Aalto University
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Agent-based modeling as an approach to evaluate price discovery process in double auction markets

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Abstract
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AGENT-BASED MODELING AS AN APPROACH TO EVALUATE PRICE DISCOVERY PROCESS IN DOUBLE AUCTION MARKETS

PURPOSE OF THE STUDY

This study investigates how agent-based modeling can be used to evaluate the price discovery process in double auction markets. The study is limited to single-unit continuous double auctions, and especially to constrained zero-intelligence (ZI-C) trader markets first introduced by Gode and Sunder (1993a).

STRUCTURE

First, I evaluate the earlier models and construct an agent-based model using the guidelines from the literature. In particular, the idea is to create an agent-based model as simple as possible, because the earlier literature in agent-based modeling lacks synthesis about the modeling principles used. After having created the model, I compare its results comprehensively against the earlier literature. In addition, I concentrate especially to evaluating the methods of Cliff and Bruten (1997) to analyze ZI-C trader markets as their ideas have influenced literature substantially, but have been recently questioned by Othman (2008).

RESULTS

The results indicate that the methods of Cliff and Bruten (1997) can be improved. Especially, it appears that the probability density functions (PDF) of bids and asks proposed by Cliff and Bruten (1997) have to be constructed in a slightly different manner than what was originally proposed. However, the results also suggest that after refining the ideas of Cliff and Bruten (1997), it is possible to describe the PDF of transaction prices in ZI-C trader markets. Generally, the results suggest that the earlier literature has overlooked the importance of the evolution in the trader population participating in the ZI-C market. In addition, the results indicate that the trading in ZI-C trader markets closely mimics a sequence of trades that would take place on the Marshallian path, which has been previously suggested, but not comprehensively analyzed by Brewer et al. (2002).

KEYWORDS

agent-based modeling, zero-intelligence, price discovery, continuous double auction

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AGENTTIPOHJAINEN MALLINTAMINEN MENETELMÄNÄ HINNANLÖYTÄMISPROSESSIN ARVIOINNISSA TUPLAHUUTOKAUPOISSA

TAVOITTEET

Tutkimuksen tavoitteena on perehtyä siihen kuinka agenttipohjaista mallintamista voidaan käyttää hinnanlöytämisprosessin tarkastelussa tuplahuutokaupoissa. Tutkimuksessa rajoitutaan tarkastelemaan jatkuvia yhden hyödykkeen tuplahuutokauppoja, ja erityisesti rajoitetun nollaälykkyyden (ZI-C) omaavia agentteja, jotka Gode ja Sunder (1993a) esittelivät ensimmäisen kerran uraauurtavassa työssään.

RAKENNE

Rakennan agenttipohjaisen mallin käyttämällä hyväkseni aiempaa kirjallisuutta ja siinä esiteltyjä metodeja. Tavoitteenani on pyrkiä rakentamaan mahdollisimman yksinkertainen malli, koska aiempi agenttipohjaista mallintamista käsittelevä kirjallisuus käsittää useita erilaisia heterogeenisiä malleja eikä yleistä mallinrakennuskehikkoa näyttäisi olevan. Arvioin rakentamaani mallia vertaamalla mallin tuloksia aiemmin kirjallisuudessa esitettyihin tuloksiin. Lisäksi tarkastelen Cliffin ja Brutenin (1997) käyttämiä metodeja analysoida ZI-C markkinoita, koska ne ovat vaikuttaneet paljon aikaisempaan kirjallisuuteen mutta toisaalta ne on kyseenalaistettu muutama vuosi sitten Othmanin (2008) toimesta.

TULOKSET

Tulokset osoittavat että Cliffin ja Brutenin (1997) käyttämiä metodeja voidaan parantaa. Tulosten perusteella näyttää ensinnäkin siltä, että myynti- ja ostotarjousten tiheysfunktiot ZI-C markkinoilla tulee muodostaa hiukan eri tavalla kuin mitä Cliff ja Bruten alun perin ehdottivat. Toisaalta tulokset tukevat myös sitä, että pienillä muutoksilla Cliffin ja Brutenin ideoita voidaan käyttää hintojen tiheysfunktioiden karakterisointiin ZI-C markkinoilla. Yleisesti ottaen tulokset osoittavat, että aiempi kirjallisuus on ylenkatsonut agenttipopulaation evoluution vaikutusta ZI-C markkinoihin. Tulosten perusteella näyttää erityisesti siltä, että kaupankäynti ZI-C markkinoilla tapahtuu likimääräisesti Marshallin polkua pitkin kuten Brewer et al. (2002) ovatkin ehdottaneet asiaa sen tarkemmin kuitenkaan analysoimatta.

AVAINSANAT

agenttipohjainen mallintaminen, nollaälykkyys, hinnanlöytämisprosessi, jatkuva tuplahuutokauppa

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1 Introduction

Market microstructure studies the microfoundations of economic theory, and is currently also an area of intense research. As an area of finance, market microstructure can be defined as the study of processes that transform investor demands into quantities and prices (Madhavan, 2000; O'Hara, 1995). The earlier literature has examined, for example, price discovery, market structure, market design and transparency of the markets (Madhavan, 2000). These areas cover different parts of real markets and reflect the fact that the general need for knowledge about markets must have been one of the main reasons why market microstructure literature has been rapidly growing during the last few decades.[1] One would also expect this trend to continue as the increasing use of different electronic marketplaces, suggested for example by Biais et al. (2005), should only increase the demand for market microstructure research in near future.

1.1 Background

The fundamental problem in the earlier economic theory is that a large part of it abstracts from the exact mechanisms of trading. This basically means that the researchers assume the market to be a black box and more importantly in many cases assume it to work efficiently (O'Hara, 1995, p. 1). However, as the downturn in end of 2000s again showed, there is demand for a deeper understanding of the markets. By studying the microstructure of different markets, one can try to find answers to some of the questions concerning, for example, investors' incentives behind trading stocks. While market microstructure research has already answered some very important questions (Biais et al., 2005), there still remains many open questions concerning real markets to be answered. Such questions relevant for this thesis concern, for example, the exact trading mechanisms used and the price discovery[2] in one particular mechanism. In general, better understanding of how markets function could constitute better regulatory frameworks and maybe even the formulation of new trading mechanisms designed for example for new electronic markets.

Theoretical models introduced in the earlier market microstructure literature have relied mostly on analytical tools. As Hommes (2006) claims, "In the traditional literature, simple analytically tractable models have been the main cornerstones and mathematics has been the main tool of analysis". Thus, mathematics has provided researchers a way to describe the behavior of a trader. At the same time, many traditional models have used assumptions, which have constrained the model designers heavily. In general,

[1] See for example the reviews by Madhavan (2000) and Biais et al. (2005); both of these surveys review an extensive amount of recent literature in market microstructure research. [2] For example Biais et al. (1999) and Madhavan and Panchapagesan (2000) define price discovery process as the price formation process where traders interact with each other and the result is the price of the asset.

the assumptions used have contributed to models, which do not describe the real markets accurately, but have instead constrained the model to answer a couple of interesting questions. An example of the assumptions used in traditional models are the assumptions for the Walrasian equilibrium: individual optimality, correct expectations, market clearing and the strong form of Walras' law of which the last one in effect means that the total value of excess supply in markets is zero (Tesfatsion, 2006). As Tesfatsion argues, one way to create an agent-based model, is to remove, for example, the traditional expectation about market clearing and replace it with something else that is dependable on the model. It is quite intuitive that such agent-based models are especially interesting from a modeling point of view, if they are able to describe the real markets more accurately.

Agent-based modeling is a quickly developing part of finance and economics and even its own definition may still change. One of the early pioneers, Tesfatsion (2006), defines agent-based computational economics to be "the computational study of economic processes modeled as dynamic systems of interacting agents". This definition well highlights the fact that agent-based models are often best understood as computer simulations of interacting agents. According to Tesfatsion, the agents can range from "active data-gathering decision-makers with sophisticated learning capabilities to passive world features with no cognitive functioning". In general, this means that different agents may be, for example, traders, consumers, workers, families, firms or governments.

Real markets can be modeled more accurately using computational methods. As an approach, the agent-based modeling uses primarily numerical methods, and the earlier literature in agent-based modeling has taken a step away from the analytic models by adding features that cannot be evaluated using analytic methods. One example of this in practice is to allow all different agents in the model to have heterogeneous expectations about the future prices.[3] As many real markets consist of a large number of heterogeneous agents interacting with each other, such a model would seem more appropriate than a traditional analytic model which assumes homogeneous agents.

However, even with the numerical methods one has to make compromises. In agent-based models this usually means that the agents have to be assumed to be boundedly rational instead of having exactly correct expectations. In effect, boundedly rational means that the agent has limited time and resources when calculating, for example, the expectations about the future (Hommes, 2006). However, in practice this may only mean that the agents form their expectations about the future using statistical methods,[4] which would at least intuitively seem to be well in line with the capabilities of human traders. In general, the analytic models are usually forced to assume one or two different groups

[3] This is a simplified example of the one explained by Chiarella et al. (2009). [4] For example Arthur et al. (1997) use inductive reasoning and statistical methods to create expectations for their agents about the future prices.

of agents, while a computer model might have hundreds of agents that all have small differences in their expectations about the future. Thus, when compared to analytic models, the agent-based models should be at least able to create such initial conditions that the resulting agent-based model resembles the real market more closely.

However, the more complicated models have also had their drawbacks. By creating more complex models, the researchers have created complex systems, which might in many cases be hard to comprehensively understand. In addition to complex models, numerous models have also used different frameworks. The number of different models and frameworks presented in the earlier research is well shown in the surveys by LeBaron (2006), Tesfatsion (2006), Duffy (2006) and Hommes (2006). All this suggests that it is possible to draw the conclusion that the agent-based modeling as a discipline is still in the middle of its development and lacks synthesis about the general principles used in modeling.

1.2 Limitations

This study will be limited to continuous double auction, henceforth referred to as CDA, markets. For example, Farmer et al. (2005) define the CDA as an auction where each trader can submit both buy and sell orders as long as the market is open, and the market is cleared after each trader has submitted a buy or a sell order. More specifically, I limit to single-unit continuous double auctions, henceforth referred to as SCDA. In a normal CDA, the traders can submit orders that correspond to any quantity of the asset, while in a SCDA all the orders are restricted to quote only a single unit. Farmer et al. also note that "continuous double auction is the most widely used method of price formation in modern financial markets" as is also suggested by the surveys of Madhavan (2000) and Biais et al. (2005). Thus, using a CDA framework should guarantee that the models created will not abstract too much from the real markets as the CDA is regarded generally a good approximation of how limit order-book trading takes place in real markets. In addition, SCDA has been used in this context in the earlier literature; see, for example, the model by Chiarella and Iori (2002). Keeping the models close to the reality is important, as, for example, the findings of Gode and Sunder (1993a) suggest that the limit order-book trading has a fundamental meaning to the efficiency[5] of the CDA markets.

However, the double auction framework also imposes challenges as it is clearly more complex than some of the frameworks assumed in the earlier market microstructure literature. Although in the literature the CDA is regarded generally as a very efficient

[5] Gode and Sunder (1993a) define the efficiency of the markets as the ratio of the surplus extracted by all the traders and the possible surplus that could have been extracted by all the traders.

allocation mechanism, characterizing the optimal behavior of the trader in such an auction seems to be very hard as noted, for example, by Huang et al. (2002). A good example of this in practice is the fact that in the double auction literature the optimal trader behavior in a double auction has only been characterized in simplified auction settings[6].

1.3 Research questions and results

This thesis will attempt to argue how a small step towards the synthesis in agent-based modeling could be taken when the interest is in price discovery. It seems evident that the agent-based models should be as tractable as possible, but at the same time they should give answers to questions, where analytic tools cannot reach. One especially intuitive way to proceed on this path is to start the modeling of the markets from a model that is as simple as possible. This thesis will try to find out whether this modeling problem could be answered by using the zero-intelligence traders, which will be henceforth referred to as ZI-traders. Such agents were first introduced by Gode and Sunder (1993a) to the agent-based modeling framework and have inspired a large amount of research afterwards.

Gode and Sunder (1993a) defined ZI-traders in SCDA markets as agents, who submit randomly buy and sell orders from independent and identical uniform distributions over an interval from 1 to 200. They named such traders as ZI-U traders, where U stands for unconstrained and highlights the fact that ZI-U traders do not take into account their valuation of the asset when trading. In contrast to ZI-U traders, Gode and Sunder (1993a) also defined ZI-C traders, who take into account their individual valuations when submitting buy and sell orders. Models in this line of research base their foundations on a set of very simple assumptions, which can be modified by adding features one at a time. Thus, using ZI-traders should be a way, which allows one to control the complexity of the model as well as it is possible with the present knowledge about the agent-based models. This could mean that the modeler would have the possibility to keep track of the model behavior while developing the model systemically in a step-by-step manner.

It also appears that ZI-trader market has to be evaluated using simulations instead constructing and analyzing an analytic model. An analytic Markov model of the ZI-trader market could be constructed, because the only factor contributing to the evolution of the trader population is the previous population of traders[7]. Thus, it seems that, for example, the ZI-C model could be analyzed analytically, although to the best of my knowledge such analysis has not been proposed yet. However, the reason for the lack of

[6] See, for example, a recent study about a double auction mechanism by Chu (2009). [7] The population of traders participating in the market at time t depends only about the population of traders that participated in the market at time $t-1$.

analytic studies is probably the fact that building such a model is problematic[8]. With already five buyers and sellers the amount of states in the Markov model equals 252, and the amount of states in a model with 75 buyers and 75 sellers[9] is approximately 10^{44}. Thus, when designing a Markov model of the ZI-C trader market, one would have to take into account this problem somehow or to limit to even simpler models than the ones presented in this thesis.

Essentially, I will add to the earlier literature by carefully examining the properties of the ZI-trader markets and comparing the price discovery process in different types of ZI-trader markets. In practice, this means that I will carefully review the earlier results about the ZI-C[10] trader markets, and then build on them when discussing the price discovery process. After that I will carefully look at the different demand and supply settings that were introduced into the ZI-C trader literature by Cliff and Bruten (1997).

The work of Cliff and Bruten (1997) has also been widely recognized as one of the strongest critique for the ZI-C trader approach. However, in contrast to Cliff and Bruten (1997), the results presented in this thesis show that the price discovery process in ZI-C trader markets is closely related to how the traders are matched to trade and how the population of traders participating in the market evolves over time. Although the ideas presented in this thesis are straightforward and actually based on the ideas and results presented already by Gode and Sunder (1993a), it seems that the earlier literature has generally overlooked the evolution of the trader population when discussing the price discovery process in ZI-C trader markets. For example, the seminal work by Cliff and Bruten (1997) concentrated on the overall price determination process, i.e. on the distribution of transaction prices, and left the evolution in the trader population aside. In this thesis I will argue how such reasoning led Cliff and Bruten (1997) to underestimate the importance of the changes in the trader population for the price discovery process.

The results suggest that by using the ideas presented by Cliff and Bruten (1997) more carefully, it appears to be possible to explain the price discovery process in the ZI-C trader markets. Especially, it appears that the probability density functions (PDFs) of bids and asks proposed by Cliff and Bruten (1997) have to be constructed in a slightly

[8] A natural way to form a Markov chain model is to define the state of the chain as the population of buyers and sellers interacting in the market at certain moment t. The idea is that when a trade takes place the state of the chain also changes. When taking into account every possible trade that can take place, such a model with n buyers and m sellers would have $\binom{n}{n-1}\binom{m}{m-1}$ possible states after one trade has taken place, because a single buyer can trade with any of the m sellers. Similarly, after two trades have taken place, there are $\binom{n}{n-2}\binom{m}{m-2}$ possible states. Thus, all in all the model would have $\sum_{i=0}^{\min(n,m)} \binom{n}{i}\binom{m}{i}$ possible states, and the amount of states in the model increases exponentially with the number of agents in the model. [9] These amounts corresponds to the amounts of agents used in the ZI-C trader models presented in this thesis. [10] ZI-U traders have been generally overshadowed in the literature by ZI-C traders, because the behavior of ZI-U traders is completely random while the behavior of ZI-C traders resembled in some ways the behavior of humans in the experiments of Gode and Sunder (1993a).

different manner than what was originally proposed. However, the results also suggest that after refining the ideas of Cliff and Bruten (1997), it is possible to describe the probability density function of transaction prices in ZI-C trader market. Generally, the results suggest that the earlier literature has overlooked the importance of the evolution in the trader population participating in the ZI-C market, which strengthens the ideas of Brewer et al. (2002), who claimed that the convergence of transaction prices in ZI-C markets is based on the fact that intramarginal[11] traders leave the market. Thus, it seems that the correct way to analyze the behavior of ZI-C markets is to look at how the population of traders changes over time and how the changes contribute to the characteristics of the market.

1.4 Structure of the thesis

This thesis is divided into six chapters. Chapter two will begin by explicitly defining the concept of single-unit continuous double auction and showing a characterizing example of the SCDA trading process taking place in real exchanges. After that the quest for a step towards the synthesis will begin with an extensive review of the previous literature. The relevant literature can be loosely divided into the following areas: market microstructure, agent-based modeling and double auctions. The methodology used will be reviewed in the fourth chapter by elaborating how the different models used in this thesis were implemented using Python programming language and, in particular, its simulation package SimPy. The fifth chapter presents the results for the ZI-trader model by beginning from the seminal results presented by Gode and Sunder (1993a), continuing with the critique for the methods of Cliff and Bruten (1997) and ending with the results for the different demand and supply settings in the spirit of Cliff and Bruten (1997). The last chapter concludes the thesis.

