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Abstract. A considerable problem in statistics and risk management is finding distributions that capture the complex behavior exhibited by financial data. The importance of higher order moments in decision making has been well recognized and there is increasing interest to modelling with distributions that are able to account for these effects. The Pearson system can be used to model a wide scale of distributions with various skewness and kurtosis. This paper provides computational examples of a new easily implemented method for selecting probability density functions from the Pearson family of distributions. We apply this method to daily, monthly, and annual series using a range of data from commodity markets to macroeconomic variables.

Key Words: Pearson system, block bootstrap, selection criteria.

1. Introduction

Deciding on which distribution to use for modelling asset prices or macroeconomic variables such as inflation is a common problem for econometricians and risk professionals. Currently there exists a considerable amount of literature on evaluating density forecast models, but being able to choose a suitable distribution even for preliminary analysis has remained as a considerable problem. Traditionally modelling has been based on the mean-variance analysis assuming a symmetric distribution, but since the research by Arditti (1967); Levy (1969); McEnally (1974), and Francis (1975), it has been well documented that return distributions are not fully captured by the first two moments of the distribution. The motivation for modelling skewness and kurtosis, especially in asset pricing, has followed from attempts to understand investor behavior and their different preferences for moments. Scott and Horvath (1980), among others, have argued that investors prefer odd moments (mean and skewness) and dislike even moments (variance and kurtosis). The evidence for various skewness and kurtosis preferences is still, however, rather inconclusive, but it has nevertheless stimulated a line of research attempting to incorporate the higher moment effects into the asset pricing frameworks such as CAPM (Hwang and Satchell, 1999).

Given the needs to allow for fat tails, skewness, and even multimodality, it has become interesting to study frameworks that are flexible enough to accommodate distributions with broad range of properties. In this paper we provide examples of a new technique for selecting distributions from the Pearson family. The reason for considering this particular approach is that the Pearson System is a parametric family of distributions with easily expressible density functions, which can be used to model a wide scale of distributions with various skewness and kurtosis. These features make the family amenable to both theoretical and empirical problems

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where density functions must be expressed explicitly. The selection approach, we are proposing in this paper, is based on the use of two criteria, which are able to discriminate between the main types of distributions and the interesting subtypes of the system with various restrictions on the support of the variable. What makes this approach useful for modelling is that the criteria are quickly computed as functions of the first four central moments, thus making the technique applicable to filtering and Bayesian modelling problems, where ability to choose a convenient distribution quickly is of great importance.

The disposition of the paper is following. In the second section we present the Pearson system and consider the method of moments estimation of the parameters. In the third section we consider how the Pearson parameters can be used to construct selection criteria and outline the steps required for distribution selection. Further, we discuss how simple bootstrapping techniques can be used to robustify the selection by testing the signs of the selection criteria. Both stationary and non-stationary bootstraps for dependent data are considered. In the fourth section we provide numerical examples of the approach using various time series ranging from commodity markets to macroeconomic variables. We conclude in the fifth section.

2. The Pearson System

Several well known distributions belong to the Pearson family; for example Gaussian, Gamma, Beta and Student's t-distributions. The system was introduced by Karl Pearson (1895), who worked out a set of four-parameter probability density functions as solutions to the differential equation

$$\frac{f'(x)}{f(x)} = \frac{P(x)}{Q(x)} = \frac{x-a}{b_0 + b_1 x + b_2 x^2} \tag{1}$$

where f is a density function and a, b_0, b_1 and b_2 are the parameters of the distribution.

What makes the Pearson's four-parameter system particularly appealing is the direct correspondence between the parameters and the central moments $(\mu_1, ..., \mu_4)$ of the distribution (Stuart and Ord, 1994)

$$\begin{aligned} b_1 = a &= -\frac{\mu_3(\mu_4 + 3\mu_2^2)}{A} = -\frac{\mu_2^{1/2}\beta_1(\beta_2+3)}{A'} \\ b_0 &= -\frac{\mu_2(4\mu_2\mu_4 - 3\mu_3^2)}{A} = -\frac{\mu_2(4\beta_2 - 3\beta_1^2)}{A'} \\ b_2 &= -\frac{(2\mu_2\mu_4 - 3\mu_3^2 - 6\mu_2^3)}{A} = -\frac{(2\beta_2 - 3\beta_1^2 - 6)}{A'} \end{aligned} \tag{2}$$

where the two moment ratios $\beta_1^2 = \mu_3^2/\mu_2^3$ and $\beta_2 = \mu_4/\mu_2^2$ denote skewness and kurtosis, respectively. The scaling parameters A and A' are obtained from

$$\begin{aligned} A &= 10\mu_4\mu_2 - 18\mu_2^3 - 12\mu_3^2 \\ A' &= 10\beta_2 - 18 - 12\beta_1^2 \end{aligned} \tag{3}$$

When the theoretical central moments are replaced by their sample estimates, the above equations define the moment estimators for the Pearson parameters a, b_0, b_1 and b_2.

As alternatives to the basic four-parameter systems various extensions have been proposed using higher-order polynomials or restrictions on the parameters. Typical extension modifies (1) by setting $P(x) = a_0 + a_1 x$

$$\frac{f'(x)}{f(x)} = \frac{P(x)}{Q(x)} = \frac{a_0 + a_1 x}{b_0 + b_1 x + b_2 x^2} \tag{4}$$

This parametrization characterizes the same distributions but has the advantage that a_1 can be zero and the values of the parameters are bound when the fourth cumulant exists (Karvanen, 2002). Several attempts to parametrize the model using cubic and quartic curves have been made already by Pearson and his coworkers, but these systems proved too cumbersome for general use. Instead the simpler scheme with linear numerator and quadratic denominator has gained broad acceptance. Thus, we prefer to use either parametrization (4) or its restricted version (1) for this study.

3. Classification and selection of distributions

There are different ways to classify the distributions generated by the roots of the polynomials in (1) and (4). Pearson himself organized the solution to his equation in a system of twelve classes identified by a number. The numbering criterion has no systematic basis and it has varied depending on the source. A convenient approach to illustrate the main types in the system is a simple moment ratio diagram as in Fig. 1. Although the classical moment ratio approach provides a description of the main types of the system, it is not easy to select between individual members of the system especially if we are to consider also the interesting subtypes in the system.

3.1. **Alternative approach for distribution selection.** In order to simplify the distribution selection, we propose another classification of distributions including transitional types, were each class is identified using two statistics that are functions of the four Pearson parameters. The scheme is presented in Tables 1 and 2, where D and λ denote the selection criteria (see Appendix A).

$$D = b_0 b_2 - b_1^2 \tag{5}$$

$$\lambda = \frac{b_1^2}{b_0 b_2} \tag{6}$$

What makes this approach useful for statistical modelling in the Pearson framework is its simplicity. Implementation is done in the following steps:

(1) Estimate moments from data.
(2) Calculate the Pearson parameters a, b_0, b_1 and b_2 using (2).
(3) Use the parameters to compute the selection criteria D and λ.
(4) Select an appropriate distribution from Tables 1 and 2 based on the criteria.

In case, one needs to robustify the selection it is easy to use bootstrapping to test the assumptions concerning the signs of selection criteria before selecting the distribution. Efron's bootstrap is a powerful tool for estimating various properties of a given statistic, most commonly its bias and variance. The idea of bootstrapping is to replicate the initial sample many times and then draw with replacement from this large sample set. If we can assume that the initial sample is representative of the population, the bootstrapped sample will be a larger representative sample of the population.

3.2. **Bootstrap statistics.** In this section we explain how bootstrap techniques can be used to test the signs of the selection criteria, but before doing so we need to discuss the limitations of this approach. The main issue we have to resolve is how to handle dependent data, since the standard bootstrap procedure fails when the observed sample points are not independent. One remedy is to use block bootstrap methods, which are able to reproduce the different aspects of the dependence structure of the observed data in the resampled data. Therefore, in this paper we have used two block bootstraps methods depending on the strength of autocorrelation in the data. For asset returns data, which are commonly autocorrelated, we use stationary bootstrap (Politis and Romano, 1994). While for levels data we use the continuous-path block bootstrap in order to account for unit roots (Paparoditis and Politis, 2001).

