Pekka Sääskilahti

ESSAYS ON THE ECONOMICS OF NETWORKS AND SOCIAL RELATIONS

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Abstract

Economic networks are ubiquitous and their influence on economic behaviour is far from trivial. This dissertation contributes to the economic theory of networks in three aspects.

1) Network models are persistently hampered by the problem of multiplicity of equilibria. Existing literature has dealt with the problem in an ad hoc manner. I carry out a comprehensive analysis on the conditions for uniqueness in a monopoly pricing model with network externalities. The conditions differ under perfect and incomplete information. Under perfect information, the relative strength of externalities has to be restricted. This implies that consumers' buying behaviour needs to be driven by non-network aspects. In contrast, the relative strength of externalities does not have to be restricted in order to obtain a unique equilibrium under incomplete information. The perfect and incomplete information regimes yield equilibria that differ qualitatively. The monopoly price is higher under incomplete information. Equilibrium profits are decreasing in uncertainty. Consumer surplus also decreases in uncertainty, but only if the absolute level of uncertainty is already low.

2) The conventional model of network externalities assumes, implicitly, a complete graph structure, i.e. total connectedness of interpersonal relations. However, the mapping of personal social contacts seldom qualifies as a complete graph. In general, some people have more contacts than others, and some contacts are more important than others. Such topological asymmetry has been overlooked in network externalities literature. The complete graph assumption greatly facilitates the analysis, but at the same time, the negligence of the topology of the network can result in a serious exaggeration of the network's value. I analyse how both the network size and topology affect monopoly pricing. The result is that the topological effect dominates the size effect, so the monopolist always incorporates the network's topology in its price. Consequently, asymmetric topologies produce distributional surplus effects thanks to the monopoly's pricing strategy.

3) Technological change is rapid in many network industries because firms regard technological leadership as a competitive advantage. The production of new technology is imperfectly appropriable as part of new knowledge spills over to rivals. Hence, the research and development (R&D) produces externalities. I fill the gap between strategic R&D and network models by analysing a duopoly model where consumers obtain network externalities, and the firms perform imperfectly appropriable R&D in order to cut down production costs. The interplay between technological and network externalities alters the general results of pure strategic R&D and networks models. In an asymmetric set-up, the disadvantaged duopolist increases its R&D efforts and lowers its price under a marginal increase in spillovers or in network compatibility. This happens when R&D and firm-specific networks carry high strategic value.

Keywords: Networks, social relations, coordination, information, heterogeneity, equilibrium uniqueness, R&D, spillovers, monopoly, duopoly.

JEL classification: D42, D82, L13, L14, L15, O32.

Essays on the Economics of Networks and Social Relations

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2. Sääskilahti, P., "Monopoly Pricing of Social Goods", an earlier version of the essay is published in HECER Discussion Papers, 2005, 66.

3. Sääskilahti, P., "Strategic R&D and Network Compatibility", forthcoming in *Economics of Innovation and New Technology*, 2006, 15(7).[1]

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Helsinki, November 2005
Pekka Sääskilahti

Contents

Introduction: Essays on the Economics of Networks and Social Relations

1 Introduction

The modern world is founded on networks. Transportation networks take people from one place to another, communications networks do the same, but virtually. Electricity and gas are supplied by networks. Family, friends, colleagues, and neighbours form our personal social networks. Livelihoods may depend less on family and friends today than in ancient agrarian times, but leisure time is at least as social as ever. Economics became serious with networks in the 1980s and the general interest in network economics was boosted during the 1990s with the popularisation of the Internet and the subsequent dotcom boom. Networks were identified as demand side economies of scale, which were further intensified by increasing returns on the supply side, caused by digitalisation. Hence, the term "new economy" was coined in business, to emphasise the perceived departure from the economic law of diminishing returns. However, it is common knowledge what happened to the major share of e-businesses and dotcom firms.

The attack on diminishing returns was based on analysis with too much abstraction. Only the role of network size was understood, whereas the role of network topology, which curbs the size effect, was overlooked. But there is nothing new in such a Ricardian vice! Five years later, the new economy is less hyped, but despite the hangover from the dotcom boom, the economic role of networks has not diminished. The most solid new economy firms grasped the true advantages of networks and digitalisation. This thesis focuses on social networks, and how they influence firm strategy.

Networks are ubiquitous, but not necessarily important in decision making. Networks become interesting when a network member's behaviour is affected in a non-pecuniary way by other members' actions, or if a member acquires monopoly power through his network position. Networks can be categorised in physical and social (virtual) networks. Physical networks are railroads, gas and electricity transmission networks, telecommunications networks, as well as Internet access and backbone networks for example. Social relations (family relations, friendships, or occupational contacts) are the purest virtual networks. A product has a social network dimension when its use depends on other users of the same or complementary goods. Other examples of virtual networks could be buyer-seller networks and R&D joint ventures. Naturally, most networks have both physical and social dimensions. For example, people (mostly) use a phone to call people they have a social relation with, but the phone call is transmitted on a physical network.

The volume of literature on the economics of networks has expanded rapidly after the seminal work by Arthur (1989), David (1985), Farrell & Saloner (1985) and Katz & Shapiro (1985). Network externalities and compatibility have attracted the bulk of the later work. On the dynamic models' front, emergence of industry standards and lock-in dynamics have interested researchers. In the late 1990s, there emerged a new wave of network models, so called economics of social relations, which understand the role of the topology of the underlying network of relations between agents. The economics of social relations study the microstructure of networks.

The most severe analytical problem in network economics has been the persistent tendency to produce multiple equilibria. In the essay "Buying decision coordination and monopoly pricing of network goods", I explore how equilibrium uniqueness is attainable endogenously.

The popular method to capture network effects is to assume a functional form for network externalities. However, this abstraction may result in an overestimation of network's value. I show in the essay "Monopoly pricing of social goods" how a monopolist's pricing strategy crucially depends on both the size and the topology of the underlying network.

Many network industries are characterised by rapid technological development. Therefore, it is important to understand how firms conduct non-price competition. In the essay "Strategic R&D and network compatibility", I analyse the interplay between technological and network related externalities in a duopoly.

In the next section, I review the economic theory of networks, starting with an overview. Sections 2.1 - 2.3 review three classic approaches. Section 2.4 introduces the second wave models of social relations. In section 3, I present the problems arising from the current theory of network economics, and summarise how I solve those problems in this thesis.

2 Economic theory of networks

The seminal papers by Arthur (1989), David (1985), Farrell & Saloner (1985), and Katz & Shapiro (1985) introduced most of the important features that make networks interesting from the economic point of view. Later research on network economics has remained faithful to the agenda laid out by these papers. Farrell & Saloner (1985) explicitly analyse how agents' preferences for coordination on technology adoption relates to network externalities. Agents must decide whether to switch to a new technology or stick with the prevailing one. Katz & Shapiro (1985) focus on supply side questions. They study what kind of strategies oligopolistic firms play when the goods exhibit network externalities. Arthur (1989) and David (1985) take a more evolutionary approach to network effects. Their interest is in how certain (de facto) standards emerge in the long run, and how early-occurring small random events affect long run dynamics. Tirole (1988: 10.6), Katz & Shapiro (1994), Economides (1996a), Shy (2001), and Gandal (2002) are surveys of the literature.

Although networks are ubiquitous in modern life, they do not always influence economic behaviour. In many situations where network effects can be identified, the actual decision making happens primarily on the basis of other information. What are then the specific features of networks that affect economic decision making?

Underlying in everything are network externalities. Network externalities mean that an additional member to a network increases the utility of all network members.[1] Hence the term "externality". Network externalities are the sole factors that differentiate network goods from ordinary goods. The most celebrated example of direct network externalities is the fax machine: the value of a single fax machine is zero, but the same machine is valuable when there are millions of other fax machines. The literature also recognises indirect network externalities that arise from complementarities. A DVD player becomes more valuable, the more DVD films there are in supply.

The term network externality has been criticised by Liebowitz & Margolis (1994). They claim that although networks are pervasive, networks seldom inflict externalities. In particular,

[1] Mas-Colell et al. (1995) define externalities as: "An externality is present whenever the well-being of a consumer or the production possibilities of a firm are directly affected by the actions of another agent in the economy." With the word "directly" they exclude price-mediated effects.

Liebowitz & Margolis (1994) object indirect network externalities. They argue that complementarities are internalised in prices. So, there are no externalities that could be associated with a market failure. The terminology is often misleading. To clarify the position of this thesis, I talk about networks that result from coordination between agents. The utility of one agent increases, the more people there are on the network. This happens regardless of the equilibrium concept, which in turn determines how the price is set in the model. In this sense, network externalities are a more primitive concept than price.

Network externalities and expectations go in hand in hand. The literature differentiates between rational and myopic behaviour. When agents are rational, their utility depends on the expected future size of the network. In equilibrium, expectations match the actual network size. Under the assumption of myopic behaviour, agents make the decision to join a network on the basis of current network sizes. Myopic behaviour makes the analysis technically less complicated, but the trade-off is a serious limitation on agents' behaviour. A side-effect of rationality is that the equilibrium is seldom unique. Basically, network externalities are strategic action complementarities, in the sense outlined by Bulow et al. (1985). An agent's payoff gain from joining a network, versus opting out, increases, the more other people choose to join the network. This property is powerful as it allows efficiency ranking of equilibria in many network models, namely in supermodular games.

From the suppliers' point of view, network externalities are demand-side economies of scale. The value of the network (i.e. the product) increases in sales. Effectively, as the size of the network increases, the price that consumers are willing to pay, rises. Mason (2000) finds support for the proposition that network externalities can be viewed as demand side economies of scale. He shows that the Coase conjecture in its strongest form fails for durable goods that induce positive network externalities.

Many networks consist of complementary parts, "hardware" and "software". But in some cases hardware does not work with certain software that is dedicated for a different brand of hardware. This means that networks are incompatible. A member of one network does not benefit (directly) from the members of incompatible networks.

Network literature has predominantly focused on the compatibility question, and its implications on firm strategy and industry performance. Since there are many factors in operation, compatibility may increase or decrease the intensity of price competition. Besen & Farrell (1994) differentiate between competition between standards (competition for market share) and competition within the standard. When networks are incompatible, firms compete to obtain high market shares for their proprietary networks. Consumers become locked-in to incompatible firm-specific networks, and the lock-in forms a switching cost, which reduces incentives for price undercutting. In order to attract consumers to switch networks, the firm must compensate consumers for the foregone network benefits of the rival network. Network switching costs exist only when there is at least some degree of incompatibility. When networks are compatible, firms compete within the market. Compatibility removes the strategic role of firm-specific networks, because consumers get full network benefits should they join any network. Consumers can also more easily switch brands as the opportunity cost of network benefits is zero; making price undercutting more effective. Besen & Farrell (1994) also claim that product variety is lower under compatibility, which induces more intense price competition. Effectively, compatibility does make products more comparable, but whether product diversity is reduced, cannot be inferred from the degree of compatibility. In contrast, the absence of market share competition has a tendency to temper price competition. There lacks a universal result on whether price competition is intensified by compatibility or not. Hence, the efficiency comparisons are not clear-cut. The far more general result is that compatibility softens price competition and increases firms' profits (Shy 2001). Compatibility also tends to be socially optimal. Whether an individual firm should pursue a proprietary incompatible standard or join an existing standard depends on case-sensitive factors. In asymmetric settings, compatibility tends to work in favour of small firms. Companies operating large networks have incentives to keep networks incompatible, because a large incompatible network gives competitive advantage over the smaller ones.

Many network industries are associated with rapid technological progress. If there is a large installed network, an entrant may be deterred from the market even if it has a superior product. This kind of lock-in to the existing technology seems disastrous in terms of efficiency, because technological change is blocked. It is ambiguous, however, if the welfare loss holds under dynamic evaluation. Buyers may strategically postpone purchases at the time there is only one technology on offer in order to buy the new technology later. The entrant can manipulate consumers to wait by announcing its intentions to enter. The value of an installed base is diluted when market penetration can occur rapidly. Creative Destruction à la Schumpeter (1975) guarantees that the dynamic inefficiency of lock-in to inferior technologies is of little concern (at least in industries with rapid technological change).

Network and digital economies are different but intertwined concepts. The most important feature of the digitalisation of goods with network dimensions is that production presents huge (infinite) economies of scale. In digital markets, reproduction can be executed infinitely quickly, so that the value of an installed customer base is nullified against entry. The examples of Napster and Google show how product adoption can be almost instantaneous in a global scale. Currently, Skype, the software that allows phone calls over the Internet, attracts over 155,000 new users per day to its existing 29 million user base.[2] The beta version of Skype was launched as recently as August 29th 2003. Even more impressive is Skype's record from the first 51 days: one and a half million registered downloads. Recently, there has been increasing interest in open standards such as the open source software licence. Open standards are likely to shorten adoption times, again diminishing the value of an existing network against entry.

Many networks derive their characteristics from specific physical assets needed to support the network. For example, railroads need tracks. Physical networks often have bottlenecks, and the owner of a bottleneck asset is able to exercise monopoly power. Network-based industries are often former government monopolies. Advances in technology have made it feasible to inject competition without duplicating the network structure, but the contestable sector is not always the bottleneck portion. Hence, the question is how to minimise the efficiency loss due to the monopoly power of the bottleneck asset owner. Adversely, the regulation of complex networks is in danger of excessive self-propagation, destroying any efficiency gains. Complicated regulatory regimes have also diverted attention from demand side questions, for example, in the economics

[2] See http://www.skype.com/company/news/2005/1m_skypeout.html (accessed on March 22nd 2005).

of telecommunications.

The first wave of network models, discussed so far, is unified by "global interaction". In other words, a network member has a need to interact with any randomly chosen person in the network. Everybody knows everybody. A case of global interaction, however, is quite special. Only recently, has there emerged a second wave of network models analysing networks where agents interact with a subset of the total population. Interaction patterns form local neighbourhoods. The second wave can be categorised in two branches. One branch studies how networks are formed, and which network structures hold in equilibrium. The other branch assumes a fixed network of relations and studies economic phenomena that are constrained by the network structure. There is a close linkage to sociological literature on networks. Applications include models of job market transactions, crime networks, village economies, and communication networks. However, industrial competition has attracted less research.

2.1 Demand side coordination

I give an overview on the approach that explicitly relates agents' preferences for coordination to network externalities in this section. Farrell & Saloner (1985) analyse a game where agents, here firms, must decide on the adoption of a new technology. The game is a standard coordination game, similar to the coordination game Γ illustrated below with sequential moves. An individual firm decides whether to upgrade to the new technology (switch) or to keep using the old one (abstain). x_{ij} (y_{ij}) is player 1's (2's) payoff from action i (j) when player 2 (1) plays j (i). If $x_{11} > x_{21}$, $y_{11} > y_{12}$, $x_{22} > x_{12}$ and $y_{22} > y_{21}$ hold true, the game Γ has two Nash equilibria (switch,switch) and (abstain,abstain). If $x_{11} > x_{22}$ and $y_{11} > y_{22}$, then (switch,switch) is the Pareto dominant equilibrium.

	switch	**abstain**
switch	x_{11}, y_{11}	x_{12}, y_{12}
abstain	x_{21}, y_{21}	x_{22}, y_{22}

Coordination Game Γ

In the game Γ, the firms play bandwagon strategies: "I switch if only if you switch." Farrell

& Saloner (1985) show that under perfect information, symmetric firms, i.e. $x_{ij} = y_{ij}$, reach the efficient equilibrium. Both either switch (if $x_{11} > x_{22}$) or abstain ($x_{11} < x_{22}$). If the firms are asymmetric, the order of movement determines the equilibrium. The first-mover has an advantage, and there is a bias in favour of switching.

Technology adoption in Γ can be inefficient under incomplete information. Symmetric "excess inertia" occurs when coordination fails. Both firms prefer to switch ($x_{11} > x_{22}$ and $y_{11} > y_{22}$), but neither firm takes the initiative in fear of being the sole adopter. Asymmetric excess inertia arises when the sum of the payoffs from switching is positive ($x_{11} + y_{11} > x_{22} + y_{22}$ and e.g. $y_{11} < y_{22}$), but in equilibrium both firms abstain. "Excess momentum" is a phenomenon when the sum of the payoffs from switching is negative ($x_{11} + y_{11} < x_{22} + y_{22}$ and e.g. $y_{11} > y_{22}$), but both firms switch because the firm which is against switching prefers switching to being stranded. Symmetric inefficiencies are alleviated by pre-game communication, but communication aggravates asymmetric inefficiencies.

Farrell & Saloner (1986) extend their (1985) model by analysing how the technology adoption game changes when there is an installed base of users. Farrell & Saloner (1986) identify that the adoption process involves two time-dependent externalities. On the one hand, firms that adopt the new technology induce a negative externality to the installed base. The installed base ceases to grow, and the supply of complementary products can diminish. On the other hand, firms who adopt in the early stages induce a positive externality on the following firms. Later arriving firms find the new technology more valuable. The installed base is a barrier to entry for the new technology. The benefit from adopting the new technology must exceed the foregone network benefits of the installed base. A preannouncement of the arrival of the new technology helps to overcome the barrier to entry by making some new adopters wait. When the new technology arrives, both the mass of initial adopters is larger and the existing installed base smaller than without preannouncements. However, welfare implications of a preannouncement can be both negative or positive.

Technology in Γ is exogenous, and neither player has any control over or financial interests in the technology (they cannot claim royalties for example). This is a special case. Firms must coordinate on a new technology that originates outside the game. The more general case discussed by Besen & Farrell (1994) is that both firms have developed proprietary technologies and must now choose whether to make technologies compatible or not. If both firms prefer an industry-wide standard (either player 1's or 2's technology), we have a similar coordination game to Γ. If "abstain" is replaced with "player 2's technology" and "switch" by "player 1's technology", and the payoff relations are kept the same, we have a coordination game where firms must decide on which technology to support for an industry standard. The payoffs from a common standard are so large that firms are willing to give up proprietary technologies (because without compatibility proprietary networks would remain too small). Assuming $x_{11} > x_{22}$ and $y_{11} < y_{22}$, firm 1 has a problem how to convince firm 2 to adopt its technology, and vice versa.

The game changes if returns from an industry-wide standard are limited so that both firms find it more desirable to stick with own proprietary technology. Networks remain incompatible in this case. Besen & Farrell (1994) discuss the possible strategies firms have in this case. They pinpoint the importance of first-mover advantage, because a larger initial customer base is a valuable asset. Firms should try to influence buyers' expectations in their favour and they could also enroll producers of complementary products in order to strengthen indirect network effects.

Finally, if the firms are asymmetric, it could be that one firm prefers compatibility at the same time that the other firm prefers incompatibility. The dominant result in the compatibility literature is that the larger firm prefers to keep networks incompatible whereas the smaller firm benefits from compatibility.

2.2 Supply side competition

In this section, I review the approach to economic networks that focuses on the supply side competition. The main character in a network model à la Katz & Shapiro (1985) is the externality function. The motivation for externalities is exogenous. A typical expected utility formulation is

$$u\left(x_i^s, p_s, \mathbb{E}_i\left(n_k\right)\right) = x_i^s + f_k\left(\mathbb{E}_i\left(n_k\right)\right) - p_s, \tag{1}$$

where x_i^s is the intrinsic utility from the product s for consumer i. The intrinsic utility represents any utility that is separate from the size of the network. The unit price for the product s allowing participation in the network k is p_s. Network externalities are captured in the function $f_k(\mathbb{E}_i(n_k))$, where $\mathbb{E}_i(n_k)$ is consumer i's expectations on the size of the network k. Externalities are positive, but the marginal benefit is, typically, non-increasing, $\frac{\partial f_k(\mathbb{E}_i(n_k))}{\partial \mathbb{E}_i(n_k)} > 0$ and $\frac{\partial^2 f_k(\mathbb{E}_i(n_k))}{\partial \mathbb{E}_i(n_k)^2} \leq 0$.

The consumers' coordination problem is incorporated in the expectations operator. Consumers maximise the expected utility (1) by choosing the supplier s (or by abstaining), but the ex post utility depends on the actual size of the network. In equilibrium, expectations are rational thus fulfilled, $\mathbb{E}_i(n_k) = n_k$ for all i. The problem of fulfilled expectations, is that they result easily in multiple equilibria. Hence, there is a possibility of a coordination failure, corresponding to the inefficient excess inertia found by Farrell & Saloner (1985). Multiplicity makes the model analytically complicated and, in the worst case, makes equilibrium analysis indeterminate. Adversely, the commonly used solution to multiplicity requires that network externalities are limited in strength. Consumers must base their decisions primarily on the intrinsic utility rather than on network-related aspects. In many dynamic games, expectations are assumed to be myopic, $\mathbb{E}_{i,t-1}(n_{k,t}) = n_{k,t-1}$ so that consumer i expects to obtain externalities at time t that match current period's $(t-1)$ observed network size. This obviously simplifies the analysis, but constrains consumer behaviour a great deal.

Typically, consumers are assumed heterogenous in terms of the intrinsic utility, but goods are homogenous across firms, $x_i^s = x_i$ for all s. On the other hand, the network externality is the same for all consumers and across firms with identically sized networks, thus $f_k(\mathbb{E}_i(n_k)) = f(\mathbb{E}(n_k))$ for all i. De Palma & Leruth (1996) consider the polar case where consumers are heterogenous in terms of the network externality. Their utility function looks like

$$u(x_i, p_s, \mathbb{E}(n_k)) = x_i f(\mathbb{E}(n_k)) - p_s, \tag{2}$$

where high consumer types (given by a high value of x_i) get more utility from the network than low types.

For incompatible goods, networks are firm-specific, $n_k = n_s$. Compatible goods form one network, $n_k = \sum_s n_s$. Compatibility can be industry-wide or partial. Compatibility may also be imperfect, $n_k = \sum_s \alpha_s n_s$, $\alpha_s \in]0, 1[$.

The externality function approach has paid much attention to the compatibility issue. Starting from Katz & Shapiro (1985), there is a lot of research on the role of network compatibility on oligopolistic competition (Economides & White 1994, Bental & Spiegel 1995, De Palma & Leruth 1996, and Baake & Boom 2001). Shy (2001) generalises that "compatibility is anticompetitive". His result is founded on the absence of market share competition under compatibility. To put it differently, perfect compatibility eliminates the strategic value of firm-specific networks. Consumers do not differentiate between firm-specific networks, but can differentiate between goods in terms of brand or product quality. Albeit dominant, Shy's (2001) result is not universal. In asymmetric industries, the general result is that the larger firm is against compatibility whereas smaller rivals prefer compatibility.

2.3 Industry-specific models

Many industries derive their characteristics from an underlying physical network. These industries include transportation (air, marine, and public city transportation, railroads, and highways), electricity, gas, telecommunications and the Internet, each comprising a substantial share of any economy. Often it is efficient to build the network so that there are bottlenecks which give the owner monopoly power. For example, telecommunication networks consist of a trunk network that links many local access networks which connect to customers in turn. Airline operators design flight routes as hub-and-spoke networks. Many of the network-based industries have a history of extensive government intervention. Economic research has focused on the industry performance, in particular on the efficiency gains that can be obtained through privatisation, regulation, and injection of competition. Even if demand side externalities are important in many network-based industries, the efficiency problems arising from the physical structure dominate the literature. Telecommunications is an example of an industry in which both the physical and the social network structure matter, but in which economic analysis has focused on regulation of the physical network. I will overview the telecommunications literature because I use (mobile) telecommunications as a real world example in the essays of this thesis.

Armstrong (1998) - Laffont et al. (1998a, 1998b) set-up is the principal framework that has been employed to study telecommunications. Laffont & Tirole (2000) and Mason & Valletti (2001) are surveys of the literature. Competition and regulation in this framework culminate on the access price for interconnection traffic. It turns out that the access price has a highly strategic value as it can be used as a device for tacit collusion. With linear pricing, the access price creates a "raise-each-other's-costs" mechanism. Telecoms operators' profits increase in the access price because higher access prices are transferred to higher retail prices. This result has been conditioned by subsequent work. For example, with two-part tariffs, the profit effect is neutralised. Higher retail prices are fully compensated by lower fixed subscription fees. The Internet, a practically inseparable industry from modern telecommunications, is a similarly interesting network as it consists of an underlying physical network and of users' personal social networks. Economic research thus far has shown limited interest in the social dimension of the Internet, as it has focused on the pricing of the backbone access (see Crémer at al. 2000, Cave & Mason 2001, and Laffont et al. 2003).

What is typical to the telecommunications models is that personal social networks are abstracted away as much as possible. Consumers have a need to call everyone, giving grounds for the so called "neutral calling pattern". This assumption is not entirely satisfactory. Making a phone call is always a social event, and people's true calling patterns tend to be highly asymmetric. Neutral call flows are disturbed if people have preferences to call, for example, mostly their family, friends, and their closest colleagues. The failure of the neutral calling pattern is indirectly verifiable from the mobile operator price plans that give reductions to calls to predetermined numbers.[3]

Because call neutrality is a critical assumption in telecommunications models, the results are potentially distorted. The most recent telecommunications research, especially Jeon et al. (2004) and Cambini & Valletti (2004), incorporate partly the social dimension of a phone call: both the caller and the receiver get utility. However, neither paper takes into account the crucial point that people may have different number of contacts and that some contacts are more important than others. Dessein (2003) proposes a model where consumers are heterogenous with respect to

[3] See e.g. the price plans "Nära & Kära Dygnet Runt" by Comviq in Sweden, "Kotisoitto" by Sonera or "Heimopalvelu" by Elisa in Finland, where calls to a small number of predetermined telephone numbers are cheaper than other calls.

call and subscription demands. The heterogeneity, however, is limited to demand parameters and does not penetrate the underlying social network. With an elegant probabilistic approach, Dessein (2003) eliminates the neutral calling pattern, but still maintains, implicitly, the assumption on a completely connected society.

The economics of telecommunications has lagged behind the industry's technological progress. The models put heavy weight on the physical networks, but technological progress has diluted the monopoly power given by asset ownership. For example, the fixed local area access network as well as the trunk network have lost value due to the uptake of improved data-transmission technology, mobile telephony, and Internet-based communication. Effectively, demand-side factors, in particular social relations that characterise people's interaction patterns, have a larger role in modern telecommunications than economic models assume.

The dilution of the physical bottlenecks does not necessarily imply that the industry becomes more competitive. In many industries, immaterial bottlenecks induce monopoly power. For example, contractual settlements such as payment card associations form bottlenecks that command monopoly power (Rochet & Tirole 2002).[4]

2.4 Models of social relations

The conventional network model in line of Farrell & Saloner (1985), Katz & Shapiro (1985) or Arthur (1989) assumes implicitly that each network member is connected to everyone else on the network. The links between players form a complete graph. Hence, an additional member increases the utility of everyone on the network. To put it differently, the value of the network follows so called Metcalfe's Law, which states that the value of the network increases approximately in the square of the number of nodes in the network.[5] The value of the network with n members is thus $n(n-1) \approx n^2$ for large n, when the benefit of each link is normalised to one "util".

The models of social networks differ from the conventional network models in that it matters who is connected to whom. Consequently, the mapping of agents' relations curb the value of

[4] Rochet & Tirole (2002) belongs to a rapidly growing literature on two-sided markets. On two-sided markets, see Armstrong (2004) and Rochet & Tirole (2003).

[5] See Mason & Valletti (2001) and Cave & Mason (2001) for dicussion on Metcalfe's Law and other characteristics of economic networks.

the network - Metcalfe's Law does not hold. Not many real networks can be characterised as complete graphs. Physical networks often have few hubs that link to many peripheral nodes which are not directly connected to each other. The complete graph is an equally poor measure of social relations. The departure from Metcalfe's Law in social networks is based on two critical observations. One, social relations are a result of long-term repetitive interaction. An existing friendship necessitates that the friends have met previously. Two, people have varying number of relations, and some relations are more important than others. No-one knows all the people in the society. In some cultures, family is the key reference group because it provides social security, whereas contacts outside family are more shallow. In other cultures, the most important social contacts are outside family, for example friendships. In general, people's social lives are characterised by close connections with a small group of people and sporadic encounters with other people. Why is this heterogeneity important? The reason is that the predictions of social interaction models differ significantly from those given by conventional models, because existing social relations are constraints to economic behaviour.

It is obvious that we are surrounded by innumerable networks with different sizes and topologies. The size factor has been thoroughly examined by the conventional models of networks, and it culminates on the Metcalfe's Law of network value. The topological factor capturing the heterogeneity in agents' links has only recently attracted interest in economics. In mathematics and physics, the topological effect has been acknowledged early. Albert & Barabási (2002) survey the development of the theory of random graphs and scale-free networks. In addition, sociology has preceded economics in its interest in asymmetric social networks. Particularly, the work on weak versus strong links by Granovetter (1973) did not find its way to economics quickly. Chwe (2000) and Morris (2000) are the first ones applying Granovetter's (1973) ideas.

There are two classes of social network models. One class is interested in how networks are formed in interaction between strategic non-atomistic players. The second class takes the network structure as given and studies economic phenomena that are constrained by the network structure. Models in my thesis belong to the latter class. However, it is worth a digression on the network formation models, before discussing exogenous social networks in detail.

Jackson (2003) is a survey of network formation games. Network formation games analyse how networks emerge from link formation decisions taken by strategic players. Forming a link is associated with a payoff, gross utility minus the cost of forming a link. Agents form links with other agents creating local neighbourhoods. The game becomes more interesting when players benefit from the links between their neighbours' neighbours. The question is, which network forms hold in equilibrium? The answer depends on the payoff structure, but in general the equilibrium network does not have to be a complete graph (Bala & Goyal 2000, Jackson 2003).

When is the network endogenous and when exogenous? The differentiating factor is whether the network arises from the particular problem analysed in the model or is formed prior to the model. In the latter case, the network is exogenous and creates a constraint. For example, a friendship means that a link has been formed at some time in history. This friendship has been formed before the friends make decisions on whether or not they should buy mobile phones in order to call each other. Social relations result from many different aspects (are we related, do we like to talk about football, do we like economics, or do we live in the same neighbourhood, etc.) and take time to develop. Most people are likely to think about with whom of their existing contacts they will interact when buying a mobile phone. Fewer people think the other way round: "How many new social contacts will I make if I buy the phone?" So, an exogenous network implies that relations are inherited from outside the model. Personal social relations fall predominantly in this category.

Endogenous network formation characterises better inter-firm relations. Companies' actions can always be measured in pure monetary terms. Consequently, the decision to form a link has consequences that can be directly evaluated with other actions. With personal social relations payoff comparisons are more complicated. In fact, for exogenous company networks we need to come up with a different motivation for the network, such as Japanese style keiretsu affiliations or R&D networks that are organised by the government.

When the link yields benefits only to the initiator, the network is directed. The polar case is when both the initiator and the receiver get benefits. In this case the network is undirected. A link's existence can require mutual consent, as in a friendship. In some networks, a link is possible to establish without an explicit agreement from the receiver. The World Wide Web is an example of that kind of network. In all, how networks are formed and sustained produces a large variety of choice. The correct mixture of features always depends on the real world application in question.

Local neighbourhood interaction means that network members may have very asymmetric equilibrium strategies. The interesting question is how a (strategically behaving) agent can take advantage of his network position? An agent, whose links are important from all network members' point of view, is able to capture higher surplus. The distribution of such critical agents affects the equilibrium structure of the network. One important equilibrium statistic is network's attack tolerance: How a network survives random, or targeted, removal of agents? Ballester et al. (2004) analyse a "key player removal" policy to fight crime. They analyse which criminals (i.e. which network positions) should be eliminated to result in maximal reduction in crime. Albert & Barabási (2002) compare attack tolerance of scale-free and random graphs. Scale-free networks are tolerant against random eliminations of nodes, but very vulnerable against a planned attack.

Schelling's (1969) model of neighbourhood segregation is an early work on social interaction. Today, models of social relations can be found in diverse fields of economics. The most active field has been labour economics. Social relations have been identified as an alternative, informal, job search channel. Calvó-Armengol & Jackson (2004) and Bramoullé & Saint-Paul (2004) analyse the interdependence between social relations and unemployment. Bentolila et al. (2004) and Labini (2004) compare wage differentials between employees who find their jobs through formal and informal channels. Ioannides & Loury (2004) cover the literature on social relations in job market transactions. Another very active field is development economics. Udry & Conley (2004) and Goldstein et al. (2002) study social networks in Ghanaian villages. Gaduh (2002) surveys work on social learning in village economies. Other work on social relations are Bramoullé & Kranton (2004) who study public good provision, particularly innovation and experimentation. Glaeser et al. (1996) and Ballester et al. (2004) study how crime rates are affected by social networks. Chwe (2000) analyses how the diffusion speed of political action depends on the social structure. Kranton & Minehart (2001) model market exchange between firms as networks. Sundararajan (2005) studies product adoption in random graphs. Goyal et al. (2003) present a model of R&D

where different levels of R&D collaboration are mapped as networks. Both Kranton & Minehart (2001) and Goyal et al. (2003) consider endogenous networks, which is in line with our earlier note that models with firms as decision makers are more suitable for endogenous link formation.

The models of social relations are related to the literature on local interaction (on local interaction see Ellison 1993, Young 1998: 6, Lee & Valentinyi 2000, Morris 2000). The unifying component is that the underlying interaction network is exogenous, and players are interested in interacting with a sub-set of the total population. Local interaction models are evolutionary in spirit, thus dynamic, and improve the research line set out by Arthur (1989). Local interaction models study how a specific coordination equilibrium (out of multiple) is selected in the long run. The prediction is that it is the risk dominant strategy which is played in the long run (almost surely). The differentiating factor between models of local interaction and social relations is that players are boundedly rational in local interaction models. Young (1998) explains how social norms and institutions emerge in situations where players have an interest to coordinate their actions. Repeated social interaction results in equilibrium dynamics where the game stays mostly in (the risk dominant) equilibrium, called the convention, but from time to time transits from one equilibrium to another. As a part of his analysis, Young (1998: 6) leverages the local interaction framework.

3 Essays

Three problems, three essays. What are the problems in network economics that I deal with? First, multiplicity of equilibria that makes the analysis of network models especially difficult. Second, network asymmetry that falsifies the implicit assumption on a complete graph structure. Third, the uncharted area of the merger of technological and network externalities.

3.1 Equilibrium uniqueness

Typical to coordination games, the technology adoption game Γ (with simultaneous moves), or the membership model with a payoff function (1) has multiple equilibria. The source of multiplicity is the combination of perfect information and homogenous agents (Herrendorf et al. 2000, Mason & Valentinyi 2003). Under these factors, the equilibrium strategy is the bandwagon strategy. Since it is optimal not to switch if the other player does not switch, (abstain,abstain) is an equilibrium. However, if the other player switches, then the best response is to switch.

Multiplicity of equilibria is a theoretically and methodologically interesting problem. Multiplicity is a problem because it complicates equilibrium analysis. In the worst case, multiplicity renders predictions on equilibrium behaviour indeterminate, because the agents' behaviour is usually equilibrium-specific. A forward-looking firm can always guarantee a unique equilibrium (in the consumers' coordination subgame), but the price it must charge in that case may not be an equilibrium price.

A solution to multiplicity is, in its simplest form, just to concentrate on the most interesting equilibrium. This means that the most interesting equilibrium is assumed to be focal on whatever grounds. I dub this the "inshallah approach". Players do what they would do in the desired equilibrium, but do not take any effort to influence the probability that the desired equilibrium emerges. Hence, we do not learn much about the players' behaviour.

A more sophisticated solution is to construct a coherent background that explains why a certain equilibrium becomes selected. Farrell & Katz (1998) discuss how exogenous behavioural cues determine which equilibrium emerges. For example, consumers have rational expectations that favour a financially strong firm, or expectations are in favour of the high quality firm. When consumers' expectations track one such factor, a unique equilibrium is selected. Farrell & Katz' (1998) approach provides compelling stories how an equilibrium is selected. The method, however, is not completely satisfactory. Why? In order to select the equilibrium, the analysis needs to resort to exogenous factors. How a particular expectational cue is formed is not explained, nor can the firms affect its formation. Principally, the emergence of a particular equilibrium is due an assumption. But then, the same assumption could be made on whatever imaginary grounds. At the end, the difference between this and the inshallah approach is superficial.

The coordination game Γ is easily formulated as a supermodular game by imposing increasing differences in the players' types, meaning that a higher type gets a higher payoff gain from choosing "switch" compared to a lower type. Supermodularity is beautiful because it guarantees the existence of a Nash equilibrium. Furthermore, if actions are strategic complements, meaning that player 1's action is increasing in player 2's action and vice versa, Nash equilibria can be Pareto-ranked. Pareto-efficiency is a better, but insufficient way to focalise an equilibrium compared to exogenous factors.

Multiplicity in network models following the externality function approach has been dealt away case-by-case, but a comprehensive analysis is missing. What the literature suggests is that uniqueness results under perfect information only if network externalities are sufficiently weak (Economides 1996b, Cabral et al. 1999 and Baake & Boom 2001). This of course is perverse. When the object of study are network effects, in order to obtain a unique equilibrium, the strength of the externalities has to be limited. This can be done by forcing high level of heterogeneity with respect to the intrinsic utility. This implies that the externalities are sufficiently limited when agents base their decisions primarily on other information than perceived network benefits.

Uniqueness is reached if agents' capacity to observe information is reduced. The global games theory shows that uniqueness can emerge endogenously in coordination games like Γ when agents' types are private but correlated information (Carlsson & van Damme 1993, Morris & Shin 2003). The global games framework provides an elegant way to obtain uniqueness in network models (in addition to my essay, see Argenziano 2004 and Farhi & Hagiu 2004). Under incomplete information, it suffices that there is a *possibility* that some agents play a strictly dominant action at the same time as some other agents play another strictly dominant action. The strength of network externalities is not bounded.

In the essay "Buying decision coordination and monopoly pricing of network goods" I give a comprehensive analysis on the requirements for equilibrium uniqueness under perfect and incomplete information.

3.1.1 Buying decision coordination and monopoly pricing of network goods

I analyse a monopoly market for social goods. The focus is on how equilibrium uniqueness is achievable and how uncertainty about the product's value affects the monopoly's pricing strategy. A product has a social (network) dimension when its use involves interaction between people. When a product enabling more efficient interaction is introduced in the market, consumers must coordinate their decisions on whether to switch to the more efficient medium or to stick with the legacy system. If two people want to interact with the help of the new device, they must both buy the product. This coordination problem is the source for network externalities. The device enables interaction usage (network benefits) and standalone usage (intrinsic utility). With perfect information and homogenous consumers, the coordination game produces multiple equilibria. Effectively, consumers play the bandwagon strategy in equilibrium. When the consumers are alike, it is impossible to predict if coordination is efficient or not. Efficiency requires that one of the consumers takes the initiative and buys, but without exogenous factors, equilibrium selection is indeterminate.

The game set-up is simple. There is a monopolist who sells a novel device that enables efficient interaction between people. In the first period of the game, consumers decide whether to buy the product or not. In the second period, those consumers who bought the product decide on the level of use. The monopolist sets a unit price in the first period, and a usage fee in the second period. If information is incomplete, consumers observe private signals of the intrinsic value of the product in the first period. Consumers know their preferences better than the firm which resorts to the prior distribution. All information is revealed in the second period, making it a standard deterministic monopoly pricing problem. The monopolist may be constrained in setting the usage fee in the second period if the first period sales are low.

Whether information is perfect or incomplete, the key to equilibrium uniqueness is player heterogeneity. There must be a group of consumers who buy as a strictly dominant strategy at the same time as another group of consumers do not buy as a strictly dominant strategy. However, the required level and nature of player heterogeneity are different under different regimes.

Multiplicity is removed by high consumer heterogeneity under perfect information. The inconvenient feature about perfect information is that consumer heterogeneity must be real, high, and independent of network benefits. Heterogeneity has to be real in the sense of sufficiently broad bandwidth of the consumer distribution. Importantly, heterogeneity must be related to the intrinsic utility making the network benefits relatively weak. One group of consumers must get relatively high utility from the standalone services, at the same time as another group relatively dislikes the standalone services. In other words, consumers must base their decisions predominantly on other features than interaction usage.

By imposing incomplete information, the conditions for consumer heterogeneity are relaxed, primarily due to a change in the nature of required heterogeneity. Under incomplete information, consumers observe private signals of the underlying "true" intrinsic value of the product. I consider a case of private values, as the signals enter directly the consumers' utility function. Hence, the truthfulness of the underlying fundamental is metaphorical. Since the private signals are correlated, I can resort to global games techniques. Real heterogeneity between consumers can be minimal, but uniqueness presupposes a *possibility* that some people have very high or low intrinsic valuations. It is in this sense that the conditions for uniqueness are less strict under incomplete information. The relative strength of network effects does not have to be limited. Incomplete information fixes the expected value of network benefits, which eventually yields straightforward equilibrium buying behaviour and pricing strategy.

Thanks to equilibrium uniqueness, equilibrium analysis gives a well-behaving demand function and determinate predictions. The optimal unit price is higher under incomplete information, because the monopoly has a bias in favour of the first period sales over the second period usage fees. The incomplete information regime yields more straightforward comparative statics on profits and consumer surplus. Profits are decreasing in uncertainty (i.e. in consumer heterogeneity). Consumer surplus increases in uncertainty, only if the level of uncertainty is already high.

3.2 Social relations

If the multiplicity of equilibria is chiefly a theoretical and methodological problem, the second issue I raise concerns very much the foundations of the economic theory of networks. The conventional network model captures the size effect but overlooks the effect network's topology has on equilibrium behaviour.

The network size factor operates as demand side economies of scale, but how does the topological factor work? The complete graph (i.e. total connectedness) maximises the value of the network for a given number of members. Whenever the topology is something less connected, the

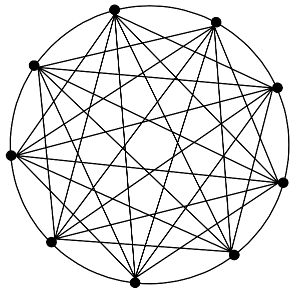

Figure 1: The Complete Graph.

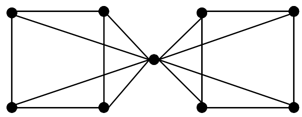

Figure 2: Centre and Islands with Four Consumers.

value of the network is lower. This result is best illustrated with an example.

Example 1 *Consider a complete graph with nine consumers illustrated in Figure* (1). *There are 72 directed links in total. This is the network structure of a conventional model with an implicit assumption on total connectedness. Adding a tenth consumer would increase the number of links (thus the value of the network when one link equals to one "util") by 18.*

What if some people are not connected with everyone else in the society? Let us assume that most people have only three friends instead of eight. In addition, let us assume that there is one person who socialises with everyone. The nine consumers are arranged into two islands and a central agent, as in Figure (2). *The size of the network drops to 32 directed links. Applying Metcalfe's Law in this network would overestimate the value of the network by 2.25 times.*

We could increase the total number of links in the network to match 72 so that the size of the network is controlled. If everyone except the centre has three links and the centre links to everyone else, the number of consumers in one island must equal to nine. We get the compensated network illustrated in Figure (3).

When the topology of the network is changed from a complete graph to centre+islands, the value

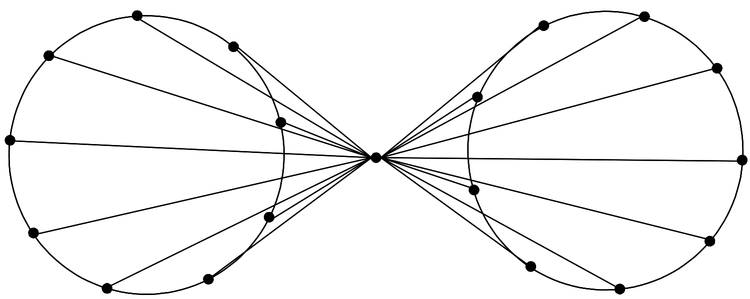

Figure 3: Centre and Compensated Islands with Nine Consumers.

of the network is reduced significantly (by 56%). Subsequently. in order to maintain the network's value, the number of network members must be increased by 10 (over double).

It is evident from Example 1 that whenever social relations are asymmetric, assuming a complete graph would overestimate the value generated in the network (with a given number of members). It is quite surprising that this, almost trivial fact has been neglected in the network economics until the arrival of the second wave models of network formation and social relations. Asymmetry in network topology also induces distributional effects between more central and peripheral agents. In the essay "Monopoly pricing of social goods", I challenge the conventional model and show how a monopolist's equilibrium behaviour is affected by both the size and topological factors.

