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SMAA-Classification - A New Method for Nominal Classification

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Abstract

We have developed a multicriteria decision aiding method for nominal classification for situations where preference information is imprecise, uncertain or absent. Such situations may appear, for instance, when the decision maker is not sure about his or her preferences or when there are multiple decision makers who have difficulties in agreeing about their common preference. The new SMAA-Classification method extends Stochastic Multicriteria Acceptability Analysis (SMAA) methodology for classification problems. The method provides the decision maker with descriptive information in the form of acceptability index for each alternative to be classified into each predefined class. We test the new method with several applications.

Keywords: multicriteria decision aiding, multiple criteria, imprecise information

1 Introduction

In many real-life situations, a decision maker (DM) (or decision makers (DMs)) faces the problem of assigning a set of given alternatives evaluated on a set of criteria into predefined classes. This task is known as a classification problem and as examples we can mention a doctor (or a group of doctors) analyzing symptoms of patients and assigning some treatment according to the results of diagnosis, or a group of engineers establishing the type of problem at hand when a paper machine is broken.

In this work, we consider a *classification problem* (also known as a supervised learning problem), where classes are predefined and well-described, to be distinguished from a *clustering problem* (also known as an unsupervised learning problem), where there is no a priori information about the classes. A central concept of the classification problem is a class. The *class* is a collection of alternatives that are more similar to each other than the alternatives in neighboring classes. The similarity measure between two alternatives and rules of assignment are subjects of discussion when dealing with different classification methods.

There are two types of classification problems: nominal and ordinal. In a *nominal classification problem*, the classes are not ordered and could be predefined with etalon alternatives. An etalon alternative is an alternative typical for a class. The classification rule in nominal classification is the following: assign an alternative into a class if that alternative is equal or roughly equal to at least one of the etalon alternatives of this class. On the other hand, in an *ordinal classification problem* the classes are ordered according to some quality. The classes might be defined with etalon alternatives like in the case of nominal classification. Then the

assignment rule is the same. However, the classes might be constrained by boundary alternatives. A boundary alternative separates two neighboring classes; it is an upper bound for a less preferred class and a lower bound for a more preferred class. The classification rule when classes are defined by boundary alternatives is the following: assign an alternative into a class if it is located between an upper and a lower boundary alternative of the class. There can be several boundary alternatives between two ordered classes.

In the present work, we develop a method for nominal classification problems in situations where there is no "exact" information about etalon alternatives, but where there is at least one assignment example for each class. These assignment examples are cases of classification that the DM(s) has in mind. This situation is typical for a real-world situation where the DM usually has some example of classification for each class. Defining etalon alternatives might be a more difficult task. For instance, in medical diagnostics doctors feel confident about clear cases with known symptoms for a certain disease, but hesitate when making decisions about assigning cases that are not clear or that have symptoms of several diseases.

There are different approaches for solving classification problems in different areas of research such as data mining, statistics and operational research. The selection of the method to be used is highly dependent on the initial information available and on requirements set on the resulting classification. Many classification and clustering methods have been developed within the framework of statistics and data mining. The idea is to observe large data sets and to find patterns of behavior common for some parts of observed data. These methods assume that the DM is not involved with the classification task and cannot influence the process. In this work, on the contrary, our interest is on methods that aim at assisting the DM in the classification problem and that are based on his or her preferences. These types of problems can be solved with multicriteria decision aiding (MCDA) [37].

A number of MCDA methods have been developed recently for nominal classification. These include the MC Filtering method [29], PROAFTN [2], a method with fuzzy integrals [10], and TRINOMFC [24]. The methods for ordinal classification include Trichotomic segmentation [31], ELECTRE TRI, [27], [39], AHP [33] adapted for classification problems [34], PAIRCLAS [7], UTADIS [40], ORCLASS, SAC [22], [23], and Dichotomic Classification [38], a method with fuzzy integrals [10], TOMASO [25], SMAA-TRI [35], and a method with rough sets [3].

Most of currently developed MCDA methods, which include classification methods, follow one of two basic approaches in MCDA: utility theory or outranking approach [37]. For instance, the MC Filtering method, Trichotomic segmentation, and ELECTRE TRI are based on the outranking relation, while AHP [33] adapted for classification problems [34] uses utility theory. Even though these two approaches assume principally different models of DM preferences, in both cases the parameters of the model should be well-defined. Usually, this means that the DM should be able to express his or her preferences in the form of model parameters, which increases the cognitive load on the DM. Some methods try to alleviate the inconvenience experienced by the DM by taking imprecision and uncertainty in initial information into account. For instance, in preference disaggregation analysis (PDA) [14], [40] the preferences are modeled based on the examples of ready decisions provided by the DM; verbal decision analysis (VDA) methods [22] allow operating with verbal data without transforming to the numerical values; while rough sets [3] develop decision rules, and fuzzy integrals assume interaction between criteria [10].

The main difficulties with extracting initial information in the form expected by the specific model may appear when parameters of the preference model need to be accepted by several DMs; or when there are no exact values but just intervals of parameter values; or when there is some distribution of parameter values; or the information about parameters is absent. The last case is the most unlikely of these, but is possible in situations where the values of parameters may change over time, or are to be obtained later on. One possible way to tackle such difficulties

is used in PDA [14], [40], where the DM is not required to define the preferences in the form of some specific model parameters. On contrary, PDA methods work in a backward manner and search for such parameters of the model that most consistently resolves the examples of ready decisions provided by the DM. In a similar way the so-called preference information free methods proceed. Such methods have been considered in [1], [4], [9], and [30]. The Stochastic Multicriteria Acceptability Analysis (SMAA) method [17–19] is also a preference information free method as well as an overall compromise criterion method developed by Bana e Costa [1]. In the present work, we develop a new SMAA-Classification method that extends the SMAA methodology for classification problems.

SMAA [17] was proposed as a decision support system for multiple DMs in the complete absence of preference information (for example, criteria weights) or in a situation of incompleteness of preference information (for example, intervals or distribution of criteria weights). SMAA allows exploring parameter space based on the Monte-Carlo simulation. The original SMAA method assumes that the DMs are rational and have a structure of preferences that might be described by a real-valued utility or value function. Thus, most of the SMAA methods search for values of parameters that maximize the value function of the alternative and make the alternative to be the best one (for the choice type of problem) or to be located at a particular rank (for the ranking type of problem). Total lack of preference information is presented, in a 'Bayesian' spirit, by a uniform distribution of parameter values. However, if there is some initial information available (i.e., partial preferences of DMs or confidence intervals for parameters values) it can be used in the form of probability distributions that provide a very general and flexible way to represent various forms of uncertainty. In the output of the SMAA methods there is descriptive information in the form of indices, such as acceptability indices that show the variety of different preferences that support the alternative; central weight vectors that represent the favoring weights with which the assignment is supported; and confidence factors that estimate the level of accuracy for such support.

