Mikko Leppämäki – Mikko Mustonen

SIGNALING WITH EXTERNALITY

Mikko Leppämäki – Mikko Mustonen

SIGNALING WITH EXTERNALITY

Signaling with Externality[1]

Mikko Leppämäki[2]
Helsinki School of Economics, RUESG and HECER

and

Mikko Mustonen[3]
Helsinki School of Economics and HECER

September 21, 2004

[2]**Address:** Department of Accounting and Finance, Helsinki School of Economics, P.O. Box 1210, FI-00101 Helsinki, Finland. e-mail: mikko.leppamaki@hkkk.fi
[3]**Address**: Department of Economics, P.O. Box 1210, FI-00101, Helsinki School of Economics, Finland. e-mail: mikko.mustonen@hkkk.fi

Abstract

We present a familiar - yet novel - signaling game where signaling in the labor market has an externality affecting the employers' profits. The externality arises due to the appearance of a free substitute (negative externalities) or a complement (positive externalities) in the product market as a result of the signaling activity. We examine in detail how the outcome of the signaling game is affected by the product market externalities and market size. It is shown first that in the case negative and mild positive externalities, the least cost separating equilibrium is standard. The level of credential, the "good" type will choose in order to separate from the "bad" type is lower (higher) with negative (positive) externality than in the case when externalities are absent. In contrast to previous literature, we show that when the magnitude of positive externalities is high enough, the least cost separating equilibrium is rather different in nature: The "good" type will choose the highest rather than the lowest possible levels of credential in order to separate from the "bad" type. Interestingly, in the case of very strong positive market externalities, the separation of workers' types is impossible, and we end up with a pooling equilibrium with maximum signaling. Finally, when considering the welfare effects, we show that under certain conditions, the private market solution may involve too little signaling compared to social optimum.

1 Introduction

In this paper we extend Spence's (1973) classical work on job market signaling into a case where the signaling activity has an externality affecting the employers' profits. The externality arises due to the appearance of a free substitute or a complement in the product market as a result of signaling activity in the labor market.

Already Spence (1974) discussed the possibility of signaling (education) increasing the labor embodied productivity, but the case where signaling has an externality effect via the consumer market is, to the best of our knowledge novel in the literature of signaling. In contrast to Spence we suggest that the potential effects of signaling are not (just) labor embodied, but instead are coming through an another channel, namely the product market. Consequently, then also the notion of a negative externality becomes meaningful and gets a natural interpretation. Our set up is thus general enough to incorporate both positive and negative market externalities within a unified framework.

We show that the introduction of product market externalities and market size effects has non-trivial effects on the outcome of the otherwise standard signaling game. When market externalities are either negative (substitute goods) or not too strong positive (complementary goods) in the least cost separating equilibrium the credential level that separates the "good" type from the "bad" type with negative (positive) externality is lower (higher) than in the case when externalities are absent. In contrast to previous literature we show that in the case of moderately strong positive market externalities, the least cost separating equilibrium is rather different in nature: the "bad" type chooses zero credential but the "good" type chooses the highest rather than the lowest possible level of credential. In the case very strong positive market externalities it turns out to be impossible to separate the types, and the signaling game has a pooling equilibrium with the highest possible levels of credential.

Since Spence it has been regarded to be the case that signaling is (almost) always purely a social waste, since it has no effect on productivity, and it's only purpose is to facilitate separation of types under asymmetric information. In our set up this is not necessarily true, and most interestingly we can show that the private market solution may actually involve too little

signaling compared to social optimum.

To get some perspective for our idea of signaling with externality consider the following examples that we believe are captured by our model. Take first the open source programming where individual programmers get involved with open source projects in order to signal their programming skills to potential employers (i.e. commercial software companies). As an outcome of successful open source programming project the freely available program will appear and coexist with commercial programs in software markets. This free program may either be a complement or a substitute to the commercial program.[4]

Another example of signaling with externality might be academic research, for example in the pharmaceuticals. Research results of the university labs, say a new molecule, contribute to the reputation of involved researchers, and are interpreted by the future (commercial) employers as a credible signal of the innate skills of researchers. At the same time, these results may either substitute or complement the potential employer's research, and thus ultimately affecting its market revenue.[5] More concretely, the research output could simply be any good research publication that might compete with or be a complement to a commercial (text)book of the potential employer interested in hiring the very person who produced the literary output.

Our third example of internal labor markets is a little bit outside of our main focus, but it is still fully consistent with the idea of signaling with externality. Consider a firm where promotion decisions are largely affected by the initiatives and innovations of current employees, and where the new ideas are regarded by the owners as credible signals of workers innate abilities. Quite naturally the new ideas and innovations can be complements

---

[4]Hann, Roberts and Slaughter (2002) provide empirical evidence of economic incentives of individual programmers within the Apache web-server open source project. Their results confirm that a higher status in a merit-based ranking does lead to significantly higher wages. A higher status in a merit-based ranking list is a credible signal of the productive capacity of a programmer, and software companies are willing to pay for high wages for the top performers. In our companion paper, Leppämäki and Mustonen (2004) we examine open source programming as a possible signaling and screening device.

[5]Stern (1999) finds that researchers in public employment are willing to accept lower wages. This he views as an evidence that the workers in public employment value reputation building opportunities.

or substitutes of those that form the base of the firm's current activities generating the market revenue. As intellectual property rights of ideas are not well defined, the workers with new ideas can leave the firm quite easily and start either competing with it in the case of substitutes or providing some complementary service.