3.2.1 Monopoly pricing of social goods

The aim of the paper is two-fold. On the one hand, I show how the conventional approach to networks exaggerates the value of the network in cases where consumers have asymmetric numbers of relations. On the other hand, I introduce two novel features to social relations literature. One, I make players' payoffs endogenous by setting a monopoly pricing problem on top of a coordination game. Two, I abandon the perfect information assumption and limit players' capacity to observe prevailing information. Perfect information makes the game unnecessarily complicated, but it provides a useful benchmark.

I analyse monopoly pricing of a social good in a market where consumers are characterised by their social relations. Consumers get utility from interacting with other people with whom they have a social relation. The mapping of the personal (local) social relations creates the (global) social network. I analyse three different social network structures, where a circle and a star are compared against the complete graph. The monopolist sells a device that enables efficient interaction and consequently, consumers need to coordinate their buying decisions, which is the source for network externalities. The device lacks any intrinsic utility.

I identify the buyer-types that have preferred positions in the networks. These types capture higher surplus than other network members. A consumer has a critical position, when his relations are important from all network members' point of view. A consumer located in a central position with high number of links is a topologically critical agent. Under perfect information, a network member can hold a critical position also due to important neighbours even in symmetric networks. He must be of low type and his neighbours high types. Hence, consumer's type matters. If consumers are very heterogenous, the monopolist wants to set a high price in order to capture the rents from the highest types. However, if the high types have only low types as neighbours, the monopolist is constrained to sell at a lower price. It must guarantee both high and low types' participation, otherwise no-one buys. This kind of agent identity is eliminated when consumers' valuations are private information. Hence, asymmetric information eliminates much of the inherent complexity prevailing under perfect information. In contrast, topologically focal positions, such as the centre in a star network, grow in importance. The monopolist always takes into account network asymmetry.

I analyse the roles of network topology and size on the monopoly price and surplus generated in the network. In markets where social relations are important, network externalities are easily exaggerated leading to distorted predictions about the monopoly price. This happens when the underlying social network is assumed as a completely connected graph, when in reality it is something less connected. I show that the topological effect works against, and dominates, the size effect.

Under asymmetric information, the monopoly prefers symmetric networks, but the social optimum is an asymmetric network. If the firm is allowed to price discriminate with respect to network location, its profits increase to the same level that it obtains in symmetric networks. Monopoly rents and consumer surplus decrease as consumer heterogeneity is increased. This does not necessarily happen under perfect information, as it depends on the network topology.

It is a popular claim that under positive network externalities, a firm should go after high volumes early, even at the cost of initial profitability. A large initial customer base would be a strategic asset against competition. Such a strategy is questioned in a set-up with asymmetric network connections. The dominant topological effect underlines the importance to enroll critical players early, rather than a random large customer base.

3.3 Networks and strategic research and development

Since the original papers by Arthur (1989), David (1985) and Farrell & Saloner (1985), technological progress has been a part of the network literature. The role of network effects in technology standards battles manifests in David's (1985) QWERTY case, even if it is disputed by Liebowitz & Margolis (1994). Farrell & Saloner (1985) provide a more rigorous analysis on adoption of new technology when the technology presents network externalities. Besen & Farrell (1994) discuss situations when firms have incentives to pursue proprietary technology standards, and the opposite cases when firms want to cooperate in setting a common industry standard. But Besen & Farrell (1994) is not a complete analysis of incentives to develop proprietary technologies. The appropriation of research and development (R&D) is often imperfect as the benefits from investments in technology spill over to rivals.

There is a large literature on imperfect appropriation of strategic R&D investments (see Spence 1984, d'Aspremont & Jacquemin 1988, 1990, Levin & Reiss 1988, Cohen & Levinthal 1989, Henriques 1990, Kamien et al. 1992, Suzumura 1992, Suzumura & Yanagawa 1993, Kesteloot & De Bondt 1993, De Bondt & Henriques 1995, Katsoulacos & Ulph 1998, Kultti & Takalo 1998, and Amir 2000). De Bondt (1997) is a survey of the literature on imperfect appropriability of R&D. Although notorious for being parameter-specific, strategic R&D models produce rather general results. R&D spillovers induce two effects: a competitive effect and a market expansion effect. The competitive effect is the diminished strategic effectiveness of R&D investments. Information leakage to rivals lowers the incentives to perform R&D. The market expansion effect, in contrast, stimulates R&D, because larger spillovers can result in higher synergies in R&D production and the whole industry operates at a lower cost level. Whether the competitive effect dominates the market expansion effect, or vice versa, depends on the model set-up and parameters. In strategic R&D games, it is the competitive effect that dominates. Firms have tendency to cut back investments when spillovers are marginally increased in equilibrium.

The intertwining of networks and technological progress calls for unified analysis on network externalities and R&D spillovers. I incorporate both externalities in the essay "Strategic R&D

and network compatibility". The result from the interplay between the externalities is more than the sum of parts. Strong network effects enforce firm behaviour that differs from the standard results.

3.3.1 Strategic R&D and network compatibility

In this essay, I analyse the effects of network externalities in a strategic R&D competition. I present a model of two firms competing with R&D investments and prices in a differentiated market. Firms choose the level of strategic R&D in the first period, and set prices in the second period. R&D is imperfectly appropriable. I focus on the competitive effect of R&D spillovers by controlling the size of the market. The good yields network benefits, which can be curbed by incompatibility. Despite a fixed number of buyers, market expansion is simulated when network compatibility is improved.

The network externality component in firms' strategies cancels out in symmetric situations due to a fixed market size. Firms only consider the level of spillovers in their R&D strategies and invest less the higher the spillovers are. Subsequently, price competition is less intense with high spillovers. Similarly, in asymmetric settings in terms of intrinsic product quality, a high degree of compatibility and large spillovers moderate price competition due to weak strategic value of firm-specific networks and R&D investments respectively.

The model yields interesting novel results when the set-up is asymmetric and R&D and firm-specific network size have high strategic value. The general result that firm-specific incentives for strategic R&D reduce as appropriability conditions are worsened can fail. Likewise, the more general, albeit not universal, result that higher network compatibility level tempers price competition can fail. The lower quality firm increases R&D and decreases its price as appropriability conditions are worsened or as compatibility is increased. This happens when both R&D spillovers and network compatibility are low simultaneously. The new results are due to network effects that dominate in firms' strategies when firms are asymmetric.

4 References

Albert, R. & Barabási, A.-L. (2002) "Statistical Mechanics of Complex Networks", **Reviews of Modern Physics, 74(1)**, 47-97.

Amir, R. (2000) "Modelling Imperfectly Appropriable R&D via Spillovers", **International Journal of Industrial Organization, 18(7)**, 1013-1032.

Argenziano, R. (2004) "Differentiated Networks, Equilibrium and Efficiency", mimeo.

Armstrong, M. (1998) "Network Interconnection in Telecommunications", **Economic Journal, 108(448)**, 545-564.

Armstrong, M. (2004) "Competition in Two-Sided Markets", mimeo.

Arthur, W.B. (1989) "Competing Technologies, Increasing Returns, and Lock-In by Historical Events", **Economic Journal, 99(394)**, 116-131.

Baake, P. & Boom, A. (2001) "Vertical Product Differentiation, Network Externalities, and Compatibility Decisions", **International Journal of Industrial Organization, 19(1-2)**, 267-284.

Bala, V. & Goyal, S. (2000) "A Noncooperative Model of Network Formation", **Econometrica, 68(5)**, 1181-1229.

Ballester, C. & Calvó-Armengol, A. & Zenou, Y. (2004) "Who's Who in Crime Networks: Wanted - The Key Player", **CEPR Discussion Paper 4421.**

Bental, B. & Spiegel, M. (1995) "Network Competition, Product Quality, and Market Coverage in the Presence of Network Externalities", **Journal of Industrial Economics, 43(2)**, 197-208.

Bentolila, S. & Michelacci, C. & Suarez, J. (2004) "Social Contacts and Occupational Choice", mimeo.

Besen, S.M. & Farrell, J. (1994) "Choosing How to Compete: Strategies and Tactics in Standardization", **Journal of Economic Perspectives, 8(2)**, 117-131.

Bramoullé, Y. & Kranton, R. (2004) "Public Goods in Networks: How Networks Can Shape Social Learning and Innovation", mimeo.

Bramoullé, Y. & Saint-Paul, G. (2004) "Social Networks and Labour Market Transitions", **IZA Discussion Paper 1215.**

Bulow, J.I. & Geanakoplos, J.D. & Klemperer, P.D. (1985) "Multimarket Oligopoly: Strategic Substitutes and Complements", **Journal of Political Economy, 93(3)**, 488-511.

Cabral, L.M.B. & Salant, D.J. & Woroch, G.A. (1999) "Monopoly Pricing with Network Externalities", **International Journal of Industrial Organization, 17(2)**, 199-214.

Calvó-Armengol, A. & Jackson, M.O. (2004) "The Effects of Social Networks on Employment and Inequality", **American Economic Review, 94(3)**, 426-454.

Cambini, C. & Valletti, T.M. (2004) "Information Creation and Competition in Telecommunications Networks", mimeo.

Carlsson, H. & van Damme, E. (1993) "Global Games and Equilibrium Selection", **Econometrica, 61(5)**, 989-1018.

Cave, M. & Mason, R. (2001) "The Economics and Regulation of the Internet", **Oxford Review of Economic Policy, 17(2)**, 188-201.

Chwe, M.S.-Y. (2000) "Communication and Coordination in Social Networks", **Review of Economic Studies, 67(1)**, 1-16.

Cohen, W.M. & Levinthal, D.A. (1989) "Innovation and Learning: The Two Faces of R&D", **Economic Journal, 99(397)**, 569-596.

Crémer, J. & Rey, P. & Tirole, J. (2000) "Connectivity in the Commercial Internet", **Journal of Industrial Economics, 48(4)**, 433-472.

D'Aspremont, C. & Jacquemin, A. (1988) "Cooperative and Noncooperative R&D in Duopoly with Spillovers", **American Economic Review, 78(5)**, 1133-1137.

D'Aspremont, C. & Jacquemin, A. (1990) "Cooperative and Noncooperative R&D in Duopoly with Spillovers: Erratum", **American Economic Review, 80(3)**, 641-642.

David, P.A. (1985) "Clio and the Economics of QWERTY", **American Economic Review, 75(2)**, 332-337.

De Bondt, R. (1997) "Spillovers and Innovative Activities", **International Journal of Industrial Organization, 15(1)**, 1-28.

De Bondt, R. & Henriques, I. (1995) "Strategic Investment with Asymmetric Spillovers", **Canadian Journal of Economics, 28(3)**, 656-674.

De Palma, A. & Leruth, L. (1996) "Variable Willingness to Pay for Network Externalities with Strategic Standardization Decisions", **European Journal of Political Economy, 12(2)**, 235-251.

Dessein, W. (2003) "Network Competition in Nonlinear Pricing", **RAND Journal of Economics, 34(4),** 593-611.

Economides, N. (1996a) "The Economics of Networks", **International Journal of Industrial Organization, 14(6)**, 673-699.

Economides, N. (1996b) "Network Externalities, Complementarities, and Invitations to Enter", **European Journal of Political Economy, 12(2)**, 211-233.

Economides, N. & White, L.J. (1994) "Networks and Compatibility: Implications for Antitrust", **European Economic Review, 38(3-4)**, 651-662.

Ellison, G. (1993) "Learning, Local Interaction, and Coordination", **Econometrica, 61(5)**, 1047-1071.

Farhi, E. & Hagiu, A. (2004) "Global Games and Networks", mimeo.

Farrell, J. & Katz, M.L. (1998) "The Effects of Antitrust and Intellectual Property Law on Compatibility and Innovation", **Antitrust Bulletin, Fall-Winter,** 609-650.

Farrell, J. & Saloner, G. (1985) "Standardization, Compatibility, and Innovation", **RAND Journal of Economics, 16(1)**, 70-83.

Farrell, J. & Saloner, G. (1986) "Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation", **American Economic Review, 76(5)**, 940-955.

Gaduh, A.B. (2002) "Information and Social Networks in Village Economies", **CSIS Working Paper WPE 063**.

Gandal, N. (2002) "Compatibility, Standardization, and Network Effects: Some Policy Implications", **Oxford Review of Economic Policy, 18(1)**, 80-91.

Glaeser, E.L. & Sacerdote, B. & Scheinkman, J.A. (1996) "Crime and Social Interactions", **Quarterly Journal of Economics, 111(2)**, 507-548.

Goldstein, M. & de Janvry, A. & Sadoulet, E. (2002) "Is a Friend in Need a Friend Indeed? Inclusion and Exclusion in Mutual Insurance Networks in Southern Ghana", mimeo.

Goyal, S. & Konovalov, A. & Moraga-González, J.L. (2003) "Hybrid R&D", **Tinbergen Institute Discussion Paper TI2003-041/1.**

Granovetter, M.S. (1973) "The Strength of Weak Ties", **American Journal of Sociology, 78(6)**, 1360-1380.

Henriques, I. (1990) "Cooperative and Noncooperative R&D in Duopoly with Spillovers: Comment", **American Economic Review, 80(3)**, 638-640.

Herrendorf, B. & Valentinyi, Á. & Waldman, R. (2000) "Ruling Out Multiplicity and Indeterminancy: The Role of Heterogeneity", **Review of Economic Studies, 67(2)**, 295-307.

Ioannides, Y.M. & Loury, L.D. (2004) "Job Information Networks, Neighborhood Effects, and Inequality", **Journal of Economic Literature, 42(4)**, 1056-1093.

Jackson, M.O. (2003) "A Survey of Models of Network Formation: Stability and Efficiency", forthcoming in **Group Formation in Economics: Networks, Clubs, and Coalitions**, Eds. Demange, G. & Wooders, M., Cambridge University Press, Cambridge United Kingdom.

Jeon, D.-H. & Laffont, J.-J. & Tirole, J. (2004) "On the “Receiver Pays” Principle", **RAND Journal of Economics, 35(1)**, 85-110.

Kamien, M.I. & Müller, E. & Zang, I. (1992) "Research Joint Ventures and R&D Cartels", **American Economic Review, 82(5)**, 1293-1306.

Katsoulacos, Y. & Ulph, D. (1998) "Endogenous Spillovers and the Performance of Research Joint Ventures", **Journal of Industrial Economics, 46(3)**, 333-357.

Katz, M.L. & Shapiro, C. (1985) "Network Externalities, Competition, and Compatibility", **American Economic Review, 75(3)**, 424-440.

Katz, M.L. & Shapiro, C. (1994) "Systems Competition and Network Effects", **Journal of Economic Perspectives, 8(2)**, 93-115.

Kesteloot, K. & De Bondt, R. (1993) "Demand-Creating R&D in a Symmetric Oligopoly", **Economics of Innovation and New Technology, 2(3)**, 171-183.

Kranton, R.E. & Minehart, D.F. (2001) "A Theory of Buyer-Seller Networks", **American Economic Review, 91(3)**, 485-508.

Kultti, K. & Takalo, T. (1998) "R&D Spillovers and Information Exchange", **Economics Letters, 61(1),** 121-123.

Labini, M.S. (2004) "Social Networks and Wages: It Is All about Connections!", **LEM Working Paper 2004/10.**

Laffont, J.-J. & Marcus, S. & Rey, P. & Tirole, J. (2003) "Internet Interconnection and the Off-Net-Cost Pricing Principle", **RAND Journal of Economics, 34(2)**, 370-390.

Laffont, J.-J. & Rey, P. & Tirole, J. (1998a) "Network Competition: I. Overview and Nondiscriminatory Pricing", **RAND Journal of Economics, 29(1)**, 1-37.

Laffont, J.-J. & Rey, P. & Tirole, J. (1998b) "Network Competition: II. Price Discrimination", **RAND Journal of Economics, 29(1)**, 38-56.

Laffont, J.-J. & Tirole, J. (2000) **Competition in Telecommunications**, The MIT Press, Cambridge Massachusetts.

Lee, I.H. & Valentinyi, Á. (2000) "Noisy Contagion without Mutation", **Review of Economic Studies, 67(1)**, 47-56.

Levin, R.C. & Reiss, P.C. (1988) "Cost-Reducing and Demand-Creating R&D with Spillovers", **RAND Journal of Economics, 19(4)**, 538-556.

Liebowitz, S.J. & Margolis, S.E. (1994) "Network Externality: An Uncommon Tragedy", **Journal of Economic Perspectives, 8(2)**, 133-150.

Mas-Colell, A. & Whinston, M.D. & Green, J.R. (1995) **Microeconomic Theory**, Oxford University Press, New York New York.

Mason, R. (2000) "Network Externalities and the Coase Conjecture", **European Economic Review, 44(10)**, 1981-1992.

Mason, R. & Valentinyi, Á. (2003) "Independence and Heterogeneity in Games of Incomplete Information", **University of Southampton Discussion Papers in Economics and Econometrics 0307.**

Mason, R. & Valletti, T.M. (2001) "Competition in Communication Networks: Pricing and Regulation", **Oxford Review of Economic Policy, 17(3)**, 389-415.

Morris, S. (2000) "Contagion", **Review of Economic Studies, 67(1)**, 57-78.

Morris, S. & Shin, H.S. (2003) "Global Games: Theory and Applications", in **Advances in Economics and Econometrics, Theory and Applications, Eight World Congress,**

1, 56-114, Eds. Dewatripont, M. & Hansen, L.P. & Turnovsky, S.J., Cambridge University Press, Cambridge United Kingdom.

Rochet, J.-C. & Tirole, J. (2002) "Cooperation among Competitors: Some Economics of Payment Card Associations", **RAND Journal of Economics, 33(4)**, 549-570.

Rochet, J.-C. & Tirole, J. (2003) "Platform Competition in Two-Sided Markets", **Journal of the European Economic Association, 1(4)**, 990-1029.

Schelling, T.C. (1969) "Models of Segregation", **American Economic Review, 59(2)**, 488-493.

Schumpeter, J.A. (1975) **Capitalism, Socialism and Democracy**, HarperPerennial, New York New York. Originally published in 1942 by Harper & Brothers.

Shy, O. (2001) **The Economics of Network Industries**, Cambridge University Press, Cambridge United Kingdom.

Spence, M. (1984) "Cost Reduction, Competition, and Industry Performance", **Econometrica, 52(1)**, 101-122.

Sundararajan, A. (2005) "Local Network Effects and Network Structure", mimeo.

Suzumura, K. (1992) "Cooperative and Noncooperative R&D in an Oligopoly with Spillovers", **American Economic Review, 82(5)**, 1307-1320.

Suzumura, K. & Yanagawa, N. (1993) "Cooperative and Non-cooperative R&D in an Oligopoly with Spillovers: Strategic Substitutes versus Strategic Complements", **Hitotsubashi Journal of Economics, 34(1)**, 1-11.

Tirole, J. (1988) **The Theory of Industrial Organization**, 9th printing (1997), The MIT Press, Cambridge Massachusetts.

Udry, C.R. & Conley, T.G. (2004) "Social Networks in Ghana", **Center Discussion Paper No. 888.**

Young, H.P. (1998) **Individual Strategy and Social Structure, an Evolutionary Theory of Institutions**, Princeton University Press, Princeton New Jersey.

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Buying Decision Coordination and Monopoly Pricing of Network Goods*

Abstract

A product has a social (network) dimension when its use involves interaction between people. When a good enabling more efficient interaction is introduced in the market, consumers must coordinate their decisions over switching to the more efficient medium or sticking with the legacy system. This coordination problem has multiple equilibria under perfect information and homogenous consumers. We do a comprehensive analysis on the conditions for equilibrium uniqueness in a model of a monopoly market for social goods. The good gives intrinsic utility and utility from its usage in interaction. The monopoly sets a two-part tariff. Multiplicity is removed by high consumer heterogeneity with respect to intrinsic utility under perfect information. Consumer heterogeneity must be real, in the sense of sufficient bandwidth of the consumer distribution. With incomplete information we can use global games techniques to eliminate multiplicity. Real heterogeneity between consumers can be minimal, but uniqueness presupposes a possibility that some people have very high or low valuations. The unit price is higher under incomplete information because the monopoly biases its tariff structure to incorporate the uncertainty over usage revenues. Under incomplete information, profits are decreasing in uncertainty. Consumer surplus increases in uncertainty, only if the level of uncertainty is already high.

Keywords: Coordination, networks, information, equilibrium uniqueness, heterogeneity, global games.

JEL classification: D42, D82, L14.

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1 Introduction

It is a well-known fact that network externalities cause multiple equilibria. Indeterminacy arising from multiplicity of equilibria has been incorporated in the theory in the forms of de facto standards and bandwagon strategy profiles. The line of these models can be traced back to Arthur (1989), David (1985), Farrell and Saloner (1985, 1986), who study technology adoption, and Katz and Shapiro (1985) who look at brand competition and compatibility between networks. Those seminal papers and subsequent literature accepting indeterminacy as a characteristic of economic networks, suggests that market structures in network industries are determined by random exogenous events. In this paper, we argue that this is partial truth. Uniqueness of equilibrium follows endogenously in a broad range of heterogenous network models, eliminating the role of random events in equilibrium selection. Drawing from the theoretical work on coordination games, we unify and improve on the ad hoc solutions to the multiplicity problem employed in the network literature by showing how (i) under perfect information, the achievable intrinsic utility, which is independent of the network size, must be the dominant criterion in the consumer's buying decision in order to obtain uniqueness, (ii) under incomplete information, uniqueness is independent of the (relative) strength of externalities, but requires so called dominance regions of strictly dominant strategies.

We analyse a monopoly pricing problem of a network product. The utility from the product increases as the number of consumers who buy increases. A version of the model with rational and homogenous consumers produces multiple equilibria. Multiplicity causes the firm's pricing strategy to be equilibrium-specific; hence, in order to derive the optimal price, the analysis must focus on one equilibrium at a time. In addition, since the arguments that select the equilibrium are inevitably exogenous to the model, we do not learn much on firm behaviour at the end. By removing multiplicity in the model, we are able to study the role of heterogeneity of consumers and uncertainty on the optimal monopoly price, and the results are determinate.

Work on network externalities has tried to overcome the problem of multiplicity often by simply analysing only the most interesting equilibrium, by resorting to arguments that are exogenous to the model, or by restricting the strength of externalities. Farrell and Katz (1998) provide a compelling discussion about different behavioural cues which align consumers' expectations in favour of a particular equilibrium. They show how a particular equilibrium is selected when consumers' expectations track e.g. product quality. The problem of multiplicity is not solved truly satisfactorily as the motivation for the selection process is exogenous. Baake and Boom (2001) analyse a quality differentiated duopoly with heterogenous consumers with respect to intrinsic utility. They identify that the Nash equilibrium is unique only if consumers evaluate the competing products chiefly in terms of quality (opposed to perceived network sizes). Cabral et al. (1999) get a unique interior equilibrium by assuming that the discount factor and the parameter measuring network externalities are not "too large". Bental and Spiegel (1995) analyse only the most interesting non-zero equilibrium. De Palma and Leruth (1996) obtain a unique equilibrium in a duopoly model, where consumer heterogeneity is related to network externalities, by allowing the firms to commit to production levels. Economides (1996) studies an oligopoly market with network externalities. In line with general results, uniqueness of equilibrium in his model also hinges on the magnitude of network externalities. Equilibrium is unique (and interior) only if the externalities function is concave with the marginal externality sufficiently small.

In other cases, where network effects, per se, are not the primary subject of analysis, multiplicity is abstracted away by assuming covered markets and high level of product differentiation. This route has been successfully used in the analyses of network-based industries such as telecommunications networks in the line of Armstrong (1998) and Laffont et al. (1998a, 1998b).

A model with demand-side network externalities is essentially a coordination game where agents' actions are strategic complements à la Bulow et al. (1985). A coordination game with perfect information and homogenous players has multiple equilibria. However, a coordination game with increasing returns to scale and perfect information can have a unique equilibrium if players are sufficiently heterogenous (Herrendorf et al. 2000). The trade-off of high heterogeneity is that we must impose a set of conditions on the magnitude of network effects. An alternative route to uniqueness is to limit agents' capacity to observe information. Recent work on global games has developed a theory that provides an elegant way to achieve uniqueness in coordination games where homogeneity between agents and common knowledge result in multiple equilibria

(Carlsson and van Damme 1993, Morris and Shin 2003).[1]

Whether information is perfect or incomplete, the key to uniqueness is the same. Uniqueness follows when one group of people play one action as a strictly dominant strategy at the same time as another group of people play a different action also as a strictly dominant strategy. The surviving equilibrium is a switching strategy with a uniquely determined cut-off point. The advantage of global games is that we can allow unlimited (relative) network externalities; with perfect information externalities must be bounded. Under perfect information, heterogeneity must be "real" in the sense that the distribution of consumers must have sufficiently broad support. In global games, uniqueness requires only a *possibility* that some consumers obtain extremely low or high signals of the underlying fundamental. Real heterogeneity can be relatively small.

In this paper, we analyse monopoly pricing under demand-side network externalities. Many network goods consist of the product itself and its usage in interaction with other people. The usage utility has been traditionally modelled as an externality function of the equilibrium installed base. We derive network externalities from interaction usage. Since the interaction can take place only after people have acquired the goods, we separate the usage stage from the acquisition stage. The second stage describes explicitly how network externalities relate to interaction on a social network of relations. The monopolist sets a two-part tariff, and is not able to commit to a tariff structure in the first stage. A fax machine, mobile phone, or on-line game console are examples of the product we have in mind.

The consumer is able to interact with the new device only with those people who have also bought the device. Hence, there is a coordination problem between consumers: whether to switch to the new medium or to stick with the legacy system. We remedy the inherent multiple equilibria problem in two alternative ways. First, we solve the problem under perfect information. We show how network effects must be relatively low if uniqueness is to be reached. In other words, consumers' decision making has to be driven by non-network attributes. For a model of network effects this constraint is troublesome. Endogenous pricing is an insufficient mean to remedy the

[1] Mason and Valentinyi (2003) derive conditions for existence of unique equilibrium in a larger set of incomplete information games that includes global games. They show that unique equilibrium exists if the (conditional) heterogeneity and correlation between consumer types are sufficiently high. Their result does not depend on strategic complementarities or dominance regions that are essential in global games.

multiplicity problem. Effectively, when network externalities are relatively low the monopolist sets a price which guarantees a unique equilibrium, but for high network externalities pricing involves multiple equilibria. The perfect information regime turns out analytically complicated, as we need to keep track about various possible states of the world. Secondly, we limit consumers' capacity to observe information. Uniqueness is derived by using global games techniques. Global games are analytically more applicable to our model as there is no limitation to the relative strength of the externalities. We also argue that the case of incomplete information better characterises a launch of a new device, because people are not able to tell how much utility the device yields to other people. This informational asymmetry is aggravated the more drastic innovation the new device is.

The benefit of uniqueness is that analysis on firm behaviour becomes clear-cut. We derive the optimal two-part tariff structure for the monopoly, and analyse the effects of a marginal change in heterogeneity (uncertainty under incomplete information) on the equilibrium under uniform prior and posterior distributions. The optimal unit price is increasing in consumer heterogeneity under perfect information. If information is incomplete, the price is independent of uncertainty (heterogeneity).

Under incomplete information, the monopolist sets a higher price in the first period in order to incorporate the possibility of a wide perception of the low quality of its goods and subsequent low usage profits. On the other hand, there is no such bias under perfect information as the monopolist is able to perfectly neutralise the usage utility. We also discuss the possibility that the second period is characterised by perfect competition. In that case, the bias to increase the first period price is more aggravated, in a standard way.

The effect of a marginal change in heterogeneity on profits and consumer surplus is ambiguous under perfect information. On the other hand, the firm's expected profits increase as uncertainty is reduced under incomplete information. The effect on expected consumer surplus depends on the absolute level of uncertainty. The effect of a marginal change in uncertainty is positive if the change is aligned with the absolute level. That is, if uncertainty is high, then further uncertainty is of good. Similarly, the expected consumer surplus increases when there is little uncertainty and we further reduce uncertainty.

The global games approach has been successfully used in a number of macroeconomic and financial problems. Morris and Shin (1998) analyse a model of speculative currency attacks. Heinemann et al. (2004) test experimentally this kind of a currency attack model. Their results support the theoretical predictions of global games. Englmaier and Reisinger (2003) apply global games to an economic development framework. Morris and Shin (2004) and Rochet and Vives (2004) study solvent but illiquid financial institutions. Morris and Shin (2004) focus on the question, how investors' beliefs affect the price of debt; whereas Rochet and Vives (2004) explain how the central bank can prevent bank runs with lender of last resort facilities. Myatt and Wallace (2002) analyse public goods provision, open source software in particular, with global games techniques. Chwe (1998) provides empirical observations that support global games' predictions. He finds that goods with social (network) externalities advertise "more on more expensive popular [TV] shows because viewers of popular shows know that many other people are also watching (Chwe 1998)".

The present paper, together with Argenziano (2004), are the first applications of global games to network economics. These models are also the first to endogenise the payoffs with a pricing problem. Argenziano (2004) studies a platform competition with pure membership externalities. Our model differs from her model in that we study a market with membership and usage decisions. We analyse a monopolist that sets two-part tariffs whereas Argenziano (2004) analyses a Bertrand duopoly with linear prices.

2 Model

2.1 Overview

The market consists of consumers and a monopoly firm. The firm sells a novel device that constitutes an efficient medium for interaction. We solve the problem of how the firm sets a price for the new product. Interaction is the source of a coordination problem: the consumer bases his buying decision on his estimate of the proportion of other people who acquire the device. Everybody agrees that all pre-innovation interaction can be mediated by the new device, and the quality of interaction is improved. In addition to interaction usage, the device also provides standalone services that are used independently of other consumers. The utility from the product is thus split into usage and intrinsic utilities. Usage utility is generated by interaction between people, and it presents positive network externalities. Standalone services yield intrinsic utility. Intrinsic utility may include also a status-enhancing type of utility, any utility derived from use with older generation services (backward compatibility), and any non-direct benefits of being a member of the network (including higher-order interaction benefits).[2]

The new device can be used in interaction only if both parties have bought the product. For example, let consumer i have a need to interact with j. If both i and j have bought the device, then they can use it. If either i or j does not have the product, then they use conventional ways to interact. Interaction is not anonymous. From consumer i's point of view, interaction with j is different compared to interaction with k. Consequently, inability to interact with j cannot be compensated by interaction with k. This is what we call an *exogenous social network structure*. Each social relation is perceived as independent from other relations and the relations have different values. We assume that each consumer is interested in interacting with the whole population.[3]

The reader may want to keep in mind the following two real world examples. First example is online gaming. Sony PlayStation 2, Microsoft Xbox and Nokia N-Gage consoles all have standalone and interaction usage features. Players can play alone against the console's computer or against other players on the same console. In addition, console manufacturers run platforms that allow people to play over the Internet against other people in different locations (Sony's Central Station, and Xbox Live; Nokia N-Gage allows people to play while connected over the air directly). The requirements are that the players buy the console, and that they have a broadband connection to the Internet. Moreover, both Xbox Live and Sony Central Station enable players to talk with each other during a game session. Firms also offer additional services and content on the platforms. One can think that the underlying social network consists of players who play the same games

[2] Higher order interaction benefits comprise utility from interaction taking place between one's friends' friends, between friends' friends' friends, and so forth.

[3] In the supplementary section, we analyse the case where each consumer is interested in interaction with only a sub-set of total population, called neighbours in the social relations literature. We show that the "global" and "local" interaction models coincide when all consumers have the same number of neighbours. See Sääskilahti (2005) for an analysis on asymmetric social relations.

or play repeatedly together. It is also usual that people swap, borrow, and trade games with their friends. A person who considers buying a console, takes into account how many games that particular console has in supply and what is the quality of the console. Another point he bears in mind is whether his friends have the same console brand so that he can play with, and against them.

The second real world example is mobile telecommunications. Mobile phones can be also used nowadays for checking latest news and e-mails, or to listen to the radio and music, and even to watch television. These features create intrinsic value for a phone in addition to the status value. The main value driver, of course, is the possibility to talk with friends and send them messages.

Consumer types are horizontally differentiated according to their perception of the intrinsic value à la Hotelling (1929). Under incomplete information, consumers derive their types through noisy signals of the underlying fundamental value of the good. Differentiation captures the idea of consumer satisfaction with product's technical performance and status-related aspects. Lower consumer types find technical performance rather poor. High types are those who like how the machine works (plus probably get high satisfaction from ownership).

Why is intrinsic utility subjected to differentiation while usage is not? On the one hand, usage utility is directly associated with the people who interact, or more precisely, with the social relation the interacting parties have. The device is a mere medium, which does not influence the value of the social relation. We assume that each consumer has equally valuable social relations and the improvement in interaction efficiency is identical for all. On the other hand, how different consumers get utility from the novel features of the device are captured in the intrinsic utility. Clearly, the capacity to use and the attitude towards new technology differ between people. Intrinsic utility does not even have to be positive in relation to older generation products for everybody. An example clarifies this issue. "Mobility" is the principal improvement of mobile telecommunications with respect to fixed line telephony. Being able to call and to be called independent of time and place is an objectively measured improvement (you can always keep the phone switched off whenever you wish!) However, mobile phones tend to be small in size and their use can therefore be very difficult for example, for elderly people. The size factor is positive

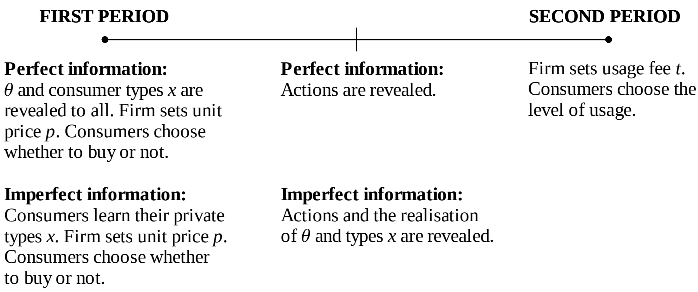

Figure 1: Time Line.

for most consumers, but it can also be negative. Alternatively, some people believe that mobile phones emit radiation harmful to the brain. Other people fear that third parties could secretly monitor the user. Whether it is due to the fear of brain tumors or malicious surveillance, some people may be reluctant to carry a mobile phone, even if they get one for free. Obviously, mobile phones have been fairly successful, and a negative intrinsic value can apply to a handful of people at most; but for our results that is enough.

2.2 Players, actions, and timing

There are I consumers in the market. We normalise $I = 1$ and treat it as continuous. The fundamental intrinsic value of the product θ is drawn from a uniform distribution $F(\theta)$. Consumer types x are distributed around the fundamental according to a uniform distribution $G(x \mid \theta)$.

Timing is summarised in figure (1). In the first period, the firm sets a unit price p, and in the second period it sets an usage fee t. The firm incurs a unit cost c_a for interaction mediation. The unit cost for manufacturing one device is c_f. Fixed costs are assumed zero. There is a trade-off in choosing the optimal price. A low unit price facilitates coordination between consumers and increases expected second period profits, but it erodes first period margins. We assume no discounting.

The first period problem for consumer $i \in I$ is to choose action $a_i \in \{B, N\}$, where $B =$ buy the device and $N =$ do not buy. If the consumer chose $a_i = B$ in the first period, then he needs to decide how much he uses it in the second period. Those consumers, who did not buy, collect the reservation utility of zero and make no further decisions. Interaction usage is possible only among any two consumers who have bought the product, but we assume that only the person paying for the usage gets utility. "Reception" is cost-less and yields zero utility, or any possible positive utility is included in the intrinsic utility.

The purchase of the device in the first period is a sunk cost to consumers, but it enables subsequent interaction usage. The firm is not able to commit to a tariff structure in the first period, thus it sets the usage charge after the consumers have made their purchasing decisions.

Social relations are unequal in terms of interaction utility. Consumers mentally arrange the whole population in descending order of desire for interaction. The person who is the most desirable interaction partner gets index 0 and the least desirable person 1. Then, consumers decide which fraction of the population they want to interact with for a given price. The underlying social network is exogenous so that interaction needs are independent of who buys the device or of the counterpart's respective ranking of interaction partners and utility. As a result, the ordering of desired usage for each consumer is exogenous, which guarantees that consumers are symmetric with respect to usage demand in the second period. Since usage is a binary operation, it is likely that in a given social relation, both consumers pay for usage and get utility.

The second period is a standard deterministic utility maximisation problem for the consumers and a deterministic profits maximisation problem for the firm. In the first period, where the coordination problem is in effect, the model is exposed to two informational regimes. First, if information is perfect to all from the beginning of the game, the game is deterministic throughout. Second, if information is incomplete in the first period, consumers observe noisy signals of θ which correspond to consumer types. The realisations of the signals are private information, but the prior distribution $F(\theta)$ and the posterior $G(x \mid \theta)$ are common knowledge. The firm observes nothing and resorts to the prior $F(\theta)$. Such informational asymmetry is because consumers know their own needs better than the firm.

3 Second period

Let $\alpha_i \in [0,1]$ be the marginal person consumer i wants to interact with for a given usage price. $\alpha_i = 1$ means that i wants to interact with the whole population. Symmetry of social relations guarantees $\alpha_i = \alpha$ for all $i \in I$. If the actualised first period demand is low, the consumer may be constrained into a sub-optimal level of usage, as he would like to interact with more people than who have bought the product. We sacrifice some generality and assume that the marginal utility is linear, but it will become evident that any function with decreasing marginal utility yields qualitatively identical results. If fraction $q \in [0,1]$ has played $a = B$ in the first period, then by the law of large numbers, q is also the probability that a particular person has bought the product. Due to exogenous social network, the net marginal utility from interaction with the social contact indexed α is $\frac{\partial \lambda(\alpha,t)}{\partial \alpha} = q(1-\alpha-t)$, where t is the price for usage. Integration gives the expected net usage utility

$$\lambda(\alpha,t,q) = q\left(\alpha - \frac{1}{2}\alpha^2 - \alpha t\right),$$

with the integration constant equal to zero. Because only the proportion q of the population has bought the product, the consumer cannot use the device with more than q people. Hence, the consumer's second period objective is

$$\max_{\alpha}\{\lambda(\alpha,t,q)\}, \text{ s.t. } \alpha \in [0,q].$$

The optimal level of usage is

$$\alpha^*(t,q) = \min\{1-t, q\}. \tag{1}$$

The firm's second period problem is to maximise usage profits by setting the usage fee $t \in [0,1]$. The per consumer demand is given by equation (1). In the second period, first period profits and the proportion of consumers who bought the product are fixed, and all uncertainty has been resolved. So, the second period profits are $\Pi_2 = q\alpha^*(t,q)(t-c_a)$, where $c_a \in [0,1]$ is the unit cost of service.

The firm always charges a usage fee such that the consumers are maintained at an efficient usage level, in the sense that an increase (decrease) in price causes a decrease (increase) in demand. To see this, assume that t is such that consumers are constrained in their usage, i.e. $1-t > q \Leftrightarrow t < 1-q$. Then, the firm could increase its price t up till point $t = 1-q$ without triggering a decrease in demand. A similar argument holds for the situation where the firm charges a price $t \geq 1$ so that demand is zero. In this case, it would pay off to reduce the price below one, $t < 1$. These observations allow us to write the firm's second period problem as

$$\max_t \{q\alpha^* (t, q) (t - c_a)\}, \text{ s.t. } t \in [1 - q, 1[.$$

The optimal usage fee is

$$t^* = \max\left\{\frac{1}{2}(1 + c_a), 1 - q\right\}, \tag{2}$$

with the interior solution $t^* = \frac{1}{2}(1 + c_a)$ satisfying second order conditions, $\frac{\partial^2 \Pi_2}{\partial t^2} = -2q < 0$. When the optimal usage fee (2) is plugged back into the second period profits, we get

$$\Pi_2^* (c_a, q) = \begin{cases} q\pi_2^{**}(c_a), & q \geq \frac{1}{2}(1 - c_a) \\ q\pi_2^*(c_a, q), & q < \frac{1}{2}(1 - c_a) \end{cases}, \tag{3}$$

where $\pi_2^{**}(c_a) = \frac{1}{4}(1 - c_a)^2$ and $\pi_2^*(c_a, q) = q(1 - c_a - q)$. Double star indicates that the monopolist is at the interior (unconstrained) solution and single star that the monopolist is at the corner solution where it is capacity constrained. Naturally, we have $\pi_2^{**}(c_a) \geq \pi_2^*(c_a, q)$.

Because the firm keeps consumers at the efficient level of usage, $\alpha^*(t^*, q) = 1 - t^*$ and $\lambda^*(\alpha^*(t^*, q), t^*, q) = \frac{1}{2}q(1 - t^*)^2$ hold when t is optimally chosen. Substituting t^* in the expected indirect usage utility, we get

$$\lambda^*(q) = \begin{cases} \frac{1}{8}q(1 - c_a)^2, & \text{if } q \geq \frac{1}{2}(1 - c_a) \\ \frac{1}{2}q^3, & \text{if } q < \frac{1}{2}(1 - c_a) \end{cases}. \tag{4}$$

This concludes the analysis of the second period. Next we study the first period when players have perfect information. In the section following, we analyse the incomplete information case.

4 First period with perfect information

The fundamental intrinsic utility θ is drawn from the uniform distribution $F(\theta)$ over the support $[-M, M]$. When θ is the realisation, consumers obtain i.i.d. private values x according to the conditional uniform distribution $G(x \mid \theta)$ over $[\theta - \epsilon, \theta + \epsilon]$. All types x and the fundamental θ are perfectly observed by the consumers and the firm at the beginning of the first period.

The expected net utility of the consumer of type x from buying $(a = B)$, when he expects a fraction q of population to buy, is $u(x, q, B) = x + \lambda^*(q) - p$, where p is the unit price. If

$x - p > 0$, then $a = B$ is strictly dominating strategy. Action N is strictly dominating strategy when $x + \lambda^* (1) - p < 0$. Action B is the best response, if the fraction of other people playing B gives a usage utility higher than the price net of intrinsic utility, $\lambda^* (q) > p - x$. Because the reservation utility from $a = N$ is zero, the payoff gain from action $a = B$ versus $a = N$ is[4]

$$v(x, q, p) = x + \lambda^* (q) - p. \tag{5}$$

Denote by Γ the coordination game of perfect information with I consumers, pure actions $a \in \{B, N\}$, and payoff (5). The payoff function (5) is continuous in its arguments, even at the cut-off point $q = \frac{1}{2}(1 - c_a)$. It is also differentiable, except at $q = \frac{1}{2}(1 - c_a)$. The payoff presents strictly increasing differences in x. Actions are strategic complements, because the payoff gain from choosing $a = B$ compared to $a = N$ is strictly higher when a larger proportion of population choose $a = B$. Since the action set $a \in \{B, N\}$ is a compact subset of $\mathbb{R}$, the complementarity and continuity properties of $v(x, q, p)$ imply that Γ is supermodular (see e.g. Vives 2001 ch.2).

Supermodularity of Γ guarantees the existence of a Nash equilibrium (NE). The equilibrium may not be unique, but it has a smallest and a largest element. Because actions are strategic complements the maximal equilibrium element is Pareto dominating.

When the consumer expects proportion $\mathbb{E}(q) = q^e$ of people to play B, he is indifferent between buying and not when his type is

$$\overline{x}(q^e, p) = p - \lambda^* (q^e). \tag{6}$$

The corresponding demand schedule is

$$q(p, q^e) = \begin{cases} 0, & \text{if } \overline{x}(q^e, p) > \theta + \epsilon \\ 1 - G(\overline{x}(q^e, p) \mid \theta), & \text{if } \theta - \epsilon \leq \overline{x}(q^e, p) \leq \theta + \epsilon \\ 1, & \text{if } \overline{x}(q^e, p) < \theta - \epsilon \end{cases}. \tag{7}$$

For a given pair (q^e, p), if there is a marginal type defined by (6), the type is unique because $v(x, q^e, p)$ is continuous and strictly increasing in x. More "optimistic" expectations reduce the marginal type, $\frac{\partial \overline{x}(q^e, p)}{\partial q^e} < 0$. This captures the correspondence between efficient coordination and

[4] The derived payoff function is essentially in line with the utility specification of Katz and Shapiro (1985), where consumers are differentiated in terms of intrinsic utility, and variable utility depending on the network size is the same for all buyers. De Palma and Leruth (1995) analyse the polar case where buyers have different valuations for the network benefits.

the Pareto-dominant maximal NE. Expectations on the number of consumers who buy are fulfilled in the equilibrium, $\mathbb{E}_i(q) = q$ for all $i \in I$.

Lemma 1 *The action profile a^* is a Nash equilibrium of Γ if*

$$\begin{cases} a^* = B, & \text{if } x \geq \overline{x}(q,p) \\ a^* = N, & \text{if } x < \overline{x}(q,p) \end{cases},$$

where expectations are fulfilled $\mathbb{E}_i(q) = q$, and $\overline{x}(q,p) = p - \lambda^*(q)$ for all $i \in I$.