The original SMAA method [17] is based on an additive value function and stochastic variables (weights or/and criteria values). SMAA-2 [18], on the other hand, allows a general value function in order to include additional preference information in a different form and to enlarge the analysis of each alternative holistically for all ranks. The SMAA-O method [19] expands the SMAA-2 in order to process ordinal and cardinal criteria values at the same time. Ref-SMAA [20] uses reference points and achievement scalarizing functions to model preferences. The SMAA-3 method [21] is based on Electre III (see [32], [37]) and allows using pseudo-criteria in SMAA-2. SMAA-Tri [35] allows analyzing the parameters' stability of the ELECTRE-TRI method (see [39]). Applications of different SMAA methods may be found in a number of papers including [12, 13, 15, 16].

The new SMAA-Classification to be introduced in this paper is an MCDA method that allows assigning a set of given alternatives evaluated on a set of criteria into a set of classes predefined with assignment examples and considers the preference information to be imprecise or uncertain. We concentrate on nominal classification.

Next, we consider a mathematical formulation of a classification problem in Section 2.1, and then we discuss the specificity of SMAA methodology in Section 2.2. After that we show how the classification problem is solved with the SMAA-Classification method and illustrate the new method with a simple example in Section 2.3. Then in Section 3 we compare the results of classification by SMAA-Classification with the results of some already developed methods. Finally, after the conclusions, some directions of future research are given.

2 The SMAA-Classification method

2.1 The model of classification problem

The mathematical formulation of a classification problem can be described as follows: it is necessary to assign the set of alternatives $X = \{x_1, x_2, \dots, x_m\}$ evaluated on the set of criteria $G = \{g_1, g_2, \dots, g_n\}$ into one class from the predefined set $L = \{l_1, l_2, \dots, l_s\}$. Estimation of the alternative x_i on the criterion g_j is denoted as $g_j(x_i)$. We assume that the set of assignment examples is available in $B = \{b_1^1, \dots, b_{t_1}^1, \dots, b_1^s, \dots, b_{t_s}^s\}$; we call them reference alternatives, where t_1 and t_s are numbers of reference alternatives for the classes l_1 and l_s , respectively. In this work we consider at least one reference alternative for each class. The SMAA method allows that the criteria evaluations and/or DM's preferences are uncertain or imprecise. That is why the criteria values of the alternative x_i can be stochastic. Thus, we use notation ξ_i for defining alternative x_i estimated with stochastic values $g_j(\xi_i)$ on each criterion g_j . The stochastic criteria evaluations are presented by joint probability distribution $f(\xi)$ in the space $R^{m \times n}$.

After the definition of classes, the actual classification procedure is considered: each alternative from the set X should be assigned into one of the classes if it is equal or roughly equal to at least one reference alternative of that class.

2.2 The SMAA-Classification method for nominal classes

If we followed the original SMAA or SMAA-2 method, a straightforward approach would be to use a value function for comparison of alternatives. However, in classification we cannot use this approach because, while the total values of the two alternatives can be roughly equal, the alternatives can still be far from each other in the criterion space. That is why other approaches are used in classification: assignment of the alternative 1) according to the maximal similarity to at least one of the reference alternatives of the class, or 2) with regards to the minimal distance between the alternative to be classified and at least one of the reference alternatives of the class. In most of the recently developed MCDA classification methods [2], [10], [24], [29], an approach based on the estimation of the similarity index is used. The similarity indices are different in different MCDA methods. For instance, in the TRINOMFC method [24], an alternative is assigned into a class based on the similarity index known as an "indifference index". Another approach to model similarity has been used in clustering analysis [11]. With regards to this approach the alternative is assigned into the class if the distance to the reference alternative of this class is minimal when compared to the distances to the reference alternatives of other classes.

The distance can be calculated with different metrics such as the Euclidean distance (L_2 -norm distance), the Mahalanobis distance (that is an Euclidean distance that takes into account correlations of the data set), the Manhattan distance (L_1 -norm distance), the Minkowski distance (L_p -norm distance) and others [8]. The most general type of the metric is the Minkowski distance, according to which the distance between the alternative to be classified ξ_i and the reference alternative b_h^q of the class l_q is:

$$d^M(\xi_i, b_h^q) = \left(\sum_{j=1}^n (|g_j(\xi_i) - g_j(b_h^q)|)^p \right)^{1/p}, \quad (1)$$

where $g_j(\xi_i)$ and $g_j(b_h^q)$ are the values of the alternatives ξ_i and b_h^q on the criterion g_j and the value of the $p \geq 1$ parameter is selectable. Thus, with $p = 1$ the Minkowski distance converts to the Manhattan distance and with $p = 2$ it becomes the Euclidean one.

We assign the alternative ξ_i into the class l_q if the distance to the reference alternative b_h^q of this class is smallest when compared to the distances to the reference alternatives of the other classes:

$$class(\xi_i) = \arg \min_{\substack{1 \leq q \leq s \\ h=1 \dots t_q}} d^M(\xi_i, w, b_h^q) = \arg \min_{\substack{1 \leq q \leq s \\ h=1 \dots t_q}} \left\{ \left(\sum_{j=1}^n (|g_j(\xi_i) - g_j(b_h^q)|)^p \right)^{1/p} \right\}. \quad (2)$$

For the case where there is information about the relative importance of the criterion values being close to reference alternative values in the form of weights we can use weighted Minkowski distance:

$$d^{WM}(\xi_i, w, b_h^q) = \left(\sum_{j=1}^n (w_j |g_j(\xi_i) - g_j(b_h^q)|)^p \right)^{1/p}. \quad (3)$$

In the present work, for the estimation of the distance between alternatives we select the weighted Euclidean distance as it is the most commonly used one:

$$class(\xi_i, w) = \arg \min_{\substack{1 \leq q \leq s \\ h=1 \dots t_q}} d^{WE}(\xi_i, w, b_h^q) = \arg \min_{\substack{1 \leq q \leq s \\ h=1 \dots t_q}} \left\{ \sqrt{\sum_{j=1}^n (w_j |g_j(\xi_i) - g_j(b_h^q)|)^2} \right\}, \quad (4)$$

where s is the number of classes and t is the number of reference alternatives in the class l_q .

The SMAA methodology assumes exploring the space of imprecise or uncertain parameters in order to find such of them that support each alternative. For example, for choice type of problem SMAA finds such parameter values that support alternative to be the best one and for ranking type of problem such parameter values that assign alternative to a particular rank. By analogy, the SMAA-Classification method searches for such parameter values that classify the alternative into a given class. For instance, in the weighted Euclidean distance one might be interested in exploring such parameters as weights.