We carry out the main analysis within a signaling model with three types of players: workers, firms and consumers, who will interact in two types of market: labor markets and product markets. We analyze workers' (who differ in their innate ability) signaling and firms' hiring and derive the perfect Bayesian equilibrium (PBE) of the signaling game. The focus on our analysis is in examining how the outcome of the signaling game is affected by the product market externalities and market size.

We show first that when externalities are either negative (substitute goods) or not too strong positive (complementary goods) the analysis of signaling and the derivation of the least cost separating equilibrium of the signaling game is standard. Then in the least cost separating equilibrium the credential level that separates the "good" type from the "bad" type with negative (positive) externality is lower (higher) than in the case when externalities are absent. Intuitively these results are due to "intensified competition" in the case of substitutes and "consumers' increased willingness to pay for the commercial good" in the case of complements. In the former case, the profits of the firms and thus the wages are reduced due to the appearance of a free substitute good in the product market. Consequently then, the worker optimally adjusts the credential level downwards. In the latter case, the firm's profits and thus the wages will go up, since consumers' willingness to pay for the commercial good has increased due to the appearance of a free complementary good. Naturally then the worker optimally adjusts the credential level upwards. It is the interaction between the commercially produced good and the freely available good in the product market, which is reflected in the level credential found in the least cost separating equilibrium of the signaling game. It is also interesting to see that, in our set up we can replicate the results of Spence (1973) as a benchmark case, when externalities and market size effects are eliminated.

Secondly, when we consider moderate positive market externalities we show that the least cost separating equilibrium is rather different in nature compared to the cases analyzed previously in the literature. In fact, it turns

5

out that as usual the "bad" type chooses zero credential, but the "good" type instead chooses the highest rather than the lowest possible level of credential. Intuitively, the reason for this is that the positive market externality effect is so strong that the increased wage balances off the disutility of attaining any credential. Clearly then the "good" type chooses the highest possible credential.

In the case of very strong positive market externalities it turns out to be impossible to separate types, since now it is beneficial even for the "bad" type to choose the highest possible credential, and eventually we end up with a pooling equilibrium with maximum signaling. Intuitively this is because under the very strong positive market externalities it becomes more and more difficult to separate types, as they are in fact becoming closer to each other. The difference in their productiveness becomes less important compared to the positive market externality effect.

Interestingly, in our case the signaling activity is not purely a social waste like in Spence's (1973) analysis, since here it will eventually materialize in a free good that consumers value. Thus our welfare analysis becomes more interesting, and we show that there is a potential conflict of interest between privately and socially optimal levels of signaling, since individual workers and firms do not internalize all the benefits and costs of signaling that are however valued by the welfare maximizing social planner. We can outline the precise conditions under which the private market solution involves too little or too much signaling compared to social optimum. The interesting new finding, of course, is that in the private market solution there may be underprovision of signaling. In addition, we show that the social welfare may actually be higher under asymmetric information with signaling than in the case of symmetric information, when there is no reason to signal in the first place.

It is evident that the papers closest to ours are the classical papers by Spence (1973, 1974). As already said Spence (1974) considered the possibility of signaling increasing the labor embodied productivity.[6] He acknowledged that acquiring education as such may increase the worker's productivity. That is, by using Spence's (1974) notation, the innate productivity of good type after attaining education level y increases from 2 to $2+y/4$.

---

[6]In Riley's (1979) informational equilibrium analysis sellers can invest in proving the quality of products they bring into the market.

In contrast to Spence, in our set up the productivity of a worker remains unchanged but the product market, where the employer operates, is affected via a positive or negative externality effect as described above. At first hand, what we are doing (especially in the case of complements) may look similar to Spence, however the mechanism how the acquired credential affects the incentives to signal and the equilibrium of the signaling game is drastically different. In particular, our approach makes it possible to examine negative externalities and the market size effects. The particular example of education increasing the labor embodied productivity that Spence (1974) considered falls safely (on purpose?) into the same category than our first case of mild positive externalities when the least cost separating equilibrium with standard properties holds

Much of the recent contributions of signaling literature are reviewed in a recent excellent survey by Riley (2001).[7] As he puts it since Spence's work, the one line of research on signaling has devoted much of it's attention on to the refinements of equilibria in the signaling games within the development of modern game theory. The other line of research has focused more on applying the idea of signaling into various economically interesting and important questions, although as Riley points out some of the applications have advanced the theory as well. Our contribution here is to introduce a new mechanism for the signaling to work, and to examine in detail how the outcome of the job market signaling game is affected by the magnitude of negative and positive product market externalities and market size.

The structure of the rest of the paper is as follows. In next section we present the model, and the main analysis is carried out in section 3. In section 4 we examine when does the private market solution involves too much or too little signaling compared to the social optimum. Section 5 concludes the paper.

2   The Model

In this section we set up the model and describe the players involved and the markets where they interact. On purpose, we follow the notation of Spence (1973, 1974) as closely as possible. We consider a model with three

---

[7]See also Spence (2002).

class of players; *workers, employers and consumers* who will interact in two markets; *labor markets* and *product markets.*

2.1 Players and Markets

We assume there exists two types of workers, who differ only in their non-verifiable ability that is also called as their productivity. We assume that the high productivity worker ("good" type) has a productivity of $\theta = 2$ and the low productivity worker ("bad" type) has a productivity of $\theta = 1$. The share of "good" types is assumed to be equal to q_1.

