Aalto University
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# Low volatility anomaly and mutual fund allocations
## Evidence from the U.S.

Finance

Master's thesis

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**Abstract of master's thesis**

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PURPOSE OF THE STUDY

The purpose of this thesis is to provide further evidence on the low volatility anomaly by examining its existence during the period from 1974 to 2013 as well as during defined periods of rising and declining stock markets. This thesis clarifies whether low volatility stocks outperform high volatility stocks in absolute return terms, or only in risk-adjusted return terms. Furthermore, this thesis studies whether there is an excess demand for high volatility stocks, a potential reason for the anomaly, by examining U.S. equity mutual funds and mutual funds' allocations. In particular, this thesis examines whether an average portfolio manager has an overweight in high volatility stocks relative to the market portfolio, and additionally, an underweight in low volatility stocks.

DATA AND METHODOLOGY

The data used to examine the low volatility anomaly and mutual fund holdings is sourced from several sources within the WRDS database. My sample comprises common stocks listed on the NYSE, AMEX and NASDAQ during the period from 1973 to 2013 consisting of 4,266 stocks on average per month. I calculate historical monthly volatilities for the stocks, and every month, sort the stocks in an ascending order into deciles based on the historical volatilities. I calculate the performance of these decile portfolios and study the relationship between volatility and expected return. The CRSP mutual fund holdings data comprises U.S. mutual fund holdings reports from 2002 to 2013. I include funds that have 80% to 100% of their total net assets invested in U.S. equities. I examine funds' exposures to the sorted volatility deciles and compare the exposures to the construction of the market portfolio to determine the relative weights in low and high volatility stocks.

RESULTS

The results indicate that low volatility stocks offer clearly higher risk-adjusted returns, measured as the Sharpe ratio, and significantly higher alphas than high volatility stocks. The bottom volatility decile also outperform the top volatility decile in absolute return terms. Furthermore, Sharpe ratios are almost monotonously decreasing with volatility. However, volatility and absolute returns are positively related except for the highest volatility deciles, for which the absolute returns are decreasing with volatility. On average, U.S. mutual funds have a moderate overweight in high volatility stocks and a significant underweight in low volatility stocks. Therefore, a moderate excess demand for high volatility stocks and a notable demand shortage for low volatility may actually be the underlying reason why the anomaly exists.

**Keywords** low volatility anomaly, risk and return relationship, mutual funds, asset allocation

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**Maisterintutkinnon tutkielman tiivistelmä**

| | |
|---|---|
| **Tekijä** Matti Saarimaa | |
| **Työn nimi** Matalan volatiliteetin anomalia ja rahastoallokaatiot Yhdysvalloissa | |
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TUTKIELMAN TAVOITTEET

Tutkielman tavoitteena on täydentää matalan riskin anomaliaa koskevaa kirjallisuutta tutkimalla matalan volatiliteetin anomaliaa vuosina 1974-2013 sekä erikseen eri markkinaolosuhteissa. Tutkin, tuottaako matalan volatiliteetin osakkeet korkean volatiliteetin osakkeita paremmin ainoastaan riskikorjatulla tuotolla mitattuna vai myös absoluuttisesti mitattuna. Lisäksi tutkin, kohdistuuko korkean volatiliteetin osakkeisiin ylikysyntää, mitä pidetään mahdollisena syynä ilmiölle. Ylikysynnän selvittämisessä käytetään tietoja yhdysvaltalaisiin osakkeisiin sijoittavista rahastoista ja niiden osakeallokaatioista. Erityisesti tutkielman tavoitteena on selvittää ylipainottavatko salkunhoitajat tyypillisesti korkean volatiliteetin osakkeita, vai yrittävätkö salkunhoitajat hyödyntää matalan volatilititeetin anomaliaa ylipainottaen matalan volatiliteetin osakkeita.

AINEISTO JA MENETELMÄT

Tutkimuksen aineistona on käytetty tietoja useasta eri lähteestä ja tiedot on kerätty WRDS-tietokannasta. Tutkimusaineisto käsittää osakkeet, jotka ovat olleet listattuina yhdysvaltalaisissa pörsseissä (NYSE, AMEX ja NASDAQ) vuosina 1973-2013. Aineisto sisältää keskimäärin 4 266 osaketta per kuukausi. Jokaisena kuukautena järjestän osakkeet kymmeneen desiiliin (1/10) historiallisen volatiliteetin mukaan, lasken desiilien tuotot ja tutkin volatiliteetin ja tuoton välistä suhdetta. Rahastoja koskeva aineisto käsittää yhdysvaltalaisten osakerahastojen omistukset vuosina 2002-2013. Tutkin rahastoja, jotka ovat sijoittaneet 80%-100% nettovaroistaan Yhdysvalloissa listattuihin osakkeisiin. Tutkin rahastojen allokaatiota määrittämissäni volatiliteettiluokissa (volatiliteettidesiilit) ja vertaan allokaatioiden kokoa suhteessa markkinaportfolion jakaumaan määrittämissäni volatiliteettiluokissa. Tällä menetelmällä määritän mitä osakkeita salkunhoitajat keskimäärin yli- tai alipainottavat suhteessa markkinaportfolioon.

TULOKSET

Tulokset osoittavat, että matalan riskin osakkeet tarjoavat korkeampaa riskikorjattua tuottoa ja korkeampaa ylituottoa. Alimman volatiliteettidesiilin absoluuttinen tuotto on myöskin korkeampi kuin ylimmän volatiliteettidesiilin tuotto – tosin volatiliteetti ja absoluuttinen tuotto on pääasiassa positiivisesti korreloituneita pois lukien ylimpiin desiileihin kuuluvien osakkeiden tuotot. Keskimäärin rahastot hieman ylipainottavat korkean volatiliteetin osakkeita ja huomattavasti alipainottavat matalan volatiliteetin osakkeita. Salkunhoitajien keskimääräiset allokaatiot saattavat selittää, miksi matalan volatiliteetin anomalia on olemassa.


**Avainsanat** matalan volatiliteetin anomalia, riskin ja tuoton välinen suhde, rahastot, sijoitukset

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# 1. INTRODUCTION

## 1.1 Background and motivation of the study

The expectation of positive reward for bearing risk is in the very core of finance. The positive relationship between risk and return was broadly accepted particularly due to the intuitive predictions of the Capital Asset Pricing Model (Sharpe (1964), Lintner (1965)) about how to measure risk and the relationship between expected return and risk. Yet, already early research on the CAMP (Black, Jensen, and Scholes (1972)) found that low risk assets provided returns that were too high relative to their risk, whereas high risk stocks provided relatively low returns. Recent academic papers (e.g. Frazzini and Pedersen (2014) and Baker and Haugen (2012) among many others) empirically show that low risk assets not only perform surprisingly well relative to their risk, but that low risk assets – either low volatility or low beta assets – actually outperform high risk assets within several different assets classes. The outperformance of low risk assets, or the so-called low risk anomaly, is observed primarily in terms of risk-adjusted returns and alphas but also in terms of absolute returns. The finding is significant because it has been observed both for beta and volatility within several asset classes and markets, and it ultimately contradicts with the core concept of finance.

As intuitive as the positive risk-return relationship may seem, the empirical evidence is at least ambiguous, and even contrary in some cases. Ilmanen (2011) provides extensive evidence on the performance of different asset classes. He notes that the empirical relation between volatility and expected return is tenuous: “Volatility and long-term average returns are positively related across assets classes.” and “However, the most volatile assets *within* each asset class – high-volatility stocks, 30-year Treasuries, and CCC-rated corporate – tend to offer low long-run returns and even worse risk-adjusted returns.” The similar conclusion was drawn by Haugen and Heins (1972). Their paper documents a negative relationship between risk and return in the U.S. stock market as well as in the U.S. bond market. However, they find that stocks as an asset class produced a much higher return than the bonds. They conclude that higher risk can be expected to produce a higher return across asset classes but not within an asset class.

A vast amount of literature extends the evidence that higher risk is not well compensated within asset classes (e.g. Frazzini and Pedersen (2014), Baker and Haugen (2012), Baker, Bradley, and Wurgler (2010), Blitz and van Vliet (2007)). Also the early research on the Capital Asset Pricing Model (CAPM) and its predictions (Black, Jensen, and Scholes (1972)) noted that high beta stocks experience lower absolute returns than suggested by the CAPM, whereas low beta stocks deliver higher absolute returns than the CAPM predicts. In other words, this means that the security market line which describes the relationship between risk and expected return was too flat relative to the CAPM.

There are several explanations for the anomaly – many of them relating to biased investor behavior. Explanations include restrictions on borrowing, inefficient investment approaches, behavioral biases and manager compensation and other agency issues among a few more, all which hypothetically would result in an excess demand for stocks and other securities with higher risk. With regards to borrowing restrictions, Frazzini and Pedersen (2014) propose that more leverage-constrained investors hold higher-beta securities than less constrained investors. They study equity portfolios of mutual funds and individual investors, both which are likely to have limited or no access to debt leverage, and find that these investors hold portfolios with average betas above 1, and therefore leverage by increasing beta risk of their portfolios. Furthermore, they find that less-constrained investors, namely leveraged buyout (LBO) funds and Berkshire Hathaway, acquire firms with average betas below 1 and apply leverage. Already in the 1970's, Black (1972) developed a modification of the CAPM assuming restrictions on borrowing, contrary to the CAPM by Sharpe (1964) and Lintner (1965) that assumes unrestricted borrowing and lending of risky assets. The restricted model suggests that the return differences would be smaller across securities with different levels of systematic risk, and that lower risk securities have higher returns and higher risk securities lower returns relative to the original CAPM.

Secondly, inefficient investment approaches have been proposed to explain the anomaly (e.g. Blitz and van Vliet (2007) and Baker, Bradley, and Wurgler (2010)). Because asset managers are compared against a market benchmark, assuming that the CAPM holds at least partially and risk-bearing results in positive expected returns, the asset managers have an incentive to tilt towards high beta or high volatility stocks (Blitz and van Vliet (2007)). Baker et al. (2010) also note that the fixed benchmarks, that are typically capitalization weighted, discourage investing in low risk stocks. Fixed benchmarks would therefore result in an excess demand for stocks with higher risk causing overpricing and reduced future expected returns. Furthermore, low risk stocks would experience the opposite effect.

Thirdly, private investors are likely to be biased in many ways. Blitz and van Vliet (2007) argue that these investors become risk-neutral of even risk-seeking with regards to asset allocation decision within a certain asset classes, opposite to risk-averse behavior with regards to asset allocation decision across asset classes. Baker et al. (2010) discuss that private investors have preference for lottery-like payoffs, and relate the bias to positive skewness in stocks returns, where large positive payoffs are more likely than large negative payoffs. Also Kumar (2009) finds that some individual clearly prefer stocks with lottery-like payoffs. Furthermore, many great investments, such as investing in Microsoft at its IPO, would have been speculative ones initially. Baker et al. (2010) argue that investors largely ignore the high rate at which these investments fail, and therefore, investors are willing to overpay for highly volatile and speculative stocks. Additionally, overconfidence of investors is proposed to explain high demand for high risk stocks (Baker et al. (2010)).

Fourth, Baker and Haugen (2012) explain that mutual fund managers have an incentive to construct a more volatile portfolio rather than a less volatile, when they have possibility to earn a bonus based on their performance. They illustrate that a compensation scheme comprising a base salary and possible bonus is comparable to an option-like payoff, and thus, a portfolio’s volatility increases the expected value of the compensation creating an excess demand for high volatility stocks. Furthermore, analysts and portfolio managers want to show that they are able to select meritorious investments, and therefore, are attracted to stocks for which they can confidently make a compelling investment case. Baker and Haugen argue that these stocks often are noteworthy due to significant amount of news flow and media attention, making it easier for analysts and portfolio managers to confidently recommend and explain the cases to fellow colleagues and clients. However, the intense news coverage and broader analyst coverage is associated with higher volatility, Baker and Haugen finds.

## 1.2 Definition of the research problem and contribution

This thesis provides evidence on the low volatility anomaly and its causes from three perspectives: first, I examine the persistence of the anomaly in the U.S. over the past 40 years and during sub-sample periods by sorting stocks into deciles based on volatility. I analyze the performance of the ten deciles in terms of absolute excess returns, alphas and Sharpe ratios, and quantify how persistent the phenomenon has been, whether low volatility stocks outperform high volatility stocks in terms of risk-adjusted returns, alphas, or even in absolute return terms. Second, I investigate the persistence of the anomaly over a stock market cycle and study the performance of the volatility sorted portfolios both during declining and rising stock market.

Finally, I obtain a large set of U.S. mutual fund holdings and calculate the aggregate positions in each decile for each fund, and determine whether mutual fund managers actually overweight high volatility stocks and underweight low volatility stocks as suggested by several explanations for the anomaly. The most central research questions can be summarized in the following way:

i) Is the low volatility anomaly persistent over time and is it a phenomenon that still exists?
ii) Do low volatility stocks earn better returns than high volatility stocks only in risk-adjusted terms or also in absolute terms?
iii) Is low volatility anomaly associated with certain market conditions, particularly with either declining or rising general stock market, or is it persistent over the cycle?
iv) Do portfolio managers overweight high volatility stocks and underweight low volatility stocks, and therefore contribute to the existence of the anomaly?

Despite the fact that the relationship between risk and return has been subject of many research papers since the introduction of the Capital Asset Pricing Model (Sharpe (1964), Lintner (1965)), the topic continues to receive a vast amount of attention from both the academics and the industry practitioners. However, the underlying reason for the anomaly remains unclear. Particularly puzzling is the fact that the anomaly has not disappeared for example through trading by arbitrageurs, although several studies (e.g. Blitz and van Vliet (2007), Baker et al. (2010), Baker and Haugen (2012)) have suggested that the anomaly is due to some sort of market inefficiency or bias.

The thesis contributes to the prior literature in several ways. The thesis confirms the results of previous studies and further clarifies whether low risk stocks have only higher risk-adjusted returns or also higher absolute returns than high risk stocks. Some studies focus on comparing the bottom and the top decile, and make conclusions based on the performance difference between these two. Therefore a reader may get an impression that both risk-adjusted and absolute returns are better for low risk stocks, although this might not be the case overall. The thesis expands the evidence on the persistence of the anomaly by looking at the returns in different market conditions. The anomaly is typically studied using fairly long sample periods that absorb the possible performance differences in volatility deciles between periods of upward trending and downward trending stock market. The information would be valuable for practitioners who are willing to exploit the anomaly. Furthermore, the thesis sheds light on mutual fund portfolio managers’ investment allocations and studies whether the allocations are in line with the suggested explanations for the anomaly. For example, Frazzini and Pedersen (2014) find that constrained investors hold portfolios with beta of above 1. However, they do not study the portfolios on security level, and therefore, it is unclear whether above 1 betas result from overweight in high beta securities or underweight in low beta securities, or both. This thesis expands the literature to cover security level analysis of mutual fund portfolios from the perspective of low volatility anomaly.

## 1.3 Research scope and limitations of the study

The thesis focuses on the U.S. common stocks listed on the NYSE, AMEX and NASDAQ during the period from 1973 to 2013. Furthermore, the thesis defines risk as standard deviation of monthly returns, and no other risk measures or data frequencies are taken into account. Some of the previous studies use beta as the risk measure, for example Frazzini and Pedersen (2014). Additionally, researchers including Malkiel and Xu (2002) and Ang, Hodrick, Xing, and Zhang (2006, 2009) have used idiosyncratic risk, or diversifiable risk, as the risk measure and study the relationship between idiosyncratic risk and future returns. However, studying the relationship between beta and expected returns as well as the relationship between idiosyncratic risk and future returns are beyond the scope of this thesis.

As for another limitation, the CRSP Mutual Funds Holdings database has an increasing amount of observations, and therefore the dataset may be more representative for more recent years. The mutual fund holdings data is available only starting from 2001 and extends through 2013. Moreover, the number of observations in 2001 is considerably low and these observations are not included in the sample. Mutual funds that are analyzed in this thesis are selected with a simple method by excluding funds that have less than 80% of net assets invested into equities. Therefore, mutual fund dataset is likely to include various mutual funds differing in terms of investment strategy, size, and other aspects.

## 1.4 Main findings

The findings suggest that investing in low risk stocks is particularly well rewarded both in terms of risk-adjusted returns and alphas. Furthermore, in terms of absolute excess returns, low risk stock perform fairly well and the returns for low volatility stocks are better than the returns for stocks in the highest 10% volatility decile in most cases. However, the stocks in between the bottom 10% and top 10% volatility decile usually have higher absolute returns than the stocks in the bottom decile.

Several explanations have been proposed for the anomaly, many of them suggesting that investors overweight risky securities and underweight low risk securities due to various reasons (see, e.g. Blitz and van Vliet (2007), Baker, Bradley and Wurgler (2010), Baker and Haugen (2012), Frazzini and Pedersen (2014)). The reasons resulting an incentive to tilt towards high risk stocks include restrictions on borrowing, use of market benchmarks, behavioral biases, fund managers' compensation schemes and other agency issues. Indeed, I find that an average mutual fund has a moderate overweight in high volatility stocks and even more significant underweight in low volatility stocks compared to the market portfolio construction. Therefore, it seems that the mutual fund allocations and portfolio managers' risk preferences contribute to the existence of the low volatility anomaly as there is an apparent excess demand for high volatility stocks and shortage of demand for low volatility stocks by the mutual funds. Moreover, both the underweight in low volatility stocks and the overweight in high volatility stocks have been persistent over the period from 2002 to 2013. I find that during this period low volatility stocks have clearly outperformed high volatility stocks in risk-adjusted return terms. Low volatility stocks have also had positive alphas, while high volatility stocks delivered roughly zero or no alpha.

### 1.5 Structure of the study

The rest of the thesis is structured as follows: Chapter 2 presents the relevant literature on the topic. The chapter highlights the central pieces of the theoretical framework of risk and expected return, empirical findings on the relationship between risk and return, the literature that focuses on the low risk anomaly, and the possible explanations for the anomaly. Chapter 3 motivates and presents the hypotheses that are tested in this thesis. Chapter 4 present the data sources and describes the data samples that are used. Additionally, the Chapter 4 also presents the methodologies that are employed. Chapter 5 presents and discusses the empirical results. Finally, Chapter 6 concludes the thesis and discusses suggestions for further study.

## 2. LITERATURE REVIEW

In this section, I briefly recap the theoretical framework of the relationship between risk and return, namely the Capital Asset Pricing Model, as well as provide summary of empirical findings on the relationship between risk and return. Furthermore, I present a detailed overview of the literature on the low risk anomaly. Lastly, I discuss what could be the underlying reasons why the low risk anomaly exists.

### 2.1 Theoretical relation between risk and return

The Capital Asset Pricing Model (CAPM) by Sharpe (1964) and Lintner (1965) is developed from the Markowitz (1959) portfolio theory and offers intuitive predictions about how to measure both risk and the relationship between expected return and risk. Under certain assumptions, the model predicts that the expected return on any asset would be

$$E(\tilde{R}_i) = R_f + \beta_i[E(\tilde{R}_m) - R_f] \tag{1}$$

where $\tilde{R}_i$ is the return on asset *i* for the period, $R_f$ is the return on a risk-free asset for the period, $\tilde{R}_m$ is the return on the market portfolio, and $\beta_i$ is the sensitivity of asset *i* to the market portfolio. The return on asset *i* equals the change in price of the asset, plus dividends, interest, and other distributions, divided by the price of the asset at the start of the period. The market sensitivity, or beta, of asset *i* is defined as follows

$$\beta_i = \frac{cov(\tilde{R}_i,\tilde{R}_m)}{var(\tilde{R}_m)} \tag{2}$$

The underlying assumptions used in deriving the CAPM are as follows[1]: (*a*) All investors have a common joint probability distribution for all assets. They have homogenous beliefs about the mean, standard deviation and correlation for all assets. (*b*) The common probability distribution that describes the possible returns on the all assets is joint normal. (*c*) All investors only care about the mean and the variance of their portfolios and choose their portfolios to maximize their expected utility of wealth at the end of the period. Investors are risk averse, meaning that their

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[1] The portfolio theory by Markowitz (1959) already includes assumptions that investors are risk averse and they care only about the mean and the variance of their one-period investment return. The CAPM by Sharpe (1964) and Lintner (1965) adds two key assumptions to the model. The first is the common agreement of joint distribution of asset returns and the second is that there is borrowing and lending at a risk-free rate.

utility function on their wealth at the end of the period increases at a decreasing rate as their wealth increases. (*d*) All investors can invest in any asset including the risk-free asset, and take a long or short position of any size in any asset. This includes the possibility that all investors may either borrow or lend any amount at the risk-free. (*e*) The markets are perfect and there are no transaction costs in investing in any of the assets.

The assumptions behind the CAPM may not be true in reality however, although some of the assumptions can approximate reality well enough. Lintner (1969) shows that relaxing the assumption (*a*) does not have a significant effect on the structure of asset prices. Furthermore, assumptions (*b*) and (*c*) are generally regarded as acceptable approximations to reality (Black, 1972). More problematic assumptions are (*d*) and (*e*). For many investors, especially the assumption that they can short any asset including borrowing at the risk-free rate is not true in reality. This reduces significantly their ability to adjust the market portfolio according to their risk preferences. Furthermore, in reality, there are transaction costs for buying and selling assets such as brokerage commissions, bid-ask spreads and taxes.

The practical implementation of the CAPM may fail also for the reason that all securities are priced relative to the market portfolio, which in reality can include all assets and not just publicly traded securities. Thus, the implementation of the CAPM is sensitive to the market portfolio choice. Roll (1977) even states that the CAPM cannot be tested because we do not know the actual market portfolio. Therefore, the CAPM may not fail due to the misspecification of the model but due to wrong measurement of the market portfolio.