[11] Intramarginal traders are either buyers, whose valuation is larger than the equilibrium price, or sellers, whose valuations is smaller than the equilibrium price.

2 Single-unit continuous double auction

A double auction has got two sides, i.e. buyers and sellers, and traders on both sides can submit offers to express their intentions instead of only one side (Friedman, 1991). This is the main difference between the double auction and the usual example of an auction, i.e. the English auction that is better known as the open ascending price auction. In an English auction only buyers are allowed to submit offers, i.e. bids, to express their willingness to buy the auctioned asset. In contrast to the English auction, in a double auction, traders submit offers, i.e. quotes, which are either bids or asks depending on the intentions of the trader. Buyers submit bids, which specify both the amount of the asset and the price that the buyer is ready to at most pay for a single asset. Similarly, sellers submit asks, which specify both the amount of the asset and the price that the seller at least demands for selling a single asset.

The two prefixes continuous and single-unit are used to limit the double auctions and also this study significantly. First, the single-unit prefix in front of the word double auction refers to the fact that in the double auctions described in this thesis bids and asks are restricted to trade a single unit of the asset at a time if not explicitly defined otherwise. This simplification helps in understanding the double auctions and makes the examples easier to follow. Such a simplification also fits the scope of this thesis well, because, in the models presented later, the traders are restricted to use only single-unit quotes. The second prefix will be elaborated in the following subsection.

2.1 Continuous clearing

A double auction may be cleared using different mechanisms, and continuous clearing is only one example of possible clearing mechanisms. In essence, the clearing mechanism is used to define how the bids submitted by the buyers are matched with the asks submitted by the sellers (Gode and Sunder, 1993b). The idea is that in a continuously cleared double auction the market is cleared each time a quote arrives at the market place (Gode and Sunder, 1993b). In practice, continuous clearing means that the traders can submit quotes at any time the market is active. The definition "any time the market is active" means in the real world that any time period market is active is comparable to an interval of positive real numbers, $\mathbb{R}_+$. This means that there is an infinite number of possible time moments in each interval for a quote to arrive at the market.

Although, for example, Gode and Sunder (1993b), Gode and Sunder (1993a) and Gode and Sunder (2004) all use term continuous double auction, their definition of the continuous double auction is different from the one given here. The essential difference is that the models by Gode and Sunder are restricted to work only in discrete time, while the definition given above does not use a discrete definition of time. Thus, the definition given here is only one possibility, but it is definitely more elaborate than, for example, the one given by Gode and Sunder; the discretization is only an approximation of the "continuous" time in the real world. To the best of my knowledge, the agent-based models presented in the literature have been based on discrete-time simulations and the same ideas are also followed when constructing models in this thesis. Thus, although there are approaches to also simulate without using the discrete definition for time, such models are not constructed in this thesis to keep the presentation as close as possible to the one in the literature.

The fact that traders may submit quotes whenever the market is active makes the continuously cleared double auction a practical way to organize trading in exchanges with a large amount of traders participating in the exchange. A good example is the change from the call-auction used in the 19th century in New York Stock Exchange (NYSE) to the continuously cleared double auction. The general difference between the two is the fact that a continuous double auction allows the traders to arrive at any time at the market place and trade, while in a call auction the traders have to be present in the market exactly at the time the call auction takes place (Kregel, 1992). According to the intuitive arguments of Kregel, the change in the mechanism was derived from the vast increase in the demand to trade in the NYSE, which in essence means that the continuous double auction fitted the exchange with increased demand better than the call-auction. The success of the continuous double auction is also highlighted by the fact that today the continuously cleared double auction is used in some of the most important stock exchanges, which include Paris Bourse, NYSE, Toronto Stock Exchange and Nasdaq (Madhavan, 2000). Thus, it appears to be essential to try to take the condition "at any time market is active" at least somehow into account when modeling the price discovery process in stock markets. One possibility is to use discrete time simulations, which are presented in this thesis.

I will next characterize how the clearing mechanism works in a continuously cleared single-unit double auction. In practice, continuous clearing means that when a quote arrives, it is immediately compared against the quotes in the limit order book. In the following presentation, I will only specify the price of each quote, because all the quotes submitted are restricted to trade only a single unit of the asset. If the quote arriving at the market is a bid, then it is matched against the asks in the limit order book. The idea in matching the submitted bid to the asks in the limit order book, is to match the submitted bid to the ask with the lowest price, i.e. the lowest ask or better known as the best ask, in the limit order book. This means that in practice a trade takes place immediately when the price in the submitted bid is larger or equal to the lowest ask price in the limit order book. If the price of the submitted bid is lower than the lowest ask, then the submitted bid is appended to the limit order book. On the other hand, if the quote arriving at the market is an ask, then it is matched against the bids in the limit order book. The idea in matching the submitted ask to the bids in the limit order book is to match the submitted ask to the bid with the highest price, i.e. the highest bid or better known as the best bid, in the limit order book. In practice, a trade takes place immediately, if the price of the submitted ask is lower or equal to the price of the highest bid. If the price of the submitted ask is larger than the highest bid, then the submitted ask is appended to the limit order book.

2.2 An example of a single-unit continuous double auction

In practice, the continuous clearing means that the limit order book has two queues: one for the bids and one for the asks. When creating a model of the limit order book, the solution is to keep the limit order books ordered by the prices and time priorities, which means that the smallest ask and the highest bid can be easily found. The ordering with time priorities in addition to the price priority is derived from the fact that usually the limit order books also obey a time priority (Chiarella and Iori, 2002). This means that the quotes that have been appended earlier to the limit order book have to leave the limit order book by trade or by cancellation before a quote with a later time priority can be cleared with a submitted quote.

Table 1: An example of a single-unit continuous double auction. It is assumed in the following that all quotes, i.e. bids and asks, correspond to single units of asset, which allows one to consider only the prices of bids and asks when defining them. At the beginning, time $t = 0$, the limit order book has got three bids at prices {1,2,3} and three asks at prices {4,5,7}. At time $t = 1$, an ask at price $p = 6$ arrives and is appended to the limit order book. At time $t = 3$, a bid at price $p = 4$ arrives and it matches the lowest ask, which means that a trade takes place.

$t = 0$		$t = 1$	$t = 2$		$t = 3$	$t = 4$	
bids	asks	ask=6	bids	asks	bid=4	bids	asks
3	7		3	7		3	7
2	5		2	6		2	6
1	4		1	5		1	5
				4			

An example of the clearing of the limit order book is given in table 1. There the limit order book at time $t = 0$ consists of bids at prices {3,2,1} and asks at prices {7,5,4}. At time $t = 2$ an asks at price 6, which was submitted at time $t = 1$, is appended to the limit order book, because there exists no matching bid in the limit order book. Such a quote is better known as a limit order in the literature, because the submitted quote does not lead immediately to a trade. However, the bid at price 4 submitted at time $t = 3$ is matched at time $t = 4$ with the lowest ask in the limit order book, because both the submitted bid and the lowest ask have the same price 4. This means that the submitted bid leads to a trade, and a quote that leads immediately to a trade is better known as a market order in the literature.

There is one important issue that is vital to understand about the limit order-book trading in a CDA. All of the quotes that are in the limit order book also have to obey the immediate clearing condition. Thus, if there existed one bid and one ask in the limit order book so that the price of the ask would be lower or equal to the price of the bid, then a trade would take place immediately and the bid and the ask would be cleared away from the limit order book. This means that assuming that the immediate clearing condition is satisfied, we know that for all bids and asks in the limit order book it has to be that the price b_i of the bid $i \in \mathbb{N}$ is always smaller than the price a_j of any ask $j \in \mathbb{N}$ in the limit order book: $b_i < a_j \forall i, j \in \mathbb{N}$.

3 Literature review

To create the foundation for this thesis, this chapter will review the most important parts of literature in agent-based modeling. The first section will briefly discuss real and artificial stock markets in general and define some of their main elements. This discussion is an important building block for the following analysis of the previous literature, because it defines most of the important concepts appearing in real markets from the agent-based modeling point of view. The discussion is also meant to give perspective for the reader on how a real stock market could be artificially constructed in a plausible way. The section following the first will continue by briefly discussing the general view about agent-based models and give a few examples of models used in the earlier agent-based modeling literature bearing in mind the concepts defined in the first section.

After the first two sections, the reader should have some idea about the agent-based modeling literature. The sections following the first two will concentrate on reviewing the ZI-trader models, which forms the backbone for the chapter concerning the ZI-trader model and results. The third section begins by taking a look at the seminal work of Smith (1962) in the field of experimental economics, which initially inspired the zero-intelligence trader framework introduced by Gode and Sunder (1993a). The fourth section describes ZI-trader models, which are the core of this thesis. The fifth section will conclude the review by examining the critique of the zero-intelligence trader model combined with the latest achievements.

3.1 About the real and artificial stock markets in general

There are certain characteristics that many real stock markets in general share, and these characteristics are naturally relevant also for constructing artificial stock markets. Boer-Sorban (2008, p. 9) identifies six main factors that appear in real markets. These characteristics also seem to be in line with the ones reported in the seminal work concerning the market microstructure of stock markets by O'Hara (1995, pp. 8-12). The factors identified by Boer-Sorban (2008, p. 9) are traded instruments, orders and quotes, market participants, trading sessions, execution systems and market rules. In the following, these concepts excluding market rules will be elaborated with respect to agent-based modeling. Market rules are left outside the contemplation, because they are considered as market specific. Thus, market rules are irrelevant for the following analysis, which tries to isolate some of the most important features common for most of the stock markets from the agent-based modeling point of view.

3.1.1 Market participants

The agents used in the artificial market models typically correspond to investors and market makers. These groups of participants can be used to form a crude approximation of the interaction between the real market participants. In general, according to Boer-Sorban (2008, p. 13) the participants in the real stock markets, i.e. traders, can be roughly divided into two distinct groups: investors and market organization. In real markets, investor can be any market participant, for example, an individual or a fund, who does not belong to the market organization (Boer-Sorban, 2008, p. 13). If one looks at the agent-based models presented in the surveys about the field by LeBaron (2006), Duffy (2006) and Hommes (2006), it seems that it has not yet been that important to exactly determine whether the investors were individuals or funds, but instead the models have only assumed some investors that interact with each other. The reason has probably been to reduce the complexity of the model as much as possible.

Similarly as investors, also the market organization in real markets can be divided into groups. Boer-Sorban (2008, p. 13) divides traders belonging to the market organization into two groups: brokers and market makers. According to her, the difference between the two groups is in their behavior: the brokers trade for their customers, while market makers are responsible for creating the prerequisites that make the trading of brokers possible. In practice, this usually means that market makers provide bid and ask quotes, and guarantee liquidity to those quotes within certain limited amounts. This is, for example, the case in NYSE, where the market makers, i.e. the specialists, quote bid and ask prices that they guarantee to hold up to some particular number of assets bought or sold (O'Hara, 1995, pp. 9-11).

In the agent-based modeling literature, one widely recognized example of a model including the market organization has been presented by Das (2001). His model has a single market maker and several investors, and the model itself appears to be a good example of the models implemented in the literature. It seems that similarly as in the model by Das, also more generally in the literature the brokers[12], integral parts of the quote-driven real exchanges, have been left outside the artificial market models. The reason has probably again been the ambition to create as simple models as possible. It also seems that leaving them out has not yet been a problem, because already such simple and crude models have proven quite good in explaining for example the stylized facts of the prices of financial assets in real markets (LeBaron, 2006; Hommes, 2006).

[12] Refer for example to the agent-based models presented in the surveys about the field by LeBaron (2006), Duffy (2006) and Hommes (2006).

3.1.2 Traded instruments

The earlier artificial market literature seems to have used on many occasions a single traded instrument, which agents have valued in different ways. In general, the traded instrument has been defined as any asset, which can be sold in real stock exchanges (Boer-Sorban, 2008, p. 11). A single representative asset seems to have been preferable over multiple different assets when looking at the earlier literature from the modeling perspective. The main reason for this preference has probably been that the researchers have tried to keep the number of parameters in the model as small as possible[13]. This cautious simplification seems to be quite appropriate, because the literature lacks even today synthesis about the other general modeling principles that should be used. Another point to make is that a single asset can also be considered a feasible assumption when compared to real markets, because the single asset may be thought of for example as a share of an exchange-traded index fund.

Another important point of view from the modeling perspective is that the assets used in models have had a certain value for the agents. This property has been probably derived from the fact that many models in the literature seem to have assumed that the willingness to trade is based on the differential between the price of the asset in the market and the value of the asset expected by the agent[14]. This assumption seems even intuitively quite feasible as also argued by Boer-Sorban (2008, p. 12), because the real markets seem to support the fact that the market price of a stock does not always correspond to the real value, for example, measured by the liquidation value of the company. Thus, the agents participating in the market may have different judgements about the correct valuation of the asset in question. In general, one can conclude that the assumption of a single representative asset used in the earlier literature seems quite appropriate, if the artificial markets presented in the literature are compared to the real markets.

3.1.3 Orders and quotes

Trading in many real markets is based on orders and quotes, which the traders use to express their intentions in the markets. These concepts have been used in some artificial market models although many models have also diverged from them and instead assumed in some cases a more simplified framework than the one specified by orders and quotes[15]. In real markets, an order is used to specify an asset, a price and an amount of it to be sold or bought, and orders can be either market or limit orders (Boer-Sorban, 2008, p. 12). The difference between the two classes or orders is following. A market order is executed

[13] See for example the model by Chiarella and Iori (2002) and their argumentation about the used assumptions. [14] See for example the surveys by Hommes (2006), Duffy (2006) and LeBaron (2006).
[15] See for example the reviews by LeBaron (2006), Hommes (2006) and Duffy (2006).

immediately at the best available price, while a limit order specifies a certain limit price, which has to satisfied for the order to be executed (Boer-Sorban, 2008, p. 12). To be more specific, limit orders are always appended to the limit order book, which contains all available limit orders, while market orders are matched against a corresponding limit order that exist in the limit order book. For example, for a market buy order, a corresponding limit order would be a limit sell order, which has a lower or equal exercise price and smaller or equal amount of assets. However, even in this simplified case the market rules would be ultimately used determine how the matching of orders would be exactly done. A general implication from these definitions for market and limit orders is that a market order can be executed only, if there exists a limit order that fulfills the market order. Also in this case, the market rules are used to decide, how the market orders, which fulfill the amount of assets for an limit order only partially, will be matched. It is good to also note that limit and market order trading also mean that the limit order book has to contain at least a single limit order for a trade to take place.

Quotes and orders are different from each other, but also share many characteristics. Quotes are, for example, used in a similar fashion as orders, and market makers quote both price and quantity at the same time (Boer-Sorban, 2008, p. 15). The important difference between a quote and an order is the fact that a quote is placed by a market maker, while an order is placed by a trader (Boer-Sorban, 2008, p. 15). A bid quote implies that the market will buy a specified amount at a certain price. Similarly an ask quote will imply that the market maker is ready to sell a specified amount at a certain price. These principles are used in major world exchanges, which include for example Paris Bourse, NYSE, Toronto Stock Exchange and Nasdaq (Madhavan, 2000).

There are a number of important concepts related to orders and quotes, which have to be defined in detail. These concepts are extensively used in real markets, which entails that it is natural to use them also when analyzing and constructing artificial stock markets. The two most important ones are two interrelated concepts: bid-ask spread and liquidity. First, the bid-ask spread for a pure limit order book can be defined as the difference between the lowest ask price and the highest bid price Boer-Sorban (2008, p. 12). Similarly, a market maker provides her customers with the bid-ask spread by quoting both her bid and ask quotes. Second, both orders and quotes are closely related to liquidity, which can be loosely defined as the ability to either buy or sell an asset at a price close to the current market price (Boer-Sorban, 2008, p. 26). Traders can be seen to offer liquidity by posting either orders or quotes and to take liquidity by accepting already available orders or quotes (Boer-Sorban, 2008, p. 12).

Agent-based models have used both orders and quotes. The decisions to use either one has depended on whether the model created has included market organization or not. At the moment, it seems that although there are also models including the market organization, it is probably wise to still try to produce models as simple as possible, which essentially means that the market organization is left more or less unnoticed. Adding the market organization to the model should at least in principal increase the complexity of the model: a pure limit order book model has only buyers and sellers, while in a model with market organization the market makers would have to be included in the model in addition to the buyers and the sellers. Although there might be models, which do not follow this argumentation, at least the present literature appears to suggest these kinds of simple differences between the structures of the models[16].

3.1.4 Execution systems, trading sessions and timing of the market

There are also a few concepts related to the structure of execution systems. According to Boer-Sorban (2008, p. 14), the execution systems in real markets can be quote-driven, order-driven or a hybrid of these two. In a quote-driven system, market makers participate in every trade and are the only source of liquidity in the market, while in a order-driven market buyers and sellers meet without intermediation. In a hybrid system, the traders can choose between the limit order book and market makers. It seems that of those few studies in agent-based modeling literature that have used a double auction setting, most have implemented either the quote-driven or the order-driven system. This seems again quite natural, because the hybrid system would again only increase the complexity of the already complex models. A general difference in agent-based models arising from these definitions is also very intuitive: the order-driven models have needed only investors, while the quote-driven models have in addition to investors needed also at least a single market maker.

The structure of the trading sessions may also be an important determinant of market prices. Earlier agent-based models have assumed different trading sessions structures, which seems understandable as the session structures differ also in real markets. The basic division between trading sessions is based on the degree of continuity of time, which refers to the fact whether trading takes place continuously or periodically. Madhavan (2000) deals the trading sessions in real markets based on the degree of continuity to two: call market sessions and continuous sessions. In a call market session, the traders trade at well-specified times, while in a continuous sessions traders are allowed to trade any time the market is open. As indicated by both Boer-Sorban (2008, p. 95) and Madhavan (2000), continuous sessions are in practice very common and used, for example, in Nasdaq, NYSE and Paris Bourse.