In order to separate from each other a worker may attain a credential y, $0 \leq y \leq y_{\max}$. It is assumed that it is easier for the "good" type to acquire a given credential, and the utilities of the workers are assumed to depend on the wage, and the disutility of attaining the credential as follows:

$$U_G = w - \frac{y}{2}, U_B = w - y.$$

In above G refers to "good", high productivity type and B to "bad", low productivity type, each attaining credential y and earning the wage w. In this paper we extend the original analysis of Spence into the case where the activity of attaining credential y creates as a by-product a good that will be freely available for consumers. Thus, this free good will interact with the commercially produced good in the product market.

In our analysis, the employer is a profit maximising monopoly supplying the commercially produced good in some product market niche. The monopolist employs a worker with productivity θ, and produces the commercial good in a project of size one. We assume that the quality of the commercial good depends on the worker's productivity and without loss of generality it is assumed to be equal to θ. The signaling activity, i.e. the effort to acquire a credential y, creates as a by-product a free good of quality $|k|y$ in the product market.

In our analysis we focus on three cases by defining that for $k = 0$, the free available good good is independent of the monopolist's market, for $k > 0$, it is a complement to the monopolist's good, and for $k < 0$, the freely available good is an incomplete substitute to the monopolist's commercial good.

As shown in the appendix the monopolist's revenues in all these three

8

cases can be expressed conveniently by a single equation:

$$R = M\left(\theta + ky\right),$$

where M captures the "market size effect", and where k may either be positive, negative or zero. Quite clearly, the case of zero externalities resembles Spence's original analysis, and indeed in section 3 we show this to be true when the market size effects are eliminated.

For later purposes it is important to notice that even if the revenue functions are identical in the cases of complement and substitute goods, the market outcomes differ substantially. For a complement, the same consumers that would have bought the commercial good anyway are (now due to the appearance of a free complementary good) willing to pay more of that very good. In the case of substitutes some consumers buy the commercial good, and the rest of the consumers acquire the freely available good.

Following tradition, we assume that the worker has all the bargaining power in the labor market, which implies that the employers will compete for the workers, and end up with zero profits

$$\pi = R - w = M\left(\theta + ky\right) - w = 0.$$

Timing of the model goes as follows. At the outset nature assigns workers' productiveness and the proportion of high and low productivity workers. Then a worker may engage himself in attaining a credential y, and thus simultaneously creating a free good that is either a complement or substitute to the commercially produced one. A monopolist hires a worker with credential level equal to y and pays out wage w. In the case of substitutes, consumers buy the commercial good or acquire the free good, and in the case of complements, the consumers are willing to pay more for the commercial good. At the end profits are realized and wages are paid out.

2.2 The Strategies and Solution Concept

In the labor market, strategies $(\overline{y}_B, \overline{y}_G, w^*)$ and a system of beliefs β^* form the perfect Bayesian equilibrium (PBE). The building block for the equilibrium is the assumption that the attained credential is regarded by firms as a credible signal of a worker's innate ability. We assume that each worker

chooses the level of credential given the wage function $w^*(\theta, y) = M(\theta + ky)$. The "bad" type faces a problem

$$\overline{y}_B \in \arg\max_y [w^*(1, y) - y],$$

and the "good" type's problem is

$$\overline{y}_G \in \arg\max_y [w^*(2, y) - \frac{y}{2}].$$

The firm hires a worker with a credential y at wage

$$w^*(\theta, y) = \beta^*(1 \mid y) M (1 + k\overline{y}_B) + (1 - \beta^*(1 \mid y)) M (2 + k\overline{y}_G) \qquad (1)$$

with beliefs β^* that are consistent with equilibrium strategies $\overline{y}_B, \overline{y}_G$. In particular, if the optimal credential levels differ, $\overline{y}_B \neq \overline{y}_G$, then if observed $y = \overline{y}_B$, $\beta^*(1 \mid y) = 1$ and if one observes $y = \overline{y}_G$, $\beta^*(1 \mid y) = 0$. Of course, in the case when the optimal credential levels coincide, $\overline{y}_B = \overline{y}_G$, then if observed credential $y = \overline{y}_B = \overline{y}_G$, $\beta^*(1 \mid y) = 1 - q_1$.

Since the predictive power of PBE is weak in a sense that it does not restrain the out of equilibrium beliefs, in the rest of the paper we use the Cho-Kreps (1987) intuitive criterion, and focus on the least cost separating equilibrium when feasible.

3   The Analysis

3.1   The Case of Substitutes and Mild Positive Externalities

We consider next the optimal behavior of workers in the labor market and demonstrate the implications of market externalities for the signaling activity. When individual workers decide on the level of credential they acquire in order to signal their capability, they anticipate the wage offer of the potential employers. That is, they choose the optimal credential levels $\overline{y}_G$ and $\overline{y}_B$ by maximizing their utilities U_G and U_B given the wage function (1)

The incentive compatibility constraints for "bad" and "good" types read as

$$M (1 + k y_B) - y_B \geq M (2 + k y_G) - y_G, \tag{2}$$

$$M (2 + k y_G) - \frac{y_G}{2} \geq M (1 + k y_B) - \frac{y_B}{2}. \tag{3}$$

From now on we consider the case when the magnitude of externalities $k \in (-\infty < k \leq \frac{1}{2M})$. That is, we allow the possibility of negative externalities (i.e. substitute goods) and not too strong positive externalities. In the next section, we in turn focus on the case of stronger positive market externalities $k > \frac{1}{2M}$ i.e. when the freely available good is a very strong (valuable) complement to the commercial good.