3.2.1. *Stationary Bootstrap.* The stationary bootstrap algorithm proposed by Politis and Romano (1994) can be defined as follows (Lahiri, 1999). Let $\{X_1, ..., X_n\}$ be a stationary sequence of the observed random variables. Assume for simplicity that the number of blocks $k = k_n$ (≥ 1) and the block length $l = l_n$ $(1 < l < n)$ are integers. Now the procedure begins by wrapping the data $X_1, ..., X_n$ around a circle to obtain a periodic extension $\{X_{0i}\}_{i \geq 1}$ where for $i \geq 1$, $X_{0i} = X_j$ if $i = mn + j$ for some integers $m \geq 0$ and $1 \leq j \leq n$. The collection of blocks with length $k \geq 1$ is defined by $\{B(i, k) = (X_{0i}, ..., X_{0i+k-1}) : i \geq 1, k \geq 1\}$.

In stationary bootstrap Politis and Romano (1994) use a random block length to generate the bootstrap sample. Let $L_{ni} \equiv L_i, i \geq 1$ be conditionally iid random variables having the geometric distribution with parameter $p = l^{-1} \in]0,1[$, that is, $P_\star(L_1 = k) = p(1-p)^{k-1}, k = 1, 2, \ldots$. Then, we obtain the stationary bootstrap resamples $K \equiv \inf\{k \geq 1 : \sum_{i=1}^{k} L_i \geq n\}$ by setting $B(I_1, L_1), \ldots, B(I_K, L_K)$, where $I_1, \ldots, I_n$ denote conditionally iid random variables drawn from discrete uniform distribution on $1, \ldots, n$. The idea behind this approach is that by resampling blocks rather than original observations we preserve the original short-term dependence structure. The block lengths are chosen using the novel technique based on the notion of spectral estimation via the flat-top lag-windows as proposed by Politis and White (2004) (see Appendix B).

3.2.2. *Continuous-Path Block Bootstrap.* However, if the data is nonstationary such as asset prices commonly are, we need to take the unit roots into account in bootstrapping. For this purpose, we have used the continous-path block bootstrap introduced by Paparoditis and Politis (2001). As before, the algorithm is carried out conditionally on the original data and implicitly defines a bootstrap probability that is capable of generating bootstrap pseudo-series of the type $\{X_t^\star, t = 1, 2, ...\}$. First we calculate the centered residuals for t=2,3,...,n

$$\hat{U}_t = X_t - X_{t-1} - \tfrac{1}{n-1} \textstyle\sum_{t=1}^{n} (X_t - X_{t-1})$$

and construct the new variables $\tilde{X}_t$ as follows:

$$\tilde{X}_t = \begin{cases} X_1 & t = 1 \\ X_1 + \sum_{j=2}^{t} \hat{U}_j & t = 2, 3, ..., n \end{cases}$$

Having chosen block length l, we take integer $k = [(n-1)/l]$ as the number of blocks and draw $I_1, ..., I_k$ i.i.d. from uniform distribution on the set $\{1, 2, ..., n-l\}$. Now construction of the first bootstrap block of l observations is as follows. Set $X_1^\star := X_1$ and

$$X_j^\star := X_1 + [\tilde{X}_{I_1+j-1} - \tilde{X}_{I_1}]$$

for $j = 2, ..., l$. The rest of the bootstrap-pseudo series $\{X^\star\}$ is obtained by setting

$$X_{ml+1+j}^\star := X_{ml+1}^\star + [\tilde{X}_{I_m+j} - \tilde{X}_{I_m}],$$

where $j = 1, ..., l$ and $m = 1, 2, ..., k$.

3.2.3. *Test statistics.* For the two-sided test $H_0 : D = D_0$ and $H_0 : \lambda = \lambda_0$ against $H_1 : D \neq D_0$ and $H_1 : \lambda \neq \lambda_0$, the test statistics are defined as

$$T_D = \tfrac{|\hat{D} - D_0|}{\hat{\sigma}_{\hat{D}}} \qquad T_\lambda = \tfrac{|\hat{\lambda} - \lambda_0|}{\hat{\sigma}_{\hat{\lambda}}}$$

where $\hat{\lambda}$ and $\hat{D}$ denote estimators of D and λ derived from the observations and $\hat{\sigma}_\lambda$ and $\hat{\sigma}_D$ are their standard deviations, respectively.

If the test is one-sided, the corresponding test statistics are obtained from

$$T_D = \tfrac{\hat{D} - D_0}{\hat{\sigma}_{\hat{D}}} \qquad T_\lambda = \tfrac{\hat{\lambda} - \lambda_0}{\hat{\sigma}_{\hat{\lambda}}}$$

Now the bootstrap is used to calculate the critical values for T_λ and T_D. Having generated the bootstrap resamples $X_b^\star$ as defined above, we use them to find bootstrap statistics $T_{\lambda,n,b}^\star$ and $T_{D,n,b}^\star$ for $b = 1, ..., k$. Next we rank the collections $T_{\lambda,n,1}^\star, T_{\lambda,n,2}^\star, ..., T_{\lambda,n,k}^\star$ and $T_{D,n,1}^\star, T_{D,n,2}^\star, ..., T_{D,n,k}^\star$ into increasing order and use the $1-\alpha$ quantiles of the test statistics as critical values. (Brown and Zoubir, 2001; Zoubir and Boashash, 1998)

4. Examples

In this section we consider an application of bootstrap techniques to estimating the proposed Pearson system selection criteria. Our data set consists of 18 time series, which were randomly chosen from five categories: commodities, US macroeconomic variables, exchange rates, equity indices, and interest rates (see Table 3 for details and Datastream symbols). In order to illustrate various types of time series the observation frequencies were allowed to vary from daily to quarterly and annual data. The only selection criterion was to ensure that the time series are of adequate length. This is important especially when considering macroeconomic variables with annual observations. Given the large number of samples necessary to evaluate a distribution, it is difficult to measure the distribution of annual, or even monthly, data. Therefore, we restricted the study to US only, where various macroeconomic indicators are available from 1960's onwards.

4.1. **Commodities.** Preliminary statistics for studentized commodity returns are furnished in Table 4. Since there is no reason to assume that the distribution would remain unchanged with different observation frequencies, we have considered both daily and weekly returns to pick up the potential aggregation effects. Starting with the daily returns reported in Panel A, we find that all of the series deviate from normal in terms of the traditional Kolmogorov-Smirnov statistics. Also the skewness and kurtosis estimates support these findings. The weekly returns presented in Panel B are, on the contrary, considerably closer to the standard log-normal distribution. These findings are inline with the aggregation effect, although many of the skewness and kurtosis test statistics are still significant at 5 percent level.

As a next step, we estimated the bootstrap test statistics for the selection criteria. The results are reported in Table 5. In order to illustrate the types of distributions associated with the estimated selection criteria, we have provided gaussian kernel densities for GS Commodity index and GSCI Excess Energy Returns for daily and weekly returns in Figures 2 and 3, respectively. Given that the null hypotheses of nonstationarity were rejected by Augmented Dickey-Fuller tests given in Table 4, we applied the stationary block bootstrap for all of the commodity returns. The optimal block lengths, which were estimated using flat-top lag windows method proposed by Politis and White (2004), varied from 1 to 8 observations. When considering the estimates for selection criteria D and λ, we found one major similarity between the different commodity index returns: in most of the cases D is consistently larger than λ. The findings are supported by the bootstrap sign tests, which indicate that D is significantly greater than zero at 5% level, whereas the null of $\lambda = 0$ is accepted throughout the table. This implies that class 10 or 11 distributions would provide the best fit within the Pearson system based on the classification provided in Tables 1 and 2. The findings are somewhat expected, since distributions in classes 10 to 11 allow for a combination of skewness and excess kurtosis, which is typically exhibited by financial data. These classes are rather general, but they include several well known distributions such as the Student's t-distribution and Pearson Type IV distribution. Of the asymmetric distributions particularly the Type IV distribution has recently gained increasing attention due to its flexible shape and ability to cover a large area of the moment ratio diagram (Figure 1).