However, Boer-Sorban (2008, p. 64) also notes that the trading sessions used in

[16] Compare for example the model by Gode and Sunder (1993a) to the model by Das (2001).

artificial markets seem to be often call market sessions instead of continuous sessions. The Best intuitive reason for this is again probably the complexity; in continuous sessions some parts of the model, like agents decision problem, become easily very complex, because there are more options to choose from than in call market sessions. In practice, there are also markets that combine the call market and continuous sessions. For example, in NYSE the trading begins with a call auction and then continues with a continuous auction during the day (O'Hara, 1995, p. 10). Implementing such a market structure in an artificial market would certainly be interesting, but currently it seems that the first step is to develop models that use continuous trading sessions instead of call market sessions, because that is one of the simplest ways to create models that mimic the real markets more accurately.

Also the exact timing of the markets, i.e. the submission and execution of orders and quotes, is important although it has been left more or less unnoticed in the agent-based modeling literature as suggested by Boer-Sorban (2008, pp. 12,78-80). It seems that it would be crucial to note that in real markets one can submit a market order, which is not executed. This may occur in real markets, although at the time of submitting the order the limit order book contained a corresponding limit order (Boer-Sorban, 2008, p. 12). Such an occasion is easy to construct theoretically, when the following conditions are met: orders are handled in the arrival order, it takes a certain constant time, say 2 time units, for the orders to move to the market place and the traders can place orders asynchronously in continuous or discrete time. These three facts together mean that when agent A sends her order at time $t = 0$, when the limit order book contains a matching limit order l, it might still be that another agent B has sent her order that matches the same limit order l in the limit order book already at time $t = -1$. If agent A had no knowledge about the order of agent B, then this would mean that agent A would think that her order will be a market order although it will actually be a limit order. Such happens, because the order of agent B would arrive at time $t = 1$ at the market place and it would be cleared with limit order l. Thus, when the order of agent A would arrive at time $t = 2$ at the market place, it would be appended to limit order book, if the order did not match any other limit order in the limit order book than limit order l.

Thus, the timing of the markets has an impact on how the trading actually takes place, because in principle every market order has a positive probability not to get executed. However, in the earlier agent-based modeling literature such non-executed market orders seem to have been mostly left outside the modeling frameworks. One important reason has probably been that there exist also other open questions, like the price discovery process discussed in this thesis, which have been seen as more important from the modeling perspective. However, it is good to note that one of the first ones to notice this difference was Boer-Sorban (2008), and as her results suggest that the timing issue seems to matter and should be considered in the future research.

3.1.5 Conclusions

The different concepts that have now been presented can be together used to form an artificial model of the stock market. As has already become apparent, different models have used some of the concepts of real stock markets and have left some untouched. Generally, the reason has probably mostly been the ambition to keep the models as simple as possible[17]. Having now tried to capture the most important elements of the real markets, it is important to notice that even if all these concepts were used in a single model, such a model would still not be a complete description of the reality. Thus, there still exist something that has not been defined: the traders decision-making mechanism. When modeling, one has to define both the market structure and the decision-making mechanisms to create a model that could at least be thought as a crude approximation of the real markets. These two aspects have been major issues in the earlier models, and so also make the main differences between different models as will be shown in the following section.

3.2 A general view on agent-based modeling

Agent-based models have begun gathering attention during the last two decades. I will in the following categorize the different models, introduce a few different models and discuss them to give a thorough view of the agent-based modeling in general.

3.2.1 Few- and many-type models

One way to categorize the different models into two subgroups is to divide them into few-type and many-type models (LeBaron, 2006). The difference between these two types lies the number of different trading strategies used by agents in the model. Trading strategy can on the high level, sufficient for now, defined as a strategy that explicitly tells the agent how to trade a risky asset. Intuitively increasing the number of strategies also increases the complexity of the model, because the number of different strategies interacting with each other increases the complexity of the model. Increased complexity of the model also creates another difference between the simplest few-type models and the many-type models: the few-type models are more analytic when compared to the many-type models, which can be only assessed using simulations (LeBaron, 2006). Thus, a

[17] This is the only argument that the previous authors, like Chiarella and Iori (2002), have used in limiting their studies.

large number of different model characteristics means that the model has to be evaluated using computational experiments, because finding any analytical solutions is impossible.

The first models introduced in the literature were few-type models according to LeBaron (2006). One of the first ones to introduce two types of different investors into models was Zeeman (1974). He defines in his paper two different groups of investors, i.e. chartists and fundamentalists, and a model for them using differential equations and catastrophe theory. Although this model is not essentially an agent-based model[18], it is a good example of the ideas used also in creating the agent-based models. In the model of Zeeman (1974), the chartists are assumed to be investors, who base their decisions on the state of the market. In contrast to the chartist, the fundamentalist base their decisions on fundamental value of the asset in question. Zeeman's model relates the proportion of chartist and the excess demand of fundamentalists at a certain moment to the rate of change of the stock index. In short, the global dynamics that the model produces for a stock exchange are following when an initial assumption used is that the fundamentalist money is inserted into the market (Zeeman, 1974):

1. Rising price of the index attracts more chartists to the market, which creates a bull market.

2. The increasing proportion of chartists will eventually make the fundamentalists leave the market as the price of the index rises too high when compared to the fundamental value.

3. When a large enough part of the fundamentalists has left the market, the chartist are no more able to create profits with the rising markets. This turns the bull markets to bear.

4. After the index has reached a sufficiently low value, a slow recovery begins as the fundamentalist start reinvesting.

This simple model presents one idea about how the bear and bull markets could be created in the markets. It also seems that Zeeman's model has been able to characterize something valuable from the point of view of researchers, because according to Hommes (2006) many similar behavioral elements have been used also in the more recent heterogeneous agent-based models.

Another carrying theme in the agent-based modeling literature has been to analyze the outputs of the artificial markets against the outputs of real markets. In practice, such

[18] This model is not an agent-based model in the sense defined for example by LeBaron (2006), because the model is not a simulation of individual agents. Instead the model uses differential equations to characterize the different behaviors today often described using agents.

experiments are done by comparing, for example, the price behavior produced by the model to the real market price behavior as presented in the reviews by LeBaron (2006), Hommes (2006) and Duffy (2006). Some authors have tried to create such markets, which would create a price behavior that mimics the real market price behavior as closely as possible. One example of such a model was presented by Day and Huang (1990), whose model was one of the first ones to create stochastically fluctuating prices and randomly switching bear and bull markets. The model is created using two different types of investors: α- and β-investors. The α-investors can be best thought of as the fundamentalist defined by Zeeman, because the α-investors try to buy when prices are below the fundamental value and sell when prices are above the fundamental value. On the other hand, the β-investors can be thought of as the "market sheep", who chase the market prices similarly as the chartist presented by Zeeman (1974). In essence, the model of Day and Huang (1990) seems to in many ways implement the ideas of Zeeman (1974) in a simulation context, and is able to create some characteristics of real market price behavior using such only a few assumptions.

The model of Day and Huang (1990) is important also from a theoretical point of view. Using their simple model, the authors are able to show, that when the strength of fundamentalist α-investors is high enough, the prices converge to a point where no net trading occurs. According to the authors, this can be seen as a situation where there is no trading on information. On the other hand, when the strength of the chartist β-investors is large enough, the model creates a sequence of irregular bull and bear markets. Unfortunately as also the authors note, the model is nowhere close to be seen as a characterization of real markets as the model misses, for example, many important endogenous feedback mechanisms.

3.2.2 Other differences between the earlier models

Few- and many-type models are only one way to categorize the different agent-based models presented in the literature. I will next give a few more categorizations to highlight the differences between the earlier models in the literature. In general, one can probably say that the most important differences between the models are in what is defined exogenously and what is left endogenously defined by the model. Market price seems to have been naturally endogenous in the earlier models. Market price also seems generally to be one of the most important issues that the earlier models have generated as many authors claim that their model is able to generate the common stylized facts for market prices (LeBaron, 2006; Hommes, 2006; Duffy, 2006).

Although in the earlier models the price is endogenously created, there are many differences between the price generation mechanisms. The branch of models based on the traditional finance literature, i.e. starting from Grossman and Stiglitz (1980), assumes a Walrasian auctioneer mechanism. With such a mechanism all the agents know that in every period the aggregate demand has to equal aggregate supply. In practice, for agents this means that they optimize their portfolio assuming that the prices have to be set so that an equilibrium prevails.

In reality, the Walrasian auctioneer mechanism can be described as a two-step procedure as suggested by O'Hara (1995, p. 7) and Tesfatsion (2006). During the first step, all the traders inform sequentially the auctioneer about their demands at each price knowing that no trade will yet take place. During the second step, the auctioneer allows the traders to trade at an equilibrium price determined during the first step. Thus, by using the Walrasian auctioneer mechanism, the earlier models have inserted an assumption of a sequential equilibrium into each agents optimization problem. This assumption has been criticized as too unrealistic, for example, by Boer-Sorban (2008), because it assumes that all the traders are present in the market at the same time. As suggested in this thesis earlier, for example, the trading in NYSE evolved from a call-auction to continuous double auction, because all the traders were not able to be present in the same place at the same time (Kregel, 1992). However, complementary mechanisms have been also introduced in the earlier literature.

LeBaron (2006) divides the different price formation mechanism into four categories: slow adjustment, equilibrium clearing, order book simulation and random trading. Slow adjustment refers to a mechanism, where the market price is changed proportionally to the excess demand by a market maker. An example of equilibrium clearing is the Walrasian auctioneer mechanism, where all agents optimize their holdings knowing that the market must clear. Order book simulation is based on simulating a order book, which includes both buy and sell orders. An example of random trading is a situation where agents randomly meet and trade, if both find it profitable.

Another difference between the models lies in the fact what models assume about trader types (LeBaron, 2006). Some models assume that fixed proportions of different agents interact in the model during the simulation. In such cases, the proportions are taken as exogenous parameters. On the other hand, some models assume that the proportions of different agents interacting in the market are determined endogenously. Such models might, for example, assume that the traders change their behavior depending on which agent type would have created best profits in the past.

A general view on agent-based models been now been presented. Hopefully, the reader has gotten a firm grip on the possible differences between the existing models. Next I will present more elaborately the ZI-trader model and its primary applications. Essentially, the reason to choose to start modeling using ZI-traders is based on the fact that the model with ZI-traders are the simplest agent-based models that the earlier literature exhibits.

3.3 Experimental economics and the seminal work of Smith (1962)

ZI-traders that will be presented in the next section are in principal based on the experimental framework presented by Smith (1962). Thus, to be able to understand ZI-markets it is good to first have a look at the framework for experimental economics presented already in the 1960s. I will next first present carefully the framework, results and methods from the article by Smith (1962). In his seminal work in experimental economics, Smith (1962) presented a framework that can be used to design, implement and asses an experiment with human subjects in a double auction market. The general idea characterizing the spirit of experimental economics is well presented by Smith (1962) when expressing caution about his experiments, which

> "are intended as simulations of certain key features of the organized markets and of competitive market generally, rather than as direct, exhaustive simulations of any particular organized exchange".

This expression of caution applies well also to the simulations presented in this thesis, because it is certainly hard to imagine a model that would capture all the relevant features of real exchanges. In his article, Smith (1962) presents results from nine different market types, which are different from each other in terms of demand and supply schedules. The following discussion will reveal how the experiments were organized in practice.

3.3.1 Framework

The experimental procedure used by Smith (1962) is based on dividing the human subjects into two subgroups: buyers and sellers. The selected buyers are informed of their private valuations, and are explained that they are not allowed to buy the asset at a price that exceeds their valuation. Similarly, the selected sellers are informed of their private valuations, and are explained that they are not allowed to sell the asset at a price that is lower than their valuation. In addition, both buyers and sellers are explained that by engaging in a transaction, they make a pure profit that is determined by the excess of their transaction price p and their valuation v. For sellers, the profit is the difference between the transaction price p and the valuation v determined as $p-v$, while for buyers the profit is the difference between the valuation v and the transaction price p determined as $v-p$. All the traders, i.e. buyers and sellers, are allowed to trade a single asset once, and after that they leave the market.

The valuations given to buyers and sellers can also be used to determine the theoretical demand and supply curves, i.e. the demand and supply schedules, and the theoretical equilibrium price and quantity. Smith (1962) emphasizes the use of the word “theoretical”, because in a real exchange none of the participants, i.e. neither traders or market organization, know the exact valuations of other participants. Another good reason to use the word theoretical is the fact that the demand and supply schedules change immediately after the first trade has taken place, because one seller and one buyer leave the market. Thus, after each transaction the demand and supply curves change. However, in an experimental setting the knowledge about the valuations appears to be very noteworthy, because, for example, the theoretical price appears to have a strong relation with the transaction prices in the experiments of Smith (1962).

In practice, the theoretical demand and supply curves follow straight from the knowledge about the valuations of individual traders. As the theoretical demand curve depicts the amount demanded at each price, it is possible to count the number of agents willing to buy at a particular price to form the demand curve. Similarly, the supply curve can be created by counting the number of agents willing to sell at a particular price. On the other hand, the theoretical equilibrium price and quantity can be determined by the intersection of the constructed demand and supply curves. The interesting results is that also the experiments reported by Smith (1962) also suggest that the transaction prices tend towards the theoretical equilibrium price as time progresses in his experiments.

In addition, the demand and supply curves seem to have certain forms. It is worth noting that as the buyers are ready to buy at any price lower than their valuation, the number of buyers willing to buy an asset can only decrease with the price. This means that the demand curve as a function of quantity is always decreasing, but not strictly. Similarly, because the sellers are ready to sell at any price higher than their valuation, the number of sellers ready to sell an asset can only increase with the price. Thus, the supply curve as a function of quantity is always increasing, but not strictly. As noted by Smith, it is worth recognizing that using these definitions for demand and supply curves, the curves stipulate the maximum amounts of bought (demand) and sold (supply) at any price in the market. This follows straight from fact that the human traders participating in the market are instructed to act according to their valuations, and from the fact that the demand and supply curves are created by using the valuations of the traders in the manner as described above.

The idea of Smith (1962) was to deal the experiments in individual trading days. In practice this meant that each of the experiments conducted by Smith (1962) lasted several trading periods, or days, which all had a time limit from 5 to 10 minutes depending on the number of participants. Smith (1962) explains that the period continued at most to the time limit, but the period also ended if the bids and asks did no longer lead to transactions. In practice, this meant that one or two final calls were made before announcing that the market was officially closed. After a period had ended, another was immediately started, and the traders were reinitialized with their initial values, which meant that all of them were again given the right to trade a single asset during the new period using the same valuations that they were given in the beginning of the whole experiment. This process continued until a certain number of completed periods, depending on the experiment, was reached. However, as Smith (1962) notes, one issue that does not appear in real markets and appears in the experimental markets is the fact that the demand and supply conditions were in most cases held constant when a new period was started; real markets in contrast are likely to experience fluctuating demand and supply. To control for this issue, Smith also experienced with markets where demand and supply were changed at some point of the experiment.

Smith (1962) also controlled the information available to the participants carefully. The idea was to keep the traders' information set as close as possible to the situation appearing in real markets. In practice, this meant that traders had no knowledge of other traders' valuations other than the transaction prices, bids and offers they witnessed appearing in the market.

3.3.2 Results

Smith (1962) measured the overall convergence of transaction prices towards the equilibrium price by introducing the coefficient of convergence α of transaction prices from the equilibrium price. To define the coefficient of convergence, I will first define the root mean squared deviation of transaction prices from the equilibrium price p_0, henceforth referred to as RMSD. The RMSD for prices p_i, $i = 1, 2, .., n$ and equilibrium price p^* can be defined as

$$\text{RMSD} = \sqrt{\sum_{i=1}^{n} \frac{(p_i - p^*)^2}{n}}. \tag{1}$$

Smith (1962) counted RMSD using all of the transaction prices available from a single period and the theoretical equilibrium price. Using the above presentation for the RMSD in equation 1, the coefficient of convergence α can be calculated by dividing the product of RMSD and 100 with the theoretical equilibrium price p^* as follows

$$\alpha = 100 \times \frac{\sqrt{\sum_{i=1}^{n} \frac{(p_i - p^*)^2}{n}}}{p^*}. \tag{2}$$

In essence, the coefficient of convergence in equation 2 measures the distance of the transaction prices p_i from the equilibrium price p^*.

The results of Smith (1962) show a strong tendency of transaction prices to tend towards the theoretical equilibrium. The results Smith (1962) presents show that the coefficients of convergence decrease monotonically in all of the other tests except test 8[19]. The experiments performed by Smith (1962) were different from each other in terms of the type of the demand and supply schedules used. In some of the tests demand and supply were symmetric in terms of a vertical line drawn at the level of equilibrium price, while in other tests the market exhibited excess demand or excess supply for the traded asset. Smith (1962) found also some evidence of the fact that the prediction of the static equilibrium requires knowledge about the shapes of the supply and demand curves and about their intersection; strongest evidence found by Smith (1962) was about the fact that a flat, i.e. perfectly elastic, supply curve leads to an empirical equilibrium price that is higher than the theoretical equilibrium price. However, Smith (1962) suggested further research on this issue.