Consumers who play the bandwagon strategy: "I buy only if you buy" are the cause of equilibria multiplicity. These consumers buy only if sufficiently many others buy. If coordination is efficient, then they buy. In a coordination failure they do not buy; only those consumers who have a strictly dominating strategy to buy, will do so. Multiplicity of equilibria is ruled out when we allow sufficient level of heterogeneity between consumers. This is done by extending the support of types distribution $G(x \mid \theta)$.

We allow negative unit prices, but we rule out prohibitively negative states θ and prohibitively high unit production costs c_f in order to exclude those cases where the firm chooses to remain inactive. If the state θ is negative, it means that the firm may have to compensate some consumers by setting a negative price. Let $p^*(\theta, c_f)$ and $q^*(\theta, c_f)$ be the optimal price and quantity respectively for state θ and costs c_f. A prohibiting state-cost pair $\left(\theta^-, c_f^+\right)$, for which first period losses outweigh second period profits, is defined implicitly by

$$\begin{aligned} 0 &\leq \pi_2^*\left(c_a, q^*\left(\theta^-, c_f^+\right)\right) < c_f^+ - p^*\left(\theta^-, c_f^+\right), \quad \text{if } q^*\left(\theta^-, c_f^+\right) < \frac{1}{2}(1-c_a) \\ 0 &\leq \pi_2^{**}(c_a) < c_f^+ - p^*\left(\theta^-, c_f^+\right), \quad \text{if } q^*\left(\theta^-, c_f^+\right) \geq \frac{1}{2}(1-c_a) \end{aligned} \tag{8}$$

Define price $\underline{p}$ as the solution to $v\left(\theta - \epsilon, q, \underline{p}\right) = 0 \ \forall q \in [0,1]$ where θ is the realisation of the fundamental. In words, the solution $\underline{p}$ is the lowest type's answer to question: "What is the lowest price that makes sure that even when everybody else buy, I still will not buy?" The answer is $\underline{p} = \theta - \epsilon + \frac{1}{8}(1-c_a)^2$ (to be precise, $\underline{p}$ leaves the lowest type indifferent). Now we have to distinguish between two cases: (i) (relatively) high network externalities and (ii) low network externalities. Network externalities are high if they dominate the distribution of the intrinsic utility in the sense $v(\theta - \epsilon, q = 1, p) > v(\theta + \epsilon, q = 0, p) \Leftrightarrow \epsilon < \frac{1}{16}(1-c_a)^2$. When network externalities are high, price $\underline{p}$ exceeds the highest type's intrinsic valuation $\theta + \epsilon - \underline{p} < 0$.

Proposition 2 *Define*

(i) *Optimal monopoly price* $p^* = \arg\max\{\Pi(p)\}$, *where*

$$\Pi(p) = \begin{cases} q(p)(p - c_f) + q(p)\pi_2^{**}(c_a), & \text{if } q(p^*) \geq \frac{1}{2}(1 - c_a) \\ q(p)(p - c_f) + q(p)\pi_2^{*}(c_a, q(p)), & \text{if } q(p^*) < \frac{1}{2}(1 - c_a) \end{cases}.$$

(ii) *Network externalities are high (low) relative to intrinsic utility when* $\epsilon \underset{(>)}{<} \frac{1}{16}(1 - c_a)^2$.

With endogenous price setting:

1. *If network externalities are high, there are always multiple equilibria.*

2. *If network externalities are low, there is always a unique equilibrium.*

Proof. *In the appendix.* ■

We sketch the proof of Proposition 2 here. (Part 1) Consider first the case where network externalities are high, $\epsilon < \frac{1}{16}(1 - c_a)^2$. Assume first that coordination among consumers is efficient so that the maximal NE emerges (for a given p). Then the lowest price the firm will ever set is $\underline{p}$, defined above, which now leaves all consumers negative intrinsic utility net of price. If coordination is efficient, the maximal NE emerges, but under total coordination failure, $q^e = 0$, no-one will buy. Both cases correspond to fulfilled expectations. In equilibrium, everybody knows which NE takes place. Hence, the firm adjusts its price downwards under the super pessimistic expectations $q^e = 0$. The optimal price is in that case $p^* < \theta + \epsilon$, which guarantees that the highest type has a strictly dominant strategy to buy. The optimal price induces still a low equilibrium associated with pessimistic expectations and a high equilibrium associated with efficient coordination. Monopoly price does not affect expectation formation, thus it is an insufficient mean to eliminate multiplicity. The NE with efficient coordination supports the highest optimal price and total coordination failure the lowest.

(Part 2) Assume low network externalities $\epsilon > \frac{1}{16}(1 - c_a)^2$. The lowest price the firm will ever set is $\underline{p} = \theta - \epsilon + \frac{1}{8}(1 - c_a)^2$. The lowest price is now uniquely determined, as for $\underline{p}$, the highest type has a strictly dominant strategy to buy. We prove in the appendix that the firm never sets a price exceeding the highest type's intrinsic utility. This gives us a closed interval for the optimal price $p^* \in [\underline{p}, \theta + \epsilon]$. Effectively, p^* guarantees that the coordination game Γ has a unique NE. The idea is that both actions are played as strictly dominating strategies simultaneously, and there is a unique marginal type who is indifferent between the actions given by (6). The firm operates on the elastic section of the demand that is, we have $\theta - \epsilon \leq \overline{x}(q,p) \leq \theta + \epsilon$, with expectations being fulfilled $q^e = q$. Demand is given by

$$q = 1 - G\left(\overline{x}\left(q, p^*\right) \mid \theta\right). \tag{9}$$

We close this section by deriving the optimal price for a case with a unique equilibrium. There are two cases to consider: (i) $q(p^*) \geq \frac{1}{2}(1 - c_a)$, and (ii) $q(p^*) < \frac{1}{2}(1 - c_a)$.

(i) Assume first that $q(p^*) \in \left[\frac{1}{2}(1 - c_a), 1\right]$. We get demand from equation (9)

$$q = \frac{\theta + \epsilon - p}{2\epsilon - \frac{1}{8}(1 - c_a)^2}. \tag{10}$$

The demand differs from the text book monopoly case, where consumers have unit demand, by the term $-\frac{1}{8}(1 - c_a)^2$ which captures the second period usage utility.

Monopoly's profits are $\Pi = q(p)(p - c_f) + q(p)\pi_2^{**}(c_a)$. First order condition gives the optimal unit price[5]

$$p^* = \frac{1}{2}\left(\theta + \epsilon + c_f - \pi_2^{**}\right). \tag{11}$$

(ii) Assume next that $q(p^*) \in \left[0, \frac{1}{2}(1 - c_a)\right]$ in the second period. In this case, profits are $\Pi(p) = q(p)(p - c_f) + q(p)\pi_2^*(c_a, q)$. It is more convenient to solve for the indirect demand $p(q)$ from equation (9), and let the firm choose optimal quantity q^*. The first order condition is[6]

$$2q^3 - 3q^2 - 2\left[2\epsilon - (1 - c_a)\right]q + \theta + \epsilon - c_f = 0. \tag{12}$$

4.1 Equilibrium analysis

When is heterogeneity between consumers sufficient for uniqueness? Or put another way: Which factor is more important in consumers' decision making: standalone value (intrinsic utility) or interaction usage (network externalities)? Whenever the condition on sufficient heterogeneity is not satisfied, buying behaviour is driven by network externalities, and demand becomes indeterminate. At best we can assume that a particular NE emerges, and then characterise the firm's

[5] The second order condition is satisfied due to our assumption on low network externalities, $\frac{\partial^2 \Pi(p)}{\partial p^2} = -\frac{1}{\epsilon - \frac{1}{16}(1 - c_a)^2} < 0$.

[6] The second order condition requires $3q(q-1) < 2\epsilon - (1 - c_a)$.

strategy in that equilibrium. We could do this for all possible equilibria, but we would lack any understanding why a particular equilibrium is selected.

Equilibrium analysis is complicated since we have to distinguish between the various possible states of θ even with low network externalities. Let us first analyse the case $q(p^*) \geq \frac{1}{2}(1-c_a)$. The optimal unit price is given by equation (11). Price increases as the consumer heterogeneity increases

$$\frac{\partial p^*}{\partial \epsilon} = \frac{1}{2}.$$

When the optimal price is plugged back into (10), we get

$$q^* = \frac{\theta + \epsilon - c_f + \frac{1}{4}(1-c_a)^2}{4\left[\epsilon - \frac{1}{16}(1-c_a)^2\right]}. \tag{13}$$

Differentiation of (13) with respect to ϵ gives

$$\frac{\partial q^*}{\partial \epsilon} \underset{(<)}{>} 0 \Leftrightarrow \theta \underset{(>)}{<} c_f - \frac{5}{16}(1-c_a)^2.$$

The above rule defines a maximum state below which demand is increasing in consumer heterogeneity. When the cut-off state $\theta = c_f - \frac{5}{16}(1-c_a)^2$ is substituted in (13), we see that if demand satisfies $\max\left\{\frac{1}{4}, \frac{1}{2}(1-c_a)\right\} < q(p^*) \leq 1$, a marginal increase in consumer heterogeneity decreases demand, $\frac{\partial q(p^*)}{\partial \epsilon} < 0$. This is a standard result due to the fact that the monopolist chooses to limit demand by pricing high and that price is increasing in heterogeneity. Since higher values of θ are associated with both higher demand and higher price elasticity, the marginal decrease in demand with respect to ϵ is larger for higher values of θ. Because consumers take into account their second period utility, an increase in heterogeneity has a stronger effect on demand than if the marginal type was solely determined by the unit price as in the textbook monopoly case. Demand increases in heterogeneity if the state is low enough relative to unit costs adjusted with usage utility. A marginal increase in ϵ has a stronger effect on demand the less heterogenous consumers are (that is the closer ϵ is to $\frac{1}{16}(1-c_a)^2$), because (relatively high) network externalities drive conformity in buying decisions. Profits are

$$\Pi(p^*, t^*) = \frac{\left[\theta + \epsilon - c_f + \frac{1}{4}(1-c_a)^2\right]^2}{8\left[\epsilon - \frac{1}{16}(1-c_a)^2\right]}.$$

Consumer surplus is given by

$$
\begin{aligned}
S &= \frac{1}{2\epsilon}\int_{x=\overline{x}(q(p^*),p^*)}^{\theta+\epsilon} x+\frac{1}{8}(1-c_a)^2 q(p^*)-p^*dx \\
&= \left\{\frac{\theta+\epsilon-c_f+\frac{1}{4}(1-c_a)^2}{4\left[\epsilon-\frac{1}{16}(1-c_a)^2\right]}\right\}^2\epsilon.
\end{aligned}
$$

A marginal change in ϵ has an ambiguous effect on profits and consumer surplus. There is a tendency for them to move in the same direction for marginal changes in ϵ. Profits increase when $\frac{\partial p^*}{\partial \epsilon}$ and $\frac{\partial q(p^*)}{\partial \epsilon}$ are both positive. In the cases where demand decreases as ϵ increases, profits tend to decrease when ϵ is close to its minimum (demand effect is stronger), and profits tend to increase when the absolute value of ϵ is large.

Assume next that costs are high so that the firm is constrained in the second period, $q(p^*) < \frac{1}{2}(1-c_a)$. The resulting optimal price p^* is higher and demand lower in this case compared to the above case where the firm is unconstrained. Also the second period price t^* is higher in this case due to lower demand in the first period. Totally differentiating the first order condition (12), gives

$$
\begin{aligned}
\frac{dq^*}{d\epsilon} &> 0 \Leftrightarrow q^* < \min\left\{\frac{1}{4},\frac{1}{2}(1-c_a)\right\} \\
\frac{dq^*}{d\epsilon} &< 0 \Leftrightarrow \frac{1}{4} < q^* < \frac{1}{2}(1-c_a).
\end{aligned}
$$

As a result, a positive marginal change in ϵ causes similar effects on demand as in the case where the firm is not constrained in the second period. Comparative statics for profits and consumer surplus are computatively more complicated but present analogous tendencies.

5 First period with incomplete information

In the previous section we derived a condition for sufficient heterogeneity between consumers that guarantees uniqueness of equilibrium. We had a trade-off between the strength of network effects and heterogeneity. If we go for unique equilibrium, network externalities must be limited. This of course is perverse, if the model is designed to study network externalities. Global games techniques require a different type of heterogeneity. We must have a *possibility* that the fundamental θ takes very low and very high values, but uniqueness does not hinge on the true heterogeneity between consumers.

The game remains otherwise unchanged from the perfect information case, except that consumers and the monopolist do not observe directly θ until at the end of period one. The second period remains intact, thus deterministic. The actual value of θ is drawn again from the uniform distribution $F(\theta)$ with support $[-M, M]$. Consumers' observations of θ are blurred by noise, whereas the firm resorts to the prior on θ in its estimates. The consumers know that the firm is uninformed, which removes all possible information about θ that might otherwise be inferred from prices (p, t).

The consumer i draws an i.i.d. signal x_i from the uniform conditional distribution $G(x \mid \theta)$ with the support $[\theta - \epsilon, \theta + \epsilon]$. The consumer who observes signal x gets an expected payoff gain from action $a = B$ versus $a = N$

$$v(x, q, p) = x + \lambda^* (q) - p. \tag{14}$$

As the signal enters directly the payoff function, uncertainty over θ corresponds to horizontal differentiation similarly to the perfect information case. The first period game is now a global game with private values. The payoff function is continuous in (x, q), even at the point $q = \frac{1}{2}(1 - c_a)$ where the indirect usage utility $\lambda^* (q)$ changes its shape. Payoffs are increasing with respect to x everywhere, $\frac{\partial v(x,q,p)}{\partial x} > 0$. We also have strategic action complementarities in the sense $\frac{\partial v(x,q,p)}{\partial q} > 0$ (everywhere outside the cut-off point $q = \frac{1}{2}(1 - c_a)$). In addition, the payoffs satisfy the "strict Laplacian state monotonicity" condition (see Morris and Shin 2003). Namely, there exists a unique $\widetilde{x}$ that solves $\int_{q=0}^{1} v(\widetilde{x}, q, p)\, dq = 0.$[7] In sum, the payoff function (14) satisfies all the conditions on strategic complementarities and continuity that global games require. The remaining condition we need to impose on payoffs in order to be able to use global games techniques is on dominance regions. For all expectations, some consumers must play $a = B$ as a strictly dominating strategy at the same time as another group plays $a = N$ as a strictly dominating strategy. Because the support of the prior is bounded and price is chosen endogenously, the existence of regions of strictly

[7] Define $z(x) = \int_{q=0}^{1} v(x, q, p)\, dq$. Integration gives $z(x) = x - p + \frac{1}{128}(1 - c_a)^2 \left[8 - (1 - c_a)^2\right]$. Hence, $\frac{\partial z(x)}{\partial x} > 0$ for all x, and there exists a unique $\widetilde{x}$ that solves $z(\widetilde{x}) = 0$. If $p \underset{(<)}{>} \frac{1}{128}(1 - c_a)^2 \left[8 - (1 - c_a)^2\right]$, then $\widetilde{x}$ is positive (negative).

dominating strategies is not trivially satisfied. The prior has to be sufficiently uninformative, as detailed in Condition 3.

Condition 3 *Dominance regions of strictly dominating strategies exist, if*

$$M > \max\left\{c_f - \frac{1}{8}(1-c_a)^2 - 2\epsilon, -\frac{1}{3}c_f + \frac{1}{6}(1-c_a)^2 - \frac{2}{3}\epsilon\right\}.$$

Proof. *In the appendix.* ■

Denote by Γ_{II} the incomplete information game with I consumers, pure actions $a \in \{B, N\}$, payoff (14), and where θ has a uniform prior, signals are i.i.d. and uniform, and where the distributions satisfy Condition 3. The coordination game Γ_{II} is supermodular since the action set is a compact subset of $\mathbb{R}$, and the payoff function (14) is continuous in its arguments and has increasing differences in x. The implications of supermodularity are familiar: (i) a pure strategy Bayesian NE exists, (ii) the equilibria set has a smallest and largest element, and (iii) if there is a unique equilibrium, it is solvable by iterated deletion of strictly dominated strategies. Furthermore, due to action complementarity, the maximal equilibrium element is Pareto dominant. This observation is, however, redundant as we will show that the game Γ_{II} has a unique Bayesian switching equilibrium denoted by Γ_{II}^*.

Proposition 4 *Let $\widetilde{x}$ be defined as the unique solution to $\int_{q=0}^{1} v(\widetilde{x}, q, p)\, dq = 0$. The game Γ_{II} has a unique switching strategy equilibrium Γ_{II}^* that survives the iterated deletion of strictly dominated strategies. The unique equilibrium strategy satisfies $a(x) = N$ for all $x < \widetilde{x}$ and $a(x) = B$ for all $x > \widetilde{x}$.*

Proof. *In the appendix.* ■

We can now compute the marginal signal $\widetilde{x}$ that acts as the cut-off rule in the equilibrium strategy. Any observed signal above (below) this cut-off gives positive (negative) payoff for $a = B$. We obtain from Proposition 4 that the consumer's expectations about the fraction of people who play $a = B$ follows a uniform distribution on the unit interval. Hence, the marginal signal is given by

$$\int_{q=0}^{\frac{1}{2}(1-c_a)} \widetilde{x} + \frac{1}{2}q^3 - pdq + \int_{q=\frac{1}{2}(1-c_a)}^{1} \widetilde{x} + \frac{1}{8}(1-c_a)^2 q - pdq = 0, \quad (15)$$

where we have taken into account the cut-off point $\alpha^*(t^*, q) = \frac{1}{2}(1-c_a)$ at which the firm reaches its interior optimal usage fee. Integration of equation (15) gives

$$\widetilde{x} = p + \tau(c_a),$$

where $\tau(c_a) = \frac{1}{128}(1-c_a)^2\left[(1-c_a)^2 - 8\right]$ captures the expected second period usage utility.

When θ is the realisation of the fundamental, the proportion of consumers who get signals higher than $\widetilde{x}$ is $q = 1 - G(\widetilde{x} \mid \theta)$, giving the first period demand

$$q(\theta, p) = \begin{cases} 1, \text{ if } \theta > \widetilde{x} + \epsilon \\ \frac{\theta + \epsilon - \tau(c_a) - p}{2\epsilon}, \text{ if } \widetilde{x} - \epsilon \leq \theta \leq \widetilde{x} + \epsilon \\ 0, \text{ if } \theta < \widetilde{x} - \epsilon. \end{cases} \tag{16}$$

Having defined the demand, we can turn to the pricing problem. Define the cut-off state $\widehat{\theta}$ as $q\left(\widehat{\theta}, p\right) = \frac{1}{2}(1-c_a)$. Whenever the true state is higher than $\widehat{\theta}$, the firm is not constrained in its second period problem, and its second period profits are $\Pi_2^* = q\pi_2^{**}(c_a)$. If the state is $\theta < \widehat{\theta}$, the optimal usage fee is at the corner solution $t^* = 1 - q$, and firm's second period profits are $\Pi_2^* = q\pi_2^*(c_a, q)$. The firm resorts to the prior in the first period, but it internalises the effect of p on the second period profits. Firm's total expected profits are

$$\begin{aligned} \mathbb{E}(\Pi) &= \mathbb{E}(\Pi_1) + \mathbb{E}(\Pi_2) \\ &= \frac{1}{2M}\left\{\int_{\theta=\widetilde{x}-\epsilon}^{\widetilde{x}+\epsilon} q(\theta,p)(p - c_f)\,d\theta + \int_{\theta=\widetilde{x}+\epsilon}^{M} p - c_f d\theta + \right. \\ &\left. + \int_{\theta=\widetilde{x}-\epsilon}^{\widehat{\theta}} q(\theta,p)\,\pi_2^*(c_a,q)\,d\theta + \int_{\theta=\widehat{\theta}}^{\widetilde{x}+\epsilon} q(\theta,p)\,\pi_2^{**}(c_a)\,d\theta + \int_{\theta=\widetilde{x}+\epsilon}^{M} \pi_2^{**}(c_a)\,d\theta\right\}. \end{aligned} \tag{17}$$

The two first integrals in equation (17) are associated with first period profits. The three last integrals capture the effect on second period profits. Maximisation of (17) gives the optimal price p^*. Denote the true state as θ^*, then the optimal price structure is as in Proposition 5.

Proposition 5 *The optimal price structure is*

$$\begin{aligned} t^* &= \max\left\{\frac{1}{2}(1+c_a), 1 - q(\theta^*)\right\} \\ p^* &= \frac{1}{2}(M + c_f) - \frac{1}{2}\tau(c_a) - \frac{1}{8}(1-c_a)^2. \end{aligned}$$

Proof. *In the appendix.* ■

The optimal prices are increasing in the usage cost,

$$\frac{dp^*}{dc_a} = \frac{1}{16}(1-c_a)\left[\frac{1}{4}(1-c_a)^2 + 3\right] \geq 0,$$

and

$$\frac{dt^*}{dc_a} = \begin{cases} \frac{1}{2}, \; q(\theta^*) \geq \frac{1}{2}(1-c_a) \\ \frac{1-c_a}{4\epsilon}\left[\frac{5}{8} - \frac{1}{32}(1-c_a)^2\right], \; q(\theta^*) < \frac{1}{2}(1-c_a) \end{cases}$$

which is positive for $0 \leq c_a < 1$, and zero if the firm is at the corner solution in the second period and $c_a = 1$.

The optimal unit price is increasing in the unit production cost, $\frac{\partial p^*}{\partial c_f} > 0$. The usage fee is independent of production cost, as long as the firm is not constrained when setting t^*. If the realised demand binds the optimal usage fee, then the usage fee increases in production costs $\frac{\partial t^*}{\partial c_f} = \frac{1}{4\epsilon} > 0$.

5.1 Role of uncertainty on profits and consumer surplus

Demand increases (decreases) in the precision of signals only if the state θ is higher (lower) than the marginal signal. Why? When the precision of the signal is high, it tells the consumer that other people observe signals very close to the one he has observed. If the realisation of θ is below the marginal signal, and if signals are relatively accurate, then the consumer infers that most people do not buy. So, if $\theta < \widetilde{x}$ and we decrease the precision of signals ($d\epsilon > 0$), then a larger proportion of people may observe signals that are above the marginal signal. Therefore, a reduction in the precision of signals when $\theta < \widetilde{x}$ increases actual demand. Opposing, if $\theta > \widetilde{x}$ and signals are relatively precise, a reduction in the precision of signals causes a larger proportion of people drawing signals that fall below the marginal signal. Therefore, a reduction in the precision of signals when $\theta > \widetilde{x}$ decreases demand. We summarise the above in Lemma 6.

Lemma 6 *Decrease in the precision of signals decreases (increases) demand when the true θ is above (below) the marginal signal,* $\frac{\partial q(\theta,p)}{\partial \epsilon} \underset{(>)}{<} 0 \iff \theta \underset{(<)}{>} p + \tau(c_a)$, $\widetilde{x} - \epsilon \leq \theta \leq \widetilde{x} + \epsilon$.

Proof. *Proof follows directly from equation* (16)*, and thus omitted.* ■

The optimal unit price p^* is independent of uncertainty over signals. This is because we have assumed uniform distributions for the prior and signals. Resulting demand is linear, which renders first period profits $\mathbb{E}(\Pi_1(p^*))$ neutral with respect to ϵ. If the firm reaches the interior solution $t^* = \frac{1}{2}(1 + c_a)$ in the second period, also the usage fee is independent of any uncertainty. However, if the firm is pushed to the corner solution, the optimal usage fee depends on the precision of signals. When the firm is constrained, we have $\left.\frac{\partial t^*}{\partial \epsilon}\right|_{q(\theta^*,p^*)<\frac{1}{2}(1-c_a)} = -\frac{\partial q(\theta^*,p^*)}{\partial \epsilon}$. The constrained optimal usage fee is higher than the interior solution. Because the firm is constrained with low values of θ, it is likely that $\frac{\partial q(\theta^*,p^*)}{\partial \epsilon} > 0$ holds. Then, if uncertainty is increased, the first period demand increases. This relaxes the capacity constraint in the second period. Consequently, the firm decreases its usage fee in order to increase its second period sales and profits.

Because there is the possibility that the true demand is low and the firm cannot charge the unconstrained optimal usage fee, expected second period profits are not independent of the precision of signals. Subsequently, the expected total profits are positively correlated with the precision of signals.

Proposition 7 *Increase in the precision of signals increases firm's profits*

$$\frac{\partial \mathbb{E}\left(\Pi\left(p^{*},t^{*}\right)\right)}{\partial \epsilon}=-\frac{\left(1-c_{a}\right)^{4}}{192M}\leq 0.$$

Proof. *In the appendix.* ■

If ϵ is increased marginally, demand increases (decreases) in states that are below (above) the marginal signal. Whenever $\theta<\widetilde{x}-\epsilon$ firm's profits are zero. Therefore states that are above the marginal signal have a larger weight in expected profits. Therefore, the negative effect on demand is dominating. The negative effect on the firm's profits comes mainly from those high states where consumers are very confident on high sales, thus on high interaction utility. When uncertainty is increased, those people have lower expectations on sales volumes, which induces lower sales and profits. The firm always benefits from more accurate information about the value of its good. Only if $c_a=1$, so that second period usage is prevented by high costs, expected profits are independent of uncertainty.

Consumer's expected surplus is

$$\begin{aligned}\mathbb{E}\left(S\right)=\frac{1}{4M\epsilon}&\left\{\int_{\theta=\widetilde{x}(p^{*})-\epsilon}^{\widehat{\theta}(p^{*})}\int_{x=\widetilde{x}(p^{*})}^{\theta+\epsilon}x+\frac{1}{2}q\left(\theta,p^{*}\right)^{3}-p^{*}dxd\theta+\right.\\&+\int_{\theta=\widehat{\theta}(p^{*})}^{\widetilde{x}(p^{*})+\epsilon}\int_{x=\widetilde{x}(p^{*})}^{\theta+\epsilon}x+\frac{1}{8}\left(1-c_{a}\right)^{2}q\left(\theta,p^{*}\right)-p^{*}dxd\theta+\\&\left.+\int_{\theta=\widetilde{x}(p^{*})+\epsilon}^{M}\int_{x=\theta-\epsilon}^{\theta+\epsilon}x+\frac{1}{8}\left(1-c_{a}\right)^{2}-p^{*}dxd\theta\right\}.\end{aligned}\tag{18}$$

To see the effect of a change in signals' precision, we differentiate (18) with respect to ϵ. The sign of $\frac{\partial\mathbb{E}(S)}{\partial\epsilon}$ depends only on c_a and ϵ. We denote the solution to $\frac{\partial\mathbb{E}(S)}{\partial\epsilon}=0$ by $\overline{\epsilon}\left(c_a\right)$. We have plotted $\epsilon=\overline{\epsilon}\left(c_a\right)$ in figure (2). Above the curve, the derivative is positive, $\frac{\partial\mathbb{E}(S)}{\partial\epsilon}>0$, and below the curve we have $\frac{\partial\mathbb{E}(S)}{\partial\epsilon}<0$.

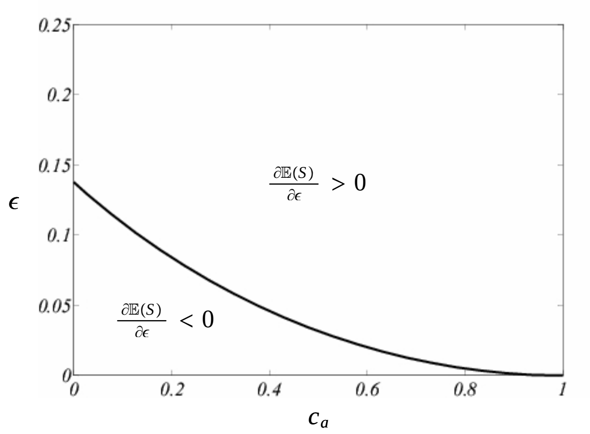

Figure 2: The sign of $\frac{\partial \mathbb{E}(S)}{\partial \epsilon}$.

Proposition 8 *A decrease in the signals' precision ($d\epsilon > 0$), induces:*

(i) *For relatively precise signals $\epsilon < \overline{\epsilon}(c_a)$, a decrease in expected consumer surplus.*

(ii) *For relatively imprecise signals $\epsilon > \overline{\epsilon}(c_a)$, an increase in expected consumer surplus.*

Unlike with profits, the absolute magnitude of ϵ plays a role in whether consumer surplus increases or decreases for marginal changes in the precision. When the signals are very precise (ϵ below the curve in figure (2)), the expected consumer surplus decreases if the precision of signals is marginally decreased ($d\epsilon > 0$). When signals are less precise (ϵ above the curve in figure (2)), consumer surplus is positively affected by a marginal increase in uncertainty. Expected surplus is affected via two effects. For a given θ, there is a change in the expected demand for the product. There is also a change in the expectation of θ.

The negative effect on surplus is foremost associated with the very high states ($\theta > \widetilde{x} + \epsilon$), where expected consumer surplus unambiguously reduces as ϵ increases. This is the segment where consumers are confident on high sales and subsequent high usage utility. The negative effect is stronger the smaller ϵ and c_a are, which shows up in that the total effect turns negative in the area $\epsilon < \overline{\epsilon}(c_a)$. For lower values of θ, there is a mixture of positive and negative effects which sum up to the result illustrated in figure (2). We have a minimum for $\mathbb{E}(S(\epsilon))$ with respect to ϵ given by $\overline{\epsilon}(c_a)$. If signals are extremely precise ($\epsilon \to 0$), so that consumers are homogeneous, the consumer benefits from the knowledge that other people are like him. In this case, network externalities have important role in decision making. When we reduce the precision of signals, the little extra uncertainty about other people hurts. However, when the precision of signals drops to a relatively low level ($\epsilon > \bar{\epsilon}(c_a)$), higher uncertainty is of good. Why? Low precision is analogous to high heterogeneity between consumers. If signals are imprecise, the consumer knows that there is a large variance in the perception of the true intrinsic value of the product within the population, and knows that other people know that everybody is equally uninformed. The consumer is then likely to base his buying decision on the intrinsic utility, rather than on the expected behaviour of other people. In this case, the consumers benefit from further knowledge ($d\epsilon > 0$) about the fact that they can base their decisions on their private values. Similarly, if signals are accurate ($\epsilon < \bar{\epsilon}(c_a)$), further information ($d\epsilon < 0$) on that perceived network externalities, which give the same utility for everyone, are driving everybody else's decisions increases expected surplus.

6 Comparison

In this section we discuss the differences between the perfect and the incomplete information regimes. We focus on the perfect information case where (i) network externalities are sufficiently low to guarantee a unique equilibrium, and (ii) the firm is not constrained in the second period giving a higher monopoly price compared with the constrained case. Let us restate the optimal prices under perfect and incomplete information for reference

$$\begin{aligned} p_{PI}^* &= \frac{1}{2}(\theta+\epsilon)+\frac{1}{2}c_f-\frac{1}{8}(1-c_a)^2 \\ p_{II}^* &= \frac{1}{2}M+\frac{1}{2}c_f-\frac{1}{8}(1-c_a)^2-\frac{1}{2}\tau(c_a). \end{aligned}$$

The term $-\frac{1}{8}(1-c_a)^2$, present in both price equations, is the effect from second period profits. The firm takes into account that high first period price reduces second period profits. This effect is eliminated if we introduce perfect competition in the second period, so that the usage fee is $t = c_a$. The optimal first period monopoly price under incomplete information when the second period is characterised by perfect competition is

$$p_C^* = \frac{1}{2}M+\frac{1}{2}c_f-\frac{1}{2}\tau_C(c_a),$$

where $\tau_C(c_a) \leq \tau(c_a) \leq 0$. Derivation of p_C^* is in the appendix.

Prices p_{PI}^* and p_{II}^* diverge in two respects. First, because the firm observes nothing under incomplete information, it takes expectations on the consumer distribution. Consequently, the price tends to be higher under incomplete information, as M replaces $\theta + \epsilon$ in the price function. Under incomplete information, the unit price is independent of the term measuring heterogeneity ϵ (i.e. independent of uncertainty), which is in contrast to the perfect information case, in which the firm increases the price for a marginal increase in consumer heterogeneity.

The second difference is the term $-\frac{1}{2}\tau(c_a) \geq 0$, which captures the firm's (accurate) perception on what are *consumers' expectations* on the second period usage utility. Because under perfect information, all players, including the firm, observe perfectly how much usage utility consumers get in the second period, the firm neutralises the effect by incorporating the expected usage utility fully in the unit price. Consumers' expectations are "fixed" under incomplete information, so there is a (potential) gap between expected and actual usage utility. This gap induces a safer pricing strategy: the firm prices high in the first period, before consumers learn the true state, at the expense of more uncertain second period profits. Thus, when the firm is uncertain about second period usage utility, it adjusts its price upwards compared to the perfect information price. This effect is aggravated, when the second period is characterised by perfect competition (with incomplete information). We have $-\frac{1}{2}\tau_C(c_a) \geq -\frac{1}{2}\tau(c_a)$ indicating that the monopoly does not have any incentives to insure second period profits by setting a low first period price. Demand, however, is higher for the (second period) competition case than for the two-period monopoly, $q(p_C^*) > q(p^*)$, because the monopoly limits supply in the second period.

The expectations mechanism affects the realised demand, and we cannot tell unambiguously, whether demand is higher under perfect or incomplete information. Numerical simulations that we have carried out tend to result in higher demand under perfect information.

The term determining real heterogeneity between consumers (and measuring uncertainty), ϵ, has an important role for coordination on a unique equilibrium under perfect information. This role is taken away if consumers' valuations are private information (but correlated). Even the smallest amount of uncertainty is sufficient to result in a unique equilibrium, whereas we had an explicit rule for minimum heterogeneity under perfect information.

The firm's preferences over heterogeneity may differ under perfect and incomplete information regimes. Let the network externalities be high and information perfect. If the market sentiment is pessimistic, so that coordination is prone to fail, the firm may prefer more heterogeneity between consumers. Higher heterogeneity facilitates coordination on the efficient NE. Under super pessimistic expectations, the firm prefers high heterogeneity which would support a (larger) unique equilibrium. If we maintain high network externalities, but impose incomplete information, coordination is unaffected by the real heterogeneity between consumers. Moreover, we know that the firm's profits increase as the precision of signals increases, $\frac{\partial \mathbb{E}(\Pi(p^*,t^*))}{\partial \epsilon} < 0$. So it prefers little heterogeneity.

7 Concluding remarks

We have analysed a market for network goods. A monopolist launches a device that enables efficient interaction between people. Hence, consumers face a coordination problem in whether to switch to using the new device or to stick with the prevailing interaction systems. This kind of coordination game has multiple Nash equilibria under perfect information and homogenous players. We have carried out comprehensive analysis how uniqueness of equilibrium can be reached. The interpretation we have given for the necessary conditions for uniqueness apply to network models in general. Uniqueness of equilibrium under perfect information requires high consumer heterogeneity. Adversely, we must limit the role of network externalities in consumers' buying decision making. Under incomplete information, uniqueness of equilibrium arises endogenously, as long as the prior distribution of the underlying economic fundamental is sufficiently dispersed.

The key to uniqueness is the same in both informational regimes. When one group of people play "Buy" as a strictly dominant strategy, at the same time as another group play "Not Buy" as a strictly dominant strategy, the resulting equilibrium is unique. Both information regimes require some level of heterogeneity, but the type of heterogeneity is different. Under perfect information, heterogeneity between consumers had to be real. Under incomplete information, uniqueness does not hinge on the real heterogeneity between people, which can be minimal. Instead of real heterogeneity, we need to raise only a *possibility* that the fundamental value of the product can be very low or very high. Hence, the model parameters are less restricted.

Uniqueness of the equilibrium allowed us to carry out comparative statics analysis. The unit price is independent of consumer heterogeneity under incomplete information, but it is increasing in heterogeneity under perfect information.

A marginal change in consumer heterogeneity has an ambiguous effect on profits and consumer surplus under perfect information. The effect is even more ambiguous if network externalities are strong, so that we have more than one Nash equilibrium in the coordination game. Under incomplete information, the firm's expected profits increase as the precision of signals improves (i.e. heterogeneity between consumers is lowered). The effect on expected consumer surplus depends on the absolute level of the signal's precision. The expected consumer surplus increases if the marginal change in the precision of signals is in-line with the way consumers base their buying decisions. If signals are precise (imprecise), further improvement (reduction) in accuracy raises surplus. In this sense, better agreement on the factor that drives decision making among the consumers is of good.

The consumers had to invest up-front to the device in order to benefit from it in the usage stage. The monopolist, on the other hand, was not able to commit to prices in the first period. It set the usage fee after consumers had made their buying decisions. We showed that under incomplete information, the monopolist biases its prices in favor of the device price at the expense of potentially lower demand in the second period. Under perfect information, such bias did not exist as the monopolist is able to perfectly incorporate second period usage utility in its first period price. The separation of stages invites further research. First, the case with credible commitment to prices in the first period by the monopolist could potentially result in a different balance between the unit price and the usage fee. Secondly, the monopolist could be allowed to sell the device in the second period to those consumers who opted for not buying in the first period. This opportunity might cause some consumers to wait in the first period, even if their net expected utility from buying was positive.

Perfect information is a strong condition. For ordinary consumable goods, the assumption on perfect information does not (necessarily) create problems. But in problems of coordination, even a marginal difference between perfect information and almost perfect information can produce strikingly different outcomes (see also Morris 2002). On the one hand, under perfect information, the coordination failure is a probable scenario. If the main selling argument is based on networking benefits, the perfect information variant is in trouble in explaining which equilibrium is the most probable one. On the other hand, under incomplete information, there is no coordination failure. The analytical easiness of the incomplete information variant of our model compared with the perfect information case, favours the limitation of people's observation capabilities from a technical point of view. More importantly, for a novel, technologically advanced, product, incomplete information regime also characterises the real world more accurately. Just think about how we are more capable of saying how much utility a fax machine or e-mail yields to other people today than, say, twenty years ago. As the product matures, information becomes more accessible. Thus, coordination failure is less of a problem compared to what earlier literature has proposed.

The incomplete information case offers a number of possible interesting extensions. We have used fairly specific distributions, and utility functions, that could be generalised. The uniqueness result also allows analysis on strategic investments that has been previously obstructed by the multiplicity problem in network models. We have assumed that consumers know their needs better than the firm. It would be interesting to allow the firm to observe something more than nothing. It could then use prices to manipulate consumers' perceptions of the value of the fundamental. This modification would give further information on when the firm has incentives to reveal information to the market about its goods, and how this improvement in the precision of the public signal affects consumer surplus.

8 References

Albert R, Barabási A-L. Statistical Mechanics of Complex Networks. Reviews of Modern Physics 2002; 74; 47-97.

Argenziano R. Differentiated Networks: Equilibrium and Efficiency. Mimeo 2004.

Armstrong M. Network Interconnection in Telecommunications. Economic Journal 1998; 108;

545-564.

Arthur WB. Competing Technologies, Increasing Returns, and Lock-In by Historical Events. Economic Journal 1989; 99; 116-131.

Baake P, Boom A. Vertical Product Differentiation, Network Externalities, and Compatibility Decisions. International Journal of Industrial Organization 2001; 19; 267-284.

Ballester C, Calvó-Armengol A, Zenou Y. Who's Who in Crime Networks: Wanted - The Key Player. CEPR Discussion Papers 2004; 4421.

Barabási A-L, Bonabeau E. Scale-free Networks. Scientific American 2003; May; 50-59.

Bental B, Spiegel M. Network Competition, Product Quality, and Market Coverage in the Presence of Network Externalities. Journal of Industrial Economics 1995; 43; 197-208.

Bentolila S, Michelacci C, Suarez J. Social Contacts and Occupational Choice. Mimeo 2004.

Bramoullé Y, Kranton R. Public Goods in Networks: How Networks Can Shape Social Learning and Innovation. Mimeo 2004.

Bramoullé Y, Saint-Paul G. Social Networks and Labour Market Transitions. IZA Discussion Papers 2004; 1215.

Bulow JI, Geanakoplos JD, Klemperer PD. Multimarket Oligopoly: Strategic Substitutes and Complements. Journal of Political Economy 1985; 93; 488-511.

Cabral LMB, Salant DJ, Woroch GA. Monopoly Pricing with Network Externalities. International Journal of Industrial Organization 1999; 17; 199-214.

Calvó-Armengol A, Jackson MO. The Effects of Social Networks on Employment and Inequality. American Economic Review 2004; 94; 426-454.

Carlsson H, van Damme E. Global Games and Equilibrium Selection. Econometrica 1993; 61; 989-1018.

Chwe MS-Y. Believe the Hype: Solving Coordination Problems with Television Advertising. Mimeo 1998.

Chwe MS-Y. Communication and Coordination in Social Networks. Review of Economic Studies 2000; 67; 1-16.

David PA. Clio and the Economics of QWERTY. American Economic Review 1985; 75; 332-337.

De Palma A, Leruth L. Variable Willingness to Pay for Network Externalities with Strategic Standardization Decisions. European Journal of Political Economy 1996; 12; 235-251.

Economides N. Network Externalities, Complementarities, and Invitations to Enter. European Journal of Political Economy 1996; 12; 211-233.

Englmaier F, Reisinger M. Information, Coordination, and the Industrialisation of Countries. Mimeo 2003.

Farrell J, Katz ML. The Effects of Antitrust and Intellectual Property Law on Compatibility and Innovation. Antitrust Bulletin 1998; 609-650.

Farrell J, Saloner G. Standardization, Compatibility, and Innovation. RAND Journal of Economics 1985; 16; 70-83.

Farrell J, Saloner G. Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation. American Economic Review 1986; 76; 940-955.

Gaduh AB. Information and Social Networks in Village Economies. CSIS Working Papers 2002; WPE 063.

Glaeser EL, Sacerdote B, Scheinkman JA. Crime and Social Interactions. Quarterly Journal of Economics 1996; 111; 507-548.

Goldstein M, de Janvry A, Sadoulet E. Is a Friend in Need a Friend Indeed? Inclusion and Exclusion in Mutual Insurance Networks in Southern Ghana. Mimeo 2002.

Goyal S, Konovalov A, Moraga-González JL. Hybrid R&D. Tinbergen Institute Discussion Papers 2003; TI2003-041/1.

Heinemann F, Nagel R, Ockenfels P. The Theory of Global Games on Test: Experimental Analysis of Coordination Games with Public and Private Information. Econometrica 2004; 72; 1583-1599.

Herrendorf B, Valentinyi Á, Waldmann R. Ruling Out Multiplicity and Indeterminacy: The Role of Heterogeneity. Review of Economic Studies 2000; 67; 295-307.

Hotelling H. Stability in Competition. Economic Journal 1929; 39; 41-57.

Katz ML, Shapiro C. Network Externalities, Competition, and Compatibility. American Economic Review 1985; 75; 424-440.

Kranton RE, Minehart DF. A Theory of Buyer-Seller Networks. American Economic Review 2001; 91; 485-508.

Labini MS. Social Networks and Wages: It Is All about Connections! LEM Working Paper 2004; 10.

Laffont J-J, Rey P, Tirole J. Network Competition I: Overview and Nondiscriminatory Pricing. RAND Journal of Economics 1998a; 29; 1-37.

Laffont J-J, Rey P, Tirole J. Network Competition II: Price Discrimination. RAND Journal of Economics 1998b; 29; 38-56.

Mason R, Valentinyi Á. Independence and Heterogeneity in Games of Incomplete Information. University of Southampton Discussion Papers in Economics and Econometrics 2003; 0307.

Morris S. Coordination, Communication, and Common Knowledge: A Retrospective on the Electronic-Mail Game. Oxford Review of Economic Policy 2002; 18; 433-445.

Morris S, Shin HS. Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks. American Economic Review 1998; 88; 587-597.

Morris S, Shin HS. Global Games: Theory and Applications. In Dewatripont M, Hansen LP, Turnovsky SJ (Eds), Advances in Economics and Econometrics: Theory and Applications, Eight World Congress vol.1. Cambridge University Press: Cambridge United Kingdom; 2003; 56-114.

Morris S, Shin HS. Coordination Risk and the Price of Debt. European Economic Review 2004; 48; 133-153.

Myatt DP, Wallace C. Equilibrium Selection and Public-Good Provision: The Development of Open-Source Software. Oxford Review of Economic Policy 2002; 18; 446-461.

Rochet J-C, Vives X. Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All? Journal of the European Economic Association 2004; 2; 1116-1147.

Sääskilahti P. Monopoly Pricing of Social Goods. HECER Discussion Papers 2005; 66.