According to the SMAA-Classification method, after the distance function is selected the Monte-Carlo simulation is organized in the following way for each alternative to be classified. Random sets of weights are generated from the nonnegative and normalized weight space, i.e.:

$$w \in W = \left\{ w \in R^n \mid w \geq 0 \text{ and } \sum_{j=1}^n w_j = 1 \right\}. \quad (5)$$

Then with regards to the selected distance function using a given weight vector the alternative to be classified is assigned into one of the classes. This procedure is repeated for the number of iterations selected. The number of iterations is selected with regards to the results' accuracy requirements. In [36] it was shown that 10000 iterations is typically enough for the reliable simulation with SMAA. As a result of the Monte-Carlo simulation the statistical information for each alternative to be assigned into each class is collected. This information includes the number of successful classifications into each class and aggregated preferences that have been simulated for such successful classifications. Then the SMAA-Classification method may calculate descriptive information in the form of acceptability index for each alternative to be classified into each class. The relation between the number of successful classifications and the total number of iterations yields an acceptability index.

The acceptability index shows the variety of different successful valuations that assign the alternative ξ_i into the class l_q :

$$a_i^q = \frac{1}{z} \sum_{k: class(\xi_i^k, w^k) = l_q} 1, \quad (6)$$

where z is total number of simulation runs and k is successful iteration at which the alternative ξ_i is assigned into the class l_q and w^k is weight vector with which such successful classification have been obtained.

The relation of the total number of runs to the number of runs in which the classification has been successful defines the acceptability index. The higher the value of the acceptability index $a_i^q \in [0, 1]$ the higher the probability for the alternative ξ_i to be assigned into the class l_q . Thus, $a_i^q = 0$ indicates that the alternative ξ_i is never assigned into the class l_q , and $a_i^q = 1$ shows that the alternative ξ_i is assigned into the class l_q with any set of simulated weights.

If additional information is desired in SMAA-Classification, we can give the information about favorable weight vectors with which successful classifications have been obtained. The favorable weight vector shows the importance of each criterion in the distance function for classification of the alternative into the class in the following way. If it is important that criterion value of alternative to be classified is close to the corresponding reference alternative value then their weight should be big.

Thus, from all simulated weight vectors the favorable once are selected in $W^q(\xi_i)$ with which the alternative ξ_i is successfully assigned into the class l_q :

$$W^q(\xi_i) = \{w \in W : class(\xi_i, w) = l_q\}. \quad (7)$$

Then a central weight vector may be calculated by averaging the set of favorable weights. The central weight vector w_i^q represents averaged weights that support classification of the alternative ξ_i into the class l_q :

$$w_i^q = \frac{\sum_{k: class(\xi_i^k, w^k) = l_q} 1}{\sum_{k: class(\xi_i^k, w^k) = l_q} \frac{1}{w^k}}, \quad (8)$$

where k is successful iteration at which the alternative ξ_i is assigned into the class l_q and w^k is weight vector with which such successful classification have been obtained.

It may be calculated as the harmonic average of all the weights that provide a successful assignment of the alternative ξ_i into the class l_q . The descriptive information is provided to the DM for further analysis.

Next we illustrate the performance of the SMAA-Classification method with a simple example. Even though the method allows criteria values to be imprecise and inaccurate, in the illustrative example and the applications presented they are exact.

2.3 An illustrative example

In order to demonstrate the SMAA-Classification method we consider an illustrative example with an artificial set of 27 alternatives evaluated on 3 criteria. This set is formed as a Cartesian product of criteria values $\{0.1, 0.2, 0.3\}$. The alternatives are classified into 2 classes that both have one reference alternative: the alternative $b_1^1 = x_6 = (0.1, 0.2, 0.3)$ is the reference alternative for the first class l_1 , and the alternative $b_1^2 = x_{13} = (0.2, 0.2, 0.1)$ is the reference alternative for the second class l_2 . We run the SMAA-Classification method with 10 and 10000 iterations in the Monte-Carlo simulation.

As an example of how the calculation is realized we show the results of a simulation with 10 iterations for the alternative $x_{15} = (0.2, 0.2, 0.3)$ presented in Figure 1. Let us follow the evaluation of this alternative at each step of the SMAA-Classification method. At the beginning the method simulates random weights with regards to some distribution, for instance, uniform, in such a way that $\sum_{j=1}^n w_j = 1$. At the first iteration we obtain the weights

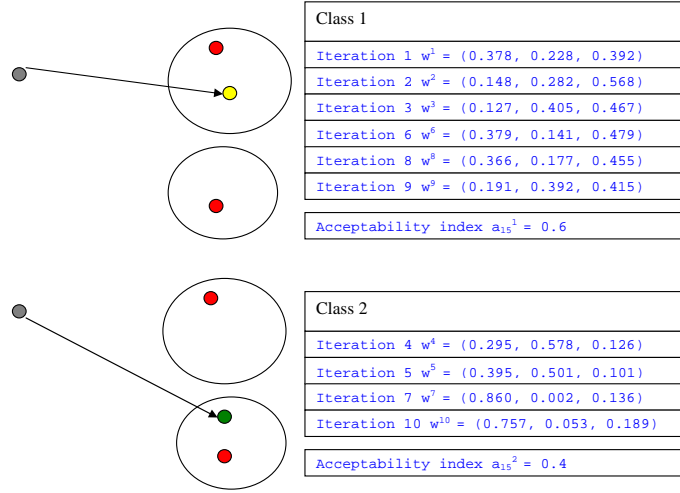


Figure 1: Random weight vectors for alternative $x_{15} = (0.2, 0.2, 0.3)$

$w_1 = (0.378, 0.228, 0.392)$. Then the distances between the alternative to be classified x_{15} and the reference alternatives x_6 and x_{13} are calculated. The alternative x_{15} is assigned into the class l_1 according to the minimal distance. Then these two steps are repeated as many times as there are iterations. After 10 iterations we have the following situation: with different simulated weights the alternative x_{15} appears 6 times in the class l_1 and 4 times in the class l_2 , which corresponds to the acceptability indices $a_{15}^1 = 0.6$ and $a_{15}^2 = 0.4$ for the two classes. (Then the central weight vectors for each class can be calculated by averaging the random weights, with which the alternative x_{15} has been assigned into each class. Thus, we can obtain the central weight vectors $w_{15}^1 = (0.264, 0.270, 0.462)$ and $w_{15}^2 = (0.576, 0.283, 0.138)$ for the classes l_1 and l_2 , correspondingly).