3.4 Models with zero-intelligence traders

Gode and Sunder (1993a) were the first ones to propose the ZI-traders in their seminal paper, which compared the efficiency of the CDA markets populated by different trader types. Essentially Gode and Sunder (1993a) showed that by replacing the human traders in a continuous double auction market by "zero-intelligence" programs, the efficiency of the CDA market may still stay close to the same level. According to them, the ZI-traders with a budget constraint are sufficient to raise the efficiency of the CDA market to a level that is comparable to the level that human subjects reach in an experimental setting. This made the authors claim that the efficiency of the continuous double auction is mainly derived from its structure, which means that the efficiency is independent of the trader's capabilities like reasoning and cognition. In essence, this would mean that a high efficiency of the markets could be achieved even with very simple trader behavior.

I will in the following first present carefully the original model of Gode and Sunder (1993a). After that I will proceed to presenting the claims and the results introduced in the article by Gode and Sunder (1993a) that were partly already visited above. This section will end with a review of the critique that the ZI-trader model has aroused in the literature.

[19] In the test 8, the double auction was changed to an auction, where only sellers were allowed to quote prices. This means that test 8 can be more or less ignored this time, because it was not about testing double auctions.

3.4.1 Framework by Gode and Sunder (1993a)

Initially Gode and Sunder (1993a) defined zero-intelligence traders as programs, which generate "random bids and offers" in the following way. At the beginning of the experiment, each agent was either chosen to be a buyer or a seller and was given an individual valuation v_i for each unit $i = 1, 2, ..., m$ to be sold or bought. During the experiment, buyers created bids and sellers offers, which were independent draws from identical uniform distributions on a range from minimum price 1 to maximum price 200. The fact that such agents do not remember, observe or seek to maximize profits induced Gode and Sunder (1993a) to name their traders as zero-intelligence traders. Gode and Sunder (1993a) named such simplest form zero-intelligence traders as ZI-U traders. Such naming convention was supposed to highlight the difference of ZI-U traders as unconstrained traders to the constrained traders introduced later.

Algorithm 1 summarizes the behavior of a simple ZI-U agent when the agent is selected to participate in the market. The ZI-U agent has to have a field indicating its own valuation v although the ZI-U agent does not use it when trading. The valuation is needed, because it is used to measure the profits the agent is able to create by trading. The buyer indicator b is used to define whether the agent is a buyer, $b = 1$, or a seller, $b = 0$, and is initialized to either of the two possible integer values accordingly. Again, although the buyer indicator is not used in algorithm 1, it still has to initialized and present, because "the market" needs to know whether the quote submitted is a bid or an ask.

Algorithm 1 ZI-U trader

Require: valuation v, buyer indicator $b \in \{0, 1\}$

1. Choose valuation $q \sim \mathrm{U}(1, 200)$
2. Submit q

Results of Gode and Sunder (1993a) suggest that the budget constraint is an important ingredient for the high allocative efficiency of the CDA markets with ZI-traders. The authors defined the ZI-traders with the budget constraint, henceforth referred to as ZI-C traders, in a similar manner as the ZI-U traders were defined above. However, the ZI-C traders do not submit entirely random bids and asks, but instead submit bids and asks with respect to their individual valuation v_i for the i'th unit. Thus, a ZI-C -buyer creates bids uniformly on a range of integers from minimum price 1 to v_i for the i'th unit, and a ZI-C -seller creates asks uniformly on a range of integers from v_i to maximum price 200 for the i'th unit. The behavior of a ZI-C trader is summarized in algorithm 2; it is in essence similar to the one presented for a ZI-U trader and the main difference is in the use of the budget constraint. The difference between the results from the markets with the constrained and unconstrained traders are the most important findings Gode and Sunder (1993a) present in their article in terms of this thesis.

Algorithm 2 ZI-C trader

Require: valuation v, buyer indicator $b \in \{0, 1\}$
1. quote $q = -1$
2. **if** b **then**
3. Choose quote $q \sim \mathrm{U}(1, v)$ {buyer}
4. **else**
5. Choose quote $q \sim \mathrm{U}(v, 200)$ {seller}
6. **end if**
7. Submit quote q

The third type of traders Gode and Sunder (1993a) analyzed were humans. The authors used 12 unique human traders, who were graduate students of business and were motivated by the fact that their course grade was dependent on their success in the markets. Thus, in practice the humans were also informed of the budget constraint, because they were given individual valuations of the asset in question. Essentially the idea of Gode and Sunder (1993a) was to compare the humans to the ZI-traders in a similar environment as the one introduced by Smith (1962).

Gode and Sunder (1993a) also made several modeling choices, which are important to note when analyzing their model. The first three decisions are such that Gode and Sunder (1993a) chose them in their own words to simplify the implementation, while the rest of the choices are reported here to introduce the characteristics of the model by Gode and Sunder (1993a) properly. First, the authors assumed that each bid, ask and transaction was valid only for a single unit. Thus, the traders were limited to trade a single unit at a time. Second, the transaction price was selected to be the price at which a bid and ask were matched. In effect, this means that the transaction price equals the price of the earlier quote, whether it is a bid or an ask. Third, they assumed that a transaction canceled all unaccepted bids and offers from the limit order book; an assumption better known in the literature as the resampling assumption[20].

When comparing the third assumption to the real markets like NYSE, it seems that the assumption is unrealistic. Because of the third assumption, the market is started over after each transaction although that does not happen in reality. Interestingly, recent literature also seems to suggest that by removing the resampling assumption, the allocative efficiency of the CDA market with ZI-traders decreases as shown by LiCalzi and Pellizzari (2008). Thus, it may well be that the resampling assumption has actually contributed a lot to the allocative efficiency of CDA markets with ZI-traders. However, it is also

[20] For more information one can see for example the discussion by LiCalzi and Pellizzari (2008)

good to keep in mind that Gode and Sunder (1993a) used a very small population of traders. Thus, to compensate for the small amount of traders it might have been a good idea to use resampling. However, LiCalzi and Pellizzari (2008) use substantially larger populations than Gode and Sunder (1993a). Another point to make is also the fact that in experiments with human subjects the resampling assumption might have contributed to keeping the human subjects active in the market.

The rest of the modeling choices reported now are more practical and concern the exact choices that have to be done when implementing the artificial market model of Gode and Sunder (1993a). The fourth assumption used was that at the beginning of the auction each trader was endowed with a right to buy one or more units of the asset being auctioned. Fifth, at the beginning of the auction each trader was given an individual valuation v_i of the i'th asset bought, which makes it possible to define the profit of a trader for selling at price p as $p - v_i$ and buying at price p as $v_i - p$. Sixth assumption was that all the traders participating in the markets were divided in each experiment evenly in the beginning into two distinct groups: buyers and sellers. Thus, although Gode and Sunder (1993a) varied the number of assets the agents were to trade and the valuations, they kept the number of buyers and sellers interacting in the market close to even amounts.

Seventh, in each experiment the authors used a population of only 12 homogeneous traders. Eight, Gode and Sunder (1993a) specified five different markets with different supply and demand schedules to support their findings in different market conditions. Apparently, these different market conditions were supposed to thoroughly cover the set of all possible market conditions. However, as noted by Cliff and Bruten (1997), there are certain market types, for example a market with fixed supply and demand, that were not considered by Gode and Sunder (1993a). All the results the authors reported from each of the markets were for homogeneous populations of traders (humans, ZI-U and ZI-C). Thus, all in all, the authors reported results for 15 different markets: for 3 populations of traders in each of the five markets. Ninth, each market was run for six periods, and each period lasted for a finite time of half a minute for machine traders and 4 minutes for human traders. Tenth, when each of the periods started, all the market variables were set to the starting values. This assumption was made to create an environment, which would resemble the trading from day to another day.

One issue has to be still defined: the arrival of traders to the market. Gode and Sunder (1993a) must have assumed something about this issue, although it is not explicitly defined in their article. Without more knowledge about their article, an educated guess would be that the authors assumed each trader to participate in the CDA market all the time. This seems to be in line with their presentation and conclusions: the authors conclude that as the CDA progresses inside the period (day), the opportunity set of the traders narrowed and caused the transaction price to tend towards the equilibrium price. If the authors did not assume that each trader participates all the time in the auction, then drawing such a conclusion would require reasoning about the probability of a trader to participate in the auction during the finite life time of the auction. It also seems that the later presentations about the same subject assumed that all the traders are all the time actively participating in the market (Cliff and Bruten, 1997; Gode and Sunder, 2004).

In addition, the authors give more precise definitions about the trading mechanism used in their two other articles concerning ZI-traders[21]. These models can be used to deduce something about the trading mechanism used in the 1993 article. The former, i.e. the 1993 article, is actually also cited in the main paper, which introduced ZI-traders for the first time to the research community. The authors cite their 1993 article in their main paper, because it is a model which yielded similar results as the 1993 model described above, but was restricted to a case where a trader had only a single unit to sell or buy. As both the 1992 paper and the 2004 paper describe a model that utilizes a continuous double auction without replacement, I suppose that assuming that also the main paper used a similar CDA without replacement is justifiable.

Thus, trading in ZI-trader markets is divided into rounds. During each round the trader population, i.e. both sellers and buyers, is sampled without replacement and the selected trader is given a chance to trade. This process is continued until there is no trader left to sample or a transaction occurs Gode and Sunder (2004). If a transaction occurs, then the limit order book is emptied and a new round begins from scratch. On the other hand, if there is no trader left to sample and no transaction has occured, then a new round is started. Such a mechanism ensures that each trader gets a chance to trade before any other trader has had two chances to trade. Now that the model of Gode and Sunder (1993a) has been thoroughly explained, it is time to have a look at its results.

3.4.2 Results concerning transaction prices by Gode and Sunder (1993a)

The results from the models presented by Gode and Sunder (1993a) seem to support their claims about the efficiency of the markets with ZI-traders. However, the credibility of their results is partly decreased by the fact that they do not support their findings by using statistical significance tests or by reporting statistical figures of the market, but instead merely report the simulation results in pictures and interpret them.

The authors report the transaction prices times series and demand-supply schedules for all of the five markets, and the results suggest that the different traders make a

[21] See Gode and Sunder (1993b) and Gode and Sunder (2004).

difference. Gode and Sunder (1993a) found out that for the human traders the transaction prices seem to tend toward the equilibrium price, while the ZI-U -traders seem to act completely randomly. The ZI-C -traders seemed to be somewhere in between the humans and ZI-U -traders with certain tend in transaction prices towards the equilibrium price. The transaction prices of ZI-C -traders are not as volatile as with ZI-U -traders. Still, the transaction prices of ZI-C traders exhibit more volatility than the transaction prices of human traders. However, the difference between ZI-C and human traders is in the periodicity of transaction prices. The results suggest that while human traders remember the last closing price from the previous period in both the experiments of Gode and Sunder (1993a) and Smith (1962), ZI-C traders do not.

Gode and Sunder (1993a) highlight three features of ZI-C transaction price time series, which are supported by the findings from all of the five markets they consider. First, neither ZI-C of ZI-U traders seem to learn anything from the earlier periods as expected. In contrast, the humans seem to continue trading with a transaction price close to the closing price of the latest period. Second, the variance of transaction prices in ZI-U markets seems to highest, and the variance of transaction prices in the human markets seems to be lowest. The ZI-C markets seem to be somewhere in the middle between these two. Third, the transaction prices in the ZI-C markets seem to tend towards the equilibrium price, while the ZI-U markets show no such development. This argument is also backed up by a presentation root mean squared deviation (RMSD) of prices from the equilibrium price for each of the five markets averaged over the six periods to strengthen their argument.

RMSD of prices p_t, $t = 1, 2, .., n$ from the equilibrium price p^* is defined as given in equation 1 above. However, in contrast to Smith (1962), it appears that Gode and Sunder (1993a) have counted RMSD for each transaction in a market using the data from the six periods they ran each of the markets. Although Gode and Sunder (1993a) have not documented their use of RMSD carefully, a later paper by Cliff (1997) presents the calculation of such a RMSD measure thoroughly in the same manner as presented above. The idea is that the transactions for each period, i.e. day, are matched by their occurrence: the first transaction of each day are used to calculate the first RMSD measure, and the second trades the second measure.

Results of Gode and Sunder (1993a) suggest that in ZI-C and human markets the transaction prices converge towards the equilibrium price. The RMSD of prices from equilibrium in each of the five markets seems to tend towards zero for ZI-C and human traders, while this cannot be said about the ZI-U trader markets. According to authors, this shows that the ZI-C -agents induce the market price to tend towards the equilibrium price although again no statistical tests are presented. The argument is backed up by a regression, which shows that the coefficients of RMSD regressed against the transaction sequence number seem to yield negative slopes in each of the markets. Unfortunately, no statistical measures, like t- or p-values for the coefficients, are reported for the regressions, which makes it quite impossible to really investigate the quality of their regression analysis. Thus, the regressions do not seem to really add much value to the more qualitative results.

The interest in the results of Gode and Sunder (1993a) is especially in the difference between the ZI-U and ZI-C traders as the transaction prices of ZI-C traders resemble more closely the transaction prices of human traders. The authors argue that the tending towards the equilibrium price in ZI-C markets can be explained by the narrowing opportunity set of ZI-C -traders. According to them, in the beginning of each period, i.e. day, the probability of seeing a bid with a price higher than equilibrium price or an ask with a price lower than the equilibrium price, is larger than in the end of the period. They heuristically explain it is most probable that a buyer with a valuation higher than the equilibrium price, i.e. an intramarginal buyer, or a seller with a valuation lower than the equilibrium price, i.e. an intramarginal seller, trade during the beginning of the period. In effect this means that the agents trading in the end of the period have valuations closer to the equilibrium value, because the intramarginal traders have already left the market, because they traded in the beginning of the period. Although such ideas are intuitively plausible, the presentation of Gode and Sunder (1993a) lacks all the quantitative proofs.

3.4.3 Results concerning the efficiency of the markets by Gode and Sunder (1993a)

After having presented the tendency of the transaction prices to converge towards the equilibrium price, Gode and Sunder (1993a) move to presenting results about the efficiency of the market. First they define the maximum total profit that can be earned by all the traders as the sum of producer and consumer surpluses that can be both counted using the knowledge about the trader's valuations and the theoretical equilibrium price. Consumer surplus can be counted iteratively by summing all the positive differences between the demand function and the equilibrium price. Similarly the producer surplus can be counted by summing the positive differences between the equilibrium price and the supply function. After that Gode and Sunder (1993a) define the allocative efficiency of a market to be "the total profit actually earned by all the traders divided by the maximum total profit that could have been earned by all the traders“.

Gode and Sunder (1993a) continue by reporting the efficiencies of the different markets. First, they note that in ZI-U markets all the possible trades took place, while in ZI-C and human markets some units were not traded. Gode and Sunder (1993a) explain this by claiming that without the budget constraint, given enough time, all the possible trades will take place, while such does not necessarily happen in ZI-C markets. In ZI-U markets the traders will eventually bid and ask such prices that trades will take place, while in ZI-C markets it may well be that there are trades that can never take place, because the budget constraint restrains the traders from quoting sufficient prices for trades to take place.

In general, the efficiencies of the five markets were highest for human trader markets, who were able to reach almost 100 percent efficiency in all markets during all periods. This finding has been suggested also in the earlier literature about human experiments, so it seems credible and supports the fact that the experiment was conducted properly. In addition, the efficiencies of ZI-C markets were close to the efficiencies of human markets. However, the efficiencies for the ZI-U markets were clearly lower when compared to two other. Using primarily these arguments Gode and Sunder (1993a) concluded that the main reason for the high allocative efficiency of double auctions is in the market discipline and not in the capabilities of individual traders.

3.4.4 Other results of Gode and Sunder (1993b, 1997)

Gode and Sunder (1993b) propose in their paper a lower limit for the efficiency of the continuous double auction markets with ZI-C traders. The market structure is otherwise exactly the same as in the paper discussed above[22], but the difference between the models is in the fact that Gode and Sunder (1993b) use a model where the traders can trade at most a single asset during a single period. To derive the expected efficiency the authors define the extramarginal traders as agents who have a valuation situated to the right from the intersection of demand and supply curves, and intramarginal traders as agents who have a valuation that situates them to the left from the intersection of demand and supply curves. The approximation for the expected efficiency of the CDA market with ZI-C traders presented by Gode and Sunder, is a function of the number of intramarginal traders participating in the market.

Result of Gode and Sunder (1993b) highlight an important factor contributing to the price discovery process. According to the results of Gode and Sunder (1993b), the efficiency of continuous double auction is derived from the differences in the proportions of intra- and extramarginal traders participating in the market. This virtue is also important to the price discovery process in ZI-C markets, because as the intramarginal traders have left the market the trading ceases; the extramarginal traders cannot trade between themselves.

This simple result can be easily shown to be true by creating an example. Assume

[22] See above the discussion about the paper by Gode and Sunder (1993a).

a market full of extramarginal buyers and sellers with an equilibrium price p. Then the valuations of all extramarginal buyers are lower than the equilibrium price, which means that ZI-C buyers can bid only prices that are lower than p. Similarly, the valuations of all extramarginal sellers are higher than the equilibrium price, which means that the ZI-C sellers can ask only prices that are larger than p. This means that no trade takes place in the market, because all the bids are by definition strictly lower than all asks. Thus, the price discovery process can take place in ZI-trader markets only as long as there are intramarginal traders left participating in the market.

In a later article, Gode and Sunder (1997) determine the allocative efficiency of different market types, which include also the continuous double auction. The authors present a number of exact formulas for the efficiency of the different market types. The importance of their results in light of this thesis is again in the fact that the results confirm and define more exactly the more earlier results presented by Gode and Sunder (1993b) mentioned above: the proportions of intra- and extramarginal traders contribute to the price discovery process.