Vives X. Oligopoly Pricing. The MIT Press: Cambridge Massachusetts; 2001.

Young HP. Individual Strategy and Social Structure, an Evolutionary Theory of Institutions. Princeton University Press: Princeton New Jersey; 1998.

9 Appendix

Proof of Proposition 2, part 1. We show that when network externalities are high, $\epsilon < \frac{1}{16}(1-c_a)^2$, (i) under efficient coordination (corresponding to the maximal NE of the game Γ), there exists a profits maximising price which exceeds the highest type's intrinsic utility; (ii) any profits maximising price p^* associated with efficient coordination induces multiple equilibria, and therefore under a coordination failure, the firm chooses a lower price than under efficient coordination. All equilibria are associated with rational expectations, $\mathbb{E}(q) = q$. From (i) - (ii) we get the result that under high network externalities, there are always multiple equilibria in the consumers' coordination subgame Γ parameterised by price. Prohibitive state-cost pairs $\left(\theta^-, c_f^+\right)$ as defined in (8) are ruled out.

(i) When coordination is efficient, we have a minimum price that the firm will ever charge

$$\underline{p} = \theta - \epsilon + \frac{1}{8}(1 - c_a)^2,$$

which corresponds to the price that leaves the lowest type indifferent between buying and not when everybody else buys. By the assumption of high network externalities, we have $\underline{p} > \theta + \epsilon$. There is also an upper boundary price $\overline{p} = \theta + \epsilon + \lambda^* (q) \geq \underline{p}$, above which it becomes dominant strategy for everyone not to buy. Hence, if there is a profits maximising price under efficient coordination, it belongs to the closed interval $p^* \in \left[\underline{p}, \overline{p}\right]$. Demand $q(p)$ given by (7) is continuous in p in the interval $\left[\underline{p}, \overline{p}\right]$ due to continuity of types x and continuity of $\lambda^* (q)$ in q. Because demand is zero at the upper extreme, $q(\overline{p}) = 0$, the firm makes zero profits $\Pi(\overline{p}) = 0$. At the low extreme, demand equals one, $q\left(\underline{p}\right) = 1$, but profits can be anything depending on cost parameters and the realisation of the state. Firm's profits are

$$\Pi(p) = \begin{cases} q(p)(p - c_f) + q(p)\pi_2^{**}(c_a), & \text{if } q(p^*) \geq \frac{1}{2}(1 - c_a) \\ q(p)(p - c_f) + q(p)\pi_2^{*}(c_a, q), & \text{if } q(p^*) < \frac{1}{2}(1 - c_a) \end{cases}. \tag{19}$$

Due to continuity of demand in $p \in \left[\underline{p}, \overline{p}\right]$, profits (19) are continuous, even at the cut-off point $q(p) = \frac{1}{2}(1 - c_a)$. By Weierstrass' Theorem, there exists $p^* \in \arg\max\{\Pi(p)\}$ in the interval $\left[\underline{p}, \overline{p}\right]$. The optimal price may be an interior solution or a corner solution.

(ii) From part (i), we know that the optimal price is bounded in the region $p^* \in \left[\underline{p}, \overline{p}\right]$ when coordination is efficient. Importantly, the lower boundary price exceeds the highest type's intrinsic utility, $\underline{p} > \theta + \epsilon$. Consider next that the firm sets price $p^* \in \left[\underline{p}, \overline{p}\right]$ expecting efficient coordination, but consumers' expectations are super pessimistic, $\mathbb{E}(q) = 0$. Since, we have $p^* > \theta + \epsilon$, all consumers now expect negative payoffs from buying, and therefore no-one buys. Any price $p^* \in \left[\underline{p}, \overline{p}\right]$ supports both an efficient coordination NE where a positive proportion of consumers buy, and a "no-one buys" NE. In equilibrium, the firm, of course, knows to which NE consumers coordinate on and adjusts its price accordingly. Under total coordination failure, $\mathbb{E}(q) = 0$, the highest price that guarantees that the "no-one buys" equilibrium is evaded is $p = \theta + \epsilon$, at which point the highest type becomes indifferent between buying and not when no-one else buys. Consequently, the firm sets a price $p^* < \theta + \epsilon$ and makes positive profits. But for $p^* < \theta + \epsilon$ there exists multiple equilibria: the low equilibrium corresponding to pessimistic expectations and the full demand equilibrium corresponding to efficient coordination.

■

Proof of Proposition 2, part 2. We prove that under low network externalities, $\epsilon > \frac{1}{16}(1-c_a)^2$, the optimal price is always bounded within $p^* \in \left[\theta - \epsilon + \frac{1}{8}(1-c_a), \theta + \epsilon\right]$. This price yields a unique equilibrium. The proof is constructed in two steps: first the lower, then the upper boundary price is derived.

Let us restate the indifferent type, when consumer expectations are $\mathbb{E}(q) \equiv q^e$ (equation (6))

$$\overline{x}(q^e, p) = p - \lambda^*(q^e),$$

which gives the demand schedule (equation (7))

$$q(p, q^e) = \begin{cases} 0, & \text{if } \overline{x}(q^e, p) > \theta + \epsilon \\ \frac{\theta + \epsilon + \lambda^*(q^e) - p}{2\epsilon}, & \text{if } \theta - \epsilon \leq \overline{x}(q^e, p) \leq \theta + \epsilon \\ 1, & \text{if } \overline{x}(q^e, p) < \theta - \epsilon \end{cases}.$$

In equilibrium, expectations are fulfilled, so that $q^e = q$. Prohibitive state-cost pairs $\left(\theta^-, c_f^+\right)$ as defined in (8) are ruled out.

(i) The lowest price the firm will ever charge is $\underline{p} = \theta - \epsilon + \frac{1}{8}(1-c_a)$. This price leaves the lowest type indifferent between buying and not when everybody else buys. By the assumption of low network externalities, for price $\underline{p} = \theta - \epsilon + \frac{1}{8}(1-c_a)$, the highest type has a strictly dominant strategy to buy. Since the highest type has strictly dominant strategy to buy, even under super pessimistic expectations ($\mathbb{E}(q) = 0$) the firm makes positive sales with price $\underline{p}$. Demand does not increase by lowering the price further below $\underline{p}$ for any given q^e.

(ii) Next we prove that the optimal price does not exceed the highest type's valuation, $p^* \leq \theta + \epsilon$. Start by assuming $q(p^*) \geq \frac{1}{2}(1-c_a)$. In this case, the demand corresponding to fulfilled expectations is given by

$$q(p)=\begin{cases}\frac{\theta+\epsilon-p}{2\left[\epsilon-\frac{1}{16}(1-c_a)^2\right]}, & \text{if } \theta-\epsilon+\frac{1}{8}(1-c_a)^2\leq p\leq\theta+\epsilon-(1-c_a)\left[\epsilon-\frac{1}{16}(1-c_a)^2\right]\\ 1, & \text{if } p<\theta-\epsilon+\frac{1}{8}(1-c_a)^2\end{cases}.$$

We have $\frac{\partial q(p)}{\partial p}<0$ in the range where the demand is elastic. So, the highest feasible price is obtained when demand is $q=\frac{1}{2}(1-c_a)$. This price is $p=\theta+\epsilon-(1-c_a)\left[\epsilon-\frac{1}{16}(1-c_a)^2\right]\leq\theta+\epsilon$ with equality at $c_a=1$.

Assume next that $q(p^*)<\frac{1}{2}(1-c_a)$ so that the firm is in the corner solution in the second period. It is now more convenient to solve for the inverse demand function

$$p(q)=\theta+\epsilon-q\left(2\epsilon-\frac{1}{2}q^2\right), \tag{20}$$

which can be increasing in $q\in\left[0,\frac{1}{2}(1-c_a)\right]$. The firm maximises profits by choosing quantity $q^*\in\left[0,\frac{1}{2}(1-c_a)\right]$. Since we are interested in the possibility of the case $p(q^*)>\theta+\epsilon$, the term in parenthesis in (20) should be negative. So, we require that $\epsilon<\frac{1}{4}q^2$ holds. As we combine this condition with the initial assumption on low network externalities, we obtain a range within the heterogeneity parameter must strictly be $\frac{1}{16}(1-c_a)^2<\epsilon<\frac{1}{4}q^2$. This condition is the least binding when demand q is at maximum. The assumption $q^*<\frac{1}{2}(1-c_a)$ gives the maximal consistent equilibrium demand level. Once this level is plugged into the condition, we end up with $\frac{1}{16}(1-c_a)^2<\epsilon<\frac{1}{16}(1-c_a)^2$, which cannot hold. Hence, if we force $\epsilon<\frac{1}{4}q^2$ to hold, we violate $\frac{1}{16}(1-c_a)^2<\epsilon$, and vice versa. Consequently, the term in the parenthesis in (20) is always positive, hence the price remains bounded from above $p(q)\leq\theta+\epsilon$. Note that price is continuous in $q\in[0,1]$. If we plug $q=\frac{1}{2}(1-c_a)$ in (20), we get $p=\theta+\epsilon-(1-c_a)\left[\epsilon-\frac{1}{16}(1-c_a)^2\right]$.

In (i) - (ii), we have established that the firm sets a price $p^*\in\left[\theta-\epsilon+\frac{1}{8}(1-c_a),\theta+\epsilon\right]$. Firm's profits are

$$\Pi(p)=\begin{cases}q(p)(p-c_f)+q(p)\pi_2^{**}(c_a), & \text{if } q(p)\geq\frac{1}{2}(1-c_a)\\ q(p)(p-c_f)+q(p)\pi_2^{*}(c_a,q), & \text{if } q(p)<\frac{1}{2}(1-c_a)\end{cases}.$$

Because profits are continuous in p there exists a profits maximising price $p^*\in\left[\theta-\epsilon+\frac{1}{8}(1-c_a),\theta+\epsilon\right]$ by Weierstrass' Theorem.

For price p^*, the lowest type gets zero payoff at maximum

$$v(\theta - \epsilon, q, p^*) \leq 0 \;\; \forall q \in [0, 1]. \tag{21}$$

At the same time, the highest type always gets at least zero payoff

$$v(\theta + \epsilon, q, p^*) \geq 0 \;\; \forall q \in [0, 1]. \tag{22}$$

Inequalities (21) and (22) establish (weak) dominance regions which together with increasing differences $\frac{\partial v(x,q,p)}{\partial x} > 0$ guarantee equilibrium uniqueness in the coordination game Γ. At the boundaries $p \in \left\{\theta - \epsilon + \frac{1}{8}(1 - c_a), \theta + \epsilon\right\}$ everybody may play the same action, but indeterminacy is restricted to the marginal (the lowest or the highest) type only. Since these are marginal cases, we can ignore them. As a result, the equilibrium is unique. ■

Existence of the dominance regions (Condition 3). The dominance regions must exist for all consumer expectations. We show that as M is sufficiently large, lower and upper dominance regions coexist.

(i) Start with the upper dominance region,

$$\exists \overline{\theta} \in \left]-M, M\right[\text{ so that } v(x, q, p) > 0 \text{ for all } q \in [0, 1] \text{ and } x \geq \overline{\theta}.$$

Assume that consumers are "optimistic" and expect full coverage $q^e = 1$. Because consumers expect $q^e = 1$, they also expect second period usage utility $\lambda^*(q^e) = \frac{1}{8}(1 - c_a)^2$. The consumer who observes x and has expectations $q^e = 1$, gets expected payoff gain $v = x + \frac{1}{8}(1 - c_a)^2 - p$. Because $v(x, q, p)$ is strictly increasing in x, we get a marginal type $\overline{x}_{q^e=1} = p - \frac{1}{8}(1 - c_a)^2$ who is indifferent between buying and not buying. The true demand schedule under expectations $q^e = 1$ is

$$q(p) = \begin{cases} 0, & \text{if } \overline{x}_{q^e=1} > \theta + \epsilon \\ \frac{\theta + \epsilon + \frac{1}{8}(1-c_a)^2 - p}{2\epsilon}, & \text{if } \theta - \epsilon \leq \overline{x}_{q^e=1} \leq \theta + \epsilon \\ 1, & \text{if } \overline{x}_{q^e=1} < \theta - \epsilon. \end{cases} \tag{23}$$

Demand (23) corresponds to the most optimistic expectations, thus it supports the highest monopoly price. Define the cut-off state $\widehat{\theta}_{q^e=1}$ below which the monopoly is constrained in the second period and above the firm reaches the interior solution. The monopoly's expected profits with expectations $q^e = 1$ are

$$\begin{aligned}\mathbb{E}(\Pi) &= \frac{1}{2M}\left\{\int_{\overline{x}_{q^e=1}-\epsilon}^{\widehat{\theta}_{q^e=1}} q(p)(p-c_f)+q(p)\pi_2^* d\theta + \right. \\ &+ \int_{\widehat{\theta}_{q^e=1}}^{\overline{x}_{q^e=1}+\epsilon} q(p)(p-c_f)+q(p)\pi_2^{**} d\theta + \\ &+ \left. \int_{\overline{x}_{q^e=1}+\epsilon}^{M} p-c_f+\pi_2^{**} d\theta \right\}.\end{aligned} \tag{24}$$

Optimisation of (24) gives price

$$p^*_{q^e=1} = \frac{1}{2}\left[M + c_f - \frac{1}{8}(1-c_a)^2\right].$$

The second order conditions are satisfied, $\frac{\partial^2 \mathbb{E}(\Pi)}{\partial p^2} = -\frac{1}{M} < 0$. Given the price $p^*_{q^e=1}$, the highest type must have a strictly dominant strategy to buy, even if no-one else buys, $M + \epsilon - p^*_{q^e=1} > 0$, where we have used $\lambda^*(q^e = 0) = 0$, which gives the following condition on the bandwidth of θ's and x's distributions

$$M + 2\epsilon > c_f - \frac{1}{8}(1-c_a)^2. \tag{25}$$

(ii) A similar line of reasoning must apply to the lower dominance region,

$$\exists \underline{\theta} \in \left]-M, M\right[\text{ so that } v(x,q,p) < 0 \text{ for all } q \in [0,1] \text{ and } x \leq \underline{\theta}.$$

Now, we look for the optimal price corresponding to the most "pessimistic" expectations $q^e = 0$. This price is the lowest price the firm will ever set. We skip the derivation of the true demand schedule corresponding to expectations $q^e = 0$, and the calculation of the respective optimal price. The procedures are identical to those explained in part (i). Given the optimal price $p^*_{q^e=0} = \frac{1}{2}\left[M + c_f - \frac{1}{4}(1-c_a)^2\right]$ corresponding to the most pessimistic expectations, the lowest type must have a strictly dominant strategy not to buy, even if everybody else buys, $-M-\epsilon+\frac{1}{8}(1-c_a)^2 - p^*_{q^e=0} < 0$. Note that we need to apply $\lambda^*(q^e=1) = \frac{1}{8}(1-c_a)^2$. As a result, the following requirement for distribution bandwidths is obtained

$$-M - \frac{2}{3}\epsilon < \frac{1}{3}\left[c_f - \frac{1}{2}(1-c_a)^2\right]. \tag{26}$$

The requirements (25) and (26) are satisfied simultaneously when we expand the support of $F(\theta)$ by increasing M sufficiently. When M is sufficiently large, the heterogeneity ϵ can afford to go to zero at the limit. ■

Proof of Proposition 4. The proof follows Morris & Shin (2003). We look for a switching equilibrium with a unique switching point. The switching strategy with a cut-off point k is a function

$$s(x) = \begin{cases} N, \text{ if } x < k \\ B, \text{ if } x > k, \end{cases}$$

where the consumer is indifferent between actions at the switching point k.

When a consumer has observed signal x, he places a (conditional) density $h(\theta \mid x)$ on any state θ. Denote $f(\theta) = \frac{1}{2M}$ as the unconditional density of the uniformly distributed underlying fundamental. When a state θ is realised, signals are also uniformly distributed, so that the density of signals is $g(x \mid \theta) = \frac{1}{2\epsilon}$ on the support $[\theta - \epsilon, \theta + \epsilon]$. The conditional density of θ, when signal x has been observed, is

$$\begin{aligned} h(\theta \mid x) &= \begin{cases} \frac{f(\theta)g(x|\theta)}{\int_{\theta=x-\epsilon}^{x+\epsilon} f(\theta)g(x|\theta)d\theta}, \text{ if } x - \epsilon \leq \theta \leq x + \epsilon \\ 0, \text{ otherwise} \end{cases} \\ &= \begin{cases} \frac{1}{2\epsilon}, \text{ if } x - \epsilon \leq \theta \leq x + \epsilon \\ 0, \text{ otherwise} \end{cases}. \end{aligned}$$

The probability that a signal higher than k is observed when the state is θ is $\mu(k) = 1 - G(k \mid \theta)$, where G is the uniform conditional distribution function of density $g(x \mid \theta)$. The probability $\mu(k)$ is decreasing in k and increasing in θ. By the law of large numbers, $\mu(k)$ equals the probability that fraction $\mu(k)$ of people at maximum get signals higher than k. The expected payoff gain from choosing action $a = B$ for a consumer who has observed signal x and knows that all other consumers will choose action $a = N$ if they observe signals less than k can be written as

$$\begin{aligned} \mathbb{E}[v(x,k,p)] &= \int_{\theta=x-\epsilon}^{x+\epsilon} h(\theta \mid x) v(x, \mu(k), p) d\theta \qquad (27) \\ &= \frac{1}{2\epsilon} \int_{\theta=x-\epsilon}^{x+\epsilon} x + \lambda^*(\mu(k), t^*) - p d\theta, \end{aligned}$$

where $\lambda^*(\mu(k), t^*)$ is the indirect usage utility, as defined in equation (4). The expected payoff (27) is continuous in x and in k despite the kink in the demand for usage.

Next we show that the expected utility $\mathbb{E}[v(x,k,p)]$ is strictly increasing in x and strictly decreasing in k everywhere. Even though usage is always at optimal level from the consumers' point of view, we need to consider two cases in order to prove $\frac{\partial \mathbb{E}[v(x,k,p)]}{\partial x} > 0$ and $\frac{\partial \mathbb{E}[v(x,k,p)]}{\partial k} < 0$. First, when the firm reaches its interior optimal fee $t^* = \frac{1}{2}(1+c_a)$, the expected payoff (27) is $\mathbb{E}[v(x,k,p)] = \frac{1}{2\epsilon}\int_{\theta=x-\epsilon}^{x+\epsilon} x + \frac{1}{8}(1-c_a)^2 \mu(k) - p d\theta$. In the second case, the firm is constrained to $t^* = 1-\mu(k)$, and the expected payoff (27) is $\mathbb{E}[v(x,k,p)] = \frac{1}{2\epsilon}\int_{\theta=x-\epsilon}^{x+\epsilon} x + \frac{1}{2}\mu(k)^3 - p d\theta$.

When consumers are at their optimum, and the firm at the interior solution, we have

$$\frac{\partial \mathbb{E}[v(x,k,p)]}{\partial x} = 1 + \frac{1}{16\epsilon}(1-c_a)^2 > 0,$$

and

$$\frac{\partial \mathbb{E}[v(x,k,p)]}{\partial k} = -\frac{1}{16\epsilon}(1-c_a)^2 < 0.$$

On the other hand, when consumers are at optimum, but the firm is at the corner solution, we have

$$\frac{\partial \mathbb{E}[v(x,k,p)]}{\partial x} = 1 + \frac{1}{16\epsilon^3}\left[\epsilon^2 + 3(\epsilon - k + x)^2\right] > 0.$$

It is equally straightforward to compute that

$$\frac{\partial \mathbb{E}[v(x,k,p)]}{\partial k} = -\frac{3}{8\epsilon^2}\int_{x-\epsilon}^{x+\epsilon} \mu(k)^2 \, d\theta < 0.$$

Combining the results of both cases with continuity of $\mathbb{E}[v(x,k,p)]$, we get that $\frac{\partial \mathbb{E}[v(x,k,p)]}{\partial x} > 0$ and $\frac{\partial \mathbb{E}[v(x,k,p)]}{\partial k} < 0$ hold everywhere. In words, the expected payoff is increasing in own type and in the number of other people playing $a = B$ (i.e. decreasing in the cut-off signal used by other people).

Let $\kappa(k)$ be a point at which $\mathbb{E}[v(x,k,p)] = \mathbb{E}[v(\kappa(k),k,p)] = 0$. This means that the best response to a switching strategy with a cut-off point k is a switching strategy with a cut-off point $\kappa(k)$. In equilibrium, we must have $\kappa(k) = k$. By induction, strategy $s(x)$ survives n rounds of iterated deletion of strictly dominated strategies if

$$s(x) = \begin{cases} N, & x < \underline{\xi}_n \\ B, & x > \overline{\xi}_n, \end{cases}$$

where $\underline{\xi}_0 = -M$ and $\overline{\xi}_0 = M$, and where $\underline{\xi}_n$ and $\overline{\xi}_n$ are defined inductively by

$$\underline{\xi}_{n+1} = \min\left\{x : \mathbb{E}\left[v\left(x, \underline{\xi}_n, p\right)\right] = 0\right\} \tag{28}$$

and

$$\overline{\xi}_{n+1} = \max\left\{x : \mathbb{E}\left[v\left(x, \overline{\xi}_n, p\right)\right] = 0\right\}. \tag{29}$$

First, let us assume that this holds for n rounds. If $a = B$ is the best response to a strategy that has survived n rounds of iterated deletion of strictly dominated strategies, then $a = B$ must be a best response to a strategy with a cut-off rule $\underline{\xi}_n$. The minimal signal x where this holds is defined as $\underline{\xi}_{n+1}$, i.e. $\mathbb{E}\left[v\left(\underline{\xi}_{n+1}, \underline{\xi}_n, p\right)\right] = 0$ holds as proposed by (28). Similarly, if strategy $a = N$ is the best response to a strategy that has survived n rounds of iterated deletion of strictly dominated strategies, then it must be the best response to a strategy with a cut-off $\overline{\xi}_n$. Cut-off point $\overline{\xi}_{n+1}$ is defined as the maximal signal for which this holds.

Since $\mathbb{E}\left[v\left(x, k, p\right)\right]$ is continuous and strictly increasing in x and strictly decreasing in k, the sequences $\underline{\xi}_n$ and $\overline{\xi}_n$ are monotone. The sequence $\underline{\xi}_n$ is increasing, with $\underline{\xi}_0 = -M < \underline{\theta} < \underline{\xi}_1$, where $\underline{\theta}$ is the boundary value for the lower dominance region defined in the proof of Condition 3. Similarly, $\overline{\xi}_n$ is a decreasing sequence, with $\overline{\xi}_0 = M > \overline{\theta} > \overline{\xi}_1$, where again, the boundary value $\overline{\theta}$ is defined as in the proof of Condition 3. As the number of iterations grows $n \to \infty$, the sequences converge $\underline{\xi}_n \to \underline{\xi}$ and $\overline{\xi}_n \to \overline{\xi}$ due to $\frac{\partial \mathbb{E}[v(x,k,p)]}{\partial x} > 0$, $\frac{\partial \mathbb{E}[v(x,k,p)]}{\partial k} < 0$, continuity of $\mathbb{E}\left[v\left(x, k, p\right)\right]$, and the construction of $\underline{\xi}$ and $\overline{\xi}$. Thus, we get $\mathbb{E}\left[v\left(\underline{\xi}, \underline{\xi}, p\right)\right] = 0$ and $\mathbb{E}\left[v\left(\overline{\xi}, \overline{\xi}, p\right)\right] = 0$.

Next we establish that $\underline{\xi}$ and $\overline{\xi}$ coincide. When the equality is true, there is a unique switching strategy with a unique switching point. The probability that a consumer observes a signal higher than k, when the true state is θ, was given by $\mu\left(k\right) = 1 - \frac{k-\theta+\epsilon}{2\epsilon}$. By the law of large numbers, the fraction of other people who observe signals higher that k is less than q when $q \geq 1 - \frac{k-\theta+\epsilon}{2\epsilon}$. This gives

$$\theta \leq k + 2\epsilon q - \epsilon. \tag{30}$$

Write the probability (the consumer assigns to the event) that a proportion less than q of other people observe signals higher than k, when the consumer has observed signal x

$$\Psi(q,x,k)=\begin{cases}1, & k+2\epsilon q-\epsilon>x+\epsilon\\ \int_{\theta=x-\epsilon}^{k+2\epsilon q-\epsilon}h(\theta\mid x)\,d\theta, & x-\epsilon\leq k+2\epsilon q-\epsilon\leq x+\epsilon\\ 0, & k+2\epsilon q-\epsilon<x-\epsilon\end{cases}\tag{31}$$

where $h(\theta\mid x)=\frac{1}{2\epsilon}$ is the density the consumer assigns to state θ when he has observed signal x. When x is observed, states farther than ϵ away from x are assigned zero density. Integration gives

$$\Psi(q,x,k)=\frac{1}{2\epsilon}(k+2\epsilon q-x),$$

for $x-\epsilon\leq k+2\epsilon q-\epsilon\leq x+\epsilon$. In equilibrium $x=k$ as the iterated deletion of strictly dominated strategies suggests. The probability becomes an identity function $\Psi(q,x,x)=q$. The probability $\Psi(q,x,x)$ is also the cumulative distribution function of q on the unit interval $[0,1]$. It is now seen that the distribution of q is uniform on support $[0,1]$, with density $\psi(q)=1$. The expected utility for the action $a=B$ versus $a=N$, when the expected fraction q of neighbours choose $a=B$, is therefore

$$\begin{aligned}\mathbb{E}[v(x,q,p)] &= \int_{q=0}^{1}\psi(q)\,v(x,q,p)\,dq\\ &= \int_{q=0}^{1}v(x,q,p)\,dq.\end{aligned}$$

The indifferent type in equilibrium is given by $\mathbb{E}[v(x,q,p)]=0$. Hence, by the fact that there is a unique solution $\widetilde{x}$ to $\int_{q=0}^{1}v(\widetilde{x},q,p)\,dq=0$ by strict Laplacian state monotonicity, the equilibrium strategy has a unique switching point $x=\widetilde{x}$, which is given by equation

$$\int_{q=0}^{1}v(\widetilde{x},q,p)\,dq=0.$$

The surviving equilibrium switching strategy is

$$s^{*}(\widetilde{x})=\begin{cases}a=B, & \text{if } x>\widetilde{x}\\ a=N, & \text{if } x<\widetilde{x}\end{cases}.$$

■

Proof of Proposition 5. Write the expected profits (17) as

$$\begin{aligned}\mathbb{E}(\Pi)=\frac{1}{2M}\Big[&\int_{\widetilde{x}-\epsilon}^{\widetilde{x}+\epsilon}q(\theta,p)(p-c_f)\,d\theta+\int_{\widetilde{x}+\epsilon}^{M}p-c_f d\theta+\\ &+\int_{\widetilde{x}-\epsilon}^{\widehat{\theta}}q(\theta,p)^2\left[1-c_a-q(\theta,p)\right]d\theta+\int_{\widehat{\theta}}^{\widetilde{x}+\epsilon}\tfrac{1}{4}(1-c_a)^2 q(\theta,p)\,d\theta+\\ &+\int_{\widetilde{x}+\epsilon}^{M}\tfrac{1}{4}(1-c_a)^2\,d\theta\Big].\end{aligned}$$

By differentiating the above expression with respect to p, we get the FOC

$$\frac{\partial \mathbb{E}(\Pi)}{\partial p} = -2p + M + c_f - \tau(c_a) - \frac{1}{4}(1-c_a)^2 = 0. \tag{32}$$

The optimal price is

$$p^* = \frac{1}{2}(M + c_f) - \frac{1}{2}\tau(c_a) - \frac{1}{8}(1-c_a)^2.$$

It is seen from the FOC (32) that the second order condition for local maximum is satisfied

$$\frac{\partial^2 \mathbb{E}(\Pi)}{\partial p^2} = -2 < 0.$$

Because the first period profits maximisation problem is unconstrained, p^* gives the global maximum. ■

Proof of Proposition 7. Write the profit function with optimal price structure as

$$\begin{aligned} \mathbb{E}[\Pi(p^*, t^*)] &= \frac{1}{2M}\left[\int_{\widetilde{x}(p^*)-\epsilon}^{\widetilde{x}(p^*)+\epsilon} q(\theta, p^*)(p^* - c_f)\, d\theta + \int_{\widetilde{x}(p^*)+\epsilon}^{M} p^* - c_f d\theta + \right. \\ &+ \int_{\widetilde{x}(p^*)-\epsilon}^{\widehat{\theta}(p^*)} q(\theta, p^*)^2 [1 - c_a - q(\theta, p^*)]\, d\theta + \\ &+ \int_{\widehat{\theta}(p^*)}^{\widetilde{x}(p^*)+\epsilon} \frac{1}{4}(1-c_a)^2 q(\theta, p^*)\, d\theta + \\ &\left. + \int_{\widetilde{x}(p^*)+\epsilon}^{M} \frac{1}{4}(1-c_a)^2 d\theta \right]. \end{aligned} \tag{33}$$

To see the effect of an increase in the precision of signals, we differentiate (33) with respect to ϵ. By applying the envelope theorem, we get the reported result

$$\frac{\partial \mathbb{E}[\Pi(p^*, t^*)]}{\partial \epsilon} = -\frac{(1-c_a)^4}{192M} < 0.$$

■

Vertical separation: perfect competition in the second period.

The introduction of competition in the second period does not change the solution process. The coordination game satisfies global game conditions.

In the second period, the price is $t = c_a$, which gives the indirect usage utility

$$\lambda^*(q) = \begin{cases} \frac{1}{2}(1-c_a)^2 q, & q \geq 1 - c_a \\ q\left(q - \frac{1}{2}q^2 - c_a q\right), & q < 1 - c_a \end{cases}.$$

The marginal signal is

$$\widetilde{x} = p + \tau_C (c_a),$$

where $\tau_C (c_a) = \frac{1}{24} (1 - c_a)^2 \left[(1 - c_a)^2 - 6 \right] \leq 0$. In calculating the marginal signal, we need to take into account the cut-off point $q = 1 - c_a$.

The firm does not take into account whether consumers are constrained in the second period or not. The firm maximises expected first period profits

$$\mathbb{E} [\Pi_1 (p)] = \frac{p - c_f}{M} (M - p - \tau_C (c_a)).$$

The monopoly price equals

$$p_C^* = \frac{1}{2} (M + c_f - \tau_C (c_a)).$$

The second order conditions are satisfied, $\frac{\partial^2 \mathbb{E}(\Pi_1(p))}{\partial p^2} = -\frac{1}{M} < 0$. If we plug the optimal price back to the demand function, we get

$$q (p_C^*) = \frac{\theta^* + \epsilon - \frac{1}{2} M - \frac{1}{2} c_f - \frac{1}{2} \tau_C (c_a)}{2\epsilon},$$

where θ^* is the realisation of the state θ. If we compare $q (p_C^*)$ with the monopoly demand of the main model

$$q (p^*) = \frac{\theta^* + \epsilon - \frac{1}{2} M - \frac{1}{2} c_f - \frac{1}{2} \tau (c_a) + \frac{1}{8} (1 - c_a)^2}{2\epsilon},$$

we see that demand is higher with competition in the second period, $q (p_C^*) > q (p^*)$. This happens because the monopoly restricts demand in the second period, which reduces expected usage utility.

■

10 Supplementary section: social relations approach

In the main analysis we adopted a "global" way of looking at network externalities. Each consumer has a need to interact with *any randomly chosen* person from the rest of the population. In other words, each consumer is linked with everyone else. In the terminology of graph theory, the population is characterised by a complete graph of social relations. As a result, we can model network externalities with a function that captures the relevant properties of interaction. A complete graph, however, generates the maximal value for a given number of network members, and therefore we risk overestimating network effects when the true network is something less connected (Sääskilahti 2005). If we confine our analysis on *symmetric* local interaction models, it turns out that the model in the main text coincides with a social relations approach. That is proved here.

The class of network models, so called economics of social relations, that has emerged as a refinement to the conventional approach to network effects starts by explicitly considering who is connected to whom. Agents have a varying number of connections, depending on the network they belong to. If we think of personal social relations, it is obvious that some people have more connections (a large family, a lot of friends) whereas some people are more introvert and maintain only few close relationships. Importantly, no-one knows every member of society. Because agents lack connections with members of the network, they cannot have a need to interact *actively* with them. Thus, criticism on the functional form approach to network externalities is valid.

There is a rapidly growing amount of literature on social relations. Applications range from job search and unemployment (Calvó-Armengol & Jackson 2004, Bramoullé & Saint-Paul 2004), to wage differentials between employees (Bentolila et al. 2004, Labini 2004), to public goods provision (Bramoullé & Kranton 2004), buyer-seller networks (Kranton & Minehart 2001) and R&D cooperation (Goyal et al. 2003), to risk sharing (Goldstein et al. 2002) and social learning (Gaduh 2002) in village economies, all the way to crime (Glaeser et al. 1996, Ballester et al. 2004). Chwe (2000) studies how political action diffuses in social networks when agents use the network to communicate their willingness to adopt a revolutionary action. His work is analogous to product diffusion, where certain consumers (rich or pro new technology) buy the product early and who are followed by mass market adoption. Our related work is a model of monopoly pricing of network goods (Sääskilahti 2005). The focus of that article is on how asymmetric social relations affect monopoly price.

Local interaction models comprise a related field of study (see e.g. Young 1998 ch.6). These models focus on the equilibrium selection in dynamic settings where boundedly rational agents interact with a subset of the total population. Agents take myopic actions in a coordination game with exogenous payoffs (no price setting problem). Agents are only imperfectly rational as an occurrence of a mutant agent (who chooses actions randomly) is positive over time.

The complete graph structure incorporated in the model in the main text usually serves as the benchmark case in the social relations literature. Models of social relations focus on (i) asymmetric networks where some agents have more connections than others, and consequently on (ii) local interaction networks.

Asymmetry in social relations affects firm strategy more than the network size. Central people with more connections are more important to the whole network, and tend to capture higher surplus than more peripheral consumers (Sääskilahti 2005). In some networks, asymmetry in the number of connections regularises as the network size grows (random graphs), whereas in other networks it magnifies (scale-free networks) (see Albert & Barabási 2002 for technical review, Barabási & Bonabeau 2003 for informal discussion, and Sääskilahti 2005 for an application to a monopoly pricing problem). Regularisation means that the network, despite being asymmetric, presents a priori regular characteristics. In particular, the number of links each node has in a random graph follows a Poisson distribution. Thus, the average number of links is well-defined. Scale-free networks lack such regular statistical properties.

Let us introduce a symmetric local interaction variant of the model in the main text. There is a mass I of consumers who are exogenously arranged on a graph $\mathcal{G}$ so that each consumer is located on a unique node of $\mathcal{G}$. We normalise $I = 1$ and treat it as continuum as in the main model. The set of undirected edges (links) on $\mathcal{G}$ is $\mathcal{E}$. Two consumers i and j are neighbours if they are connected by an edge, $\{i, j\} \in \mathcal{E}$. The edges are undirected so that, if $(i, j) \in \mathcal{E} \Rightarrow (j, i) \in \mathcal{E}$. The set of consumer i's neighbours is $\mathcal{N}_i$, with $\mathcal{N}_i \neq \emptyset$ so that there are no isolated nodes (i.e. the network is completely connected).[8]

Assumption A1 The graph $\mathcal{G}$ is completely connected, symmetric and n-dimensional.

Symmetry means that everyone has the same number of neighbours, and the graph dimension defines the number of neighbours. Thus, Assumption A1 means that every consumer has $n \in]0, 1]$ neighbours. Consumer i's neighbourhood $\mathcal{H}_i$ is defined as a collection of i and the set of his

[8] Note the difference between complete graph (= everybody is linked with everyone else) and completely connected network (= there is a path between any two network members expanding one or more links).

neighbours, $\mathcal{H}_i = \{i, \mathcal{N}_i\}$. We can allow each consumer's neighbourhood to consist of all other consumers, $\mathcal{H}_i = \{i, \mathcal{G} \diagdown i\}$. This case corresponds to the global interaction model presented in the main text. Since the graph is infinitely large (because consumers are weightless), link configuration with everybody holding identical number of links is guaranteed to exist.

Opposed to the main model, each consumer is now interested only in interacting with his own neighbours only. A link is said to be potentially active if both end nodes have bought the product. Interaction between two consumers is represented by the activation of the link between them. Activation is a directed process, so that both end nodes can activate the same link simultaneously, but only if both have got the product. Reception of an activated link is automatic, free of charge, and gives no utility. Activation yields utility which presents decreasing marginal utility.

The problem for consumer $i \in \mathcal{G}$ is to choose action $a_i \in \{B, N\}$, where $B =$ buy the device and $N =$ do not buy. If he chooses $a_i = B$, then he needs to decide which links he activates in the second period. Define an active link between agents i and j as $e_{ij} = 1$. If only one agent buys or neither buy, the edge cannot be activated: $e_{ij} = 0$. A potentially active link is activated if the consumer pays the activation fee t. Because social relations are exogenous, the activation need is independent of the number of neighbours who buy. Let $\alpha_i = \frac{\sum_{j \in \mathcal{N}_i} e_{ij}}{n}$ be the fraction of active links per total number of neighbours. We have $\alpha_i \in [0, 1]$. $\alpha_i = 1$ means that all links are activated. Symmetry and exogeneity of social relations guarantee $\alpha_i = \alpha$ for all $i \in \mathcal{G}$. This formulation makes it possible that the consumer would like to activate more links than there are potentially active links.

Marginal usage utility is $\frac{\partial \lambda(\alpha, t)}{\partial \alpha} = q(1 - \alpha - t)$, where $q \in [0, 1]$ is the probability that neighbour indexed α has bought the product in the first period and t is the per link activation fee. By law of large numbers, q is the proportion of consumers who bought the device in the first period. Consumer's second period objective is to maximise expected usage utility,

$$\max_{\alpha} \{\lambda(\alpha, t)\}, \text{ s.t. } 0 \leq \alpha \leq q.$$

The optimal level of usage is

$$\alpha^*(t, q) = \min\{1 - t, q\}.$$

Because all consumers are symmetric, firm's second period problem is identical to the one in the main model. In the second period, first period profits and the proportion of consumers who bought the product are fixed, so, we can write second period profits as

$$\Pi_2 = q\alpha^* (t, q) (t - c_a),$$

where $c_a \in [0, 1]$ is the per link activation cost.

The firm charges an activation price such that the consumers are maintained at an efficient level of link activation (see the main text). We can write the firm's second period problem as

$$\max_t \{q\alpha^* (t, q) (t - c_a)\}, \text{ s.t. } t \in [1 - q, 1[.$$

The optimal usage fee is

$$t^* = \max \left\{ \frac{1}{2} (1 + c_a), 1 - q \right\}.$$

We have arrived to identical optimal levels α^* and t^* as in the main model. These give us the value functions $\lambda^* (t^*, q)$ and $\Pi_2^* (t^*, q)$ which match equations (4) and (3) respectively.

For perfect information regime, we have to assume that all neighbourhoods are identical in order to have all the arguments of the main text go through. Identical neighbourhoods mean that in each neighbourhood, consumer types are distributed uniformly over $[\theta - \epsilon, \theta + \epsilon]$. This constraint is quite strong, and it reduces the applicability of the model under perfect information. On the contrary, in the incomplete information regime, we do not need to make any additional assumptions to the main model. If Condition 3 (dominance regions) holds, all the arguments of the main model go through.

3.11.2005

Monopoly Pricing of Social Goods*

Abstract

A product has a social network dimension when its use involves interaction between people. We analyse monopoly pricing in a market where consumers are characterised by their social relations. Consumers get utility from interacting with other people with whom they have a social relation. The monopolist sells a device that enables efficient interaction. We study both symmetric and asymmetric social networks, and analyse the roles of network topology and size on the monopoly price and surplus generated in the network.

This paper introduces two novel features to social relations literature. One, we make players' payoffs endogenous by setting a monopoly pricing problem on top of a coordination game. Two, we abandon the perfect information assumption by limiting players' capacity to observe prevailing information. Asymmetric information eliminates much of the complexity inherent in the perfect information variant: the role of consumer identity is eliminated, while the role of network structure is maintained.

In markets where social relations are important, the implicit assumption on total connectedness in conventional network externalities models exaggerates the value of the network. The topological effect works against, and dominates the size effect. Therefore, the monopolist incorporates network topology in its price. Under asymmetric information, the monopoly prefers symmetric networks, but the social optimum is an asymmetric network. If the firm is allowed to price discriminate, its profits increase to the same level that it obtains in symmetric networks. Monopoly profits and consumer surplus decrease as consumer heterogeneity is increased in symmetric and asymmetric networks. This does not necessarily happen under perfect information; it depends on the network topology.

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1 Introduction

In network economics theory, network effects have been predominantly synthesised in positive externalities: an agent's utility increases as an additional member joins the network. What this approach has overlooked are the effects arising from network topology. In this paper, we analyse how monopoly pricing and welfare depend on the network size (positive externalities) and topology, in markets for network goods such as personal (tele)communications equipment, PC software, and online game consoles.

People have a varying number of social relations. Some people maintain a small number of close relations, whereas some people have a large number of more shallow acquaintances. In some collectivist cultures, family constitutes the main social reference group, whereas in more individualist cultures the most important social relations can be friends outside the family. On group level, cooperation in Japanese keiretsu-groups exceeds that of pure supplier-buyer relationships. In high tech industries, firms form R&D collaboration bodies such as the GSM Association and Open Mobile Alliance. Conventional economic models of networks have abstracted this kind of diversity away. The conventional externalities model building on the seminal work by Farrell & Saloner (1985, 1986), Katz & Shapiro (1985), David (1985) and Arthur (1989) assumes a functional form for network effects: a network member's utility increases directly, the more people join the network.[1] This approach implicitly takes the underlying relations network as a completely connected graph. What it means is that any kind of heterogeneity in terms of social relations is absent. Network members are symmetric in terms of connectedness, therefore, in markets where (asymmetric) social relations are important, conventional models fall short and need to be corrected.

Recent work on the economics of social relations has moved beyond the traditional functional forms of network externalities. Modern models consider richer forms of underlying networks with an emphasis on the network topology. In these models it becomes important to know who is

[1] Network externalities appear most commonly as a linear function of the number of network members. Consider the example by Mason & Valletti (2001): Assume that a link between two network members corresponds to utility equal to 1. When a member indexed n joins the network with $n-1$ existing members, the total utility generated in the network increases by amount $2(n-1)$. The total utility equals $n(n-1)$ in the network of n members. When n is large, we have $n(n-1) \approx n^2$. This corresponds to the famous Metcalfe's Law, which states that the value of the network equals the square of the number of network members.

connected to whom. There can be well-connected members and members with very little relations leading to asymmetric behaviour.

Social relations literature focuses on games that are structured in a form of network. The underlying network is (usually) fixed so that agents inherit their social characteristics from outside the model. On top of the network, agents play a game of perfect information. The interesting question is how network members can benefit from their network position. Our model differs from the previous work in two aspects. First, we introduce a monopoly pricing decision which makes players' payoffs endogenous. The question we are interested in is how an external player (the firm) can take advantage of the network structure. Secondly, we introduce imperfect information.

Communications networks, rural village economies, and job markets are the most obvious examples of markets where social relations have a non-trivial role. Schelling's (1969) model of neighbourhood segregation is an early work on social interaction. Goldstein et al. (2002) and Udry & Conley (2004) study different overlapping social networks including mutual insurance and information sharing in Ghanaian villages. Gaduh (2002) surveys work on social learning networks in village economies. There is a rapidly growing amount of literature on the role of social networks in labour economics. Applications include Calvó-Armengol & Jackson (2004) and Bramoullé & Saint-Paul (2004) who analyse the interdependence between social relations and unemployment. Bentolila et al. (2004) and Labini (2004) compare wage differentials between employees who find their jobs through either formal or informal channels (social relations). Ioannides & Loury (2004) is a survey of the literature on social relations in labour markets. Bramoullé & Kranton (2004) study public good provision, particularly innovation and experimentation, in different network settings. Kranton & Minehart (2001) discuss buyer-seller networks, and how buyers (sellers) can hedge against too high dependency on a single subcontractor (client) by forming links with other subcontractors (clients). Goyal et al. (2003) present a model of R&D where different levels of R&D collaboration are mapped as networks. Glaeser et al. (1996) and Ballester et al. (2004) study how crime rates are affected by social relations. Chwe (2000) analyses how the diffusion speed of political action depends on the social network. Glaeser & Scheinkman (2003) discuss various models of non-market social interactions.