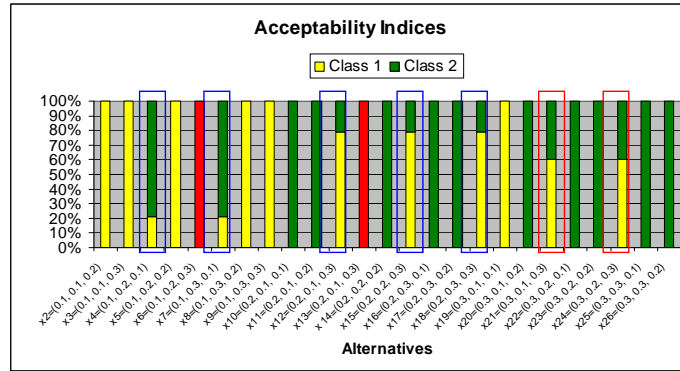


Figure 2: Diagram of alternatives with acceptability indices

Usually, 10 iterations is not enough to get a complete picture of all possible random situations. That is why we have run the same example with 10000 iterations. The results are presented in Figure 2 in the form of a diagram with acceptability indices calculated for 27 alternatives. Here each bar shows the percentage of acceptability for assigning the alternative into each class. For instance, from the very first bar on the left we can see that the alternative $x_2 = (0.1, 0.1, 0.2)$ is assigned into the class l_1 with 100% of random weight vectors. This means that the alternative x_2 is assigned into the class l_1 with any simulated weight vector. A similar situation appears with the alternatives x_3, x_5, x_8, x_9 . For the alternatives $x_{10}, x_{11}, x_{14}, x_{16}, x_{17}, x_{20}, x_{22}, x_{23}, x_{25}, x_{26}$ there is an unambiguous classification into the class l_2 . For the rest of the alternatives the assignment is not so obvious. For instance, the alternative $x_4 = (0.1, 0.2, 0.1)$

is assigned into the class l_1 with 21% of simulated weight vectors and into the class l_2 with 79% of cases.

Alternative	Class 1	Class 2
$x_4=(0.1, 0.2, 0.1)$	$a_4^1 = 0.21$	$a_4^2 = 0.79$
$x_7=(0.1, 0.3, 0.1)$	$a_7^1 = 0.21$	$a_7^2 = 0.79$
$x_{12}=(0.2, 0.1, 0.3)$	$a_{12}^1 = 0.79$	$a_{12}^2 = 0.21$
$x_{15}=(0.2, 0.2, 0.3)$	$a_{15}^1 = 0.79$	$a_{15}^2 = 0.21$
$x_{18}=(0.2, 0.3, 0.3)$	$a_{18}^1 = 0.79$	$a_{18}^2 = 0.21$
$x_{21}=(0.3, 0.1, 0.3)$	$a_{21}^1 = 0.6$	$a_{21}^2 = 0.4$
$x_{24}=(0.3, 0.2, 0.3)$	$a_{24}^1 = 0.6$	$a_{24}^2 = 0.4$

Table 1: Alternatives with acceptability indices different from 0 and 1

Table 1 presents the acceptability indices for the alternatives with acceptability indices different from 0 or 1. The DM may be interested to see the classes where each alternative may be classified. It is up to the DM to define the value of the acceptability index that is sufficient for crisp assignment of an alternative into a class. Alternatively, the DM may pay more attention to such alternatives.

3 Numerical experiments

In order to test the performance of the SMAA-Classification method we have done several numerical experiments with data sets widely used in MCDA publications. We compare the results of SMAA-Classification to the results obtained with ELECTRE TRI and TRINOMFC. Even though ELECTRE TRI is an ordinal classification method, it is possible to compare the results obtained with it to the results obtained with nominal classification methods such as SMAA-Classification. In this case we have to assume that in the data provided for analysis the order of classes is unknown and the reference alternatives for each class are available. On the other hand, in this work we have used the testing data sets with precise criteria values of the alternatives to be classified, while the SMAA-Classification method can also perform assignment on the data sets with imprecise and uncertain criteria values.

The following applications are considered: "Diagnosis of the health of the firms" [6] which has been used for estimation of the ELECTRE TRI method in [28]; "Credit granting in the banking sector" [39] used in [26] for testing ELECTRE TRI; and "Identification of accident type" [24] used in the evaluation of the TRINOMFC method. We ran the SMAA-Classification method with 10000 iterations for each application.

The results of testing the three applications with SMAA-Classification, ELECTRE TRI and TRINOMFC are presented in the Appendix. Tables A1, B1 and C1 introduce the reference alternatives for each problem. In these tables the first columns define the alternatives estimated on the sets of criteria and the second columns show the assignment class. The alternatives to be classified and the results of classification by different methods are presented in Tables A2-A4, B2-B4 and C2. In these tables the first columns present the alternatives to be classified estimated on the set of criteria and the rest of the columns show the classes where the alternatives have been assigned by different methods. For the SMAA-Classification method we have presented the acceptability indices for the alternative to be assigned into each class. The cells highlighted in bold face indicate the highest acceptability indices when compared to the rest of classes.

The first application "Diagnosis of the health of the firms" involves 40 firms evaluated on 7 criteria and assigned into 3 ordered classes defined with the set of 3 assignment examples (one for each class) [6]. We have compared the performance of SMAA-Classification to the results of the classification given by ELECTRE TRI and presented in [28]. In Tables A2-A4 (see Appendix) the first column indicates the alternative to be assigned, the second and the third columns show the classes for the indicated alternative according to the assignment with the ELECTRE TRI optimistic and pessimistic procedures, respectively. And the last columns show the classes and acceptability indices for each alternative to be assigned into each class obtained with the SMAA-Classification method.

The results of the SMAA-Classification method are compatible with the results obtained by the ELECTRE TRI method. For both of these methods the class l_1 is empty. The rest of the classification differs in the following ways. In cases where assignment according to the optimistic and pessimistic procedures of ELECTRE TRI is different, SMAA-Classification usually has the highest acceptability into one of them (for instance, the alternative x_1 is assigned into class l_4 by the ELECTRE TRI optimistic procedure and also by SMAA-Classification, and into l_5 by the ELECTRE TRI pessimistic procedure). The general picture of assignment for SMAA-Classification and ELECTRE TRI is the same.

The second application "Credit granting in the banking sector" involves 45 alternatives evaluated on 7 criteria and assigned into 3 ordered classes with the set of 13 assignment examples (several for each class) [39]. We have compared the work of SMAA-Classification to the ELECTRE TRI results presented in [26]. The results of SMAA-Classification are similar to the ones of ELECTRE TRI (see Tables B2-B4 in Appendix).

The third application "Identification of accident type" is very small: there are 3 workers evaluated on 3 criteria and assigned into 3 not ordered classes with the set of 6 assignment examples (several for each class). We have compared the results of TRINOMFC presented in [24] with the results obtained by SMAA-Classification. As we can see from Table C2 in Appendix the first two alternatives are assigned similarly by both methods, while the third one is assigned into the class l_3 by SMAA-Classification and into the class l_2 by TRINOMFC (although the similarity index for the class l_3 is also high).

Let us finally emphasize that even though the new SMAA-Classification method does not require a great deal of parametrical information from the DM (such as thresholds or/and weights) and, thus, it does not load him or her cognitively, still it provides the same results as the methods that assume that the DM is able to define all the parameters precisely. Thus, SMAA-Classification is more flexible, while being as effective. On the other hand, SMAA-Classification can be used as a tool for modeling and solving classification tasks in situations where only imprecise or uncertain information about parameter values is available. At the same time SMAA-Classification provides a general picture of the chances for each alternative to be assigned into each possible class.