It turns out that in the former case we can use the standard methods to derive the equilibrium that has the usual properties.[8] As we are focusing on the least cost separating equilibrium, $\overline{y}_B \neq \overline{y}_G$ and since getting a credential is costly, it is optimal for the "bad" type not to get one, since $\frac{dU_B}{dy} < 0$ for $-\infty < k \leq \frac{1}{2M}$

$$\overline{y}_B = 0.$$

The "good" type chooses the lowest level of credential that allows him to separate from the "bad" type. That is simply due to the fact that $\frac{dU_G}{dy} \leq 0$ for $-\infty < k \leq \frac{1}{2M}$. The level of credential the "good" type will choose can be solved from the "bad" type's binding IC -constraint (2)

$$\overline{y}_G = \frac{M}{1 - Mk}.$$

Consequently the wages are $w_B^* = M$, $w_G^* = M \left(2 + k \frac{M}{1 - Mk} \right)$. In the least cost separating equilibrium, the utilities of the workers are

$$\overline{U}_B = M,$$

$$\overline{U}_G = 2M + \frac{(2kM - 1) M}{2 (1 - Mk)}.$$

---

[8]The best of our knowledge in the literature of signaling the papers have focused so far solely on cases where the least cost separating equilibrium always implies the lowest possible levels of signaling that allows separation of types. See e.g. Riley (2001).

As we have now derived the least cost credential levels, we state the following result with regard to the market externality:

Proposition 1 *The least-cost credential level that separates the "good" type from the "bad" type with negative (positive) externality is lower (higher) than in the case when externalities are absent,* $\overline{y}_G^s < \overline{y}_G^i < \overline{y}_G^c$.

Proof. It is enough to notice that in the case of substitutes($k < 0$), $\overline{y}_G^s = \frac{M}{1-Mk}$ is smaller than than $\overline{y}_G^i = M$ in the case of independent goods ($k = 0$), which in turn is smaller than $\overline{y}_G^c = \frac{M}{1-Mk}$ in the case of complements ($k > 0$). ∎

Intuition for the result is straightforward. When the signaling activity creates a substitute good to the monopolist's commercial good, the high productivity type suffers from this: The intensified competition in the product market lowers the monopolist's profits and thus the wage the "good" type receives once hired by the firm. The worker fully internalizes the negative externality effect and optimally adjusts the credential level downwards. In the case of complementary goods the worker in turn adjusts the credential level upwards.

In previous section we claimed that when externalities and market size effects are absent our analysis reproduces the results of Spence. To see this formally, let us assume for a moment that externalities are absent i.e. that $k = 0$, and that the market size effects are normalized to $M = 1$. In this benchmark case, the least cost separating equilibrium has exactly the same features as in Spence (1973, 1974):

Corollary 2 *In the absence of product market externalities and market size effects, our results coincide with those of Spence (1973, 1974):* $\overline{y}_B = 0$, $\overline{y}_G = 1$, $\overline{U}_B = 1$ and $\overline{U}_G = \frac{3}{2}$.

Proof. Just plug in $k = 0$ and $M = 1$ into expressions of $\overline{y}_G, \overline{U}_B$,and $\overline{U}_G$, and the results will follow immediately. ∎

The results of Proposition 1 seem to imply that there may be a conflict of interest between privately and socially optimal levels of signaling. In the case of substitutes the worker does not take into account the benefits that accrue to the consumers who have an access for the freely available good. Interestingly, in the case of complementary goods the worker does internalize

some but not all of the realized benefits. The magnitude of conflict of interest seems to depend on the underlying market structure. We'll come back to this issue in section 4 where we focus on the welfare effects.

We also find that the level of credential the "good" type has to choose in order to signal his capability in the least cost separating equilibrium increases in the magnitude of market externality. This, of course, is reflected in his utility, and we have the following result:

Corollary 3 *The "good" type's utility in the case of complementary goods is higher than in the case of substitute goods.*

Proof. This is clear, since $\frac{\partial \overline{U}_G}{\partial k} > 0$. ∎

Although quite simple and straightforward, the above result has interesting economic implications. Namely, given that the worker can decide how to allocate his attention in terms of attaining credential it is clear that he will devote attention towards the signaling activity that will produce as by-product a free complementary good. More generally, this of course implies that one should observe signaling activity occurring more often in such areas where the outcome of the signaling activity has some complementariness with the commercially produced goods. Clearly this is an interesting empirical question that, of course, is beyond the current paper, and thus left for future research.

3.2   The Case of Strong Positive Externalities

Above we assumed that the magnitude of market externalities was bounded from above. How does our analysis extend into the cases where positive market externalities are far more stronger i.e. when the goods become more and more valuable complements? It is useful to examine the effects of stronger externalities in two steps. First, we analyze the case when the positive externalities are moderately strong, $\frac{1}{2M} < k < \frac{1}{M}$, and show that there exists a least cost separating equilibrium where the "bad" type chooses zero credential and the "good" type chooses the maximum credential. Finally, we consider very strong positive market externalities, $k \geq \frac{1}{M}$, and show that then the separation of types is impossible, and we have a (unique) pooling equilibrium with maximum credential levels.

Once the positive market externalities are moderately strong, the incentive problem of the "bad" type remains the same, and since $\frac{dU_B}{dy} < 0$ for

$-\infty < k \leq \frac{1}{M}$ he prefers to choose $\overline{y}_B = 0$, and receives $w_B^* = M$. In fact, the "bad" type's IC constraint is not binding any more, since the "good" type will choose now even a higher credential level than before. That is, now the "bad" type is strictly better off with choosing $\overline{y}_B = 0$ than the credential level that will be chosen by the "good" type.