There exists two classes of social relations models.[2] One class treats network structures exogenous to the model, and the other studies endogenous network formation. Jackson (2003) is a survey on endogenous (undirected) network formation models. When link formation is endogenous, it must comprise of all relevant aspects. Therefore, endogenous network formation models tend to be less applicable to problems associated with personal social relations. The economic dimension in a personal, say a friendship, link formation is often marginal and difficult to isolate. In contrast, if the network of social relations is exogenous, we can immediately focus on a specific economic problem, such as whether to buy or not a mobile phone. The applied fixed network structure can reflect personal relations which give utility that is hard to measure against utility from consumption of mobile services. Moreover, social relations in many cases exist prior to the decision making. When we think about buying a mobile phone, we think about with whom we can use it; not how many new contacts we can make when using it. Hence, there is a reason for separating the social aspects from economic decision making and taking them as exogenous parameters. But, the separation of social relations from economic decisions does not mean that they are irrelevant. In models where the network does not characterise personal relations, such as firm-level R&D networks, endogenous link formation fits well, because all link formation decisions involve payoff of the same kind. Indeed, Goyal et al. (2003) and Kranton & Minehart (2001), who analyse firms as decision makers, consider endogenous network formation.

In this paper, we depart from the implicit assumption of complete graphs of conventional network externalities models. Consumers are characterised by their exogenous personal social networks. Each person is interested only in interaction with a subset of the population, called his neighbourhood, with social relations being determined outside of the model. A social link between consumers could mean for example that they are friends, relatives, or colleagues that tend to do things together and thus have a need to interact. We analyse a monopoly market for pure coordination goods which do not carry any standalone value. Consumers must decide whether to switch to the new efficient good or whether to stay using the legacy system. Because all

[2] Local interaction models form a related class of dynamic games (see Ellison 1993, Young 1998 ch.6, Lee & Valentinyi 2000 and Morris 2000). Local interaction models analyse how a particular equilibrium play becomes adopted in the long run. Key features of local interaction games are fixed network structures, imperfect rationality of agents, and exogenous payoffs.

utility is generated in interaction between people (efficient interaction is possible only if all parties have the device), consumers need to coordinate their purchases. Consumers are heterogenous with respect to the valuation of the new good. What is important is that consumers cannot tailor their actions vis-à-vis each neighbour. They take a single action that applies to every neighbour. This way, consumers must consider the overall network structure, rather than each particular link separately. The firm decides on an (introductory) price.

We consider two informational regimes. One, where information is perfect. In the other case, buyers' valuations of the goods are private information. We give general characterisations of both cases and apply them to three network topologies: complete graph, circle, and star. The complete graph and the circle are symmetric networks, whereas the star is asymmetric.

We show how the topology of the social network affects the firm's pricing strategy and total surplus generated in the network. Under perfect information, the monopolist covers the whole market even if it is unable to price discriminate in some network structures. In identical networks, except in terms of who is connected to whom, the firm may choose to limit supply. It is shown how some agents have preferential roles through their connections. These critical agents are able to capture higher surplus than other agents. Critical positions exist in asymmetric and symmetric networks under perfect information. In symmetric networks, critical positions are due to consumer heterogeneity. Agents who have links *with* high types are critical, as opposed to the high types themselves. When information is reduced to asymmetric, critical agents lose their market power in symmetric networks. On the other hand, in asymmetric networks, the topologically central agents always capture higher utility. This is not true necessarily with perfect information.

There are three main findings in the paper. One, network topology matters in pricing. Two, the implicit complete graph assumption of the conventional network externality model risks seriously overestimating the value of network effects. Three, with private information, asymmetric networks yield lower monopoly profits, but higher total surplus, than symmetric networks of a given link value. However, the firm can match the profits generated in symmetric networks if price discrimination according to the network position is allowed, but this reduces total surplus.

In section 2, we formulate the utility function and formalise the social network. In section 3, we study the perfect information case. In section 4, we analyse the asymmetric information case. In section 5, we present extensions to the basic model. We conclude in section 6.

2 Network structure and actions

2.1 Overview

The firm launches an innovative new device that constitutes an efficient medium for interaction. The product supersedes earlier generations of products serving similar interaction needs. The product has no intrinsic value as it is used only when two people are interacting with each other. As a consequence, a potential buyer needs to estimate what proportion of other people buy it. People are heterogeneous with respect to attainable network benefits. For example, some people like to write letters (the conventional way to interact), whereas some people prefer to send e-mails (the novel product). An exogenous network of nodes and links characterises the population with each node hosting one consumer. A link between two consumers (nodes) represents a social relation. Its origin is in e.g. family ties, friendships, or occupational contacts. The mapping, or graph, of all social relations gives information about who is interested in interacting with whom. If both end nodes buy the new product, we say that the link between them becomes active, which represents efficiently mediated interaction between consumers. The firm on the other hand has to decide on the price of the product. A low price may help solving buyers' coordination problems, but it erodes margins.

We think of products such as the fax machine that is a relatively drastic innovation in the sense that it is not compatible with earlier generations of products (postal and courier services). Other examples are PC and mobile phone operating systems, software applications such as spreadsheet that enable collaboration, and the fixed line telephony in the late 1800's.[3]

There tends to be multiple equilibria, because like any network externalities model, our model is inherently a coordination game.[4] A coordination failure occurs whenever coordination fails to

[3] See Gandal (1994) for an analysis of network externalities in the spreadsheet software.

[4] Our related paper, Sääskilahti (2005), focuses on solving the multiplicity problem. In that paper, we analyse how equilibrium uniqueness is attainable in a monopoly model of network goods under perfect and incomplete information. Perfect information requires that network externalities are sufficiently low. Under incomplete information, uniqueness is reachable via global games theory.

reach the Pareto-efficient outcome. Our model relates coordination failures to situations where ex post demand falls short of the forecasted demand. From a dynamic perspective, a coordination failure occurs when demand fails to grow above a critical level above which network externalities self-propagate demand. The firm's problem is how to bridge this "chasm" between low and high equilibria.[5] We justify the use of the maximal coordination equilibrium by observing that the underlying coordination game is supermodular with positive spillovers. Consequently, the maximal coordination equilibrium Pareto-dominates other equilibria, which, we argue, focalises the equilibrium.

2.2 Model

The timing of events is that consumers draw their types θ, then the firm sets its price p, after which consumers decide on buying. Let the population of individuals $\mathcal{I} = (1, ..., I)$, $I \in \mathbb{N}$ be located on the graph $\mathcal{G}$ so that there is a unique individual located on each node of the graph. The set of undirected links, or edges, between the nodes of $\mathcal{G}$ is $\mathcal{E}$. An edge represents a social relation. Two consumers i and j are neighbours if they are connected by an edge, $\{i, j\} \in \mathcal{E}$. Undirectedness of all edges guarantees symmetry so that, if $(i, j) \in \mathcal{E} \Rightarrow (j, i) \in \mathcal{E}$. The set of neighbours of consumer i is $\mathcal{N}_i = \{j \in \mathcal{I} \setminus i\}$, with $\mathcal{N}_i \neq \emptyset$ so that there are no isolated nodes. The consumer cannot be his own neighbour, $i \notin \mathcal{N}_i$. The graph $\mathcal{G}$ is completely connected so that there exists a path between any two nodes. The neighbourhood $\mathcal{H}_i = \{i, \mathcal{N}_i\}$ of consumer $i \in \mathcal{G}$ is defined as a collection of agent i himself and the set of his neighbours $\mathcal{N}_i$. Consumer i has an interest in interacting only with the people in his neighbourhood.

The network inherits its structure from outside the model. Links represent personal relationships with e.g. family members, friends and colleagues. These connections have been formed prior to the launch of the product. Because the network is stable over time, the firm is able to acquire information about its topology and size. As a result, we assume that the structure of the graph $\mathcal{G}$ is common knowledge.

The problem for the consumer $i \in \mathcal{G}$ is to choose action $a_i \in \{B, N\}$, where $B =$ buy the new

[5] The taxonomy of bridging "the chasm" between early and mass market adoption is due Moore (1999).

device and $N =$ do not buy. The activity of link between i and j is represented by $e(a_i, a_j) \equiv e_{ij}$. A link between neighbours becomes automatically active if both end nodes buy the goods, defined as $e_{ij} = 1$. If only one agent buys or neither buy, the edge remains inactive, $e_{ij} = 0$.

The value of an inactive link is normalised to zero. This value represents the utility from interaction with the help of older generation systems. Interaction generates positive utility when it is facilitated by the new device. This can be thought as an efficiency gain or additional utility obtained from the types of interaction not previously possible. Consumer i gets utility θ_i from each activated link. The value θ_i is an i.i.d. random variable across consumers $i \in \mathcal{G}$. It is drawn from a uniform distribution $F(\theta)$ with the support $[\theta^-, \theta^+]$, with $\theta^- \geq 0$. We assume that the valuation θ_i of consumer $i \in \mathcal{G}$ is independent of the network location he occupies. Why is this? The social relations are formed prior to the model and they are independent of the value the consumer puts on the new device. Exogeneity of the network rules out those cases where the new device would create a new link with a formerly unknown person.[6] We analyse two informational regimes. Under perfect information, all types θ are revealed to everybody, including the firm, before the firm sets the price. Under asymmetric information, types θ are private information, and the firm observes nothing. This asymmetry is based on the assumption that consumers know their own needs better than the firm. The distribution $F(\theta)$ is common knowledge.

The question whether a link is active or not, builds another (technical) layer on top of the inherent (social) network. This way, we differentiate between the exogenous social network and the endogenous technical network. The following definition characterises the degree of activity on the technical network.

Definition 1 *Technical network is said to be*

(i) *a complete network, when* $a_i = B$ *for all* $i \in \mathcal{G}$.

(ii) *an empty network, when* $a_i = N$ *for all* $i \in \mathcal{G}$.

(iii) *a partial network, when* $a_i = B$ *for at least one* $i \in \mathcal{G}$ *and* $a_j = B$ *for at least one* $j \in \mathcal{N}_i$, *and* $a_k = N$ *for at least one* $k \in \mathcal{G}$, $k \neq i, j$ *simultaneously.*

All interaction is mediated by the new product in a complete network. A partial network is a network where some interaction is mediated by the new product. Note that under perfect

[6] See section 5.2 for discussion on more complex utility specifications.

information, the minimum number of consumers which buys is always two (neighbours), whereas under asymmetric information, there can be isolated consumers who buy the good while none of his neighbours buy. In the empty network, no-one uses the new product.

Throughout the paper we are interested in the role of the social network's structure on the activity level on the technical network. We consider three network topologies:

- Complete graph, where each consumer is connected to everybody else, $\mathcal{N}_i = \{\mathcal{G} \setminus i\}$ for all $i \in \mathcal{G}$. The complete graph is the structure used implicitly by conventional network externalities models.

- Circle, where each consumer is connected to exactly two neighbours. When agents are indexed in ascending order, the consumer labelled i has neighbours $\mathcal{N}_i = \{i-1, i+1\}$. The links form a circle, as consumer labelled I is connected to consumers $I-1$ and 1.

- Star, where one consumer is a central agent with connections to everybody else, and where peripheral agents are linked only to the centre. Centre's set of neighbours is $\mathcal{N}_C = \{\mathcal{G} \setminus C\}$, where C is the index for the centre. A peripheral consumer's only neighbour is the centre, $\mathcal{N}_i = \{C\}$, $i \in \{\mathcal{G} \setminus C\}$.

The network is symmetric if all consumers have identical number of links. Network symmetry implies that any two neighbourhoods are symmetric, but the reverse is not necessarily true. The complete graph and the circle are symmetric networks, whereas the star is asymmetric.

The link e_{ij} comprises two directed links (i,j) and (j,i). With I consumers, the complete graph has $I(I-1)$ directed links, whereas the circle has $2I$ and the star $2(I-1)$ directed links. We can do a comparison across different networks either by fixing the number of agents or the number of links. By construction, the complete graph generates the highest maximal link value for a given number of consumers. If we control for the link value of the network, we need to compensate the less connected networks by increasing the number of nodes. With a given total number of directed links in the complete graph, $I(I-1)$, the corresponding compensated number of consumers in the circle is $\frac{I(I-1)}{2}$. Respectively, the compensated star has $1 + \frac{I(I-1)}{2}$ consumers.

In the real world, we can observe almost infinite number of different network structures. Instead of analysing more complex topologies, we opt for the most primitive ones in order to obtain clear-cut results. This approach is not different from the social relations literature. It is typical that any large-scale network turns out to be analytically cumbersome. In spite of primitivity, the three example topologies bring out topological effects missing in conventional externalities models. We discuss how our results generalise to two more complex network types, namely the random network and the scale-free network, in section 5.3.

3 Perfect information

We start with the case where all information is revealed to all before the firm sets the price. The consumer i receives utility $u_i(\theta_i, B)$ if he buys the product

$$u_i(\theta_i, B) = \sum_{j \in \mathcal{N}_i} e_{ij}\theta_i - p, \tag{1}$$

where $e_{ij} = \{0, 1\}$ captures link activity, and p is the unit price for the device.[7] If the consumer does not buy, he receives zero utility. At the margin, the agent is indifferent between buying and not when his valuation is

$$\widetilde{\theta}_i = \frac{p}{\sum_{j \in \mathcal{N}_i} e_{ij}}.$$

The better connected the agent is, the lower is his marginal value.

The coordination game Γ consists of consumers $\mathcal{I}$ arranged on the graph $\mathcal{G}$, pure actions $a \in \{B, N\}$, and payoffs $u_i(N) = 0$ and $u_i(B)$ given by equation (1) for all $i \in \mathcal{G}$, and it is parameterised by the unit price p. Let $\mathbf{a}_{\mathcal{N}_i} = (a_j \mid j \in \mathcal{N}_i)$ be the vector of actions taken by consumer i's neighbours. The consumer i's best response is $a_i^* \in \arg\max_{a_i \in \{B,N\}} u_i(\theta_i, a_i, \mathbf{a}_{\mathcal{N}_i})$. Nash equilibrium (NE) of Γ is the strategy profile $\mathbf{a}^* = (a_1^*, ..., a_I^*)$ which maximises the consumer's utility, $u_i(\theta_i, a_i^*, \mathbf{a}_{\mathcal{N}_i}^*) \geq u_i(\theta_i, a_i, \mathbf{a}_{\mathcal{N}_i}^*)$ for all $i \in \mathcal{G}$.

[7] To be precise, e_{ij} indicates if the link becomes active when i buys the good, given that j buys the good. If $e_{ij} = 1$, the link between i and j is potentially active, and it becomes active when i buys as expectations on e_{ij} are fulfilled in equilibrium. We can also write the utility with social relations explicitely expressed, $u_i(\theta_i, B) = \sum_{j \in \{\mathcal{G} \setminus i\}} g_{ij} e_{ij} \theta_i - p$, where $g_{ij} = \{0, 1\}$ indicates whether i and j are neighbours ($g_{ij} = 1$) or not ($g_{ij} = 0$).
If we write the utility as $u_i(\theta_i, B) = \alpha + \sum_{j \in \mathcal{N}_i} e_{ij}\theta_i - p$, where $\alpha = 0$ is the intrinsic utility from the good, we see that the utility function is of the type where consumers have differentiated valuation of network benefits. Such utility formulation has been used by de Palma & Leruth (1996). Compare this with Katz & Shapiro (1985) specification where consumers are differentiated according to the intrinsic utility α.

Lemma 2 *The action profile* $\mathbf{a}^* = (a_1^*, ..., a_I^*)$ *is a Nash equilibrium with perfect information, if*

$$\begin{array}{l} a_i^* = B \Leftrightarrow \theta_i \geq \widetilde{\theta}_i \\ a_i^* = N \Leftrightarrow \theta_i < \widetilde{\theta}_i \end{array},$$

where $\widetilde{\theta}_i = \frac{p}{\sum_{j \in \mathcal{N}_i} e_{ij}^*}$ *and* $e_{ij}^* = e\left(a_i^*, a_j^*\right)$ *for all* $i \in \mathcal{G}$.

The coordination game has multiple equilibria. In particular, the empty network is always NE. A total coordination failure occurs when all consumers expect that no-one will buy, they play $a = N$ "stubbornly" irrespective of valuations in other words.[8] Equilibria impaired with coordination failure of smaller sets of consumers (than the total population) are also possible due to an exogenous network structure. We argue that equilibrium selection is likely to favour efficient coordination, because it corresponds to Pareto efficient NE.

Lemma 3 *The coordination game* Γ *is supermodular with positive spillovers (action complementarity).*

Proof.

(i) *Action set* $a = \{B, N\}$ *is a compact subset of* $\mathbb{R}$.

(ii) *The payoffs show increasing differences. If proportion* $k = |a_j = B|$, $j \in \mathcal{N}_i$ *of* i*'s neighbours play* B, *the number of active links is* $\sum_{j \in \mathcal{N}_i} e_{ij} = k$ *when* i *plays also* B. *The payoff of* $a_i = B$ *versus* $a_i = N$ *is* $v_i(\theta_i, k) = u_i(\theta_i, k, B) - u_i(\theta_i, k, N) = k\theta_i - p$. *Then* i*'s payoff gain* $v_i(\theta_i, k)$ *is strictly increasing in* θ_i *for all* $i \in \mathcal{G}$.

(iii) *The payoff function* $u_i : \{B, N\} \times \theta \to \mathbb{R}$ *is continuous.*

(iv) *The payoff gain* $v_i(\theta, k)$ *is strictly increasing in* k.

Steps (i)-(iii) prove the supermodularity of the game Γ. *Positive spillovers result from (iv).* ■

Topkis' theorem guarantees that the supermodular game Γ has the largest and the smallest NE elements (Vives 2001, p. 33). The smallest NE is the empty network. The largest equilibrium, on the other hand, depends on price p and corresponds to efficient coordination. Due to positive spillovers, the largest NE is Pareto-dominating (Vives 2001, p. 34). Supermodularity with positive spillovers applies to both symmetric and asymmetric social networks.

We assume that Pareto-dominance makes the equilibrium focal. Especially, allowing pre-game communication, efficient coordination should be more likely although it is not guaranteed. Since

[8] Consider a duopoly competing in introducing of new products. Products are differentiated by quality. Farrell & Katz (1998) call consumers' expectations "stubborn in favor of firm k" when a consumer expects that all other consumers prefer firm k's product irrespective of current market prices. All consumers buy always from firm k, except in the cases where the rival l's quality advantage is large enough to overcome the expected network benefits from the total network. Motivation for such stubborn expectations is in exogenous conditions, e.g. when firm k has a strong financial position compared to the rival or it has a good reputation. Note that consumers are perfectly rational, and the resulting equilibrium belongs to the class of fulfilled expectations.

the consumers' social relations have been established prior to the coordination game, it is likely that consumers use the social network to communicate their buying intensions. Also the firm could help coordination by advertising, for example. There could be other focal points that favour efficient coordination as well. Some neighbours might be known to work in the high tech industry or be otherwise pro new technology. Alternatively, macrofactors such as a technology boom could trigger efficient outcomes. On the other hand, technology antagonism works against efficient coordination.

When considering the firm's problem, we focus on the maximal NE. Denote $b(p)$ as the largest possible number of consumers who buy (in the maximal NE). The function $b(p)$ is confined in the interval $b(p) \in [0, I]$, and it is decreasing in p with possible large discontinuities (drops).

The firm observes the realisations of θ and sets the price p. It cannot price discriminate between consumers. If price discrimination was allowed, the firm would capture all surplus from every consumer. The resulting technical network would always be a complete network. The pricing problem becomes interesting when the firm must choose one price that applies to everyone.

The firm's problem is to maximise profits $V = b(p)(p - c)$. Marginal cost is constant $c \geq 0$, and there are no fixed costs. The optimal price is given by equation (2).

$$p^* = \arg\max_p \{b(p)(p - c)\} \tag{2}$$

We have now characterised the model under perfect information. Next we apply the general framework to the complete graph, circle and star. We do a comparison across networks in section 3.4. Detailed analyses of the cases discussed in the comparison are provided in appendix 8.1.

3.1 Complete graph

The specific location of a consumer on the underlying social network is irrelevant in a complete graph because each consumer is connected to everybody else. Utility for the consumer i can be written as $u_i(\theta_i, B) = \sum_{j \in \{\mathcal{G} \setminus i\}} e_{ij}\theta_i - p$. NE of the coordination game is expressed in lemma 4.

Lemma 4 *The action profile* $\mathbf{a}^*$ *is a NE, if for all* $i \in \mathcal{G}$,

$$a_i^* = N \Leftrightarrow \theta_i < \frac{p}{\sum_{j \in \{\mathcal{G} \setminus i\}} e_{ij}^*}$$
$$a_i^* = B \Leftrightarrow \theta_i \geq \frac{p}{\sum_{j \in \{\mathcal{G} \setminus i\}} e_{ij}^*}$$

All network forms are sustainable in equilibrium, conditional on price p and the realisations of θ. Empty network is NE when all agents face $\mathbf{a}_{\mathcal{N}_i} = (N, ..., N)$ or, if for all $i : \theta_i < \frac{p}{I-1}$. Complete network is a feasible NE only if for all $i : \theta_i \geq \frac{p}{I-1}$. Partial network is a feasible NE if for at least two agents $\theta_{i,j} \geq \frac{p}{\sum_{k \in \{\mathcal{G} \setminus i,j\}} e_{ik,jk}}$, $i \neq j$, and at least one agent has $\theta_h < \frac{p}{\sum_{l \in \{\mathcal{G} \setminus h\}} e_{hl}}$, $h \neq i, j$, simultaneously. The game can produce multiple equilibria in the price range $p \in \left[(I-1)\,\theta^-, (I-1)\,\theta^+\right]$, where θ^- and θ^+ are the lower and upper boundary of the distribution $F(\theta)$. The maximal NE is the Pareto efficient NE.[9]

The firm maximises profits $V = b(p)(p - c)$ with price $p^* \in \left[(I-1)\,\theta^-, (I-1)\,\theta^+\right]$. Function $b(p)$ gives the largest number of agents who buy for a given price p. The function $b(p)$ is decreasing in p, with a ceiling $b\left((I-1)\,\theta^-\right) = I$ and a floor $b\left((I-1)\,\theta^+ + \varepsilon\right) = 0$, where $\varepsilon > 0$ is small. Price $p = (I-1)\,\theta^-$ guarantees that all agents buy in the maximal NE, and $p = (I-1)\,\theta^+ + \varepsilon$ guarantees that nobody buys. Example 25 in the appendix analyses how the firm sets price in a four consumer complete graph.

3.2 Circle

In the circle each consumer has exactly two neighbours. Utility from $a = B$ can be written as $u_i(\theta_i, B) = (e_{i,i-1} + e_{i,i+1})\,\theta_i - p$. We obtain a three-partition of types. Low types are consumers who never buy. Medium types are those who buy only if both of their neighbours buy. High types are those who buy if at least one of their neighbours buys.

Lemma 5 *Let a low type have a valuation $\theta < \frac{1}{2}p$. Similarly, let a medium and high type have valuations $\frac{1}{2}p \leq \theta < p$ and $\theta \geq p$ respectively. Then the action profile $\mathbf{a}^*$ constitutes a NE if*

(i) $\mathbf{a}^*_{\mathcal{N}_i} = (N, N) \Rightarrow a^*_i = N$ *for all* $i \in \mathcal{G}$.

(ii) $\begin{cases} \mathbf{a}^*_{\mathcal{N}_i} = (B, N) \text{ or } (N, B) \Rightarrow a^*_i = N \text{ for low and medium types.} \\ \mathbf{a}^*_{\mathcal{N}_i} = (B, N) \text{ or } (N, B) \Rightarrow a^*_i = B \text{ for high types.} \end{cases}$

(iii) $\begin{cases} \mathbf{a}^*_{\mathcal{N}_i} = (B, B) \Rightarrow a^*_i = N \text{ for low types.} \\ \mathbf{a}^*_{\mathcal{N}_i} = (B, B) \Rightarrow a^*_i = B \text{ for medium and high types.} \end{cases}$

Lemma 5 shows that all activity levels are feasible as NE, conditional on price p and realisations of θ. It is also evident that network structure matters more than in the case of a complete network.

[9] Consider a complete graph of four agents with valuations $\theta_1 < \theta_2 < \theta_3 < \theta_4$. Let $\theta_3 > p$, and assume that θ_3 and θ_4 buy. If $3\theta_1 > p$, then the Pareto optimal NE is with all four agents buying. However, if also $2\theta_2 < p$ holds, then we have two possible non-empty NE (and the empty network NE). One where all four agents buy, and the other where only agents θ_3 and θ_4 buy.

The consumer's action depends on the fact which types his neighbours happen to be. As with the complete graph, also the circle can produce multiple equilibria.[10]

The firm maximises profits $V = b(p)(p - c)$ with price $p^* \in [2\theta^-, 2\theta^+]$. Function $b(p)$ gives the number of buyers in the maximal equilibrium. It is decreasing in p, with upper bound, $b(2\theta^-) = I$, and lower bound $b(2\theta^+ + \varepsilon) = 0$ (ε small and positive). See example 26 in the appendix for an example how the monopolist sets the price in a four consumer circle.

3.3 Star

The star formation is asymmetric with a single central agent who is connected to $I - 1$ peripheral agents. The peripheral consumers are connected only to the centre. Centre's utility from buying is $u_C(\theta_C, B) = \sum_{i \in \mathcal{N}_C} e_{Ci}\theta_C - p$, $\mathcal{N}_C = \{\mathcal{G} \setminus C\}$, where C stands for "centre". Peripheral consumer's utility is $u_i(\theta_i, B) = e_{iC}\theta_i - p$, for all $i \neq C$.

Lemma 6 *The action profile* $\mathbf{a}^*$ *is a NE if*

(i) *For centre* $C \in \mathcal{G}$:

$$\mathbf{a}^*_{\mathcal{N}_C} = (N)^{\mathcal{N}_C} \Rightarrow a^*_C = N.$$
$$\mathbf{a}^*_{\mathcal{N}_C} = (a_i)^{i \in \mathcal{N}_C}, \text{ and not all } a^*_i = N \Rightarrow a^*_C = N \text{ if } \theta_C < \frac{p}{\sum_{i \in \mathcal{N}_C} e^*_{Ci}}.$$
$$\mathbf{a}^*_{\mathcal{N}_C} = (a_i)^{i \in \mathcal{N}_C}, \text{ and not all } a^*_i = N \Rightarrow a^*_C = B \text{ if } \theta_C \geq \frac{p}{\sum_{i \in \mathcal{N}_C} e^*_{Ci}}.$$

(ii) *For all peripheral agents* $i \in \{\mathcal{G} \setminus C\}$:

$$a^*_C = (N) \Rightarrow a^*_i = N.$$
$$a^*_C = (B) \Rightarrow a^*_i = N \text{ if } \theta_i < p.$$
$$a^*_C = (B) \Rightarrow a^*_i = B \text{ if } \theta_i \geq p.$$

The firm has to set the price low enough to attract the central agent and at least one peripheral consumer to buy. Let $b_C(p)$ be centre's quasi-demand, and $b(p)$ the largest number of peripheral agents who buy for a given price p. Centre's quasi-demand is a step-function

$$b_C(p) = \begin{cases} 0, \text{ if } p > \overline{u}_C \\ 1, \text{ if } p \leq \overline{u}_C \end{cases},$$

where $\overline{u}_C = b(p)\theta_C$ is the utility from active links. The lower and upper bounds for $b(p)$ are $b(\min\{\theta^+, (I-1)\theta_C\} + \varepsilon) = 0$, and $b(\theta^-) = I - 1$, which take into account the centre's and periphery's topological differences. Between the limits, the function $b(p)$ is decreasing in p with

[10] As an example of multiplicity of equilibria, consider a sequence of four agents of a circle, and assume that the price is $p \in (2\theta^-, \theta^+)$. Assume that the agents at the ends of the sequence are high types and they play B in equilibrium, and the middle agents are of medium type. Then, the middle agents can either both play B or N. Both $(..., B, B, B, B, ...)$ and $(..., B, N, N, B, ...)$ constitute NE, with all buy NE being the Pareto dominant.

possible large drops. In order to evade the empty network, the firm must set $b_C(p) = 1$. Hence, the firm's problem is to maximise profits, $V = [1 + b(p)](p - c)$ subject to $p \leq \overline{u}_C$. See example 27 in the appendix how the monopolist sets the price in a four consumer star.

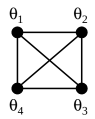
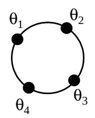
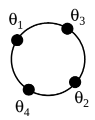
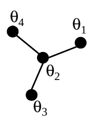
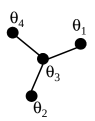
3.4 Comparison of networks

In this section, we study the differences between the complete graph, circle, and star. It is a matter of substance whether we should take the number of consumers or the link value as the primitive of the model. In most cases, a fixed number of consumers is the appropriate set-up, since it is the consumer who makes the decision. However, the comparison across different network types when the number of consumers is fixed, comprises the size effect (number of links) and the topological effect (link wiring). If we fix the value of the network, we can isolate the topological effect. Due to the overwhelming number of different cases under perfect information, a comparison of compensated networks is unfeasible to carry out. For example, a complete graph with four consumers corresponds to a compensated circle with six consumers. A circle of six consumers has 720 permutations (of which half are mirror images). Fortunately, it is easy to distinguish between the topological effect and the size effect.

Consider a complete graph, a circle and a star of four agents with valuations $\theta_1 \leq \theta_2 \leq \theta_3 \leq \theta_4$. We assume $c = 0$ for expositional reasons. Table 1 gives firm's profits in the maximal NE for different social networks. The social networks are given in the rows, columns correspond to the technical networks (activity level). Tables 2 and 3 in the appendix 8.2 present consumer surplus and total surplus (consumer surplus plus profits). The different network examples are detailed in appendix 8.1. We summarise the results from the comparison in remarks 7-12.[11]

[11] We label the observations from numerical examples as *remarks*, in order to distinguish them from analytical *propositions* with formal proofs.

Table 1: Profits, $\theta_1 \leq \theta_2 \leq \theta_3 \leq \theta_4$

	Complete network	3-buyer network	2-buyer network
Complete graph 	$4\,(3\theta_1)$	$3\,(2\theta_2)$	$2\,(\theta_3)$
Circle A 	$4\,(2\theta_1)$	$3\,(\theta_2)$	$2\,(\theta_3)$
Circle B 	$4\,(2\theta_1)$	$3\,(\min\{2\theta_2, \theta_3\})$	Dominated
Star, 2 as centre 	$4\,(\theta_1)$	$3\,(\min\{2\theta_2, \theta_3\})$	Dominated
Star, 3 as centre 	$4\,(\theta_1)$	$3\,(\theta_2)$	$2\,(\theta_3)$

From table 1 we see that alternative social structures support different optimal monopoly prices. Optimal monopoly price is affected by the number of links, the topology of the social network, and agent configuration (which types are connected, e.g. circle A and B have the same topology, but are different configurations).

Remark 7 *Monopoly profits and price are (weakly) increasing in the number of links.*

When a link is added to the network, there are no consumers whose utility would be negatively affected by the addition, prior any price modifications. The addition may increase utility of some consumers, which is the reason why the firm can potentially increase its price. A price increase is feasible if the added link is not redundant so that the link effectively eases the pricing constraint. The monopolist's capacity to capture the increase in network value depends on the network topology and agent configuration.

Remark 8 *(i) Complete graph generates the highest total surplus. (ii) Consumer surplus and total surplus are maximised in a complete network in all social network topologies.*

The more links there are in the network, the higher is the generated value in the network. Part (ii) of remark 8 is an implication of supermodularity and action complementarity of the coordination game. Since profits are just transfers from consumer surplus, total surplus is maximised when the maximal number of links is activated. Consumer surplus is maximised in the complete network because the price is the lowest in the complete network.

Remarks 7 and 8 comprise the size effect. A more complex issue is how the network topology and agent configuration affect the price level. Remarks 9-12 summarise these effects.

Remark 9 *Agent configuration is irrelevant in pricing if the social network is a complete graph. In other social network topologies, configuration matters.*

Compare the circles A and B, and assume that a 3-buyer network maximises profits. The networks differ only in the way who is connected to whom. Still, the monopolist makes higher profits in B. Consumer θ_2 benefits from the links with high types θ_3 and θ_4, and the firm is able to capture some (or all) of this rent. In the circle A, the fact that low types are neighbours leads to lower profits. When we compare 2-buyer networks, we see that in the circle B and the star with θ_2 at the centre, 2-buyer networks are always dominated (in terms of profits) by 3-buyer networks. On the contrary, in the circle A and the star with θ_3 at the centre, they do not have to be, because high types are clustered. If a low valuation consumer is in a focal position (θ_2 in the circle B or in star's centre), the firm may be forced to sell at a lower price in order to guarantee his participation.

There are two types of critical consumers who have connections that are important from all network members' perspective. One type are focal topology-wise, e.g. the centre in a star. The second, more subtle, type is focalised by high heterogeneity between the critical consumer's and his neighbours' valuations. Consumer θ_2 in the circle B is an example of this type. His position is critical and constrains pricing if his type is sufficiently low ($2\theta_2 < \theta_3$), otherwise the connections with high types θ_3 and θ_4 are redundant in the sense that his participation is guaranteed. If $\theta_3 < 2\theta_2$, price is not constrained due to him, and his position is actually beneficial to the firm.

Remark 10 *Critical agents have (i) topologically central positions (e.g. centre in star), (ii) important connections (low types with high type neighbours). The existence of critical agents can increase profits or constrain the optimal price depending on the network topology and agent configuration.*

Under perfect information, the optimal monopoly price is determined by the combination of consumer heterogeneity and social network structure. Consider the circles A and B again. Let the complete network be optimal in A. This means that $8\theta_1 > \max\{3\theta_2, 2\theta_3\}$. If we also have $8\theta_1 < 6\theta_2$ and $2\theta_2 > \theta_3$ it is optimal for the firm to choose the 3-buyer network in B. Why? The firm finds it profitable to increase the price so that θ_1 opts out. At the same time, the high types θ_3 and θ_4 induce their common neighbour θ_2 to purchase. Hence, in some graphs the monopolist limits supply whereas in other graphs that are identical save the configuration of agents, it covers the whole market. Full coverage is more likely when consumers' relative valuations are close together. In more heterogenous markets, the monopolist is better off by excluding the lowest types from the market by setting a sufficiently high price.

Remark 11 *The firm excludes low types in (relatively) heterogeneous markets. Homogeneous markets are completely covered.*

When consumers are homogeneous, the firm prefers to have high types dispersed in the network. Dispersed high types support the purchases of lower types. On the other hand, if the valuations are highly heterogeneous, so that the firm prefers to exclude the low types, the dispersion of high types hurts the firm as the price is constrained by the critical (low) types.

Remark 12 *In homogeneous markets, the dispersion of high types is good for the firm. In heterogeneous markets, the dispersion of high types constrains the firm.*

Figure (1) illustrates how the monopoly's choice affects the total surplus created in the network.[12] We have used valuations $\theta_1 = 1$, $\theta_2 = 2$, $\theta_3 = 3$, and $\theta_4 = 4$. The network size effect

[12] In the figure, total surplus is optimal profits plus consumer surplus. The maximum value equals total surplus in the complete network.

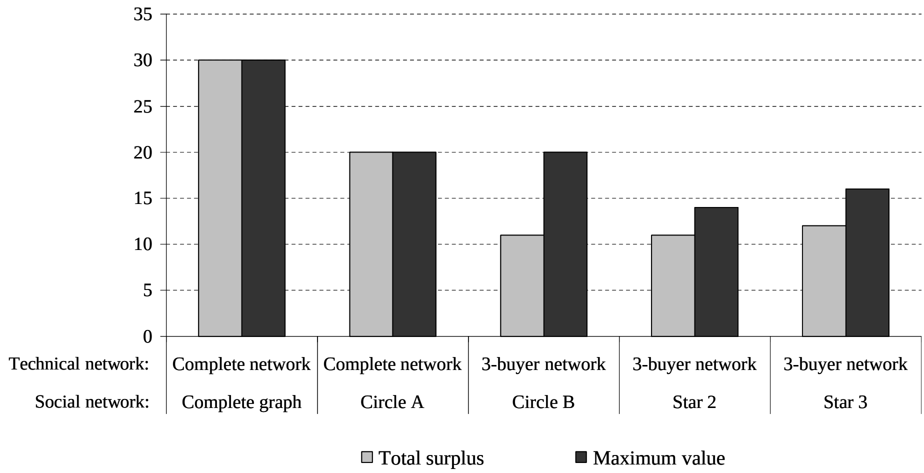


Figure 1: Total surplus and maximal generated value.

is clearly visible, as maximal value increases in the number of links. Topological effects come through in two ways. First, the star with consumer θ_3 at the centre generates higher maximum value than star with consumer θ_2 at the centre. What drives the difference is supermodularity of the payoff function. Second, the total surplus in the circle B is only 55% of the total surplus in the circle A. In A, the firm chooses complete network, which maximises total surplus, but in B, the firm excludes θ_1 from the network. This happens because, in B, the critical consumer θ_2 is the common neighbour to high types θ_3 and θ_4 who require only one neighbour who buys. As a result, θ_1 is rendered redundant in the circle B and it pays off to exclude him. The drop in the total surplus is due to the reduction in consumer surplus. In the circle B, the firm makes only 13% higher profits compared with the circle A, but consumer surplus is reduced by 83%.

A change in the opposite direction can be observed in the comparison between the stars 2 and 3. There is a slight increase in total surplus in moving from star 2 to star 3. As the consumer θ_2 loses his preferential position as the centre, the firm lowers its price (from 3 to 2) in order to include θ_2 in the network. The total surplus in the star 2 is 92% of total surplus in the star 3. The increase in consumer surplus offsets the decrease in profits.

Next, let us increase the valuation θ_3 from 3 to 3.5, while maintaining everything else. What this apparently positive change does, is that it increases the maximal value in all networks. The total surplus is increased in all networks except in the star with θ_3 as the centre. The total surplus in star 3 is significantly lowered, by 38%. Why? In the new situation the top two consumers have valuations sufficiently higher than the two bottom ones. The new optimal network structure for the firm is a 2-buyer network in star 3 (when with $\theta_3 = 3$ it was a 3-buyer network). Exclusion of both θ_1 and θ_2 increases firm's profits by 17%. At the same time, the consumer surplus is reduced by 92%, which dominates the increase in profits. Due to θ_2's critical position, he is not excluded in the star 2 or the circle B. The complete graph and the circle A remain fully covered.

The comparison of profits and consumer surplus has illustrated how they crucially depend on the underlying social relations. The comparison has revealed how the strength of network externalities can be overestimated. An assumption on a complete graph as the prevailing social structure, when the true social structure is something less connected, produces significantly exaggerated estimates for consumer surplus and monopoly rents.

We close the analysis on perfect information with a counter-example to remark 11 saying that the monopolist excludes low types in heterogenous markets. This illustrates the complexity perfect information creates, and the importance of network topology in pricing. Consider a modified star network: "insiders-outsider" illustrated in figure (2) with valuations $\theta_1 < \theta_2 < \theta_3 < \theta_4$. The consumer θ_1 has obviously a preferential position. Let the firm prefer a 2-buyer network over a 3-buyer network, i.e. $V_2 = 2(\theta_2) > V_3 = 3(2\theta_1)$. But, if the outsider θ_4 has a very high valuation $3\theta_1 < \theta_4$, the firm may prefer the complete network over the 2-buyer network, even if buyers' valuations are very heterogeneous. Let $3\theta_1 < \theta_2 < 6\theta_1 \Rightarrow V_4 = 4(3\theta_1) > V_2 = 2(\theta_2) > V_3 = 3(2\theta_1)$. When this holds, types θ_3 and θ_4 can differ significantly from θ_1 and θ_2 (high heterogeneity), and the firm still covers the whole market. This is possible thanks to two factors. One, θ_3 and θ_4 are not neighbours, so the firm cannot sell only to them. Two, θ_1 has many links which compensate his low valuation.

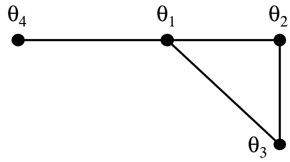


Figure 2: Insiders - Outsider

4 Asymmetric information

In this section, we limit the players' ability to observe their opponents' valuations. The valuations θ are now private information. Because θ's are i.i.d., the buyers are ex ante symmetric but ex post heterogenous. The social network structure $\mathcal{G}$ and distribution $F(\theta)$ remain common knowledge.

Write π_{ij} as the probability consumer i puts on the event that his neighbour j buys the device. The expected payoff from the link between i and j is independent from any other link i has. Consequently, the expected payoff from link $\{i,j\}$ to i is just $\pi_{ij}\theta_i$, and $\pi_{ji}\theta_j$ to his neighbour j. The consumer i's expected utility from $a_i = B$ is the sum over all his links

$$\mathbb{E}\left[u_i\left(\theta_i, B\right)\right] = \sum_{j\in\mathcal{N}_i} \pi_{ij}\theta_i - p. \tag{3}$$

If the expected payoff from buying the product exceeds the reservation value of zero, the agent makes the purchase. At the margin, the consumer's valuation is

$$\widetilde{\theta}_i\left(\pi_{\mathcal{N}_i}\right) = \frac{p}{\sum_{j\in\mathcal{N}_i}\pi_{ij}}. \tag{4}$$

Pure strategy for consumer i is $a_i : \left[\theta^-, \theta^+\right] \to \{B, N\}$, and his best response is the switching strategy $a_i^* = B$, if $\theta_i \geq \widetilde{\theta}_i\left(\pi_{\mathcal{N}_i}\right)$ and $a_i^* = N$, if $\theta_i < \widetilde{\theta}_i\left(\pi_{\mathcal{N}_i}\right)$. The probability that consumer i buys, given his beliefs over his neighbours' actions π_{ij} and price p, is

$$\pi_i = 1 - F\left(\min\left\{\theta^+, \widetilde{\theta}_i\left(\pi_{\mathcal{N}_i}\right)\right\}\right).$$

The coordination game with asymmetric information Γ_{AI} consists of consumers $\mathcal{I}$ arranged on graph $\mathcal{G}$, pure actions $a = \{B, N\}$, types $(\theta_i)_{i=1}^{I}$ with prior distribution $F(\theta)$, and payoffs $u_i(N) = 0$ and $\mathbb{E}\left[u_i(B)\right]$ given by equation (3). The game Γ is parameterised by price p. Bayesian Nash equilibrium (BNE) of Γ_{AI} is characterised in lemma 13.

Lemma 13 *The action profile* $\mathbf{a}^* = (a_1^*, ..., a_I^*)$ *is a Bayesian Nash equilibrium of the asymmetric information game* Γ_{AI} *if*

$$\begin{array}{c} a_i^* = B \Leftrightarrow \theta_i \geq \widetilde{\theta}_i\left(\pi^*_{\mathcal{N}_i}\right) \\ a_i^* = N \Leftrightarrow \theta_i < \widetilde{\theta}_i\left(\pi^*_{\mathcal{N}_i}\right) \end{array},$$

where $\widetilde{\theta}_i\left(\pi^*_{\mathcal{N}_i}\right) = \frac{p}{\sum_{j\in\mathcal{N}_i}\pi^*_{ij}}$ *and* $\pi_i^* = 1 - F\left(\min\left\{\theta^+, \widetilde{\theta}_i\left(\pi^*_{\mathcal{N}_i}\right)\right\}\right)$ *for all* $i \in \mathcal{G}$ *and* θ_i.

Supermodularity carries over to the asymmetric information regime.

Lemma 14 *The coordination game with asymmetric information* Γ_{AI} *is supermodular with positive spillovers.*

Proof.

(i) *The set* $\pi_i \in [0,1]$ *is a compact subset of* $\mathbb{R}$.

(ii) *The payoffs exhibit increasing differences. Write the expected utility of action* $a = B$ *versus* $a = N$ *as* $\mathbb{E}[v_i(\theta_i, \pi_{ij})] = \mathbb{E}[u_i(\theta_i, B)] - \mathbb{E}[u_i(\theta_i, N)] = \mathbb{E}[u_i(\theta_i, B)]$ *for all* $i \in \mathcal{G}$ *and* $j \in \mathcal{N}_i$, *where* $\mathbb{E}[u_i(\theta_i, B)]$ *is given by equation* (3)*. We have* $\mathbb{E}\left[v_i\left(\theta_i', \pi_{ij}\right)\right] \geq \mathbb{E}[v_i(\theta_i, \pi_{ij})]$ *for all* $\theta_i' > \theta_i$.