For weekly returns, the distributions appear to be closer to log-normal, but the effects of combined skewness and excess kurtosis are still strongly manifested and we end-up often choosing a distribution from classes 10 or 11. For example in the case of GSCI Energy Excess Return the distribution shifted from class 10 to the asymmetric distribution in class 11 when moving from daily returns to weekly. As implied by the increased skewness statistics in Table 4, one potential explanation for these shifts is that the role of skewness has become more pronounced than kurtosis. Therefore accommodating asymmetry could improve the fit for these distributions considerably. For GS Commodity index and London Brent it is, however, difficult to say whether class 10 or 11 should be used: the sample estimates for D and λ point for class 11, but the bootstrap sign statistics favor class 6 or 10. These discrepancies could be explained by a large variance in λ or skewness in its distribution. Thus, although the sample estimate for λ gets a positive value between 0 and 1, a fat negative tail in its bootstrap distribution can lead us to reject the null of non-negativity even though the values would be positive on average.

These findings have important implications for estimation of standard econometric models such as GARCH. Given that some of the underlying distributions are asymmetric, the consistency of QMLE estimators is no longer guaranteed if symmetric distributions are applied in estimation. Newey and Steigerwald (1996) show that the identification condition holds if both the assumed and true density functions are unimodal and conditionally symmetric about zero. However, if these conditions fail, even the correct specification of the conditional mean and variance is not sufficient to ensure consistency.

4.2. **Macroeconomic variables and other series.** The summary statistics and bootstrap estimates for macroeconomic data and exchange rates are provided in Tables 6 and 7, respectively. The gaussian kernel estimates for US unemployment rate, monetary velocity, real private consumption, household savings rate, Euro to USD exchange rate and SP500 are displayed in Figures 4 to 7. Motivation for considering this broad range of variables and allowing some of the time series to be nonstationary is to illustrate the amenability of our approach to data that exhibits completely different characteristics from standard financial data such as multimodality and heavy skewness. These issues are important if we are to propose application of the technique to support non-gaussian filtering or blind source signal separation.

A typical difference between the macroeconomic variables and returns data discussed in previous section is the lower kurtosis of macroeconomic data combined with still significant skewness as highlighted in Table 6. This is, of course, well expected as the observation frequencies of macroeconomic data range from monthly to quarterly and annual data, whereas asset returns are commonly daily. Furthermore, the macroeconomic data can be often nonstationary especially, if we want to model data without taking the necessary transformations to achieve stationarity. This is the main reason why we have considered it worthwhile to apply the continuous-path block bootstrap proposed by Paparoditis and Politis (2001) to estimate the bootstrap sign tests. Although the algorithm is slightly slower than standard block bootstrap methods, it has the advantage of being designed particularly to account for unit roots and thereby it allows us to work with a broad range time series.

Based on the bootstrap sign tests reported in Table 7, it appears that majority of the series in Panel A falls into classes 4 or 8. This is quite an interesting finding as the best known distributions that characterize these classes are Beta I and Beta II type distributions. The key advantages of Beta distributions are that they are able to capture multimodality and establish limits on the support of the variable. This could be of interest for example when modelling the household savings rate or non-accelerating inflation rate of unemployment. The Beta distributions can also accommodate a wide variety of tail-thickness and permit skewness as well. Recently income distributions have been often modelled by Beta I and II type densities (Bordley et al., 1996) and sometimes also asset returns, but with less success. Also in the case of exchange rates and interest rates we find that the distribution categories 10 to 11 are more applicable than Beta densities.

However, before we move to conclusions, there is one important issue that demands attention: in a number of cases we find that not only the bootstrap tests but also the critical values are close to zero. This implies that the variance associated with the selection criteria in these cases is really large compared to the statistic itself. Why this should be the case is, however, a puzzle. The large variances could follow from a number of sources. One potential explanation is distribution uncertainty, which could be manifested both as parameter uncertainty and variability of the whole functional form. If the higher moments, skewness and kurtosis, vary strongly over time we may expect also the distribution parameters to change. However, we could also find that not only the parameters change but the whole structure shifts from one class to another. One interesting working paper by Kacperczyk (2003) has discussed distribution uncertainty in the context of asset allocations, where he defined the concepts of distribution and parameter uncertainty as follows. Under distribution uncertainty, new information may add uncertainty regarding the values of the parameters of a new distribution and the type of distribution. Whereas, under parameter uncertainty each additional piece of information adds uncertainty only about the parameters.

So far, we have focused on the static framework, where only overall fit of distributions matters for the selection process. The naive sample moment estimates ignore completely the potential dynamic effects, which could be now reflected as unstable selection criteria. Therefore, it must be noted that the proposed distribution selection approach depends strongly on the accuracy at which we are able to model the first four moments. Allowing for time-varying higher moments would lead to time-varying selection criteria, which would permit us to account for possible structural breaks. To our knowledge, however, there are not many papers where the problem of distribution uncertainty is discussed.

5. Conclusions

In this paper, we introduced an alternative methodology for selecting candidate distributions from the finite set of density functions defined by the Pearson's differential equation. The framework is fast to implement and requires little memory. Once the values for the first four sample moments and selection criteria have been obtained, a glance at the distribution classification table is enough to provide the functional form for the density that fits the data best within the Pearson system. The main appeal of this approach is in its simplicity and applicability to support filtering algorithms and source signal separation techniques (Karvanen, 2002). The approach is particularly well fitted for Bayesian modelling paradigm as it readily treats both parameters and distributions as random quantities. Thus, since we do not know the true density function we propose a technique to approximate it by choosing among a set of candidates, which at most can estimate its best approximation.

In order to illustrate how the framework can be implemented we have provided several computational examples with randomly selected time series ranging from commodity markets to macroeconomic variables. While doing so, we made a few interesting findings, which provide challenges for future research. The most important issue concerns distribution uncertainty. In some cases we found the selection criteria to exhibit considerable variability, which was reflected as bootstrap test statistics getting values close to zero. One plausible theory is to consider the high variances as a consequence of parameter instability or uncertainty about the functional form of the distribution. In our approach these issues are strongly related to the accuracy at which we can approximate the first four moments. However, so far we have ignored the higher moment dynamics completely by focusing only on the static framework, which aims at selecting a candidate distribution to describe the whole time series. Therefore, as an issue for future research we would propose search for techniques to model the time-varying third and fourth moments. Taking these dynamics into account would enable analysis of time-varying selection criteria. Another intriguing challenge would be to gauge whether the approach can be extended to the multivariate context.

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APPENDIX A: Note on Pearson equation

Pearson equation (1) defines a separable first order differential equation with solution

$$f(x) = C \exp\left\{ \int \frac{P(x)}{Q(x)} dx \right\}$$

where C is a scaling constant.

For example, assuming $P(x) = a_0 + a_1 x$ and $Q(x) = b_2 x^2 + b_1 x + b_0 = b_2(x-\alpha)(x-\beta)$, $b_2 = 1$. By writing

$$\frac{P(x)}{Q(x)} = \frac{a_0 + a_1 x}{(x-\alpha)(x-\beta)} = \frac{m}{x-\alpha} + \frac{n}{x-\beta}$$

where

$$m = \frac{-a_0 - a_1\alpha}{\beta - \alpha}, \qquad n = \frac{a_0 + a_1\beta}{\beta - \alpha}$$

we get

$$f(x) = C|x-\alpha|^m |x-\beta|^n.$$

Since f(x) has discontinuities at α and β, the only possible supports of x are $[-\infty, \alpha]$, $[\alpha, \beta]$ and $[\beta, \infty]$. Due to symmetry, the first and third cases lead to the same type of a distribution.

