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Based on Demand Response

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BASED ON DEMAND RESPONSE**

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REDEFINING LOSS AVERSE AND GAIN SEEKING CONSUMER PRICE BEHAVIOR BASED ON DEMAND RESPONSE

Abstract

Traditionally loss aversion in price behavior has been defined on the basis of changes in perceived value. In the spirit of marketing we propose a complementary definition, one based on changes in demand response.

It is commonly accepted that consumers use a reference price when evaluating prices in a category. Researchers have extensively studied if existing data reveal loss aversion in value functions, i.e. that consumers put more emphasis on prices above the reference price (perceived losses) than prices below it (gains). If the utility change owing to a price increase is greater (smaller) than the change owing to an equal decrease, loss aversion (gain seeking) in value is detected. Reference price and loss aversion are corner stones in Kahneman and Tversky's prospect theory. Numerous studies that attempt to identify such loss averse or gain seeking behavior estimate an additive utility function employing a multinomial or conditional logit model for choice.

We redefine loss averse and gain seeking behavior based on response in demand rather than in value. Hence, loss aversion in demand takes place if the expected fall of demand resulting from a price increase from the reference level is greater than the rise of demand due to an equal price decrease. We point out that loss aversion in value does not imply loss aversion in demand and vice versa. Assuming the consumers' preferences are given by a random utility model and the choice model is McFadden's conditional logit, we develop a simple formula to check the character of the price behavior of a segment. The formula provides novel insights in price behavior revealing an unexpected key player, the market share of the product under consideration, and advising to pay increased attention to gain seeking behavior. We also give an explanation to why the market share is so significant. In the results prospect theory plays only a minor part.

Keywords: reference price, loss aversion, conditional logit, brand choice

1. Introduction

We begin with an example. Hardie, Johnson and Fader (1993) study loss aversion and reference dependence of brand choice. Using scanner panel data for six brands of refrigerated orange juice purchases they estimate alternative models for household utilities. Such utility depends, among other things, on the price of the brand and on a reference price. Estimation results of their preferred model indicate significant *loss aversion in value*; i.e., marginal loss in utility for a price larger than the reference price is 66 percent larger than marginal gain in utility due to a lower price. Therefore, one might expect *loss aversion in demand* as well; i.e. that a price increase by e.g. 10 percent from reference level would change demand more than a price decrease of 10 percent would. Incorrect! Consumers in this case are in fact *gain seeking in demand*; i.e., demand response is stronger for price decrease than for price increase. We return to this example later.

The reference price concept is today well accepted in marketing theory. According to it consumers evaluate the prices of their choice alternatives not only in absolute values but against a reference price. Prices above the reference price are considered as perceived losses and below the reference price as perceived gains. Researchers have noted that consumers in their perceived value often reveal higher sensitivity to price increases than to decreases. In this case we say the behavior is *loss averse in value*. In the opposite case the behavior is *gain seeking in value*. Loss averse and gain seeking behavior represent asymmetric responses to price changes whereas a symmetric reaction is equal in size for both price rise and fall from the reference level. Similarly, Kahneman and Tversky's (1979 and 1991) prospect theory involves a reference level (for price, for instance) and a value function, which reflects higher sensitivity to losses than to gains.

In marketing literature, choice modelers have tested the reference effects, especially the existence of loss aversion in perceived value, typically using scanner panel data. Mixed evidence of loss aversion has been reported. The model of choice employed in the studies is commonly the multinomial or conditional logit (McFadden 1974) and the utility function estimated is typically additive which, in addition to price variables, can include e.g. brands, loyalty to the brands, promotional variables and quality as explanatory variables (Kalwani et al 1990, Kalyanaram and Little 1994, Krishnamurti, Mazumdar and Raj 1992, Lattin and Bucklin 1989, Hardie et al. 1993, Rajendran and Tellis 1994). Aggregate models were often estimated but in some of the studies the results are considered by segments or in different groups (Erdem, Mayhew and Sun 2001, Mazumdar and Papatla, 1995 and 2000, Bell and Lattin 2000). Thanks to new efficient estimation methods, recently research including individual models, and thus greater heterogeneity, estimated has come out (Klapper, Ebling and Temme 2005, Terui and Dahana 2006).

In all the work mentioned above the comparison of loss and gain in utilities is used to test whether the price behavior is loss averse or gain seeking. However, since the choice behavior is explicitly included in the estimation, another natural criterion is to compare changes in the expected demand of a product (as is done in Halme and Somervuori, 2009). If the demand decrease in case of a price increase is greater than the demand increase under an equal price decrease, then the price behavior is *loss averse in demand*, while in the opposite case the behavior is *gain seeking in demand*. As will be shown later, definitions based on value and on demand are different; for instance, loss

aversion in value may appear simultaneously with gain seeking in demand. In other words, to identify loss averse price behavior in demand it is not sufficient to observe loss aversion in value.

In the above setting, if the utility function of a segment has been estimated, then for any product, the model can be employed to calculate expected demand for alternative prices and thereby to test loss averse or gain seeking behavior in demand. We develop a simple formula for such tests. It needs as inputs the loss and gain in utilities caused by a price changes from a reference level. Moreover, a rewarding feature in the formula are the novel insights and interpretations it offers. A striking news is that the product's market share prevailing at its reference price plays a significant role in detecting loss averse or gain seeking behavior in demand. Also in the new light it offers, the study of gain seeking behavior gains additional interest. We conclude that, perhaps contrary to intuition, prospect theory only plays a minor role in understanding loss aversion in demand.