4 Conclusions and Future Research

In the present work we have introduced the SMAA-Classification method for nominal classification. The method assists in the assignment of alternatives when no DM(s) preferences are available or when they are inaccurate or imprecise. The possibility of the criteria values to be inaccurate is also taken into account. The method, based on the Monte-Carlo simulation, calculates, at each iteration, the distances between the alternatives to be classified and the reference alternatives. At each iteration the method assigns the alternative into the class if the distance to the reference alternative is minimal when compared to the distances to the refer-

ence alternatives of the other classes. Based on the analysis of statistical information obtained at the end of the simulation process the method shows the level of acceptability of the same alternative into different classes. If additional information is needed the method can also define central weight vectors with which such assignment is supported. This descriptive information is provided to the DM for evaluation. The task of the DM is to define a value for the acceptability index that is sufficient to classify each alternative into only one class and to resolve ambiguous cases of high level of acceptability into several classes.

We have tested our method with the results obtained by applying ELECTRE TRI on two different data sets and by classifying with TRINOMFC. The results of such experiments speak for the compatibility and effectiveness of the SMAA-Classification method when compared to these other earlier developed methods.

The SMAA-Classification method is greatly affected by the set of assignment examples or reference alternatives chosen. The closer this set to the set of etalon alternatives (that are brightest representatives of classes) is the better the resulting classification. Thus, it would be interesting in the future to test, for the SMAA-Classification method, the possibilities of interactive improvement of the set of reference alternatives in a similar way as it was done for the ELECTRE TRI method in [5]. The other interesting question concerns resolving contradictions among reference alternatives that might appear, for instance, if several DMs assign the same alternative to different classes or if the DM is inconsistent. Similar work has been done with ELECTRE TRI in [28].

In the future, we also plan to develop SMAA-Classification for more general situations, in which there is no a priori information about classes i.e., we will be dealing with clustering problem.

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Appendix

Table A1: "Diagnosis of the health of the firms" application: Reference Alternatives

Reference Alternative x	Class
$b_1^1 = (-12.0, -62.5, 92.5, 29.5, 42.5, 0.5, 0.0)$	l_1
$b_1^2 = (-5.0, -50.0, 82.5, 25.5, 36.0, 1.5, 1.0)$	l_2
$b_1^3 = (4.0, -10.0, 67.5, 20.5, 27.0, 3.0, 2.5)$	l_3
$b_1^4 = (16.5, 25.0, 47.5, 14.0, 18.0, 4.5, 3.5)$	l_4
$b_1^5 = (30.5, 48.5, 27.0, 5.0, 7.0, 5.0, 4.5)$	l_5

Table A2: "Diagnosis of the health of the firms" application: Assignments given by ELECTRE TRI and SMAA-Classification

Alternative x	ELECTRE		SMAA-Classification	
	PES	OPT	Class	Acceptability index
$x_0=(35.8, 67.0, 19.7, 0.0, 0.0, 5.0, 4.0)$	l_5	l_5	l_5	$a_0^5=1$
$x_1=(16.4, 14.5, 59.8, 7.5, 5.2, 5.0, 3.0)$	l_4	l_5	l_4 l_5 l_3	$a_1^4=0.916$ $a_1^5=0.084$ $a_1^3=0.00001$
$x_2=(35.8, 24.0, 64.9, 2.1, 4.5, 5.0, 4.0)$	l_3	l_5	l_5 l_4 l_3	$a_2^5=0.698$ $a_2^4=0.301$ $a_2^3=0.001$
$x_3=(20.6, 61.7, 75.7, 3.6, 8.0, 5.0, 3.0)$	l_3	l_5	l_4 l_5 l_3	$a_3^4=0.668$ $a_3^5=0.312$ $a_3^3=0.02$
$x_4=(11.5, 17.1, 57.1, 4.2, 3.7, 5.0, 2.0)$	l_3	l_5	l_4 l_5 l_3	$a_4^4=0.844$ $a_4^5=0.124$ $a_4^3=0.031$
$x_5=(22.4, 25.1, 49.8, 5.0, 7.9, 5.0, 3.0)$	l_4	l_5	l_4 l_5	$a_5^4=0.712$ $a_5^5=0.288$
$x_6=(23.9, 34.5, 48.9, 2.5, 8.0, 5.0, 3.0)$	l_4	l_5	l_5 l_4	$a_6^5=0.513$ $a_6^4=0.487$
$x_7=(29.9, 44.0, 57.8, 1.7, 2.5, 5.0, 4.0)$	l_4	l_5	l_5 l_4	$a_7^5=0.811$ $a_7^4=0.189$
$x_8=(8.7, 5.4, 27.4, 4.5, 4.5, 5.0, 2.0)$	l_3	l_5	l_4 l_5 l_3	$a_8^4=0.708$ $a_8^5=0.271$ $a_8^3=0.021$
$x_9=(25.7, 29.7, 46.8, 4.6, 3.7, 4.0, 2.0)$	l_3	l_5	l_4 l_5 l_3	$a_9^4=0.656$ $a_9^5=0.334$ $a_9^3=0.01$
$x_{10}=(21.2, 24.6, 64.8, 3.6, 8.0, 4.0, 2.0)$	l_3	l_5	l_4 l_5 l_3	$a_{10}^4=0.829$ $a_{10}^5=0.111$ $a_{10}^3=0.06$
$x_{11}=(21.2, 24.6, 64.8, 3.6, 8.0, 4.0, 2.0)$	l_3	l_5	l_4 l_5 l_3	$a_{11}^4=0.756$ $a_{11}^5=0.226$ $a_{11}^3=0.019$
$x_{12}=(21.2, 24.6, 64.8, 3.6, 8.0, 4.0, 2.0)$	l_3	l_5	l_5 l_4 l_3	$a_{12}^5=0.588$ $a_{12}^4=0.404$ $a_{12}^3=0.008$
$x_{13}=(21.2, 24.6, 64.8, 3.6, 8.0, 4.0, 2.0)$	l_3	l_5	l_4 l_3 l_5	$a_{13}^4=0.852$ $a_{13}^3=0.132$ $a_{13}^5=0.017$
$x_{14}=(10.4, 9.3, 80.9, 1.4, 4.1, 4.0, 2.0)$	l_2	l_5	l_4 l_3 l_5 l_2	$a_{14}^4=0.598$ $a_{14}^3=0.317$ $a_{14}^5=0.085$ $a_{14}^2=0.00001$
$x_{15}=(17.7, 19.8, 52.8, 7.9, 6.1, 4.0, 4.0)$	l_4	l_5	l_4 l_5	$a_{15}^4=0.88$ $a_{15}^5=0.12$

Table A3: Cont. "Diagnosis of the health of the firms" application: Assignments given by ELECTRE TRI and SMAA-Classification