The selection criteria are given by

$$D = b_0 b_2 - b_1^2 = \alpha\beta - (\alpha+\beta)^2, \quad \lambda = \frac{b_1^2}{b_0 b_2} = \frac{(\alpha+\beta)^2}{\alpha\beta}$$

where the signs of D and λ are obtained for different supports of x as follows: (1) If $x \in [\alpha, \beta]$ and $\alpha < 0 < \beta$, then $\alpha\beta < 0$ leading to $\lambda < 0$ and $D < 0$, (2) If $x \in [-\infty, \alpha]$, $\alpha < \beta < 0$ or $x \in [\beta, \infty]$, $0 < \alpha < \beta$, then $0 < \alpha\beta < (\alpha+\beta)^2$ causing $\lambda > 0$ and $D < 0$.

APPENDIX B: Block length selection

Politis and White (2004) have proposed the following spectral estimation technique for automatic block-length selection for stationary bootstrap.

Suppose $X_1, \ldots, X_N$ are strictly stationary with mean $\mu = EX_t$ and autocovariance $R(s) = E(X_t - \mu)(X_{t+|s|} - \mu)$. Then, the estimator for the expected block size choice b_{opt} is given by:

$$\hat{b}_{opt,SB} = \left(\frac{2\hat{G}^2}{\hat{D}_{SB}}\right)^{1/3} N^{1/3}$$

where $\hat{G} = \sum_{k=-M}^{M} \lambda(k/M)|k|\hat{R}(k)$ is the sample estimator of the infinite sum $\sum_{k=-\infty}^{\infty} |k|R(k)$ and $\hat{R}$ is the sample estimator of autocovariance. The smoothing function $\lambda(t)$ has a trapezoidal shape symmetric around zero, i.e.,

$$\lambda(t) = \begin{cases} 1 & \text{if } |t| \in [0, 1/2] \\ 2(1-|t|) & \text{if } |t| \in [1/2, 1] \\ 0 & \text{otherwise.} \end{cases}$$

The quantity $\hat{D}_{SB}$ is obtained from

$$\hat{D}_{SB} = 4\hat{g}^2(0) + \frac{2}{\pi}\int_{-\pi}^{\pi} (1 + \cos w)\hat{g}^2(w) dw$$

where $\hat{g}(w) = \sum_{k=-M}^{M} \lambda(k/M)\hat{R}(k)\cos(wk)$ is the sample estimate of spectral density.

The bandwith M for the flat-top lag window is chosen based on inspection of the correlogram, i.e., the plot of $\hat{R}(k)$ vs. k. First we look for the smallest integer $\hat{m}$ such that $\hat{R}(k) \simeq 0$ for $k > \hat{m}$. After identifying $\hat{m}$ on the correlogram, the recommendation is to set $M = 2\hat{m}$.

Table 1. Pearson distributions. The table provides a classification of the Pearson distributions $f(x)$ satisfying the differential equation $(1/f)df/dx = P(x)/Q(x) := (a_0 + a_1x)/(b_0 + b_1x + b_2x^2)$. The signs and values for selection criteria, $D := b_0b_2 - b_1^2$ and $\lambda := b_1^2/(b_0b_2)$, are given in columns three and four.

$P(x) = a_0,\ Q(x) = 1$

	Restrictions	D	λ	Support	Density
1.	$a_0 < 0$	0	0/0	$\mathbb{R}^+$	$\gamma e^{-\gamma x}$ $\gamma > 0$

$P(x) = a_0,\ Q(x) = b_2x(x + \alpha)$

	Restrictions	D	λ	Support	Density
2(a).	$\alpha > 0$	< 0	∞	$[-\alpha, 0]$	$\frac{m+1}{\alpha^{m+1}}(x+\alpha)^m$ $m < -1$
2(b).	$\alpha > 0$	< 0	∞	$[-\alpha, 0]$	$\frac{m+1}{\alpha^{m+1}}(x+\alpha)^m$ $-1 < m < 0$

$P(x) = a_0,\ Q(x) = b_0 + 2b_1x + x^2 = (x - \alpha)(x - \beta),\ \alpha < \beta$

	Restrictions	D	λ	Support	Density
3(a).	$a_0 \neq 0$ $0 < \alpha < \beta$	< 0	> 1	$[\beta, \infty]$	$\frac{(\beta-\alpha)^{-(m+n+1)}}{B(-m-n-1,n+1)}(x-\alpha)^m(x-\beta)^n$ $m > -1,\ n > -1,\ m \neq 0,\ n \neq 0,\ m = -n$
3(b).	$a_0 \neq 0$ $\alpha < \beta < 0$	< 0	> 1	$[-\infty, \alpha]$	$\frac{(\beta-\alpha)^{-(m+n+1)}}{B(-m-n-1,m+1)}(x-\alpha)^m(x-\beta)^n$ $m > -1,\ n > -1,\ m \neq 0,\ n \neq 0,\ m = -n$
4.	$a_0 \neq 0$ $\alpha < 0 < \beta$	< 0	< 0	$[\alpha, \beta]$	$\frac{\alpha^{2m}\beta^{2n}}{(\alpha+\beta)^{m+n+1}B(m+1,n+1)}(x-\alpha)^m(x-\beta)^n$ $m > -1,\ n > -1,\ m \neq 0,\ n \neq 0,\ m = -n$

$P(x) = a_0 + a_1x,\ Q(x) = 1$

	Restrictions	D	λ	Support	Density
5.	$a_1 \neq 0$	0	0/0	$\mathbb{R}$	$\frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/2\sigma^2}$

$P(x) = a_0 + a_1x,\ Q(x) = x - \alpha$

	Restrictions	D	λ	Support	Density
6.	$a_1 \neq 0$	< 0	∞	$[\alpha, \infty]$	$\frac{k^{m+1}}{\Gamma(m+1)}(x-\alpha)^{-m}e^{-k(x-\alpha)}$ $k > 0$

TABLE 2. Pearson distributions (continued).

$P(x) = a_0 + a_1 x$, $Q(x) = b_0 + 2b_1 x + x^2 = (x-\alpha)(x-\beta)$, $\alpha \neq \beta$

	Restrictions	D	λ	Support	Density
7(a).	$a_1 \neq 0$	< 0	> 1	$[\beta, \infty]$	$\frac{(\beta-\alpha)^{-(m+n+1)}}{B(-m-n-1,n+1)}(x-\alpha)^m(x-\beta)^n$
	$0 < \alpha < \beta$				$m > -1$, $n > -1$, $m \neq 0$, $n \neq 0$,$m \neq -n$
7(b).	$a_1 \neq 0$	< 0	> 1	$[-\infty, \alpha]$	$\frac{(\beta-\alpha)^{-(m+n+1)}}{B(-m-n-1,m+1)}(x-\alpha)^m(x-\beta)^n$
	$\alpha < \beta < 0$				$m > -1$, $n > -1$, $m \neq 0$, $n \neq 0$,$m \neq -n$
8.	$a_1 \neq 0$	< 0	< 0	$[\alpha, \beta]$	$\frac{\alpha^{2m}\beta^{2n}}{(\alpha+\beta)^{m+n+1}B(m+1,n+1)}(x-\alpha)^m(x-\beta)^n$
	$\alpha < 0 < \beta$				$m > -1$, $n > -1$, $m \neq 0$, $n \neq 0$,$m \neq -n$

$P(x) = a_0 + a_1 x$, $Q(x) = b_0 + 2b_1 x + x^2 = (x-\alpha)(x-\beta)$, $\alpha = \beta$

	Restrictions	D	λ	Support	Density
9.	$a_1 > 0$	0	1	$[\alpha, \infty]$	$\frac{\gamma^{m-1}}{\Gamma(m-1)}(x-\alpha)^{-m}e^{-\gamma/x}$
	$\alpha = \beta$				$\gamma > 0$, $m > 1$