We will neither address some commonly studied reference price problems such as which kind of reference price to use, nor do we consider how to define in detail the utility function to be estimated. McFadden's conditional logit and our work consider a segment level model but we also discuss how individuals' preferences can differ within this segment. Generalization to an individual level model are straightforward, i.e. if individual utility functions and multinomial logit as the choice model are employed, our results are readily applicable.

We proceed by reviewing the conditional logit model in Section 2. We begin Section 3 by defining loss averse and gain seeking price behavior separately based on perception in value and on response in demand. Then criteria for price behavior being loss averse (gain seeking) in demand are given. We discuss in examples the new spices the result offers us in viewing loss averse or gain seeking price behavior. Further we investigate the implications of the result in two special cases. In the first one value functions are based on prospect theory and even then it is no exception to identify gain seeking behavior in demand response. In the second case linear or smooth value functions are viewed. Section 4 discusses implications in our framework in the case where demand functions are assumed to be linear. In Section 5 we return to the assumptions underlying the conditional logit model of McFadden and discuss in some detail how it includes consumer heterogeneity.

2. Conditional Logit Model

Employing the conditional logit model by McFadden (1974), we aim to analyze consumer behavior under price variations of a given product.

Consider customers in a given population facing a finite set of alternative choices $i = 0, 1, 2, \dots$. For $i > 0$, the alternatives refer to products which may be substitutes to each other. Alternative $i = 0$ refers to not choosing any of the products.

Customers are denoted by $\omega \in \Omega$, where Ω is the entire population under consideration. We interpret population Ω as a market segment. Each customer makes a choice either to buy one of the products ($i > 0$) or not to buy any ($i = 0$).

Customer preferences are assumed to be specified by an *additive random utility* model as follows. For each customer $\omega \in \Omega$, the utility $u_i(\omega)$ from choosing alternative i is given by

$$u_i(\omega) = v_i + e_i(\omega), \quad (1)$$

where v_i is common to all customers in Ω and $e_i(\omega)$ accounts for individual preferences.

We refer to component v_i as the *value function* of the random utility, and interpret the components $e_i(\omega)$ as realizations of a random variable ϵ_i in Ω . As in McFadden (1974), assume that the random variables ϵ_i are independent and identically Gumbel distributed¹, for all i .

Furthermore, assume that both common value components v_i and individual components $e_i(\omega)$ may account for product price but the distributions of random terms ϵ_i are independent of prices.

Given utility functions in equation (1), alternative i is preferred by the customer ω to other alternatives $j \neq i$ if

$$u_i(\omega) \geq u_j(\omega) \quad \text{for all } j \neq i.$$

Assume that the utility function (1) is scaled such that the standard deviation of ϵ_i is $\sigma_i = \pi/\sqrt{6} \approx 1.28$. With this scaling the probability of a randomly chosen customer $\omega \in \Omega$ choosing product i is

$$p_i = \frac{\exp(v_i)}{\sum_j \exp(v_j)}. \quad (2)$$

Hence, for product $i > 0$, p_i is interpreted as expected *relative demand*, while $p_i/(1-p_0)$ is the *market share*.

3. Pricing Impacts

Consider one particular product $i > 0$. Let prices h_K , for $K = N, H, L$ denote three alternative price levels for product i . Here N refers to normal price, which may be the current market price. Price h_N is considered as the reference price. Index H refers to high (an increased price) and L to low (a decreased price).

While keeping the prices constant for other products $j \neq i$, we now aim to investigate the sensitivity of demand of product i with respect to its price increase and decrease. For notational convenience, suppose the price increase $h_H - h_N$ is equal to the price decrease $h_N - h_L$. For the price alternatives N, H and L , denote the common value component v_i of product i by v_N, v_H and v_L , respectively. Denote

$$\delta_L = v_L - v_N, \quad \delta_H = v_N - v_H \quad \text{and} \quad \delta = \delta_H - \delta_L, \quad (3)$$

the gain in value due to price decrease, the loss due to price increase, and their difference, respectively. We assume $\delta_L \geq 0$ and $\delta_H \geq 0$.

Definition A. Given δ in (3), market segment Ω , with respect to price changes of product i , is *loss averse in value* if $\delta > 0$, *gain seeking in value* if $\delta < 0$, and *symmetric in value* if $\delta = 0$.

¹One might assume normal distribution instead of Gumbel distribution. In this case only numerical analysis is possible instead of the main result in (2).

Given price levels N , H and L for product i , the respective expected relative demand of product i obtained from (2) is denoted by p_N , p_H and p_L . Price sensitivity of demand for i is measured by changes in expected relative demand, given a price increase or decrease while maintaining prices of other products unchanged.

Let $\Delta_H = p_N - p_H$ denote the decrease in demand and $\Delta_L = p_L - p_N$ the increase due to price increase and decrease, respectively. Denote

$$\Delta = \Delta_H - \Delta_L = 2p_N - p_H - p_L. \quad (4)$$

Definition B. Given Δ in (4), market segment Ω with respect to price changes of product i is *loss averse in demand* if $\Delta > 0$, *gain seeking in demand* if $\Delta < 0$, and *symmetric in demand* if $\Delta = 0$.

