRI2001< -0.5, IRI2001 – IRI2000< -0.5, or IRI2002 – IRI2000< -0.5. Figure 2b shows the remaining data after this operation. Obviously, the data is much less distorted by this procedure as compared to the procedure shown in Figure 2a.
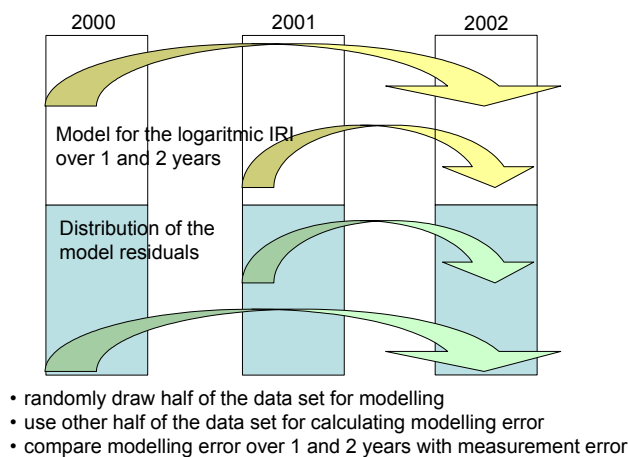
a)   b)
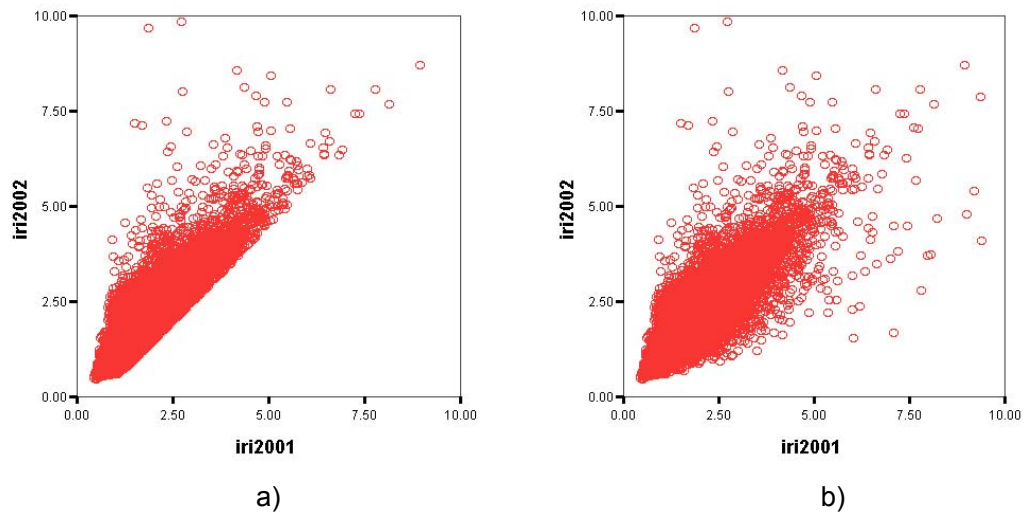
Figure 2   Scatter plots based on the data from 2001 and 2002: a) The disregarded method of excluding any single observation with IRI2002 – IRI2001 < -0.5; b) The adopted method of excluding observations only when measurements from two or more consecutive 100-meter sections exhibit IRI2002 – IRI2001 < -0.5.

3 Methods

3.1 Modelling

To compare the accuracy of current measurements and forecasted values based on previous years' measurements, cross-validation methods were used: The data set was randomly divided into two parts of approximately equal size (Figure 3). One half of the data set was used for developing regression models that predict the condition of a 100-meter section in the year 2002 based on its measured condition in the year 2000, 2001 or both. The identified models were then tested using the other half of the data set, which was not used for model determination. The residuals between the measured and predicted values were calculated to determine modelling accuracy which was then compared with the accuracy of the measurement.