$P(x) = a_0 + a_1 x$, $Q(x) = b_0 + 2b_1 x + x^2$, complex roots

	Restrictions	D	λ	Support	Density
10.	$a_0 = 0$, $a_1 < 0$	> 0	0	$\mathbb{R}$	$\frac{\alpha^{2m-1}}{B(m-1/2,1/2)}(x^2+\beta^2)^{-m}$
	$b_1 = 0$, $b_0 = \beta^2$				$m > 1/2$
	$\beta \neq 0$				
11.	$a_0 \neq 0$, $a_1 < 0$	> 0	$0 >$	$\mathbb{R}$	$c(b_0 + 2b_1 x + x^2)^{-m}e^{-\nu \arctan((x+b_1)/\beta)}$
	$b_1 \neq a_0/a_1$		< 1		$m > 1/2$, $\beta = \sqrt{b_0 - b_1^2}$

TABLE 3. Data

Symbol	Name	Frequency	Period
Commodities			
CGSYSPT	Goldman Sachs commodity price index	Daily	2/8/1995-2/8/2005
RECMDTY	Reuter's commodity price index	Daily	2/8/1995-2/8/2005
CRBPRMI	Reuter's CRB Precious metals index	Daily	2/8/1995-2/8/2005
GSENEXR	GSCI Energy excess return index	Daily	2/8/1995-2/8/2005
LCRINDX	London Brent crude oil index	Daily	2/8/1995-2/8/2005
NPXAVRF	Nordpool - electricity spot price	Daily	1/5/1996-11/14/2005
US macroeconomic variables			
USUN%TOTQ	Unemployment rate	Quarterly	9/15/1954-8/15/2004
USOCFNUN	NAIRU	Annual	1965-2005
USCNFBUSQ	ISM purchasing managers index	Quarterly	9/15/1954-8/15/2004
USOCFMVL	Monetary velocity	Annual	1960-2005
USOCFHRB	Average hours worked per year	Annual	1960-2005
USOCFPCN	Real private consumption	Annual	1960-2005
USOCFSVR	Household savings rate	Annual	1960-2005
Other series			
USOCFIST	US short rate	Annual	1960-2005
JAPAYE$	Japanese Yen to USD	Daily	12/31/1993-2/8/2005
EUDOLLR	Euro to USD	Daily	12/31/1998-2/8/2005
USDOLLR	USD to UK pound	Daily	2/8/1990-2/8/2005
S&PCOMP	SP500 price index	Daily	9/23/1994-9/23/2004

TABLE 4. Summary statistics for commodity returns. The table reports 2-tailed skewness and kurtosis tests along with Kolmogorov-Smirnov statistics for the null hypotheses of normal (KS_n) and t-distributions (KS_t). Degrees of freedom used in KS_t tests are given in parenthesis. Bold denotes rejection at 5% level. See Table 3 for data symbols.

Panel A: Studentized daily returns							
Symbol	Skew.	Kurt.	Skew-test	Kurt-test	ADF	KS_n	KS_t
Goldman Sachs commodity index	-0.245	5.258	**-5.055**	**11.913**	**-72.839**	**0.053**	**0.037** (7)
Reuter's commodity index	0.172	8.732	**3.572**	**18.041**	**-73.725**	**0.067**	**0.034** (5)
Precious metals index	0.243	11.613	**5.008**	**20.649**	**-73.947**	**0.068**	**0.042** (5)
GSCI Energy excess ret.	-0.226	5.444	**-4.672**	**12.407**	**-73.010**	**0.045**	**0.029** (7)
London Brent index	-0.308	6.390	**-6.304**	**14.391**	**-52.993**	**0.054**	**0.049** (5)
Electricity spot	1.217	40.224	**16.446**	**23.189**	**-50.151**	**0.137**	**0.105** (5)
Panel B: Studentized weekly returns							
Symbol	Skew.	Kurt.	Skew-test	Kurt-test	ADF	KS_n	KS_t
Goldman Sachs commodity index	-0.702	5.243	**6.030**	**5.593**	**-32.233**	0.034	0.028 (7)
Reuter's commodity index	-0.006	3.350	-0.054	1.585	**-26.257**	0.030	0.025 (20)
Precious metals index	0.221	5.587	**2.067**	**6.022**	**-34.572**	0.044	0.023 (7)
GSCI Energy excess ret.	-0.592	4.505	**5.210**	**4.460**	**-31.934**	0.041	0.049 (9)
London Brent index	-0.559	4.893	**-4.954**	**5.098**	**-32.558**	0.052	0.044 (10)
Electricity spot	0.176	15.704	1.337	**9.025**	**-26.888**	**0.097**	0.064 (5)

TABLE 5. Selection criteria and bootstrap tests for commodity returns. The table reports one-tailed and two-tailed bootstrap test statistics for D and λ with 1000 resamplings. The chosen block lengths are denoted by b and Class gives the selected distribution category using the classification presented in Tables 1 and 2. Critical values for test statistics are given in parenthesis. Bold denotes rejection of null hypothesis at 5% level. See Table 3 for data symbols.

Panel A: Studentized daily returns								
	Estimates		Bootstrap tests for D		Bootstrap tests for λ			
	D	λ	$D = 0$	$D \leq 0$	$\lambda = 0$	$\lambda \geq 0$	b	Class
Goldman Sachs commodity price index	0.075	0.045	**11.539**	**13.351**	0.679	**0.723**	1	10
			[2.463]	[1.005]	[1.236]	[1.533]		
Reuter's commodity index	0.082	0.010	**13.072**	**12.914**	0.218	**0.219**	2	10
			[2.169]	[0.923]	[2.896]	[1.536]		
Precious metals index	0.082	0.016	**11.531**	**9.882**	0.179	**0.219**	2	10
			[1.930]	[1.053]	[2.191]	[1.973]		
GSCI Energy excess ret.	0.077	0.036	**9.794**	**9.557**	0.724	**0.480**	2	10
			[2.309]	[0.677]	[1.964]	[2.469]		
London Brent index	0.078	0.052	**5.529**	**5.074**	0.658	**0.400**	3	10
			[2.497]	[0.935]	[2.028]	[3.649]		
Electricity spot	0.061	0.253	1.227	**1.313**	0.432	**0.466**	8	10;11
			[2.473]	[0.552]	[1.713]	[2.020]		
Panel B: Studentized weekly returns								
	Estimates		Bootstrap tests for D		Bootstrap tests for λ			
	D	λ	$D = 0$	$D \leq 0$	$\lambda = 0$	$\lambda \geq 0$	b	Class
Goldman Sachs commodity price index	0.031	0.570	0.418	**0.492**	**0.831**	**0.013**	1	5
			[3.917]	[0.215]	[0.036]	[2.416]		
Reuter's commodity index	0.039	0.000	**0.819**	**1.104**	0.000	**0.000**	1	10
			[0.428]	[0.332]	[0.176]	[0.501]		
Precious metals index	0.078	0.032	**3.659**	**3.888**	0.018	**0.133**	2	10
			[1.793]	[0.447]	[1.830]	[1.603]		
GSCI Energy excess ret.	0.026	0.593	0.518	0.255	**0.089**	0.026	1	11
			[1.851]	[0.873]	[0.089]	[0.023]		
London Brent index	0.045	0.367	0.619	**0.943**	0.222	**0.002**	1	5;10
			[0.939]	[0.369]	[0.442]	[0.759]		
Electricity spot	0.082	0.007	**2.617**	**2.636**	0.022	**0.015**	6	10
			[1.884]	[1.221]	[2.084]	[1.898]		

Table 6. Summary statistics for macroeconomic series. The table reports 2-tailed skewness and kurtosis tests along with Kolmogorov-Smirnov statistics for the null hypotheses of normal (KS_n) and t-distributions (KS_t). Degrees of freedom used in KS_t tests are given in parenthesis. Bold denotes rejection at 5% level. L, D, and R denote levels, differences, and relative changes, respectively.