Next we introduce the conditions for price behavior that is loss averse/gain seeking in demand. Its proof is in Appendix A. In the sequel, loss averse/gain seeking behavior refers to Definition B (to loss averse/gain seeking in demand) if not otherwise indicated.

Proposition 1. *Define*

$$p^* \equiv \frac{\exp(\delta_H) + \exp(-\delta_L) - 2\exp(\delta_H - \delta_L)}{2[\exp(\delta_H) - 1][1 - \exp(-\delta_L)]}. \quad (5)$$

Then the price behavior of market segment Ω is loss averse in demand if $p_N > p^$, gain seeking in demand if $p_N < p^*$, and symmetric in demand if $p_N = p^*$.*

The market share of the product under consideration is $p_N/(1 - p_0)$, where p_0 is the expected share of population which chooses not to buy any of the products. Hence, by Proposition 1, the segment is loss averse with respect to the product under consideration if its market share is larger than $p^*/(1 - p_0)$. For example, if $p_0 = .80$ and $p^* = .1$, then a market share larger than $.1/(1 - .8) = 50\%$ implies loss aversion. Note that for suitable values of δ_H and δ_L , p^* is negative. In this case the market segment is loss averse for any level of p_N . On the other hand, p^* can be larger than one so that the segment is gain seeking.

Figure 1 presents p^* as a function of δ_H and δ_L . When δ_H is large compared with δ_L , p^* is negative which means loss averse price behavior whereas in the opposite case p^* exceeds 1 indicating gain seeking behavior.

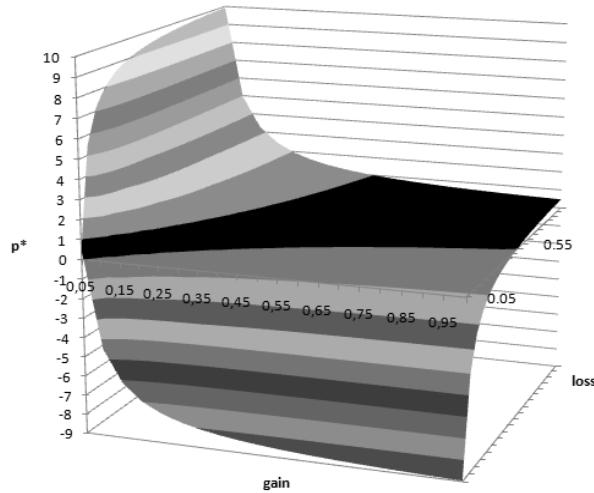
In the region of special interest in Figure 1 we have $0 < p^* < 1$. Figure 2 illustrates the values of δ_L and δ_H for p^* between zero and one. For $p^* = 0$, (5) yields $\delta_L = \log[2 - \exp(-\delta_H)]$. Hence, δ_L increases with δ_H and approaches $\log 2$ for large δ_H . Symmetrically for $p^* = 1$, (5) yields $\delta_H = \log[2 - \exp(-\delta_L)]$, so that also δ_H approaches $\log 2$ for large δ_L .

We illustrate the result in more detail by two examples.

Example 1. Table 1 presents the values of p^* when $\delta_L = .35$.² In this demonstration,

²The interpretation of the levels of value gain δ_L and loss δ_H relates to the standard deviation of the random variable ϵ_i of the random utility model. To obtain (2) from the utility function (1), the standard deviation of ϵ_i is $\pi/\sqrt{6} \approx 1.28$. Hence, $\delta_L = .35$ refers to a value loss, which is about 27 percent of the standard deviation 1.28. Hence, $\delta_L = .35$ refers to a value loss, which is about 27 percent of the standard deviation 1.28.

Figure 1: Limit p^* in (5) under alternative levels of value loss δ_H and gain δ_L . The black area refers to the region, where $0 \leq p^* \leq 1$.



we set $p_0 = 0$ referring to a case where choice alternative $j = 0$ is non-existent, and the relative demand p_N is the market share of product i .

If $\delta_H = \delta_L = .35$, then the price behavior is symmetric in value, and loss aversion in demand appears if the market share p_N with the prevailing price exceeds $p^* = .5$. This requirement is hard to meet in almost any product category. In the opposite case with $p_N < .5$, price behavior is gain seeking in demand.

Next, let us look at the case where the loss δ_H is strictly larger than the gain δ_L ; i.e. the behavior is loss averse in value. By intuition, one might expect that price behavior is loss averse in demand as well. However, it depends on the value of p^* , and in fact, the behavior may well be gain seeking in demand. For example, in case $\delta_H = .5 > \delta_L = .35$, Table 1 yields $p^* = .08$. Consequently, the price behavior is loss averse in demand only if $p_N > .08$, and in the opposite case it is gain seeking in demand. Hence, if the market share is small ($p_N < .08$), a price fall raises the demand more than an equal price increase decreases it even if the price increase causes a value loss δ_H greater than the value gain δ_L in the price fall.