(iii) *The payoff function* $\mathbb{E}(u_i) : \{B, N\} \times \theta \to \mathbb{R}$ *is continuous.*

We conclude from (i)-(iii) that the game Γ_{AI} *is supermodular. Positive spillovers arise because the payoff gain is strictly increasing in neighbours' strategies,* $\frac{\partial \mathbb{E}[v_i(\theta_i,\pi_{ij})]}{\partial \pi_{ij}} > 0$ *for all* $j \in \mathcal{N}_i$. ■

The implications of supermodularity are familiar. It guarantees that there exists the largest and the smallest equilibrium element. The smallest BNE is the empty network where $\pi_i^* = 0$ for all $i \in \mathcal{G}$. On the other hand, the structure of any non-empty BNE depends on the price and social network topology. Positive spillovers mean that the largest BNE is Pareto-dominating, which, we argue, focalises the efficient equilibrium.

The firm maximises expected profits $\mathbb{E}(V) = \sum_{i\in\mathcal{G}} \pi_i^* [p(\boldsymbol{\pi}^*) - c]$. The firm cannot choose the activity level directly, as it could with perfect information. Instead, we let the firm maximise profits by choosing quantity, i.e. the probability π_i^*. The inverse demand $p(\boldsymbol{\pi}^*)$ is derived from the BNE of the coordination game Γ_{AI}.

We have now characterised the asymmetric information case. Next we apply the general framework to a complete graph, a circle, and a star.

4.1 Symmetric networks

Asymmetric information makes all symmetric networks analytically the same. It only needs to understand that a symmetric social network coupled with private information on θ makes all agents ex ante symmetric.

Lemma 15 *With asymmetric information, each consumer $i \in \mathcal{G}$ has an identical probability to buy in a symmetric network.*

Proof. *Let $n \in [1, I-1]$ be the number of neighbours for consumer i in a population of $\mathcal{I}$ that is arranged on a symmetric graph $\mathcal{G}^{sym}$. By symmetry n is the number of neighbours for all consumers. Assume first that the probabilities are different so that for all other consumers except i, the probability to buy is π and for i it is $\pi_i < \pi$. We can write consumer i's expected utility from $a_i = B$ as*

$$\begin{aligned}\mathbb{E}\left[u_i\left(\theta_i, B\right)\right] &= \sum_{k=0}^{n} \binom{n}{k} \pi^k \left(1-\pi\right)^{n-k} k\theta_i - p \\ &= \pi n \theta_i - p.\end{aligned}$$

Similarly, the expected payoff for consumer $j \neq i$ is

$$\begin{aligned}\mathbb{E}\left[u_j\left(\theta_j, B\right)\right] &= \sum_{k=0}^{n-1} \binom{n-1}{k} \pi^k \left(1-\pi\right)^{(n-1)-k} k\theta_j + \pi_i \theta_j - p \\ &= \left[\left(n-1\right)\pi + \pi_i\right]\theta_j - p.\end{aligned}$$

The equilibrium condition that consumer i buys is $z_i\left(B\right) = 1 - F\left(\min\left\{\theta^+, \frac{p}{n\pi}\right\}\right)$, and for all other consumers except i it is $z_{-i}\left(B\right) = 1 - F\left(\min\left\{\theta^+, \frac{p}{(n-1)\pi+\pi_i}\right\}\right)$. The functions $z_i\left(\pi\right)$ and $z_{-i}\left(\pi_i, \pi\right)$ are increasing in π and in $\left(\pi, \pi_i\right)$ respectively. If the initial assumption $\pi_i < \pi$ holds, then it must be that $z_i\left(\pi\right) > z_{-i}\left(\pi_i, \pi\right)$ which leads to a contradiction. The case $\pi_i > \pi$ leads to a corresponding contradiction. Hence, in the equilibrium it must be that $\pi_i = \pi$ for all $i \in \mathcal{G}^{sym}$. ■

Both the complete graph and the circle are symmetric networks. We work through a generalised version where all agents have n neighbours. For the complete graph $n = I - 1$ and for the circle $n = 2$. Note that some configurations are impossible. For example, it is impossible to construct a symmetric network with five agents each having three neighbours. The generalised version does apply to complete graphs and circles of any number of consumers, however.

The expected payoff from $a_i = B$ can be written as $\mathbb{E}\left[u_i\left(\theta_i, B\right)\right] = n\pi\theta_i - p$. The common system of probabilities satisfies

$$\pi = 1 - F\left(\min\left\{\theta^+, \frac{p}{n\pi}\right\}\right). \tag{5}$$

The introduction of incomplete information in the model has reduced the number of equilibria. There can be maximum of three different equilibria. Firstly, the empty network is BNE. To see that the empty network is a BNE, substitute $\pi = 0$ in equation (5), and it is immediate that all agents play $a = N$ with probability one. In addition, there can be at most two positive equilibria. In the interval $\pi \in \left]0, \frac{p}{n\theta^+}\right]$, there are no equilibrium values. To check the existence of positive equilibria, we solve the equation $\pi = 1 - F\left(\frac{p}{n\pi}\right)$ for π. Real roots exist when $\left(\theta^+ n\right)^2 - 4\left(\theta^+ - \theta^-\right)np \geq 0$.

In the cases where there are two positive equilibria, the higher value is the Pareto-dominating maximal BNE. Lower equilibria are associated with coordination failure.

The equilibrium condition (5) gives the inverse quasi-demand $p(\pi)$. In the area where $\theta^{+} \leq \frac{p}{n\pi}$ price is indeterminate, and the probability to buy is zero $\pi = 0$. The firm operates in the region where the price is determinate. The inverse demand is

$$p = n\pi \left[\theta^{+} - \left(\theta^{+} - \theta^{-}\right)\pi\right].$$

The firm maximises profits by choosing the optimal level of π. Firm's expected profits are $\mathbb{E}(V) = I\pi\left[p(\pi) - c\right]$. The first order condition gives the standard monopoly mark-up rule

$$\frac{p(\pi^{*}) - c}{p(\pi^{*})} = \frac{1}{\eta}, \tag{6}$$

where π^{*} is the optimal value and $\eta = -\frac{\partial \pi^{*}}{\partial p}\frac{p(\pi^{*})}{\pi^{*}}$ the elasticity of the quasi-demand.

Consider the special case of zero unit costs, $c = 0$. Equation (6) gives

$$\pi^{*} = \frac{2\theta^{+}}{3\left(\theta^{+} - \theta^{-}\right)},$$

and

$$p(\pi^{*}) = \frac{2\left(\theta^{+}\right)^{2}}{9\left(\theta^{+} - \theta^{-}\right)}n, \tag{7}$$

which satisfy second order conditions.[13] The derived values represent the desired, maximal, equilibrium for the firm. When the obtained equilibrium price $p(\pi^{*})$ is plugged back into equation (5), we can solve again for the corresponding equilibrium probabilities. As suggested, there exist two positive equilibria

$$\pi = \frac{\theta^{+} \pm \frac{1}{3}\theta^{+}}{2\left(\theta^{+} - \theta^{-}\right)}.$$

Denote the larger value, associated with the maximal BNE, as π^{*}_{+}. Firm's expected profits are in that case

$$\mathbb{E}\left(V^{*}_{+}\right) = \frac{4}{27}\left(\frac{\theta^{+}}{\theta^{+} - \theta^{-}}\right)^{2}\theta^{+} I n.$$

The difference in realised profits between the maximal BNE and the lower (positive) equilibrium is $\left(\pi^{*}_{+} - \pi^{*}_{-}\right)p(\pi^{*}) = \frac{1}{2}\mathbb{E}\left(V^{*}_{+}\right)$. The empty network yields zero profits of course.

[13] $\left.\frac{\partial^{2}\mathbb{E}(V)}{\partial \pi^{2}}\right|_{\pi = \frac{2\theta^{+}}{3\left(\theta^{+} - \theta^{-}\right)}} = -2\theta^{+} I n < 0.$

The maximal and empty network are Cournot tâtonnement stable BNE, whereas the lower positive equilibrium is an unstable one. The checks for stability are provided in appendix 8.3. Because the low equilibrium is an unstable one, convergence occurs towards zero or the maximal BNE, unless the tâtonnement process begins exactly at the lower equilibrium.

Total expected consumer surplus in the maximal BNE is given by

$$\begin{aligned}\mathbb{E}(CS) &= I\int_{\widetilde{\theta}}^{\theta^+} f(\theta)\left[n\pi_+^*\theta - p^*\right]d\theta \\ &= \frac{4}{27}\left(\frac{\theta^+}{\theta^+-\theta^-}\right)^2\theta^+ In\end{aligned}$$

Expected consumer surplus equals expected profits, $\mathbb{E}(CS) = \mathbb{E}\left(V_+^*\right)$.

We are ready to compare the asymmetric information model (with $c = 0$) with the results from the perfect information case (remarks 7-12). We see from equation (7) that the monopoly price is increasing in the number of links (agrees with remark 7). For a given number of consumers, the complete graph supports the highest price. This result follows from the size effect, which holds that the network is more valuable the mode links it has.

Symmetry has removed the preferential roles that existed under perfect information. Agent configuration is irrelevant since consumers are ex ante symmetric (disagrees with remarks 9 and 10(ii)). How the highest types are positioned in the network does not affect expected profits under asymmetric information, because the firm cannot distinguish between agents. Thus, its behaviour is independent of the dispersion of high consumer types (disagrees with remarks 11 and 12).

Proposition 16 *Monopoly price increases as the number of neighbours increases. Agent configuration does not affect monopoly price under asymmetric information.*

Proof. *Follows directly from equation* (7) *and lemma 15.* ■

It is obvious now that the complete graph corresponds to the conventional network externalities model where the underlying social structure is abstracted away. When we take the probability π as the fraction of the total population who buy, we arrive at a basic membership externality model where the agent's utility increases with the number of people joining the network.

Complete graph generates highest equilibrium profits and consumer surplus, thus total surplus $\left(\mathbb{E}\left(V_+^*\right) + \mathbb{E}(CS)\right)$, which agrees with results from the perfect information case (remark 8). This brings up the problem of overestimation of network externalities. If we use the complete graph, when the true social network is something less connected, we end up exaggerating the value generated in the network.

To see what the impact of heterogeneity is, we apply a mean-preserving spread $\left[\theta^- - x, \theta^+ + x\right]$ on the uniform distribution of types $F(\theta)$. Increased heterogeneity reduces the probability to buy $\frac{\partial \pi_+^*}{\partial x} = -\frac{2\left(\theta^+ + \theta^-\right)}{3\left(\theta^+ - \theta^- + 2x\right)^2} < 0$. This leads the monopoly to reduce its price in general, $\frac{\partial p^*}{\partial x} = -\frac{4\left(\theta^+ + x\right)\left(\theta^- - x\right)}{9\left(\theta^+ - \theta^- + 2x\right)^2}$ which is negative when $\theta^- > x > 0$, but positive with $\theta^- = 0$. An increase in heterogeneity causes two effects. First, higher heterogeneity induces higher monopoly price as in the standard case of monopoly pricing with unit demand. Second, higher heterogeneity increases the uncertainty about neighbours' buying decisions. The second effect induces the monopoly to reduce its price. The total effect is negative, in general, as the differentiation proves. We can write the expected consumer surplus in the maximal BNE, which equals maximal BNE profits, as

$$\mathbb{E}(CS) = \frac{4}{27}\left(\frac{\theta^+ + x}{\left(\theta^+ + x\right) - \left(\theta^- - x\right)}\right)^2 \left(\theta^+ + x\right) In. \tag{8}$$

Since the spread increases uncertainty about neighbours' purchasing decisions, expected consumer surplus decreases. For the firm, higher uncertainty leads to lower demand and lower price, thus lower profits. The firm cannot distinguish between networks where high types are clustered and networks where high types are dispersed. Hence, it is incapable of taking advantage of clusters of high types, as it could with perfect information (disagrees with remarks 11 and 12).

Proposition 17 *Surplus effects:*

(i) *Complete graph supports the highest expected consumer surplus and profits in the maximal BNE.*

(ii) *Increased heterogeneity decreases expected consumer surplus and profits in the maximal BNE.*

Proof.

(i) *Both $\mathbb{E}(CS)$ and V_+^* are strictly increasing in n. So the maximum is reached at complete graph $n = I - 1$.*

(ii) *By differentiating (8) we get $\frac{\partial \mathbb{E}(CS)}{\partial x} = \frac{\partial \mathbb{E}\left(V_+^*\right)}{\partial x} = \frac{4}{27} In \frac{\left(\theta^+ + x\right)^2}{\left(\theta^+ - \theta^- + 2x\right)^3}\left[-\left(\theta^+ + x\right) - 3\left(\theta^- - x\right)\right]$, which is negative when $\theta^- \geq 0$ and x is small.*

■

The asymmetric information case is analytically easier to handle than the perfect information case, because agent configuration plays no role. Some of the predictions of the perfect information case hold, but some are invalidated. The asymmetric information case is more suitable for large social networks, where each agent has many connections. The adverse possibility to overestimate network value is more serious in larger networks, however.

4.2 Star

For the star, we obtain an equilibrium system that comprises two distinct probabilities for buying. One is for the centre and the other for peripheral agents. The firm has to choose a price that applies to all consumers, creating a price bias in favour of the centre. We allow price discrimination in section 4.3.

Consumers' utilities are, $\mathbb{E}\left[u_C\left(\theta_C, B\right)\right] = \sum_{j\in\mathcal{N}_C} \pi_{Ci}\theta_C - p$ for the centre, and $\mathbb{E}\left[u_i\left(\theta_i, B\right)\right] = \pi_{iC}\theta_i - p$ for peripheral agent $i \in \{\mathcal{G}\diagdown C\}$. Since peripheral consumers are a priori symmetric, by lemma 15, their behaviour is characterised by a common probability. The centre places probability $\pi_{Ci} = \pi$ on the event that a peripheral consumer $i \in \{\mathcal{G}\diagdown C\}$ buys. Each peripheral consumer $i \in \{\mathcal{G}\diagdown C\}$ places probability $\pi_{iC} = \pi_C$ that the centre buys.

Lemma 18 *BNE in a star is characterised by (π_C, π), where π_C is the probability that the centre buys and π is the probability that a peripheral agent buys. The equilibrium satisfies*

$$\begin{array}{l} \pi_C = 1 - F\left(\min\left\{\theta^+, \frac{p}{(I-1)\pi}\right\}\right) \\ \pi = 1 - F\left(\min\left\{\theta^+, \frac{p}{\pi_C}\right\}\right) \end{array} \tag{9}$$

Proof. *Proof follows directly from lemma 15 and uses the symmetry property.* ■

As in symmetric networks, asymmetric information eliminates the role of agent (type) configuration. From system (9) we get the market clearing price and the centre's probability to buy as a function of π.

$$\begin{aligned} p(\pi) &= \pi_C(\pi)\left[\theta^+ - \left(\theta^+ - \theta^-\right)\pi\right] \\ \pi_C(\pi) &= \frac{\theta^+ (I-1)\pi}{\theta^+ + (I-2)\left(\theta^+ - \theta^-\right)\pi} \end{aligned}$$

Periphery's and centre's strategies are complements in the sense $\frac{\partial \pi_C}{\partial \pi} > 0$. The difference between probabilities is $\pi_C - \pi = \frac{\left(\theta^+ - \left(\theta^+ - \theta^-\right)\pi\right)\pi}{\theta^+ + \left(\theta^+ - \theta^-\right)\pi}$ which is always non-negative, saying that the probability that the centre buys is higher than the probability that a peripheral agent buys.

The firm maximises expected profits $\mathbb{E}(V) = [\pi_C(\pi) + (I-1)\pi][p(\pi) - c]$ by choosing the probability π. The FOC gives a modified inverse elasticity rule

$$\frac{p(\pi^*) - c}{p(\pi^*)} = \frac{1}{\eta}\left\{\frac{\left[2\theta^+ + (I-2)\left(\theta^+ - \theta^-\right)\pi^*\right]\left[\theta^+ + (I-2)\left(\theta^+ - \theta^-\right)\pi^*\right]}{\left(\theta^+\right)^2 + \left[\theta^+ + (I-2)\left(\theta^+ - \theta^-\right)\pi^*\right]^2}\right\}, \tag{10}$$

where $\eta = -\frac{\partial\pi^*}{\partial p}\frac{p(\pi^*)}{\pi^*}$ is the price elasticity of the quasi-demand of a peripheral agent.

Because the result (10) is difficult to use analytically, let us consider the specific case with zero unit costs and a uniform distribution $\theta \sim Unif[0,1]$ with a non-degenerate star ($I \geq 3$). In this case, there is only one real root to the equation (10) in the range $\pi \in (0,1)$, which yields positive profits, and the corners $\pi = \{0,1\}$ yield zero profits.[14] Hence, the only real root in the range $\pi \in (0,1)$ is the global maximum. Because the derivative $\frac{\partial\mathbb{E}(V)}{\partial\pi}$ at point $\pi = \frac{2}{3}$ $\left(\pi = \frac{1}{3}\right)$ is positive (negative), the optimal π must be in the range $\frac{1}{3} < \pi^* < \frac{2}{3}$. So, the probability to buy for a peripheral consumer is less than the probability to buy in symmetric graphs. Respectively, the monopoly achieves a higher mark-up associated with the periphery than the standard monopoly mark-up associated with symmetric graphs. We summarise the above in the proposition 19.

Proposition 19 *A consumer in the periphery has a lower probability to buy, and the centre has a higher probability to buy, compared with a consumer in a symmetric network.*

Proposition 19 states that the topological effect on the monopoly price is never latent under asymmetric information. The firm's pricing strategy resembles those cases of perfect information where the centre is a binding constraint to pricing (because the centre has sufficiently low valuation). In the case of asymmetric information, the firm always takes into account the topologically focal centre by guaranteeing him a higher probability to buy, but compensates with a higher mark-up for the periphery.

We have verified numerically that the optimal π^* is decreasing in I, whereas the optimal $\pi_C^* = \pi_C(\pi^*)$ is growing in I. The centre benefits the more people join his neighbourhood, but a peripheral consumer is negatively affected by an additional peripheral consumer, even though the additional agent does not affect his neighbourhood directly. Why? The centre's probability to buy increases as a peripheral agent is added. The firm can compensate this addition by increasing

[14] Second order conditions for maximal profits are satisfied for the non-zero equilibrium. This can be checked numerically for the particular case $c = 0$, $\theta^+ = 1$, $\theta^- = 0$. We have $\frac{\partial^2\mathbb{E}(V)}{\partial\pi^2} < 0$ for $I \geq 3$. A stability check for the equilibrium is in the appendix 8.3.

the price. The price increase, however, does not capture the whole increase in the centre's utility. By leaving more surplus to the centre, thus increasing the centre's probability to buy, the firm indirectly increases the periphery's expected utility. The price increase, however, is high enough that an individual peripheral consumer gets a negative effect in total. As I grows very large, the optimal π^* approaches $\frac{1}{2}$, and the optimal π_C^* approaches $\frac{I-1}{I} \approx 1$. In the minimal case where $I = 3$, the optimal values are $\pi^* \approx 0.5971$ and $\pi_C^* \approx 0.7478$. The larger the periphery is, the higher is the centre's market power thus larger the surplus he captures. This happens regardless the centre being actually a pricing constraint in the respective perfect information game or not. The monopoly price is the lowest at $I = 3$, where it equals $p(\pi^*) \approx 0.3012$. As the periphery becomes very large, the optimal price approaches $\frac{1}{2}$.

Profits and the total expected consumer surplus (centre's surplus plus periphery's surplus) increase in the number of peripheral consumers. An individual peripheral consumer becomes less important the more there are peripheral consumers. On the other hand, the centre's relative position against the periphery increases in importance. As a result, the difference between the centre's and the periphery's expected surpluses becomes larger the more numerous the periphery is. We summarise the results from the numerical run in remark 20.

Remark 20 *The effect of changes in the size of periphery:*

(i) *The centre benefits the larger the periphery is.*

(ii) *A peripheral consumer is adversely affected by an addition of a new peripheral consumer.*

(iii) *Total consumer surplus increases as the periphery grows.*

(iv) *The price and the monopoly's expected profits increase as the number of peripheral agents increases.*

Remark 20 agrees with the size effects of perfect information regime, as well as, with the symmetric networks case under asymmetric information. It also shows how network topology can have distributional effects on consumer surplus by affecting the monopoly's price strategy.

Consider a spread $\theta \sim Unif[-x, 1+x]$, $x > 0$ and small.[15] A numerical run shows that the firm increases the optimal price for a small x. This is in contrast with the result from the

[15] Although we now give the lowest type a negative valuation, it does not affect the results as long as the spread we consider is small since the lowest type already had a dominant strategy $a = N$ for any given positive price p.

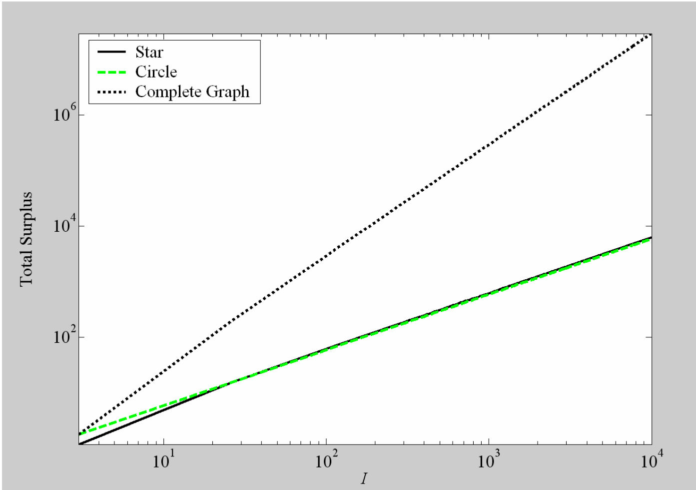


Figure 3: Uncompensated total surplus (log scale), $\theta \sim Unif\,[0,1]\,,\ c = 0$.

symmetric network case. An increase in heterogeneity induces a price increase in the standard way, as in the symmetric networks case. The negative effect of higher uncertainty about neighbours' buying decisions is now weaker thanks to asymmetric network topology. The firm is able to limit the negative effect by contrasting the centre and the periphery, which results to a positive price change in total. However, the increase in uncertainty has a negative effect on profits and consumer surplus in total.

Remark 21 *Small increase in uncertainty decreases equilibrium profits, and total consumer surplus associated with the periphery and the centre.*

4.3 Comparison and price discrimination

We close the analysis of the asymmetric information variant with a comparison of the symmetric networks and the star. We ignore the integer problem in order to get results easily illustrated, thus $I \geq 3$ and continuous. Let the distribution of θ be uniform over $[0,1]$ and costs zero $c = 0$. We first confirm the results about the size factor.

Figure (3) illustrates how the complete graph (dotted line) generates far higher total surplus (profits plus consumer surplus) than the *uncompensated* circle (dashed line) or star (solid line) do. This is because each additional consumer induces $2\,(I-1)$ new links in the complete graph whereas only two links in the circle and the star.

Remark 22 *The complete graph generates the highest total surplus.*

For small numbers of consumers, the circle produces higher total surplus compared to the star, but for large networks, the star generates higher total surplus. The solid line crosses the dashed line just before the number of consumer reaches $I = 30$. The circle always has 2 links (one two-directional link) more than the star, which returns higher consumer surplus for small networks. The star, however, supports inherently lower price than symmetric networks, thus for large networks consumer surplus is higher in the star. Since the firm maintains its strategy constant with respect to the number of consumers in the circle, but adjusts its price in the star as the number of consumers is increased, the relation between the two surpluses changes. In small star networks, the firm is more pressed to set a low price in order to attract the centre. As the periphery grows in number, the firm increases its price as it compensates (negatively) for larger periphery. Because the firm maintains a lower price in the star than in a symmetric network, and because there are less links in the star network, firm's profits are the lowest in the star for a given number of consumers.

We can isolate the topological effect by comparing *compensated* networks. The comparison of compensated networks shows how the monopoly price changes when network topology changes, while the link value of the network is kept constant. Let us fix the maximal value generated in the complete graph of size I. A compensated circle has $I_C = \frac{I(I-1)}{2}$ consumers and a compensated star $I_s = \frac{I(I-1)+2}{2}$ consumers.

Remark 23 *(i) The firm prefers the symmetric compensated network topology. (ii) Total surplus is maximised in the compensated star (asymmetric network).*

From picture (4) we can read that the firm is worse off in the star network of compensated size. The asymmetric network structure constrains the firm as it has to leave more surplus to the centre by setting a relatively lower price. As a result, consumer surplus is higher in the compensated star than in the circle or the complete graph. Total surplus is higher in the compensated star, since higher consumer surplus dominate lower profits. This is seen in figure (5).

In the star, there is a bias in favour of the centre. This raises the question whether the firm could benefit by price discriminating with respect to the network position. With price discrimination,

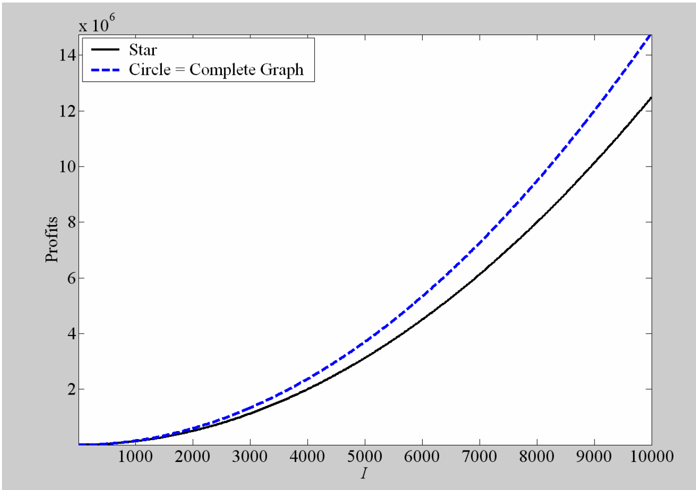


Figure 4: Compensated profits, $\theta \sim Unif\,[0,1]\,,\, c = 0.$

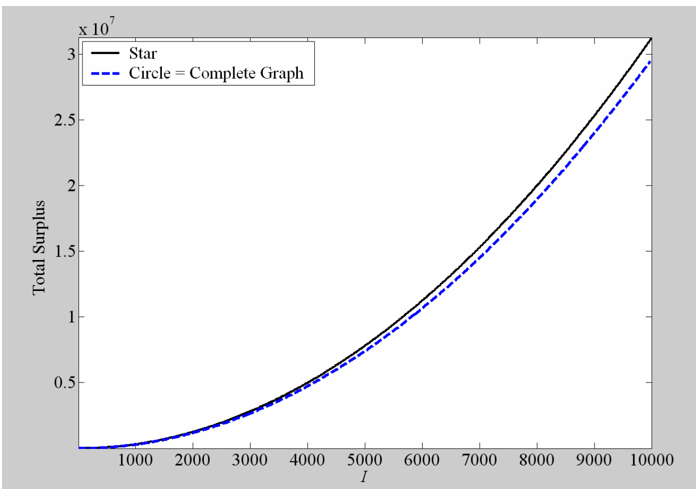


Figure 5: Compensated total surplus, $\theta \sim Unif\,[0,1]\,,\, c = 0.$

the equilibrium probability system in the star is

$$\begin{aligned} \pi_C &= 1 - F\left(\min\left\{\theta^+, \tfrac{p_C}{(I-1)\pi}\right\}\right) \\ \pi &= 1 - F\left(\min\left\{\theta^+, \tfrac{p}{\pi_C}\right\}\right) \end{aligned},$$

where p_C is the price for the centre and p for the periphery. The firm maximises expected profits $\mathbb{E}(V) = \pi_C [p_C(\pi_C, \pi) - c] + (I-1)\pi[p(\pi_C, \pi) - c]$ by choosing (π, π_C).

For zero unit costs, $c = 0$, the optimal probabilities are

$$\pi_C^* = \pi^* = \frac{2\theta^+}{3(\theta^+ - \theta^-)}.$$

Proposition 24 *Price discrimination with respect to network location removes the bias in favour of the centre.*

By price discriminating, the firm of course captures a larger share of the maximal value generated in the network. If we consider the case $\theta \sim Unif[0,1]$, $c = 0$, and compensated networks. It is straightforward to calculate that price discrimination increases the firm's profits to the same level as in the compensated symmetric networks. Respectively, total consumer surplus falls to the level of symmetric networks.

Two important insights can be drawn from this section. One, the complete graph generates the highest surplus for a given number of consumers. This means that use of a complete graph as a mapping of social relations calls for caution. There is a possibility to overestimate the value of the network, when the true network is something less-connected. Two, network topology matters for price strategy, and consequently, for how the surplus is split between players. For compensated networks, the social optimum is the star network, where the monopoly power of the firm is reduced due to the asymmetric topology. The centre captures a large part of the rents, which happens at the periphery's expense. The monopoly, on the other hand, prefers a symmetric network. If the firm is able to price discriminate, it can increase its profits to the same level as in the symmetric networks. However, price discrimination leads to an efficiency loss as the consumer surplus is reduced more than the firm gains.

5 Extensions

We conclude the analysis with few extrapolations on the basic model.

5.1 Intrinsic utility and multiplicity of equilibria

Symmetric networks produce multiple equilibria under asymmetric information. If the firm was certain that the maximal equilibrium is the correct one, it would be willing to invest up to $\mathbb{E}\left(V_+^*\right)$ to enter the market. More generally, we could assume that the firm holds beliefs $\beta_k \in [0,1]$ on the possible equilibrium k, with $\sum_k \beta_k = 1$. Maximum acceptable sunk cost to enter the market is then $\kappa = \sum_k \beta_k \mathbb{E}\left(V_k^*\right)$, where $\mathbb{E}\left(V_k^*\right)$ are the expected profits from equilibrium k.

In growth industries, in the early development stages, forecasting future states of the world involve highly qualitative and subjective metrics that make it difficult to estimate β_k. Therefore, it should not come as a surprise that many (most) of the dotcoms that founded their business models on increasing returns found themselves insolvent in a short time. Secondly, dotcoms' business models were often "eyeball game" strategies where network externalities were assumed to generate demand automatically. Such business models did not take into account consumers' local and asymmetric social relations, which, as we have shown, reduce the strength of network effects.

In the main model, we have assumed that there is no intrinsic utility associated with the goods. Inclusion of intrinsic utility would not change the results qualitatively. It could facilitate analysis, by removing some possible equilibria. In fact, equilibrium uniqueness is reachable if we impose sufficient heterogeneity between consumers with respect to intrinsic utility (Herrendorf et al. 2000). The key to uniqueness is that we have one group of consumers who buy as a strictly dominant strategy, independent of other people's strategies, at the same time as another group does not buy as a strictly dominant strategy. Under perfect information, heterogeneity needs to be real in the sense of sufficiently broad distribution of the intrinsic utility between consumer types. Under asymmetric information, the model is easily turned into a game of correlated private values. Such a set-up allows to use global games techniques to derive a unique equilibrium. With asymmetric information, heterogeneity does not have to be real in the above sense, but we must have a possibility that some consumers are sufficiently high and low types. Our related paper Sääskilahti (2005) does a comprehensive analysis on equilibrium uniqueness in a model of monopoly pricing of network goods under perfect and incomplete information.

In the current model, multiplicity of (positive) equilibria emerged only with symmetric networks under asymmetric information. The star produced a unique positive equilibrium in addition to the empty network. This result is due to heterogeneity with respect to network locations. Since most real world networks are asymmetric, multiplicity of equilibria could be less of a problem than what the conventional network externalities models predict.

5.2 Direct externalities and interaction propagation

The main model presents a utility function in which each link generates value θ for the consumer. In other words, utility is independent of the counter party of the social relation, and it only depends on the number of neighbours. A consumer induces an indirect externality to his neighbourhood. His own high probability to buy increases neighbours' probabilities to buy. The motivation for this specification is that high types enjoy more of the novel device from each social relation they have. For example, high type can be a synonym for a technically able person. Such a person is likely to get more out of new technology than those people who find new gadgets difficult to use.

However, it would not be unrealistic that the value of the new device also depended on the social relation. Some contacts could be more important than others. Ideally, we could impose a value distribution from which the value of each link is drawn. Allowing this kind of heterogeneity between links, inevitably necessitates expanding the model to discuss also usage decisions in addition to the plain buying decision. This is an area that calls for further research.[16]

Alternatively, utility could be a function of both parties' valuations. This way, high types induce a direct positive externality to their neighbours, in addition to the indirect externality. For example, a sharing rule of the following kind could capture the desired "propagation" dynamics. Consider the link between consumers i and j. Consumers contribute agent-specific values θ_i and θ_j to an active link. The active link generates total utility of $\upsilon_{ij}(\theta_i, \theta_j)$. In the simplest form, the generated utility presents constant returns when $\upsilon_{ij}(\theta_i, \theta_j) = \theta_i + \theta_j$. Utility generation of course can present decreasing ($\frac{\partial \upsilon(\theta_i,\theta_j)}{\partial \theta_k} < 1$, $k = i, j$) or increasing returns ($\frac{\partial \upsilon(\theta_i,\theta_j)}{\partial \theta_k} > 1$, $k = i, j$). Total utility is shared by the consumers according to a rule, by which a share $r_{ij} = r(\upsilon_{ij})$

[16] For a model with a exogenous *symmetric* social network structure and buying and usage decisions, see Sääskilahti (2005).

goes to consumer i and $1 - r_{ij}$ goes to j. Basically, the sharing rule could be anything. The sharing rule complicates the analysis a great deal as it does away with symmetry properties. The consumer must now think about what types his neighbours' neighbours are, what their neighbours' neighbours' neighbours are, potentially ad infinitum. The solution to this problem might require a limitation on consumer rationality (e.g. myopic consumers), which in general is undesirable, or a limitation on the capacity to collect information.

There is an interesting connection between increasing returns interaction and telecommunications models. Cambini & Valletti (2005) present a conventional telecommunications model with call propagation features. Their model builds on the Armstrong (1998) - Laffont et al. (1998) paradigm. Interaction network is assumed a complete graph where every consumer has a social relation with all other consumers. When consumers interact pair-wise, interaction propagates in the sense that the more one consumer calls the other, the more the other consumer calls back. Hence, their model incorporates the direct externality, but abstracts away heterogeneity in social relations.

5.3 Random and scale-free networks

The complete graph, circle, and star obviously do not characterise fully any real social network. This warrants a discussion on more complex and general network topologies. There are two classes of networks that are of particular interest: random networks (Erdős-Rényi model) and scale-free networks (see Albert & Barabási (2002) for technical review and Barabási & Bonabeau (2003) for informal discussion).

Random networks theory associates graphs with some specified probabilistic characteristics. The nice feature about random networks is that despite randomness, they present a large degree of regularity. Regularity is captured in the probabilistic characteristics of the network, in particular, in the average number of links a member has. The number of links, or the degree n_i, of a node i is given by a binomial distribution $\Pr\{n_i = n\} = \binom{I-1}{n}\pi^n(1-\pi)^{I-1-n}$ for a population of I individuals. π is the connection probability between two (randomly chosen) nodes. The apparent a priori regularity of random graphs, makes them potentially very applicable. The players would then take expectations on the valuation as well as on the number of links. It is straightforward to see that the results of our model carry on to random network settings. Because consumers are a priori symmetric, the firm sets prices in similar fashion to the symmetric networks under asymmetric information. Sundararajan (2005) constructs a model of local interaction where the underlying social network is a random graph, but focuses on the equilibria of the network members' game without the monopoly pricing problem.

Scale-free networks lack the regularity of random networks. They cannot be characterised by the average number of links. The unifying characteristic of scale-free networks is that the degree of the network follows a power distribution $P(\lambda) \sim \lambda^{-\beta}$, where the degree λ captures the connectedness of a network member. The power law tells that in a scale-free network there are only a handful of members who have very many links, and a very large number of members with only very few links. Scale-free networks are common, as they characterise certain social interaction networks (sexual relationships, academic collaboration), the Internet, even protein interaction networks in the human body. The star we have analysed approximates scale-free networks.

A scale-free network is very tolerant towards random elimination of links, but very vulnerable towards targeted removal of the topologically focal hubs. Ballester et al. (2004) show how crime is best prevented by a "key player removal" policy. The elimination of the hub criminals destroys the crime network in the most efficient way. Respectively, the diffusion of diseases or innovations occurs very rapidly in scale-free networks, because the hubs spread information very efficiently. This can be compared with random networks, where information travels with far less speed. In our model, the firm can remove the empty network equilibrium by providing free goods to some consumers. Provision of free goods corresponds to "piloting" where the firm tests the novel device with a selected group of consumers before the commercial launch. Piloting has two functions. One, pilot users spread information about the goods (create latent demand). Two, they help in product development by testing the product in real life situations. In the star network, the firm should target the centre for the most rapid deployment of information about a new device. On the other hand, the centre is a potential source for large rents, so it may be more profitable to provide free goods to a few peripheral agents instead. The firm could then rely on indirect information transmission through the centre to other peripheral agents.

Chwe (2000) presents a model where agents coordinate their actions with the help of a communication network. Chwe's (2000) primary interest is in political action, but the model has important implications on diffusion of new products as well. His analysis on how agents time their actions is particularly valuable. The analysis allows for the categorisation of agents into social roles like "early adopters" and "followers". He analyses how the network structure affects the diffusion of an action.[17] Chwe (2000) also discusses how the initial seeding of agents who are biased towards revolting affects the diffusion speed. This is an analogy of how to choose the pilot customers: whether they should be dispersed in the network or clustered.

Granovetter (1973) introduced the concept of weak and strong links. A strong link network means that most of one consumer's neighbours are also his neighbours' neighbours: "My friends' friends are also my friends". News or innovations travel faster in weak link networks where neighbourhoods overlap little. Differentiation between weak and strong link networks has two implications to our model. If the consumer has not bought the device and his neighbours have not bought it either, then having weak links is better, since the impact of someone outside the neighbourhood who has bought the good travels a greater distance. On the other hand, if the consumer has bought the device, then it is better to have strong links. Why? Strong links are good because neighbours observe that the consumer has bought the device, plus they know that their neighbours also observe that the consumer has bought the device. Since neighbours' neighbours tend to also be the buyer's neighbours, the purchase induces an effective positive externality in a strong link neighbourhood.

6 Conclusions

We have analysed monopoly pricing of social goods when the market is characterised by buyers' social relations. The model presents a stylised version of coordination goods such as mobile phones, fax machines, e-mail clients, or online game consoles. We have showed that in markets where social relations are important the conventional models of network externalities fall short and need to be

[17] Gladwell (2000) discusses social roles in networks. He reports how political revolt, crime, or product penetration hinges on the information reaching the critical agents (i.e. social roles) at a proper time.

refined. In particular, the implicit assumption of a completely connected graph that does away all topological asymmetries can result in serious overestimation of network effects. Consequently, both achievable monopoly rents and total surplus generated in the network become exaggerated. Our model is an improved treatment of markets of communication goods, where social relations determine the patterns of interaction, and consequently determine the demand for the good.

Agents who have important connections capture a higher surplus compared to more peripheral agents. Under perfect information, focal positions can arise in symmetric and asymmetric networks. A preferential position is either due to central network position (network topology) or due to important neighbours (agent configuration). Once agents' valuations are private information, the preferential positions in symmetric networks are eliminated. However, topologically central agents in asymmetric networks always capture higher surplus than peripheral agents. This contrasts the results from the perfect information regime where a preferential position is always dependent on other agents. In particular, a topologically central position can be redundant if the agent occupying that location does not constrain the firm by having a relatively low valuation.

Under perfect information, total surplus is maximised in the complete network (full activity). However, private (monopoly's) incentives to cover the market tend to differ from social optimum. The monopoly's price strategy depends on the network topology, and on the configuration and heterogeneity of agents. The firm chooses partial coverage in cases where consumer heterogeneity is high and high types form tight clusters. Heterogeneity between consumers can benefit the firm. Profits increase as heterogeneity increases if the high type consumers are clustered. When the high types are each others' neighbours, the firm can charge a high price from them and exclude the low types. If the high types are dispersed in the network, low types' participation is also needed, and therefore the firm does not benefit from increased heterogeneity.

Asymmetric information turned out to be analytically more straightforward. Much of the complexity of perfect information was eliminated as the role of agent configuration lost all importance. The number of neighbours (each member has) is the only network-specific parameter which affects the optimal price. Because agent configuration is irrelevant, the lowest types are ex ante excluded by monopoly pricing. Monopoly price is increasing in the number of links. Higher heterogeneity equals higher uncertainty that reduces profits and consumer surplus.

In asymmetric networks, the firm chooses a price that guarantees a higher probability to buy for the topologically central agents under asymmetric information. Monopoly price is increasing in the size of the periphery, but the centre and the periphery get opposite surplus effects as the number of peripheral agents is increased. An additional peripheral consumer increases the expected utility of the centre. A peripheral agent is not directly affected by the additional consumer, however, increased price decreases his expected utility.

When we compare the compensated networks under asymmetric information, we see that the star is the social optimum, but the firm prefers a symmetric network. If the firm is allowed to price discriminate with respect to network location, its profits rise to the level it obtains in the symmetric networks. Price discrimination reduces total surplus as consumer surplus drops more than profits increase.

We assumed that the size and topology of the underlying social network are common knowledge. While the assumption on size is easily received, the assumption on topology is relatively strong, especially for asymmetric networks. If we assume that the neighbourhoods are private information, consumers have to take expectations on the sizes of their neighbours' neighbourhoods, neighbours' neighbours' neighbourhoods, and so on. On the other hand, if the firm observes nothing, it applies same expectations on all neighbourhoods. The complexity level may not be overly increased for networks that present regularity, such as the random graphs, with independently drawn neighbourhood sizes. This is an interesting area for future research.

We have focused on the static properties of social networks. An obvious extension would be to expand the model in time. A multi-period model would shed light on optimal price paths. It would be interesting to see what is the order of purchases in asymmetric networks. We have analysed three primitive network topologies with a simple utility specification. The next step is to consider richer topologies and utility specifications in order to generalise our results from the numerical examples. Finally, allowing transfers or communication between consumers would introduce signalling aspects to the coordination game under asymmetric information. The interesting question is then how the firm could benefit from high types' preferences to signal their types to the neighbours. It would also be interesting to understand how the firm could use two-part tariffs for screening.

7 References

Albert, R. & Barabási, A.-L. (2002) "Statistical Mechanics of Complex Networks", **Reviews of Modern Physics, 74(1)**, 47-97.

Armstrong, M. (1998) "Network Interconnection in Telecommunications", **Economic Journal, 108(448)**, 545-564.

Arthur, W.B. (1989) "Competing Technologies, Increasing Returns, and Lock-In by Historical Events", **Economic Journal, 99(394)**, 116-131.

Ballester, C. & Calvó-Armengol, A. & Zenou, Y. (2004) "Who's Who in Crime Networks: Wanted - The Key Player", **CEPR Discussion Paper 4421**.

Barabási, A.-L. & Bonabeau, E. (2003) "Scale-free Networks", **Scientific American, May**, 50-59.

Bentolila, S. & Michelacci, C. & Suarez, J. (2004) "Social Contacts and Occupational Choice", mimeo.

Bramoullé, Y. & Kranton, R. (2004) "Public Goods in Networks: How Networks Can Shape Social Learning and Innovation", mimeo.

Bramoullé, Y. & Saint-Paul, G. (2004) "Social Networks and Labour Market Transitions", **IZA Discussion Paper 1215**.

Calvó-Armengol, A. & Jackson, M. O. (2004) "The Effects of Social Networks on Employment and Inequality", **American Economic Review, 94(3)**, 426-454.

Cambini, C. & Valletti, T. (2005) "Information Exchange and Competition in Communications Networks", mimeo.

Chwe, M.S.-Y. (2000) "Communication and Coordination in Social Networks", **Review of Economic Studies, 67(1)**, 1-16.

David, P.A. (1985) "Clio and the Economics of QWERTY", **American Economic Review, 75(2)**, 332-337.

De Palma, A. & Leruth, L. (1996) "Variable Willingness to Pay for Network Externalities with Strategic Standardization Decisions", **European Journal of Political Economy, 12(2),** 235-251.

Ellison, G. (1993) "Learning, Local Interaction, and Coordination", **Econometrica, 61(5)**, 1047-1071.

Farrell, J. & Katz, M.L. (1998) "The Effects of Antitrust and Intellectual Property Law on Compatibility and Innovation", **Antitrust Bulletin, Fall-Winter,** 609-650.

Farrell, J. & Saloner, G. (1985) "Standardization, Compatibility, and Innovation", **RAND Journal of Economics, 16(1)**, 70-83.

Farrell, J. & Saloner, G. (1986) "Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation", **American Economic Review, 76(5)**, 940-955.

Fudenberg, D. & Tirole, J. (1991) **Game Theory**, The MIT Press, Cambridge Massachusetts.