Alternative x	ELECTRE		SMAA-Classification	
	PES	OPT	Class	Acceptability index
$x_{16}=(14.8, 15.9, 27.9, 5.4, 1.8, 4.0, 2.0)$	l_3	l_5	l_4 l_5 l_3	$a_{16}^4=\mathbf{0.664}$ $a_{16}^5=0.32$ $a_{16}^3=0.015$
$x_{17}=(16.0, 14.7, 53.5, 6.8, 3.8, 4.0, 4.0)$	l_4	l_5	l_4 l_5	$a_{17}^4=\mathbf{0.82}$ $a_{17}^5=0.18$
$x_{18}=(11.7, 10.0, 42.1, 12.2, 4.3, 5.0, 2.0)$	l_3	l_5	l_4 l_5 l_3	$a_{18}^4=\mathbf{0.936}$ $a_{18}^5=0.036$ $a_{18}^3=0.027$
$x_{19}=(11.0, 4.2, 60.8, 6.2, 4.8, 4.0, 2.0)$	l_3	l_5	l_4 l_3 l_5	$a_{19}^4=\mathbf{0.8}$ $a_{19}^3=0.165$ $a_{19}^5=0.035$
$x_{20}=(15.5, 8.5, 56.2, 5.5, 1.8, 4.0, 2.0)$	l_3	l_5	l_4 l_5 l_3	$a_{20}^4=\mathbf{0.846}$ $a_{20}^5=0.093$ $a_{20}^3=0.061$
$x_{21}=(13.2, 9.1, 74.1, 6.4, 5.0, 2.0, 2.0)$	l_2	l_5	l_3 l_4 l_5 l_2	$a_{21}^3=\mathbf{0.64}$ $a_{21}^4=0.342$ $a_{21}^5=0.017$ $a_{21}^2=0.001$
$x_{22}=(9.1, 4.1, 44.8, 3.3, 10.4, 3.0, 4.0)$	l_3	l_5	l_4 l_5 l_3	$a_{22}^4=\mathbf{0.812}$ $a_{22}^5=0.118$ $a_{22}^3=0.07$
$x_{23}=(12.9, 1.9, 65.0, 14.0, 7.5, 4.0, 3.0)$	l_3	l_5	l_4 l_3 l_5	$a_{23}^4=\mathbf{0.879}$ $a_{23}^3=0.119$ $a_{23}^5=0.002$
$x_{24}=(5.9, -27.7, 77.4, 16.6, 12.7, 3.0, 2.0)$	l_2	l_4	l_3 l_4 l_2	$a_{24}^4=\mathbf{0.98}$ $a_{24}^5=0.019$ $a_{24}^3=0.00001$
$x_{25}=(16.9, 12.4, 60.1, 5.6, 5.6, 3.0, 2.0)$	l_3	l_5	l_4 l_3 l_5	$a_{25}^4=\mathbf{0.702}$ $a_{25}^3=0.258$ $a_{25}^5=0.04$
$x_{26}=(16.7, 13.1, 73.5, 11.9, 4.1, 2.0, 2.0)$	l_3	l_5	l_3 l_4 l_5 l_2	$a_{26}^3=\mathbf{0.66}$ $a_{26}^4=0.334$ $a_{26}^5=0.006$ $a_{26}^2=0.00001$
$x_{27}=(14.6, 9.7, 59.5, 6.7, 5.6, 2.0, 2.0)$	l_3	l_5	l_3 l_4 l_5 l_2	$a_{27}^3=\mathbf{0.51}$ $a_{27}^4=0.471$ $a_{27}^5=0.019$ $a_{27}^2=0.00001$
$x_{28}=(5.1, 4.9, 28.9, 2.5, 46.0, 2.0, 2.0)$	l_1	l_5	l_3 l_4 l_2 l_5 l_1	$a_{28}^3=\mathbf{0.635}$ $a_{28}^4=0.292$ $a_{28}^2=0.046$ $a_{28}^5=0.006$ $a_{28}^1=0.001$

Table A4: Cont. "Diagnosis of the health of the firms" application: Assignments given by ELECTRE TRI and SMAA-Classification

Alternative x	ELECTRE		SMAA-Classification	
	PES	OPT	Class	Acceptability index
$x_{29}=(24.4, 22.3, 32.8, 3.3, 5.0, 3.0, 4.0)$	l_4	l_5	l_5 l_4 l_3	$a_{29}^5=0.765$ $a_{29}^4=0.228$ $a_{29}^3=0.007$
$x_{30}=(29.5, 8.6, 41.8, 5.2, 6.4, 2.0, 3.0)$	l_4	l_5	l_4 l_5 l_3	$a_{30}^4=0.562$ $a_{30}^5=0.278$ $a_{30}^3=0.16$
$x_{31}=(7.3, -64.5, 67.5, 30.1, 8.7, 3.0, 3.0)$	l_2	l_5	l_3 l_4 l_2 l_1 l_5	$a_{31}^3=0.844$ $a_{31}^4=0.132$ $a_{31}^2=0.023$ $a_{31}^1=0.001$ $a_{31}^5=0.00001$
$x_{32}=(23.7, 31.9, 63.6, 12.1, 10.2, 3.0, 3.0)$	l_3	l_5	l_4 l_3 l_5	$a_{32}^4=0.875$ $a_{32}^3=0.122$ $a_{32}^5=0.003$
$x_{33}=(18.9, 13.5, 74.5, 12.0, 8.4, 3.0, 3.0)$	l_3	l_5	l_4 l_3 l_5	$a_{33}^4=0.626$ $a_{33}^3=0.372$ $a_{33}^5=0.002$
$x_{34}=(13.9, 3.3, 78.7, 14.7, 10.1, 2.0, 2.0)$	l_2	l_5	l_3 l_4 l_2 l_5	$a_{34}^3=0.873$ $a_{34}^4=0.122$ $a_{34}^2=0.005$ $a_{34}^5=0.00001$
$x_{35}=(-13.3, -31.1, 63.0, 21.2, 29.1, 2.0, 1.0)$	l_2	l_3	l_2 l_3 l_1	$a_{35}^2=0.822$ $a_{35}^3=0.177$ $a_{35}^1=0.001$
$x_{36}=(6.2, -3.2, 46.1, 4.8, 10.5, 2.0, 1.0)$	l_2	l_4	l_3 l_4 l_2 l_5	$a_{36}^3=0.643$ $a_{36}^4=0.318$ $a_{36}^2=0.028$ $a_{36}^5=0.011$
$x_{37}=(4.8, -3.3, 71.1, 8.6, 11.6, 2.0, 2.0)$	l_3	l_4	l_3 l_4 l_2 l_5	$a_{37}^3=0.871$ $a_{37}^4=0.128$ $a_{37}^2=0.001$ $a_{37}^5=0.00001$
$x_{38}=(0.1, -9.6, 42.5, 12.9, 12.4, 1.0, 1.0)$	l_2	l_4	l_3 l_2 l_4	$a_{38}^3=0.678$ $a_{38}^2=0.212$ $a_{38}^4=0.11$
$x_{39}=(13.6, 9.1, 76.0, 17.1, 10.3, 1.0, 1.0)$	l_2	l_5	l_3 l_2 l_4 l_5	$a_{39}^3=0.692$ $a_{39}^2=0.245$ $a_{28}^4=0.062$ $a_{28}^5=0.001$