Similarly, consider the case $\delta_H = .3 < \delta_L = .35$, where the behavior is gain seeking in value. Now in Table 1 we have $p^* = .74$. Again, perhaps against intuition, we conclude that the price behavior is loss averse in demand provided that $p_N > .74$. Hence, even if value loss is smaller than gain, in very favorable market conditions the market is more sensitive to an increase in the price than to a decrease that would further attempt to increase the already high market share.

As stated above, the condition for loss averse behavior in demand is $p_N > .5$ in case the value loss δ_H and gain δ_L are equal in size. Therefore, perhaps growing attention should be paid to empirical studies concerning gain seeking price behavior in demand. See also the more general results for linear and smooth value functions below.

Figure 2: Levels of value loss δ_H and gain δ_L with $0 \leq p^* \leq 1$.

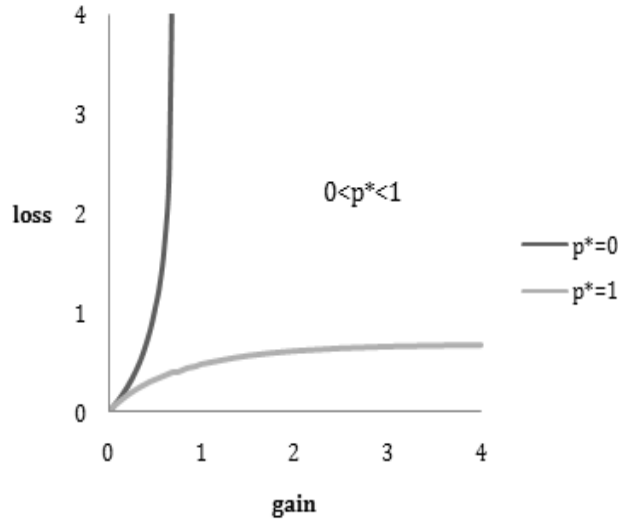


Table 1: Limits p^* under alternative levels of value loss δ_H given the level of value gain $\delta_L = 0.35$.

δ_H	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65
p^*	1.57	1.07	.74	.50	.32	.19	.08	-.01	-.08	-.15

Example 2. The literature contains abundantly examples on the estimation of linear value functions with the price as an argument. They use almost exclusively scanner panel data and thus model real marketplace choice. The value functions estimated for a segment then may readily be employed to evaluate p^* for a brand and compare it with the market share. To illustrate this we review the case of Hardie et al. (1993). Using scanner panel data for six brands of refrigerated orange juice, they estimate several models for household utilities. Here we concentrate on their preferred model *RBD*. In this model, the utility of a particular household depends among other things on the brand j in consideration and on a reference brand i , the last brand purchased. For illustrative purposes, we use price and quality data as well as parameter estimates for the utility model given in Tables 1 and 2 of Hardie et al (1993). The other factors in *RBD* related to promotion and brand loyalty are omitted in our analysis.

Let v_{ij} be the value of brand j , given reference brand i . For example, for $i = 1$ (Citrus Hill) and $j = 2$ (Minute Maid), the reference price is \$1.83, the price of Minute Maid is \$1.98, and the quality measures are .395 and .474, respectively. The increment in the value function due to quality gain is $1.904 \times (.474 - .395) = .150$. The increment

due to price increase is $1.660 \times 1.911 \times (1.83 - 1.98) = -.476$. Hence, given the reference brand is Citrus Hill, then the value of Minute Maid is $v_{12} = .150 - .476 = -.326$.

If π_i is the share of consumers with reference brand i , then the value function v_j in (1) is $v_j = \sum_i \pi_i v_{ij}$, and the market share p_i of i is given by (2). Given v_{ij} for all i and j , and assuming $p_i = \pi_i$, we may solve for market share p_i . Consequently, the market shares p_i for brands $i = 1, 2, \dots, 6$ are 21, 17, 15, 19, 21, and 7 per cent, respectively.

Next view Table 2 that presents results from price variations up and down by 2 % and 10 %. For each brand, the loss in utility δ_H for a price increase is larger than the gain δ_L due a price decrease. Hence the results indicate loss aversion in value, for each brand. However, for a price change of 10 percent, the change in demand is greater in the case of a price decrease than in the case of increase. Hence, in this case the consumers are gain seeking in demand, for each brand. For a price change of 2 percent, the market is gain seeking in demand for brands $i = 2$ and $i = 6$, and loss averse in demand for other brands. A more general result concerning loss aversion and small price changes is presented in Section 3.1.

Table 2: Results in Example 2 given price changes of two and ten per cent: For each brand i , δ_H = value loss, δ_L = value gain, Δ_H = demand decrease, Δ_L = demand increase, p^* = limit defined by (5), and p_N = market share of brand i under normal prices.

i	δ_H	δ_L	Δ_H	Δ_L	p^*	p_N
relative price change 2 %						
1	0.105	0.095	0.017	0.016	0.017	0.210
2	0.122	0.114	0.016	0.017	0.196	0.169
3	0.092	0.079	0.011	0.010	-0.382	0.152
4	0.089	0.076	0.013	0.012	-0.468	0.189
5	0.058	0.051	0.009	0.008	-0.685	0.208
6	0.143	0.139	0.009	0.010	0.396	0.073
relative price change 10 %						
1	0.532	0.430	0.075	0.080	0.282	0.210
2	0.610	0.555	0.069	0.092	0.421	0.169
3	0.486	0.385	0.053	0.056	0.233	0.152
4	0.479	0.380	0.063	0.065	0.234	0.189
5	0.289	0.254	0.044	0.045	0.264	0.208
6	0.717	0.696	0.036	0.064	0.480	0.073