• randomly draw half of the data set for modelling
• use other half of the data set for calculating modelling error
• compare modelling error over 1 and 2 years with measurement error

Figure 3   Illustration of the modelling and validation methodology

The form of the linear regression model that predicts the logarithmic IRI value in a particular year (lnY), based on the previous years' logarithmic measurements (lnX$_j$, j = 1, 2, ..., k) is

$$\ln Y = \beta_0 + \beta_1 \ln X_1 + \beta_2 \ln X_2 + ... + \beta_k \ln X_k + \varepsilon, \tag{1}$$

where β_0 is the regression constant;
$\quad\quad\quad \beta_j$ the regression coefficients of the independent variables, j = 1, 2, ..., k;
$\quad\quad\quad \varepsilon$ the error term.

In this study, the following regression models were considered:

$$\ln Y = \beta_0 + \beta_1 \ln X_2 + \varepsilon \tag{2a}$$
$$\ln Y = \beta_0 + \beta_1 \ln X_1 + \varepsilon \tag{2b}$$
$$\ln Y = \beta_0 + \beta_1 \ln X_1 + \beta_2 \ln X_2 + \varepsilon, \tag{2c}$$

where lnY is the logarithmic IRI in year 2002 and lnX$_1$ and lnX$_2$ are one and two year old logarithmic IRI-measurements, respectively.

3.2 Comparing the accuracy of the modelled and measured values

When using any of the equations (2a), (2b) or (2c) to predict the logarithmic IRI of a 100-meter section based on previous years' measurements, uncertainty is associated with each individual prediction. This uncertainty can be described by the standard deviation estimate s_Y of the predicted values, which in the case of one independent variable (equations 2a and 2b) is (see e.g. Pindyck & Rubinfeld 1997):

$$s_Y = s\sqrt{1 + \frac{1}{n} + \frac{(\ln x_0 - \overline{\ln x})^2}{SS_x}} \quad, \tag{3}$$

where s is the standard deviation estimate of the error term (ε) in the regression equation;
$\quad\quad\quad$ n the number of observations;
$\quad\quad\quad \ln x_0$ the value of the independent variable;
$\quad\quad\quad \overline{\ln x}$ the mean of the independent variable; and
$\quad\quad\quad SS_x$ the sum of squares of the independent variable lnx indicating the total variation of that variable, $SS_x = \sum_{i=1}^{n}(\ln x_i - \overline{\ln x})^2$.

In this case, the number of observations is so large (n = 32796) that both of the latter terms under the square root are close to zero. Therefore, the value of the square root is close to unity and the standard deviation of the predicted values can be estimated directly based on s, the standard deviation estimate of the error. We found this approximation to be correct to the fourth decimal.

The standard deviation estimate of the error (s in equation 3) is calculated from the residuals between the measured and predicted values of the logarithmic IRI in year 2002. The residuals e$_i$; i = 1, 2, ..., n, are calculated from the other half of the data set not used for fitting the models as follows:

$$e_i = \ln(\text{IRI2002}) - \ln(\widehat{\text{IRI2002}}), \qquad\qquad (4)$$

where $\ln(\text{IRI2002})$ is   the measured logarithmic IRI-value in 2002; and
      $\ln(\widehat{\text{IRI2002}})$ the predicted logarithmic IRI-value for 2002.

In short, the accuracy of modelling is considered based on the standard deviation estimate of the predicted values (i.e. s_Y given by equation 3), and is approximated by the standard deviation s of residuals e_i from equation (4). It is then compared with the standard deviation of the measured values of logarithmic IRI, calculated as explained in the remaining part of this section. An analogous analysis is performed in the case when there are two independent variables (equation 2c), i.e. when the logarithmic IRI value in the year 2002 is predicted based on the measured logarithmic IRI from both of the previous two years.

Due to the extent of the whole measurement program (approximately 30000 km annually) several (4 or 5, depending on the year) measurement vehicles have been used for production measurements, so that each 100-meter section is measured by one vehicle. The measurement accuracy is here understood as the standard deviation of the measured logarithmic IRI values. Since there is only one measurement per each 100-meter section and each vehicle, the accuracy of a single measurement is not known. However, an estimate was developed based on the standard deviation of the difference between the logarithms of the measured IRI values using two vehicles that measure the same 100-meter sections.