Under the CAPM framework investors only pay for systematic risk, in other words, risk that is economy wide and correlated with the market. Non-systematic risk (also referred as idiosyncratic risk or diversifiable risk) will not be priced into security prices according to the CAPM framework because it can be eliminated by holding a diversified portfolio. Due to the assumptions of the CAPM presented earlier, in theory, all investors end up holding a well-diversified portfolio, which is the market portfolio that includes all assets and is the tangency portfolio on the mean-variance efficient frontier defined in the modern portfolio theory of Markowitz (1959). Figure 1 provides an illustrative presentation of the efficient frontier, the market portfolio and the tangency line of risk-free rate and the market portfolio, also called as the Capital Market Line. The market portfolio has the highest expected return per unit of risk, or Sharpe ratio, as described by the Capital Market Line. Although all investors may not have similar utility functions and therefore risk preferences, in theory all investors still end up holding the market portfolio and a certain amount of the risk-free asset – either long or short position – so that the combination suits to one's risk preferences. In other words, depending on an investor's risk preferences, he/she would end up holding a combination of the market portfolio and the risk-free asset so that the combination is located somewhere on the Capital Market Line described in the Figure 1 and has the same expected Sharpe ratio as all the other combinations of the market portfolio and the risk-free asset.

The Capital Asset Pricing model's prediction about the relationship between beta risk and expected return is presented in the Figure 2. According to the framework, the relationship between risk and expected return is linear and higher risk is compensated with higher expected return. Furthermore, idiosyncratic risk is not priced and does not affect asset prices. In the following section, I cover the empirical findings of previous research on the CAPM framework and the relationship between expected return and risk.

**Figure 1: Capital Market Line and the efficient frontier**

Figure 1 provides an illustrative presentation of the theoretical Capital Market Line and the mean-variance efficient frontier for risky assets. In theory, the market portfolio is located on the efficient frontier and the Capital Market Line is the tangency line of the risk-free rate and the market portfolio. The figure illustrates that the market portfolio is the best possible combination of risky assets for obtaining the highest expected return per unit of risk.

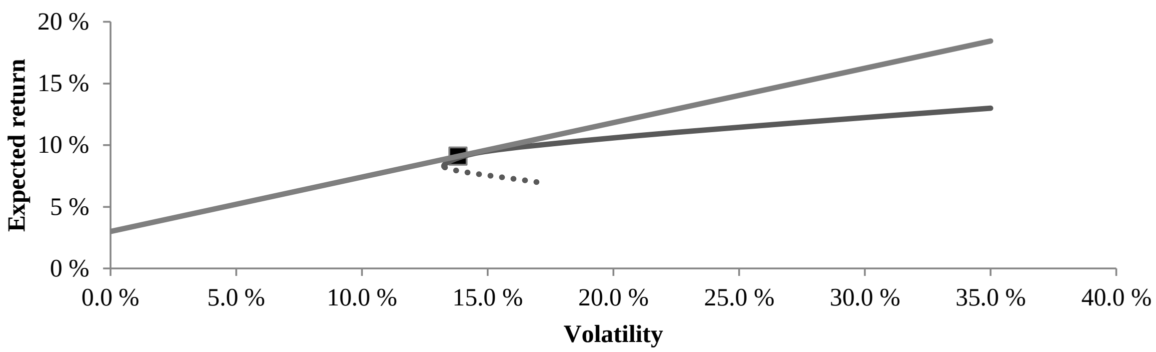
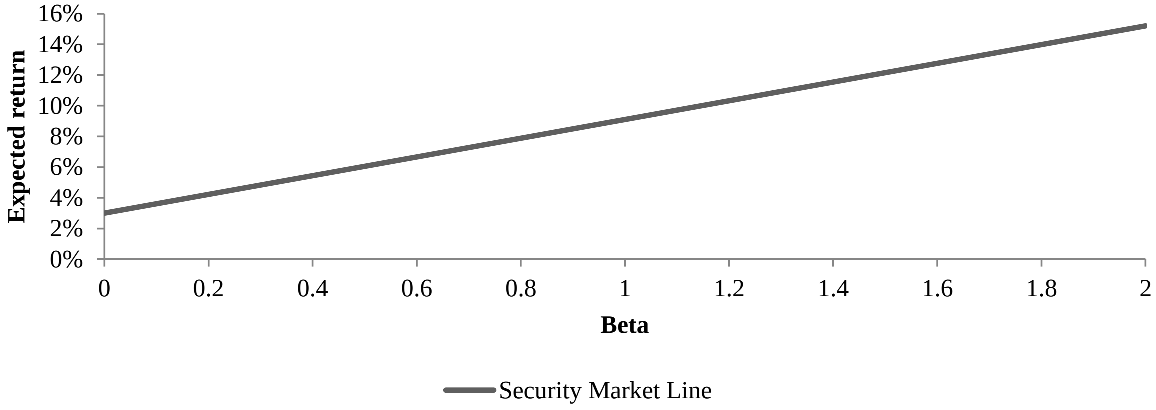


**Figure 2: Security Market Line**

Figure 2 illustrates the theoretical linear relation between the expected return and beta. The market risk-free rate has beta of zero and the market portfolio has beta of one.




## 2.2 Empirical findings on the CAPM

As briefly discussed in the previous section, many of the assumptions of the CAPM are not likely to hold in reality. Broad empirical evidence suggests that the CAPM fails to measure risk and predict the relationship between risk and expected return (see e.g. Black, Jensen, and Scholes (1972), Black (1993), Baker, Bradley, and Taliaferro (2014), Frazzini and Pedersen (2014)). The researchers typically find that the relationship between beta and expected returns is flatter than the theory predicts – some studies even report a negative relationship between the two. Fama and French (2004) note the empirical findings contradicting with the theory may be due to the simplifying assumptions of the CAPM or due to difficulties in implementing valid tests of the model. Further research on the relationship between risk and expected return documents that there is a fairly flat or even negative relationship also between volatility and future returns (Falkenstein (1994), Blitz and van Vliet (2007), Ang, Hodrick, Ying, and Zhang (2006, 2009), Baker and Haugen (2012)), and therefore the phenomenon is not limited only to beta and future returns but is a broader low risk anomaly.

Early on after the CAPM was introduced, Black, Jensen, and Scholes (1972) found that the security market line that describes the relation between securities' systematic risk and their expected returns, was too flat for U.S. stocks during the 35-year period from 1931 to 1965. In short, the result indicates that safer assets provided returns that were too high relative to the

CAPM, whereas riskier assets provided returns that were too low relative to their systematic risk. More recently, Frazzini and Pedersen (2014) show the relative flatness of the security market line for U.S. equities during the period from 1926 to 2012. Furthermore, they find that the security market line is not flatter than predicted by the CAPM only for U.S. equities, but also for equities in 18 of 19 international markets, in Treasury markets, for corporate bonds sorted by maturity and by rating, and in futures. Baker, Bradley, and Wurgler (2010) also conclude that their empirical results indicate that the relation between risk and return has not just flattened, but inverted.

The fact that the empirical findings of the relationship between risk and return contradict with the theory may be traced back to possibly unrealistic assumptions of the CAPM. Black (1972) developed the CAPM by Sharpe (1964) and Lintner (1965) further and presented a model with restricted borrowing. The restricted model first assumes that there is no risk-free asset and that no riskless borrowing or lending is allowed. Secondly, it assumes that there is a risk-free asset but that only long positions in the risk-free asset are allowed. Both cases predict a linear relation between the expected return on a risky asset and the systematic risk of the risky asset. However, when there are restrictions on borrowing, the slope of the security market line is predicted to be smaller than without restrictions as in the standard CAPM. The empirical findings of Black, Jensen, and Scholes (1972) and Frazzini and Pedersen (2014) are more consistent with the CAPM with restricted borrowing, thereby indicating that the assumption of the original CAPM that all investors can invest in any asset including the risk-free asset, and take a long or short position of any size in any asset does not hold in reality.

The standard CAPM also argues that only systematic, or market risk, should be incorporated into asset prices and non-systematic, or idiosyncratic risk, will not be priced into security prices because it can be eliminated through diversification by holding the market portfolio. However, Malkiel and Xu (2002) notes that also idiosyncratic risk may play a role in asset pricing, if some investors are unable to hold the market portfolio. Furthermore, if some investors fail to hold the market portfolio, also the remaining investors will not be able to hold the market portfolio. Therefore, an investor should not only be rewarded for market risk but also for idiosyncratic risk.

Some early studies on the relationship between idiosyncratic risk and return (Lintner (1965), Douglas (1969)) already found that idiosyncratic risk may be priced. The early research finds that the relation between the two is either positive or statistically insignificant. For example, Lintner (1965) finds a positive coefficient on idiosyncratic volatility in cross-sections. However, as Malkiel and Xu (2002) notes, the early empirical evidence that supported the role of idiosyncratic risk was disregarded after the study by Fama and MacBeth (1973) that rejected the role and provided a more powerful cross-sectional test. Later, Lehmann (1990) again found a statistically significant, positive coefficient on idiosyncratic volatility. The finding was supported with more recent study by Malkiel and Xu (2002), who found that idiosyncratic volatility is useful in explaining cross-sectional expected returns under both the Fama and MacBeth (1973) and Fama and French (1992) testing frameworks. Tinic and West (1986) and Malkiel and Xu (2002) also conclude that portfolios with higher idiosyncratic volatility earn higher average returns, yet without reporting any significance levels for their results. Later, Fu (2009) reported statistically significant and positive relationship between estimated conditional idiosyncratic volatilities and expected returns.

Eventually, based on several recent studies, it seems that idiosyncratic risk may be priced in. However, there is an ongoing debate about whether idiosyncratic risk carries a positive or negative coefficient. Malkiel and Xu (2002) state an explicit role for idiosyncratic risk in pricing assets based on their results. They find that high idiosyncratic risk, measured by idiosyncratic volatility, is associated with high returns on average. Furthermore, their findings are robust for size and value effect as well as controlling for liquidity. Positive relationship was reported also by Fu (2009). Ang, Hodrick, Ying, and Zhang (2006) present critique to the findings of Malkiel and Xu (2002), and state that they do not examine firm-level idiosyncratic volatility because they only use the idiosyncratic volatility of one of the 100 beta/size portfolios to which a stock belongs to proxy for the stock's idiosyncratic risk. On contrary, Ang et al. (2006) find that stocks with high idiosyncratic volatility relative to the Fama and French (1993) model have "abysmally" low average returns. Later, Ang et al. (2009) present the same result that stocks with high idiosyncratic volatility have low average returns also in 23 developed markets, and not only in the U.S. markets. Lastly, Bali and Cakici (2008) have come up with a finding that again states that no robustly significant relationship exists between idiosyncratic volatility and expected returns. Additionally, he concludes that the negative relationship disappears, when small stocks are excluded from the sample.

To summarize, the empirical findings on the CAPM suggest that the relationship between risk and expected return is definitely less positive than the CAPM predicts. The relative flatness of the Security Market Line is observed by many researchers possibly due to unrealistic assumptions of the CAPM. Issues such as restrictions on borrowing and that idiosyncratic risk may be priced due to the fact that investors do not hold well-diversified portfolios are among the explanations why the framework fails. Furthermore, some studies state that the relationship is flat and no significant return differences exist between high and low risk assets, while at one extreme, some researchers propose an inverted relationship, whereby low risk stocks would have higher returns than high risks assets. Based on these studies, low risk stocks indeed seem to earn attractive returns outperforming high risk stocks in many occasions. In the following section, I elaborate the literature on the relationship between risk and return further by introducing more recent papers on the topic which also address several possible explanations for the anomalous relationship between risk and return.

## 2.3 Literature on the low risk anomaly

As the relationship between risk and return was found to be less positive than predicted, and further, as some studies even proposed a negative relationship, the recent research has concentrated on examining whether low risk stocks and other low risk assets actually outperform their riskier counterparts. Following a decent amount of evidence on the negative relationship, several papers have extended the research on this so called low risk anomaly and studied its persistence, its existence in numerous markets, and most importantly, its magnitude and the significance of the return differences. The low risk anomaly appears to be substantial in risk-adjusted terms, it has been persistent over decades and extends to several different markets and asset classes. The low risk anomaly also covers different risk measures, and both low beta and low volatility anomalies have been observed. Typically, the studies employ a methodology, whereby stocks are sorted based on their historical volatilities or betas, and then assigned into deciles (one-tenth), quintiles (one-fifth) or quartiles (one-fourth) in an ascending order. The returns for these portfolios are usually measured for the following month after the portfolio formation and the portfolios are assumed to be rebalanced every month.

To start with, Blitz and van Vliet (2007) report a clear volatility effect that low risk stocks experience significantly higher risk-adjusted returns than the market portfolio. They also find that high risk stocks significantly underperform on a risk-adjusted basis. Furthermore, they find that the effect holds globally as well as within the U.S., European and Japanese markets in isolation, and it is robust for size, value, and momentum effects. The Sharpe ratios for low and high volatility stocks are 0.72 and 0.05 in the global results, respectively, and 0.58 and 0.10 in the U.S. results, respectively. Additionally, Blitz and van Vliet report alpha for portfolios ranked on volatility and beta, although considerably less for beta ranked portfolios. The annual alpha spread between global low volatility decile and high volatility decile portfolios is reported to be 12% over the period from 1986 to 2006. Later Blitz, Pang, and van Vliet (2012) study returns in emerging markets and the results are consistent with the findings for developed equity markets. Blitz et al. find a flat or even negative relationship between volatility and returns. They also report Sharpe ratios that are clearly higher for low volatility stocks than for high volatility stocks.

Similarly, Baker, Bradley, and Wurgler (2010) find that low volatility and low beta portfolios have earned higher average absolute returns and also experienced smaller drawdowns than high volatility and high beta portfolios. Baker et al. focus on value-weighted portfolios and study both volatility and beta sorted portfolios in a full-sample of U.S. stocks as well as in a sample where only top 1000 stocks based on market capitalization are included. The annualized return spread is clearly positive between the lowest risk and highest risk quintiles (low risk minus high risk) ranging from 2.7% to 6.9% in all the samples except in the volatility sorted sample comprising top 1000 stocks based on market capitalization, for which the return spread is negative. The strongest low risk effect is observed in volatility sorted portfolios comprising the full sample without any market cap limitations. In addition, Baker et al. also emphasize two aspects that make the low risk portfolios look even more attractive than the high risk portfolios. Firstly, they find that the low risk portfolios' paths to their higher dollar values have been much smoother, and thus, they have had genuinely lower risk. Secondly, the high risk portfolios have higher transaction costs of monthly rebalancing which means that the reported outperformance of low volatility and low beta portfolios is understated. The finding of Baker et al. that low volatility and low beta portfolios have earned higher absolute returns indicates that the security market line could actually be inverted during their sample period from 1968 through 2008. Yet, also the findings in this thesis show that period from 1974 to 2013 was very good period in absolute return wise for low volatility stocks compared to high volatility stocks, and especially in value-weighted portfolios. Therefore, it is crucial to have a look on results in other studies that are based on different sample periods and preferably supported with findings in other equity markets.

Baker and Haugen (2012) extend research of Baker et al. (2010) to cover several additional equity markets outside the U.S. They study stock markets in 21 developed countries and in 12 emerging markets over the period from 1990 to 2011. The paper supports the evidence that greater risk cannot be expected to produce a greater reward but rather bearing relative risk in the equity markets yields an expected negative return in all developed countries and emerging markets. They calculate the volatility of total return for each company over the previous 24 months and rank the stocks by volatility into deciles, quintiles, and halves in each country. Their sample includes non-survivors and the sample consists of 99.5% of the capitalization in each country. The results in Baker and Haugen's paper indicate that the low volatility effect is not limited to only to the U.S. equity market but is a rather global phenomenon. Both in developed countries and emerging markets low volatility stocks show significant outperformance over high volatility stocks: in addition to 10% to 25% reduction in portfolio volatility, the absolute returns are clearly higher for the lowest volatility decile compared to the highest volatility decile. As a result, the Sharpe ratio differences between low and high volatility stocks across countries are dramatic and clearly in favor of low volatility stocks. Baker and Haugen summarize that there are times when high volatility outperforms, but they are "few and far between".

Baker, Bradley, and Taliaferro (2013) decompose the low beta anomaly into micro and macro components in an attempt to find out more precisely what drives the anomaly. With regards to micro component they mean the selection of low beta stocks, while macro component refers to either low beta country or industry selection. They find that actually both the micro-level stock selection and macro-level industry and country selection contribute to the low beta anomaly. Interestingly, they state that micro selection of low beta stocks leads to a significant reduction in risk with only a modest difference in return. Therefore, micro selection presents an opportunity to form lower risk portfolios that do not suffer lower returns. Secondly, macro selection of low beta industries or countries has little impact on risk but leads increases in return. Baker et al.'s point out that countries that are identified high risk ex ante have distinctly lower future returns.

Recent paper by Frazzini and Pedersen (2014) studies betting-against-beta factor, which is long leveraged low-beta assets and short high-beta assets, and finds that the factor produces significant positive risk-adjusted returns. They also find that high beta is associated with low alpha within U.S. equities and in 20 international equity markets but also within Treasury bonds, corporate bonds, and futures. In each asset class, alphas and Sharpe ratios are almost monotonically declining in beta. This indicates an additional evidence that the relative flatness of the security market line is not isolated to the U.S. stock market but that it is a global phenomenon and exists across various asset classes. Their findings are also robust for market, value, size, momentum, and liquidity factors.

The results presented in the literature for both the low volatility anomaly and low beta anomaly are clearly inconsistent with theoretical models of risk and return such as the CAPM, which predicts a positive risk-return relationship. Interesting question is whether the anomalous relationship can be attributed to market mispricing or to compensation for higher systematic risk. Examination by Li, Sullivan, and Cargia-Feijoo (2013) suggests that the anomalous returns are more likely attributable to market mispricing rather than to compensation for some hidden risk factor. Furthermore, the mispricing seems to be connected to volatility as a stock characteristic, i.e. investors appear to have some particular preference for high volatility stocks over low volatility stocks.

In summary, several studies report attractive returns for low risk stocks and also for other low risk securities. However, not so much critique has been presented to these results. In their more recent study, Li, Sullivan, and Cargia-Feijoo (2014) are among the few to present some counter arguments to the low volatility anomaly. They state that the existence and effectiveness of the anomaly are more limited than widely believed. Although their findings suggest that the returns are anomalous over the following month of portfolio formation, they find that holding low volatility stocks beyond the first month produces little or no alpha, and therefore exploiting the anomaly requires frequent trading which diminishes the returns significantly due to transaction costs. Additionally, they find that the low risk effect is completely eliminated for equal-weighted portfolios within a sample where stocks with price less than $5 are excluded. However, Li, Sullivan, and Cargia-Feijoo (2014) study the effect for idiosyncratic volatility similar to Ang et al. (2006 and 2009), and as earlier discussed, the evidence on the relationship between idiosyncratic volatility and expected returns is mixed. Therefore, far-reaching conclusions should not be made before similar results are reported for simple volatility and beta. For now, broad evidence suggest that low volatility and low beta stocks outperform high volatility and high beta stocks and the results are robust to various factors such as value, size, momentum and liquidity. The follow section presents the possible explanations for the anomaly from previous studies.

## 2.4 Explanations for the low risk anomaly

The low risk anomaly is puzzling because investing in low volatility and low beta assets has provided attractive returns across asset classes, the result can be observed in long samples, and the phenomenon has become stronger in more recent sample periods. Furthermore, the phenomenon cannot be explained by any other anomaly and the result is significant after controlling for size, value, momentum and liquidity effects. Blitz, Falkenstein, and van Vliet (2013) provide an excellent summary of various explanations that have been proposed in the previous literature. These include for example leverage constraints, constraints on short-selling, use of pre-specified benchmark indexes, agency issues, preference for skewness and representativeness bias among other explanations. In the following, I describe the proposed explanations in detail.

### 2.4.1 Restrictions on borrowing

Already in the 1970s, the CAPM assumptions and the prediction of linear relationship between the systematic risk of a risky asset and the expected return on the risky assets were challenged. Black (1972) developed a model that assumed restrictions on borrowing, contrary to the CAPM that assumes unrestricted borrowing and lending of risky assets. The result of his model was that, when there are restrictions on borrowing, the slope of the security market line – the relationship between beta and expected return – is smaller than what the CAPM suggests. Therefore, the CAPM with restricted borrowing suggests that the return differences are smaller across assets with different levels of systematic risk. However, the level of systematic risk for different assets is still the same, although the differences in expected returns are suggested to be smaller. With respect to risk-adjusted returns, the CAPM with restricted borrowing makes assets with lower risk to look more attractive as flatter security market line means that more risky assets have lower Sharpe ratios or less risky assets have higher Sharpe ratios, or both, compared to predictions of the traditional CAPM. As a result, borrowing or leverage restrictions can be a plausible explanation why low risk assets perform so much better, particularly in risk-adjusted terms, and therefore an explanation for the low risk anomaly. Leverage restrictions have been noted as an explanation for the low risk anomaly also in more recent research, namely by Blitz and van Vliet (2007) and Frazzini and Pedersen (2014). Frazzini and Pedersen discuss that borrowing restrictions cause investors to leverage by overweighting risky assets, and thus, bidding the prices of risky assets up resulting in lower expected return. They also find that high beta is associated with low alpha, and the finding applies to U.S. equities, 20 international equity markets, Treasury bonds, corporate bonds, and futures.