Gaduh, A.B. (2002) "Information and Social Networks in Village Economies", **CSIS Working Paper WPE 063**.

Gandal, N. (1994) "Hedonic Price Indexes for Spreadsheets and an Empirical Test for Network Externalities", **RAND Journal of Economics, 25(1)**, 160-170.

Gladwell, M. (2000) **The Tipping Point**, Back Bay Books, New York New York.

Glaeser, E.L. & Sacerdote, B. & Scheinkman, J.A. (1996) "Crime and Social Interactions", **Quarterly Journal of Economics, 111(2)**, 507-548.

Glaeser, E. & Scheinkman, J.A. (2003) "Nonmarket Interactions", in **Advances in Economics and Econometrics, Theory and Applications, Eight World Congress, 1**, 339-369, Eds. Dewatripont, M. & Hansen, L.P. & Turnovsky, S.J., Cambridge University Press, Cambridge United Kingdom.

Goldstein, M. & de Janvry, A. & Sadoulet, E. (2002) "Is a Friend in Need a Friend Indeed? Inclusion and Exclusion in Mutual Insurance Networks in Southern Ghana", mimeo.

Goyal, S. & Konovalov, A. & Moraga-González, J.L. (2003) "Hybrid R&D", **Tinbergen Institute Discussion Paper TI2003-041/1**.

Granovetter, M.S. (1973) "The Strength of Weak Ties", **American Journal of Sociology, 78(6)**, 1360-1380.

Herrendorf, B. & Valentinyi, A. & Waldman, R. (2000) "Ruling Out Multiplicity and Indeterminancy: The Role of Heterogeneity", **Review of Economic Studies, 67(2)**, 295-307.

Ioannides, Y.M. & Loury, L.D. (2004) "Job Information Networks, Neighbourhood Effects, and Inequality", **Journal of Economic Literature, 42(4)**, 1056-1093.

Jackson, M.O. (2003) "A Survey of Models of Network Formation: Stability and Efficiency", forthcoming in **Group Formation in Economics: Networks, Clubs and Coalitions**, Eds. Demange, G. & Wooders, M., Cambridge University Press, Cambridge United Kingdom.

Katz, M.L. & Shapiro, C. (1985) "Network Externalities, Competition, and Compatibility", **American Economic Review, 75(3)**, 424-440.

Kranton, R.E. & Minehart, D.F. (2001) "A Theory of Buyer-Seller Networks", **American Economic Review, 91(3)**, 485-508.

Labini, M.S. (2004) "Social Networks and Wages: It Is All about Connections!", **LEM Working Paper 2004/10**.

Laffont, J.-J. & Rey, P. & Tirole, J. (1998) "Network Competition: I. Overview and Nondiscriminatory Pricing", **RAND Journal of Economics, 29(1)**, 1-37.

Lee, I.H. & Valentinyi, A. (2000) "Noisy Contagion without Mutation", **Review of Economic Studies, 67(1)**, 47-56.

Mason, R. & Valletti, T.M. (2001) "Competition in Communication Networks: Pricing and Regulation", **Oxford Review of Economic Policy, 17(3)**, 389-415.

Moore, G.A. (1999) **Inside the Tornado: Marketing Strategies from Silicon Valley's Cutting Edge**, HarperCollins, New York New York.

Morris, S. (2000) "Contagion", **Review of Economic Studies, 67(1)**, 57-78.

Schelling, T.C. (1969) "Models of Segregation", **American Economic Review, 59(2)**, 488-493.

Sundararajan, A. (2005) "Local Network Effects and Network Structure", mimeo.

Sääskilahti, P. (2005) "Buying Decision Coordination and Monopoly Pricing of Network Goods", **HECER Discussion Papers, 64**.

Udry, C.R. & Conley, T.G. (2004) "Social Networks in Ghana", **Center Discussion Paper, 888.**

Vives, Xavier (2001) **Oligopoly Pricing,** MIT Press, Cambridge Massachusetts.

Young, H.P. (1998) **Individual Strategy and Social Structure, an Evolutionary Theory of Institutions**, Princeton University Press, Princeton New Jersey.

8 Appendix

8.1 Perfect information examples

Example 25 (Complete graph) *Consider a complete graph with four consumers. Let the valuations be* $\theta_1 < \theta_2 < \theta_3 < \theta_4$, *and* $c < 3\theta_1$ *so that costs do not constrain firm's decisions. Complete network is feasible only if* $p \leq 3\theta_1$. *Partial network with three buyers is feasible if* $\max\{3\theta_1, \theta_3\} < p \leq 2\theta_2$, *and with two buyers if* $\max\{3\theta_1, 2\theta_2\} < p \leq \theta_3$. *We omit the uninteresting case of the empty network.*[18] *Firm's profits are* $V_4 = 4(3\theta_1 - c)$, $V_3 = 3(2\theta_2 - c)$, *and* $V_2 = 2(\theta_3 - c)$ *respectively. Depending on the relative values of* θ_1, θ_2 *and* θ_3 *(the highest type*

[18] *Price* $p > \max\{3\theta_1, 2\theta_2, \theta_3\}$ *guarantees an empty network in the maximal NE. Firm's profits are* $V_0 = 0$ *in this case.*

does not matter), the firm chooses between a complete network and a partial network of either 2 or 3 buyers. A comparison of profits suggests that complete network is chosen if valuations are close together.

- *Complete network:* $V_4 > V_3$ *and* $V_4 > V_2 \Leftrightarrow \theta_1 > \max\left\{\frac{1}{2}\theta_2 + \frac{1}{12}c, \frac{1}{6}\theta_3 + \frac{1}{6}c\right\}$.

Partial 3-buyer network is chosen when middle valuations θ_2 *and* θ_3 *are close together, but significantly higher than* θ_1.

- *3-buyer network:* $V_3 > V_4$ *and* $V_3 > V_2 \Leftrightarrow \theta_2 > \max\left\{2\theta_1 - \frac{1}{6}c, \frac{1}{3}\theta_3 + \frac{1}{6}c\right\}$.

Partial 2-buyer network is chosen when there is a large difference between the two lowest and the two highest valuations.

- *2-buyer network:* $V_2 > V_4$ *and* $V_2 > V_3 \Leftrightarrow \theta_3 > \max\left\{6\theta_1 - c, 3\theta_2 - \frac{1}{2}c\right\}$.

When the market is relatively homogenous in terms of consumers' valuations, the firm will choose a complete network in the equilibrium. It benefits from high sales volumes. Even if the monopolist is unable to price discriminate, it may choose to cover the whole market. On the other hand, if agents are heterogenous, then it pays off to exclude low types from the market by charging a high price.

Example 26 (Circle) *Consider a circular network with four consumers with valuations* $\theta_1 < \theta_2 < \theta_3 < \theta_4$, *and* $c < 2\theta_1$ *so that costs do not interfere pricing, and focus on the maximal equilibrium. Immediately, we can see that there are two cases that yield different results. In the case A, the high valuation types* θ_3 *and* θ_4 *are neighbours (a circle where* θ_1 *has neighbours* θ_2 *and* θ_3, *and where his neighbours are* θ_2 *and* θ_4 *yield identical results). In the case B, they are not. The network structure sets limits to the firm's choices in the case B. Consumer* θ_2 *located between* θ_3 *and* θ_4, *holds a critical position. Any non-empty equilibrium must include him. In both cases, complete network occurs if* $2\theta_1 \geq p$, *and firm's profits are* $V_4 = 4(2\theta_1 - c)$. *Partial network with three buyers is feasible in the case A if* $2\theta_1 < p \leq \theta_2$, *and in the case B if* $2\theta_1 < p \leq \min\{2\theta_2, \theta_3\}$. *Partial network with two buyers is feasible in the case A if* $\max\{2\theta_1, \theta_2\} < p \leq \theta_3$. *Two buyer network is always dominated by other structures in the case B. We skip the uninteresting case of the empty network.*[19] *The firm chooses the complete network only when consumers' valuations are sufficiently close together.*

- *Complete network in the case A:* $V_4 > V_3^A$ *and* $V_4 > V_2^A \Leftrightarrow \theta_1 > \max\left\{\frac{3}{8}\theta_2 + \frac{1}{8}c, \frac{1}{4}\theta_3 + \frac{1}{4}c\right\}$.
- *Complete network in the case B:* $V_4 > V_3^B \Leftrightarrow \theta_1 > \frac{3}{8}\min\{2\theta_2, \theta_3\} + \frac{1}{8}c$.

The firm chooses a three buyer network in both cases if the lowest type has a significantly lower valuation, and the other consumers have valuations not too different from each other.

- *3-buyer network in the case A:* $V_3^A > V_4$ *and* $V_3^A > V_2^A \Leftrightarrow \theta_2 > \max\left\{\frac{8}{3}\theta_1 - \frac{1}{3}c, \frac{2}{3}\theta_3 + \frac{1}{3}c\right\}$.
- *3-buyer network in the case B:* $V_3^B > V_4 \Leftrightarrow \min\{2\theta_2, \theta_3\} > \frac{8}{3}\theta_1 - \frac{1}{3}c$.

In three buyer networks, the firm benefits if the high types (θ_3 *and* θ_4*) are dispersed in the network. We have* $V_3^A < V_3^B$ *always. High types support the purchases of their common neighbour* θ_2, *so that network configuration relaxes firm's pricing constraint.*

The firm chooses a two buyer network in the case A when the two highest types have significantly higher valuations compared with the two lowest types. In the case B, two buyer network is always dominated either by the complete network or the three buyer network.

- *2-buyer network in the case A:* $V_2^A > V_4$ *and* $V_2^A > V_3^A \Leftrightarrow \theta_3 > \max\left\{4\theta_1 - c, \frac{3}{2}\theta_2 - \frac{1}{2}c\right\}$.

[19] *In the case A, price* $p > \max\{2\theta_1, \theta_3\}$ *yields an empty network. In the case B, empty network is produced with price* $p > \max\{2\theta_1, \theta_M\}$, *where* $\theta_M = \min\{2\theta_2, \theta_3\}$. *Profits are zero in both cases.*

In general when the agents have valuations close together, the firm prefers the complete network, and if the high types have significantly higher valuations than the low types, the firm chooses a partial network. This relation is disturbed by the way consumers are arranged on the network. Segregation between two highest types and two lowest types may be blocked by the agent configuration, as it happens in the case B.

When the difference in valuations of two highest and two lowest types grow large, so that we have $\theta_3 > \max\left\{4\theta_1 - c, \frac{3}{2}\theta_2 - \frac{1}{2}c\right\}$, *the firm strictly prefers two buyer network. In the case A, this causes no problems to the firm as it can exclude* θ_1 *and* θ_2 *from the market. However, in the case B, the network structure may constrain the firm. It is forced to sell to three consumers, which yields lower profits if* $\theta_2 < \frac{1}{3}\theta_3 + \frac{1}{6}c$. *In this case, the firm prefers the case where* θ_3 *and* θ_4 *are neighbours. Respectively, if we have* $\theta_3 < \max\left\{4\theta_1 - c, \frac{3}{2}\theta_2 - \frac{1}{2}c\right\}$, *then the firm is better off if high types are dispersed in the network.*

Example 27 (Star) *Consider the following four consumer example with a centre and three peripheral agents. Let the peripheral consumers' valuations be* $c < \theta_1 < \theta_2 < \theta_3$.

(i) Complete network is optimal if $\begin{cases} \min\{\theta_1, 3\theta_C\} > \frac{3}{4}\left(\min\{\theta_2, 2\theta_C\}\right) + \frac{1}{4}c \\ \min\{\theta_1, 3\theta_C\} > \frac{1}{2}\left(\min\{\theta_3, \theta_C\}\right) + \frac{1}{2}c \end{cases}$.

(ii) 3-buyer network is optimal if $\begin{cases} \min\{\theta_2, 2\theta_C\} > \frac{4}{3}\left(\min\{\theta_1, 3\theta_C\}\right) - \frac{1}{3}c \\ \min\{\theta_2, 2\theta_C\} > \frac{2}{3}\left(\min\{\theta_3, \theta_C\}\right) + \frac{1}{3}c \end{cases}$.

(iii) 2-buyer network is optimal if $\begin{cases} \min\{\theta_3, \theta_C\} > 2\left(\min\{\theta_1, 3\theta_C\}\right) - c \\ \min\{\theta_3, \theta_C\} > \frac{3}{2}\left(\min\{\theta_2, 2\theta_C\}\right) - \frac{1}{2}c \end{cases}$.

From (i)-(iii) we see that higher heterogeneity in θ *supports partial networks, whereas if consumers are sufficiently homogeneous in terms of* θ, *the firm chooses a complete network. The topologically important position of the centre is illustrated. The firm must guarantee his participation, thus its price may be constrained.*

8.2 Consumer surplus and total surplus under perfect information

Table 2: Consumer surplus, $\theta_1 \leq \theta_2 \leq \theta_3 \leq \theta_4$

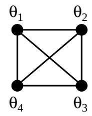
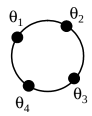
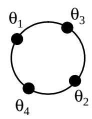
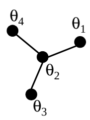
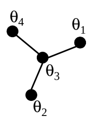
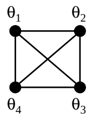
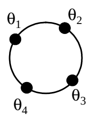
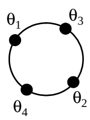
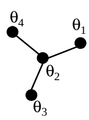
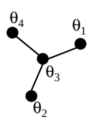
	Complete network	3-buyer network	2-buyer network
Complete graph 	$3\,(\theta_2 + \theta_3 + \theta_4) - 9\theta_1$	$2\,(\theta_3 + \theta_4) - 4\theta_2$	$\theta_4 - \theta_3$
Circle A 	$2\,(\theta_2 + \theta_3 + \theta_4) - 6\theta_1$	$(2\theta_3 + \theta_4) - 2\theta_2$	$\theta_4 - \theta_3$
Circle B 	$2\,(\theta_2 + \theta_3 + \theta_4) - 6\theta_1$	$\max\{(\theta_3 + \theta_4) - 4\theta_2,$ $(2\theta_2 + \theta_4) - 2\theta_3\}$	Dominated
Star, 2 as centre 	$(3\theta_2 + \theta_3 + \theta_4) - 3\theta_1$	$\max\{(\theta_3 + \theta_4) - 4\theta_2,$ $(2\theta_2 + \theta_4) - 2\theta_3\}$	Dominated
Star, 3 as centre 	$(\theta_2 + 3\theta_3 + \theta_4) - 3\theta_1$	$(2\theta_3 + \theta_4) - 2\theta_2$	$\theta_4 - \theta_3$

Table 3: Total surplus, $\theta_1 \leq \theta_2 \leq \theta_3 \leq \theta_4$

<table>
<tr><th></th><th>Complete network</th><th>3-buyer network</th><th>2-buyer network</th></tr>
<tr><td>Complete graph
</td><td>$3\,(\theta_1+\theta_2+\theta_3+\theta_4)$</td><td>$2\,(\theta_2+\theta_3+\theta_4)$</td><td>$\theta_3+\theta_4$</td></tr>
<tr><td>Circle A
</td><td>$2\,(\theta_1+\theta_2+\theta_3+\theta_4)$</td><td>$\theta_2+2\theta_3+\theta_4$</td><td>$\theta_3+\theta_4$</td></tr>
<tr><td>Circle B
</td><td>$2\,(\theta_1+\theta_2+\theta_3+\theta_4)$</td><td>$2\theta_2+\theta_3+\theta_4$</td><td>Dominated</td></tr>
<tr><td>Star, 2 as centre
</td><td>$\theta_1+3\theta_2+\theta_3+\theta_4$</td><td>$2\theta_2+\theta_3+\theta_4$</td><td>Dominated</td></tr>
<tr><td>Star, 3 as centre
</td><td>$\theta_1+\theta_2+3\theta_3+\theta_4$</td><td>$\theta_2+2\theta_3+\theta_4$</td><td>$\theta_3+\theta_4$</td></tr>
</table>

8.3 Stability of equilibria under asymmetric information

We provide checks for equilibria stability based on a Nash tâtonnement process (see e.g. Fudenberg & Tirole 1991). This process checks equilibrium stability against small perturbations.

8.3.1 Symmetric networks

The equilibrium condition can be deconstructed into two equations $\pi = z_i$ (the 45-degree line) and $z_i = 1 - F\left(\min\left\{\theta^+, \widetilde{\theta}_i\right\}\right)$, which must be equal in the equilibrium for all $i \in \mathcal{G}$. The condition for asymptotic stability is $\left|\frac{\partial \pi}{\partial z_i}\right| \left|\frac{\partial z_i}{\partial \pi}\right| < 1 \; \forall i \in \mathcal{G}$. The deconstructed equilibrium condition is

$$\begin{cases} \pi = z \\ z = 1 - F\left(\frac{p}{n\pi}\right) \end{cases} \quad \text{for positive equilibria}$$

$$\begin{cases} \pi = z \\ z = 0 \end{cases} \quad \text{for the empty network.}$$

We have for the maximal BNE $\left|\frac{\partial \pi}{\partial z}\right| \left|\frac{\partial z}{\partial \pi}\right|_{\pi=\pi^*_+} = \frac{1}{2}$, and the equilibrium is stable. For the lower positive BNE the same check returns $\left|\frac{\partial \pi}{\partial z}\right| \left|\frac{\partial z}{\partial \pi}\right|_{\pi=\pi^*_-} = 2$, which indicates that the equilibrium is unstable. The empty network is also stable since $\left|\frac{\partial \pi}{\partial z}\right| \left|\frac{\partial z}{\partial \pi}\right|_{\pi=0} = 0$.

8.3.2 Star

In the region where a non-zero positive equilibrium can exist, the equilibrium conditions are

$$\pi_C = 1 - F\left(\frac{p}{(I-1)\pi}\right)$$
$$\pi = 1 - F\left(\frac{p}{\pi_C}\right)$$

We study only the case $c = 0$, $\theta^+ = 1$, and $\theta^- = 0$, which we discuss in the main text. Since the model does not give out explicit equilibrium values that would be easily applied to the stability check $\left|\frac{\partial \pi}{\partial \pi_C}\right| \left|\frac{\partial \pi_C}{\partial \pi}\right| < 1$, we resort to a numerical test. When the equilibrium values π^*_C, π^* and $p\left(\pi^*_C, \pi^*\right)$ are substituted in the stability equation, we can plot the curve $s = \left|\frac{\partial \pi}{\partial \pi_C}\right| \left|\frac{\partial \pi_C}{\partial \pi}\right|$ for different values of I. It turns out that s remains below one for $I \geq 3$, and it approaches zero as I grows very large. Hence, the equilibrium is a stable one. The other equilibrium, namely the empty network, is obviously a stable one as well.

3.11.2005

Strategic R&D and Network Compatibility*

Abstract

We analyse the effects of network externalities in strategic R&D competition. We present a model of two firms competing with R&D investments and prices in a differentiated consumer market. Buyers form firm-specific networks which can be compatible. A high degree of compatibility and large spillovers moderate price competition due to weak strategic value of firm-specific networks and R&D investments respectively. Asymmetry in product qualities brings out network effects that cancel out in conventional symmetric settings. The lower quality firm increases R&D and decreases its price as spillovers or network compatibility is increased. This happens when R&D and firm-specific network size have high strategic value.

Keywords: R&D, spillovers, networks, compatibility.

JEL Classification: L13, L15, O32.

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1 INTRODUCTION

Network structures are pervasive in modern economies: people spend increasing amounts of time and money on internet services, organisations link themselves to other organisations with various cooperational relationships. At the same time, competition in many network industries is undertaken on various levels that mix strategic investments and price competition. This complexity generates interesting competitive firm behaviour. In this paper, we examine the effects of demand side network externalities on strategic cost reducing R&D investments. We augment a standard horizontal differentiation model by introducing network externalities into the demand side as well as involuntary spillovers into R&D production. Technological change has been an element in network economics since Farrell & Saloner (1985), David (1985) and Arthur (1989), but the feature that technological investments are imperfectly appropriable has been overlooked. The merger of strategic R&D and networks brings up new kinds of firm behaviour that refines the results of earlier literature. The model we construct allows us to study R&D, R&D spillovers, network externalities and compatibility - all in the same framework. Moreover, the model enables us to study also asymmetric settings, which turn out to produce more diverse results compared with symmetric cases.

Imperfect appropriability of R&D and its consequences on industrial competition, as well as the performance of different forms of R&D cooperation, in terms of welfare, have been studied by, among others: Spence (1984), d'Aspremont & Jacquemin (1988, 1990), Henriques (1990), Kamien *et al.* (1992), Suzumura (1992), Suzumura & Yanagawa (1993), Kultti & Takalo (1998), and Amir (2000). De Bondt (1997) is a survey of R&D appropriability literature. In strategic investment games, the total effect of R&D spillovers consists of a market expansion effect that encourages R&D investments as well as a competitive effect that discourages R&D. De Bondt (1997) generalises that the competitive effect dominates the market expansion effect, and therefore, spillovers tend to have a discouraging effect on an individual firm's willingness to invest in strategic R&D. Yet, De Bondt (1997) claims that the negative relationship may reverse with a small number of rivals and sufficient product differentiation. Asymmetry between firms can result in increasing R&D in spillovers. For example, if one firm is better in appropriating R&D than other firms, it may increase its efforts when spillovers increase. Foros (2004) studies a vertically integrated Internet service provider's incentives to upgrade upstream quality when it faces Cournot duopoly downstream. He shows that if the firm is unregulated to set an access charge for the downstream input, investments in quality are increasing in downstream spillovers. This happens when the downstream competitor is better at utilising the quality improvement. On the other hand, effective R&D that measures total cost reduction for a firm, i.e. firm-specific R&D plus the spillover benefits from other firms, tends to be increasing up to a certain spillovers threshold. When spillovers exceed the threshold, also the effective R&D decreases in spillovers. Product differentiation raises the threshold.

Even though the negative association between firm-specific R&D and spillovers is the general result in the strategic R&D literature, other results have a tendency to be dependent on the chosen model set-up. Hence, many results appear not to be too robust. For example, Amir (2000) reports that the effective R&D level increases in the d'Aspremont & Jacquemin (1988, 1990) set-up when spillovers are relatively small, and decreases when spillovers are large. But, the set-up used by Kamien *et al.* (1992) produces decreasing effective R&D for all spillover levels.

Network incompatibility may arise from many sources such as technical product features or personal tastes. Following Katz & Shapiro (1985), the compatibility literature has focused on the technical interpretation and analysed private and social incentives for compatibility. Incompatibilities create implicit switching costs. Beggs & Klemperer (1992) show that consumer prices are higher with switching costs than without. Besen & Farrell (1994) interpret this result as a tendency of incompatibility to tone down price competition. Incompatibility represents a degree of consumer lock-in, which allows the firm to charge above the price of perfectly compatible goods. Even though Besen & Farrell (1994) leave some room for doubt, they report the literature generally agreeing on that incompatibilities reduce (price) competition. Bental & Spiegel (1995) analyse a network competition model with income-wise differentiated consumers. Wealthier consumers are willing to pay more for a network of a given size. They show that consumers prefer compatible networks as market coverage is the highest and price the lowest under compatibility. The result, however, comes from free entry of firms. In contrast, Shy (2001) shows how "*compatibility is* *anticompetitive*". The key to his conclusion is that, with incompatible goods, market share competition is the dominating feature, which drives prices down. Incompatibility means that a price cut by a firm gains market share in the standard way. In addition, there is a reinforcement effect as a larger network size increases the value of the firm's product. Shy (2001) explains that, with incompatibility, consumers care about the price difference between goods and about the sizes of firm-specific networks. Under perfect compatibility, price competition is relaxed, since the reinforcement effect is absent. The sizes of firm-specific networks are irrelevant as all consumers attain the benefits of the whole network. Firm-specific market shares carry strategic value only under incompatibility. Kristiansen (1998) shows that the competition intensifying nature of incompatibility extends to dynamic R&D games. He demonstrates how duopolists have incentives to agree on a common standard rather than compete by introducing incompatible technologies early.

It is interesting to expose the results of R&D spillover models to network externalities. The combination is more than the sum of parts. In this paper, we employ demand side network externalities that resemble the telecommunications interconnection traffic in Laffont *et al.*'s (1997, 1998) and Armstrong's (1998) models. We extend the game with a stage in which firms choose R&D investments that can be imperfectly appropriable. Our focus is on cost-reducing (process) R&D and its implications on price competition.[1] The main item of interest in our model is consumer interaction; each consumer gets utility from interaction with others. Two firms each serve a network of consumers. Networks are vertically differentiated and they may be linked with different degrees of compatibility. Compatibility is linked chiefly to consumer preferences as opposed to the technical interpretation that has been extensively studied. Firms choose R&D investment levels in the first period. In the second period, they set prices. Our model differs from Kristiansen (1998) in that we allow R&D spillovers, and in that Kristiansen (1998) considers a flow of consumers. His model characterises an emerging market whereas we analyse a mature market.

Network externalities are prone to produce multiple equilibria. Multiplicity causes that equi-

[1] For a network model à la Armstrong (1998) - Laffont *et al.* (1997, 1998) with product R&D that improves the quality of the good, see Valletti & Cambini (2005).

librium analysis does not yield determinate predictions. We overcome the multiplicity problem by differentiating consumers à la Hotelling (1929). With sufficient price insensitivity, we get a unique interior equilibrium. We also consider a conceptually more interesting vertical differentiation of networks, which makes possible the analysis of asymmetric set-ups.

We show how asymmetric equilibria differ from a symmetric one, and when the asymmetric equilibria involve firm behaviour which disagrees with the standard results of strategic R&D and network compatibility models. Our model adds to the complexity in the results of conventional models, rather than generalising them. The main findings are:

1. We derive a case where a firm increases firm-specific R&D investments under a marginal increase in spillovers: the firm with lower quality product tends to increase R&D under an increase in spillovers if R&D and firm-specific network size have high strategic value. This happens when networks are (almost) perfectly incompatible and spillovers small. Demand-side network externalities are driving this result, whereas Foros' (2004) result that a vertically integrated firm's investments are increasing in downstream spillovers is due to a difference in the capability to utilise new technology for the non-investing firm's advantage.

2. We characterise conditions where Shy's (2001) result "*compatibility is anticompetitive*" fails in its strongest form: the firm with lower quality product tends to decrease its price with an increase in compatibility, again, if R&D and firm-specific network size have high strategic value. The price decreases because the firm chooses to increase R&D investments in order to cut costs.

Under competition, private and social incentives for network compatibility are aligned, but they differ for R&D appropriability in general. In the situations where we have the unorthodox behaviour detailed in 1 and 2 above, consumer surplus and the higher quality firm's profits move together and dominate the opposite change in the lower quality firm's profits with marginal changes in compatibility and R&D appropriability.

The findings from a symmetric model agree with the existing literature. Because asymmetry can produce unorthodox results, literature which focuses on symmetric equilibria fails to capture all the prevailing effects.

The model is constructed in section 2. Equilibrium is derived in section 3. Then the equilibrium is subjected to comparative statics analysis in section 4. We illustrate the unorthodox results with a numerical example in section 5. After that we bring forth some issues on total surplus in section 6. We conclude with a discussion. All proofs of results are straightforward and are relegated to the appendix.

2 MODEL

We are interested in industries that present demand side network externalities and some degree of product compatibility. We will later formally propose a compatibility measure that incorporates personal tastes as well as pure technical compatibility.

Personal computer (PC) software is an example of our model. The PC software is an archetypical case for network externalities: utility from a specific type of software increases as its user base grows. A possibility to share files with other people, using the software, combined with interoperability are typical sources of network benefits. The intensity of network externalities depends on software compatibility. For example, users of a particular brand of software may use files created by different brands. Brands may not be perfectly compatible so that some functionalities do not work thoroughly. Some users may dislike the way information is handled in one brand of software, and therefore may swap files less with users of that brand.

The PC operating system (OS) market is famously dominated by Microsoft, especially after the launch of Windows 95, which was the landmark of marginalisation of competing operating systems. Despite overwhelming network benefits from adopting Windows, the market has not tipped entirely in favour of Microsoft. Apple MacOS and Linux hold minor market shares, chiefly because of brand loyal users. On top of the OS layer, there exist a large number of software applications that present similar network externalities. An important example is spreadsheet software. Gandal (1994) confirms that network externalities exist with spreadsheet software as consumers are willing to pay a premium for compatibility and links to external databases. Media player software, such as Windows media player, RealOne player, or Winamp, is an analogous example. Compatibility between players is less a technical feature, but depends more on the user's experience. The players are usually pre-set at operating on proprietary file formats, but the settings can be switched to use compatible standards. Instant messaging (IM) software allows people to communicate over the Internet. A user of IM software benefits when there are more people on the IM network. Yahoo!, MSN or AOL IM do not interoperate, but users of Gaim, Adium or Trillian IM can communicate with the users of (practically) any other IM software.[2] In addition to basic instant messaging, some IM software include more advanced communication functionalities (including voice services), file swapping, and entertainment features. The choice of IM software depends heavily on the versatility of the software as well as on the brand factor (e.g. some consumers have preferences to use only open-source rather than proprietary software). But if software is incompatible, then also the size of the user network matters. Usually the basic versions of IM software are free of charge, but more advanced features have to be purchased.

Another example is game consoles (PlayStation 2, Xbox, and Game Cube) that allow playing against other people over an Internet connection. Consoles tend to support little interoperability. A particular game seldom works on more than one brand, even if there exists many versions of the game (which cover the whole console spectrum).

The final example is mobile telecommunications. Here, the demand side externalities arising from person to person communication are obvious, but brand preference in interaction is (almost) negligible.[3]

We proceed with the construction of the model. There are two firms in the market, A and B. The number of consumers is fixed and they are uniformly distributed over a unit line according to their subjective taste preferences. Each firm is exogenously located at one extreme of the line and the locations are inherited from outside this model, so that firm A is located at 0 and firm B at 1.

[2] Some multi-medium software, such as Trillian, actually allows the users to log-on simultaneously to "host" IM networks of Yahoo!, MSN or AOL (among others), rather than being standalone IM networks. However, the multi-medium IM software incorporate features that are not supported by the host networks making them more than pure adapters.

[3] We assume that firms charge simple flat rate fees without interconnection payments. This reduces applicability of our model to voice telecommunications. However, SMS and MMS services and similar new services are a good fit.

Consumers have unit demand. The product yields intrinsic utility and utility contingent on all other consumers who have bought the product. The idea is that utility is driven by peer-to-peer types of services that enable interaction between consumers, and interaction utility can be split into two parts. First, the consumer gets utility from consumers who have bought from the same firm as he has. This network of consumers is referred to as a "home network" and the utility is labelled as an "intra-network utility". Secondly, the consumer may also get utility from the consumers who have bought from the other firm. This network is referred to as a "rival network", and the utility as an "inter-network utility". Interaction utility depends on the sizes of home and rival networks and on the compatibility between networks. Consumers who have not bought any product give zero utility to those that have. If the consumer located at $s \in [0,1]$ on the Hotelling beach, buys the product, he gets intrinsic utility $V(s) = v - l(s)z$, where $v > 0$ is a fixed base utility, and $l(s)$ is the distance to the supplier. Intrinsic utility is derived from standalone usage that is independent of other people's usage. The transportation cost parameter z measures how well the product matches the consumer's subjective preferences. Consumers are assumed not to be constrained by their budgets, and v is large enough so that all consumers opt for purchasing the product independent of other consumers' decisions. As a result, the consumer's problem is reduced to choosing from which firm he buys.

Firm i charges a flat rate p_i. Net utility for a consumer located at s who buys from firm $i = A, B$, $i \neq j$ is then

$$U_i(s) = v - l(s)z + n_i v_i + n_j \overline{v}_i - p_i. \tag{1}$$

The principal item of interest in equation (1) is the interaction utility given by $n_i v_i + n_j \overline{v}_i$. For the consumer that has decided to buy from firm i, intra-network utility is $n_i v_i$, where n_i is the number of consumers on the home network. Parameter v_i measures the objective value associated with each network member. It gives the usage value of services used in interaction. This objective valuation is shared by all consumers and it is independent of subjective taste. A consumer located in the middle of the Hotelling beach can be indifferent between the goods in terms of subjective attractiveness but strictly prefer one good to the other in terms of objective quality. Term $n_j \overline{v}_i$ gives the respective inter-network utility from a rival network of size n_j

with objective value $\overline{v}_i$. The utility function is inherently of the Katz & Shapiro (1985) type where consumers are heterogeneous with respect to intrinsic utility and homogeneous with respect to network externalities. Our specification of network externalities implicitly assumes that the underlying social network characterising consumers' relationships is a completely connected graph. Each consumer is connected to everyone else.[4]

The consumer gains at most equal utility from a single rival network member compared to a home network member. The rationale behind this assumption is that it is likely that similar types of consumers choose to buy from the same firm, and that consumers interact more with people similar to them, i.e. more with people on the home network. In the absence of usage fees, the rival network is less regularly accessed. Furthermore, there might be technical incompatibilities between rival goods which hamper inter-network interaction. From the perspective of a consumer, the assumption that an individual consumer on the home network is at least valuable as one on the rival network translates into a condition $v_i \geq \overline{v}_i$, which can be parameterised as $v_i \geq v_i(1-t)$, where $t \in [0, 1]$ measures network compatibility. The proposed concept of network compatibility should be understood arising from consumers' tastes and from technical compatibility features. The following example clarifies this. A consumer, who has bought from firm A, located at s interacts more with people located close to s (call these people s's friends). He also has a bias towards interacting more with friends who are on his home network. At the margin, where half of the consumer's friends are on his home network and half on the rival network, the bias in favour of home network is captured in t. If there is no bias between networks, we have $t = 0$. If, however, network brand determines perfectly with whom he interacts with, $t = 1$.[5]

Our assumption of fixed market size, removes the network expansion effects of R&D. An improvement in network compatibility, however, captures some forms of market expansion. Define the efficient network size as $n_i v_i + n_j v_i (1-t)$. As t is lowered, the effective size grows.

[4] See Sääskilahti (2005) for a model with asymmetric network topologies.

[5] Note that we do not need to make any formal assumptions on the balance of access between networks since consumers make a single flat rate payment to the firm. Compare this with telecommunications industry models by Armstrong (1998) or Laffont *et al.* (1997, 1998) who assume a neutral calling pattern. Dessein (2003) makes an important extension to the Armstrong (1998) - Laffont *et al.* (1997, 1998) paradigm by assuming heterogeneous consumers. Consumers are differentiated both horizontally and vertically. Neutral calling patter is disturbed since the horizontal and vertical differentiation are independent. In Dessein (2003), it is the consumer types that are vertically differentiated, whereas in our model the products are vertically differentiated.

If networks are perfectly incompatible ($t = 1$), the rival network does not yield utility. In the typology of Besen & Farrell (1994), perfect incompatibility makes the firms compete for the market. In this case, firm-specific market shares are important in consumers' decision making. The polar case, perfect compatibility ($t = 0$), produces competition within the market. Here, consumers get the full benefits of the total network, and firm-specific network sizes are irrelevant in consumers' decision making.

Consumers' expectations on network sizes are fulfilled in the equilibrium. The indifference condition $U_A(s) = U_B(s)$ determines market shares uniquely. The market share for firm A is

$$s = \frac{z - p_A + p_B + (1-t)\, v_A - v_B}{2z - t\,(v_A + v_B)}. \tag{2}$$

The firms' problem is to maximise profits by choosing R&D investments and setting unit prices. Investments in R&D reduce unit production costs capturing the idea of process R&D. Let x_i be the autonomous, or firm-specific, output of firm i's R&D investment. We eliminate the case in which R&D spillovers would flow only from the R&D leader to the laggard by interpreting x_i so that it represents both R&D output and total input including all trials and errors. A fraction of R&D output, ξx_i, is spilled over to the rival without any cost or compensation. The spillover parameter, $\xi \in [0, 1]$, is symmetric between firms. The effective cost reduction for firm i is then $X_i = x_i + \xi x_j$. Productivity of R&D is independent of spillovers, and the firm's own and the rival's R&D are substitutes. The potentially undesirable possibility that one firm can benefit passively from the rival's R&D is not dealt with. A firm can enjoy cost reduction even if it does not invest in R&D at all. The technological process characterises an incrementally cumulative generation of new knowledge. The firms have equal access to prevailing production technology and they are equally capable of implementing new findings.[6] The unit cost per sale for firm i is $C_i = c - X_i$, where $c > 0$ is assumed to be symmetric between firms.

[6] Different approaches to modeling R&D and spillovers are abundant: for example, Levin & Reiss (1988) and Kesteloot & De Bondt (1993) assume imperfect substitutability of autonomous R&D and an industry-wide pool of R&D. Of other variants Cohen & Levinthal (1989) (variable learning capacities and intra-industry R&D spillovers), De Bondt & Henriques (1995) (asymmetric spillovers and R&D absorption capabilities) and Katsoulacos & Ulph (1998) (endogenous spillovers) are among the most interesting ones. See also Amir (2000) who looks at the differences between d'Aspremont & Jacquemin's (1988, 1990) and Kamien *et al.*'s (1992) formulations of R&D productivity. However, we find symmetry and the absence of industry R&D pools in our set-up appropriate considering the small number of firms and the nature of the incremental technological progress. See section 7 for a discussion on a more detailed account of the technological process.

The objective functions for firm A and B are

$$\begin{aligned}\pi_A &= s\left[p_A - (c - x_A - \xi x_B)\right] - \tfrac{1}{2}x_A^2 \\ \pi_B &= (1-s)\left[p_B - (c - x_B - \xi x_A)\right] - \tfrac{1}{2}x_B^2,\end{aligned} \tag{3}$$

where R&D costs are given by $-\frac{1}{2}x_i^2$, $i = A, B$.

3 EQUILIBRIUM

The firms maximise profits (3) in two stages. In the first stage, they choose simultaneous R&D investments (x_A, x_B), and in the second stage they set prices (p_A, p_B) simultaneously. We solve the problem for a sub-game perfect Nash equilibrium (NE).

Second stage best responses are

$$p_i(p_j) = \frac{1}{2}\left[p_j + c - (x_i + \xi x_j) + (1-t)v_i - v_j + z\right]. \tag{4}$$

Reactions (4) are upward sloping, characteristic of Bertrand price competition. They cross correctly to produce a stable equilibrium.[7] Note that the reaction to a marginal increase in the value of rival product is tougher than to an increase in the value of the firm's own product.

NE prices are

$$p_i^{NE} = c + z - \frac{1}{3}\left[(2+\xi)x_i + (1+2\xi)x_j - (1-2t)v_i + (1+t)v_j\right]. \tag{5}$$

The firm's own R&D causes a larger drop in price than the rival's R&D does, $\frac{\partial p_i^{NE}}{\partial x_i} \leq \frac{\partial p_i^{NE}}{\partial x_j}$, with equality at $\xi = 1$. Hence, all things constant, an increase in R&D by one firm increases its market share.

First stage best responses are

$$x_i(x_j) = \frac{(1-2t)v_i - (1+t)v_j + 3z - (1-\xi)x_j}{A - (1-\xi)}, \tag{6}$$

where $A = \frac{9}{2(1-\xi)}\left[2z - t(v_A + v_B)\right]$. First stage reaction functions have a cut-off point where R&D investments change from being strategic complements to strategic substitutes. Strategic

[7] Stability is ensured since $\left|\frac{\partial p_i}{\partial p_j}\right| = \frac{1}{2} < 1$. Second stage second order conditions require that $z > \frac{1}{2}t(v_A + v_B)$.

complementarity, however, is ruled out by the second order condition for a maximum.[8] Since the reaction functions are linear, they cross at most once. Thus if there is an interior equilibrium, it is unique and corresponds to the fulfilled expectations equilibrium of the consumers' problem.

NE investments are given by equation (7).

$$x_i^{NE} = \frac{(1-2t)\,v_i - (1+t)\,v_j + 3z - \frac{2}{3}(1-\xi)^2}{A - 2(1-\xi)}. \tag{7}$$

NE investments can be presented as $x_A^{NE} = \frac{2}{3}(1-\xi)\,s^{NE}$ and $x_B^{NE} = \frac{2}{3}(1-\xi)\left(1-s^{NE}\right)$. In the case of symmetric market shares, investments are equal. If one firm has smaller market share, it also invests less in R&D. Industry-wide R&D effort depends only on spillovers, $x_A^{NE} + x_B^{NE} = \frac{2}{3}(1-\xi)$, underlining the absence of the market expansion effect. The industry-wide effective cost reduction is $(1+\xi)\left(x_A^{NE} + x_B^{NE}\right) = \frac{2}{3}\left(1-\xi^2\right)$. Obviously, both industry-wide investment and effective output are decreasing on the whole range $\xi \in [0,1]$, and they drop to zero with perfect spillovers. There are no incentives to do R&D when it does not yield competitive advantage.

A general condition for equilibrium stability is $\left|\frac{\partial x_i(x_j)}{\partial x_j}\right|\left|\frac{\partial x_j(x_i)}{\partial x_i}\right| < 1$ (Fudenberg & Tirole 1991). This condition requires in the current model that $A - 2(1-\xi) > 0$. Effectively, this condition sets a lower limit for the transportation costs. The condition guarantees also that second order conditions for maximum hold. Hence, the following condition (A1) is required to hold in equilibrium.

Assumption (A1) Minimum price insensitivity: $z > \frac{1}{2}\left[t\,(v_A + v_B) + \frac{4}{9}(1-\xi)^2\right]$.

Using NE prices and investments, firm i's NE market share can be expressed as

$$s_i^{NE} = \frac{1}{2} - \frac{(v_j - v_i)\left(1 - \frac{1}{2}t\right)}{\frac{2}{3}(1-\xi)\left[A - 2(1-\xi)\right]}, \tag{8}$$

so that $s_A^{NE} = s^{NE}$ and $s_B^{NE} = 1 - s^{NE}$. Note that the denominator in equation (8) is positive by Assumption (A1).

[8] Strategic complementary and substitutability are defined as in Bulow et al. (1985). First stage second order conditions require that $z > \frac{1}{2}t\,(v_A + v_B) + \frac{1}{9}(1-\xi)^2 \Leftrightarrow A > (1-\xi)$, which is more stringent condition than the second stage second order conditions. Now, the second order conditions for the first stage rule out the case in which $\frac{\partial x_A}{\partial x_B}$ would be positive. Consequently, investments are always strategic substitutes. Unique strategic substitutability is a simplification of the general tendency of mixed strategic substitutability and complementarity in strategic R&D investments. De Bondt (1997) states that, in general with quadratic payoffs, first stage investments are strategic substitutes when spillovers are below a certain critical level, and strategic complements with spillovers that exceed the critical level. Here, there is no critical threshold in that sense.

4 COMPARATIVE STATICS

4.1 Efficiency benchmark

A competitive duopoly produces industry-wide R&D levels which are lower than the social optimum. We show this with a comparison between the competitive industry and a Ramsey benchmark. In the Ramsey case, we maximise consumer surplus conditional on the industry breaking even. When qualities are asymmetric, this may involve transfers between firms. Consumer surplus is

$$CS = v + \int_0^s \left[sv_A + (1-s)(1-t)v_A - p_A - zl \right] dl + \\ + \int_s^1 \left[s(1-t)v_B + (1-s)v_B - p_B - z(1-l) \right] dl \quad (9)$$

The Lagrangian of the Ramsey maximisation problem is

$$\mathcal{L} = CS - \lambda(\pi_A + \pi_B), \quad (10)$$

where λ is the Lagrange multiplier. Second stage optimisation of (10) gives the socially optimal prices $\left(p_A^R, p_B^R\right)$. Prices are the same if qualities are identical, otherwise they differ. First stage optimisation of (10) gives socially optimal R&D levels $\left(x_A^R, x_B^R\right)$. The Ramsey industry-wide R&D equals to

$$x_A^R + x_B^R = 1 + \xi. \quad (11)$$

In the competitive duopoly, the industry-wide R&D level is

$$x_A^{NE} + x_B^{NE} = \frac{2}{3}(1-\xi). \quad (12)$$

It is evident that the competitive industry always produces socially too little R&D.

4.2 Symmetric qualities

We start the analysis of the competitive duopoly equilibrium with a case of symmetric qualities ($v_A = v_B = \widetilde{v}$). Symmetric firms split the market 50/50. Market share effects due to changes in ξ and t are neutralised with symmetric qualities, $\frac{ds_{SYM}^{NE}}{d\xi} = \frac{ds_{SYM}^{NE}}{dt} = 0$. With symmetry, the first stage NE degenerates into a simple relationship between R&D and spillovers,

$$x_{SYM}^{NE} = \frac{1}{3}(1-\xi). \quad (13)$$

The two comparative statics that are of interest, namely $\frac{dx_{SYM}^{NE}}{d\xi}$ and $\frac{dx_{SYM}^{NE}}{dt}$, are trivial. Both firm-specific and effective R&D are decreasing in $\xi \in [0,1]$. On the other hand, $\frac{dx_{SYM}^{NE}}{dt}$ is zero. In this case, neither firm can take advantage of the other firm's network due to symmetric behaviour that cancels out.