Table B1: "Credit granting in the banking sector" application: Reference Alternatives

Reference Alternative x	Class
$b_1^1=x_{19}=(13.02, 15.74, 18.02, 7.24, 79.21, 42.63, 79.32)$	l_1
$b_2^1=x_{23}=(13.66, 11.01, 14.11, 70.55, 69.01, 18.77, 42.39)$	l_1
$b_3^1=x_{41}=(13.04, 7.99, 22.44, 7.24, 31.4, 14.83, 58.65)$	l_1
$b_4^1=x_{49}=(13.48, 1.05, 18.02, 6.45, 31.4, 18.77, 100.0)$	l_1
$b_5^1=x_{68}=(9.91, 7.99, 14.11, 7.24, 12.92, 3.02, 58.65)$	l_1
$b_6^1=x_5=(14.43, 11.01, 18.02, 29.25, 22.16, 31.73, 39.44)$	l_1
$b_1^2=x_{29}=(13.39, 11.01, 18.02, 17.36, 22.16, 8.91, 39.44)$	l_2
$b_2^2=x_{55}=(12.14, 7.99, 18.02, 5.46, 22.16, 3.02, 39.44)$	l_2
$b_3^2=x_{62}=(11.07, 7.99, 18.02, 5.46, 12.92, 3.02, 39.44)$	l_2
$b_4^2=x_{66}=(10.25, 7.99, 14.11, 15.58, 12.92, 8.91, 20.24)$	l_2
$b_1^3=x_{69}=(10.65, 11.01, 10.47, 3.69, 3.76, 8.91, 20.24)$	l_3
$b_2^3=x_{84}=(8.26, 7.99, 10.47, 3.69, 3.76, 3.02, 20.24)$	l_3
$b_3^3=x_{94}=(3.86, 1.96, 3.02, 3.69, 3.76, 3.02, 20.24)$	l_3

Table B2: "Credit granting in the banking sector" application: Assignments given by ELECTRE TRI SMAA-Classification

Alternative x	ELECTRE PES	SMAA-Classification	
		Class	Acceptability index
$x_6=(14.15, 11.01, 18.02, 19.13, 49.87, 14.83, 58.65)$	l_1	l_1	$a_6^1=\mathbf{0.908}$
		l_2	$a_6^2=0.092$
$x_9=(14.54, 14.02, 18.02, 4.65, 12.92, 3.02, 58.65)$	l_1	l_1	$a_9^1=\mathbf{0.837}$
		l_2	$a_9^2=0.161$
		l_3	$a_9^3=0.002$
$x_{12}=(14.42, 11.01, 18.02, 4.65, 12.92, 8.91, 58.64)$	l_1	l_1	$a_{12}^1=\mathbf{0.832}$
		l_2	$a_{12}^2=0.15$
		l_3	$a_{12}^3=0.018$
$x_{19}=(13.02, 15.74, 18.02, 7.24, 79.21, 42.63, 79.32)$	l_1	l_1	$a_{19}^1=\mathbf{1}$
$x_{21}=(13.6, 4.02, 22.44, 4.65, 31.4, 31.73, 58.65)$	l_1	l_1	$a_{21}^1=\mathbf{1}$
$x_{23}=(13.66, 11.01, 14.11, 70.55, 69.01, 18.77, 42.39)$	l_1	l_1	$a_{23}^1=\mathbf{1}$
$x_{25}=(13.04, 7.99, 22.44, 40.19, 40.64, 6.96, 100.0)$	l_1	l_1	$a_{25}^1=\mathbf{0.922}$
		l_2	$a_{25}^2=0.078$

Table B3: Cont. "Credit granting in the banking sector" application: Assignments given by ELECTRE TRI and SMAA-Classification

Alternative x	ELECTRE PES	SMAA-Classification	
		Class	Acceptability index
$x_{26}=(12.97, 7.99, 18.02, 29.66, 31.4, 18.77, 100.0)$	l_1	l_1 l_2	$a_{26}^1=\mathbf{0.996}$ $a_{26}^2=0.004$
$x_{41}=(13.04, 7.99, 22.44, 7.24, 31.4, 14.83, 58.65)$	l_1	l_1	$a_{41}^1=\mathbf{1}$
$x_{49}=(13.48, 1.05, 18.02, 6.45, 31.4, 18.77, 100.0)$	l_1	l_1	$a_{49}^1=\mathbf{1}$
$x_{58}=(10.41, 1.96, 18.02, 19.13, 22.16, 18.77, 100.0)$	l_1	l_1 l_2 l_3	$a_{58}^1=\mathbf{0.922}$ $a_{58}^2=0.074$ $a_{58}^3=0.004$
$x_{68}=(9.91, 7.99, 14.11, 7.24, 12.92, 3.02, 58.65)$	l_1	l_1	$a_{68}^1=\mathbf{1}$
$x_{73}=(8.65, 1.96, 18.02, 71.48, 59.11, 7.88, 60.12)$	l_1	l_1 l_2 l_3	$a_{73}^1=\mathbf{0.996}$ $a_{73}^2=0.003$ $a_{73}^3=0.001$
$x_{81}=(7.58, 1.96, 18.02, 71.48, 49.87, 7.88, 60.12)$	l_1	l_1 l_2 l_3	$a_{81}^1=\mathbf{0.996}$ $a_{81}^2=0.004$ $a_{81}^3=0.001$
$x_{87}=(6.46, 4.98, 18.02, 50.78, 12.92, 6.96, 100.0)$	l_1	l_1 l_2 l_3	$a_{87}^1=\mathbf{0.927}$ $a_{87}^2=0.043$ $a_{87}^3=0.03$
$x_2=(14.64, 14.02, 18.02, 5.46, 22.16, 3.02, 39.44)$	l_2	l_2 l_1 l_3	$a_2^2=\mathbf{0.998}$ $a_2^1=0.001$ $a_2^3=0.001$
$x_8=(14.71, 14.02, 18.02, 6.43, 12.92, 3.02, 39.44)$	l_2	l_2 l_3 l_1	$a_8^2=\mathbf{0.997}$ $a_8^3=0.02$ $a_8^1=0.01$
$x_{13}=(14.65, 14.02, 18.02, 6.43, 22.16, 14.83, 39.44)$	l_2	l_2 l_1 l_3	$a_{13}^2=\mathbf{0.904}$ $a_{13}^1=0.095$ $a_{13}^3=0.001$
$x_{27}=(13.51, 14.02, 18.02, 29.25, 22.16, 8.91, 39.44)$	l_2	l_2 l_1	$a_{27}^2=\mathbf{0.739}$ $a_{27}^1=0.261$
$x_{29}=(13.39, 11.01, 18.02, 17.36, 22.16, 8.91, 39.44)$	l_2	l_2	$a_{29}^2=\mathbf{1}$
$x_{37}=(13.52, 11.01, 14.11, 6.43, 22.16, 3.02, 39.44)$	l_2	l_2 l_1 l_3	$a_{37}^2=\mathbf{0.986}$ $a_{37}^1=0.013$ $a_{37}^3=0.001$
$x_{38}=(13.39, 11.01, 18.02, 5.46, 22.16, 3.02, 39.44)$	l_2	l_2	$a_{38}^2=\mathbf{1}$
$x_{40}=(13.4, 7.99, 14.11, 6.43, 22.16, 8.91, 39.44)$	l_2	l_2 l_3 l_1	$a_{40}^2=\mathbf{0.974}$ $a_{40}^3=0.018$ $a_{40}^1=0.008$
$x_{48}=(13.24, 4.98, 14.11, 6.43, 22.16, 20.32, 39.44)$	l_2	l_2 l_1 l_3	$a_{48}^2=\mathbf{0.782}$ $a_{48}^1=0.214$ $a_{48}^3=0.004$