### 2.4.2 Inefficient investment approach

Second potential explanation for the low risk anomaly could be inefficient decentralized investment approach that is often used in the investment industry, argue Blitz and van Vliet (2007). The authors refer to a process where a CIO or an investment committee first makes the asset allocation decision followed by capital allocation to managers who buy securities within different asset classes. Binsbergen et al. (2008) argue that this kind of investment approach cause uncertainty for the CIO about the managers' risk appetites increasing the costs of decentralized investment management and the value of an optimally designed benchmark. The problem with the approach is that the asset managers are usually compared against a market benchmark, and as a result, the managers have an incentive to tilt towards high beta or high volatility stocks because it is a way to generate above average returns, if the CAPM holds at least partially (Blitz and van Vliet (2007)). Tilting towards high risk stocks may cause these stocks to become overpriced reducing the expected future return for these stocks, whereas low risk stocks may become underpriced together with improved expected future return. Also Baker et al. (2010) note that many institutional investors have fixed benchmark mandates, typically capitalization weighted, which discourage investing in low risk stocks. They state that it is unlikely that asset managers would exploit this mispricing because the low risk strategy involves holding assets with more or less similar long-term returns, thus resulting in no improvement in excess returns, but with different risks, which increases tracking error, and as a result, decreases information ratio, a measure of an asset manager's performance. Furthermore, they emphasize that the low risk anomaly will likely persist due to the ever-increasing importance of institutional investors with fixed benchmarks.

### 2.4.3 Behavioral biases of investors

Thirdly, the low risk anomaly can be explained by behavioral biases of private investors. Blitz and van Vliet (2007) discuss about behavioral portfolio theory that describes private investors thinking in terms of a two-layer portfolio, where the first layer is the asset allocation decision and the second is the allocation decision with a certain asset class. Shefrin and Statman (2000) call the first layer as a low aspiration layer which is designed to avoid poverty, whereas the second layer is called a high aspiration layer designed for a road to riches. Blitz and van Vliet suppose that private investors are rational risk-averse investors with regards to the asset allocation decision but become risk-neutral or even risk-seeking with regards to the allocation decision within a certain asset class. If this is the case, investors are willing to overpay for risky stocks which are perceived to be similar to lottery tickets. Furthermore, diversification within an asset class destroys upside potential, while buying a few volatile stocks leaves upside potential intact. This is consistent with the finding that most private investors only hold about one to five stocks in their portfolio. Blitz and van Vliet argue that deviations from risk-averse behavior may cause high-risk stocks to be overpriced and low-risk stocks to be underpriced.

Also Baker et al. (2010) identify the behavioral biases of private investors as an explanation for the low risk anomaly. The irrationality biases they list include preference for lotteries, representativeness and overconfidence biases. With regards to the preference for lotteries, they note that the bias is more about positive skewness in stock returns, where large positive payoffs are more likely than large negative payoffs, than it is about volatility. Kumar (2009) also finds that some individual investors show clear preference for stocks with lottery-like payoffs.

The representativeness bias is related to the characteristics of "great investments". For example, buying Microsoft at its IPO in 1986 would have been a great investment, yet a speculatively one. Therefore, an investor – if not the most sophisticated one – may conclude that the road to riches is paved with speculative investments. Here, an investor would however largely ignore the high rate at which speculative investments fail, and thus, is willing to overpay for risky stocks (Baker et al. (2010)).

The third behavioral bias causing the preference for high volatility stocks is overconfidence (Baker et al. (2010)). Experimental evidence shows that most people form too narrow confidence interval, or in other words, are overconfident about the accuracy of their judgment. Valuing stocks also includes making judgment about the future, for example estimating revenues. Baker et al. write that overconfident investors are likely to disagree about the future estimates. In addition, they are likely to agree to disagree, and therefore stick with the false precision of his or her estimate. The higher the uncertainty of the outcome the higher is the extent of disagreement. Thus, volatile stocks will elicit a wider range of opinions. There is one additional assumption needed to connect overconfidence to the demand for high risk stocks: pessimists must act less aggressively in markets than optimists. Already, the empirically evident scarcity of short sales among individual investors and even institutional investors compared long positions limits the actions of pessimists and allows optimists to act more aggressively. Miller (1977) finds that prices are generally set by optimists, and therefore stocks with a wider range of opinions will have more optimists as shareholders and sell for higher prices resulting in lower expected future returns.

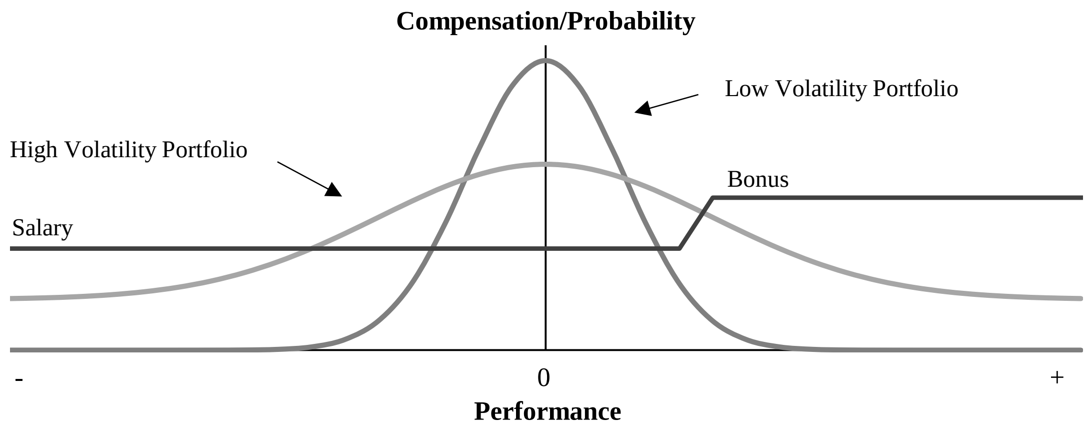
### 2.4.4 Manager compensation and agency issues

Fourth explanation for the low volatility anomaly provided in the previous literature relates to portfolio managers' compensation and agency issues (Baker and Haugen (2012)). The agency issues are noted to exist both between professional investment managers within an organization and also between these professionals and their clients.

Firstly, Baker and Haugen (2012) explains that the nature of portfolio manager compensation has an influence on how volatile portfolio is constructed by the portfolio manager. Typically, a manager's compensations consists of a base salary and an additional bonus when the portfolio's performance is sufficiently high. This sort of a compensation schedule is described in the Figure 3 below.

**Figure 3: Option-like portfolio manager compensation**

Figure 3 describes Baker and Haugen's (2012) illustration of a manager compensation schedule whereby the manager is paid a base salary and then a bonus when performance is sufficiently high. The figure also includes two probability distributions: one for a portfolio with high volatility and the other for a portfolio with lower volatility.



The figure 3 also presents two probability distributions describing the probabilities of different performance levels for a low volatility and a high volatility portfolios. Assuming that a manager aims to maximize the expected value of his/her compensation, and that there are no immediate downside risks for the manager him-/herself (such as termination of employment etc.), he/she is likely to build a more volatile, rather than a less volatile portfolio. Due to the fact that a typical compensation schedule has an option-like payoff, the higher the portfolio's volatility is, the higher is the expected value of manager's compensation. This sort of compensation mechanism should therefore result in an increased demand for stocks with higher volatility.

Similar conclusion was drawn by Falkenstein (1994) and Karceski (2002) that asymmetric payoff creates an incentive for mutual fund managers to tilt their portfolios toward high volatility and high beta stocks. Karceski develops an agency model that first hypothesizes that mutual fund investors chase returns through time and that the cash inflows to mutual funds are unusually large just after strong market runups. Secondly, Karceski's model assumes that mutual fund investors chase returns cross-sectionally across funds so that the best-performing funds capture the largest share of aggregate inflows into mutual funds. Indeed, Karceski shows empirically that the market returns have a large economic impact on the subsequent aggregate cash flows into mutual funds. Furthermore, Sirri, and Tufano (1998) report that net cash inflows into mutual funds are relatively insensitive to fund performance, except for the best performing funds. Karceski's model further hypothesizes that these two performance-fund flow relationships create an asymmetry in the payoffs for mutual fund managers, whereby fund managers mostly care about outperforming peers during bull markets. Since high beta stocks tend to outperform during rising markets, active fund managers have an incentive to overweight high beta stocks, decreasing the expected future returns for these stocks. Additionally, Falkenstein (1994) reports that open-ended mutual funds clearly prefer stocks that are liquid, well-known, and highly volatile.

In addition to the issues related to fund manager compensation and the performance-fund flow relationships, Baker and Haugen (2012) highlight that there is a significant agency issue also amongst the portfolio managers themselves that creates excess demand for more volatile stocks. They describe that a typical investment process includes periodic investment committee meetings that are part of the process of building a model portfolio guiding the construction of individual portfolios for clients. During these meetings, a team of analysts, each with a specific focus area, are typically asked to make a case for stocks they think should be included in the model portfolio. Baker and Haugen describe that the analysts have a natural need to impress the Chief Investment Officer (CIO) and their fellow analysts with their ability to select meritorious investments. As a result, the analysts are attracted to stocks for which they can confidently make a compelling case. These stocks tend to be noteworthy and typically receive notable media attention. However, because of the noteworthiness of these compelling and attractive cases, the related flow of new information is fairly intense resulting in higher than average volatility. As a result, Baker and Haugen expect that based on the analysts' desire to impress their colleagues, stocks with higher than average volatility would be likely to be included in the model portfolio. Furthermore, Baker and Haugen argue that the interesting nature of these stocks help the professional money managers to explain changes in the model portfolio to their clients. In summary, they hypothesize that these agency issues create excess demand for highly volatile stocks by both professional investors and their clients.

Baker and Haugen study roughly the largest 1,000 stocks in the United States over the period from 2000 through 2009. They look at the percentage of a stock's total market capitalization held by institutional investors and conclude that institutional investors as a group do indeed have a higher ownership share in more volatile stocks. The result applies for all of the ten size deciles based on market capitalization expect for the very smallest stocks (smallest 10% of stocks). Moreover, they empirically find that more volatile stocks do indeed have both a significantly greater analyst coverage and a more intense new coverage. Consequently, Baker and Haugen conclude that these findings support their hypotheses that both the typical portfolio manager compensation where sufficiently high performance is rewarded with bonus and the agency issues are responsible for creating excess demand for volatile stocks resulting in their over-pricing and production of poor returns in the future.

# 3. HYPOTHESES

## 3.1 Introduction to hypotheses

This section describes the hypotheses of the study. The first hypothesis addresses the existence and persistence of the low volatility anomaly over the past 40 years. In addition to hypothesizing a strong low volatility anomaly over the 40-year period from 1974 to 2013, I expect that the anomaly has been a persistent phenomenon also during shorter sample periods, particularly over the four 10-year sub-sample periods.

The second and third hypotheses are related to whether low volatility stocks actually earn higher absolute returns than high volatility stocks, or whether the low volatility anomaly exists only in risk-adjusted return basis. Following the confirmation of the existence and persistence of the low volatility anomaly and clarifying the significance of it, I shed new light on the topic by studying the anomaly over periods both when the stock market is rising and when the market is declining. The fourth hypothesis addresses the question whether the low volatility anomaly is driven by either declining or rising market conditions, or does the anomaly persist over stock market cycles.

The fifth hypothesis relates to the holdings of mutual funds and whether the mutual fund portfolio managers invest in low volatility stocks and harvest the presumably attractive returns given by low volatility stocks. The fifth hypothesis has theoretically and empirically contradicting background: As the low volatility anomaly has been recognized since the early research of Black, Jensen, and Scholes (1972), one could expect that sophisticated investors such as mutual fund managers would be exploiting the anomaly by investing in low volatility stocks – or at least not investing against the phenomenon by underweighting low volatility stocks. On the other hand, there is a question why any mutual fund manager would want to invest excessively in low risk stocks given the intuitive prediction of the CAPM that higher returns result from taking additional risk, not the other way round. The fifth hypothesis takes a view on this contradictory issue.

## 3.2 Risk and return relationship

The first three hypotheses are associated with the basic premise of the capital asset pricing theory that risk and return should be correlated (Sharpe (1964) and Lintner (1965)). If assets are priced correctly, then the expected return for one asset would be higher compared to the expected return of another asset only if the former asset has higher risk than the latter one. However, previous studies have shown that stocks with lower risk have earned better risk-adjusted returns (e.g. Frazzini and Pedersen (2014), Blitz and van Vliet (2007)) and better absolute returns (e.g. Baker, Bradley, and Wurgler (2010), Baker and Haugen (2012)) than stocks with higher risk. Studying the period from 1926 to 2012, Frazzini and Pedersen have also shown that high risk is associated with low alpha and that alphas and Sharpe ratios are almost monotonically declining in risk within several separate asset classes.

The Capital Asset Pricing Model (CAPM) by Sharpe (1964) and Lintner (1965) was introduced in the 1960's with predictions of the mean-variance efficiency of the market portfolio and the Efficient Market Hypothesis assuming informationally efficient markets. However, already early research on the CAPM notes that the security market line, which describes the relation between securities' expected returns and their systematic risk, was too flat for U.S. stocks (Black, Jensen, and Scholes (1972)) – safer assets provided returns that were too high relative to the CAPM, whereas riskier assets provided returns that were too low relative to their systematic risk. More recently, Frazzini and Pedersen (2014) showed the relative flatness of the security market line for U.S. equities during the period from 1926 to 2012.

Many explanations for the phenomenon have been provided including restrictions on borrowing and leveraging through overweighting riskier assets (Black (1972), Blitz and van Vliet (2007), Frazzini and Pedersen (2014)), the use of market benchmarks creating an incentive to tilt towards high beta and high volatility stocks (Blitz and van Vliet (2007), Baker et al. (2010)), behavioral biases resulting in deviations from risk-averse behavior that causes investors to overpay for risky stocks which are perceived similar to lottery tickets (Blitz and van Vliet (2007), Baker et al. (2010)), and portfolio manager compensation together with agency issues that create excess demand for stocks with above average volatility (Baker and Haugen (2012)). All the previous hypotheses are supported with empirical evidence, although it remains unclear which of them has the greatest impact on the existence of the low volatility anomaly.

Assuming that these issues have been persistent and significant since the introduction of the CAPM in the 1960's and for most of the time during the period from 1974 to 2013, my first hypothesis is the following:

**H1.** *Low volatility anomaly has been a persistent phenomenon over the past 40 years and during the four 10-year sub-sample periods.*

In order to test the first hypothesis, all the stocks in the main U.S. stocks exchanges are sorted based on their past volatility into ten deciles. I calculate both equally weighted and value-weighted returns for the portfolios. I also calculate alphas against different market models and Sharpe ratios for each decile.

Despite the general statement that low risk stocks have outperformed high risk stocks, not all studies report that this conclusion holds in absolute return terms in addition to risk-adjusted return terms. For example, based on the research by both Frazzini and Pedersen (2014) and Blitz and van Vliet (2007), the low volatility anomaly seems to be a matter of low risk stocks outperforming high risk stocks only in risk-adjusted terms, whereas Baker, Bradley, and Wurgler (2010) and Baker and Haugen (2012) find low volatility anomaly in absolute returns.

Empirically it seems to be the case that risk-adjusted returns for low risk stocks are consistently higher than the risk-adjusted returns for high risk stocks. However, only a few studies report that also absolute returns would be higher for low risk stocks compared to high risks stocks. Additionally, most studies focus on comparing the performance of the lowest volatility (or beta) decile to the performance of the highest volatility decile, while for example Frazzini and Pedersen (2014) report increasing absolute returns when moving from the lowest volatility decile up to 6$^{th}$ decile. They also report higher absolute returns for the 7$^{th}$, 8$^{th}$, 9$^{th}$ and the 10$^{th}$ deciles compared to the lowest volatility decile. However, in Frazzini and Pedersen (2014), the return differences are fairly small (monthly returns range from 0.91% to 1.10%) compared to the alpha differences (monthly alpha spread ranges from 0.37% to 0.89% between low beta and high beta) and volatility differences (high beta decile has annual volatility of 41.7% compared to 15.7% for the lowest beta decile). I expect that the low volatility anomaly in risk-adjusted returns is primarily driven by large differences in volatilities, while the differences in absolute returns actually mitigate the anomaly. Following this, my second and third hypotheses are as follows:

**H2.** *Low volatility anomaly is a persistent phenomenon primarily in risk-adjusted terms, and it is driven by large differences in volatilities and small differences in absolute returns.*

**H3.** *There is not a clear and consistent low volatility anomaly observable in absolute return terms.*

In order to test the second and third hypotheses, I follow similar methodology as for testing the first hypothesis. Again, all the stocks in the main U.S. stocks exchanges are sorted based on their past volatility into ten deciles. I calculate both equally weighted and value-weighted returns for the portfolios. I also calculate alphas against different market models – the simple market model, Fama-French three-factor model, Carhart four-factor model and Pastor-Stambaugh five-factor model – and Sharpe ratios for each decile.

### 3.3 Association with market conditions

The low volatility anomaly is typically studied using long samples and fairly long sub-sample periods, where timeframes range from 20 years up to some 80 years. Therefore, one of the contributions of this paper is to study whether the low volatility anomaly is driven by periods when the stock market is either rising or declining, or is it a persistent phenomenon over the stock market cycle.

The fourth hypothesis is therefore associated with the movements of the general stock market portfolio. Blitz and van Vliet (2007) finds that the low risk portfolios underperform the market during up market months, while outperforming the market during down market months. Furthermore, they find that the underperformance during up months is considerably smaller than the outperformance during down months. Yet, this effect is countered to some extent by more frequent occurrence of up months, 59% of up months versus 41% of down months in their sample. Based on their findings, my fourth hypothesis is the following:

**H4.** *Low volatility anomaly is not a persistent phenomenon over the stock market cycle and it is primarily observable when the stock market is declining. When the stock market is rising, low volatility anomaly becomes weaker, or even non-existent.*

For testing the fourth hypothesis, I use similar methodology as for testing the first hypothesis, but instead of using the whole sample period or arbitrarily selected sub-sample periods, I classify the month-observations into five separate data sets based on the trend in the market index. I study the data between 1994 and 2013, and select five separate sample periods that are determined as follows: the period starting from January 1994 and ending in March 2000, when the stock market peaks, the second periods starts from the market peak experienced in March 2000 and ends in September 2002, when the stock market bottoms, and so on. As a result, I end up studying three separate periods of rising stocks market and two separate periods of declining stock market during the past 20 years.

### 3.4 Mutual fund holdings

The main contribution of this paper is to study how mutual fund managers invest relative to the composition of a benchmark index – in this case, relative to the CRSP value-weighted market index. In particular, I study mutual fund holdings – focusing on funds that mainly invest in the NYSE, AMEX and NASDAQ listed stocks – in the ten deciles that are created by sorting stocks in the NYSE, AMEX and NASDAQ based on their historical volatility, and then comparing a mutual fund’s aggregate exposure to a particular decile to that decile’s weight in the market portfolio. As a result, it can be determined whether mutual funds generally overweight or underweight stocks that have either low or high volatility.

One of the hypothesized causes for the low volatility anomaly is the use of benchmark indices in the fund management industry. Therefore, to understand the anomaly more thoroughly, it is useful to study how mutual fund managers invest relative to the benchmark. The theory that the use of benchmark indices drives the excess demand for stocks with higher risk proposes that, in an attempt to beat a benchmark index, mutual fund managers overweight the stocks with higher risk and underweight the stocks with lower risk due to an expectation that the CAPM holds well enough. However, a broad literature has already noted that the CAPM does not hold well in reality (see e.g. Black, Jensen, and Scholes (1972), Black (1993), Blitz and van Vliet (2007), Baker and Haugen (2012), Baker, Bradley, and Taliaferro (2014), Frazzini and Pedersen (2014)). Despite this, the financial industry relies on the CAPM framework to some extent, which gives a reason to believe that fund managers' portfolios can also be influenced by the CAPM's proposition of risk and return.

Second explanation for the low volatility anomaly that relates to mutual fund portfolio composition is the notion of portfolio manager compensation and agency issues by Baker and Haugen (2012). Firstly, they argue that a typical compensation schedule of base salary added with possible bonus, when the performance is sufficiently high, can be seen as an option-like payoff mechanism resulting in an incentive for a portfolio manager to increase the volatility of his/her portfolio because the portfolio volatility increases the expected value of his/her compensation. This would then result in an excess demand for the stocks with higher volatility. Furthermore, additional agency issues emerge in the process of building a model portfolio that guides the construction of individual portfolios for the clients of an asset manager. In short, during the process, analysts are asked to make a case for stocks they believe should be included in the model portfolio. To be able to impress colleagues and enhance their own career advancement possibilities, analysts tend to be attracted to noteworthy stocks for which they can confidently make a compelling case. However, these noteworthy stocks tend to be volatile ones due to both significant analyst coverage and intense news coverage. Moreover, it is easier for portfolio managers to explain the changes in the model portfolio to their clients, when a stock is interesting and receives attention from both analysts and media. Baker and Haugen (2012) argues that these agency issues create excess demand for stocks with above average volatility, in addition to manager compensation issues.