Interestingly, the situation with the "good" type changes compared to the case of the previous section. In the least cost separating equilibrium he will not any more prefer to choose the lowest level of credential in order to separate from the "bad" type. Rather, he chooses the highest possible amount $y_{\max}$, since $\frac{dU_G}{dy} > 0$ for $k > \frac{1}{2M}$. In essence, this means that the positive market externality effect is so strong that the increased wage balances off the disutility of attaining any credential y. Clearly then the "good" type chooses as high y as possible. In other words, there is no upper limit for y, and in equilibrium the "good" type sets $\overline{y}_G = y_{\max}$, and thus we have the following result:

Proposition 4 *In the case of complementary goods with moderately strong externalities, $\frac{1}{2M} < k < \frac{1}{M}$, we have a least cost separating equilibrium with $\overline{y}_B = 0$ and $\overline{y}_G = y_{\max}$.*

Proof. Given $y = 0$ has been observed the monopolist updates beliefs $\beta^*(1 \mid 0) = 1$, and if the observed $y = y_{\max}$, then $\beta^*(1 \mid 0) = 0$. Given these equilibrium beliefs, the wage function is

$$w^*(\theta, y) = \beta^*(1 \mid y)M + (1 - \beta^*(1 \mid y))M(2 + ky_{\max}).$$

Given the wage function it is optimal for the "bad" type to choose $y = 0$, since

$M > M(2 + ky_{\max}) - y_{\max}$ for $\forall \frac{1}{2M} < k < \frac{1}{M}$. That is, he is strictly better of by choosing $\overline{y}_B = 0$ and receiving M instead of choosing $y_{\max}$ and receiving the higher wage $M(2 + ky_{\max})$, since we know that $y_{\max} > \frac{M}{1-Mk}$. From above, we already know that the "good" type prefers to set $\overline{y}_G = y_{\max}$ receiving then $M(2 + ky_{\max}) - y_{\max}$ instead of getting only M by choosing zero credential for $\forall \frac{1}{2M} < k < \frac{1}{M}$. ∎

Let us now move to the case of very strong positive market externalities $k \geq \frac{1}{M}$. For the "good" type the analysis remains as above, since now it is even more beneficial for him to choose as high y as possible, and

thus he sets $\overline{y}_G = y_{\max}$. Interestingly, now even for the "bad" type, the positive market externality effect is strong enough that the increased wage balances off the disutility of attaining any credential y. Consequently, the "bad" type prefers to mimic the "good" type, and sets $\overline{y}_B = y_{\max}$, since $\frac{dU_B}{dy} > 0$ for $\forall \ k > \frac{1}{M}$. Thus we have to consider the possibility of pooling equilibrium where $\overline{y}_B = \overline{y}_G = y_{\max}$ and the expected wage is equal to $(1 - q_1)M(1 + ky_{\max}) + q_1 M(2 + ky_{\max})$. Thus we have a pooling equilibrium with maximum signaling:

Proposition 5 *In the case of complementary goods with very strong market externalities, $k \geq \frac{1}{M}$, we have a pooling equilibrium with maximum signaling* $\overline{y}_B = \overline{y}_G = y_{\max}$.

Proof. We prove our claim with $k = \frac{1}{M}$. If the main result will hold then it will be true for any $k > \frac{1}{M}$ as well. After plugging in k in the wage function we have $w^*(\theta, y) = M(1 + q_1) + y_{\max}$. Given that $\overline{y}_B = \overline{y}_G = y_{\max}$ it is enough to consider downward deviations. The "bad" type's IC constraint reduces to $M(1 + q_1) > M$. That is, on the LHS we have the utility under the pooling contract, and on the RHS the utility of the "bad" type in the case when he would choose deviate by choosing $y < y_{\max}$ in which case the monopolist would update the beliefs that the worker is the "bad" type, and would set wage equal to $M + y$. Similarly the "good" type has no incentives to deviate from $y_{\max}$ either, since by deviating downwards he would get $M + \frac{y}{2}$ which is always lower than $M(1 + q_1) + \frac{y_{\max}}{2}$ he gets under the pooling contract, since $y < y_{\max}$. ∎

Another way to understand the result is to think some chosen credential $\widetilde{y} < y_{\max}$, and the consider this $\widetilde{y}$ as a potential pooling equilibrium. When trying to apply for this potential pooling equilibrium the Cho-Kreps intuitive criterion we see immediately that in contrast to the normal case (of zero externality), now actually both types would like to deviate from $\widetilde{y}$ upwards, say up to another $\widehat{y}$. In short, this means that IC constraint is not binding for neither types. By using again the intuitive criterion for $\widehat{y}$ we see that the same thing happens; also now both types prefer to deviate upwards. Ultimately one ends up to $y_{\max}$, and from there neither type prefers to deviate (downwards).[9]

---

[9]Notice that the above result would be valid even if we had assumed that there is no

Intuition for the effects of very strong positive market externalities is rather straightforward. Once k increases it becomes more and more profitable to signal, and the market externality effect dominates the disutility of attaining credential even for the "bad" type. The key thing is that it becomes more and more difficult to separate types, as they are in fact becoming closer to each other when k increases; the difference in their productiveness becomes less important compared to the market effect of signaling.

Finally, it is useful to notice that it is actually a combination of some critical values of $|k|$ and M that determine to which type of equilibrium we end up. For instance, for very large markets (i.e. high M) even relatively low level of positive market externality (i.e. low k) is enough to destroy the least cost separating equilibrium. Similarly, for very small markets (i.e. low M), a rather high level of negative or positive market externality (i.e. high $|k|$) is compatible with the standard least cost separating equilibrium.[10]

4   Welfare Analysis

4.1   Symmetric vs. Asymmetric Information

We have previously learned from Spence's model that all signaling is social waste, since it has no effect on productivity, and it's only purpose is to facilitate separation of types under asymmetric information. Therefore, under symmetric information when both the firm and the worker know the productivity of the latter, there is no incentives to signal, and the worker is paid according to his known productivity. Clearly then welfare is higher under symmetric information than under asymmetric information.