Panel A: US macroeconomic variables							
	Skew.	Kurt.	Skew-test	Kurt-test	ADF	KS_n	KS_t
Unemployment rate(D)	0.548	4.724	**5.207**	**5.134**	**-25.924**	**0.15**	**0.15** (8)
NAIRU(L)	-0.607	1.972	-1.710	1.363	-1.178	0.120	0.119 (100)
ISM index(R)	0.092	5.532	0.930	**6.330**	**-29.685**	0.055	0.038 (7)
Monetary velocity(R)	-0.165	2.448	-0.505	0.858	**-6.531**	0.083	0.084 (100)
Aver. work hours(R)	-0.334	2.876	-1.010	0.243	**-5.615**	0.051	0.052 (100)
Aver. work hours(L)	0.962	2.472	**2.662**	0.405	-0.849	**0.281**	**0.281**
Real consumption(R)	-0.628	3.225	-1.830	0.769	**-5.392**	0.054	0.054 (64)
Savings rate(L)	-0.873	3.087	**-2.353**	0.584	-2.609	0.129	0.128 (100)
Panel B: Interest rates, exchange rates, and equity indices							
US short rate(D)	-0.121	2.617	-0.373	0.624	**-5.594**	0.058	0.058 (100)
YEN/USD(R)	-0.599	8.372	**-12.236**	**18.528**	**-74.216**	**0.065**	**0.039** (6)
EUR/USD(R)	-0.204	3.922	**-3.303**	**5.387**	**-55.642**	**0.035**	0.033 (11)
USD/GBP(R)	-0.156	5.695	**-3.973**	**15.894**	**-81.670**	**0.054**	**0.034** (7)
S&P500(R)	-0.109	6.320	**-2.281**	**14.412**	**-72.398**	**0.061**	**0.046** (8)

Table 7. Selection criteria and bootstrap tests for macroeconomic series. The table reports one-tailed and two-tailed bootstrap test statistics for D and λ with 1000 resamplings. The chosen block lengths are denoted by b and Class gives the selected distribution category using the classification presented in Tables 1 and 2. Critical values for test statistics are given in parenthesis. Bold denotes rejection of null hypothesis at 5% level. L, D, and R denote levels, differences, and relative changes, respectively.

Panel A: US macroeconomic variables								
	Estimates		Bootstrap tests for D		Bootstrap tests for λ			
	D	λ	$D=0$	$D\leq 0$	$\lambda=0$	$\lambda\geq 0$	b	Class
Unemployment rate(D)	0.042	0.392	0.572	**0.163**	0.001	**0.068**	11	5;10
			[1.229]	[-0.043]	[0.551]	[0.183]		
ISM index(R)	0.080	0.006	**4.116**	**2.894**	0.014	**0.022**	4	10
			[1.247]	[0.615]	[1.853]	[2.732]		
NAIRU(L)	-1.176	-0.425	0.000	-0.004	0.848	**-0.211**	4	5;4;8
			[0.000]	[0.000]	[1.035]	[0.759]		
Monetary velocity(R)	-0.319	-0.070	0.000	-0.000	0.020	**-0.028**	1	5;4;8
			[0.087]	[0.000]	[1.186]	[0.066]		
Aver. work hours(L)	-10579.873	-1.017	**0.022**	-0.984	**2.575**	**-0.314**	4	4;8
			[0.000]	[0.000]	[1.176]	[1.791]		
Aver. work hours(R)	-0.122	-0.557	0.000	-0.000	**0.432**	**-0.160**	2	4;8
			[0.097]	[-0.000]	[0.172]	[0.053]		
Real consumption(R)	-0.259	-1.777	0.000	-0.000	**0.159**	**-0.223**	2	4;8
			[0.000]	[0.000]	[0.155]	[0.530]		
Savings rate(L)	-18.395	-1.329	0.003	-0.014	**0.261**	**-0.019**	4	4;8
			[0.005]	[0.000]	[0.048]	[0.239]		
Panel B: Interest rates, exchange rates, and equity indices								
	Estimates		Bootstrap tests for D		Bootstrap tests for λ			
	D	λ	$D=0$	$D\leq 0$	$\lambda=0$	$\lambda\geq 0$	b	Class
US short rate(D)	-0.136	-0.055	0.000	-0.000	0.014	**-0.072**	1	5
			[0.029]	[-0.000]	[2.360]	[0.042]		
YEN/USD	0.071	0.148	**6.409**	**6.370**	0.958	**0.920**	2	10
			[2.586]	[0.743]	[2.768]	[3.805]		
EUR/USD(R)	0.058	0.074	**5.040**	**3.692**	0.325	**0.704**	1	10
			[3.676]	[0.978]	[1.852]	[2.077]		
USD/GBP(R)	0.080	0.015	**19.125**	**19.989**	0.460	**0.497**	2	10
			[2.384]	[1.092]	[1.879]	[4.941]		
SP500(R)	0.082	0.006	**14.647**	**15.611**	0.159	**0.200**	1	10
			[2.242]	[0.854]	[2.861]	[2.278]		

FIGURE 1. Moment ratio diagram for the Pearson curves. Skewness and kurtosis are denoted by $\beta_1^2 = \mu_3^2/\mu_2^3$ and $\beta_2 = \mu_4/\mu_2^2$ respectively. Limit for all distributions is line $\beta_2 - \beta_1^2 - 1 = 0$. The Latin numbers refer to the traditional classification of Pearson distributions. Types I and II are beta distributions of first kind. Notation I(J,U) refers to J- and U-shaped distributions and I(M) to unimodal. The boundary of I(J,U) is line $4(4\beta_2 - 3\beta_1^2)(5\beta_2 - 6\beta_1^2 - 9)^2 = \beta_1^2(\beta_2 + 3)^2(8\beta_2 - 9\beta_1^2 - 12)$. Type III (Gamma distributions) limit is $\beta_2 - 3/2\beta_1^2 - 3 = 0$. Type VI denotes the beta distributions of the second kind. Type V is defined by $\beta_1^2(\beta_2 + 3)^2 = 4(4\beta_2 - 3\beta_1^2)(2\beta_2 - 3\beta_1^2 - 6) = 0$. Type IV is obtained when $b_0 + b_1 + b_2x^2 = 0$ has complex roots and Type VII includes Student's t-distribution.

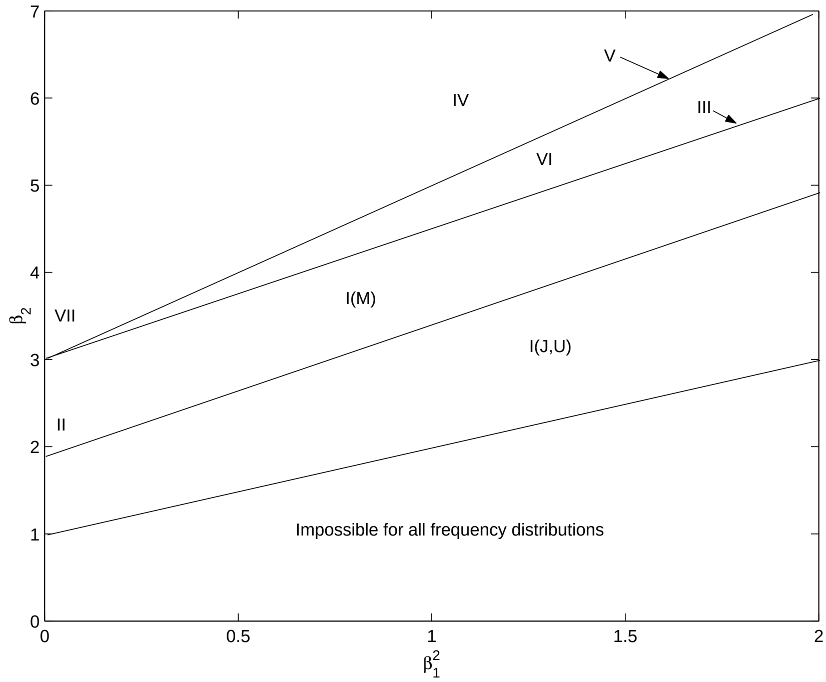

FIGURE 2. Normal kernel estimates of daily returns for GS Commodity Index and Energy Excess Ret.