Based on the explanations related to benchmarking and portfolio manager compensation and agency issues, my fifth hypothesis is as follows:

**H5.** *Mutual fund managers overweight the stocks with higher volatility, and conversely, underweight the stocks with lower volatility.*

In order to test this hypothesis, I have obtained mutual fund holdings data from the CRSP Mutual Fund Database. I study the portfolios with majority of their investments made into the

NYSE, AMEX and NASDAQ listed equities. In particular, I focus on the portfolios with at least 80% of a portfolio's total net assets invested into U.S. listed common stocks. I exclude levered portfolios for which over 100% of a portfolio's total net assets has been invested into equities. Each month, I classify every common stock in each mutual fund portfolio to one of the ten volatility deciles defined based on the historical volatilities of individual stocks in the NYSE, AMEX and NASDAQ. Next, I calculate the percentage of each mutual fund's total net assets by volatility decile in each observation month and determine monthly decile exposures for the mutual funds. I also calculate similar monthly decile exposures for the market portfolio, i.e. for the CRSP value-weighted market portfolio. Following these, I compare the monthly decile exposures in each mutual fund to the decile exposures of the market portfolio and compute mutual funds' relative positions – whether overweight or underweight - in each decile compared to the market portfolio. Consequently, I obtain results that provide evidence whether mutual fund portfolio managers overweight high volatility stocks and underweight low volatility stocks relative to the market portfolio.

# 4. DATA AND METHODOLOGY

This chapter presents the data used in the empirical part of the study. There are several data sets and data sources used in the study. Data comprises monthly stock returns, delisting returns for non-survivors, trading volumes, closing prices, market capitalizations, market portfolio returns, monthly risk-free rates and other market factors including Fama-French factors, Carhart's momentum factor, Pastor-Stambaugh traded liquidity factor. All data is at monthly frequency. In addition, with regards to U.S. mutual fund holdings, I have obtained data of portfolio constituents and constituents' share of a portfolio's total net assets.

The data are collected from the Center for Research in Stock Prices (CRSP) and the CRSP Mutual Fund Database, both in the Wharton Research Data Services (WRDS) database at the Wharton School of the University of Pennsylvania. Fama-French factors, market excess return, risk-free interest rate (one month Treasury Bill rate), and momentum factors are originally from Kenneth French's website at Darthmouth, though also obtained from the WRDS. Pastor-Stambaugh traded liquidity factors are obtained from the WRDS.

In addition to the data description, this chapter describes the methodology used in the study. The description of the methodology includes calculation methodology of stock volatilities and construction of volatility ranked portfolios, and estimation of alphas controlled for the market, size, value, momentum and liquidity. In addition, I describe the methodology used to determine mutual funds' relative exposures to volatility deciles.

## 4.1 Stock return data

The stock return data comprises ordinary common stocks (CRSP share codes 10 and 11) traded on the NYSE, AMEX and NASDAQ (CRSP exchange codes 1, 2 and 3) during the period from January 1973 to November 2013. I exclude ADR's, REIT's, financials, closed-end funds, foreign shares, and stocks with no trading volume during a month. I calculate historical volatilities using the monthly returns over the past 12, 36 and 60 months. The dataset that is used to calculate 12-month historical volatilities begins from January 1973, whereas datasets that are used to calculate 36-month and 60-month volatilities begin from January 1971 and January 1969, respectively. The return observation period for all the three datasets is from January 1974 to November 2013.

The sample using 12-month volatility as the measure of risk consists of 4,266 companies on average in each month. The lowest number of company-observations, 1,995 in total, is observed in December 1976, while the highest number of company-observations, 6,594 in total, is observed in November 1997. The dataset beginning from January 1974 and ending in November 2013 has 479 month-observations.

In this study, I use monthly stock return data. Monthly excess returns over the risk-free rate are used for calculating portfolio returns and stock volatilities. The return data for the general market index is also obtained from the CRSP at monthly frequency. The used market index is the CRSP value-weighted market portfolio that excludes ADR's. The CRSP value-weighted market portfolio takes dividends and other distributions into account. The market portfolio excess return data is used for comparison of the performance between the market and the decile portfolios. The risk-free rate is originally from Kenneth French's website, though also obtained from the WRDS. The risk-free rate is at monthly frequency and is the one month U.S. Treasury Bill rate.

## 4.2 Fama-French factors and other market factors

I use the basic Capital Asset Pricing Model (Sharpe (1964) and Lintner (1965)), Fama-French three-factor model (Fama and French (1993)), Carhart four-factor model (Carhart (1997)) and five-factor model (Pastor and Stambaugh (2003)), which is the four-factor model with added liquidity factor, to calculate the alphas for each of the portfolios. The factors other than the Pastor-Stambaugh traded liquidity factors are originally from Kenneth French's website, though obtained from the WRDS. The Pastor-Stambaugh traded liquidity factors are obtained from the WRDS. All the factors are at monthly frequency.

The factors in the models include the market excess return over the risk-free rate, small market capitalization minus big market capitalization factor (SMB), high book-to-market minus low book-to-market factor (HML), the momentum factor (UMD), and the traded liquidity factor (TLF). The SMB factor describes the returns from a strategy of buying the companies with a small market capitalization and selling the companies with a high market capitalization capturing the small firm effect. On the other hand, the HML factor describes the returns of a strategy of buying the companies with a high book-to-market valuation and selling the companies with a low book-to-market valuation capturing the value premium. Furthermore, the momentum factor describes the effect of buying the winners based on the share price performance 12 months prior to portfolio formation less the return over the most recent month and selling the losers based on the same performance measurement. Lastly, the Pastor-Stambaugh liquidity factor captures the effect of buying the stocks with high sensitivities to aggregate liquidity and selling the stocks with low sensitivities to aggregate liquidity describing the liquidity premium.

### 4.3 Mutual fund holdings

The mutual fund holdings data is obtained from CRSP Mutual Fund database. The database comprises portfolio specific holdings on a security level. The data is available from the beginning of 2001. However, the number of records is considerably low for 2001, and thus, I exclude the data for 2001 and use the data from 2002 to 2013 for the analysis.

I obtain fund specific holdings on a security level and securities' percentages of total net assets. The data that I use includes all the portfolios that have at least 80% but not more than 100% of a fund's assets invested in the U.S. equities (stocks listed in the NYSE, AMEX and NASDAQ). The dataset comprises 9,097 mutual fund holdings reports per year on average. The lowest number of mutual fund holdings reports, 403 in total, is observed in 2002, while highest number of mutual fund holdings reports, 22,231 in total, is observed in 2012.

### 4.4 Methodology

Previous studies (most recently Baker and Haugen (2012), Baker, Bradley, and Taliaferro (2014), Frazzini and Pedersen (2014)) have examined the relationship between risk and return in the stock markets by measuring stocks' riskiness either with beta or volatility. Then, based on the risk measure, stocks are ranked based on the risk and sorted into deciles, quintiles or quartiles. These portfolios are then used to measure the performance of assets having different levels of risk, and ultimately to examine how risk and return are associated. I follow similar methodology in this study by sorting stocks into deciles based on their historical volatility.

In the following, I present the methodology used in the calculation of stock volatilities, construction of the decile portfolios and in the estimation of the decile portfolio alphas controlled for the market performance, size, value, momentum and liquidity effects are presented. Lastly, I describe the methodology used in determining the mutual fund portfolios' relative exposure to the volatility deciles. For comparison, Table 1 presents the methodologies that have been used in the previous low risk anomaly research.

**Table 1: Methodologies in the previous research**

This table shows the methodologies used in the previous research. The key components that vary between studies on low risk anomaly include sample period, sample market, limitations that are applied to the sample selection, risk measure and risk measurement period, data frequency and portfolio construction methodology.

| Methodologies used in previous research | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Paper** | **Sample period** | **Market(s)** | **Sample selection limitations** | **Risk measure(s)** | **Risk measurement period** | **Data frequency** | **Portfolio construction** |
| **Blitz and van Vliet (2007)** | 1986-2006 | U.S., European and Japanese stocks | Large caps | Beta and volatility | 36 months, and 12 months as robustness test | Weekly returns | Deciles; equally weighted portfolios |
| **Baker, Bradley, and Wurgler (2010)** | 1968-2008 | U.S. stocks | All / Top 1000 based on market cap | Beta and volatility | 60 months but at least 24 months | Monthly returns | Quintiles; value-weighted portfolios |
| **Baker and Haugen (2012)** | 1990-2011 | 21 developed and 12 emerging stocks markets | 99.5% of the capitalization in each country | Volatility | 24 months | Monthly returns | Deciles; value-weighted for developed, equal-weighted for emerging markets |
| **Baker, Bradley, and Taliaferro (2013)** | 1968-2012 | U.S. stocks | No limitations | Beta | 60 months but at least 12 months | Monthly returns | Quintiles; value-weighted portfolios |
| **Frazzini and Pedersen (2014)** | 1926-2012 | U.S. stocks | No limitations | Beta | 12 months for volatilities, 60 months for correlations | Daily returns | Deciles; equally weighted portfolios |

### 4.4.1 Volatility estimation

In this study, historical monthly volatility is used as the measure of risk. I calculate 12-, 36- and 60-month volatilities using monthly excess returns for each stock, and based on these, construct decile portfolios to empirically study the relationship between risk and return. I focus on the results based on 12-month volatility, and as a robustness check, report the key results also based on 36- and 60-month volatilities. The volatilities are calculated as follows:

$$\sigma_{i,t}^{N} = \sqrt{\frac{1}{N}\left[\left(r_{i,t-1}-\mu\right)^2 + \left(r_{i,t-2}-\mu\right)^2 + \cdots + \left(r_{i,t-N}-\mu\right)^2\right]} \quad (3)$$

where

$N = number\ of\ months\ used\ to\ estimate\ volatility,$

$i = security\ for\ which\ the\ volatility\ is\ estimated\ ,$

$t = month\ over\ which\ the\ stock's\ return\ is\ measured,$

$r = monthly\ excess\ return\ for\ stock\ i\ at\ time\ T$, and

$\mu = \frac{1}{N}\left(r_{i,t-1} + r_{i,t-2} + \cdots + r_{i,t-N}\right).$

### 4.4.2 Construction of volatility ranked portfolios and calculating portfolio returns

At the end of each month, I assign stocks into ten decile portfolios, each comprising 10% of all stocks, by ranking stocks based on estimated volatility. The stocks that belong to the lowest 10% based on their past volatility are assigned into the Decile 1, D1, and so on. The decile portfolios are monthly rebalanced. For each decile portfolio, I calculate the return in excess of the U.S. Treasury bill rate over the month following the portfolio formation. Both equally weighted and value-weighted returns are calculated. I primarily focus on the results based equally weighted portfolios constructed based on 12-month historical volatilities. I also report the results for the value-weighted portfolios based on 12-month historical volatilities as well as the results based on 36-month and 60-month volatilities for both equally weighted and value-weighted portfolios. Using the resulting time series, I calculate the average excess returns, standard deviations, Sharpe ratios and alphas for the decile portfolios.

### 4.4.3 Portfolio alphas controlled for market, value, size, momentum, and liquidity

To control for the other well-known anomalies such as value, size and momentum as well as for liquidity effect, I follow the methodology used in Frazzini and Pedersen (2014) and regress alphas against the CAPM (Sharpe (1964) and Lintner (1965)), Fama-French (1993) three-factor model, Carhart (1997) four-factor model, and lastly, against Pastor-Stambaugh (2003) five-factor model. The CAPM alpha is the abnormal return over the market portfolio, three-factor alpha is over the market portfolio, the small company effect and the value effect, the four-factor alpha is over the previous effects as well as over the momentum effect, and lastly, the five-factor alphas takes also the illiquidity effect into account.

The alphas are estimated using the following regressions

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + \varepsilon_i \tag{4}$$

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + s_i SMB_t + h_i HML_t + \varepsilon_i \tag{5}$$

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + s_i SMB_t + h_i HML_t + u_i UMD_t + \varepsilon_i \tag{6}$$

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + s_i SMB_t + h_i HML_t + u_i UMD_t + l_i TLF_t + \varepsilon_i \tag{7}$$

where $R_{i,t}$ is the excess return over the risk-free rate on a decile portfolio *i* at time *t*, $R_{m,t}$ is the excess return on the CRSP value-weighted market portfolio at time *t*, $SMB_t$ is the Fama-French small-minus-big size factor at time *t*, $HML_t$ is the Fama-French high-minus-low value factor at time *t*, $UMD_t$ is the Carhart momentum factor at time *t*, $TLF_t$ is the Pastor-Stambaugh traded liquidity factor at time *t*, and $\beta_i$, $s_i$, $h_i$, $u_i$ and $l_i$ are the estimated factor exposures, $\alpha_i$ is the alpha adjusted for control factors and $\varepsilon_i$ is the error term.

### 4.4.4 Mutual funds' relative exposures to volatility deciles

The CRSP mutual fund holding data includes portfolio holdings on a security-level. I analyse the portfolios that have at least 80% but not more than 100% of a fund's assets invested in the U.S. equities (stocks listed in the NYSE, AMEX and NASDAQ) to make comparison between portfolio allocations and the CRSP value-weighted index meaningful. As previously mentioned, I obtain the data of all the stocks in the NYSE, AMEX and NASDAQ for creating volatility deciles and examining the relationship between volatility and return in these markets. Because this data practically includes all the stocks in these three stock markets, and therefore in the CRSP value-weighted index, I can determine how the market portfolio is distributed across the volatility deciles using volatility decile portfolios and their construction. Thereafter, I use this information to calculate the mutual funds' allocations relative to the CRSP value-weighted index, or the market portfolio.

Each month and for every mutual fund, when sufficient amount of holdings reports is available, I calculate the aggregate percentages that have been allocated into stocks in the volatility deciles. If no investments have been made into stocks in a particular decile, a fund's allocation to that decile is naturally zero. Then, I compare a mutual fund's allocation in a particular decile to the market portfolio's allocation in that decile. I compute decile overweights by subtracting the market portfolio's allocation in a particular decile from a mutual fund's allocation in the same decile at certain point of time. If a mutual fund has a greater allocation, say in the first decile, than the market portfolio, then the mutual fund has an overweight in that decile, and vice versa. Using this methodology for all the mutual funds, I obtain a dataset comprising mutual funds' relative allocations in the volatility deciles from 2002 to 2013.

# 5. RESULTS

This chapter presents and discusses the results from empirical analyses regarding the key research questions of the thesis. The first part examines the relationship between risk and expected return over the whole sample period from 1974 to 2013 as well as during four separate 10-year sub-sample periods. The second part concentrates on the existence of the low volatility anomaly under different market conditions by separating the periods of declining and rising stock market, and then discusses whether the anomaly is persistent over a stock market cycle or does it diminish when the stock market trend is upward. The third part of this chapter examines the mutual fund holdings and mutual funds' relative holdings in ten volatility deciles in order to find whether mutual fund portfolio managers typically overweight volatile stocks relative to the market portfolio while underweighting stocks with lower volatility. Finally, I provide results confirming the robustness of the findings in the earlier sections with regards to the volatility estimation period and to the weights used in the portfolio construction.

## 5.1 The relationship between risk and return

The focus of this section is on the existence and persistence of the low volatility anomaly over the past 40 years and four separate 10-year sub-sample periods. The first subsection concentrates on the whole sample, whereas the second subsection examines the sub-sample periods.

### 5.1.1 Low volatility anomaly in the full sample

The results in this subsection contribute to the evidence on the existence of the anomaly. The subsection tests hypotheses H1, H2 and H3 over a long timeframe from 1974 to 2013. The next subsection returns to the same hypotheses but examines these over a shorter timeframes. An extensive set of literature has reported confirmatory results that low risk stocks have higher expected returns than high risk stocks (e.g. Blitz and van Vliet (2007), Baker et al. (2010), Baker and Haugen (2012), Frazzini and Pedersen (2014)). Literature also finds that it is persistent (Frazzini and Pedersen (2014), Baker and Haugen (2012)) – existing now and far back in time.  Furthermore, it is remarkable because it is not apparent only in the U.S. equity markets but across equity markets both in developed countries and emerging markets, in

Treasury bonds, corporate bonds, and futures (Frazzini and Pedersen (2014), Baker and Haugen (2012)).

With a sample of 479 month-observations comprising 4,266 common stocks on average per month, I find that low volatility stocks have higher risk-adjusted returns than high volatility stocks and the risk-adjusted returns, or Sharpe ratios, are almost monotonically decreasing with volatility. Table 2 present the excess returns, alphas, realized betas, volatilities and Sharpe ratios for volatility sorted portfolios. Over the period from 1974 to 2013 in the main U.S. equity markets, the lowest volatility decile experiences much higher and more significant alpha than the higher volatility deciles in all four testing frameworks. Furthermore, the excess return differences between the volatility sorted deciles are fairly small, expect for the 10% highest volatility, compared to the differences in alphas and volatilities. Therefore, I find strong evidence supporting the first three hypotheses concerning the low volatility anomaly.

### *i) Excess returns*

With regards to excess returns for the equally weighted decile portfolios, there is neither a clearly positive nor negative relationship between volatility and excess returns. Monthly excess returns range from 0.63% to 0.92%, except for the highest volatility decile, for which the excess return is close to zero at 0.01%. Highest excess returns are experienced by the third and fourth deciles, monthly excess return at 0.92% for the both deciles. The deciles from the second lowest volatility up to the sixth decile experience somewhat higher excess returns than the lowest volatility decile. Interestingly, the lowest volatility decile outperforms the four highest volatility deciles presenting anomalous returns between low and high volatility stocks. However, the low volatility anomaly is not consistent throughout the different portfolios as the risk-return relationship is positive in the low volatility end. Therefore, it cannot be concluded that the low volatility anomaly would be fully observable in terms of absolute returns – a result that is in line with the hypothesis H3.

## Table 2: U.S. Equity Returns, 1/1974-11/2013

This table shows the returns of volatility sorted portfolios: at the beginning of each calendar month stocks are ranked in ascending order on the basis of their estimated past 12-month volatility at the end of the previous month. The ranked stocks are assigned to decile portfolios. Panel A shows the results for portfolios in which all stocks are equally weighted, whereas the Panel B shows the results for portfolios in which all stocks are given weights based on their market capitalization. The portfolios are rebalanced every month. This table includes all available common stocks on the CRSP database between January 1974 and November 2013. The rightmost column (MKT) reports returns of the CRSP value-weighted market portfolio. Excess returns are over the risk-free rate. Alpha is the intercept in a regression of monthly excess return. The explanatory variables are the monthly returns from Fama and French (1993) mimicking portfolios, Carhart (1997) momentum factor and Pastor and Stambaugh (2003) traded liquidity factor. Returns and alphas are in monthly percent, t-statistics are shown below the coefficient estimates. Beta (realized) is the regression coefficient on the market portfolio. Volatilities and Sharpe ratios are annualized.

**Stocks sorted by 12-month volatility - full sample 1974-2013**

| **Panel A: Equal-weights** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** | **MKT** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1974-2013 | Low volatility | | | | | | | | High volatility | | Diff. | |
| Excess return | 0.81 | 0.85 | 0.92 | 0.92 | 0.89 | 0.87 | 0.77 | 0.68 | 0.63 | 0.01 | 0.80 | 0.48 |
| **CAPM alpha** | **0.51** | **0.45** | **0.47** | **0.45** | **0.40** | **0.37** | **0.27** | **0.18** | **0.16** | **-0.37** | **0.88** | |
| | (5.65) | (4.93) | (4.63) | (4.04) | (3.14) | (2.40) | (1.46) | (0.83) | (0.59) | (-1.10) | | |
| **3-factor alpha** | **0.28** | **0.18** | **0.17** | **0.13** | **0.07** | **0.03** | **-0.05** | **-0.14** | **-0.18** | **-0.73** | **1.01** | |
| | (3.84) | (2.79) | (2.58) | (1.96) | (0.99) | (0.34) | (-0.45) | (-0.98) | (-0.98) | (-2.97) | | |
| **4-factor alpha** | **0.28** | **0.21** | **0.23** | **0.21** | **0.21** | **0.22** | **0.20** | **0.17** | **0.19** | **-0.42** | **0.70** | |
| | (3.88) | (3.30) | (3.52) | (3.27) | (3.00) | (2.63) | (2.09) | (1.27) | (1.13) | (-1.75) | | |
| **5-factor alpha** | **0.26** | **0.20** | **0.22** | **0.20** | **0.19** | **0.20** | **0.18** | **0.16** | **0.17** | **-0.42** | **0.68** | |
| | (3.59) | (3.09) | (3.30) | (3.11) | (2.76) | (2.46) | (1.88) | (1.19) | (1.04) | (-1.72) | | |
| Beta (realized) | 0.56 | 0.77 | 0.88 | 0.97 | 1.05 | 1.14 | 1.24 | 1.35 | 1.44 | 1.52 | -0.96 | 1.00 |
| Volatility | 11.25 | 14.16 | 16.13 | 17.65 | 19.45 | 21.61 | 24.22 | 27.39 | 30.62 | 35.18 | -23.93 | 16.08 |
| Sharpe ratio | 0.87 | 0.72 | 0.68 | 0.63 | 0.55 | 0.48 | 0.38 | 0.30 | 0.25 | 0.00 | 0.86 | 0.36 |

| **Panel B: Value-weights** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** | **MKT** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1974-2013 | Low volatility | | | | | | | | High volatility | | Diff. | |
| Excess return | 0.59 | 0.49 | 0.60 | 0.60 | 0.45 | 0.33 | 0.44 | 0.17 | 0.01 | -0.61 | 1.19 | 0.48 |
| **CAPM alpha** | **0.26** | **0.08** | **0.12** | **0.08** | **-0.08** | **-0.21** | **-0.12** | **-0.38** | **-0.51** | **-1.10** | **1.35** | |
| | (2.69) | (1.01) | (1.75) | (1.19) | (-0.90) | (-1.76) | (-0.82) | (-2.04) | (-2.16) | (-3.86) | | |
| **3-factor alpha** | **0.20** | **0.02** | **0.05** | **0.04** | **-0.13** | **-0.18** | **-0.08** | **-0.38** | **-0.53** | **-1.18** | **1.38** | |
| | (2.62) | (0.33) | (0.84) | (0.57) | (-1.42) | (-1.63) | (-0.65) | (-2.46) | (-2.79) | (-5.29) | | |
| **4-factor alpha** | **0.18** | **0.00** | **0.08** | **0.08** | **-0.06** | **-0.05** | **0.01** | **-0.24** | **-0.32** | **-1.03** | **1.20** | |
| | (2.27) | (-0.04) | (1.24) | (1.11) | (-0.68) | (-0.45) | (0.08) | (-1.58) | (-1.72) | (-4.58) | | |
| **5-factor alpha** | **0.18** | **-0.01** | **0.06** | **0.07** | **-0.06** | **-0.04** | **0.03** | **-0.20** | **-0.27** | **-0.95** | **1.14** | |
| | (2.37) | (-0.08) | (0.93) | (1.01) | (-0.61) | (-0.35) | (0.20) | (-1.32) | (-1.43) | (-4.23) | | |
| Beta (realized) | 0.66 | 0.86 | 1.00 | 1.11 | 1.21 | 1.32 | 1.39 | 1.52 | 1.64 | 1.70 | -1.04 | 1.00 |
| Volatility | 12.80 | 14.84 | 16.91 | 18.57 | 20.57 | 23.06 | 24.77 | 28.17 | 31.76 | 34.65 | -21.85 | 16.08 |
| Sharpe ratio | 0.55 | 0.40 | 0.42 | 0.39 | 0.26 | 0.17 | 0.21 | 0.07 | 0.01 | -0.21 | 0.76 | 0.36 |

For the value-weighted portfolios, similar conclusions can be drawn: highest excess returns are experienced by the third and fourth deciles, although the returns are lower, at 0.60%, compared to the situation when equal-weights are applied. Yet, when the portfolios are constructed using value-weights, the lowest volatility decile is close to being the top performer in terms of absolute excess return, return difference being only 0.01% per month compared to returns for the third and fourth deciles. Again, the differences in monthly excess returns are fairly small except for the highest volatility deciles: the returns fall significantly for the eighth, ninth and tenth deciles.