Does the introduction of market externalities change this clear-cut result? We will argue that it does, and next we develop our claim that under certain conditions, the welfare may actually be higher under asymmetric in-

---

upper limit for y. All that matters for our result is that both types would prefer to choose as high y as possible since $\frac{dU_i}{dy} > 0$ for $\forall\, k > \frac{1}{M}$, $i = B, G$.

[10]It is worthwhile to notice that for example in the software industry the market size effects can be huge. For instance, Apache web server is used in 63% of the world's over 100 million webserves.

formation with signaling than in the case of symmetric information without signaling. That is, in our set up society as such would in fact prefer private information about workers' types not be revealed immediately to employers. Rather, the private information about workers' types should be revealed via signaling.

To understand this result recall that in our model the signaling activity has both negative and positive welfare effects. The effort is costly and as such a loss just as Spence already pointed out. However, the signaling activity in our paper creates as a by-product a good that will be freely available for consumers, and clearly this is valuable for the society. We compare next welfare between symmetric information and asymmetric information with signaling, and derive exact conditions for the latter to be higher.

Consider first the case of asymmetric information. When the firm employs the "good" type, the social welfare, measured as the net of market surplus (see appendix) and cost of signaling, is in the case of a complement, $k > 0$,

$$W_C = \frac{3}{2} M \left(2 + k \overline{y}_G\right) - \frac{\overline{y}_G}{2}. \tag{4}$$

And in the case of a substitute, $k < 0$, the welfare is equal to

$$W_S = \frac{3}{2} M \left(2 - \frac{1}{3} k \overline{y}_G\right) - \frac{\overline{y}_G}{2}. \tag{5}$$

From earlier we know that in the case of substitute goods, the firm's profit and thus wages are decreasing in the level of signaling credential but the welfare as captured by (5) is increasing in k. This is simply because some consumers acquire the freely available good instead of buying the commercial good.

Under symmetric information, both the firm and the worker know the productivity of the worker, and the worker is paid according to his productivity. The firm then just employs the "good" type and develops a good that yields a welfare of $W^* = 3M$, which is the sum of the firm's profit paid as wage $(2M)$ and the consumer surplus. Thus we can express our main claim as follows

Proposition 6 *Welfare under asymmetric information with signaling is higher than under symmetric information when either $k < -\frac{1}{M}$ or $k > \frac{1}{3M}$.*

Proof. Consider first the case of substitutes when $k < 0$. In this case we know that $\overline{y}_G = \frac{M}{1-Mk}$. What we want show is that $W_S(\overline{y}_G) > W^*$. After plugging $\overline{y}_G$ into W_S we can develop the inequality to hold as long as $k < -\frac{1}{M}$. Similarly, in the case of complements when $0 < k \leq \frac{1}{2M}$ we know that $\overline{y}_G = \frac{M}{1-Mk}$. Now we want to derive the condition for $W_C(\overline{y}_G) > W^*$. Once $\overline{y}_G = \frac{M}{1-Mk}$ is substituted in W_C, we see that the inequality holds when $k > \frac{1}{3M}$. From the main analysis we know that if the externality is any higher, $y_{\max} > \overline{y}^G$ is chosen, and thus $k > \frac{1}{3M}$ holds trivially. ∎

To get intuition for the results notice that if the freely available good - that is appearing as a by-product of the signaling activity - is a strong substitute with $k < -\frac{1}{M}$ or a strong complement with $k > \frac{1}{3M}$, the product market impact of signaling is so strong that the additional welfare accruing to the firm as profit and thus to the worker as wage and to the consumers as increased surplus exceeds the cost of the individual worker from engaging in the signaling activity. To understand somewhat asymmetric effects of substitutes and complements in the welfare comparison one has to notice that in the case of substitutes the externality effect reduces the wages the "good" type earns, which in welfare calculation is compensated in terms of increased consumer surplus. Once the freely available good becomes stronger substitute the latter effect starts to dominate, and for that reason the absolute level of threshold value of $|k|$ is higher in the case of substitutes than it is in the case of complements. Notice finally that when the freely available good is a complement, the wage of the "good" type and welfare are both increasing in k.

4.2   Too Much or Too Little signaling?

In section 3.1 we already mentioned briefly that there may be a conflict of interest between privately and socially optimal levels of signaling under asymmetric information. In this section we develop this point further, and derive the conditions under which the private market solution may involve too little or too much signaling compared to the level chosen by the welfare maximizing social planner.