3.4.5 Critique by Cliff and Bruten (1997)

Probably one of the hardest critiques for the ZI-trader model has been presented by Cliff and Bruten (1997)[23]. The results presented by Cliff and Bruten (1997) have been cited in several publications ever since[24], although the analysis of Cliff and Bruten itself has not been widely questioned in the citing publications. In essence, the results of Cliff and Bruten (1997) reject the convergence of ZI-C markets to the equilibrium price in certain market types that will be defined below. The authors present both mathematical analysis and simulation experiments, which both lead to the same conclusion. According to them more than zero-intelligence is in general required from the trading agents to make the markets behave as if the traders were humans. In addition, according to Cliff and Bruten (1997), only the chosen parameters of the models in the ZI-C markets presented by Gode and Sunder (1993a) guaranteed the convergence to the equilibrium.

The heart of the argument by Cliff and Bruten (1997) is the analysis of probability density functions for bids and asks in ZI-C markets. Generally, the probability density function (PDF) of a random variable describes the relative likelihood of that random variable to have a certain value. More formally, define the probability space as $(\Omega, \mathcal{F}, P)$, where Ω corresponds to the sample space, $\mathcal{F}$ to the sigma algebra and P to the probability measure. Then, a continuous random variable X can be defined as a function X from the sample space to real numbers, $X : \Omega \to \mathbb{R}$, if for all $x \in \mathbb{R}$ we have that $\{\omega \in \Omega :$

[23] See also the more elaborated article about the same subject by Cliff (1997)

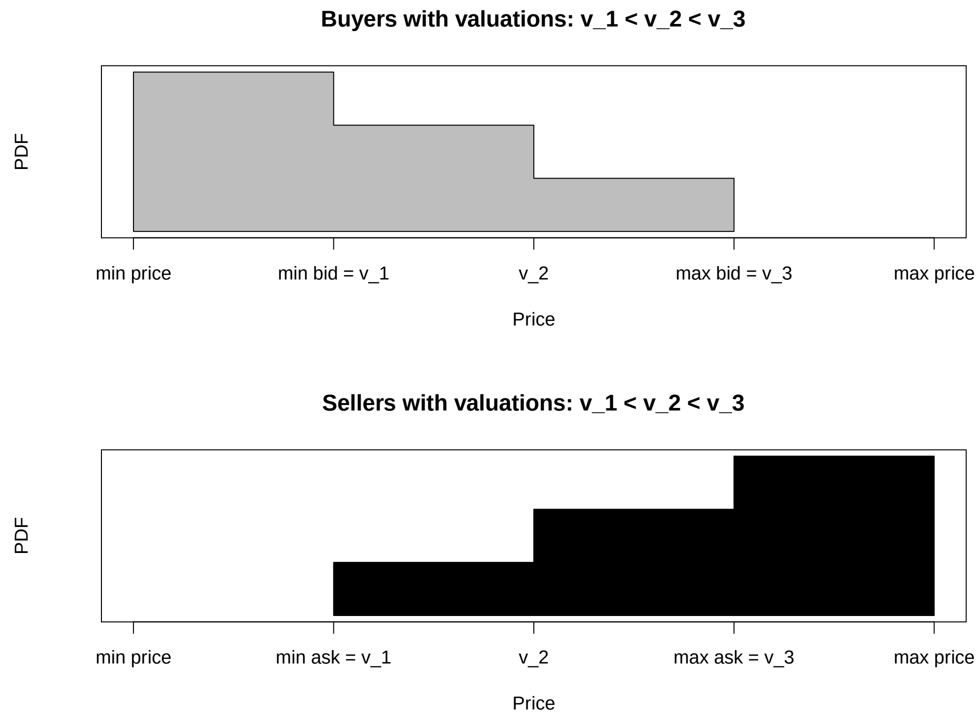
[24] For example Google Scholar gave 9th of March, 2011 216 citations for the paper by Cliff (1997).

$X(\omega) = x\} \in \mathcal{F}$. Now, a PDF of a random variable X is defined as a function $f_X \geq 0$ such that for any set $B \subset \mathbb{R}$ we have $P(X \in B) = \int_B f_X(x)dx$ and the integral over the whole space $\int f_X(x)dx$ equals one.

Cliff and Bruten (1997) derive the market wide PDFs for bids and asks by first defining the PDFs for both ZI-C sellers and buyers. To make the following presentation as simple as possible, it will be assumed in the following that each agent can trade at most a single asset as was done also in the previous section. Again, this should not be a problem, because already the results by Gode and Sunder (1993a) were assured using agents, who were allowed to trade at most a single good during the experiment. Generally, for ZI-C agents the PDFs are uniform distributions, which essentially means that the PDF is constant over its support. For ZI-C buyer the support is defined as an interval of real numbers from minimum price 1 to valuation v, and for a ZI-C seller the support is defined as an interval of real numbers from valuation v to maximum price 200. Qualitative versions of such PDFs are given in figure 1.

Figure 1: Qualitative PDFs of quotes of a ZI-C buyer and seller as proposed by Cliff and Bruten (1997). The range of possible prices in the market is determined by minimum price (min price) and maximum price (max price), while the valuation $v_i, i \in \mathbb{N}$ is an agent specific variable. The distributions of ZI-C agents are uniform distributions, which essentially means that the PDF is constant over its support. For a ZI-C buyer the support is defined as an interval of real numbers from 1 to valuation v_i, and for a ZI-C seller the support is defined as an interval of real numbers from valuation v_i to maximum price 200.

Figure 2: Qualitative PDFs of quotes in a market with three ZI-C buyers and sellers with unequal valuations. The range of possible prices in the market is determined by minimum price (min price) and maximum price (max price), while the valuations $v_1 < v_2 < v_3$, are agent specific. The minimum bid price (min bid) corresponds to lowest valuation in the group of all buyers in the market, and the maximum bid price (max bid) corresponds to the highest valuation in the group of all buyers in the market. The minimum ask price (min ask) corresponds to the lowest valuation in the group of all sellers in the market, and the maximum ask price (max ask) corresponds to highest valuation in the group of all buyers in the market. The PDF of bids decreases from valuation v_1 to v_3, because the number of buyers willing to bid at a higher price decreases as the price increases. Similarly the PDF of asks increases from valuation v_1 to v_3, because the number of sellers ready to sell at a higher price increases as the the price increases.



Assuming that all the valuations of buyer agents are not the same, means in effect that the PDF of bids market wide is a decreasing function in price. This is derived from the fact that the number of buyer agents willing, i.e. having a positive probability, to bid with at a certain price decreases as the the price increases. For example[25], assume that there exist three buyer agents in the market: agent 1, agent 2 and agent 3 and that they have valuations v_1, v_2 and v_3 correspondingly. If all the three agents demand a single good and the valuations are not equal, then the agent with the higher valuation is always also ready to buy at a price that is accepted by the agent with the lower valuation, but this is not the case the other way around. Thus, if $v_1 < v_2 < v_3$, then there exists a single agent, i.e. agent 3, in the market who has got a positive probability to bid a price, which

[25] See figure 2 for an illustration of this example.

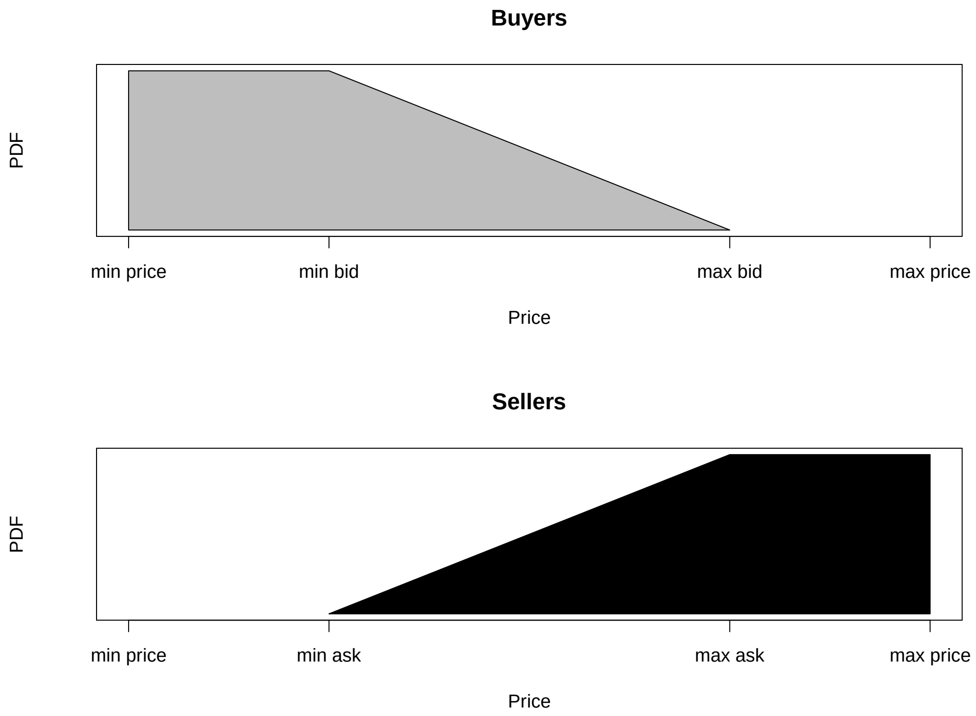
is in the range (v_2, v_3). Similarly, there exists two agents in the market who have got a positive probability to bid a price, which is in the range (v_1, v_2). Thus, the market wide PDF is decreasing with the price. Similarly, when the valuations of all seller agents are not the same, then one can easily see that the PDF for market wide asks is an increasing function in price using similar arguments. The qualitative versions of such PDFs for market wide bids and asks are presented below in figure 2 for a market with three buyers and three sellers with valuations $\{v_1, v_2, v_3\}$.

By increasing the number of agents participating in the market, the step size in the market wide PDF for bids and asks decreases. According to Cliff and Bruten (1997) this should result in probability density functions for bids and asks that have constant slopes. Examples of such qualitative probability density functions are given in figure 3. At first sight such result might appear to be correct, but actually the argumentation is not sufficient to really characterize the probability density functions for quotes in ZI-C markets. I will address this issue further in the Models-section and for now it is enough to know that the correct probability density functions for quotes defined later are similar in their characteristics to the ones proposed by Cliff and Bruten (1997).

The essence of the argument of Cliff and Bruten (1997) is that the probability density function for the all transaction prices during a SCDA is given by the intersection of market wide probability density functions for bids and asks. This argument is based on the heuristic that according to Cliff and Bruten (1997) for an ask and a bid to be valid, it has to be that the transaction prices are determined by the intersection of the probability density functions for the quotes. The intersection for the market wide probability density functions for bids and asks is presented in figure 4. Cliff and Bruten (1997) use the intersection argument also to derive analytic measures of the expected value of the transaction price, and compare the derived expected values to the theoretical equilibrium price and average transaction prices. Especially, Cliff and Bruten (1997) claim that the expected transaction price is different from the equilibrium price when the supply and demand schedules are changed radically from the ones presented by Gode and Sunder (1993a). Thus, according to Cliff and Bruten (1997) the results of Gode and Sunder (1993a) were based on appropriately chosen demand and supply schedules

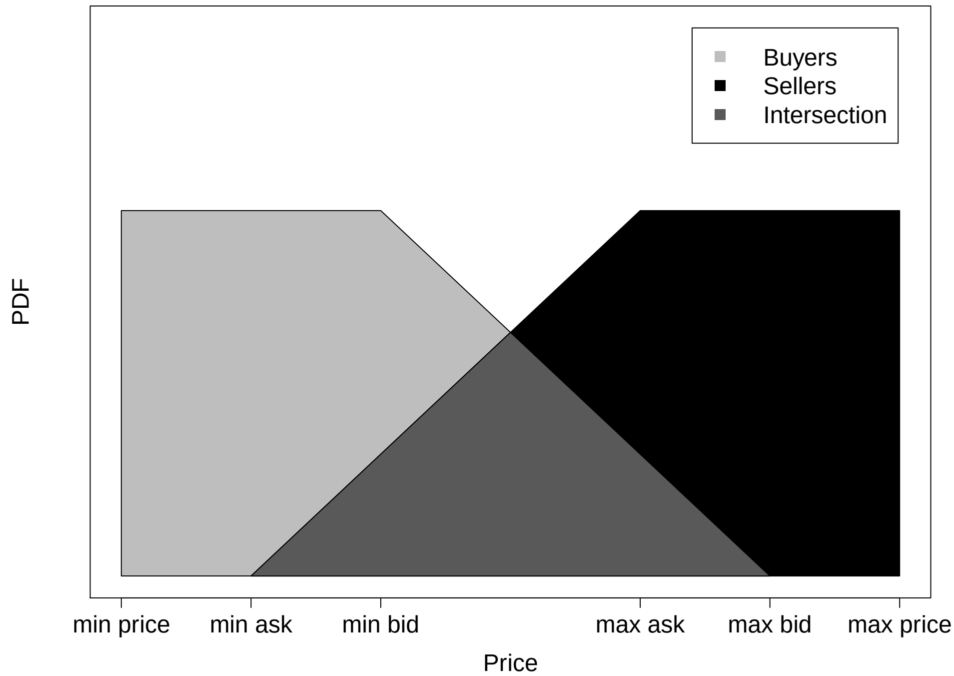
Cliff and Bruten (1997) criticize Gode and Sunder (1993a) that all the markets Gode and Sunder reviewed in their study were in terms of demand and supply schedules in similar. Cliff and Bruten (1997) derive the expected value of the transaction prices for four market types, which are according to them different in terms of supply and demand schedules. The differences between the market types reviewed by Cliff and Bruten (1997) are summarized in figure 5. Market A presented in the top left corner of figure 5 can be characterized as symmetric in terms of demand and supply. Term symmetric is derived

Figure 3: Qualitative PDFs of quotes in a market with ZI-C buyers and sellers as proposed by Cliff and Bruten (1997). The range of possible prices in the market is determined by minimum price (min price) and maximum price (max price), while the valuations are agent specific variables. The minimum bid price (min bid) corresponds to lowest valuation in the group of all buyers in the market, and the maximum bid price (max bid) corresponds to the highest valuation in the group of all buyers in the market. The minimum ask price (min ask) corresponds to the lowest valuation in the group of all sellers in the market, and the maximum ask price (max ask) corresponds to highest valuation in the group of all buyers in the market. The probability to see a bid decreases from price min bid to max bid, because the number of buyers willing to bid at a higher price decreases as the price increases. Similarly, the probability to see an asks increases from price min ask to max ask, because the number of sellers ready to sell at a higher price increases as the the price increases. See figure 2 for a market with three sellers and three buyers.



from the fact that demand and supply are geometrically symmetric in terms of a horizontal line, if such would be drawn at price p_0 for market A. I will use henceforth use the term symmetric demand and supply schedules to refer to a market type as the one now presented for market A. Market B, the top right corner in figure 5, corresponds to a situation where all the sellers have the same valuation. This means that the supply curve is flat. Market C, the bottom left corner in figure 5, corresponds to a situation where both demand and supply curves are flat, but in addition there exists excess demand in the market. In the bottom right market D in figure 5, both demand and supply curves are also flat, but this time the market exhibits excess supply. It is good to notice that

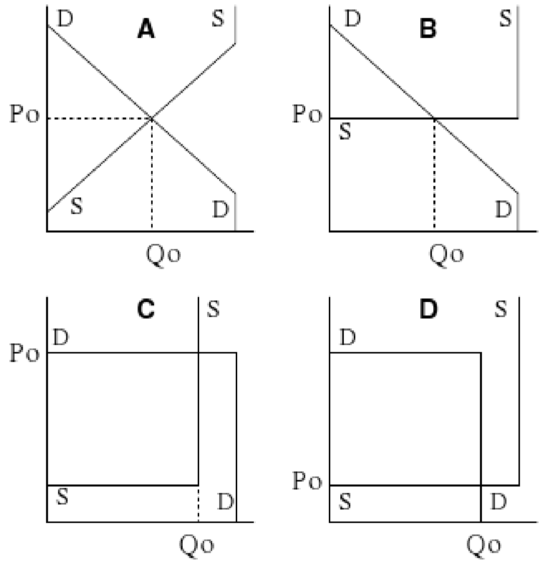
Figure 4: Intersection of qualitative PDFs of quotes in a market with ZI-C buyers and sellers as proposed by Cliff and Bruten (1997). The range of possible prices in the market is determined by minimum price (min price) and maximum price (max price), while the valuations are agent specific variables. The minimum bid price (min bid) corresponds to lowest valuation in the group of all buyers in the market, and the maximum bid price (max bid) corresponds to the highest valuation in the group of all buyers in the market. The minimum ask price (min ask) corresponds to the lowest valuation in the group of all sellers in the market, and the maximum ask price (max ask) corresponds to highest valuation in the group of all buyers in the market. Cliff and Bruten (1997) argue heuristically that the intersection corresponds to the probability density function of the transaction prices, because according to their the intersection defines all the valid bids and asks during the CDA.



each of the markets have a unique theoretical equilibrium point characterized by the intersection of demand and supply curves. In a market with multiple equilibrium prices the characterization of an equilibrium price could be problematic, while in a market with a single equilibrium there is no such problem.