For the value-weighted portfolios, similar conclusions can be drawn: highest excess returns are experienced by the third and fourth deciles, although the returns are lower, at 0.60%, compared to the situation when equal-weights are applied. Yet, when the portfolios are constructed using value-weights, the lowest volatility decile is close to being the top performer in terms of absolute excess return, return difference being only 0.01% per month compared to returns for the third and fourth deciles. Again, the differences in monthly excess returns are fairly small except for the highest volatility deciles: the returns fall significantly for the eighth, ninth and tenth deciles.

Previous studies that conclude superior performance for the low risk stocks in terms of absolute excess returns (e.g. Blitz and van Vliet (2007), Baker et al. (2010), Baker and Haugen (2012)), typically compare the performance of the lowest risk decile or quintile to that of the highest risk decile or quintile, while ignoring the returns for the portfolios in between these two. Based on the results in this paper, highest excess returns are experienced by the deciles with median or slightly below the median volatility. By looking only at the low and high volatility ends, my results also support the statement that the stocks with the lowest 10% volatility deliver higher absolute returns than the stocks with the highest 10% volatility.

### *ii) Alphas*

In equally weighted portfolios, alphas for the lowest volatility decile are statistically highly significant and clearly positive, monthly alphas ranging from 0.26% to 0.51%. The CAPM alphas and 3-factor alphas decline almost monotonically with risk, being negative for the highest volatility decile. Monthly alpha-spreads between the lowest and the highest volatility deciles range from 0.68% to 1.01% depending on the testing framework. However, the alpha for the lowest volatility decreases, when the SML and HML factors are added to the CAPM framework, which indicates that the positive performance of the low volatility stocks can be attributed to either size or value effect to some extent. Still, the good performance is not fully linked to the size and value effects, and even less to momentum and illiquidity premium because the alpha remains positive and statistically significant after adding all these factors. Therefore, low volatility stocks earn positive alpha beyond size, value and momentum effects, and beyond illiquidity premium. Furthermore, the alphas decrease also for all the other deciles when the size and value factors are included indicating that there are notable amount of stocks in each decile that earn either size or value premium.

With regards to value-weighted portfolios, similar conclusions can be drawn. Yet, the alphas for the lowest risk decile are somewhat lower – although still statistically significant and positive – in the value-weighted portfolios than in equally weighted portfolios. Even more remarkable are the deeply negative alphas for the highest risk decile, alphas being statistically significant ranging from -0.95% to -1.18% per month. The same applies for all the other deciles that alphas are notably lower when the portfolios are constructed using value-weights instead of equal-weights. This indicates that the size effect indeed has a notable positive effect on stock returns and the effect can be found in stocks with different levels of volatility. Lastly, the alphas for the lowest five deciles decrease further when the SML and HML factors are added to the CAPM framework. This is likely to be attributable to the value effect as the alphas were already deteriorated by the value-weighting making the size effect less dominant.

Consistent with the above results, the previous literature unanimously finds positive and mostly significant alpha for the lowest risk decile or quintile. Furthermore, the alpha-spread between the lowest risk and the highest risk has been found to be substantial (Frazzini and Pedersen (2014), Baker, Bradley, and Taliaferro (2013), Baker, Bradley, and Wurgler (2010), Blitz and van Vliet (2007), Ang et al. (2006)). The reported annual alpha-spreads are in favor of the lowest risk, spreads ranging from 2.56% up to 16.20% varying based on the sample period, testing framework, whether beta or volatility is used as a risk measure and the universe of stocks that has been studied. Typically CAPM alpha-spreads are higher than 3-factor alpha-spreads, whereas 3-factor alpha-spreads are higher than 5-factor alpha-spreads (Frazzini and Pedersen (2014), Blitz and van Vliet (2007), Ang et al. (2006)). Moreover, the alpha-spread is higher when all the stocks are included without market cap limitations compared to alpha-spread for top 1,000 largest stocks for example (Baker et al. (2010)).

*iii) Sharpe ratios*

As mentioned above, I find that low volatility stocks have higher risk-adjusted returns than high volatility stocks. Additionally, the risk-adjusted returns are decreasing with volatility almost linearly. Equally weighted portfolio of the lowest volatility decile delivered Sharpe ratio of 0.87 over the sample period, while the Sharpe ratio for the highest volatility decile was practically zero. The Sharpe ratio for the lowest volatility deciles was almost 2.5 times higher than that of the market portfolio showing a significance outperformance for the low volatility stocks in risk-adjusted terms. The results are roughly similar for the value-weighted portfolios: Sharpe ratios are again decreasing with volatility almost linearly and the Sharpe ratio spread between the lowest volatility and the highest volatility deciles is significant at 0.76. In comparison, the spread is 0.86 for the equally weighted portfolios. However, the outperformance compared to the value-weighted market portfolio is not as dramatic as it is in the case of equally weighted portfolios. The Sharpe ratio of 0.55 for the low volatility decile compared to that of the market portfolio, 0.36, is 1.5 times higher. These findings, that low risk stocks significantly outperform high risk stocks, are in line with the research by Blitz and van Vliet (2007), Baker et al. (2010), Baker and Haugen (2012) and Frazzini and Pedersen (2014).

I previously noted the excess return differences are fairly small and not the highest for the lowest volatility decile. By definition, volatility is lowest for the first decile and highest for the tenth decile. The difference in annual volatility between the first and the tenth decile is around 24% in the equally weighted portfolios (22% in the value-weighted portfolios). The finding that Sharpe ratios decline almost linearly with volatility is therefore driven by large differences in volatilities and very small differences in absolute excess returns – or by the fact that absolute returns do not increase with risk as predicted by the finance theory. In summary, I find a clear evidence supporting both the hypothesis H1 and the H2.

### 5.1.2 Low volatility anomaly in the sub-sample periods

In this part, I examine the relationship between risk and expected return over the sub-sample periods. The whole sample period is split into four 10-year periods: 1/1974-12/1983, 1/1984-12/1993, 1/1994-12/2003 and 1/2004-11/2013. I use equally weighted portfolios to study the relationship during these periods. The Table 3 present the results for the four sub-sample periods, Panel A for the first period, Panel B for the second, Panel C for the third, and Panel D for the last sub period. The results in this subsection extend the knowledge of the low volatility anomaly and its persistence over the past 40 years. The subsection further tests hypotheses H1, H2 and H3 by examining shorter sub-periods within the whole sample from 1974 to 2013.

With four sub-sample periods, each having 120 month-observations except the last one that comprises 119 month-observations, I find that stocks with lower volatility have generally higher risk-adjusted returns than stocks with higher volatility. Yet, the negative relationship between volatility and risk-adjusted returns is not as linear for all the sub-sample periods as it is for the whole sample. Again, the lowest volatility decile is typically among the portfolios that experience the highest alpha and the positive alphas are typically most significant for the lower volatility deciles. Interestingly, the excess return differences between the volatility sorted deciles vary more across the sub-sample periods and the lowest volatility decile actually delivers the highest absolute excess return of all of the portfolios over two separate sub-sample periods. The lowest volatility decile also has higher absolute excess return than the highest volatility decile in all the sub-samples, although in the first and the last sub-sample the return for the first decile is lower than the return for the second to ninth volatility decile.

## Table 3: U.S. Equity Returns, Sub-sample periods

This table shows the returns of volatility sorted portfolios: at the beginning of each calendar month stocks are ranked in ascending order on the basis of their estimated past 12-month volatility at the end of the previous month. The ranked stocks are assigned to decile portfolios. Panel A shows the results for equally weighted portfolios during 11/1994-12/2000, Panel B shows the results for equally weighted portfolios during 1/2001-12/2006, and Panel C shows the results for equally weighted portfolios during 1/2007-11/2012. The portfolios are rebalanced every month. The results are based on all available common stocks on the CRSP database between November 1994 and November 2012. The rightmost column reports returns of the CRSP value-weighted market portfolio. Excess returns are over the risk-free rate. Alpha is the intercept in a regression of monthly excess return. The explanatory variables are the monthly returns from Fama and French (1993) mimicking portfolios, Carhart (1997) momentum factor and Pastor and Stambaugh (2003) traded liquidity factor. Returns and alphas are in monthly percent, t-statistics are shown below the coefficient estimates, and 5% statistical significance is indicated in bold. Beta (ex ante) is the weighted average estimated 12-month beta at portfolio formation. Beta (realized) is the regression coefficient on the market portfolio. Volatilities and Sharpe ratios are annualized.

**Stocks sorted by 12-month volatility, equally weighted portfolios**

| **Panel A: Subperiod 1** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** | **MKT** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1974-1983 | Low volatility | | | | | | | | High volatility | | Diff. | |
| Excess return | 0.82 | 1.09 | 1.21 | 1.29 | 1.35 | 1.43 | 1.39 | 1.43 | 1.30 | 0.62 | 0.20 | 0.27 |
| **CAPM alpha** | **0.60** | **0.80** | **0.90** | **0.97** | **1.01** | **1.11** | **1.07** | **1.12** | **1.02** | **0.47** | **0.13** | |
| | (2.80) | (4.07) | (4.08) | (4.09) | (3.90) | (3.49) | (3.09) | (2.82) | (2.26) | (0.84) | | |
| **3-factor alpha** | **0.07** | **0.19** | **0.16** | **0.14** | **0.12** | **0.02** | **-0.14** | **-0.22** | **-0.48** | **-1.38** | **1.45** | |
| | (0.43) | (1.58) | (1.42) | (1.47) | (1.07) | (0.14) | (-1.06) | (-1.29) | (-2.24) | (-4.96) | | |
| **4-factor alpha** | **0.33** | **0.41** | **0.31** | **0.26** | **0.22** | **0.10** | **-0.02** | **-0.17** | **-0.47** | **-1.48** | **1.81** | |
| | (2.50) | (4.16) | (3.04) | (2.82) | (2.01) | (0.78) | (-0.17) | (-0.98) | (-2.11) | (-5.14) | | |
| **5-factor alpha** | **0.35** | **0.39** | **0.26** | **0.23** | **0.18** | **0.04** | **-0.01** | **-0.21** | **-0.51** | **-1.42** | **1.77** | |
| | (2.61) | (3.89) | (2.56) | (2.47) | (1.64) | (0.30) | (-0.09) | (-1.17) | (-2.24) | (-4.88) | | |
| Beta (realized) | 0.71 | 0.90 | 1.02 | 1.08 | 1.16 | 1.19 | 1.29 | 1.36 | 1.41 | 1.50 | -0.79 | 1.00 |
| Volatility | 14.63 | 17.30 | 19.46 | 20.61 | 22.31 | 23.77 | 25.80 | 27.81 | 29.69 | 33.27 | -18.63 | 17.27 |
| Sharpe ratio | 0.67 | 0.76 | 0.75 | 0.75 | 0.72 | 0.72 | 0.65 | 0.62 | 0.52 | 0.22 | 0.45 | 0.18 |

| **Panel B: Subperiod 2** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** | **MKT** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1984-1993 | Low volatility | | | | | | | | High volatility | | Diff. | |
| Excess return | 0.75 | 0.67 | 0.66 | 0.59 | 0.44 | 0.26 | 0.21 | -0.02 | -0.16 | -0.17 | 0.92 | 0.63 |
| **CAPM alpha** | **0.35** | **0.15** | **0.09** | **0.00** | **-0.15** | **-0.33** | **-0.37** | **-0.59** | **-0.67** | **-0.58** | **0.93** | |
| | (2.79) | (1.13) | (0.60) | (0.03) | (-0.63) | (-1.11) | (-1.15) | (-1.52) | (-1.42) | (-0.96) | | |
| **3-factor alpha** | **0.23** | **0.11** | **0.13** | **0.07** | **-0.02** | **-0.16** | **-0.14** | **-0.40** | **-0.50** | **-0.55** | **0.78** | |
| | (2.23) | (1.52) | (1.91) | (0.97) | (-0.23) | (-1.11) | (-0.92) | (-1.72) | (-1.59) | (-1.23) | | |
| **4-factor alpha** | **0.18** | **0.12** | **0.17** | **0.13** | **0.07** | **-0.03** | **-0.01** | **-0.24** | **-0.32** | **-0.42** | **0.59** | |
| | (1.69) | (1.57) | (2.35) | (1.71) | (0.76) | (-0.20) | (-0.04) | (-1.05) | (-1.01) | (-0.91) | | |
| **5-factor alpha** | **0.17** | **0.11** | **0.16** | **0.12** | **0.07** | **-0.03** | **0.00** | **-0.24** | **-0.31** | **-0.41** | **0.58** | |
| | (1.66) | (1.55) | (2.33) | (1.68) | (0.75) | (-0.20) | (-0.01) | (-1.04) | (-0.98) | (-0.88) | | |
| Beta (realized) | 0.59 | 0.81 | 0.92 | 0.97 | 1.01 | 1.07 | 1.07 | 1.10 | 1.06 | 0.99 | -0.41 | 1.00 |
| Volatility | 10.35 | 13.69 | 15.53 | 16.83 | 18.22 | 20.10 | 20.68 | 22.56 | 24.29 | 27.49 | -17.14 | 15.74 |
| Sharpe ratio | 0.87 | 0.59 | 0.51 | 0.42 | 0.29 | 0.16 | 0.12 | -0.01 | -0.08 | -0.08 | 0.95 | 0.48 |

| **Panel C: Subperiod 3** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** | **MKT** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1994-2003 | Low volatility | | | | | | | | High volatility | | Diff. | |
| Excess return | 1.15 | 1.04 | 1.05 | 1.04 | 0.94 | 0.80 | 0.89 | 0.63 | 0.67 | -0.05 | 1.20 | 0.52 |
| **CAPM alpha** | **0.91** | **0.75** | **0.70** | **0.65** | **0.51** | **0.34** | **0.39** | **0.17** | **0.21** | **-0.34** | **1.26** | |
| | (4.56) | (3.70) | (3.13) | (2.63) | (1.72) | (0.93) | (0.81) | (0.27) | (0.29) | (-0.37) | | |
| **3-factor alpha** | **0.55** | **0.34** | **0.25** | **0.22** | **0.07** | **0.00** | **0.19** | **0.09** | **0.26** | **-0.01** | **0.56** | |
| | (3.82) | (2.73) | (1.86) | (1.36) | (0.39) | (0.00) | (0.53) | (0.20) | (0.53) | (-0.02) | | |
| **4-factor alpha** | **0.59** | **0.42** | **0.38** | **0.40** | **0.36** | **0.38** | **0.73** | **0.77** | **1.01** | **0.68** | **-0.09** | |
| | (4.01) | (3.36) | (2.93) | (2.70) | (2.31) | (1.88) | (2.61) | (2.16) | (2.52) | (1.18) | | |
| **5-factor alpha** | **0.59** | **0.41** | **0.37** | **0.39** | **0.34** | **0.37** | **0.70** | **0.75** | **0.98** | **0.66** | **-0.07** | |
| | (3.93) | (3.26) | (2.83) | (2.60) | (2.16) | (1.81) | (2.48) | (2.08) | (2.40) | (1.12) | | |
| Beta (realized) | 0.32 | 0.46 | 0.58 | 0.69 | 0.83 | 1.01 | 1.22 | 1.40 | 1.68 | 1.87 | -1.54 | 1.00 |
| Volatility | 9.19 | 10.70 | 12.61 | 14.62 | 17.53 | 21.29 | 26.95 | 32.24 | 38.30 | 46.45 | -37.26 | 16.30 |
| Sharpe ratio | 1.50 | 1.17 | 1.00 | 0.86 | 0.64 | 0.45 | 0.40 | 0.24 | 0.21 | -0.01 | 1.51 | 0.38 |

| **Panel D: Subperiod 4** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** | **MKT** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2004-2013 | Low volatility | | | | | | | | High volatility | | Diff. | |
| Excess return | 0.54 | 0.61 | 0.75 | 0.77 | 0.86 | 1.02 | 0.64 | 0.73 | 0.78 | -0.33 | 0.88 | 0.52 |
| **CAPM alpha** | **0.20** | **0.13** | **0.22** | **0.19** | **0.25** | **0.37** | **0.00** | **0.05** | **0.09** | **-0.99** | **1.18** | |
| | (1.94) | (1.20) | (1.64) | (1.27) | (1.42) | (1.72) | (-0.01) | (0.15) | (0.23) | (-2.25) | | |
| **3-factor alpha** | **0.18** | **0.10** | **0.17** | **0.13** | **0.19** | **0.30** | **-0.08** | **-0.04** | **-0.01** | **-1.09** | **1.27** | |
| | (1.86) | (1.14) | (1.84) | (1.41) | (1.57) | (1.88) | (-0.41) | (-0.15) | (-0.03) | (-2.81) | | |
| **4-factor alpha** | **0.17** | **0.10** | **0.20** | **0.16** | **0.23** | **0.35** | **-0.01** | **0.05** | **0.10** | **-1.00** | **1.17** | |
| | (1.77) | (1.21) | (2.32) | (1.89) | (2.34) | (2.62) | (-0.05) | (0.24) | (0.38) | (-2.75) | | |
| **5-factor alpha** | **0.17** | **0.10** | **0.18** | **0.14** | **0.21** | **0.33** | **-0.05** | **0.06** | **0.09** | **-0.98** | **1.15** | |
| | (1.73) | (1.14) | (2.12) | (1.65) | (2.11) | (2.42) | (-0.36) | (0.29) | (0.34) | (-2.66) | | |
| Beta (realized) | 0.62 | 0.91 | 1.03 | 1.14 | 1.22 | 1.30 | 1.40 | 1.56 | 1.65 | 1.73 | -1.11 | 1.00 |
| Volatility | 10.08 | 14.27 | 16.30 | 18.13 | 19.52 | 21.21 | 23.06 | 26.15 | 28.63 | 30.82 | -20.75 | 15.12 |
| Sharpe ratio | 0.64 | 0.51 | 0.56 | 0.51 | 0.53 | 0.57 | 0.33 | 0.34 | 0.33 | -0.13 | 0.77 | 0.41 |

### *i) Excess returns*

The excess return pattern varies from period to period to the extent that neither a clear positive nor negative relationship between volatility and excess return can be found. With regards to the first subperiod from 1974 to1983 and the last subperiod from 2004 to 2013, excess returns are increasing with volatility up to the sixth decile, after which the returns deteriorate with volatility. Whereas for the second and the third subperiods, the pattern shows clearly negative – although not linear – relationship between volatility and excess returns: in the low volatility end, excess returns decrease slightly with volatility while decreasing more towards the high volatility end. One remarkable finding that applies across the subperiods is that the lowest volatility decile outperforms the highest volatility decile. Additionally, the outperformance is substantial over the last three subperiods, differences in excess returns between the lowest and the highest volatility deciles ranging from 0.88% to 1.20% per month. In the first subperiod, the outperformance is less dramatic, excess return difference being 0.20% per month. Although the results do not show a clear volatility anomaly in terms of absolute excess returns, which is in line with the hypothesis H3, they do not support the traditional theory that risk bearing can be expected to produce a reward either.

*ii) Alphas*

The low volatility stocks appear to deliver positive alpha over all the subperiods. However, the statistical significance is somewhat weaker than in the full sample and slightly below 5% significance under certain testing frameworks. In most cases, the lowest three volatility deciles are among the portfolios with the highest and most significant alpha except for the most recent period, where the middle deciles experience the highest alphas. Furthermore, the highest volatility deciles perform the worst in terms of alpha, and alpha for the highest volatility decile is typically negative. These findings extend the evidence that low volatility stocks earn a premium that cannot be explained by the traditional factors including market, size, value, momentum and liquidity factors.