In addition to the production measurements, separate quality control measurements were taken using another vehicle for 3148 selected 100-meter sections. Each vehicle used for production measurements has also been used as a control vehicle for another one of the vehicles (Hätälä and Ruotoistenmäki 2002). The logarithmic difference d of the measured values from any two vehicles is given by

$$d = \ln(\text{IRI}_1) - \ln(\text{IRI}_2), \qquad\qquad (5)$$

where IRI_1 is the measured IRI value from production vehicle; and
      IRI_2 the measured IRI value from control vehicle.

In this analysis, the logarithmic differences from all pairs of vehicles have been combined into one data set. This was done by appending the measurement sequences from different roads so that the original order of 100-meter sections was retained. In this data set, the autocorrelation of the logarithmic differences d is slightly negative (r= –0.10; excluding those pairs where the successive 100-meter sections come from non-adjacent road sections), so that it has only a minor effect on the variance estimate of the logarithmic difference between measured values from any two vehicles, which is given by

$$\text{Var}[\ln(\text{IRI}_1) - \ln(\text{IRI}_2)] = \text{Var}[\ln(\text{IRI}_1)] + \text{Var}[\ln(\text{IRI}_2)] - 2\text{Cov}[\ln(\text{IRI}_1), \ln(\text{IRI}_2)], \qquad (6)$$

where Var denotes variance and Cov denotes covariance.

In the following it is assumed that 1) the variances of the logarithmic IRI are the same for all vehicles, i.e. $\text{Var}[\ln(\text{IRI}_1)] = \text{Var}[\ln(\text{IRI}_2)]$ for each individual 100-meter section, and that 2) the covariance term disappears for each individual 100-meter section, i.e. measurement errors of two vehicles are uncorrelated. In addition, it is assumed that 3) the difference $d = \ln(\text{IRI}_1) - \ln(\text{IRI}_2)$ is stationary across different 100-meter sections, i.e. it has constant mean and variance. In our data set these assumptions are approximately fulfilled, and thus the variance of the logarithmic IRI for a single measurement can be calculated as

$$\text{Var}\big[\ln(\text{IRI})\big] = \frac{\text{Var}\big[\ln(\text{IRI}_1) - \ln(\text{IRI}_2)\big]}{2} . \qquad (7)$$

As the standard deviation is calculated as the square root of variance, the standard deviation of the logarithmic IRI for a single measurement is given by

$$\text{Stdev}\big[\ln(\text{IRI})\big] = \frac{\text{Stdev}\big[\ln(\text{IRI}_1) - \ln(\text{IRI}_2)\big]}{\sqrt{2}} . \qquad (8)$$

4 Results and discussion

The accuracy of the different models (equations 3 and 4) and the measuring accuracy (equation 8) are presented in Table 2.

Table 2   The accuracy of modelling and measuring roughness

Regression model		Modelling accuracy	Measurement	Measuring accuracy
Independent variable	Dependent variable(s)			
ln(IRI2002)	ln(IRI2000)	0.1677	Quality control measurements in 2002	0.1213
	ln(IRI2001)	0.1560		
	ln(IRI2000) & ln(IRI2001)	0.1466		

The results show that the standard deviation of error is increased from 0.1213 when the logarithmic IRI is measured, to 0.1677 when a prediction model based on two-year old measurements is used. The logarithmic differences and the corresponding standard deviations can be interpreted as percentages. Thus the change is 4.6 %-units (0.1677 – 0.1213 = 0.0464). Similarly, the standard deviation of error is increased to 0.1560 (by 3.5 %-units) when a prediction model over one year is used and to 0.1466 (by 2.5 %-units) when the model predicting the logarithmic IRI is based on the measured logarithmic IRI from both of the previous two years.