Cliff and Bruten (1997) also present simulation results for all of the four markets, and the results appear to support their arguments. Only the results from market A with symmetric demand and supply schedules show that the equilibrium price is equal to the expected transaction price. In all of the other markets, the equilibrium price differs from the expected transaction price, and the empirical average transaction price seems to be in all cases in line with the expected transaction price. For a careless reader, such results would suggest that in the market other than A, the ZI-C traders do not converge to

Figure 5: Different market types presented as by Cliff and Bruten (1997) in their figure 1. In each of the four markets depicted, horizontal axis corresponds to quantity and the vertical axis corresponds to price. DD and SS curves correspond to demand and supply curves, while P_0 and Q_0 refer to equilibrium price and quantity. Equilibrium price and quantity are determined by the intersection of DD and SS curves in each of the four markets.



equilibrium. However, it is questionable that Cliff and Bruten (1997) report only average transaction prices and leave out from their contemplation, for example, the closing prices, which could have shown how strong the tendency towards the equilibrium really is.

This is important as the results of Gode and Sunder (1993a) were in principal about the tend of the transaction prices towards the equilibrium price during a trading day and not about the average correctness of equilibrium price as an forecast of the transaction prices. This problem is noted also by Cliff and Bruten (1997), but for some reason the exact results have been left to the more elaborate version of the same study by Cliff (1997). In the elaborated version, Cliff (1997) reports the RMSD of transaction prices from the equilibrium price as suggested first by Smith (1962) and used by Gode and Sunder (1993a) to measure the convergence towards the equilibrium. The results are in other ways similar to the ones presented using averages, but when using RMSD also the market B seems to show some tendency of transaction prices towards the equilibrium price (Cliff, 1997).

The general plausibility of the arguments expressed above was first questioned by Othman (2008). The argumentation presented by Cliff and Bruten (1997) has a blind spot, because the authors do not have explicitly covered the reasons why the PDF of transaction prices should be the intersection of PDF's for bids and asks (Othman, 2008).

Essentially, Othman (2008) presents a counter example that shows how at least in one market type the intersection of the PDFs of bids and asks does not determine the PDF for transaction prices in the market correctly. In addition, Othman (2008) also characterizes the seemingly complex probability density function for the transaction prices, which is clearly more complex than what Cliff and Bruten (1997) proposed. Thus, there are certainly reasons to question at least parts of the presentation of Cliff and Bruten (1997).

Another important problem with the method by Cliff and Bruten (1997) is that their analysis considers only the first round of SCDA. Thus, actually their analysis of the intersection tries only to characterize the expected value of the transaction price of the first trade after the market is started. The reason for this is that the demand and supply schedules change after two traders, i.e. a seller and a buyer, are removed from the market, because they have traded all they were allowed to trade. Cliff and Bruten (1997) also self note vaguely this issue, but claim that as their empirical simulation results seem to suggest their theory to be close to correct, there is no need to start revising the theory. However, the results of Othman (2008) seem to suggest that actually the theoretical results are not correct, but should be instead revised and properly reassessed.

It is also interesting that the analysis of Cliff and Bruten (1997) takes no view on the claim of Gode and Sunder (1993a) about the narrowing range of feasible transaction prices. This is especially interesting, because the buyers and sellers with the intramarginal valuations in the SCDA market are the ones that are most likely to trade during the beginning of the day, while the buyers and sellers with extra marginal valuations seem to trade closer to the end of the day according to heuristic arguments of Gode and Sunder (1993a). Thus, it would seem interesting to look quantitatively how the probability density functions for the bids and asks change during the day when the amount of traders in the market changes. As it is clear that this issue is important in understanding ZI-C markets, it will be taken into consideration in the empirical part of this thesis. To the best of my knowledge, no other author has previously taken a quantitatively look at it although Brewer et al. (2002) have proposed similar ideas as will be discussed next.

3.4.6 Critique by Brewer et al. (2002)

The critique by Brewer et al. (2002) was pointed towards the convergence of transaction prices in ZI-C traders markets. As the first authors in this line of literature, Brewer et al. (2002) defined the properties of transaction price convergence towards the equilibrium price explicitly as follows:

1. Initial transaction prices are further from the equilibrium than final prices.

2. Variance of transaction prices decreases over time.

3. If a parameter change moves the equilibrium price, then the transaction prices move towards the new equilibrium.

Although, for example, the study of Gode and Sunder (1993a) lacked similar definition, Gode and Sunder used similar arguments to argue about the convergence of ZI-markets. I will essentially use the first two bullet points of the above definition to measure the convergence of transaction prices towards the equilibrium price in the following. The third bullet point is left out of the contemplation, because changes in demand and supply schedules are outside of the scope of this thesis.

The essential contribution by Brewer et al. (2002) was to define a setting, which does not allow the transaction prices of ZI-C trader markets to converge towards the equilibrium price. The essential ingredient was to create a market, where the amount of intramarginal traders stays relatively constant. Brewer et al. (2002) named such a market as the continuously refreshed supply and demand (CRSD) market. Intuitively, in such a market with ZI-C traders, the transaction prices do not converge to the equilibrium price, because every time an intramarginal trader leaves the market, a new one arrives to fill in. Brewer et al. (2002) showed experimentally that in a CRSD market the ZI-C traders are not able to show transaction price converge towards the equilibrium price.

Brewer et al. (2002) also introduced the idea of a Marshallian path in the context of ZI-C traders. A Marshallian path is a sequence of trades such that traders are paired from left to right along supply and demand curves (Brewer et al., 2002). According to Brewer et al. (2002), trading in ZI-C markets as suggested by Gode and Sunder (1993a) takes place in a manner that resembles the Marshallian path, and their idea in the first place was to design the CRSD markets so that the Marshallian path does not lead to transaction price convergence. In terms of experimental economics, the results of Brewer et al. (2002) essentially show that the ZI-C traders are too simplistic to really describe human behavior, because they showed that with human traders CRSD markets exhibit transaction price convergence, while with ZI-C traders no such convergence is present.

The idea in this thesis is to explicitly and quantitatively show that the price convergence in ZI-C markets can be explained using the idea of Marshallian path first introduced to ZI-C traders context by Brewer et al. (2002). Essentially, I will experimentally show the proposition by Brewer et al. (2002) that the probability that an intramarginal buyer trades with an intramarginal seller is higher than any other combination of intra- and extramarginal traders. In addition, it would be interesting to know the probabilities to trade between the traders inside the groups of intramarginal buyers and sellers. This is interesting, because it is reasonable to expect that, for example, a intramarginal buyer with the highest valuation is the most probable intramarginal trader to trade during a single round.

3.4.7 Critique by Gjerstad and Shachat (2007)

Gjerstad and Shachat (2007) criticize the conclusions made by Gode and Sunder (1993a) from two perspectives. First, according to Gjerstad and Shachat (2007), the transaction prices do not converge to equilibrium values in the simulations of ZI-C trader markets presented by Gode and Sunder. The main argument is that the ZI-C traders do not remember anything from the previous periods, which makes them to start the converge towards the equilibrium always from scratch when the market is restarted. However, the arguments about the convergence made by Gode and Sunder (1993a) were mainly about the convergence inside the periods. Thus, actually the first critique made by Gjerstad and Shachat (2007) concerns mainly the definition of convergence, which is not explicitly defined in either studies.

The second critique posed by Gjerstad and Shachat (2007) concerns the definition of zero-intelligence. Gjerstad and Shachat show in their paper that the definition of budget constraint by Gode and Sunder (1993a), is actually an individual rationality constraint. The difference in the economic sense is that a budget constraint is only a constraint in the maximization problem, while an individual rationality constraint means that an agents takes part only in transactions, which increase or leave her utility constant. Thus, also the other critique by Gjerstad and Shachat (2007) is mainly directed towards the loose definitions of the concepts that Gode and Sunder (1993a) used in their presentation.

3.4.8 ZI-trader model today

Ladley and Schenk-Hoppé (2009) presented another model of ZI-C traders, where the traders were allowed to enter and exit the market with certain probabilities. In addition, compared to the original model by Gode and Sunder (1993a), the model by Ladley and Schenk-Hoppé (2009) also incorporated an order book mechanism that allowed the modified ZI-C traders to also trade more than a single unit at a time. Interestingly, such an extended model was able to create many of the stylized facts of the order-book, like the shape of the order book, size of spreads and conditional probabilities of order submissions that are exhibited by the real markets (Ladley and Schenk-Hoppé, 2009).

As a conclusion, I view that the ZI-C trader model should be evaluated more quantitatively than has been done in the past. The present literature seems to suggest that the ZI-C traders are a simple approach to create some of the stylized facts appearing in real markets. Thus, it also seems to be important to explain why the markets with ZI-C traders exhibit such characteristics. In the following, I will explain how an agent-based model for ZI-traders can be created and what methods will be used to assess the model more quantitatively than has been done in the previous literature.

4 Methods

The following presentation will explain how the models used were created and take a look at the learning process that was went through when implementing the agent-based model in practice. There are a number of issues that had to be taken into account when designing and implementing an agent-based model.

4.1 Building an agent-based model using SimPy and Python

One of the first decisions to make when implementing the model in practice was the selection of the programming language used. Initially I preferred Python over other possible languages, because it offers a forceful and minimalistic way to present complex structures. For example, when comparing Python and its standard library implementation of list to Java and its comparable standard library class ArrayList, the use of list in Python requires less lines and definitions. Such abilities in general make the designing of complex structures simpler in Python than, for example, in Java. As Python also offers a general discrete time simulation package SimPy, it seemed wise to choose Python as the programming language to be used.

SimPy also seemed to suit the agent-based context well, because it provided the basics needed for a simulation of simultaneously interacting agents. This means that the selection of SimPy as the framework for agent-based modeling did not constrict the implementation of different models. This was an important issue when choosing the framework, because I had no earlier experience from implementing agent-based models in practice. There are a number of different programs used in different fields of science[26], and, without a doubt, all of them have their good and bad sides.

Simultaneously interacting agents are implemented in SimPy using Python generators. This property allows, for example, the agent process to be interrupted at a certain point in time. After that the generators offer the possibility for the agent to continue the processing from exactly the same point where the processing was interrupted. In the agent-based modeling context, this means that many agents may interact simultaneously with each other. Such a possibility is especially interesting if the agents are supposed to interact with each other without central coordination. In such a simulation, it may, for example, happen that an agent, say agent A, tries to acquire a resource, for example, a limit order book, but another agent, agent B, has already reserved it. This means that agent A has to take another decision. A simulation environment that does not allow simultaneous interaction of agents cannot simulate such collisions.

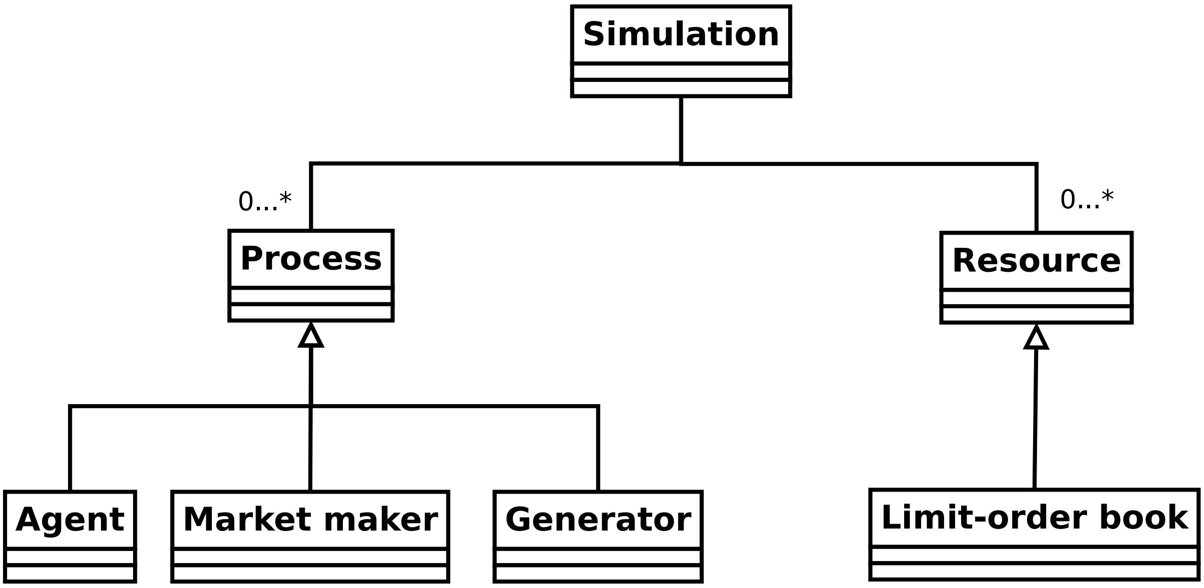
[26] A good example of the large number of different environments used in agent-based modeling is the list provided by the agentlink.org: `http://eprints.agentlink.org/view/type/software.html`, which was visited 30 January, 2011.

However, the downside of using SimPy is naturally efficiency. Essentially, it means that the number of agents that can be used in simulations are smaller than with more efficient approaches. This is derived from the fact that the simulation in an environment allowing the simultaneous interaction agents, like SimPy, needs more steps than a simulation in an environment that does not allow the simultaneous interaction of agents. This is quite an intuitive issue, because the simulation of interacting agents requires the simulation environment at each time step to check the status of each agent somehow. An environment not supporting simultaneous interaction can proceed without such checks, which means that the latter needs less steps. At first place, this did not seem to be a problem and the results provided in this thesis are certainly comparable to the results of Gode and Sunder (1993a) in terms of the number of agents used. However, if one wanted to increase the number of agents dramatically, then it would be necessary to change from SimPy to a simpler model, if possible. In the heuristic tests done using SimPy and a home desktop, it seemed that increasing the number of agents from 150 to 500 increased the running time of the model dramatically. Thus, in case one wanted to simulate a larger amount of agents, it would be necessary to optimize the interaction of agents.

It is also good to note that the use of SimPy caused the implementation of a simple ZI-trader model to become more complex than would have been needed. The ZI-traders presented by Gode and Sunder (1993a) are centrally coordinated, which means that there is no need for the simultaneous interaction of agents. Thus, all the checks that SimPy does for the agents implicitly could have been optimized away. However, in the case of the models that are based on simultaneous interactions of agents, like the one presented by Boer-Sorban (2008), one needs to use a package like SimPy or otherwise such models cannot be implemented. As this thesis has in general been a learning process, it was certainly a good decision to choose a simulation framework that did not restrict the implementation of different models. However, in practice, it seems that the ZI-trader models used in this thesis could have been implemented in a more straight forward and computationally efficient manner than will be presented in the following.

From the point of view of the user, a SimPy simulation builds primarily on three different types of classes: Process, Resource and Simulation. To create the first ZI-trader model, agent, market maker and generator were implemented as subclasses of the Process class, the limit order books were implemented as subclasses of the Resource class and the simulation itself was implemented as a subclass of the Simulation class as is presented in figure 6. The following subsection will present the most important decisions regarding the design of the used classes, while the whole source code can be found from the Appendix A. The following presentation is supposed to introduce the SimPy to the reader at the same time with the simplest ZI-trader model.

Figure 6: A unified modeling language class diagram for the implemented SimPy simulation. Agent, market maker and generator were implemented as subclasses of the Process class, the limit order books were implemented as subclasses of the Resource class and the simulation itself was implemented as a subclass of the Simulation class.



4.1.1 Agent and Market maker

Implemented Agent class has two most important methods: __init__() and work(), while the rest of the methods implemented are used to simplify the implementation of the work()-method. Agents are initalized using the __init__()-methods, which is called by the generator when initializing the agents. Generator initializes all of the agents in the beginning of the simulation. After that, it does not have any meaning in the simulation, which is also the reason why it is only briefly mentioned here. __init__()-method initializes the characteristics of agents by choosing their type to be a seller or a buyer, and after that according to their type chooses their valuation. When initializing the agents, __init__()-method also counts the demand and supply curves for the market according to the valuations of the agents that it initializes.

The work()-method is the main method of the Agent class, and implements the actions of both seller and buyer agents. Principals that govern the actions of both buyers and sellers are very similar as both of the agents try to trade a single asset once and after that leave the market. The actions of an agent are restricted in time by the actions of both other agents and the market maker. The agents and the market maker use global fields, selectedBuyer, selectedSeller, buyerQueue and sellerQueue, to communicate with between themselves about the status of the market; communication is needed to somehow coordinate the market.

When the simulation starts, all of the agents are initialized and after that all the agents enqueue themselves either to buyerQueue or sellerQueue. The market maker waits for all the agents to be in the two queues, and after that selects randomly one agent from the queues to trade. The traders see the market maker's selection by looking at the two fields: selectedBuyer and selectedSeller. After one of the fields is changed, all the traders check the field corresponding to their type and see if they were selected. The selected trader then checks, if there exists a limit order that satisfies her bid/ask. If such a limit order exists, then she submits a market order by trading at the limit price. If the trader does not find a satisfiable limit order, she leaves an own limit order at her bid/ask price. After she has traded or left a limit order, the trader sets the corresponding selectedSeller or selectedBuyer to a value, which informs the other traders that the selected trader is ready. After the selected trader is ready, the market starts a new round.

MarketMaker class is used to centrally coordinate the interaction of the traders and to select the trader to trade at each round. The class has only a single method work(), which is executed as long as the market is functioning. After the market maker is initialized, she waits until all of the agents are initialized and appended into the buyer and seller queues. Next, the market maker chooses the agent that is selected to trade. The agent is selected randomly from the group of agents, who have not traded in the ongoing round; when all agents have traded on the ongoing round, the market maker starts a new round. After selecting the trader, the market maker waits until she has traded and after that starts the while loop from the beginning.

4.2 Random number generation

Random number generation is naturally always an important part of a simulation experiment. However, the earlier literature in agent-based modeling has not in all cases documented the used pseudo random number generators extensively. For example, Gode and Sunder (1993a) do not even mention the random number generation and its implications. On the other hand, for example, Cliff and Bruten (1997) use a cookbook algorithm from the numerical recipes textbook[27]. Although that algorithm might have been state of the art in the end of 1990s, today there are better solutions available.