Table B4: Cont. "Credit granting in the banking sector" application: Assignments given by ELECTRE TRI and SMAA-Classification

Alternative x	ELECTRE PES	SMAA-Classification	
		Class	Acceptability index
$x_{55}=(12.14, 7.99, 18.02, 5.46, 22.16, 3.02, 39.44)$	l_2	l_2	$a_{55}^2=1$
$x_{59}=(11.01, 7.99, 18.02, 17.36, 22.16, 3.02, 39.44)$	l_2	l_2 l_3 l_1	$a_{59}^2=0.995$ $a_{59}^3=0.005$ $a_{59}^1=0.00001$
$x_{76}=(8.76, 4.98, 14.11, 17.36, 22.16, 3.02, 39.44)$	l_2	l_2 l_3 l_1	$a_{76}^2=0.93$ $a_{76}^3=0.056$ $a_{76}^1=0.014$
$x_{88}=(6.65, 4.98, 14.11, 5.46, 3.76, 14.83, 39.44)$	l_2	l_2 l_3 l_1	$a_{88}^2=0.636$ $a_{88}^3=0.343$ $a_{88}^1=0.021$
$x_{90}=(4.06, 1.05, 18.02, 39.78, 12.92, 3.02, 39.44)$	l_2	l_2 l_1 l_3	$a_{90}^2=0.554$ $a_{90}^1=0.319$ $a_{90}^3=0.127$
$x_{91}=(4.64, 1.96, 6.74, 5.46, 3.76, 3.02, 39.44)$	l_2	l_2 l_3 l_1	$a_{91}^2=0.529$ $a_{91}^3=0.471$ $a_{91}^1=0.00001$
$x_{66}=(10.25, 7.99, 14.11, 15.58, 12.92, 8.91, 20.24)$	l_3	l_3	$a_{66}^3=1$
$x_{69}=(10.65, 11.01, 10.47, 3.69, 3.76, 8.91, 20.24)$	l_3	l_3	$a_{69}^3=1$
$x_{77}=(9.4, 7.99, 10.47, 3.69, 3.76, 3.02, 20.24)$	l_3	l_3	$a_{77}^3=1$
$x_{84}=(8.26, 7.99, 10.47, 3.69, 3.76, 8.91, 20.24)$	l_3	l_3	$a_{84}^3=1$
$x_{85}=(8.45, 7.99, 6.74, 3.69, 3.76, 8.91, 20.24)$	l_3	l_3	$a_{85}^3=1$
$x_{92}=(5.11, 4.98, 3.02, 3.69, 3.76, 3.02, 20.24)$	l_3	l_3	$a_{92}^3=1$
$x_{93}=(3.68, 1.96, 6.74, 15.58, 3.76, 3.02, 20.24)$	l_3	l_3	$a_{93}^3=1$
$x_{94}=(3.86, 1.96, 3.02, 3.69, 3.76, 3.02, 20.24)$	l_3	l_3	$a_{94}^3=1$
$x_{95}=(2.56, 1.96, 6.74, 15.58, 3.76, 14.83, 20.24)$	l_3	l_3	$a_{95}^3=1$
$x_{96}=(2.81, 1.96, 3.02, 3.69, 5.41, 3.02, 20.24)$	l_3	l_3	$a_{96}^3=1$
$x_{97}=(1.04, 1.05, 14.11, 49.9, 3.76, 3.02, 20.24)$	l_3	l_3 l_1 l_2	$a_{97}^3=0.755$ $a_{97}^1=0.242$ $a_{97}^2=0.003$
$x_{98}=(1.48, 1.05, 3.02, 15.58, 3.76, 3.02, 20.24)$	l_3	l_3	$a_{98}^3=1$
$x_{99}=(1.68, 1.96, 3.02, 15.58, 5.41, 3.02, 20.24)$	l_3	l_3	$a_{99}^3=1$
$x_{100}=(0.6, 1.05, 0.71, 3.69, 5.41, 3.02, 20.24)$	l_3	l_3	$a_{100}^3=1$

Table C1: "Identification of accident type" application: Reference Alternatives

Reference Alternative x	Class
$b_1^1=(11, 17, 15)$	l_1
$b_2^1=(11, 18, 11)$	l_1
$b_1^2=(10, 13, 18)$	l_2
$b_1^3=(15, 14, 12)$	l_3
$b_2^3=(15, 16, 11)$	l_3
$b_3^3=(16, 15, 14)$	l_3

Table C2: "Identification of accident type" application: Assignments by TRINOMFC and SMAA-Classification

Alternative x	TRINOMFC		SMAA-Classification	
	Class	Similarity index	Class	Acceptability index
$x_0=(16, 14, 14)$	l_3	$SI_0^3=\mathbf{0.79}$	l_3	$a_0^3=\mathbf{1}$
	l_2	$SI_0^2 = 0.48$	l_2	$a_0^2=0$
$x_1=(10, 18, 11)$	l_1	$SI_1^1=\mathbf{0.7}$	l_1	$a_1^1=\mathbf{0.991}$
	l_3	$SI_1^3 = 0.21$	l_3	$a_1^3=0$
	l_2	$SI_1^2 = 0.1$	l_2	$a_1^2=0.009$
$x_2=(15, 17, 18)$	l_2	$SI_2^2=\mathbf{0.72}$	l_2	$a_2^2=\mathbf{0.17}$
	l_3	$SI_2^3 = 0.65$	l_3	$a_2^3=0.54$
	l_1	$SI_2^1 = 0.3$	l_1	$a_2^1=0.29$

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