3.1 Prospect Theory

Employing the normal price h_N as the reference price for product i , prospect theory suggests that the marginal value of the component v_i jumps down at the normal price. Hence, given small values of $\delta_H = v_N - v_H$ and $\delta_L = v_L - v_N$, prospect theory suggests

$$\delta_H > \delta_L \quad (6)$$

so that the behavior is loss averse in value. Of course, Proposition 1 applies in this case as well. Because (5) does not imply that $p^* \leq 0$, it is possible to observe gain seeking behavior even if preferences conform to prospect theory. For δ_H and δ_L small, we employ first order Taylor approximation for the exponent functions in (5) to obtain

$$p^* \approx \frac{[\delta_L - \delta_H]}{2\delta_H\delta_L}. \quad (7)$$

Because $\delta_L - \delta_H < 0$ by (6), loss aversion condition $p_N > p^*$ always holds. Consequently, we have the result:

Proposition 2. *If δ_H and δ_L are small (i.e., the price changes are small), then prospect theory implies loss aversion in demand for market segment Ω .*

Proposition 2 concerns infinitely small price changes only. Next, we aim to illustrate the entire range of price changes for which prospect theory implies loss aversion for any level of p_N ; i.e., $p^* \leq 0$ in Proposition 1.

Assume that the shape of the value function is the one given in Kahneman and Tversky's pioneering article (1979). This functional form is copied to Figure 3, and completed by our scales for monetary gain and value gain. Monetary gain is measured relative to the reference price. In Figure 3, the monetary gain ranges from -20% to 20% of the reference price. The value is expressed by the value gain relative to $\sigma = 1.28$, where σ is the standard deviation of ϵ_i of the utility function. The range of value gain relative to σ is from -1.5 to 1. At the reference price (where monetary gain is zero) there is a kink in the value function and the marginal value jumps down by factor 2.6.

The top-left region in Figure 2 reveals combinations (δ_L, δ_H) of gain and loss such that $p^* \leq 0$; i.e., combinations for which the market is loss averse for any p_N . In algebraic form, $p^* \leq 0$ if and only if

$$\delta_L \leq \log[2 - \exp(-\delta_H)]. \quad (8)$$

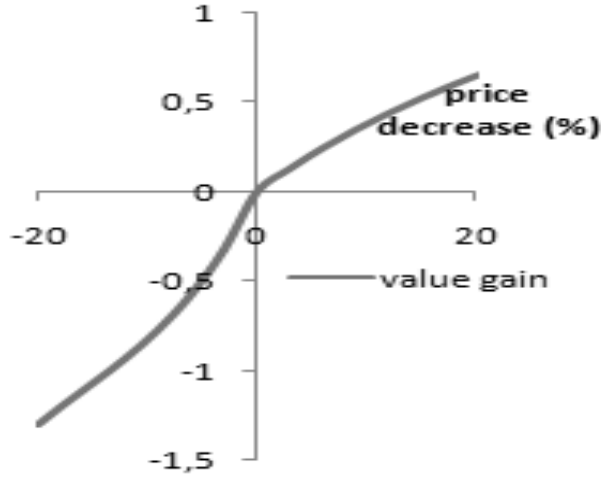
In Figure 3, a price increase of 18% yields a value loss $\delta_H/\sigma = 1.2$, so that the right hand side in (8) divided by σ is .45. But a price decrease of 18% yields a value gain $\delta_L/\sigma = .6$ violating condition (8). Hence, $p^* > 0$, and the market is loss averse or gain seeking according to Proposition 1. Similarly we may inspect other levels of price change and discover that $p^* \leq 0$ if the price change up and down is at most 14%. For such changes the market is always loss averse in demand.

As another example, consider a modification of the value function in Figure 3 such that at the kink of the revised value function the marginal value jumps down by factor 1.5 only, instead of the original factor 2.6. This is accomplished by scaling the gain up by factor $\sqrt{2.6/1.5}$ and loss down by the same factor. Again, by inspection we find a price range satisfying (8). The result is that $p^* \leq 0$ and the segment is loss averse in

demand for all p_N if the price change up and down is at most 5%. As expected, this range is smaller than the range obtained with a larger jump of marginal utility at the kink.

Finally consider a piece-wise linear value function (approximating the function of Figure 3.) with a kink at the reference price. Let θ denote the relative price change, and suppose the loss is $\delta_H/\sigma = 8\theta$ and gain $\delta_L = \delta_H/2.6$. Hence, the marginal utility jump factor again is 2.6. Then condition (8) holds if the price change up and down is at most 15%. If the jump factor is reduced to 1.5, then condition (8) requires the price change to be at most 12%.

Figure 3: A value function of prospect theory.