Python offers a good random number generator from the standard library. According

[27] For that algorithm refer to the numerical recipes textbook (Press et al., 1992).

to the documentation for Python's version 2.6.5[28], those Python's functions that are used in this thesis and are implemented the module "random", use an algorithm called Mersenne Twister. It was initially developed by Matsumoto and Nishimura (1998), and is today by far one of the best pseudo-random number generators available, which means that the results should stand out in a comparison with any other well-known pseudo-random number generators available today (L'Ecuyer, 2001).

[28] See the webpage `http://docs.python.org/release/2.6.5`.

5 Results

This chapter presents the results from of the models created. I will first present a model of ZI-traders to benchmark my results to the previous literature[29]. After that, the second section will critically review the methods of Cliff and Bruten (1997) and introduce a few additional methods to analyze the ZI-C markets. After that using the created methods, an analysis of the convergence of bids, asks and transaction prices in symmetric ZI-C markets is presented. Finally, the third section uses the most essential methods presented in the second section to briefly review the different market types introduced by Cliff and Bruten (1997).

5.1 ZI-model with symmetric demand-supply schedule

The first model is a replication of the model and results presented first by Gode and Sunder (1993a) and later by Cliff and Bruten (1997). However, to create a model as simple as possible, the traders are restricted to trade only a single asset; after that they leave the market. This assumption should not be a problem, as the study by Gode and Sunder (1993b) suggests that this change should not affect the results. On the other hand, when using this assumption, it is possible to argue how the population of traders evolves throughout the continuous double auction, which appears to be a very important factor contributing to the price discovery process. The analysis presented in the end of this section is largely based on the fact that the traders leave the market after they have traded once during a single period of trading.

The model created exhibits similar characteristics for different output measures as the one presented by Gode and Sunder (1993a). These different characteristics are divided to qualitative differences, efficiency of the markets and transaction price time series characteristics in a similar way as was done by Gode and Sunder (1993a). In the following, the characteristics are reviewed and compared at the same time to the results of Gode and Sunder (1993a). The results presented in this section are based on the same assumptions regarding the used market type as described by Cliff and Bruten (1997) with the symmetric case. The selected valuations for both buyers and sellers were exactly the same in all of the runs of the model and can be defined by using the following arithmetic sequence:

$$p^j = p^0 + j\delta, \qquad (3)$$

where $p^0 = 26$, $\delta = 2$ and $j = 0, 1, 2, ..., 74$. Thus, the arithmetic sequence presented in equation 3 defines the valuations as $26, 28, ..., 172, 174$. This sequence has 75 distinct

[29] As suggested by Davis et al. (2007), it is important to try to verify that the created simulation model works correctly and one way to do this in practice is to compare the results to the earlier results.

valuations, and each of the valuations was given to two agents, i.e. to one buyer and to one seller. This means that the market consisted of 150 agents, which were equally divided to buyers and sellers. As the market type was exactly the same for both ZI-U and ZI-C traders, the differences in the results next presented can be argued to be derived from the differences in the capabilities of the two trader types.

According to Cliff and Bruten (1997) the symmetric market type is the one that was chosen by Gode and Sunder (1993a) to arrive at the expected results. It is still a good starting point, because the results from a symmetric market can be easily compared to the results of Gode and Sunder. After the basic results have been confirmed, it is possible to evaluate also the more complex results.

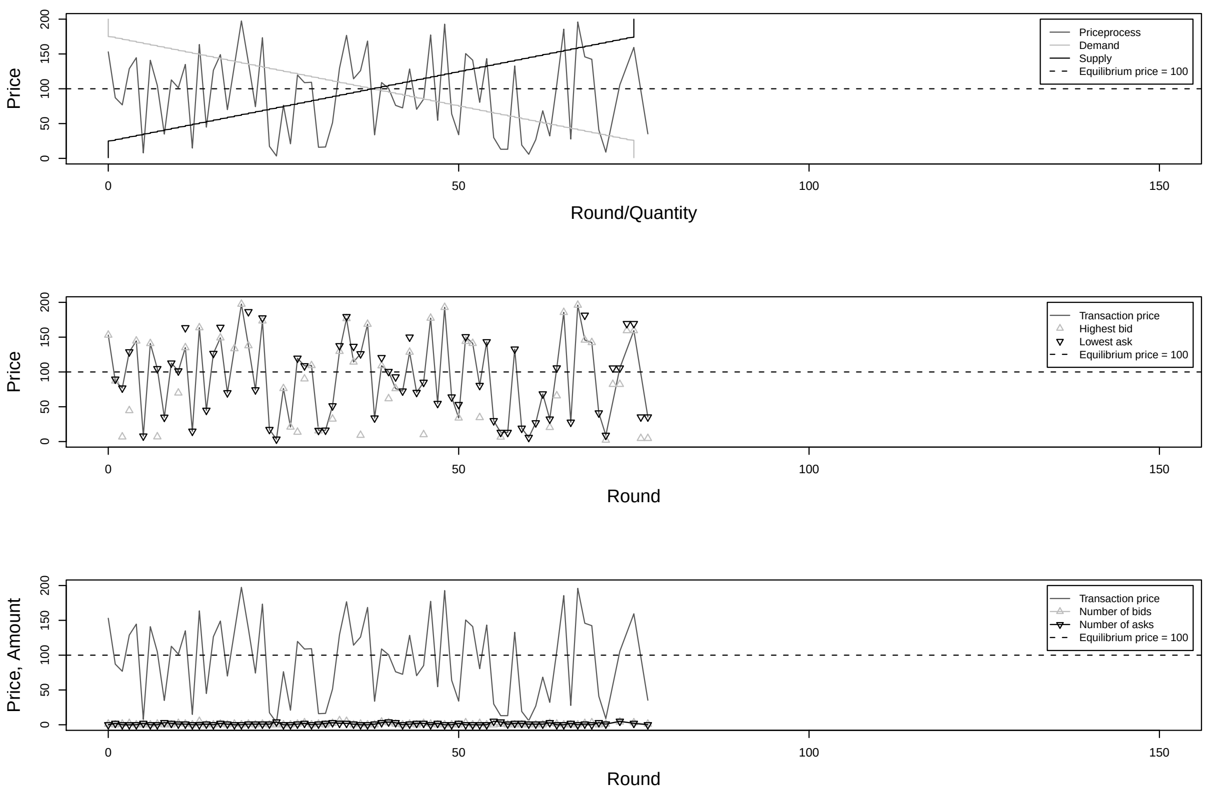
5.1.1 Qualitative differences

A qualitative view to the results is best acquired by eye-balling the transaction price time series and demand-supply schedules from single runs of the ZI-C and ZI-U models. The transaction price time series and demand-supply schedules for a ZI-U market with 150 traders in markets that lasted for 150 rounds are presented in the top panel of figure 7. A similar graph for a ZI-C market lasting for 150 rounds with 150 traders is presented in the top panel of figure 8.

Certain qualitative issues seem to be clear already from the two figures presented. First of all, the traders in the ZI-U markets seem to trade more than ZI-C traders. In the ZI-U market presented in figure 7, a transaction seems to take place at regular intervals of rounds as long as the market is active, while in the ZI-C markets presented in figure 8 transactions take place only during the first 50 rounds. In the particular ZI-U market presented in figure 7, all the traders participating in the market traded, and the market was closed after round 77 as all of the traders had left the market. In contrast to this, in the ZI-C market presented in figure 8, the market was not closed until the maximum number of rounds, 150, was reached. This happened, because there were still ZI-C buyers and sellers left in the market trying to trade. However, although the market was active as long as possible with ZI-C traders, no trades took place after the last transaction that took place on round number 71.

Table 2 quantitatively shows these differences for the ZI-U and ZI-C markets. There the first column reports some measures from the ZI-U market presented in figure 7, while the second column reports the same measures for the ZI-C market presented in figure 8. The table shows that the trading in ZI-U market did not stop before the 75th transaction. This means that the maximum number of possible transactions with 150 traders were undertaken, because a single transaction always need two counterparts, one buyer and one seller. However, in ZI-C markets, the trading ceased after the 41st transaction had

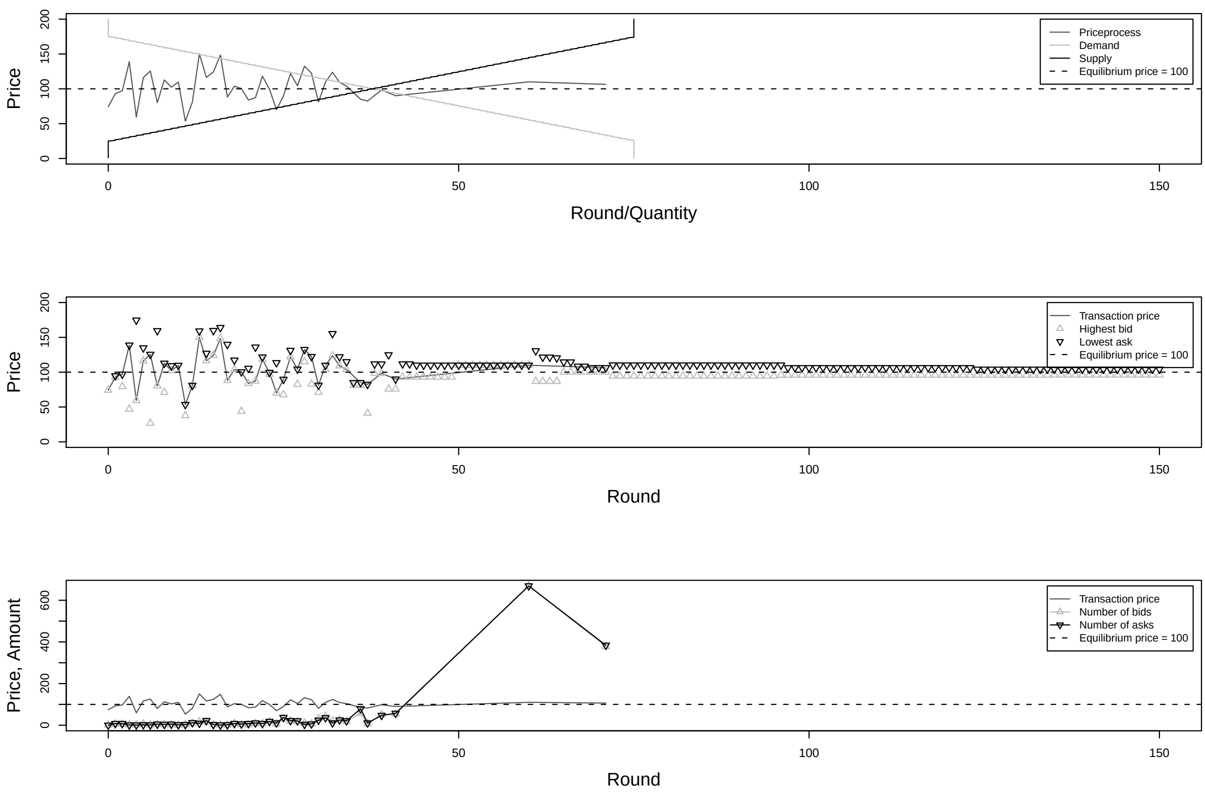
Figure 7: Transaction price time series, symmetric demand-supply schedules, best quotes and the amount of bids and ask in the limit order book for a single run of ZI-U market with 150 traders and 150 rounds. The traders were divided into buyers and sellers equally, and the valuations are specified in equation 3. In all of the three panels, the transaction price times series is presented by a dark gray solid line as a function of rounds, and the equilibrium price is presented by black dashed line. In the top panel, demand as a function of quantity is presented in light gray, and supply as a function of quantity is presented in black. Demand and supply functions were counted using the valuations of individual traders, and the equilibrium price was determined by the intersection point of demand and supply functions. In the middle panel, the "best" quotes in each round are presented; the best quotes are defined as the highest bid and the lowest ask in each round. Highest bids are reported by light gray triangles and the lowest asks are reported by black triangles. In the bottom panel, the number of bids is depicted by light gray line with triangles, while the number of asks is presented by black line with triangles. The number of both bids and asks are reported for each transaction that took place during the single run of the model.



taken place.

The statistics from 100 runs of both ZI-U and ZI-C markets also seem to support these findings from single markets. Table 2 also presents the overall results from 100 runs of both ZI-U and ZI-C markets. All of the runs reported in table 2 were executed using unique seeds for the random number generator, which should guarantee that the runs were not identical in the sense that the traders would have been participating in the

Figure 8: Transaction price time series, symmetric demand-supply schedules, best quotes and the amount of bids and ask in the limit order book for a single run of ZI-C market with 150 traders and 150 rounds. The traders were divided into buyers and sellers equally, and the valuations are specified in equation 3. In all of the three panels, the transaction price times series is presented by a dark gray solid line as a function of rounds, and the equilibrium price is presented by black dashed line. In the top panel, demand as a function of quantity is presented in light gray, and supply as a function of quantity is presented in black. Demand and supply functions were counted using the valuations of individual traders, and the equilibrium price was determined by the intersection point of demand and supply functions. In the middle panel, the "best" quotes in each round are presented; the best quotes are defined as the highest bid and the lowest ask in each round. Highest bids are reported by light gray triangles and the lowest asks are reported by black triangles. In the bottom panel, the number of bids is depicted by light gray line with triangles, while the number of asks is presented by black line with triangles. The number of both bids and asks are reported for each transaction that took place during the single run of the model.



market in exactly the same manner in all of the runs. The results are reported in the same way as in Cliff and Bruten (1997) by using averages and the standard deviations. The results from the 100 runs of the ZI-U market show that the number of transaction in all of the runs is constantly 75, because the mean is exactly 75 and the standard deviation is equal to zero. In the ZI-C markets, the average number of transactions was 41.5, and the standard deviation was 1.26. These results also suggest that the number of transactions taking place in the ZI-C markets, was in all of the runs just above 40 transactions.

Table 2: Descriptive statistics from 100 runs of the model for both ZI-U and ZI-C markets with symmetric demand-supply schedules. The traders were divided into buyers and sellers equally. All the runs were done using the same symmetric demand and supply schedule, which are depicted in both of the figures 7 and 8. The first two columns summarize statistics for two single runs of the models; the same runs are also depicted in the same figures 7 and 8. The results for the 100 runs are presented in the following four columns by using averages and standard deviations.

	SINGLE RUN		AVERAGES			
			ZI-U		ZI-C	
	ZI-U	ZI-C	Average	St.dev.	Average	St.dev.
Equilibrium price	100.0	100.0	100.0	0.00	100.0	0.0
Efficiency (%)	0.0	98.6	0.0	0.00	96.6	0.016
Number of transactions	75	41	75.0	0.00	41.43	1.29
Mean of prices	93.0	102.6	101.6	6.40	100.6	2.34
Median of prices	101.4	103.8	101.8	9.99	100.8	2.48
Maximum price	197.4	150.2	197.5	2.22	149.5	10.49
Minimum price	3.5	53.6	3.8	2.49	51.6	10.60
Standard deviation of prices	56.5	21.9	57.3	2.72	21.7	2.62
Kurtosis of prices	1.8	2.81	1.82	0.13	3.12	0.58
Skewness of prices	0.01	0.05	-0.02	0.15	-0.01	0.38
25 percentile	35.1	87.3	53.4	10.27	88.2	3.33
75 percentile	139.3	116.4	150.0	8.97	113.3	3.79
Coefficient of Convergence	56.6	21.8	57.3	2.64	21.6	2.56

The results presented until now suggests that in the experiments conducted the ZI-U traders were given an environment, where it was very likely that all of the traders would trade. Exactly the same results was reported by Gode and Sunder (1993a) as they pointed out that in their ZI-U markets

> "the maximum possible number of units (equal to the lower of the total units sellers are allowed to sell and the total units the buyers are allowed to buy) is always traded."

The results for the ZI-C markets seem to also be similar to those reported by Gode and Sunder (1993a): ZI-C traders trade only as long as their constrained behavior is possible and a feasible counterpart is found. These results also suggest that the chosen maximum number of rounds, i.e. 150, used is enough for both the ZI-U and the ZI-C traders to find the feasible trades and trade, because the results appear to be similar to the ones given by other authors.

5.1.2 Efficiency

An important and widely recognized result presented by Gode and Sunder (1993a) is the difference in the observed efficiencies of ZI-U and ZI-C market models especially when compared to the efficiencies of human markets. Gode and Sunder reported that ZI-C markets attained efficiencies close to the levels that the human traders achieved in their experiments, while in the markets of ZI-U traders the efficiency levels were clearly lower. Figure 9 shows that the efficiency of ZI-C markets was is in all of the 100 runs of the model consistently close to nearly 100 percent, while the efficiency of ZI-U market was constantly 0.0 percent. Table 2 shows quantitatively the same results: the average efficiency of ZI-U markets was 0.0 with a standard deviation of 0.0 while the average efficiency of ZI-C markets was 96.4 with a standard deviation of 0.016. These results together confirm the results of Gode and Sunder: the ZI-C markets seem to be clearly more efficient than ZI-U markets.

The fact that the efficiency of ZI-U markets is constant is consistent with the results of Gode and Sunder (1993a). They found out that the efficiency of the ZI-U markets depended only on the initial demand-supply schedules. The reason for the constant behavior is the fact that all possible trades always take place in ZI-U markets. Thus, both the trades that create surplus and the trades that do not create surplus take place, which means that the efficiency does not depend on how the traders are matched to trade. In addition, if the demand-supply schedules used are symmetric, then the efficiency of the ZI-U markets is also close to zero as will be next shown quantitatively.