In what follows is the analysis of how the worker's and the social planner's choice of signaling activity compare. To provide a complete comparison of the privately and socially optimal levels of signaling under asymmetric

information we consider all the possible levels of externalities i.e. $-\infty < k < \infty$. Before explaining in detail our findings, it is useful to summarize our findings as follows:

Proposition 7 *Once comparing privately and socially optimal levels of signaling we find that (i) for $k < -\frac{1}{M}$ private market solution involves too little signaling, (ii) for $-\frac{1}{M} \leq k < \frac{1}{3M}$ private market solution involves too much signaling, (iii) for $\frac{1}{3M} \leq k < \frac{1}{2M}$ private market solution involves too little signaling,(iv) for $\frac{1}{2M} \leq k < \frac{1}{M}$ private and socially optimal levels of signaling coincide, and (v) for $k \geq \frac{1}{M}$ private market solution involves excess signaling.*

Proof. To prove point (i) notice that welfare is increasing in the level of signaling, and thus the social planner prefers the highest possible credential y^s. However, the "good" type's individual rationality (IR) constraint, $U_G = M(2 + ky) - \frac{y}{2} \geq 0$ puts the upper limit for such y^s. In fact, it must be that $y^S \leq \frac{2M}{\frac{1}{2} - Mk}$. Now given that $k < \frac{1}{M}$, we see immediately that such $y^S > \overline{y}^G = \frac{M}{1 - Mk}$ that is the private market solution involves too little signaling.

To prove (ii) it is enough to recall from previous section that when $-\frac{1}{M} < k < \frac{1}{3M}$ equations (4) and (5) reach a maximum with $y = 0$. On the other hand we know from section 3.1 that the privately optimal solution $\overline{y}^G = \frac{M}{1 - Mk}$ is valid here, and thus the private market solution involves clearly too much signaling.

To see point (iii) we know from section 3.1 that the privately chosen level of signaling $\overline{y}_G = \frac{M}{1 - Mk}$. However, from the social point of view this is too low, since the welfare is increasing in y, and thus the social planner would choose $y_{\max}$. That is, the private market solution involves too little signaling.

Under point (iv) when $\frac{1}{2M} < k < \frac{1}{M}$ we know from section 3.2 that the privately chosen level of signaling $\overline{y}_G = y_{\max}$. Since welfare is increasing in y the social planner would choose the highest possible level of signaling as well i.e. $\overline{y}_G = y^s = y_{\max}$.

To see point (v) notice that when $k > \frac{1}{M}$, the private market solution involves $\overline{y}_B = \overline{y}_G = y_{\max}$ compared to $\overline{y}_G = y_{\max}$ that would be chosen by the social planner. That is, the private market solution involves excess signaling, since also the "bad" type will choose $y_{\max}$. ∎

To get some further economic intuition for the above results, and for why there may exist a conflict it is useful to recall that when choosing the level of signaling the worker takes only into account that the signaling activity affects the firm's revenue and thus the wages paid out. The worker does not take into account any of those welfare effects that accrue to consumers in the form of consumer surplus from the freely available complementary or substitute goods. These welfare effects are, of course, valued by the social planner, and thus it is rather clear that in most cases the privately chosen and socially optimal levels of signaling do not coincide. In fact this simple reasoning is valid in points (i) - (iv). The final point is simply due to the duplication of signaling activities, and the signaling activity of the "bad" type is there simply a social waste.

5 Conclusion

In this paper we have extended Spence's classical job market signaling model into the situation where the signaling activity itself creates interesting externality effects that are coming through product markets. In particular, we advocated the idea where a freely available good will appear as a by-product of signaling activity. The free good can either be a substitute or a complement to the commercially produced good, and it is the interaction between the goods in the product market, that will be reflected in the signaling behavior of individual workers'.

When externalities are either negative (substitute goods) or not too strong positive (complementary goods) the analysis of signaling and the derivation of the least cost separating equilibrium of the signaling game was shown to follow standard methods. It was pointed out that in the least cost separating equilibrium the credential level that separates the "good" type from the "bad" type with negative (positive) externality is lower (higher) than in the case when externalities are absent.

Interestingly, we showed that the least cost separating equilibrium is quite different than previously analyzed in the literature when we consider moderately strong positive market externalities. In particular, we proved that there exists a least cost separating equilibrium, where the "bad" type chooses zero credential but where the "good" type chooses the maximum credential. Finally, in the case complementary goods and very strong market

20

externalities it was shown that the signaling game has a pooling equilibrium with maximum signaling.

Finally, we also showed that under certain conditions the private market solution may involve too little or too much signaling compared to the social optimum. This result is simply due to the fact that individual workers and firms do not internalize all the benefits and costs of signaling that are however valued by the welfare maximizing social planner. Interestingly, we also showed that the social welfare may actually be higher under asymmetric information with signaling than in the case of symmetric information, when there is no reason to signal in the first place.

6 Appendix: Derivation of Revenue Functions and Welfare

The market model used in this paper is based on Mustonen (2003, 2004) where the author uses it to model interaction between commercial and free open source programs in the consumer market for computer software.

At the outset we assume that there exist $4M$ consumers who differ only in their willingness to pay for the goods, M is the measure of market size. When only the firm's commercial good is available ($k = 0$), consumers' valuation of it are evenly distributed on the interval $[0, \theta]$. If the signaling activity has resulted in a free complement to the firm's good ($k > 0$), consumers valuation of the firm's good is increased and are on the interval $[0, \theta + ky]$. Finally, if the signaling activity produces as by-product a free substitute good ($k < 0$), consumers' valuations of the commercial good are on the interval $[0, \theta]$ and the valuations of the freely available good are on the interval $[0, -ky]$. We assume that that the freely available good is an incomplete substitute to the commercial good i.e. $-ky < \theta$, and that the ratio of valuations is equal for all consumers.