3.2 Linear or Smooth Value Function

Next, suppose v_i is an affine function of the price of product i . Then the marginal value with respect to price is constant, and consequently, the behavior is symmetric in value with $\delta_H = \delta_L$. In this case, $p^* = .5$ and loss aversion condition $\Delta > 0$ reduces to

$$p_N > .5. \tag{9}$$

Next, consider the case where v_i is differentiable with respect to price at the normal price. Then for small price variations $\delta_H \approx \delta_L$, and again, loss aversion condition $p_N > p^*$ reduces to (9). Hence, we conclude

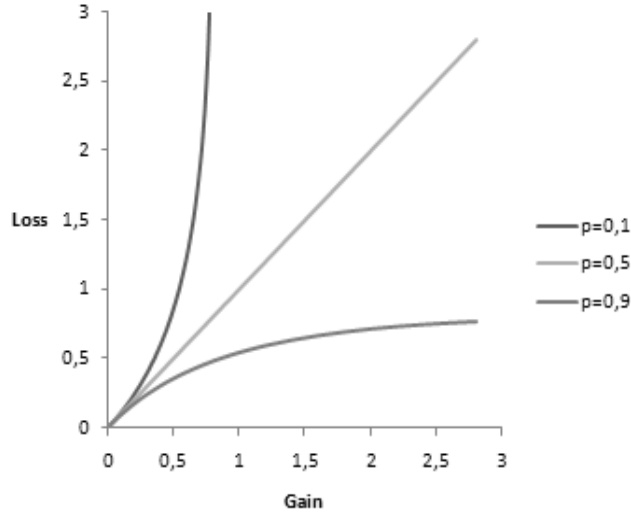
Proposition 3. *For market segment Ω , if the value function is smooth and the price changes are small or if the value function is affine, then $p_N > .5$ implies loss averse, $p_N < .5$ gain seeking, and $p_N = .5$ symmetric behavior in demand.*

By Proposition 3, the segment is loss averse if market share of the product under consideration is larger than $.5/(1-p_0)$. Hence, for $p_0 = .20$, a market share larger than $.5/(1-.2) = 62.5\%$ implies loss aversion.

4. Linear Demand Function

Linear models, where change in demand is proportional to change in price, are frequently used in text book examples, in applied market models, and in marketing research. We now examine the implications to our model in the case of such linear demand function.

Figure 4: Linear demand function: Value loss δ_H and gain δ_L .



Consider market segment Ω and price changes for product i . Let h_N denote the reference price and θ a decrease in price. Parameter θ is the *monetary gain* due to price change. It is positive for price decreases and negative for increases. For $\theta < 0$, the increased price is $h_H = h_N - \theta$ and the decreased price is $h_L = h_N - \theta$, for $\theta > 0$.

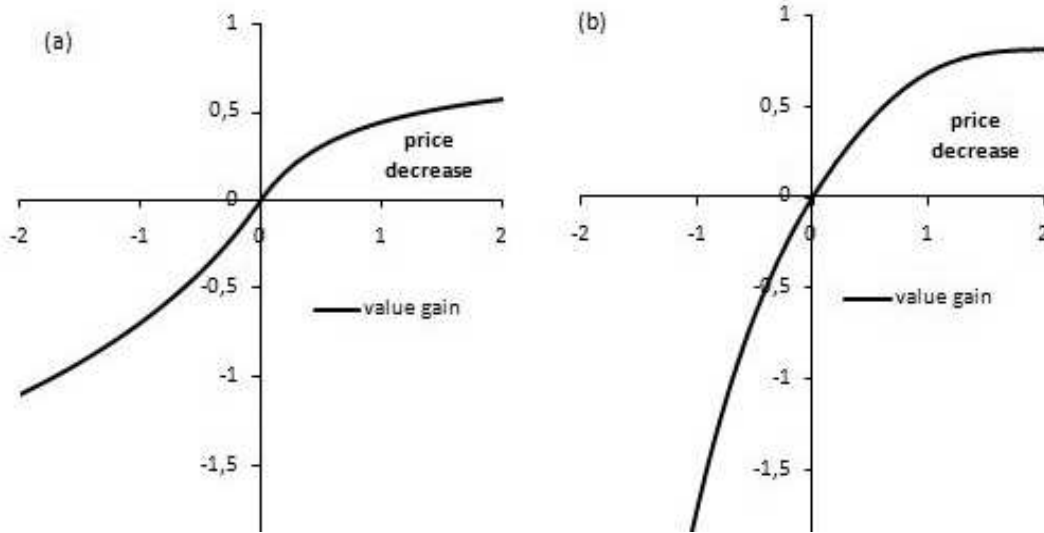
By Definition B, the price response is symmetric in demand if $\Delta = 0$ in (4). If the demand (response) function is linear, then the price response is symmetric for all price changes up and down which are equal in absolute value. Then (15) in Appendix A with $p = p_N$ and $q = 1 - p$ implies

$$1/[p + q \exp(\delta_H)] + 1/[p + q \exp(-\delta_L)] = 2. \quad (10)$$

Solving for value gain δ_L , yields

$$\delta_L = \log \left[\frac{2q + (p - q) \exp(-\delta_H)}{q - p + 2p \exp(-\delta_H)} \right]. \quad (11)$$

Figure 5: Linear demand function: Value gain as a function of price decrease.