*iii) Sharpe ratios*

The sub-sample results provide further evidence on the persistence of low volatility anomaly in risk-adjusted terms: the low volatility end has significantly higher Sharpe ratios than the high volatility end, and the Sharpe ratios are almost consistently decreasing with volatility in all the sub-sample periods, supporting the hypothesis H1. Furthermore, the difference in Sharpe ratios between the lowest volatility decile and the highest volatility decile ranges from 0.45 up to 1.51.

Generally, the volatility differences are driving the anomaly together with the relatively small differences in absolute returns for the neighboring deciles, consistent with the hypothesis H2. Volatility differences between bottom and top decile are within the range from around 17% up to 37%. The anomaly is further strengthened by the good absolute excess returns experienced by the low volatility stocks – the lowest volatility portfolio even has the highest absolute return in the second and third sub-sample period. Lastly, there is usually are dramatic drop in risk-adjusted performance when moving from the ninth decile to the highest volatility decile.

## 5.2 Low volatility anomaly in different market conditions

The focus of this section is on the performance of volatility sorted portfolios during periods when the market declines, and secondly, when the market rises. The rising and declining periods are defined over the period from 1994 to 2013 for the reason that prior to 1994 the general market was trending mostly upwards. The subsection tests the hypothesis H4 whether the anomaly is a persistent phenomenon over a stock market cycle, or is it primarily associated with either declining or rising market conditions. Table 4 presents excess returns, alphas, realized betas, volatilities and Sharpe ratios for the volatility sorted portfolios separately for periods of declining and rising stocks market.

Blitz and van Vliet (2007) finds that the low risk portfolios underperform the market during up market months, while outperforming the market during down market months. Furthermore, they find that the underperformance during up months is considerably smaller than the outperformance during down months. Yet, this effect is countered to some extent by more frequent occurrence of up months, 59% of up months versus 41% of down months in their sample.

### 5.2.1 Returns during declining stock markets

There are two separate periods when the stock market was clearly trending downwards during the period from 1994 to 2013: the first market decline occurred from April 2000 to September 2002 following the dot-com bubble. The second period when the general market trended downwards, was from November 2007 to February 2009 and it was triggered by a large decline in home prices in the U.S. that lead to subprime borrowers defaulting on their mortgage payments and devaluation of housing-related securities – known as the U.S. subprime mortgage crisis that resulted in the collapse of Lehman Brothers, a former global financial services firm. Panels B and D of the Table 4 present the performance of the volatility sorted portfolios during declining stock markets.

*i) Excess returns*

Based on the finance theory, it would be natural that the performance of more volatile stocks is worse in terms of absolute excess returns compared to the stocks with lower volatility. This also appears to be the case based on the data. The excess returns are declining almost monotonically with volatility, the monthly return differences between the bottom and the top decile being 2.62% for the period associated with subprime crisis and staggering 6.08% for the period following the dot-com bubble. The performance is not only less negative for the low volatility deciles but actually highly positive in the aftermath of the dot-com bubble, the bottom decile experiencing 1.48% monthly excess return over the 30-month period of market decline. This indicates that the stock market was willing to overpay significantly for certain stocks during the bubble resulting in very poor future returns for these stocks – some of these companies are likely those dot-com firms that went bankrupt, while other stocks were significantly undervalued and not of interest to a broad investor base.

*ii) Alphas*

Not surprisingly, knowing the great absolute performance of the low volatility deciles after the dot-com bubble, the low volatility stocks earned highly significant alpha during the period from April 2000 to September 2002. The alphas are positive and clearly significant for the bottom three deciles, whereas the higher volatility deciles have positive but insignificant alphas. During the second period from November 2007 to February 2009, all 3-, 4- and 5-factor alphas are negative for all deciles, yet mostly insignificant. Alpha-spreads between the bottom and top decile range from 0.85% up to 1.50% per month during the first market decline and from 1.67% up to 3.58% per month during the decline causes by the subprime crisis.

*iii) Sharpe ratios*

The Sharpe ratio patterns across the deciles for the two separate periods are contradictory: during the first observation period, Sharpe ratios are clearly declining in volatility and the Sharpe ratio spread between the bottom and top decile is extremely high, at 2.92. For the second observation period, however, Sharpe ratios are the lowest for the bottom two volatility deciles, while the deciles five and nine have the least negative Sharpe ratios. Thus, there is no clear relationship between risk-adjusted returns and volatilities during this period. Therefore, the findings are mixed and support the hypothesis H4 only partly.

Compared to the first period, all the Sharpe ratios are within a fairly narrow range from -2.35 for the bottom decile to -1.70 for the ninth decile. Furthermore, the difference in risk-adjusted returns between the bottom and top decile is 0.39 in favor of the top decile. So far, the results have consistently supported the previous findings that low risk stocks have generally higher risk-adjusted returns than high risk stocks. The observations for the period following the burst of subprime crisis contradict with the evidence supporting the existence of the low volatility anomaly. Furthermore, these findings do not fully support the conclusion by Blitz and van Vliet (2007) that low risk portfolios outperform the market during down market months.

### 5.2.2 Returns during rising stock markets

There are three separate periods when the stock market was clearly trending upwards during the period from 1994 to 2013: the first from January 1994 to March 2000, the second from October 2002 to October 2007, and the last from March 2009 to November 2013. Panels A, C and E of the Table 4 present the performance of the volatility sorted portfolios during rising stock markets.

*i) Excess returns*

The excess return pattern is consistent over the three periods of rising stock market. Generally, the bottom volatility decile experiences the lowest absolute excess return when the stock market goes up, and additionally, the excess return increases with volatility up to the ninth decile. The top decile typically has an excess return that is in between that of the bottom and the ninth deciles. Again, as during the periods of declining stock market, the relationship between absolute return and volatility – except for the top volatility decile – is positive as predicted by the theory. However, the relationship is too flat to justify the increase in volatility as observed from the pattern of declining Sharpe ratios towards the high volatility end.

*ii) Alphas*

Although the relationship between absolute excess return and volatility is positive when the market goes up, interestingly, low volatility deciles deliver higher and more significant alphas than the higher volatility portfolios. This result applies for the two most recent periods when the market trended upwards – during the upward trend from January 1994 to March 2000, the alphas for all deciles were smaller and mostly insignificant.

The finding that low volatility portfolios actually experience positive and significant alphas is contradicting with the conclusion by Blitz and van Vliet (2007) that these portfolios underperform the market during up market months. The good performance of low volatility portfolios when the market goes up cannot be explained by the traditional market, size, value, momentum and liquidity factors. The finding is remarkable for the reason that the outperformance of low volatility stocks is expected to be related to downward trends in the market rather than to the upward trends in the market. The finding is also an indication that the low volatility anomaly may not be limited only the periods when the stock market declines but is potentially a phenomenon that is persistent over stock market cycle.

*iii) Sharpe ratios*

Indeed, low volatility stocks perform remarkably well in risk-adjusted terms when the general stock market rises. Sharpe ratios for the bottom volatility decile over the three periods are 0.89, 1.88 and 2.30 compared to the Sharpe ratio of 0.36 for the market portfolio over the whole 40-year sample period. Furthermore, Sharpe ratios are almost consistently decreasing in volatility, the top volatility decile having the lowest risk-adjusted return in all three sample periods. The spread in risk-adjusted returns between the bottom and top decile range from 0.50 up to 1.52 in favor of the bottom decile. These findings contradict with the hypothesis H4 and show that the low volatility anomaly is actually more consistent during periods of rising stocks market.

In addition to the positive and significant alphas for the low volatility deciles when the market trends upward, the risk-adjusted returns are higher for lower volatility stocks too. The result confirms that low volatility anomaly is not limited periods when the stock returns are low, and the general market is declining, but it appears to be a persistent phenomenon over stock market cycle. Additionally, low volatility portfolios are less volatile when the market is rising compared to periods when the market is declining. This contributes further to the extremely good risk-adjusted performance of the low volatility stocks.

### Table 4: U.S. Equity Returns, Association with market performance

This table shows the returns of volatility sorted portfolios: at the beginning of each calendar month stocks are ranked in ascending order on the basis of their estimated past 12-month volatility at the end of the previous month.

| Stocks sorted by 12-month volatility, equally weighted portfolios | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Panel A: Rising market 1** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** | **MKT** |
| 1/1994-3/2000 | Low volatility | | | | | | | | High volatility | | Diff. | |
| Excess return | 0.71 | 0.77 | 0.79 | 0.97 | 0.90 | 0.91 | 1.13 | 1.25 | 1.34 | 1.17 | -0.46 | 1.38 |
| **CAPM alpha** | **0.08** | **-0.04** | **-0.14** | **-0.05** | **-0.22** | **-0.26** | **-0.15** | **-0.05** | **-0.02** | **0.00** | **0.08** | |
| | (0.32) | (-0.14) | (-0.56) | (-0.18) | (-0.64) | (-0.59) | (-0.26) | (-0.07) | (-0.02) | (0.00) | | |
| **3-factor alpha** | **0.10** | **-0.01** | **-0.11** | **0.01** | **-0.12** | **-0.12** | **0.03** | **0.16** | **0.24** | **0.32** | **-0.23** | |
| | (0.60) | (-0.10) | (-0.68) | (0.07) | (-0.56) | (-0.46) | (0.10) | (0.39) | (0.52) | (0.47) | | |
| **4-factor alpha** | **0.11** | **0.18** | **0.16** | **0.35** | **0.27** | **0.25** | **0.46** | **0.57** | **0.69** | **0.64** | **-0.53** | |
| | (0.62) | (1.28) | (1.15) | (1.99) | (1.45) | (0.98) | (1.48) | (1.33) | (1.47) | (0.88) | | |
| **5-factor alpha** | **0.11** | **0.18** | **0.17** | **0.37** | **0.30** | **0.28** | **0.48** | **0.60** | **0.70** | **0.64** | **-0.53** | |
| | (0.63) | (1.30) | (1.23) | (2.10) | (1.62) | (1.11) | (1.53) | (1.42) | (1.47) | (0.87) | | |
| Beta (realized) | 0.47 | 0.60 | 0.70 | 0.77 | 0.86 | 0.91 | 1.03 | 1.07 | 1.17 | 1.18 | -0.71 | |
| Volatility | 9.63 | 11.07 | 12.21 | 13.68 | 15.41 | 17.64 | 21.30 | 24.42 | 28.78 | 36.50 | -26.88 | 14.06 |
| Sharpe ratio | 0.89 | 0.83 | 0.77 | 0.85 | 0.70 | 0.62 | 0.64 | 0.61 | 0.56 | 0.38 | 0.50 | 1.18 |
| **Panel B: Declining market 1** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** | **MKT** |
| 4/2000-9/2002 | Low volatility | | | | | | | | High volatility | | Diff. | |
| Excess return | 1.48 | 0.98 | 0.73 | 0.25 | -0.33 | -1.03 | -1.59 | -2.76 | -3.30 | -4.61 | 6.08 | -2.02 |
| **CAPM alpha** | **1.98** | **1.86** | **1.99** | **1.93** | **1.74** | **1.84** | **2.11** | **1.46** | **1.88** | **0.48** | **1.50** | |
| | (4.83) | (4.31) | (3.59) | (3.16) | (2.24) | (1.98) | (1.55) | (0.87) | (1.08) | (0.22) | | |
| **3-factor alpha** | **1.19** | **0.90** | **0.78** | **0.69** | **0.34** | **0.57** | **1.18** | **0.72** | **1.63** | **0.34** | **0.85** | |
| | (3.63) | (3.46) | (2.04) | (1.43) | (0.50) | (0.64) | (0.86) | (0.45) | (1.03) | (0.19) | | |
| **4-factor alpha** | **1.14** | **0.86** | **0.70** | **0.58** | **0.16** | **0.35** | **0.84** | **0.32** | **1.28** | **0.07** | **1.07** | |
| | (3.75) | (3.81) | (2.30) | (1.62) | (0.38) | (0.58) | (0.88) | (0.29) | (1.04) | (0.05) | | |
| **5-factor alpha** | **1.13** | **0.83** | **0.67** | **0.51** | **0.04** | **0.26** | **0.71** | **0.12** | **1.09** | **-0.04** | **1.17** | |
| | (3.63) | (3.69) | (2.19) | (1.50) | (0.12) | (0.44) | (0.75) | (0.11) | (0.91) | (-0.03) | | |
| Beta (realized) | 0.27 | 0.41 | 0.56 | 0.73 | 0.88 | 1.22 | 1.55 | 1.85 | 2.30 | 2.57 | -2.30 | |
| Volatility | 8.76 | 10.78 | 14.30 | 17.33 | 21.43 | 28.14 | 37.57 | 45.55 | 52.82 | 61.53 | -52.77 | 18.91 |
| Sharpe ratio | 2.02 | 1.09 | 0.61 | 0.17 | -0.19 | -0.44 | -0.51 | -0.73 | -0.75 | -0.90 | 2.92 | -1.28 |

| Panel C: Rising market 2 | D1 | D2 | D3 | D4 | D5 | D6 | D7 | D8 | D9 | D10 | D1-D10 | MKT |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10/2002-10/2007 | Low volatility | | | | | | | | | High volatility | Diff. | |
| Excess return | 1.01 | 1.10 | 1.36 | 1.51 | 1.71 | 1.99 | 1.99 | 2.04 | 2.50 | 1.71 | -0.70 | 1.11 |
| **CAPM alpha** | **0.47** | **0.34** | **0.45** | **0.46** | **0.41** | **0.47** | **0.27** | **0.14** | **0.21** | **-0.75** | **1.22** | |
| | (2.61) | (1.99) | (2.20) | (2.05) | (1.50) | (1.49) | (0.73) | (0.29) | (0.34) | (-1.00) | | |
| **3-factor alpha** | **0.35** | **0.22** | **0.33** | **0.35** | **0.27** | **0.39** | **0.28** | **0.15** | **0.28** | **-0.54** | **0.89** | |
| | (2.42) | (1.92) | (3.30) | (3.21) | (2.01) | (2.24) | (1.34) | (0.46) | (0.59) | (-0.85) | | |
| **4-factor alpha** | **0.36** | **0.21** | **0.33** | **0.35** | **0.31** | **0.44** | **0.34** | **0.27** | **0.47** | **-0.31** | **0.67** | |
| | (2.41) | (1.84) | (3.25) | (3.18) | (2.59) | (2.76) | (1.78) | (0.99) | (1.20) | (-0.56) | | |
| **5-factor alpha** | **0.40** | **0.24** | **0.37** | **0.37** | **0.39** | **0.47** | **0.41** | **0.42** | **0.62** | **-0.22** | **0.61** | |
| | (2.56) | (1.98) | (3.64) | (3.17) | (3.19) | (2.85) | (2.10) | (1.51) | (1.53) | (-0.38) | | |
| Beta (realized) | 0.47 | 0.67 | 0.79 | 0.90 | 1.13 | 1.30 | 1.53 | 1.72 | 2.08 | 2.48 | -2.01 | |
| Volatility | 6.48 | 7.92 | 9.35 | 10.58 | 13.14 | 15.21 | 17.79 | 20.94 | 25.90 | 31.06 | -24.57 | 9.93 |
| Sharpe ratio | 1.88 | 1.67 | 1.75 | 1.71 | 1.56 | 1.57 | 1.34 | 1.17 | 1.16 | 0.66 | 1.22 | 1.34 |

| Panel D: Declining market 2 | D1 | D2 | D3 | D4 | D5 | D6 | D7 | D8 | D9 | D10 | D1-D10 | MKT |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11/2007-2/2009 | Low volatility | | | | | | | | | High volatility | Diff. | |
| Excess return | -2.59 | -3.35 | -3.52 | -3.76 | -3.82 | -3.97 | -4.43 | -4.67 | -4.50 | -5.20 | 2.62 | -3.49 |
| **CAPM alpha** | **-0.28** | **0.10** | **0.11** | **0.38** | **0.36** | **0.13** | **-0.08** | **-0.27** | **0.08** | **-1.95** | **1.67** | |
| | (-0.79) | (0.21) | (0.17) | (0.64) | (0.47) | (0.15) | (-0.08) | (-0.23) | (0.06) | (-1.09) | | |
| **3-factor alpha** | **-0.35** | **-0.36** | **-0.60** | **-0.33** | **-0.61** | **-0.81** | **-1.26** | **-1.40** | **-1.44** | **-3.48** | **3.13** | |
| | (-0.83) | (-0.88) | (-1.08) | (-0.62) | (-0.90) | (-0.99) | (-1.19) | (-1.18) | (-1.03) | (-1.97) | | |
| **4-factor alpha** | **-0.26** | **-0.27** | **-0.46** | **-0.24** | **-0.45** | **-0.69** | **-1.06** | **-1.28** | **-1.24** | **-3.30** | **3.05** | |
| | (-0.73) | (-0.80) | (-1.12) | (-0.49) | (-0.84) | (-0.89) | (-1.13) | (-1.07) | (-0.92) | (-1.86) | | |
| **5-factor alpha** | **-0.31** | **-0.39** | **-0.65** | **-0.60** | **-0.80** | **-1.09** | **-1.75** | **-1.73** | **-1.77** | **-3.89** | **3.58** | |
| | (-0.82) | (-1.16) | (-1.72) | (-1.85) | (-1.96) | (-1.58) | (-2.84) | (-1.47) | (-1.35) | (-2.15) | | |
| Beta (realized) | 0.63 | 0.97 | 1.03 | 1.17 | 1.19 | 1.20 | 1.33 | 1.40 | 1.39 | 1.28 | -0.64 | |
| Volatility | 13.21 | 20.01 | 21.60 | 24.31 | 25.18 | 25.57 | 29.09 | 30.55 | 31.78 | 31.81 | -18.59 | 19.98 |
| Sharpe ratio | -2.35 | -2.01 | -1.96 | -1.85 | -1.82 | -1.86 | -1.83 | -1.83 | -1.70 | -1.96 | -0.39 | -2.10 |

| Panel E: Rising market 3 | D1 | D2 | D3 | D4 | D5 | D6 | D7 | D8 | D9 | D10 | D1-D10 | MKT |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3/2009-11/2013 | Low volatility | | | | | | | | | High volatility | Diff. | |
| Excess return | 1.75 | 2.17 | 2.46 | 2.50 | 2.79 | 3.15 | 2.86 | 3.53 | 3.56 | 2.13 | -0.38 | 2.02 |
| **CAPM alpha** | **0.57** | **0.38** | **0.33** | **0.15** | **0.23** | **0.35** | **0.02** | **0.21** | **-0.04** | **-1.28** | **1.84** | |
| | (3.67) | (2.35) | (1.54) | (0.59) | (0.76) | (0.96) | (0.04) | (0.39) | (-0.06) | (-1.84) | | |
| **3-factor alpha** | **0.58** | **0.42** | **0.38** | **0.22** | **0.31** | **0.48** | **0.13** | **0.35** | **0.15** | **-1.06** | **1.63** | |
| | (3.79) | (3.88) | (2.73) | (1.57) | (1.56) | (1.81) | (0.42) | (0.81) | (0.27) | (-1.83) | | |
| **4-factor alpha** | **0.58** | **0.42** | **0.38** | **0.22** | **0.31** | **0.47** | **0.12** | **0.33** | **0.13** | **-1.07** | **1.65** | |
| | (4.44) | (3.87) | (2.95) | (1.90) | (1.95) | (2.50) | (0.63) | (1.19) | (0.34) | (-2.15) | | |
| **5-factor alpha** | **0.56** | **0.40** | **0.38** | **0.19** | **0.28** | **0.45** | **0.14** | **0.25** | **0.10** | **-1.19** | **1.75** | |
| | (4.26) | (3.68) | (2.87) | (1.70) | (1.80) | (2.39) | (0.70) | (0.92) | (0.24) | (-2.42) | | |
| Beta (realized) | 0.56 | 0.85 | 1.02 | 1.15 | 1.24 | 1.33 | 1.40 | 1.60 | 1.79 | 1.89 | -1.33 | |
| Volatility | 9.11 | 13.36 | 16.13 | 18.18 | 19.83 | 21.70 | 23.07 | 27.04 | 30.84 | 32.72 | -23.60 | 14.98 |
| Sharpe ratio | 2.30 | 1.95 | 1.83 | 1.65 | 1.69 | 1.74 | 1.49 | 1.57 | 1.38 | 0.78 | 1.52 | 1.62 |

## 5.3 Mutual funds' relative exposures to volatility deciles

The focus of this section is on the mutual fund holdings and relative equity positions in the volatility sorted deciles compared to the construction of the market portfolio. The results in this section contribute to the explanations for the low volatility anomaly. In particular, the section tests the hypothesis H5. The data of mutual fund holdings covers only the recent period from 2002 to 2013 due to data limitations. The mutual fund holdings data is from the CRSP Mutual Fund Holdings database.

The literature has provided several explanations for the anomaly, many of them relating to institutional investors' tendency to overweight risky stocks. Firstly, the CAPM assumes unrestricted borrowing and lending of risky assets, although regular mutual funds typically have restrictions on borrowing for investment purposes. Frazzini and Pedersen (2014) argue that borrowing restrictions cause investors to leverage by overweighting risky assets, and thus, the prices of risky assets are inflated resulting in lower expected return.

Secondly, Blitz and van Vliet (2007) raise a problem related to decentralized investment approach and market benchmarks. They describe that an asset manager is typically responsible for making capital allocation decision within certain asset class after asset allocation decision across asset classes is made by an investment committee. Furthermore, the success of the capital allocation made by the manager is measured against a market benchmark which creates an incentive to tilt towards high beta or high volatility stocks, they argue, because it would be a way to generate above average returns, if the CAPM holds at least partially. Baker et al. (2010) also raise the same issue and note that many institutional investors are measured against fixed benchmarks, typically capitalization weighted, which discourage investing in low risk stocks.