In the case of independent goods, the marginal consumer's net valuation of the firm's commercial good is zero, $V_m - p = 0$, where V_m is the marginal consumer's valuation of the firm's good, and p stands for the price of it. The distribution of willingness to pay of consumers implies that the number of consumers that have a higher willingness to pay than V_m is equal to $\frac{\theta - V_m}{\theta} 4M$. Maximization of profit function $p \left(\frac{\theta - p}{\theta} 4M \right)$ yields the optimal price $\frac{\theta}{2}$ and output $2M$. In the case of complements, the consumers' valuations are increased compared to the preceding case. Now the number of consumers with a willingness to pay higher than V_m is $\frac{(\theta + ky) - V_m}{\theta + ky} 4M$. Analogously, the optimal price is $\frac{\theta + ky}{2}$ and output $2M$. Signaling activity may also create a substitute to the firm's commercial good ($k < 0$), and in that case the firm has to take into account the competing freely available good. The surpluses of the marginal consumer when consuming the firm's commercial good or the freely available good have to be equal, $V_m - p = V_{mf} - 0$, where V_{mf} is the marginal consumer's valuation of the free good. We know that $\frac{V_m}{V_{mf}} = \frac{\theta}{-ky}$. Developing the marginal condition yields $V_m = p \frac{\theta}{\theta + ky}$. As in the case of independent goods, the number of consumers with higher willingness to pay for the firm's commercial good than V_m is $\frac{\theta - V_m}{\theta} 4M$. After

substitution, profit maximization yields the same optimal price $\frac{\theta+ky}{2}$ and output $2M$. We can thus conclude that in our analysis we can simply use the reduced form revenue function $M(\theta + ky)$, where $k \lesseqgtr 0$.

It is useful and important to notice that the market model used in above is by no means the only one that will generate the derived revenue function. Consider for instance M homogeneous consumers with utility functions $U = V - p$, where V is the value of the firm's commercial good to the consumer. When a freely available good is independent $V = \theta$. In the case of a complement, the value is $V = \theta + ky$. The optimal price is thus in both cases $p = \theta + ky$. If the freely available good is a substitute, then there are two goods for consumers to choose from with values $V = \theta$ and $V_f = -ky$. The firm has to set the price in such a way that consumers prefer the commercial good i.e. $V - p \geq V_f - 0$. The optimal price is again $p = \theta + ky$, and the revenue function in all cases is $M(\theta + ky)$.

Let us go back to the case considered first and calculate the expression for the welfare in the product market, which is measured as a sum of firm's profit and consumer surplus. In the case of independent goods, the social welfare is simply $W_i = \frac{3}{2}M\theta$. With a freely available complementary good, both profit and consumer surplus are increased, and $W_c = \frac{3}{2}M(\theta + ky)$. If the freely available good is a substitute to the firm's commercial good $(k < 0)$, the sum of profit and consumer surplus from the commercial good is equal to that in the case of an independent good. However, those consumers who do not buy the firm's commercial good, acquire the freely available good and enjoy the consumer surplus from it, and thus $W_s = \frac{3}{2}M(\theta - \frac{1}{3}ky)$. In the following picture the welfare is expressed in both cases.

welfare
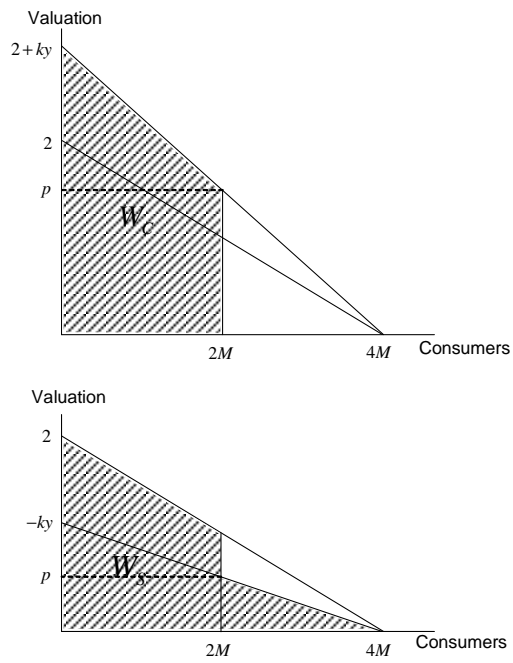
Figure 1: Welfare in the case of complementary and substitute goods.

References

[1] Cho, I-K and D. Kreps 1987: Signaling in Games and Stable Equilibria, Q*uarterly Journal of Economics 102.* 179-221.

[2] Hann I., Roberts J., Slaughter S. and Fielding R., 2002: Delayed Returns to Open Source Participation: An Empirical Analysis of the Apache HTTP Server Project, Graduate School of Industrial Organization, Carnegie Mellon University, mimeo.

[3] Leppämäki, M. and M. Mustonen 2004: Signaling and Screening with Open Source Programming. *Helsinki School of Economics, Working Papers.*

[4] Mustonen M., 2003: Copyleft - The Economics of Linux and Other Open Source Software. *Information Economics and Policy,* 15(1), 99-122.

[5] Mustonen M. 2004: When Do Firms Support The Development of Substitute Copyleft Programs? *Journal of Economics and Management Strategy*, forthcoming.

[6] Riley, J. 1979: Informational Equilibrium, *Econometrica 47.* 331-360.

[7] Riley, J. 2001: Silver Signals: Twenty-Five Years of Screening and signaling. *Journal of Economic Literature XXXIX.* 432-478.

[8] Spence, M. 1973: Job Market Signaling, Q*uarterly Journal of Economics 87.* 355-74.

[9] Spence, M. 1974: Market Signaling, Harvard University press, Cambridge, USA.

[10] Spence, M. 2002: Signaling in Retrospect and the Informational Structure of Markets. *American Economic Review 92.* 434-459.

[11] Stern, S. 1999: Do Scientists Pay to Be Scientists? *NBER Working Paper 7410.*