For $p = 0.5$, (11) yields $\delta_L = \delta_H$. For $p < .5$, it can be shown that δ_L in (11) is an increasing and concave function of δ_H , and for large δ_H , δ_L approaches $\log[2q/(q - p)]$. In this case value loss is larger than gain.

Equivalently, solving for value loss δ_H from (10), yields

$$\delta_H = \log \left[\frac{2p + (q - p) \exp(-\delta_L)}{p - q + 2q \exp(-\delta_L)} \right] \quad (12)$$

Again, for $p = 0.5$, we have $\delta_L = \delta_H$. For $p > .5$, δ_H in (12) is an increasing and concave function of δ_L . For large δ_L , δ_H approaches $\log[2p/(p - q)]$, and value loss is smaller than gain. The functions (11) and (12) are depicted in Figure 4 for $p = .1$, $p = .5$ and $p = .9$.

The *value function* as a function of monetary gain θ is $v_i = v_N + v(\theta)$. For $\theta \geq 0$, $v(\theta) = \delta_L$ is the gain due to price decrease, $v(\theta)$ is non-negative, increasing with θ , and $v(0) = 0$. Hence, v_N is the value v_i at the reference price. For $\theta \leq 0$, $-v(\theta) = \delta_H$ is the loss due to price increase, and $v(\theta)$ is non-positive and increasing with θ .

The interrelationship between δ_H and δ_L is given by (11) and (12). We now illustrate the value function v_i by two examples, both with $p_N = .1$ for which gain δ_L as a function of loss δ_H is depicted in Figure 4.

In the first example, suppose that $v(\theta)$, for $\theta \leq 0$, due to price increase is convex. In particular, consider losses such that $\delta_H = v(\theta) = -\log(1 - \theta)$. This together with (11) determines the gain $\delta_L = v(\theta)$ for price decreases with $\theta > 0$. The resulting increasing value function v_i is depicted in Figure 5a. For price increases with $\theta < 0$, v_i is strictly convex, and for price decreases it is strictly concave. Such properties are some of the main characteristics of the value function of prospect theory as well. However, unlike in prospect theory, here the value function is smooth at the reference price.

In the second example, suppose that the loss function is concave and $\delta_H = v(\theta) = 1 - \exp(-\theta)$, for $\theta \leq 0$. Again, (11) helps to determine the gain $\delta_L = v(\theta)$ for $\theta > 0$, and the resulting value function v_i is depicted in Figure 5b. In this case v_i is increasing and strictly concave at all levels of the monetary gain θ . Hence, v_i may be interpreted as von Neumann-Morgenstern utility with risk averse preferences.

5. Customer Heterogeneity Considerations

Next we illustrate what kind of heterogeneity of households is allowed in the model that we use. Preferences of one particular customer $\omega \in \Omega$ are specified by utility $u_i(\omega) = v_i + e_i(\omega)$ from choosing alternative i , where v_i is common to all customers and $e_i(\omega)$ accounts for individual preferences. Alternative i is preferred to other alternatives $j \neq i$, if $u_i(\omega) \geq u_j(\omega)$ for all $j \neq i$. Hence, the probability of customer ω choosing i is either zero or one, unless there is a tie among two or more choices.

Consider one particular product $i > 0$ and three alternative price levels h_N , h_H and h_L for product i . Again, index N stands for a normal price, H for an increased price and L for a decreased price.

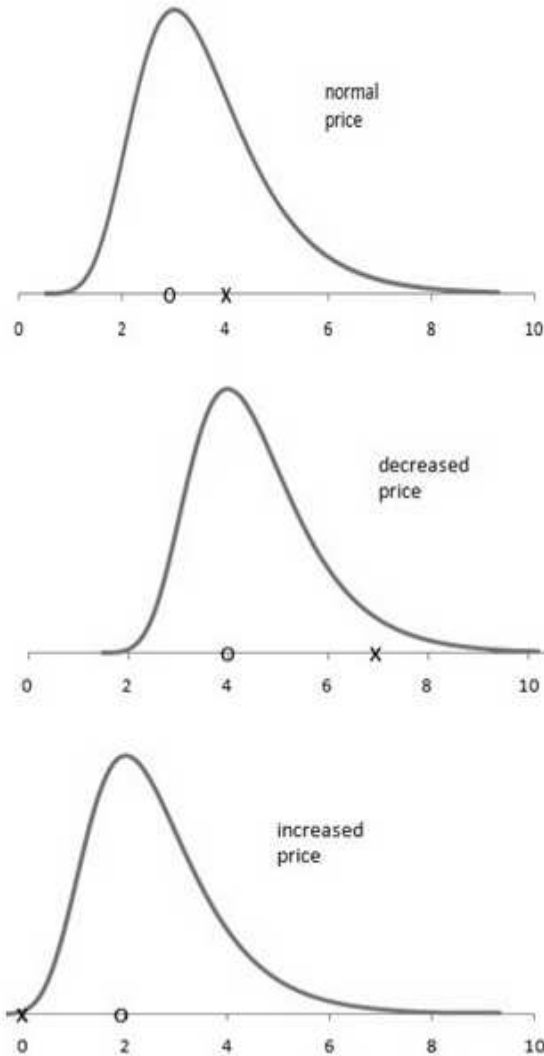
Both value components v_i and $e_i(\omega)$ may account for product price. For price levels h_N , h_H and h_L , let the respective values of v_i be v_N , v_H and v_L . Similarly, the values of $e_i(\omega)$ are denoted by e_N , e_H and e_L . Then the values of individual utility $u_i(\omega)$ are $u_N = v_N + e_N$, $u_H = v_H + e_H$ and $u_L = v_L + e_L$.