The practical consequences of these results should be evaluated by considering how the increased uncertainty in the input affects the output of the decision-making process. The cost of measuring the IRI for the data set used in this study was in the neighbourhood of 17 € / km. Thus the cost of measuring the data set of 65 592 observations was approximately 111 500 €.

The effect of measuring can also be expressed as € / %-unit savings per kilometre. By measuring one kilometre of road, an improvement of 4.6 %-units in the accuracy of the logarithmic IRI is achieved, compared to modelling the logarithmic IRI value based on two-year old measurements. The cost of measurements (17 € / km) divided by the improvement in accuracy leads to a range of 3.7 - 6.7 € / %-unit savings per kilometre, depending on the model.

The benefits of the investment in the measurements should exceed the cost in order to justify the investment. The benefits may include improved accuracy of fund allocation and the use of data for other purposes such as design and quality control of procured maintenance works. The costs of wrong decisions due to the inaccuracy of the condition data could also be quantified, and used as lost or negative benefits in the analysis.

The time interval between the measurements has a slight but clear effect on the accuracy of the condition data. The results indicate that the condition of the road network should be predicted using as new measurements as possible. Also, if measurements from several years are available, they should be considered for prediction.

The modelling error includes the measurement error. Both the measurement error (each year) and the modelling error (from year to year) are far greater than the rate of deterioration of IRI. The following model illustrates this:

$$\ln(IRI2002) = 0.0656 + 0.9277\ln(IRI2001) \tag{9}$$

If the IRI of a road section is, for example, the median value 1.4 mm/m in year 2001, then according to equation (9), it is 1.46 mm/m in year 2002, which shows an increase of only 0.06 mm/m per year, or only 0.041 (=4.1 %) in logarithmic values. The standard deviation of the logarithmic IRI values based on the control measurements in 2002 is 0.1213, which is approximately three times the rate of deterioration! This demonstrates the continuing need to improve the measuring accuracy, as also noted, among others, by Virtala et al. (2004).

It has to be noted, that the numerical results depend on the data set that was used and more specifically, on the measurement equipment used for collecting the data. Therefore the estimates of the accuracy of the measured and predicted values or the models themselves cannot be generalised to other networks measured using other equipment. The method, however, can be transferred for use on any road network and any kind of equipment.

5 Summary and conclusions

The condition of a road network is an essential input to maintenance planning. The current or future condition and the deterioration rate of the road network are estimated using measurements and statistical models, both of which include variation. In this paper, these effects were studied using a data set of IRI-measurements from three years (2000-2002) drawn from the road condition data base of the Finnish Road Administration (Finnra). Two particular questions were addressed: How does predicting road network condition based on previous years' measurements, instead of measuring the condition, affect the accuracy of the condition distribution? How does the time interval between the measurements (1 to 3 years) affect the accuracy of the condition distribution? The main results of this study are the following:

- The standard deviation of the error in the logarithmic IRI value is increased by 2.5 – 4.6 %-units, when a model is used for predicting the current condition from previous years' measurements instead of measuring it. The amount of increase in the standard deviation depends on the age of the previous measurement and the corresponding model used.
- The practical consequences should be evaluated by considering how the increased uncertainty in the input affects the output of the decision-making process. A cost-benefit analysis of the condition measurements is recommended.
- The benefit from measurements can be expressed as € / %-unit savings per kilometre. The cost of measurements (17 € / km) divided by the improvement in accuracy leads to a range of 3.7 - 6.7 € / %-unit savings per kilometre, depending on the model.
- The results indicate that the condition of the road network should be predicted based on as new measurements as possible. Also, if measurements from several years are available, they should be considered for prediction.

- When determining the measurement error, one needs to consider the accuracy of the equipment, differences between several pieces of equipment and operators and conditions prevailing at the time of the measurement. This total variation in the measured data can be quantified from control measurements.
- The modelling error includes the measurement error. Both the measurement error (each year) and the modelling error (from year to year) are far greater than the rate of deterioration of IRI. Therefore, the greatest need still is to improve the quality of the measured data.

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