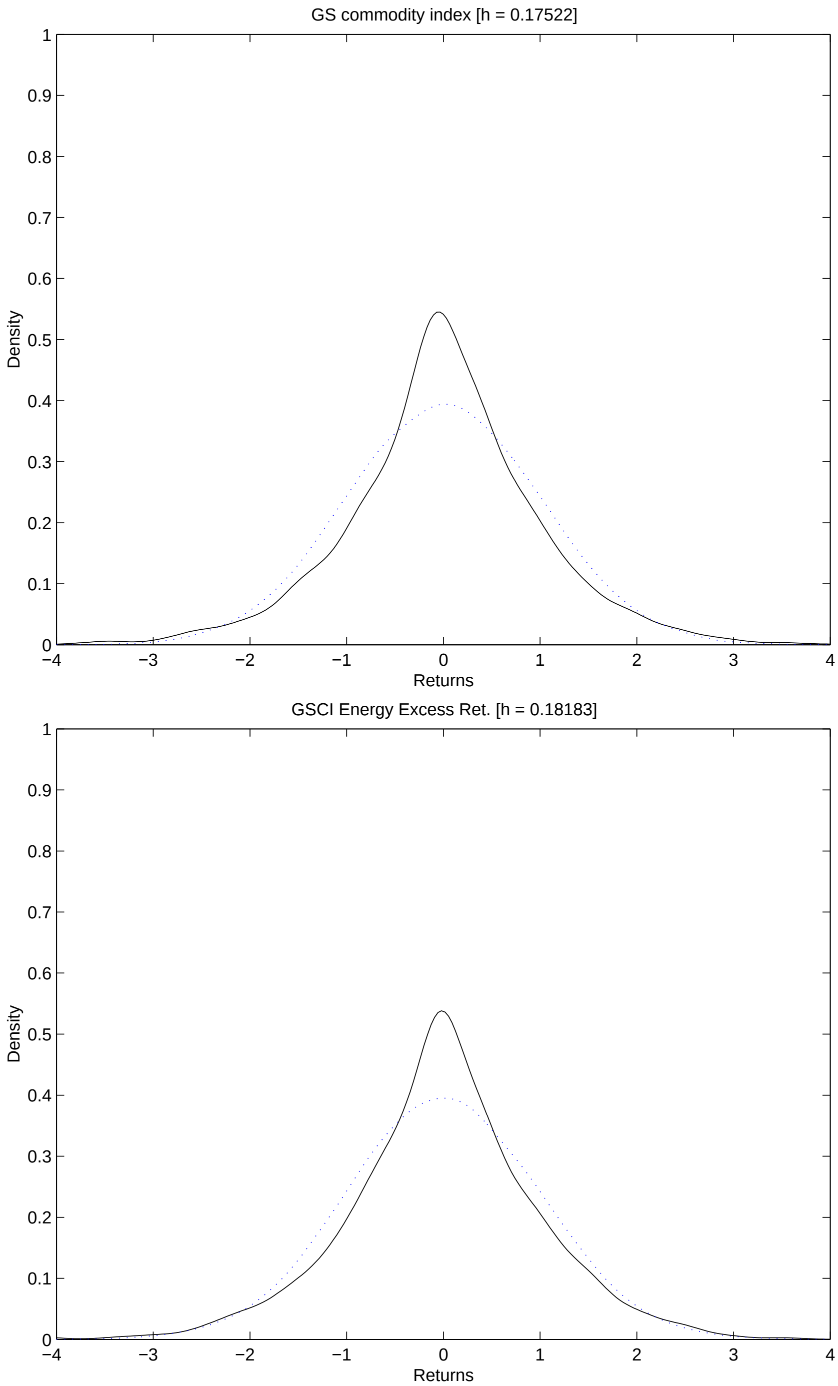

FIGURE 3. Normal kernel estimates of weekly returns for GS Commodity Index and Energy Excess Ret.