As an illustrative example, consider price levels $h_N = 10$, $h_H = 12$ and $h_L = 8$ for product i . Suppose the common value component v_i as a function of the price h is given by $v_i = 8 - .5 \times h$ so that $v_N = 3.0$, $v_H = 2$ and $v_L = 4$. For customer ω , suppose the individual value component $e_i(\omega)$ is concave and piece-wise linear in price so that $e_i(\omega) = -2 \times (h - 11)$, if $h \geq 11$ and $e_i(\omega) = 1 \times (11 - h)$, if $h \leq 11$. Hence there is a kink in the individual utility function $u_i(\omega)$, and price level $h = 11$ may be interpreted as an individual reference price in prospect theory.

The resulting values of $e_i(\omega)$ are $e_N = 1$, $e_H = -2$ and $e_L = 3$. Consequently, the utility levels $u_i(\omega)$ are $u_N = 4$, $u_H = 0$, and $u_L = 7$. Figure 6 shows the three cases. At the normal price the utility of customer ω is 33 % above the average. For price decrease, the individual utility of customer ω is 75 % above average. Instead, for a price increase, the individual utility is below average. Price changes from the normal level increment the common value component v_i up and down by 1, whereas the utility of customer ω is incremented up by 3 and down by 4. Hence, customer ω is more sensitive to price changes than customers on the average.

As pointed out above and indicated in Figure 6, the distribution of individual utility margins is independent of price level. One might argue that customer heterogeneity implies that the spread of individual margins $e_i(\omega)$ is influenced by price changes. Similarly, and also contrary to the assumption in McFadden's conditional logit model, such spread may depend on the choice alternative i as well. However, in this article we have adopted McFadden's model the same way as is done, for instance, by Bell and Lattin (2000), Krishnamurthi, Mazumdar and Raj (1992) and Kumar, Karande and Reinartz (1998). Incorporating standard deviations, which depend on choice alternative and on price, remains subject to further research and thereby is outside the scope of the present article.

Figure 6: Three distributions of utility function values. Notation: $\circ = v_N, v_L, v_H$, $\times = u_N, u_L, u_H$.



6. Conclusions

Numerous studies inspect empirically the presence of loss averse or gain seeking behavior around the reference price. They frequently employ an additive utility model and the multinomial or conditional logit model of choice. They define the existence of loss averse or gain seeking behavior in value using the estimated value function. We introduce loss averse and gain seeking behavior based on responses in expected demand due to price changes.

We employ the random additive utility model and the conditional logit, and derive general conditions for a market segment to be loss averse or gain seeking in demand. A formula to check the character of price behavior is introduced and its interpretation

is discussed. We note that even when the value function is consistent with prospect theory the existence of loss aversion in demand is not self-evident. The product's market share prevailing at the reference price has a powerful effect on the result. It is no surprise that a utility gain has a different effect on demand in the cases of a very low or very high market share. It seems plausible that a low market share can easily be boosted by a price decrease whereas it is tough to push up a market share already exceeding 50 per cent. For the main conclusions, assumptions from prospect theory are not essential.

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Appendix A: Proof of Proposition 1

For brevity, denote $p = p_N > 0$, $q = 1 - p > 0$, $x = \exp(\delta_H) > 1$ and $y = \exp(-\delta_L) < 1$. Then, given the normal price level N for product i , (2) yields expected relative demand p_N for product i , and

$$\frac{q}{p} = \frac{\sum_{j \neq i} \exp(v_j)}{\exp(v_N)}. \quad (13)$$

Then for any price level K ($K = N, H, L$) of product i , (2) and (13) yield expected relative demand

$$p_K = \frac{1}{1 + \exp(-v_K) \sum_{j \neq i} \exp(v_j)} = \frac{p}{p + q \exp(v_N - v_K)}. \quad (14)$$

Hence, from (3), (4) and (14) we get

$$\Delta = p \left[2 - \frac{1}{p + qx} - \frac{1}{p + qy} \right]. \quad (15)$$

In (15), $\Delta > 0$ if and only if

$$\begin{aligned} & 2(p + qx)(p + qy) - (p + qx) - (p + qy) \\ &= (p + qx)(p + qy - 1) + (p + qy)(p + qx - 1) \\ &= (p + qx)q(y - 1) + (p + qy)q(x - 1) > 0 \end{aligned}$$

or

$$\begin{aligned} & [p(1 - x) + x](y - 1) + [p(1 - y) + y](x - 1) \\ &= p[2(x - 1)(1 - y)] - x - y + 2xy > 0. \end{aligned}$$

Because $(x - 1)(1 - y) > 0$, this implies the loss aversion condition $p_N > p^*$ in Proposition 1. A non-positive nominator in (5) implies loss aversion for any p_N . However, the nominator can be strictly positive in which case $p_N < p^*$ implies gain seeking behavior, and if $p_N = p^*$, the price behavior is symmetric.