Thirdly, Baker and Haugen (2012) explain that typical manager compensation schedule and other agency issues related to portfolio construction have a similar effect causing a portfolio manager to construct a portfolio that is likely to be more volatility rather than less volatile. At first they argue that the expected value of a manager's compensation increases with portfolio volatility due to an option-like compensation payoff in a situation where the compensation consists of a base salary and a bonus when the performance is sufficiently high. Moreover, they raise other agency issues that further inflate the demand for high volatility stocks. One explanation is that analysts tend to recommend noteworthy stocks for which they can confidently make a compelling investment case in order to impress the CIO and fellow analysts. However, these stocks tend to receive a good amount of both news and analyst coverage which both increase the volatility of these stocks, Baker and Haugen argue. Moreover, the second argument is that it is easier for a portfolio manager to explain the changes in a model portfolio, when the newly included stocks are interesting by nature, supported with newsworthiness and a decent amount of news and analyst coverage – all which tend to increase the volatility of these stocks.

Taking all these explanations into account, I expect that mutual fund managers overweight risky, high volatility stocks and underweight stocks with lower volatility. Table 5 shows the average market portfolio construction per year as well as the average mutual fund allocations per year. Table 6 presents relative positions of equity-dominated, non-levered U.S. mutual funds against a value-weighted market portfolio. In other words, Table 6 shows which are the volatility deciles that mutual fund portfolio managers typically underweight and overweight. The Figure 4 is a graphical presentation of this. Each month and for every mutual fund, when sufficient amount of holdings reports is available, I calculate the aggregate percentage that has been allocated into stocks in a particular volatility decile. Then, I compare the average allocation percentages each month to the market portfolio's allocation. The results are presented on an annual level. Market portfolio distribution and mutual fund holdings in the volatility deciles are presented as averages of individual observations per year. The mutual funds' relative overweights (or underweights) per decile are presented both as averages and medians of all observations per year. Based on the data, the largest proportion of the total market capitalization in the U.S. equity markets fall into the bottom three deciles meaning that the least volatile stocks typically include companies with high market capitalization. On contrary, the high volatility deciles include companies with low market capitalization.

I find that, over the period from 2002 to 2013, mutual fund managers significantly underweight the stocks in the bottom deciles. Since 2004, the average underweight in the bottom decile has varied between -9.3 and -13.5 percentage points relative to the market portfolio's allocation. Median underweight in the bottom decile has been even deeper and the underweight has varied between -11.2 and -14.6 percentange points since 2004. Furthermore, mutual fund managers overweight the stocks with higher volatility, in particular the stocks in the top six deciles, relative to the market portfolio. The average overweight on these portfolios has been between 0.1 and 2.0 percentage points most of the time. However, the median overweight in the top six deciles is lower varying -1.0 and 1.0 most of the time, indicating that some portfolio managers overweight the high volatility stocks more than the others. Interestingly, the results show that rather than significantly overweighting high volatility stocks, most of the mutual fund portfolio managers actually prefer to underweight low volatility stocks.

In addition to the evident underweight in low volatility stocks, the underweight has been fairly persistent over the period from 2002 to 2013. This also shows that mutual fund managers do not decrease the risk of their portfolios below the market portfolio during market turmoil and when the general trend is downward, for example in 2008, when the market was declining in the aftermath of the subprime crisis. In 2002, after the burst of the dot-com bubble, however, the underweight in low volatility stocks was less severe than during the following years.

In summary, I find clear evidence that supports the hypothesis H5. The results present an evidence that portfolio managers indeed cause a slight excess demand to high volatility stocks and even more significant demand shortage to low volatility stocks. As a result, if the market impact of mutual fund managers' trades is significant, this partly explains why high volatility stocks are overpriced and have too low future returns relative to risk, and why low volatility stocks are underpriced and have too high future returns relative to risk. The results are in line with the proposed explanations for the anomaly including restrictions on borrowing that is assumed to cause investors and portfolio managers to leverage by increased risk-taking (Blitz and van Vliet (2007), Frazzini and Pedersen (2014)), benchmarking that may cause portfolio managers to tilt their portfolios towards high volatility stocks (Blitz and van Vliet (2007), Baker et al. (2010)), and manager compensation and agency issues that encourage investing in stocks with higher volatility (Falkenstein (1994), Karceski (2002), Baker and Haugen (2012)). The finding provides significant evidence on the low volatility anomaly and is among first studies to report that mutual fund portfolios managers, on average, underweight low volatility stocks significantly, and additionally, overweight high volatility stocks as proposed in the previous literature.

### Table 5: Market portfolio distribution and mutual fund holdings

This table shows both the market portfolio's construction and mutual fund holdings in volatility sorted deciles. First, Panel A shows the market portfolio's construction as averages of monthly observations, i.e. the share of market capitalization of CRSP value-weighted index per volatility sorted decile. Volatility breakpoints are defined for the universe comprising stocks in the NYSE, AMEX and NASDAQ. CRSP value-weighted index comprises stocks listed in the NYSE, AMEX and NASDAQ. Second, Panel B shows annual average values of mutual fund holdings in stocks listed in the NYSE, AMEX and NASDAQ. Mutual funds include portfolios that invest 80% to 100% of net assets in the stocks listed in the three market places.

**Market portfolio distribution, average decile weights**

| **Panel A: 2002-2013** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| % of the market | Low volatility | | | | | | | | High volatility | | Diff. |
| 2002 | 15.9 | 25.4 | 18.3 | 15.1 | 8.0 | 7.4 | 5.6 | 1.9 | 1.4 | 0.9 | 15.0 |
| 2003 | 19.9 | 19.2 | 15.7 | 15.1 | 9.7 | 10.1 | 4.8 | 2.6 | 1.6 | 1.1 | 18.8 |
| 2004 | 31.4 | 23.2 | 13.1 | 12.0 | 7.1 | 5.7 | 3.4 | 2.0 | 1.3 | 0.8 | 30.6 |
| 2005 | 32.8 | 17.9 | 13.8 | 14.2 | 7.8 | 5.1 | 3.9 | 2.5 | 1.4 | 0.6 | 32.1 |
| 2006 | 24.4 | 24.3 | 19.3 | 9.6 | 6.9 | 5.9 | 4.9 | 2.5 | 1.5 | 0.7 | 23.7 |
| 2007 | 27.8 | 24.7 | 15.7 | 9.8 | 8.0 | 5.5 | 3.6 | 2.5 | 1.7 | 0.6 | 27.2 |
| 2008 | 33.2 | 20.8 | 13.0 | 9.2 | 7.6 | 5.3 | 4.4 | 3.8 | 1.8 | 0.9 | 32.2 |
| 2009 | 37.0 | 20.8 | 11.4 | 8.7 | 6.6 | 4.8 | 4.0 | 2.3 | 2.9 | 1.6 | 35.4 |
| 2010 | 31.4 | 20.2 | 15.6 | 10.7 | 8.6 | 4.7 | 3.2 | 2.6 | 2.2 | 0.9 | 30.5 |
| 2011 | 29.9 | 20.9 | 13.7 | 12.1 | 9.0 | 5.6 | 3.3 | 3.0 | 1.7 | 0.8 | 29.1 |
| 2012 | 30.6 | 21.2 | 13.1 | 9.9 | 7.4 | 6.6 | 5.3 | 3.3 | 1.8 | 0.8 | 29.8 |
| 2013 | 30.0 | 21.4 | 14.3 | 10.5 | 8.7 | 6.5 | 3.9 | 2.2 | 1.7 | 0.8 | 29.2 |
| 2002-2013 average | 28.7 | 21.7 | 14.7 | 11.4 | 7.9 | 6.1 | 4.2 | 2.6 | 1.8 | 0.9 | 27.8 |
| 2002-2013 median | 30.3 | 21.1 | 14.1 | 10.6 | 7.9 | 5.7 | 3.9 | 2.5 | 1.7 | 0.8 | 29.5 |

**Mutual fund holdings, average decile weights**

| **Panel B: 2002-2013** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| % of net assets | Low volatility | | | | | | | | High volatility | | Diff. |
| 2002 | 11.0 | 19.4 | 16.2 | 14.4 | 8.4 | 7.7 | 6.6 | 2.9 | 2.4 | 1.3 | 9.7 |
| 2003 | 12.7 | 14.0 | 14.4 | 14.0 | 10.4 | 9.8 | 6.5 | 4.2 | 2.7 | 1.4 | 11.4 |
| 2004 | 19.0 | 17.0 | 12.9 | 12.1 | 9.1 | 7.8 | 5.3 | 3.5 | 2.3 | 1.1 | 17.9 |
| 2005 | 20.2 | 14.1 | 12.7 | 13.3 | 9.4 | 7.3 | 5.5 | 4.1 | 2.2 | 0.9 | 19.3 |
| 2006 | 15.1 | 17.0 | 15.2 | 10.5 | 8.7 | 8.3 | 7.0 | 4.1 | 2.8 | 1.1 | 14.0 |
| 2007 | 18.0 | 18.1 | 14.3 | 10.6 | 9.1 | 7.2 | 5.3 | 3.9 | 2.3 | 0.7 | 17.3 |
| 2008 | 21.7 | 17.0 | 12.6 | 10.0 | 8.3 | 6.6 | 5.5 | 4.2 | 2.3 | 1.1 | 20.6 |
| 2009 | 23.5 | 18.4 | 12.4 | 10.0 | 7.8 | 6.3 | 5.1 | 3.1 | 2.7 | 1.7 | 21.8 |
| 2010 | 20.8 | 17.7 | 14.6 | 11.0 | 9.2 | 6.0 | 4.3 | 3.1 | 2.4 | 1.1 | 19.6 |
| 2011 | 18.5 | 17.7 | 13.2 | 12.2 | 9.7 | 7.0 | 4.7 | 3.9 | 2.5 | 1.1 | 17.5 |
| 2012 | 18.5 | 16.9 | 13.1 | 10.4 | 8.5 | 7.8 | 5.9 | 4.6 | 2.9 | 1.1 | 17.4 |
| 2013 | 18.9 | 16.5 | 14.0 | 10.9 | 9.4 | 7.3 | 5.2 | 3.4 | 2.4 | 0.9 | 18.0 |
| 2002-2013 average | 18.2 | 17.0 | 13.8 | 11.6 | 9.0 | 7.4 | 5.6 | 3.8 | 2.5 | 1.1 | 17.0 |
| 2002-2013 median | 18.7 | 17.0 | 13.6 | 11.0 | 9.1 | 7.3 | 5.4 | 3.9 | 2.4 | 1.1 | 17.6 |

## Table 6: Relative allocations of mutual funds

This table shows the differences in decile weights between mutual fund holdings and the market portfolio distribution, i.e. mutual funds' overweight (underweight) per decile per year. Panel A present the average overweights (underweights) based individual observations per year per decile. Panel B present the median overweights (underweights) based individual observations per year per decile.

**Mutual fund holdings vs. market portfolio distribution, average decile overweights**

| **Panel A: 2002-2013** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| %-pts. difference | Low volatility | | | | | | | | | High volatility | Diff. |
| 2002 | -4.9 | -6.0 | -2.1 | -0.7 | 0.4 | 0.3 | 1.0 | 1.0 | 0.9 | 0.4 | -5.3 |
| 2003 | -7.2 | -5.1 | -1.3 | -1.1 | 0.7 | -0.3 | 1.7 | 1.5 | 1.0 | 0.2 | -7.5 |
| 2004 | -12.4 | -6.2 | -0.2 | 0.1 | 2.0 | 2.1 | 1.9 | 1.5 | 1.0 | 0.3 | -12.7 |
| 2005 | -12.6 | -3.8 | -1.1 | -0.9 | 1.6 | 2.2 | 1.6 | 1.6 | 0.8 | 0.2 | -12.8 |
| 2006 | -9.3 | -7.3 | -4.0 | 0.9 | 1.8 | 2.4 | 2.1 | 1.7 | 1.3 | 0.4 | -9.6 |
| 2007 | -9.8 | -6.6 | -1.4 | 0.8 | 1.0 | 1.7 | 1.7 | 1.4 | 0.6 | 0.2 | -10.0 |
| 2008 | -11.4 | -3.8 | -0.4 | 0.9 | 0.7 | 1.2 | 1.1 | 0.4 | 0.5 | 0.2 | -11.6 |
| 2009 | -13.5 | -2.4 | 1.0 | 1.3 | 1.3 | 1.5 | 1.1 | 0.8 | -0.2 | 0.1 | -13.6 |
| 2010 | -10.6 | -2.5 | -1.0 | 0.3 | 0.6 | 1.3 | 1.1 | 0.6 | 0.2 | 0.2 | -10.9 |
| 2011 | -11.3 | -3.3 | -0.5 | 0.1 | 0.7 | 1.3 | 1.4 | 0.9 | 0.8 | 0.3 | -11.6 |
| 2012 | -12.1 | -4.4 | 0.0 | 0.5 | 1.1 | 1.1 | 0.6 | 1.3 | 1.1 | 0.3 | -12.4 |
| 2013 | -11.1 | -5.0 | -0.4 | 0.4 | 0.8 | 0.9 | 1.3 | 1.1 | 0.7 | 0.1 | -11.2 |
| 2002-2013 average | -10.5 | -4.7 | -0.9 | 0.2 | 1.1 | 1.3 | 1.4 | 1.2 | 0.7 | 0.2 | -10.8 |
| 2002-2013 median | -11.2 | -4.7 | -0.8 | 0.4 | 0.9 | 1.3 | 1.4 | 1.2 | 0.8 | 0.2 | -11.5 |

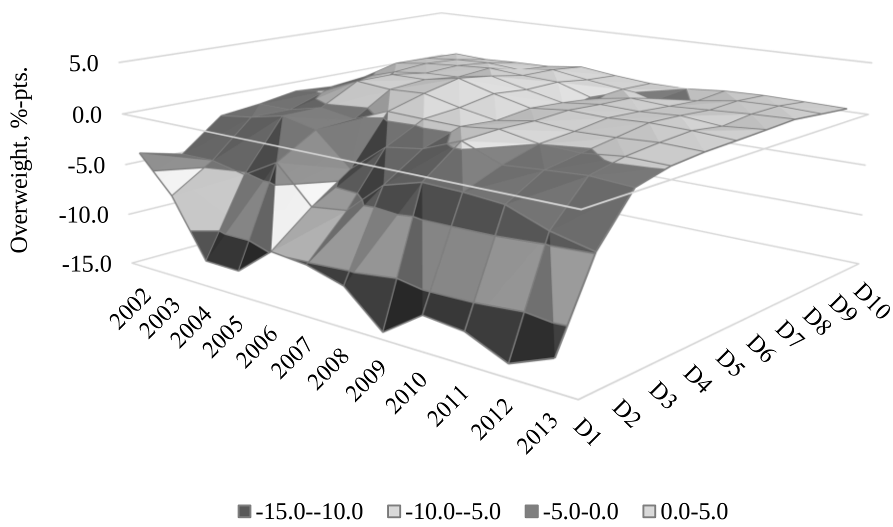
**Mutual fund holdings vs. market portfolio distribution, median decile overweights**

| **Panel B: 2002-2013** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| %-pts. difference | Low volatility | | | | | | | | | High volatility | Diff. |
| 2002 | -4.7 | -5.0 | -1.6 | -1.1 | -0.5 | -0.7 | -0.4 | -0.6 | -0.7 | -0.7 | -4.0 |
| 2003 | -7.6 | -4.7 | -1.1 | -1.1 | 0.1 | -0.9 | 0.4 | 0.0 | -0.6 | -0.7 | -6.9 |
| 2004 | -13.3 | -5.3 | -0.4 | -0.3 | 1.5 | 1.4 | 0.3 | -0.4 | -0.7 | -0.8 | -12.6 |
| 2005 | -14.2 | -3.4 | -1.0 | -0.8 | 1.2 | 1.2 | 0.4 | 0.1 | -0.6 | -0.6 | -13.5 |
| 2006 | -11.8 | -7.2 | -4.2 | 0.4 | 1.3 | 1.8 | 1.1 | -0.1 | -0.4 | -0.7 | -11.1 |
| 2007 | -11.2 | -6.5 | -1.4 | 0.3 | 0.6 | 0.8 | 0.2 | -0.1 | -0.8 | -0.5 | -10.7 |
| 2008 | -12.0 | -3.3 | -0.9 | 0.3 | 0.0 | 0.1 | 0.0 | -0.6 | -0.6 | -0.6 | -11.4 |
| 2009 | -14.6 | -2.3 | 0.5 | 0.6 | 0.5 | 0.6 | 0.2 | -0.2 | -0.9 | -0.7 | -13.9 |
| 2010 | -12.1 | -2.4 | -1.0 | 0.0 | 0.1 | 0.3 | -0.1 | -0.5 | -0.7 | -0.6 | -11.4 |
| 2011 | -12.5 | -2.7 | -0.7 | -0.2 | 0.3 | 0.6 | 0.1 | -0.3 | -0.5 | -0.6 | -11.9 |
| 2012 | -14.2 | -4.1 | -0.5 | 0.2 | 0.5 | 0.7 | 0.0 | 0.1 | -0.2 | -0.7 | -13.5 |
| 2013 | -12.5 | -4.9 | -0.7 | 0.1 | 0.4 | 0.4 | 0.2 | -0.2 | -0.4 | -0.6 | -11.9 |
| 2002-2013 average | -11.7 | -4.3 | -1.1 | -0.1 | 0.5 | 0.5 | 0.2 | -0.2 | -0.6 | -0.6 | -11.1 |
| 2002-2013 median | -12.3 | -4.4 | -0.9 | 0.0 | 0.4 | 0.6 | 0.2 | -0.2 | -0.6 | -0.7 | -11.6 |

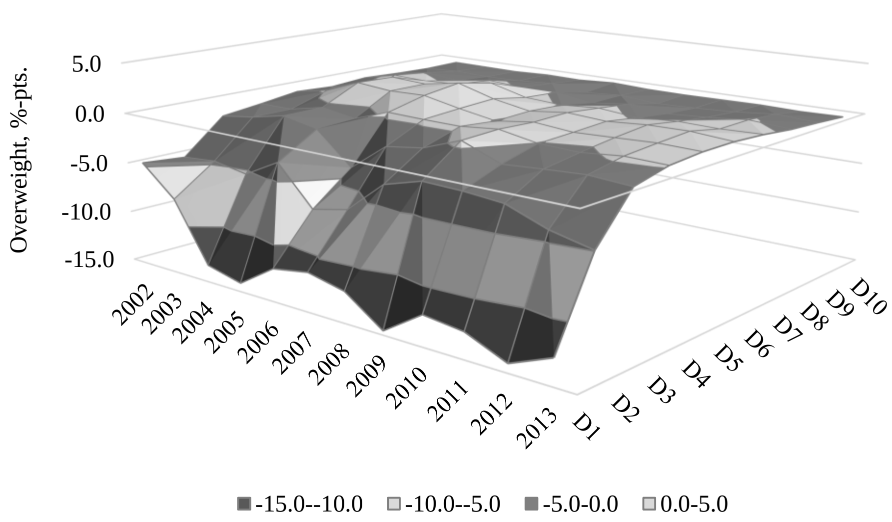
**Figure 4: Mutual fund holdings relative to market portfolio**

Figure 4 shows an illustration of equity-dominated, non-levered U.S. mutual funds' allocations relative to market portfolio, the CRSP value-weighted market index, over the period from 2002 to 2013. The figure is a graphical presentation of the data in Table 6. Allocations are presented as relative overweights (or underweights when the sign is negative) in terms of percentage point difference compared to the market portfolio. Both average and median overweights per year are presented. For example, if common stocks within the lowest 10% volatility (D1) comprise 20% of the market portfolio, and a mutual fund has allocated 15% of its assets into these stocks at certain point of time, the relative overweight for the fund is -5% (i.e. underweight in this case). Equity-dominated means that between 80% and 100% of a mutual fund's total assets are invested in equities, while non-levered means that a mutual fund do not use leverage, i.e. a fund neither borrows money nor has short positions in any stocks.

**Average decile overweights**




**Median decile overweights**



## 5.4 Robustness to weights in the portfolio construction

The analysis in this thesis is primarily based on the portfolios that are constructed using equal-weights for the portfolio constituents. Additionally, the results for the full sample period from 1974 to 2013 are also calculated using value-weights, i.e. market capitalization weights, for the constituents in order to verify the consistency of results in both settings. Panel B of the Table 2 in the section 5.1.1 presents the results for value-weighted portfolios that are created using 12-month volatility estimation period. Panel B's of the Appendices 2 and 3 present the results for value-weighted portfolios but using 36-month and 60-month volatility estimation periods, respectively.

The results are relatively similar for both equal-weighted and value-weighted portfolios: in both settings, the lowest volatility decile clearly outperforms the highest volatility decile in absolute excess return terms, while the best performance is observed for some of the portfolios from third to sixth decile. Yet, equal-weighting appears to be better for all the deciles and the excess returns are clearly higher for portfolios constructed using equal-weights instead of value-weights across. Secondly, the low volatility deciles deliver positive and statistically significant alpha, whereas high volatility deciles show negative or zero alpha in both settings. The statistical significance of low volatility portfolios' alphas, however, is clearly higher, when the portfolios are constructed using equal-weights. Lastly, for both equal-weighted and value-weighted portfolios, the Sharpe ratios are almost monotonically declining with volatility. Similar to excess returns, the Sharpe ratios are clearly higher for equal-weighted portfolios, which is obviously driven by the higher excess returns for these portfolios. In addition, equal-weighted portfolios also appear to have mostly lower volatilities than value-weighted portfolios. In summary, the low volatility anomaly is present both in equal-weighted and value-weighted portfolios, although it is stronger when equal portfolio weights are applied.