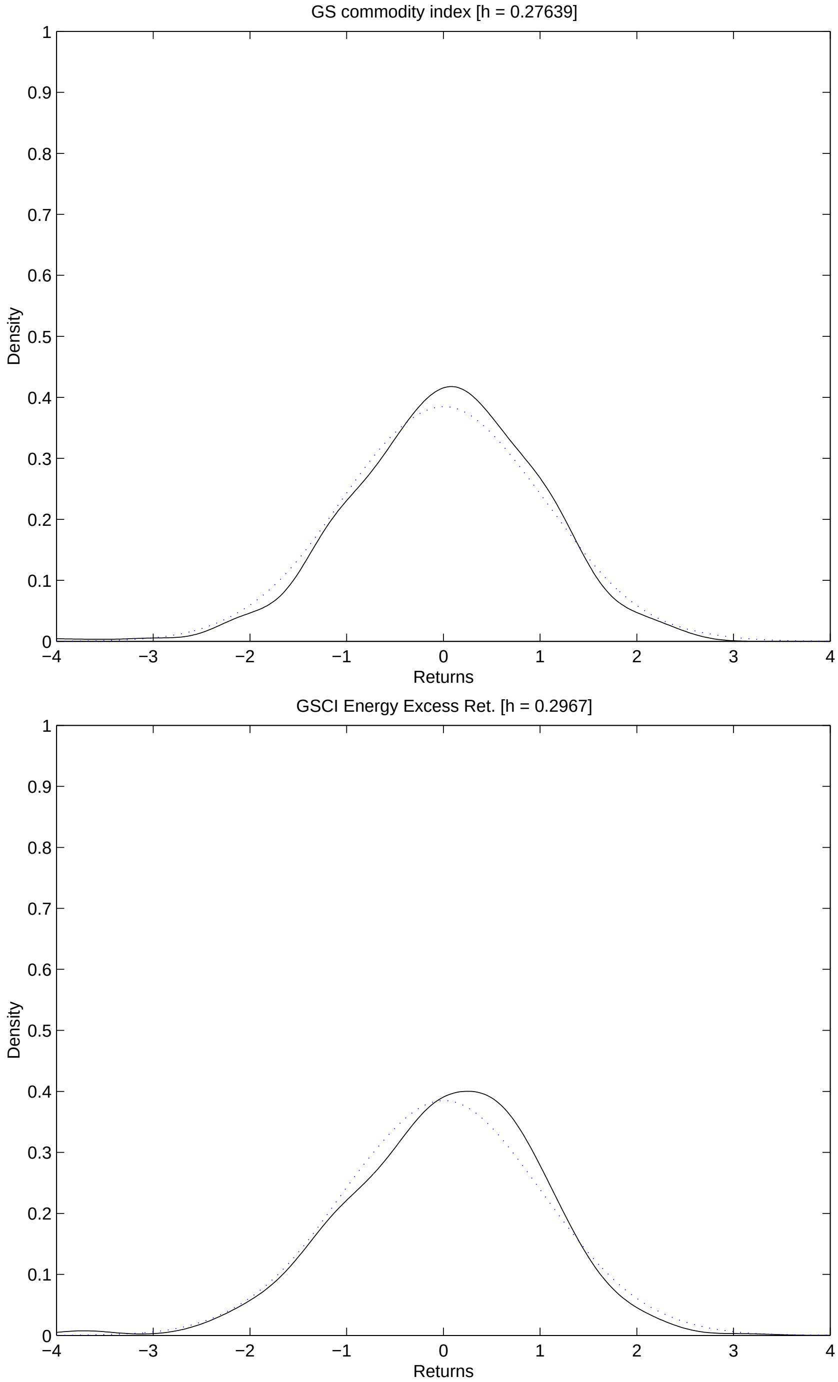

FIGURE 4. Normal kernel estimate of changes in US unemployment rate

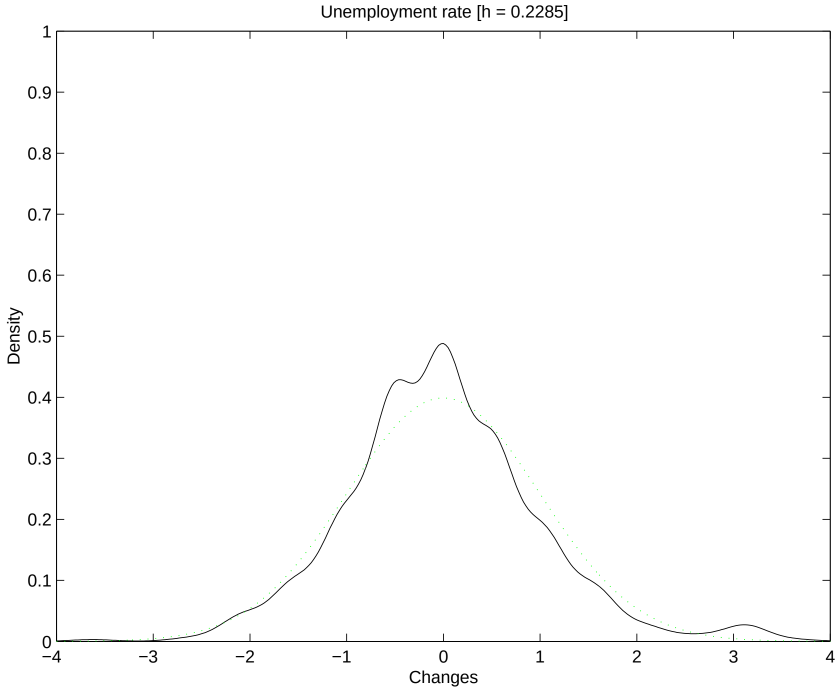

FIGURE 5. Normal kernel estimate of relative changes in US monetary velocity and real private consumption

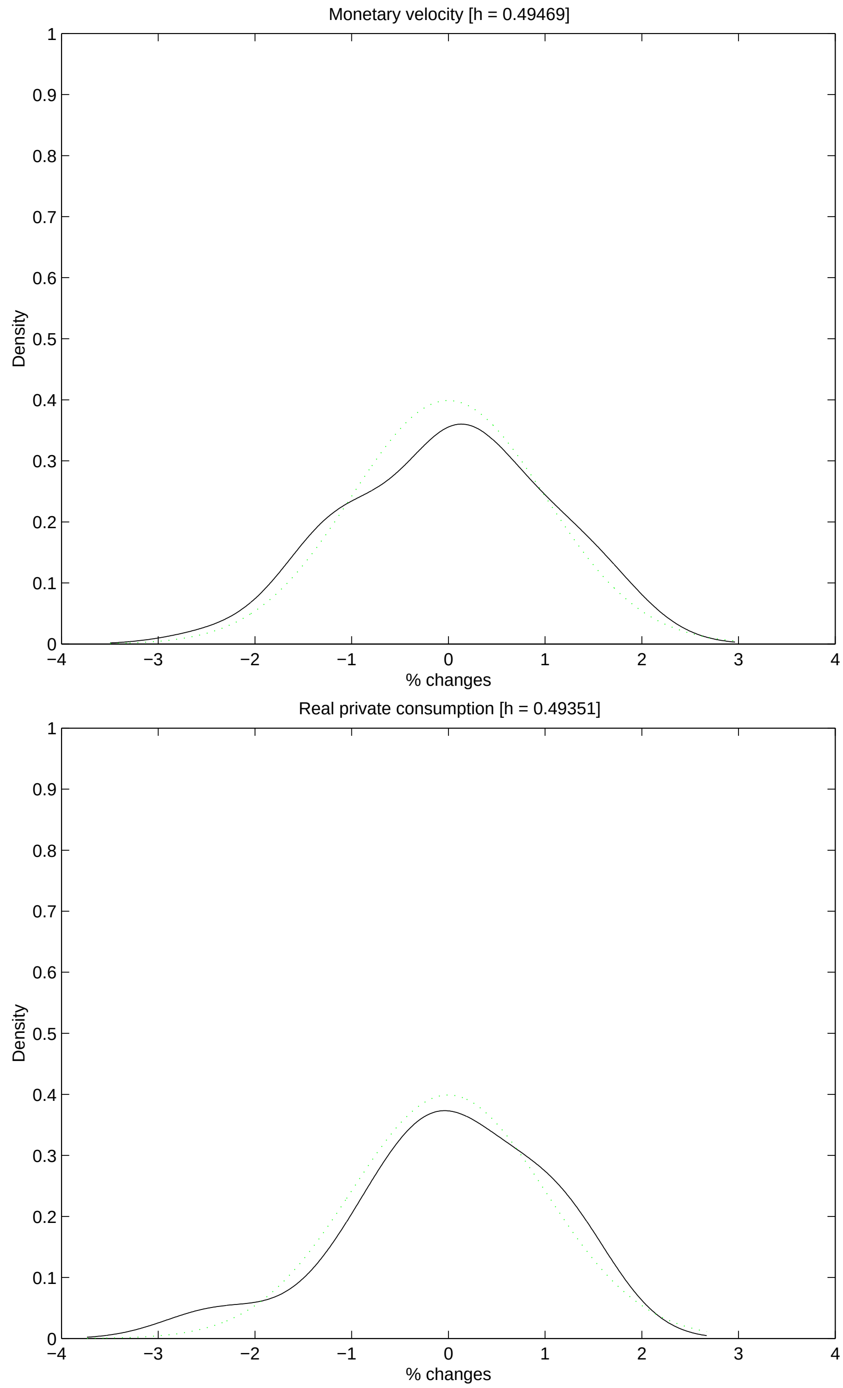

FIGURE 6. Normal kernel estimate of US household savings rate

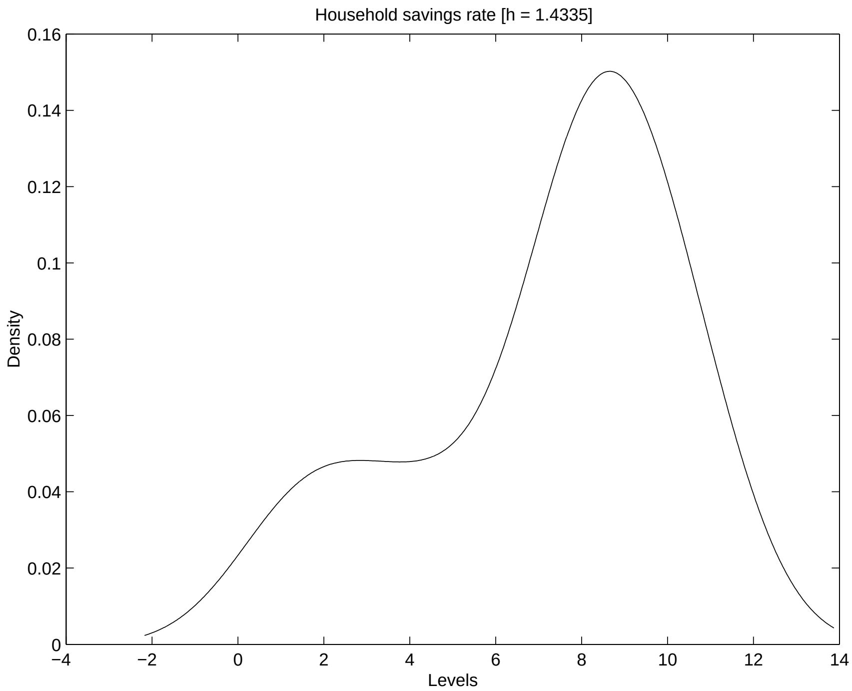

Figure 7. Normal kernel estimate of returns for Euro to US exchange rate and SP500 equity index