NE prices are

$$p_{SYM}^{NE} = c + z - t\widetilde{v} - \frac{1}{3}\left(1 - \xi^2\right). \tag{14}$$

The last term in equation (14) equals effective R&D. Not surprisingly, NE prices increase as spillovers increase. This is due to decreasing investments in the first stage. Profits increase in spillovers because of smaller outlays for R&D.

More interestingly, higher levels of network compatibility are associated with higher prices. With some degree of incompatibility ($t > 0$) firms are involved in competition for market share. This competition pushes prices down, despite the fact that incompatibility represents a degree of lock-in of consumers, which softens price competition, but only imperfectly. Price competition is at the most intense level when consumer networks are completely incompatible ($t = 1$), whereas the highest profits are attained with perfectly compatible goods ($t = 0$). Despite consumer utility looks likely to increase as parameter t decreases; the benefit is offset by the increase in prices. In this sense, the parameter t acts as a device of tacit collusion, similar to the interconnection charge in telecommunications models (à la Laffont *et al.* 1998). The firms have incentives to negotiate high compatibility. This could be arranged through industry standards bodies, such as Java Community Process (JCP) for Java software development, or Third Generation Partnership Project (3GPP) and Open Mobile Alliance (OMA) in mobile telecommunications.

4.3 Asymmetric qualities

Like the NE investment functions point out, market share dynamics have a central role in the model. Therefore, it is useful to derive comparative statics for the NE market shares. Lemmas 1 and 2 summarise these statics. Due to a covered market, the market share effects for firms A and B have opposite signs.

Lemma 1 *(i) The market share of the firm with a lower quality good is increasing in spillovers*

$$\frac{ds^{NE}}{d\xi} > 0 \Leftrightarrow v_A < v_B.$$

(ii) The market share of the smaller firm is increasing in spillovers.

Lemma 2 *(i) With sufficiently high price sensitivity,* $\frac{1}{2}t(v_A + v_B) + \frac{2}{9}(1-\xi)^2 < z < v_A + v_B + \frac{2}{9}(1-\xi)^2$*, the market share of the firm with a lower quality good is increasing in network compatibility*

$$\frac{ds^{NE}}{dt} < 0 \Leftrightarrow v_A < v_B.$$

(ii) With sufficiently low price sensitivity, $z > v_A + v_B + \frac{2}{9}(1-\xi)^2$ *, the market share of the firm with a lower quality good is decreasing in network compatibility*

$$\frac{ds^{NE}}{dt} > 0 \Leftrightarrow v_A < v_B.$$

The Assumption (A1) imposes the lower limit for z in part (i) of Lemma 2.

Because network compatibility is foremost associated with consumer's utility, and firms' profits depend on it only indirectly, it is worthwhile to study what kind of impact a change in the parameter t has on intra- and inter-network utilities, which equal to $sv_A + (1-s)(1-t)v_A$ for firm A's customers.

First, if the firm's market share is decreasing in network compatibility, then intra-network utility is also decreasing in compatibility. Smaller home network yields less intra-network utility.

Second, a decrease in compatibility has both a direct effect and an indirect effect on inter-network utility. These two effects are $\frac{d[(1-s)(1-t)v_A]}{dt} = \underbrace{-(1-s)v_A}_{\text{direct effect}} \underbrace{-(1-t)v_A\frac{ds}{dt}}_{\text{indirect effect}}$ for firm A. The direct effect is always negative. If a decrease in compatibility increases home network size ($\frac{ds}{dt} > 0$), then the indirect effect is negative as well. Smaller rival network yields less inter-network utility.

Consider a case with high price sensitivity (as defined in Lemma 2) and a reduction in network compatibility ($dt > 0$). The market share of the lower quality good firm decreases. Its customers get a negative utility effect through intra-network utility. They also get a negative direct effect through the inter-network utility. Negative effects are partly compensated by a positive indirect inter-network effect due to the increase in the size of the rival network. If networks are relatively incompatible, the positive effect is not very strong. It becomes stronger with higher compatibility levels. At the extreme, with perfectly compatible networks, the intra-network effect is cancelled by the positive indirect effect of inter-network utility. Perfect compatibility eliminates the strategic role of firm-specific network size.

4.3.1 *Comparative statics with respect to spillovers*

Conventionally, in strategic investment games, the dominant competitive effect of spillovers guarantees that firm-specific R&D unambiguously decreases as spillovers increase. Since, the market expansion effect is absent in the current model, the competitive effect should guarantee a negative relationship between R&D investments and spillovers. Even if this relation still holds in most cases with asymmetric qualities as well as in the symmetric case, it is not universally true. With asymmetry in the product qualities and high strategic value of R&D and firm-specific networks, the disadvantaged firm increases its investments under a marginal increase in spillovers.

Direct differentiation of x_A^{NE} with respect to ξ gives the following formula

$$\frac{dx_A^{NE}}{d\xi} = -\frac{2}{3}s^{NE}\left[1 + \frac{1-\xi}{\xi}\varepsilon_\xi^s\right], \tag{15}$$

where $\varepsilon_\xi^s = -\left(\frac{\xi}{s^{NE}}\right)\left(\frac{ds^{NE}}{d\xi}\right)$ is the elasticity of market share with respect to spillovers.[9] Increase in spillovers always induces a direct effect to reduce investments. There is also an indirect effect through market share. The firm with higher market share always cuts back investments since its market share decreases as spillovers increase, as given by Lemma 1. Smaller firm's market share is growing in spillovers. When its market share is sufficiently elastic, the positive effect can dominate, and the firm increases its R&D investments with an increase in spillovers.

Proposition 3 *(i) The firm increases its autonomous NE R&D investments with a marginal increase in the level of R&D spillovers, if the elasticity of its market share with respect to spillovers is sufficiently high*

$$\frac{dx_A^{NE}}{d\xi} > 0 \Leftrightarrow \varepsilon_\xi^s < \frac{\xi}{\xi - 1}.$$

(ii) The firm with a higher market share always decreases R&D investments under a marginal increase in spillovers.

(iii) The positive relation $\frac{dx_A^{NE}}{d\xi} > 0$ *is more likely with high quality difference, and with low absolute levels of spillovers. The positive relation is also more likely with low levels of network compatibility, conditional on sufficiently high price sensitivity.*

The intuition in Proposition 3 is that the smaller firm can take advantage of the possibility to grow its home network (when t is large). The larger firm always invests more in R&D than

[9] Since consumers have unit demand, the elasticity of market share corresponds to the elasticity of demand.

its rival, but once spillovers are increased, it wants to limit the leakage. It reduces R&D, which increases its costs and subsequently drives its price up. As the larger firm becomes relatively less attractive, the smaller firm can afford to attack. It invests more. In the new situation, network externalities generated by a larger home network outweigh the quality disadvantage, though the smaller firm remains smaller. Higher network externalities compensate for lower quality.

The comparative static for the NE price of firm A is given by equation (16).

$$\frac{dp_A^{NE}}{d\xi} = \frac{-\left[\left(1-2s^{NE}\right)-2\xi\left(2-s^{NE}\right)\right]\left[2z-t\left(v_A+v_B\right)\right]-\frac{3}{4}\xi\left(1-\xi\right)^2}{\left(1-\xi\right)\left[A-2\left(1-\xi\right)\right]}. \tag{16}$$

Proposition 4 *There exists (at least one) threshold level ξ^*, above which both firms increase their NE prices under a marginal increase in spillovers. Below the threshold, the smaller firm (in terms of market share) reduces and the larger firm increases its NE price.*

Since firm size is directly related to the quality difference of the goods, the firm with the lower quality good decreases its price under a marginal increase in spillovers when $\xi < \xi^*$. Otherwise both firms increase price due to smaller outlays in R&D.

4.3.2 *Comparative statics with respect to network compatibility*

In the symmetric case, network compatibility is neutralised in the investment decisions. The situation becomes more interesting with asymmetric qualities, which reintroduce the competitive nature of network size and network compatibility into the game. By differentiating firm A's NE investments, we get

$$\frac{dx_A^{NE}}{dt} = \frac{2}{3}\left(1-\xi\right)\frac{ds^{NE}}{dt}. \tag{17}$$

By using Lemma 2, we arrive at Proposition 5 which gives the equilibrium behaviour of the lower quality firm. The investment behaviour of the high quality firm is of opposite sign.

Proposition 5 *(i) With sufficiently high price sensitivity, $\frac{1}{2}t\left(v_A+v_B\right)+\frac{2}{9}\left(1-\xi\right)^2 < z < v_A + v_B + \frac{2}{9}\left(1-\xi\right)^2$, the low quality firm decreases R&D under a marginal decrease in network compatibility*

$$\frac{dx_A^{NE}}{dt} < 0 \Leftrightarrow v_A < v_B.$$

(ii) With sufficiently low price sensitivity, $z > v_A + v_B + \frac{2}{9}\left(1-\xi\right)^2$, the low quality firm increases R&D under a marginal decrease in network compatibility

$$\frac{dx_A^{NE}}{dt} > 0 \Leftrightarrow v_A < v_B.$$

Consider an asymmetric case where $\frac{ds^{NE}}{dt} < 0$. An increase in network compatibility results in a gain in intra-network utility for firm A's customers due to the increase in firm A's market share. In addition, the direct effect of the inter-network utility is positive as well, but the indirect effect is negative. However, the negative indirect effect never dominates the positive effects. Hence, firm A's customers face a positive impact on their utility in total. The competitive position of firm A is improved. This opportunity to increase the net value of the product through network externalities motivates the firm to increase investments.

Although the firms increase prices with an increase in network compatibility in general, the opposite reaction is possible. Firm A's price response to a decrease in network compatibility is

$$\frac{dp_A^{NE}}{dt} = -\frac{1}{3}\left[\frac{2}{3}(1-\xi)^2 \frac{ds^{NE}}{dt} + 2v_A + v_B\right]. \tag{18}$$

With high price sensitivity defined as in Lemma 2, Proposition 6 summarises price changes.

Proposition 6 *Let $v_A < v_B$, then under a marginal decrease in network compatibility ($dt > 0$):*

(i) High quality firm
- *decreases its price when price sensitivity is high*
- *decreases its price when price sensitivity is low and* $0 < \frac{ds^{NE}}{dt} < \frac{2v_B+v_A}{\frac{2}{3}(1-\xi)^2}$
- *increases its price when price sensitivity is low and* $\frac{ds^{NE}}{dt} > \frac{2v_B+v_A}{\frac{2}{3}(1-\xi)^2}$

(ii) Low quality firm
- *decreases its price when price sensitivity is high and* $0 > \frac{ds^{NE}}{dt} > -\frac{2v_A+v_B}{\frac{2}{3}(1-\xi)^2}$
- *increases its price when price sensitivity is high and* $\frac{ds^{NE}}{dt} < -\frac{2v_A+v_B}{\frac{2}{3}(1-\xi)^2}$
- *decreases its price when price sensitivity is low*

The interesting result is that for some parameter values, the firm decreases its price for an increase in compatibility. The price of the lower quality firm is decreasing in compatibility if price sensitivity is high and its market share is sufficiently elastic with respect to compatibility. In this case, the firm increases its R&D efforts for a marginal increase in compatibility, resulting in lower costs and price. Note that the higher are R&D spillovers, i.e. the less effective R&D is, the greater must the change in market share be in order to obtain the positive result $\frac{dp_A^{NE}}{dt} > 0$.

5 NUMERICAL EXAMPLE

We have derived two results that work against the general findings. The first one is that a firm can increase its autonomous R&D investments if spillovers are marginally increased. The second result is that a firm can decrease its price when network compatibility is marginally increased.

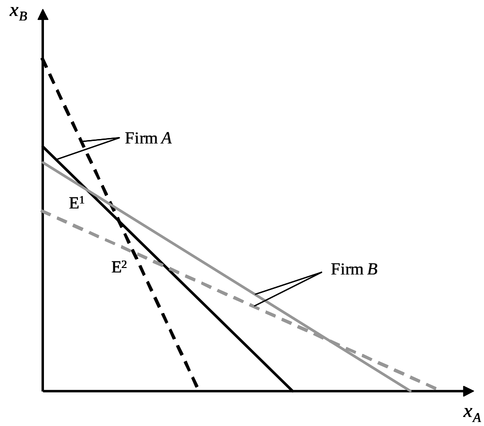

Figure 1: First stage reactions, $t = 1$.

The origin of both results is in network externalities and asymmetric qualities. The unorthodox behaviour appears only when R&D and firm-specific networks have high strategic value.

We can clarify the results with a numerical example. Consider a duopoly with the following set of parameters: $v = 2$, $v_A = 0.72$, $v_B = 0.8$, $z = 1$, $c = 0.6$. Firm A has a 10% disadvantage in terms of product quality. Consumers are relatively price sensitive as the transportation cost is set at a low level (as defined in Lemma 2) motivating the firms to engage in harsher price competition as undercutting is more effective. The parameter configuration characterises a market with a dominant player and a challenger. The challenger's product suffers from early development phase problems, so that its objective quality is slightly lower than the dominant player's. However, the challenger has established an equally attractive brand (captured in the uniform distribution on Hotelling beach).

5.1 Changes in spillovers

We first demonstrate the case in which $\frac{dx_i^{NE}}{d\xi} > 0$ holds for the smaller firm. In the first situation, consumer networks are incompatible, $t = 1$. This is the archetypical case for competition for the market as categorised by Besen & Farrell (1994). Figures 1 and 2 show the reaction functions in both stages as spillovers change from $\xi_1 = 0.05$ to $\xi_2 = 0.15$.

The initial equilibrium E^1, corresponding to $\xi_1 = 0.05$, is given by the crossing of the solid

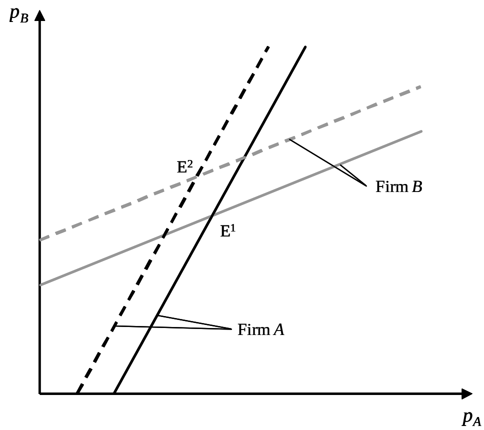

Figure 2: Second stage reactions, $t = 1$.

reaction curves. The equilibrium after the change is E^2, at the crossing of the dashed lines. Firm A has increased its investments. Firm B reduces its investments because an invested unit of R&D becomes strategically less effective. Since firm B invests more in absolute terms, its own investments dominate its behaviour in the second stage. Reduced investments lead to a higher price for firm B. The positive spillover effect caused by an increase in firm A's investments never dominates firm B's investments. Firm A decreases its price due to an increase in investments. Because networks are incompatible, home network has a high strategic value which compensates for (low) quality. Firm A realises the possibility to increase home network size as firm B becomes less attractive. In sum, firm A increases its market share by cutting its price, in the standard way; but also through the reinforcement effect that increases the value of the (incompatible) home-network.

In the initial situation, firms' profits are $\pi_A^1 = 0.0306$ and $\pi_B^1 = 0.1251$, and firm A has a market share of $s^{NE^1} = 0.3310$. After the change in appropriability conditions, firms' profits are $\pi_A^2 = 0.0553$ and $\pi_B^2 = 0.1089$, and firm A has increased its market share to $s^{NE^2} = 0.4161$.

In the symmetric case, when networks were fully compatible, price competition was relaxed as firm-specific network sizes became irrelevant. Hence, it would be logical to expect that as network compatibility increases, the unorthodox aggressive behaviour of the underdog illustrated in Figures 1 and 2 would soften. This in fact is true. Only the extreme case $t = 1$ and its proximate values produce the unorthodox behaviour of firm A. The typical relation of decreasing R&D in spillovers emerges with higher network compatibility. As the parameter t is decreased, the elasticity of market share gets closer to zero, and any market share expansion triggered by a change in spillovers becomes too small to justify aggressive investment behaviour. With higher levels of network compatibility, home network size has less strategic value, and therefore the underdog wins more by concurring with the dominant firm's strategy.

At the extreme when networks are perfectly compatible ($t = 0$), we have the archetypical case for competition within the market. Consumers do not distinguish between home and rival networks, eliminating any strategic motives for market share competition. Consider the same parameter set and perfect compatibility. Again, the change in spillovers is from $\xi_1 = 0.05$ to $\xi_2 = 0.15$. Firm A cuts its investments from $x_A^{NE^1} = 0.3061$ to $x_A^{NE^2} = 0.2743$. Firm B's investments decrease from $x_B^{NE^1} = 0.3272$ to $x_B^{NE^2} = 0.2923$. Firm A's market share increases from $s^{NE^1} = 0.4833$ to $s^{NE^2} = 0.4841$. This happens because it benefits from a larger share of rival's R&D. Profits increase for both firms due to savings in R&D and increased price level (tacit collusion effect). For firm A, profits go up from $\pi_A^{NE^1} = 0.4204$ to $\pi_A^{NE^2} = 0.4311$, and for firm B from $\pi_B^{NE^1} = 0.4804$ to $\pi_B^{NE^2} = 0.4895$. Hence, firm behaviour is regular.

5.2 Changes in compatibility

The case where both firms raise prices if compatibility is increased, is the most common case, which underlines the general result "*compatibility is anticompetitive*" by Shy (2001). Still, we can construct situations where the underdog firm has incentives to decrease its price under a marginal increase in compatibility. This happens only when R&D and firm-specific networks have high strategic value. Price drop results from a boost in R&D output. Consider the same parameter set, as in previous section, with zero spillovers, $\xi = 0$. Table 1 gives the model output for cases $t = 1$, $t = 0.95$, and $t = 0.9$.

TABLE 1 Changes in compatibility

	x_A^{NE}	x_B^{NE}	p_A^{NE}	p_B^{NE}	π_A^{NE}	π_B^{NE}	s^{NE}	$1-s^{NE}$
$t=1$	0.0833	0.5833	0.5767	0.4367	0.0040	0.1974	0.1250	0.8750
$t=0.95$	0.2497	0.4170	0.5586	0.5308	0.0468	0.1306	0.3745	0.6255
$t=0.9$	0.2812	0.3855	0.5854	0.5800	0.0729	0.1370	0.4218	0.5782

Firm A prices above firm B, even though its product has lower quality. It can do so thanks to brand loyal customers (horizontal differentiation). It is not worth battling over consumers located in the middle of the Hotelling beach; it is more profitable to charge a high price for brand loyal consumers. Competitive pressure from firm B though destroys its profits.

As we increase network compatibility marginally (to $t = 0.95$), we open up competition within the market, which benefits firm A because of the large size of the rival network. Firm A's offering becomes more attractive, despite lower quality. Firm A can increase market share significantly by lowering its price. At the same time, firm B increases its price in order to temper price competition. If we increase compatibility even further (to $t = 0.9$), then firm A's aggressive behaviour is moderated and it concurs with firm B by increasing its price. It is no more profitable to fiercely compete against the higher quality firm B. Note that firm B's profits are not monotonic in t. The large firm prefers either full incompatibility or high level of compatibility, as there is a dip in profits initially when we depart from perfect incompatibility.

If compatibility was further increased, firms would further increase prices (tacit collusion effect) and profits would be driven up. Firm A's aggressive pricing characterised above would be eliminated also if spillovers were large. R&D would be relatively too expensive with regard to the potential market share gain.

6 SURPLUS

We conclude our analysis with comments on surplus. It was earlier shown that competitive industry produces too little R&D compared with the social optimum. The Ramsey surplus for symmetric qualities ($v_A = v_B \equiv \widetilde{v}$) is obtained from the optimisation problem (10)

$$CS^R = \widetilde{v}\left(1 - \frac{1}{2}t\right) + \frac{1}{2}\xi + \frac{1}{4}\xi^2 + \frac{7}{4}. \tag{19}$$

CS^R equals total surplus generated in the industry. It is increasing in compatibility and in spillovers.

In the competitive symmetric duopoly, NE profits of a firm are

$$\pi_{SYM}^{NE} = \frac{1}{2}\left[z - t\widetilde{v} - \frac{1}{9}(1-\xi)^2\right]. \tag{20}$$

It is easy to see now that industry profits increase as spillovers increase. This is because of cut-backs in R&D outlays and subsequently higher prices. Industry profits increase as network compatibility increases. This is due to relaxed price competition. Hence, private (firms') incentives for compatibility and R&D appropriability are aligned with the Ramsey case.

Consumer surplus in the symmetric competitive duopoly is

$$CS_{SYM}^{NE} = v + \widetilde{v}\left(1 + \frac{1}{2}t\right) - c - \frac{5}{4}z + \frac{1}{3}\left(1 - \xi^2\right). \tag{21}$$

It is straightforward to see that consumer surplus decreases as spillovers increase or as network compatibility increases. Both features are consequences of firms' decisions to raise prices.

Total surplus in the competitive duopoly is given by $W_{SYM}^{NE} = CS_{SYM}^{NE} + 2\pi_{SYM}^{NE}$, which equals to

$$W_{SYM}^{NE} = v - \frac{1}{4}z - c + \widetilde{v}\left(1 - \frac{1}{2}t\right) + \frac{2}{9}(1-\xi)(1+2\xi). \tag{22}$$

When considering the total surplus, the sign of the comparative statics with respect to spillovers is not constant. Total surplus is maximised with $\xi = \frac{1}{4}$. When spillovers are below the cut-off point of $\frac{1}{4}$, increase in spillovers increases total surplus. In this region, firms' profits increase more than the consumer surplus decreases. With spillovers above the cut-off point, a marginal increase in spillovers causes a reduction in total surplus. Firms' incentives for R&D appropriability differ from social incentives under competition.

Total surplus increases in network compatibility. The increase in firms' profits outweighs the decrease in consumer surplus. Hence, total surplus is maximised with $t = 0$. Private and social incentives for compatibility match. However, consumers would be better off with incompatible networks and intense price competition.

The asymmetric qualities case examined with the numerical example provides similar results. Consumer surplus decreases in spillovers and network compatibility everywhere due to relaxed price competition. Total surplus tends to increase in network compatibility because the positive change in profits dominates. There is also a spillovers threshold that maximises total surplus. The only region where these results break is the area where R&D is (almost) perfectly appropriated ($\xi = 0$) and networks (almost) completely incompatible ($t = 1$). This is the region where the lower quality firm has unconventional behaviour illustrated in the numerical example. When spillovers or network compatibility is increased in that region, the lower quality firm aggressively increases R&D and lowers its price. The lower quality firm's profits go up, but the positive effect is dominated by a decrease in higher quality firm's profits despite that it raises its price. Consumers belonging to the underdog firm's network gain from a lower price, whereas the dominant firm's customers lose from a higher price. The total effect on consumer surplus is negative, as normal. In sum, there is a double surplus loss on aggregate as both profits and consumer surplus decrease in this parameter range.

7 CONCLUSIONS

How spillovers and compatibility jointly affect firm behaviour in strategic games with network externalities has been overlooked in existing literature. Network effects have also been overlooked by focusing on symmetric equilibria. We studied the interplay between demand side network effects and strategic R&D. We constructed a duopoly model with horizontal differentiation and exogenously given product qualities. Horizontal differentiation provided us means to derive a unique equilibrium, which is needed to obtain determinate comparative statics. Exogenously given quality difference helped to study situations where firms start in asymmetric positions. An alternative, but principally analogous way to establish asymmetric positions, is via fixed installed customer bases à la Crémer *et al.* (2000).[10]

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[10] Crémer *et al.* (2000) is a network competition model with endogenous quality and fixed captive customer bases. They analyse internet backbone operator competition, where the quality of interconnection is chosen by the

We focused on the roles of R&D spillovers and network compatibility in price competition. Higher spillovers have a tendency to reduce incentives to invest in R&D and push up prices. Network compatibility tends to moderate price competition by reducing market share competition. Both effects have a similar background. Spillovers reduce the competitive advantage a firm can achieve by investing more than its rival in R&D. Network compatibility reduces the strategic value of firm-specific customer networks. Thus, the tacit collusion effects, dominant in pure price setting models, carry over to two-stage games in general. We also showed how a firm may take up reverse actions as R&D appropriability or network compatibility conditions are altered compared to what the standard models predict.

Symmetry was found to support regular firm behaviour. This is natural as the strategic positions of the firms are levelled, which inflicts symmetric behaviour that cancels out. Hence, asymmetry is required if a firm's behaviour is to differ from the norm. Once we considered asymmetric firms, we found cases where the underdog firm takes reverse actions compared to conventional predictions. As R&D spillovers or network compatibility increases, the underdog firm increases its investments and decreases its price. However, this unorthodox behaviour is not universal, even in asymmetric settings. The strategic variables must have sufficient power. Only when spillovers and compatibility level are low, such reverse behaviour exists. In that case, network externalities intensify the effect induced by strategic investments. In contrast, if compatibility (or spillovers) is very high, the strategic value of firm-specific market shares (or R&D investments) is diminished, supporting regular behaviour.

The main model assumes symmetry between firms in terms of knowledge production, usage, and appropriation. Yet, in many industries such symmetry is not found and firms differ in terms of access to new knowledge as well as in implementation capabilities. This kind of asymmetry traces back to the question how new knowledge is created and implemented. Antonelli (2003a) analyses how the R&D incentives are affected by knowledge complexity and fungibility. Complexity means that knowledge is accumulated by combining different complementary pieces of new information. Importantly, combining information from different sources (i.e. the R&D process)

operators.

exhibits supermodularity yielding increasing incentives to perform R&D. On the other hand, high knowledge fungibility means that a new finding has a large number of applications, thus yielding increasing returns through economies of scope. Both complexity and fungibility are likely to have a non-monotonic relation with spillovers, thus R&D incentives. Once knowledge complementarity and fungibility are incorporated in the model, relevant R&D input, coordination, and transaction costs have to be also figured in. The relative input prices have composition effects that can lead to further asymmetries between firms (Antonelli 2003b). Extending the analysis with a more detailed account on the technological process calls for further research.

We have focused on a mature market where growth is driven by usage and replacement purchases, rather than by new or early adopters. It would be interesting to understand how the results hold in a dynamic model with an inflow of new adopters. In particular, could compatibility be beneficial for both firms and consumers when the market is large with fast growth, even if compatibility is always linked to higher prices? Network externalities are associated with supermodularity of the utility function. Incompatibility curbs supermodularity, therefore, as the compatibility level is increased, increasing marginal network externalities could dominate the negative effect of an increase in price, resulting in higher profits and consumer surplus. Kristiansen (1998) shows that compatibility eliminates excessive R&D rivalry, indicating that profits are likely to increase as compatibility increases also in more dynamic R&D set-ups. However, his model does not include spillovers, raising an interesting area for further research, especially when combined with a more detailed account on the technology process.

Although the newly found unconventional results are less than universal, their existence is interesting. Asymmetry between firms is more common than symmetry. Consequently, firm behaviour in industries with network externalities can differ from conventional predictions. In order to find *unsurprising* results, the reverse behaviour should be seen as a refinement to theory.

8 REFERENCES

Amir, R. (2000) Modelling Imperfectly Appropriable R&D via Spillovers. *International Journal of Industrial Organization*, **18**(7), 1013–1032.

Antonelli, C. (2003a) Knowledge Complementarity and Fungeability: Implications for Regional Strategy. *Regional Studies*, **37**(6-7), 595–606.

Antonelli, C. (2003b) The Digital Divide: Understanding the Economics of New Information and Communication Technology in the Global Economy. *Information Economics and Policy*, **15**(2), 173–199.

Armstrong, M. (1998) Network Interconnection in Telecommunications. *Economic Journal*, **108**(448), 545–564.

Arthur, W.B. (1989) Competing Technologies, Increasing Returns, and Lock-In by Historical Events. *Economic Journal*, **99**(394), 116–131.

Beggs, A. & Klemperer, P. (1992) Multi-Period Competition with Switching Costs. *Econometrica*, **60**(3), 651–666.

Bental, B. & Spiegel, M. (1995) Network Competition, Product Quality, and Market Coverage in the Presence of Network Externalities. *Journal of Industrial Economics*, **43**(2), 197–208.

Besen, S.M. & Farrell, J. (1994) Choosing How to Compete: Strategies and Tactics in Standardization. *Journal of Economic Perspectives*, **8**(2), 117–131.

Bulow, J.I. & Geanakoplos, J.D. & Klemperer, P.D. (1985) Multimarket Oligopoly: Strategic Substitutes and Complements. *Journal of Political Economy*, **93**(3), 488–511.

Cohen, W.M. & Levinthal, D.A. (1989) Innovation and Learning: The Two Faces of R&D. *Economic Journal*, **99**(397), 569–596.

Crémer, J. & Rey, P. & Tirole, J. (2000) Connectivity in the Commercial Internet. *Journal of Industrial Economics*, **48**(4), 433–472.

D'Aspremont, C. & Jacquemin, A. (1988) Cooperative and Noncooperative R&D in Duopoly with Spillovers. *American Economic Review*, **78**(5), 1133–1137.

D'Aspremont, C. & Jacquemin, A. (1990) Cooperative and Noncooperative R&D in Duopoly with Spillovers: Erratum. *American Economic Review*, **80**(3), 641–642.

David, P.A. (1985) Clio and the Economics of QWERTY. *American Economic Review*, **75**(2), 332–337.

De Bondt, R. (1997) Spillovers and Innovative Activities. *International Journal of Industrial Organization*, **15**(1), 1–28.

De Bondt, R. & Henriques, I. (1995) Strategic Investment with Asymmetric Spillovers. *Canadian Journal of Economics*, **28**(3), 656–674.

Dessein, W. (2003) Network Competition in Nonlinear Pricing. *RAND Journal of Economics*, **34**(4), 593–611.

Farrell, J. & Saloner, G. (1985) Standardization, Compatibility, and Innovation. *RAND Journal of Economics*, **16**(1), 70–83.

Foros, Ø. (2004) Strategic Investments with Spillovers, Vertical Integration and Foreclosure in the Broadband Access Market. *International Journal of Industrial Organization*, **22**(1), 1–24.

Fudenberg, D. & Tirole, J. (1991) *Game Theory.* Cambridge, MA.: The MIT Press.

Gandal, N. (1994) Hedonic Price Indexes for Spreadsheets and an Empirical Test for Network Externalities. *RAND Journal of Economics*, **25**(1), 160–170.

Henriques, I. (1990) Cooperative and Noncooperative R&D in Duopoly with Spillovers: Comment. *American Economic Review*, **80**(3), 638–640.

Hotelling, H. (1929) Stability in Competition. *Economic Journal*, **39**(153), 41–57.

Kamien, M.I. & Müller, E. & Zang, I. (1992) Research Joint Ventures and R&D Cartels. *American Economic Review*, **82**(5), 1293–1306.

Katsoulacos, Y. & Ulph, D. (1998) Endogenous Spillovers and the Performance of Research Joint Ventures. *Journal of Industrial Economics*, **46**(3), 333–357.

Katz, M.L. & Shapiro, C. (1985) Network Externalities, Competition, and Compatibility. *American Economic Review*, **75**(3), 424–440.

Kesteloot, K. & De Bondt, R. (1993) Demand-Creating R&D in a Symmetric Oligopoly. *Economics of Innovation and New Technology*, **2**(3), 171–183.

Kristiansen E.G. (1998) R&D in the Presence of Network Externalities: Timing and Compatibility. *RAND Journal of Economics*, **29**(3), 531–547.

Kultti, K. & Takalo, T. (1998) R&D Spillovers and Information Exchange. *Economics Letters*, **61**(1), 121-123.

Laffont, J.-J. & Rey, P. & Tirole, J. (1997) Competition between Telecommunications Operators. *European Economic Review*, **41**(3-5), 701–711.

Laffont, J.-J. & Rey, P. & Tirole, J. (1998) Network Competition: I. Overview and Nondiscriminatory Pricing. *RAND Journal of Economics*, **29**(1), 1–37.

Levin, R.C. & Reiss, P.C. (1988) Cost-Reducing and Demand-Creating R&D with Spillovers. *RAND Journal of Economics*, **19**(4), 538–556.

Shy, O. (2001) *The Economics of Network Industries.* Cambridge, U.K.: Cambridge University Press.

Spence, M. (1984) Cost Reduction, Competition, and Industry Performance. *Econometrica*, **52**(1), 101–122.

Suzumura, K. (1992) Cooperative and Noncooperative R&D in an Oligopoly with Spillovers. *American Economic Review*, **82**(5), 1307–1320.

Suzumura, K. & Yanagawa, N. (1993) Cooperative and Non-cooperative R&D in an Oligopoly with Spillovers: Strategic Substitutes versus Strategic Complements. *Hitotsubashi Journal of Economics*, **34**(1), 1–11.

Sääskilahti, P. (2005) Monopoly Pricing of Social Goods. *HECER Discussion Papers*, 66.

Valletti, T.M. & Cambini, C. (2005) Investments and Network Competition. *RAND Journal of Economics,* **36**(2), 446–467.

9 APPENDIX

Proof of Lemma 1. Differentiation of equation (8) gives $\frac{ds^{NE}}{d\xi} = \frac{2}{A-2(1-\xi)}\left(1-2s^{NE}\right)$. The term $\frac{2}{A-2(1-\xi)}$ is positive by Assumption (A1). Therefore, the derivative $\frac{ds^{NE}}{d\xi}$ is positive only if $s^{NE} < \frac{1}{2}$ holds. Since we have $s^{NE} = \frac{1}{2} - \frac{(v_B - v_A)\left(1-\frac{1}{2}t\right)}{\frac{2}{3}(1-\xi)[A-2(1-\xi)]}$, the firm's market share is less than half, only if its quality is lower than rival's quality, $s^{NE} < \frac{1}{2} \Leftrightarrow v_A < v_B$. Hence, we have that, if $v_A < v_B$ holds, then $\frac{ds^{NE}}{d\xi} > 0$ follows. By symmetry, a respective rule holds for firm B. ■

Proof of Lemma 2. Differentiation of equation (8) yields

$$\frac{ds^{NE}}{dt} = \frac{(v_B - v_A)\left[z - (v_A + v_B) - \frac{2}{9}(1-\xi)^2\right]}{\frac{1}{3}\left\{\frac{2}{3}(1-\xi)\left[A - 2(1-\xi)\right]\right\}^2}. \tag{23}$$

The sign of the numerator depends now on the relative qualities (v_A, v_B), on the transportation cost z, and on spillovers ξ. For a given pair (v_A, v_B), the sign of the derivative $\frac{ds^{NE}}{dt}$ is different for high and low price sensitivity. Equation (23) gives the following rule

$$(i)\ \frac{1}{2}t(v_A + v_B) + \frac{2}{9}(1-\xi)^2 \quad < \quad z < v_A + v_B + \frac{2}{9}(1-\xi)^2 \Rightarrow \begin{cases} v_A < v_B \Rightarrow \frac{ds^{NE}}{dt} < 0 \\ v_A > v_B \Rightarrow \frac{ds^{NE}}{dt} > 0 \end{cases}$$

$$(ii)\ z \quad > \quad v_A + v_B + \frac{2}{9}(1-\xi)^2 \Rightarrow \begin{cases} v_A < v_B \Rightarrow \frac{ds^{NE}}{dt} > 0 \\ v_A > v_B \Rightarrow \frac{ds^{NE}}{dt} < 0 \end{cases},$$

where Assumption (A1) gives the lower limit for z in part (i). ■

Proof of Proposition 3. (i) Part one of Proposition 3 follows directly from equation (15).

(ii) Write the comparative statics as $\frac{dx_A^{NE}}{d\xi} = -\frac{2}{3}s^{NE} + \frac{2}{3}(1-\xi)\frac{ds^{NE}}{d\xi}$. The RHS comprises the direct effect and the indirect effect of a marginal change in R&D spillovers. The direct effect equals to $-\frac{2}{3}s^{NE}$, which is always non-positive. The indirect effect equals to $-\frac{2}{3}(1-\xi)\frac{ds^{NE}}{d\xi}$. By Lemma 1, we have $\frac{ds^{NE}}{d\xi} > 0 \Leftrightarrow v_A < v_B$.

(iii) Define

$$h(\cdot) = -\frac{2}{3}s^{NE} + \frac{2}{3}(1-\xi)\frac{ds^{NE}}{d\xi}. \tag{24}$$

Differentiating $h(\cdot)$ with respect to quality relation $\alpha = \frac{v_B}{v_A}$ gives $\frac{\partial h}{\partial \alpha} = \frac{2}{3}(1-\xi)\frac{\partial^2 s^{NE}}{\partial \xi \partial \alpha} - \frac{2}{3}\frac{\partial s^{NE}}{\partial \alpha}$. Next, we differentiate s^{NE} with respect to α. This gives $\frac{ds^{NE}}{d\alpha} = \frac{v_A\left[-(1+t)+3ts^{NE}\right]}{\frac{2}{3}(1-\xi)[A-2(1-\xi)]} > 0 \Leftrightarrow s^{NE} > \frac{1+t}{3t}$. Hence firm A's market share increases with an increase in the quality of the rival good only if its market share is higher than $\frac{1+t}{3t}$. Term $\frac{1+t}{3t}$ is decreasing in t, and with perfect incompatibility it becomes $\frac{2}{3}$. When the positive relation $\frac{dx_A^{NE}}{d\xi} > 0$ holds, we know that $s^{NE} < \frac{1}{2}$ from part (ii). Consequently, if $\frac{dx_A^{NE}}{d\xi} > 0$ holds, then $\frac{\partial s^{NE}}{\partial \alpha} < 0$. Next we compute $\frac{\partial^2 s^{NE}}{\partial \xi \partial \alpha} = \frac{9(1-\xi)^{-1} t v_A \left(1-2s^{NE}\right) - 4[A-2(1-\xi)] \frac{ds^{NE}}{d\alpha}}{[A-2(1-\xi)]^2}$. We get the result $s^{NE} < \frac{1}{2} \Rightarrow \frac{\partial^2 s^{NE}}{\partial \xi \partial \alpha} > 0$. Hence, if $s^{NE} < \frac{1}{2}$ holds, then we have $\frac{\partial h}{\partial \alpha} > 0$, which implies that higher quality difference makes the positive relation $\frac{dx_A^{NE}}{d\xi} > 0$ more likely.

Next, we differentiate $h(\cdot)$ with respect to spillovers. We get $\frac{\partial h}{\partial \xi} = \frac{2}{3}(1-\xi)\frac{\partial^2 s^{NE}}{\partial \xi^2} - \frac{4}{3}\frac{\partial s^{NE}}{\partial \xi}$. The second derivative is $\frac{\partial^2 s^{NE}}{\partial \xi^2} = \frac{-2\left(\frac{A}{1-\xi}+2\right)\left(1-2s^{NE}\right) - 4[A-2(1-\xi)] \frac{ds^{NE}}{d\xi}}{[A-2(1-\xi)]^2}$. By Lemma 1, we have $\frac{\partial s^{NE}}{\partial \xi} > 0 \Leftrightarrow s^{NE} < \frac{1}{2}$, which implies that if $s^{NE} < \frac{1}{2} \Rightarrow \frac{\partial^2 s^{NE}}{\partial \xi^2} < 0$. In total, if $s^{NE} < \frac{1}{2}$ holds, then we have $\frac{\partial h}{\partial \xi} < 0$, which implies that lower spillovers level makes the positive relation $\frac{dx_A^{NE}}{d\xi} > 0$ more likely.

Finally, we differentiate $h(\cdot)$ with respect to t. We get $\frac{\partial h}{\partial t} = \frac{9(v_A+v_B)(1-\xi)^{-1}\left(1-2s^{NE}\right) - 4[A-2(1-\xi)] \frac{ds^{NE}}{dt}}{[A-2(1-\xi)]^2}$. When transportation costs are sufficiently low (as defined in Lemma 2), $z < v_A + v_B + \frac{2}{9}(1-\xi)^2$, $\frac{\partial h}{\partial t} > 0$ holds if $v_A < v_B$. With higher transportation costs, the sign of $\frac{\partial h}{\partial t}$ becomes ambiguous. Hence, conditional on sufficiently low transportation costs, if $s^{NE} < \frac{1}{2}$, then the case $\frac{dx_A^{NE}}{d\xi} > 0$ is more likely with low levels of network compatibility. ■

Proof of Proposition 4. Start with symmetric qualities, thus with equal NE firm sizes. Equation (16) can be expressed as $\frac{dp_A^{NE}}{d\xi} |_{v_A=v_B} = \frac{2}{3}\xi \geq 0$. Differentiation of the numerator of equation (16) with respect to s yields $2(1-\xi)[2z - t(v_A+v_B)]$, which is always positive. Hence, the numerator increases in s. This proves that, the firm with larger market share always increases its price under a marginal increase in spillovers.

Term $-\frac{3}{4}\xi(1-\xi)^2$ in equation (16) is always negative, but the sign of the first term depends on parameter values. If $s^{NE} < \frac{1}{2}$, the numerator is negative for low levels of spillovers. With zero spillovers, the numerator equals $-\left(1-2s^{NE}\right)[2z - t(v_A+v_B)] < 0$. For high levels of spillovers, the numerator is positive. With perfect spillovers, the numerator becomes $3[2z - t(v_A+v_B)] > 0$. Hence, there exists at least one spillover level, ξ^*, below which smaller firm decreases its price under a marginal increase in spillovers, and above which it increases its price as spillovers are marginally increased. ■

Proof of Proposition 5. Proof follows directly from Lemma 2. ■

Proof of Proposition 6. Let $v_A < v_B$. Firm A's NE price can be expressed as

$$p_A^{NE} = c + z - \frac{1}{3}\left[\frac{2}{3}(1-\xi)(1+2\xi) + \frac{2}{3}(1-\xi)^2 s^{NE} - (1-2t)v_A + (1+t)v_B\right]. \tag{25}$$

The derivative of p_A^{NE} with respect to t is

$$\frac{dp_A^{NE}}{dt} = -\frac{1}{3}\left[\frac{2}{3}(1-\xi)^2\frac{ds^{NE}}{dt} + 2v_A + v_B\right]. \tag{26}$$

First observation is that whenever $\frac{ds^{NE}}{dt}$ is positive, the sign of $\frac{dp_A^{NE}}{dt}$ is always negative. In such a case, firm A always increases its NE price if network compatibility is marginally increased.

$$\frac{dp_A^{NE}}{dt} < 0 \Leftrightarrow \frac{ds^{NE}}{dt} > 0.$$

Next, consider the case when $\frac{ds^{NE}}{dt}$ is negative. In this case, firm A may increase or decrease its NE price conditional on the magnitude of the change in its market share. By rearranging the square bracketed term in equation (26), the following rule is established

$$\begin{aligned}\frac{dp_A^{NE}}{dt} &< 0 \Leftrightarrow 0 > \frac{ds^{NE}}{dt} > -\frac{2v_A + v_B}{\frac{2}{3}(1-\xi)^2}\\ \frac{dp_A^{NE}}{dt} &> 0 \Leftrightarrow \frac{ds^{NE}}{dt} < -\frac{2v_A + v_B}{\frac{2}{3}(1-\xi)^2}.\end{aligned}$$

Firm B's market share moves in the opposite direction. Its NE price is

$$p_B^{NE} = c + z - \frac{1}{3}\left[\frac{2}{3}(1-\xi)(2+\xi) - \frac{2}{3}(1-\xi)^2 s^{NE} - (1-2t)v_B + (1+t)v_A\right]. \tag{27}$$

The derivative with respect to t is

$$\frac{dp_B^{NE}}{dt} = -\frac{1}{3}\left[-\frac{2}{3}(1-\xi)^2\frac{ds^{NE}}{dt} + 2v_B + v_A\right], \tag{28}$$

which yields respective comparative statics

$$\begin{aligned}&\tfrac{ds^{NE}}{dt} < 0 \Rightarrow \tfrac{dp_B^{NE}}{dt} < 0\\ &0 < \tfrac{ds^{NE}}{dt} < \tfrac{2v_B + v_A}{\frac{2}{3}(1-\xi)^2} \Rightarrow \tfrac{dp_B^{NE}}{dt} < 0\\ &\tfrac{ds^{NE}}{dt} > \tfrac{2v_B + v_A}{\frac{2}{3}(1-\xi)^2} \Rightarrow \tfrac{dp_B^{NE}}{dt} > 0.\end{aligned}$$

■

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