## 5.5 Robustness to the volatility estimation period

In this thesis, the analysis is primarily based on the portfolios that are created by sorting stocks by the volatility of total monthly returns over the past 12 months. I also calculate the full sample results using extended volatility estimation periods. In particular, both 36-month and 60-month estimation periods are used to obtain supplementary results and check whether the findings are robust to different estimation periods. The results using 36- and 60-month volatility estimation periods are presented in Panel A's in the appendices 2 and 3, respectively.

The results using 36-month and 60-month volatility estimation periods are similar to the results using 12-month volatility estimation periods: the excess returns are gradually increasing with volatility, except for the three top volatility deciles. Lowest volatility decile also outperforms the highest volatility decile regardless of the estimation period, although the volatility estimation period extension weakens the outperformance slightly the longer the estimation period is. Furthermore, the low volatility portfolios exhibit positive and significant alphas even if the estimation period is extended from 12 months to 36 and 60 months. The statistical significance of the alphas decreases only slightly when a longer estimation period is used. Lastly, the relationship between volatility and Sharpe ratios remains negative across the different volatility estimation period setups. In summary, the length of the volatility estimation period has surprisingly small effect on the results, results being only slightly stronger for the shortest estimation period.

## 6. CONCLUSION

The aim of this thesis was to study the persistence and the significance of the low volatility anomaly during the past 40 years. The thesis examined the phenomenon over a long 40-year sample period, over shorter sub-sample periods as well as separately during different market conditions. In addition, motivated by the previous research suggesting that mutual funds overweight risky assets (e.g. Blitz and van Vliet (2007), Baker, Bradley and Wurgler (2010), Baker and Haugen (2012), Frazzini and Pedersen (2014)), the thesis studied U.S. mutual fund holdings and examined whether portfolio managers actually overweight high volatility stocks, and therefore, cause excess demand, overpricing and low future returns for these stocks.

The findings in this thesis challenge the standard asset pricing models such as the CAPM by Sharpe (1964) and Lintner (1965). The findings with regards to stocks returns over the past 40 years show that risk bearing is not well rewarded, and the highest volatility deciles have lower absolute excess returns and lower Sharpe ratios than the lowest volatility deciles. Moreover, the alphas for the highest volatility deciles are insignificant or even negative. At the same time, low volatility stocks have had surprisingly good absolute excess returns, clearly positive and significant alphas, and remarkably high Sharpe ratios. In addition, the Sharpe ratios are not only higher for the low volatility stocks but decrease almost monotonically with volatility. These findings are in line with the previous research and many recent papers report similar results (e.g. Blitz and van Vliet (2007), Baker et al. (2010), Baker and Haugen (2012) and Frazzini and Pedersen (2014) for beta and returns). It is worth noting, however, that the absolute excess returns are not monotonically decreasing with volatility but the absolute returns are the highest for the stocks with median or slightly below median volatility – a finding that has not been made very clear in the previous literature.

The findings for the sub-sample periods are similar to the findings based on the full sample period. For the three most recent sub-sample periods, Sharpe ratios are the highest for the lowest volatility decile and the Sharpe ratios decrease almost monotonically with volatility. In the first sub-sample period, the lowest volatility decile does not quite deliver the highest Sharpe ratio but is still among the best performing deciles. Again, the lowest volatility decile has higher absolute excess returns than the highest volatility decile, although the best absolute performance is experienced by the stocks close to median volatility. In two out of four sub-sample periods, alphas are the highest for the lowest volatility decile and decrease with volatility, while for the remaining two sub-sample periods, portfolios with median or below median volatility deliver the highest and most significant alphas – alphas for the lowest volatility stocks not far behind though. To conclude the analysis, the low volatility anomaly exists primarily in risk-adjusted terms and risk-adjusted returns decrease almost monotonically with volatility. Furthermore, low volatility stocks earn positive and significant alphas, while high volatility stocks earn zero or even negative alphas. The findings concerning alphas and the relationship between volatility and risk-adjusted returns are clearly persistent over the whole sample period and mostly over the sub-sample periods. The negative relationship between volatility and risk-adjusted returns appear to be driven by large differences in volatilities and fairly small differences in absolute returns. However, although the relationship between volatility and absolute returns is mostly positive except at the high volatility end, also negative relationship between these two is observed during two of the four sub-sample periods, thus strengthening the negative relationship between volatility and risk-adjusted returns further during these periods. Lastly, in terms of absolute returns, the lowest volatility decile outperforms the highest volatility decile in all sample periods and the anomalous relationship between these two deciles actually exists. However, absolute returns are generally the highest for the stocks with median or close to median volatility, and therefore the anomalous relationship is not as consistent as it is in terms of risk-adjusted returns.

With regards to the existence of the anomaly in different market conditions and over the stock market cycle, it appears that the anomaly is not attributable especially to either upward or downward trending market but is observed during both periods. The relationship between volatility and risk-adjusted returns is consistently negative for all the three periods of rising stock market. During the two periods of declining stock market, I observe one period with negative and one with positive relationship between volatility and risk-adjusted returns. Therefore, rather than being driven by periods of declining stocks market, the low volatility anomaly appears to exist especially during the rising stock market periods, when low volatility stocks have fairly high absolute returns and very low volatilities.

Lastly, and most importantly, the thesis sheds light on the U.S. mutual fund holdings and provides new information on the topic by studying mutual fund allocations relative to a market benchmark from the perspective of the low volatility anomaly. The previous literature had proposed that investors and mutual fund managers are, for varying reasons, likely to tilt their portfolios towards high volatility and high beta stocks (e.g. Blitz and van Vliet (2007), Baker, Bradley and Wurgler (2010), Baker and Haugen (2012), Frazzini and Pedersen (2014)), therefore causing excess demand and overpricing for the high risk stocks. Indeed, I was able to show that an average mutual fund portfolio manager has a moderate overweight in high volatility stocks and additionally a significant underweight in low volatility stocks. Therefore, this thesis shows that the mutual fund allocations and risk preferences may actually be the underlying reason why the anomaly exists.

This thesis provided an overview how mutual fund managers invest across the volatility deciles in the U.S. As this thesis, to my best knowledge, was the first study to show how investments in mutual fund portfolios are distributed across different volatility levels, future research needs to confirm if similar patterns exist in other markets, where the anomaly has been observed already. Additionally, because mutual funds are only one class of investors, though a dominant one (Blitz and van Vliet (2007)), it would be important to conduct a similar study covering the portfolios of other investor classes, such as portfolios of private investors or pension funds, to find out which investors prefer risky investments and which are the ones who rather invest in low risk stocks. Another suggestion for further research is to study the characteristics of the companies in the lowest volatility deciles. For example, are the low volatility stocks ones that have good and stable dividend yields from year to year but not so high growth prospects, therefore being stocks that are not that attractive for fund managers who are seeking growth instead of stable cash flow. Together with studying the stock characteristics, it would be valuable to study how the performance of stocks within one decile varies. Do all the stocks in the lowest volatility decile have good risk-adjusted returns and decent absolute returns, or are there certain stocks that should be excluded anyway from a low volatility portfolio.

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# APPENDICES

## Appendix 1: Glossary of definitions

| | |
|---|---|
| **Alpha** | Alpha is the return for a stock or a portfolio in excess of the return for benchmark. For example, benchmarks can include the market portfolio's return and other common risk factors such as the returns for size and value strategies. Technically, alpha is the regression intercept of an asset pricing model such as the CAPM or the Fama-French three-factor model. |
| **Beta** | Beta is a risk measure that describes a stock's sensitivity to the market portfolio. If a security's beta is above one, say 1.5, and market portfolio goes up by 1.0%, then that security is expected to go up by 1.5%. The market portfolio has beta of one by definition. |
| **CAPM** | The Capital Asset Pricing Model (CAPM) developed by Sharpe (1964) and Lintner (1965) is a model that describes the relationship between risk and expected return under certain assumptions. |
| **Decile** | 10 percent grouping of stocks. In this study, stocks are sorted by their historical volatilities and assigned into ten deciles. |
| **Excess return** | Absolute return in excess of the risk-free rate. |
| **Equal-weighted** | In an equal-weighted portfolio, the same amount of money is invested in each stock. |
| **Idiosyncratic volatility** | Idiosyncratic volatility is the volatility of a stock's returns that cannot be explained by the market risk (beta) or other common risk factors, and is typically defined as the standard deviation of the regression residual of the CAPM or Fama-French three-factor model. Idiosyncratic volatility is also referred as non-systematic risk or diversifiable risk, and it will not be priced into security |

| | |
|---|---|
| | prices according to the CAPM framework because it can be eliminated by holding a diversified portfolio. |
| **Market portfolio** | In theory, market portfolio refers to the portfolio of all available assets. In this study, market portfolio also refers to the CRSP value-weighted market index. |
| **Quartile** | 25 percent grouping of stocks. |
| **Quintile** | 20 percent grouping of stocks. |
| **Risk-adjusted return** | See Sharpe ratio. |
| **Sharpe ratio** | Sharpe ratio describes a security's return relative to its risk and it is calculated by dividing a security's excess return by the security's volatility. Also called as the risk-adjusted return. |
| **Value-weighted** | In a value-weighted portfolio, the amount of money that is invested in each stock is determined based on the market capitalizations of the stocks in the portfolio, i.e. higher the market capitalization, the higher is the weight. |
| **Volatility** | Volatility refers to standard deviation of returns, a statistical measure of the variability of returns over a defined period of time. |

## Appendix 2: U.S. Equity Returns, 36-month volatility estimation period, 1/1974 - 11/2013

This table shows the returns of volatility sorted portfolios: at the beginning of each calendar month stocks are ranked in ascending order on the basis of their estimated past 36-month volatility at the end of the previous month. The ranked stocks are assigned to decile portfolios. Panel A shows the results for portfolios in which all stocks are equally weighted, whereas the Panel B shows the results for portfolios in which all stocks are given weights based on their market capitalization. The portfolios are rebalanced every month. This table includes all available common stocks on the CRSP database between January 1974 and November 2013. The rightmost column (MKT) reports returns of the CRSP value-weighted market portfolio. Excess returns are over the risk-free rate. Alpha is the intercept in a regression of monthly excess return. The explanatory variables are the monthly returns from Fama and French (1993) mimicking portfolios, Carhart (1997) momentum factor and Pastor and Stambaugh (2003) traded liquidity factor. Returns and alphas are in monthly percent, t-statistics are shown below the coefficient estimates. Beta (realized) is the regression coefficient on the market portfolio. Volatilities and Sharpe ratios are annualized.

**Stocks sorted by 36-month volatility - full sample 1974-2013**

| **Panel A: Equal-weights** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** | **MKT** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1974-2013 | Low volatility | | | | | | | | High volatility | | Diff. | |
| Excess return | 0.77 | 0.84 | 0.82 | 0.84 | 0.85 | 0.88 | 0.85 | 0.73 | 0.59 | 0.11 | 0.65 | 0.48 |
| **CAPM alpha** | **0.48** | **0.44** | **0.39** | **0.38** | **0.36** | **0.38** | **0.33** | **0.23** | **0.14** | **-0.25** | **0.73** | |
| | (5.38) | (5.05) | (3.98) | (3.50) | (2.82) | (2.39) | (1.77) | (1.00) | (0.49) | (-0.71) | | |
| **3-factor alpha** | **0.25** | **0.19** | **0.09** | **0.06** | **0.02** | **0.02** | **-0.02** | **-0.11** | **-0.19** | **-0.56** | **0.81** | |
| | (3.55) | (2.97) | (1.40) | (0.88) | (0.31) | (0.20) | (-0.16) | (-0.76) | (-0.98) | (-2.12) | | |
| **4-factor alpha** | **0.26** | **0.21** | **0.15** | **0.14** | **0.15** | **0.20** | **0.23** | **0.19** | **0.15** | **-0.18** | **0.44** | |
| | (3.57) | (3.21) | (2.33) | (2.24) | (2.13) | (2.48) | (2.26) | (1.43) | (0.83) | (-0.71) | | |
| **5-factor alpha** | **0.25** | **0.21** | **0.15** | **0.14** | **0.14** | **0.19** | **0.22** | **0.17** | **0.13** | **-0.21** | **0.45** | |
| | (3.36) | (3.14) | (2.29) | (2.14) | (2.00) | (2.31) | (2.12) | (1.28) | (0.70) | (-0.80) | | |
| Beta (realized) | 0.53 | 0.76 | 0.86 | 0.96 | 1.05 | 1.15 | 1.26 | 1.37 | 1.44 | 1.54 | -1.01 | 1.00 |
| Volatility | 10.83 | 13.95 | 15.73 | 17.44 | 19.54 | 21.99 | 24.74 | 27.70 | 31.28 | 36.26 | -25.43 | 16.08 |
| Sharpe ratio | 0.85 | 0.72 | 0.63 | 0.58 | 0.52 | 0.48 | 0.41 | 0.31 | 0.23 | 0.04 | 0.81 | 0.36 |

| **Panel B: Value-weights** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** | **MKT** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1974-2013 | Low volatility | | | | | | | | High volatility | | Diff. | |
| Excess return | 0.52 | 0.55 | 0.53 | 0.52 | 0.50 | 0.47 | 0.55 | 0.34 | -0.17 | -0.58 | 1.11 | 0.48 |
| **CAPM alpha** | **0.18** | **0.11** | **0.04** | **-0.01** | **-0.04** | **-0.10** | **-0.02** | **-0.22** | **-0.71** | **-1.07** | **1.26** | |
| | (2.18) | (1.52) | (0.56) | (-0.08) | (-0.43) | (-0.77) | (-0.09) | (-1.11) | (-2.92) | (-3.78) | | |
| **3-factor alpha** | **0.16** | **0.07** | **-0.03** | **-0.03** | **-0.03** | **-0.12** | **0.02** | **-0.24** | **-0.75** | **-1.12** | **1.27** | |
| | (2.32) | (1.09) | (-0.44) | (-0.35) | (-0.32) | (-1.09) | (0.15) | (-1.52) | (-3.94) | (-5.29) | | |
| **4-factor alpha** | **0.12** | **0.04** | **0.01** | **0.02** | **0.04** | **-0.01** | **0.14** | **-0.06** | **-0.55** | **-0.96** | **1.08** | |
| | (1.78) | (0.69) | (0.11) | (0.23) | (0.40) | (-0.06) | (1.01) | (-0.41) | (-2.91) | (-4.53) | | |
| **5-factor alpha** | **0.13** | **0.05** | **0.01** | **0.03** | **0.04** | **-0.04** | **0.16** | **-0.03** | **-0.50** | **-0.93** | **1.07** | |
| | (1.93) | (0.78) | (0.10) | (0.35) | (0.44) | (-0.35) | (1.19) | (-0.17) | (-2.63) | (-4.37) | | |
| Beta (realized) | 0.68 | 0.90 | 1.05 | 1.16 | 1.25 | 1.40 | 1.47 | 1.59 | 1.73 | 1.71 | -1.03 | 1.00 |
| Volatility | 12.62 | 15.50 | 17.88 | 19.52 | 21.65 | 24.45 | 26.80 | 29.60 | 33.24 | 34.80 | -22.18 | 16.08 |
| Sharpe ratio | 0.50 | 0.42 | 0.36 | 0.32 | 0.27 | 0.23 | 0.24 | 0.14 | -0.06 | -0.20 | 0.70 | 0.36 |

## Appendix 3: U.S. Equity Returns, 60-month volatility estimation period, 1/1974 - 11/2013

This table shows the returns of volatility sorted portfolios: at the beginning of each calendar month stocks are ranked in ascending order on the basis of their estimated past 60-month volatility at the end of the previous month. The ranked stocks are assigned to decile portfolios. Panel A shows the results for portfolios in which all stocks are equally weighted, whereas the Panel B shows the results for portfolios in which all stocks are given weights based on their market capitalization. The portfolios are rebalanced every month. This table includes all available common stocks on the CRSP database between January 1974 and November 2013. The rightmost column (MKT) reports returns of the CRSP value-weighted market portfolio. Excess returns are over the risk-free rate. Alpha is the intercept in a regression of monthly excess return. The explanatory variables are the monthly returns from Fama and French (1993) mimicking portfolios, Carhart (1997) momentum factor and Pastor and Stambaugh (2003) traded liquidity factor. Returns and alphas are in monthly percent, t-statistics are shown below the coefficient estimates. Beta (realized) is the regression coefficient on the market portfolio. Volatilities and Sharpe ratios are annualized.

**Stocks sorted by 60-month volatility - full sample 1974-2013**

| **Panel A: Equal-weights** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** | **MKT** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1974-2013 | Low volatility | | | | | | | | | High volatility | Diff. | |
| Excess return | 0.75 | 0.83 | 0.82 | 0.85 | 0.86 | 0.97 | 0.94 | 0.94 | 0.74 | 0.34 | 0.42 | 0.48 |
| **CAPM alpha** | **0.46** | **0.43** | **0.39** | **0.39** | **0.38** | **0.47** | **0.42** | **0.43** | **0.28** | **-0.02** | **0.48** | |
| | (5.14) | (5.00) | (3.94) | (3.57) | (2.90) | (3.01) | (2.27) | (1.92) | (1.02) | (-0.06) | | |
| **3-factor alpha** | **0.24** | **0.19** | **0.08** | **0.06** | **0.01** | **0.09** | **0.05** | **0.08** | **-0.08** | **-0.34** | **0.57** | |
| | (3.30) | (2.90) | (1.32) | (0.88) | (0.14) | (1.05) | (0.47) | (0.56) | (-0.41) | (-1.25) | | |
| **4-factor alpha** | **0.24** | **0.21** | **0.14** | **0.13** | **0.13** | **0.27** | **0.27** | **0.34** | **0.21** | **0.06** | **0.18** | |
| | (3.33) | (3.27) | (2.24) | (2.05) | (1.91) | (3.50) | (2.72) | (2.67) | (1.22) | (0.24) | | |
| **5-factor alpha** | **0.23** | **0.22** | **0.15** | **0.13** | **0.13** | **0.27** | **0.26** | **0.34** | **0.19** | **0.03** | **0.20** | |
| | (3.16) | (3.32) | (2.37) | (2.03) | (1.87) | (3.43) | (2.61) | (2.61) | (1.07) | (0.11) | | |
| Beta (realized) | 0.54 | 0.77 | 0.87 | 0.96 | 1.05 | 1.13 | 1.26 | 1.33 | 1.42 | 1.49 | -0.95 | 1.00 |
| Volatility | 11.03 | 14.05 | 15.85 | 17.45 | 19.45 | 21.60 | 24.48 | 27.18 | 30.58 | 35.90 | -24.88 | 16.08 |
| Sharpe ratio | 0.82 | 0.71 | 0.62 | 0.59 | 0.53 | 0.54 | 0.46 | 0.41 | 0.29 | 0.11 | 0.71 | 0.36 |

| **Panel B: Value-weights** | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D1-D10** | **MKT** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1974-2013 | Low volatility | | | | | | | | | High volatility | Diff. | |
| Excess return | 0.51 | 0.53 | 0.56 | 0.45 | 0.50 | 0.68 | 0.60 | 0.23 | -0.04 | -0.35 | 0.85 | 0.48 |
| **CAPM alpha** | **0.16** | **0.10** | **0.07** | **-0.07** | **-0.04** | **0.12** | **0.03** | **-0.32** | **-0.58** | **-0.83** | **0.99** | |
| | (1.93) | (1.18) | (0.93) | (-0.83) | (-0.39) | (0.89) | (0.18) | (-1.73) | (-2.59) | (-2.89) | | |
| **3-factor alpha** | **0.13** | **0.04** | **0.01** | **-0.07** | **-0.05** | **0.11** | **0.09** | **-0.36** | **-0.65** | **-0.89** | **1.02** | |
| | (2.00) | (0.58) | (0.16) | (-0.95) | (-0.52) | (0.99) | (0.63) | (-2.48) | (-4.33) | (-4.08) | | |
| **4-factor alpha** | **0.09** | **0.03** | **0.05** | **-0.07** | **0.07** | **0.20** | **0.17** | **-0.24** | **-0.44** | **-0.67** | **0.76** | |
| | (1.31) | (0.51) | (0.76) | (-0.91) | (0.72) | (1.81) | (1.26) | (-1.65) | (-3.07) | (-3.10) | | |
| **5-factor alpha** | **0.10** | **0.05** | **0.06** | **-0.09** | **0.05** | **0.18** | **0.19** | **-0.20** | **-0.44** | **-0.64** | **0.73** | |
| | (1.44) | (0.73) | (0.86) | (-1.07) | (0.55) | (1.63) | (1.38) | (-1.37) | (-3.00) | (-2.92) | | |
| Beta (realized) | 0.69 | 0.91 | 1.05 | 1.16 | 1.27 | 1.36 | 1.47 | 1.57 | 1.67 | 1.69 | -1.01 | 1.00 |
| Volatility | 12.74 | 15.91 | 17.79 | 19.58 | 22.03 | 23.95 | 26.94 | 28.94 | 31.66 | 34.70 | -21.96 | 16.08 |
| Sharpe ratio | 0.48 | 0.40 | 0.38 | 0.28 | 0.27 | 0.34 | 0.27 | 0.10 | -0.01 | -0.12 | 0.60 | 0.36 |