Table 3: A simple example about the efficiency of ZI-U markets with symmetric demand-supply schedules. The table defines a ZI-trader market with six agents, who are equally dealt to buyers and sellers. Each agent is allowed to trade a single asset. In addition, the valuations of the traders are chosen so that the demand-supply schedule is symmetric. The valuations are defined as positive real numbers so that there is one buyer and one seller for each valuation $v_i \in \mathbb{R}_+, i = 1, 2, 3$.

Buyers		Sellers	
Name	Valuation	Name	Valuation
A	v_1	D	v_1
B	v_2	E	v_2
C	v_3	F	v_3

The efficiency of ZI-U markets with symmetric demand-supply schedules can be inspected more closely by using an example. First assume that the valuations of buyers and sellers are as given in table 3. In the ZI-trader markets, the surplus of a buyer can be defined as the difference between buyer b's valuation v_b and the transaction price p in the following way: $v_b - p$. Similarly the surplus extracted by the seller can be defined as the difference between the transaction price p and the seller s's valuation v_s in the following way: $p - v_b$. By using these two results, the surplus extracted from a single trade, where one buyer and seller are matched to trade, can be defined as the sum of the surpluses of the buyer and the seller participating in the trade

$$v_b - p + p - v_s = v_b - v_s. \tag{4}$$

As equation 4 shows, the surplus extracted from a single trade depends only on the valuations of the buyer and the seller participating in the trade. This suggests that actually the only important issues contributing to the efficiency of the ZI-trader market are the way how the traders are matched to trade and the way how the valuations of the traders are specified. The different ways to match the buyers and sellers given in table 3 are defined in table 4, which shows why the overall efficiency of a market with symmetric demand and supply schedules is zero when all the traders in the ZI-U market trade. Table 4 shows that the only issue contributing to the overall efficiency of the ZI-U market are the differences in the valuations of the traders participating in the market. In a completely symmetric case, as presented in tables 3 and 4, each of the valuations v_1, v_2, v_3 is used once on both sides, i.e. buyer and seller sides, in each of the matchings of traders, which means that the overall efficiency is in all cases equal to zero. Thus, all the different ways to match the traders participating in the market lead to the same overall efficiency result, which depends solely on the demand-supply schedules.

In addition, it also seems that, if there are $n \in \mathbb{N}$ buyers with valuations $v_b^1, v_b^2, ..., v_b^n$ and if there are n sellers with valuations $v_s^1, v_s^2, ..., v_s^n$, then the efficiency of the ZI-U markets with all n trades taking place can be defined as the difference between the sum of valuations for buyers and the sum of valuations for sellers as

$$\sum_{i=1}^{n} v_b^i - \sum_{i=1}^{n} v_s^i \tag{5}$$

The claim seems very intuitive, because when all the traders trade once, then also all of the buyer and seller valuations "contribute" once to the overall surplus measure extracted from the market. This simple example has shown that the way to match the traders does not contribute in any way to the efficiency in ZI-U trader markets, if we assume that all traders trade in ZI-U markets. Thus, the only factor contributing to the efficiency of ZI-U markets is the initial market type defined by demand and supply schedules.

A statistical test was also made to measure the credibility of the results presented in figure 9 and table 2. The directional version of the Wilcoxon signed-rank test[30] was made

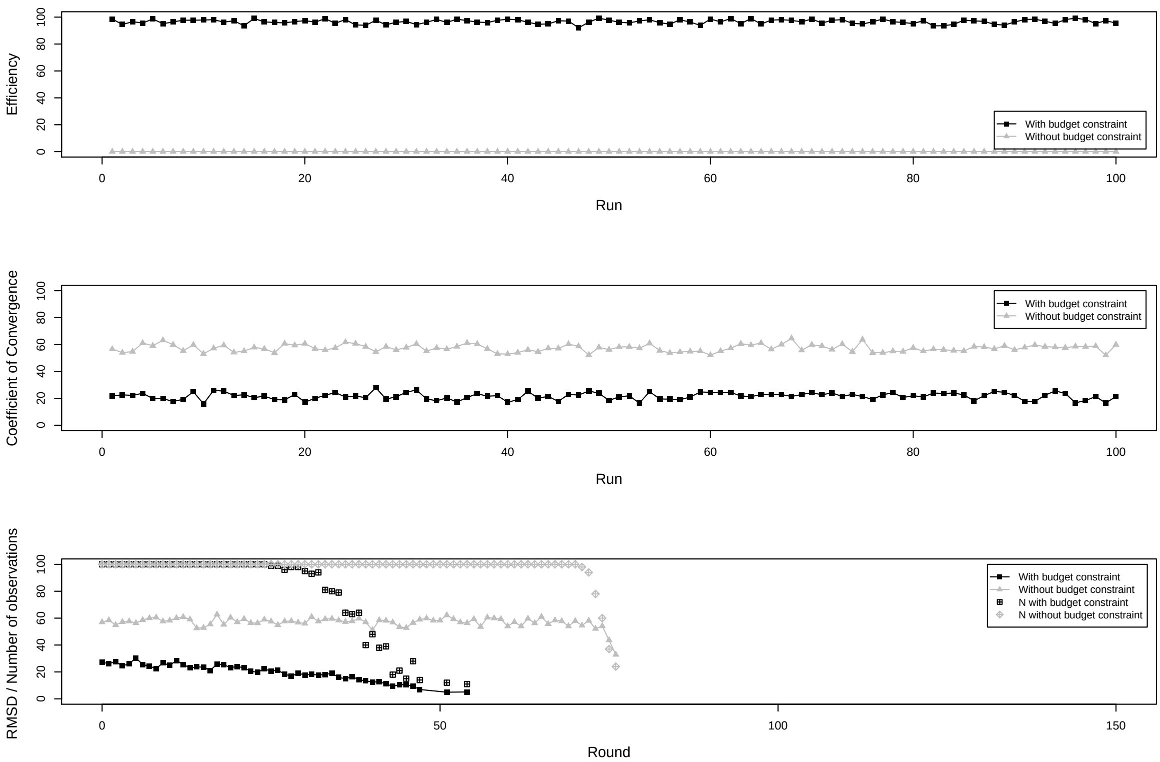
[30] See for example the original paper by Wilcoxon (1945).

Table 4: The different ways to match the traders participating in the ZI-trader market given in table 3. The extracted surplus of each trade for all possible six ways to match the traders into three pairs are reported. In addition, the overall surplus extracted from the market when all of trades take place is also reported as the Total-measure for all of the six matchings. As each of the valuations v_1, v_2, v_3 is used twice the tree trades that take place, the overall efficiency is in all cases equal to zero. The valuations $v_i \in \mathbb{R}_+, i = 1, 2, 3$.

Pair		Extracted surplus
Buyer	Seller	
A	D	$v_1 - v_1$
B	E	$v_2 - v_2$
C	F	$v_3 - v_3$
Total		0
A	D	$v_1 - v_1$
C	E	$v_3 - v_2$
B	F	$v_2 - v_3$
Total		0
A	E	$v_1 - v_2$
B	D	$v_2 - v_1$
C	F	$v_3 - v_3$
Total		0
A	E	$v_1 - v_2$
B	F	$v_2 - v_3$
C	D	$v_3 - v_1$
Total		0
A	F	$v_1 - v_3$
B	D	$v_2 - v_1$
C	E	$v_3 - v_2$
Total		0
A	F	$v_1 - v_3$
B	E	$v_2 - v_2$
C	D	$v_3 - v_1$
Total		0

with the null hypothesis that the location shift should be smaller or equal to zero when efficiencies from ZI-C markets and ZI-U markets are compared in this order, because figure 9 suggests that the location shift should be positive. This test was not used in this context by Gode and Sunder (1993a) as their work generally lacked statistical significance tests. However, LiCalzi and Pellizzari (2008) use the Wilcoxon signed-rank test in a similar context to strengthen the credibility of their results. To be able to make the test, the observations for the efficiency had to be paired. The pairing was done by using unique

Figure 9: Efficiency, coefficient of convergence and root mean squared deviation of transaction prices from the equilibrium price for 100 runs of ZI-C and ZI-U markets with 150 traders and 150 rounds. The traders were divided into buyers and sellers equally. All the runs were done using the same symmetric demand and supply schedule, which are depicted in both of figures 7 and 8, while the statistics for the runs are presented in table 2. In all of the three graphs, the results for ZI-U markets are presented in light gray line and triangles, while the results for the ZI-C markets are presented in black line and squares. The efficiency of the CDA markets is presented in the top panel. It is determined as the ratio of the total profit the traders actually earned in the market and the total profit the traders could have earned in the market. In the bottom panel, the root mean square deviation of transaction prices from the equilibrium price (RMSD) in ZI-U markets as a function of rounds is presented in light-gray, while the RMSD in ZI-C markets as a function of rounds is presented in black. Both measures were counted for each round using the data of transaction prices of either ZI-C markets of ZI-U markets. In addition to RMSDs, the bottom panel also depicts the number of observations, i.e. the number of transaction prices from 100 simulations, on each round. The plot does not show RMSDs in rounds, which had fewer than 10 observations. The number of observations from the ZI-U markets are depicted in light gray diamonds and the number of observations from ZI-C markets are depicted in black squares. In the middle panel, the coefficient of convergence is presented for each run of the model. The coefficient of convergence was initially presented by Smith (1962) and is defined as the ratio of RMSD and equilibrium price.



seeds so that each unique seed was used always twice: once on a ZI-U market model and once on a ZI-C market model. This way the only difference between the two efficiency observations is in the abilities of the traders. The test yielded a p-value of 1.97e-18, which suggests that the null hypothesis is rejected with a very high probability and the location shift from ZI-C markets to ZI-U markets is larger than zero. This means that the test confirms the result that the difference between the efficiencies of ZI-C and ZI-U markets is greater than zero with a large probability. Thus, the result that the efficiency of ZI-C markets is greater than that of ZI-U markets when the demand-supply schedule is symmetric is also confirmed using the Wilcoxon signed-rank test.

5.1.3 Transaction price time series characteristics

The two figures 7 and 8 also seem to comply with the two of the three features that Gode and Sunder (1993a) reported from their experiments with the ZI-market and human market for the transaction price time series. One feature, the lack of memory for ZI-traders, is deliberately this time left unnoticed, because, by definition, the zero-intelligence traders do not remember anything from the past periods. Thus, it is unimportant to run the market for several periods instead of independent reruns to investigate this issue. In the context of Gode and Sunder (1993a), the reruns were justified, because the reruns were used to compare the results from the ZI-markets to the results from the human markets.

The first important transaction price time series characteristics is that the standard deviation of the transaction prices seems to be greater in the ZI-C market when compared to the ZI-U markets. This can be easily seen from the figures by eye-balling, but it can be also confirmed by looking at table 2. The table clearly shows that the standard deviation of the prices in ZI-U markets, 57.3, is significantly greater than the same measure for ZI-C markets, 21.7. The results also suggest that the standard deviations of the two markets seem to vary in similar proportions as measured by the standard deviation of the standard deviations. Thus, these results seem to comply with the results of Gode and Sunder (1993a), who found that the ZI-C market transaction price time series was more volatile than its human market counterpart, while the ZI-U market transaction price time series was clearly the most volatile of the three.

There is also a second important feature of the transaction price time series that seems to be present in the top panels of figures 7 and 8. It seems that as time progresses in the ZI-C market, the transaction prices tend to converge towards the theoretical equilibrium price. However, the ZI-U market does not seem to show similar convergence towards the theoretical equilibrium price, but instead the transaction prices oscillate wildly around the equilibrium price inside the range of all possible transaction prices throughout the rounds that the market is active. A glance to the same convergence seems to also be present in the middle panel of figure 8 as the highest bids and the lowest asks presented seem to converge close to the equilibrium value in the ZI-C trader market. Again, a similar graph in the middle panel of the figure 7 shows that it is hard to see any such convergence in ZI-U markets.

The convergence can also be evaluated by using the root mean squared deviation (RMSD) of transaction prices from the equilibrium as the previous authors have done. Both Gode and Sunder (1993a) and Cliff and Bruten (1997) use the RMSD of transaction prices from the equilibrium price to evaluate the convergence to equilibrium inside the trading period. Essentially, the idea is to measure how the transaction prices evolve around the equilibrium price as time progresses. The bottom panel of figure 9 shows the RMSD of transaction prices from the equilibrium price in the ZI-C and the ZI-U markets for 100 runs of both markets. These results clearly indicate that the RMSD of transaction prices seems to be lower for the ZI-C markets than for the ZI-U markets throughout the time the market is open. The bottom panel of figure 9 also appears to show that the RMSD decreases in ZI-C markets as the rounds increase, while in ZI-U markets no such development can be found before the few last rounds that the market is active. Such a result would also be in line with the claims of Gode and Sunder (1993a), as by using the convergence of RMSD as their principal argument, Gode and Sunder reported that the ZI-C market converges towards the equilibrium within each trading period.

However, one should not draw hasty conclusions from the results now presented. Actually, only the result about the ZI-U markets not converging towards the equilibrium price seems certain, while the convergence in ZI-C markets can be partly questioned, because of the decreasing number of observations as shown by the light gray diamonds and black boxes for ZI-U and ZI-C markets correspondingly in the bottom panel of figure 9. As Gode and Sunder (1993a) do not report the number of observations for each round, it is impossible to say whether their results would have endured similar challenge. From the bottom panel of figure 9 it seems actually that the RMSD of ZI-C markets starts to decrease towards the equilibrium exactly at the same time as the number of observations starts to decrease. The decreasing number of observations could be partly explained by the fact that the number of observations start to decrease at the same time as the number of quotes needed for a trade start to increase. This seems a plausible explanation, because it seems that in ZI-C markets most of the trades take place during the first 40 rounds, and from then on it seems less likely to see a trade taking place.[31] However, still the slight decrease of RMSD is heavily questioned by the decreasing number of observations. In general, it seems that the convergence issue is more or less unsolvable using the methods presented until now by Gode and Sunder (1993a) and Cliff and Bruten (1997).

There is still one measure left that was utilized by Gode and Sunder (1993a) and Cliff and Bruten (1997) to evaluate the convergence to the theoretical equilibrium. Smith

[31] This is only a plausible explanation that is based on the fact that the results presented in figure 8 are a characterizing example of the results for all of the 100 runs that are reported in table 2.

(1962) defined the coefficient of convergence[32] as a measure that shows how close to the theoretical equilibrium price the transaction prices were on average during the experiment. Especially, Smith used a single coefficient of convergence to measure all of the transaction prices in a single experiment. Thus, originally coefficient of convergence was used to compare different experiments with each other. The middle panel of figure 9 depicts the coefficients of convergence for all of the 100 runs of both ZI-U market and ZI-C market. These results seem to clearly support the fact that the ZI-C markets trade at prices closer to the theoretical equilibrium price than the ZI-U markets.

The difference in closeness of transaction prices to the equilibrium price in ZI-C and ZI-U markets is also statistically significant. The significance was confirmed using a directional version of the Wilcoxon signed-rank test. To be able to make the test, the observations of the coefficients of convergence were paired in a similar manner as mentioned earlier with efficiencies. The pairing was done by using unique seeds in simulations so that each seed was used always twice: once on a ZI-U model and once on a ZI-C model. This way the only difference between the two markets that provided the efficiency observations shown in figure 9 was in the traders abilities. The null hypothesis used was that the difference between coefficients of convergence when compared from ZI-C to ZI-U is greater or equal to zero, because figure 9 suggests that the difference should be negative. The test yielded a p-value of 1.98e-18, which suggest with a very high probability that the null hypothesis is wrong and that the real difference in negative. This is also exactly what figure 9 and table 2 also suggest qualitatively as the coefficients of convergence seem to be clearly smaller in ZI-C markets when compared to the ZI-U markets. Thus, the results presented now suggest that the ZI-C traders make transaction closer to the equilibrium price than the ZI-U traders. However, it is quite hard to statistically say anything proper about actual convergence towards the equilibrium.

5.1.4 A more detailed analysis of the convergence to equilibrium

It seems clear that the convergence of transaction prices towards the equilibrium price should be analyzed further. The methods presented until now do not really look at the convergence during the trading period, but instead compare the closeness of transaction prices from the equilibrium price. The analysis now presented is based on the idea that Gode and Sunder (1993a) explained the convergence to the equilibrium in ZI-C markets by the progressive narrowing of the opportunity sets of ZI-C traders. However, Gode and Sunder did not give evidence about the progressive narrowing in their article, but instead were satisfied to describe it only. In the following, I will try to argue how the progressive narrowing of the opportunity sets of ZI-C traders can be quantitatively shown to be

[32] Refer to the literature review for the exact definition.

main driving force of transaction prices in ZI-C markets. This presentation is supposed to clarify and extend the ideas of Brewer et al. (2002), who heuristically explained the idea of the progressively narrowing opportunity sets of ZI-C traders and compared the evolution of the ZI-C market to the Marshallian path.

In essence, the idea is to explain why the intramarginal traders are the most probable traders to trade during the beginning of the continuous double auction. The middle panel of figure 8 seems to support the progressive narrowing argument: both the highest bids and the lowest asks seem to tend towards the equilibrium price as time progresses. This behavior was repeated in all of the 100 simulations of the ZI-C market, although only the characterizing example is presented in figure 8. Another view at this same issue can be taken by looking at the number of quotes in the limit order book before a transaction takes place. A graphical presentation of the number of quotes in the limit order book is given in the bottom panel of figure 7 for ZI-U markets and in the bottom panel of figure 8 for ZI-C markets. Figure 7 for ZI-U markets suggests that the number of bids and asks stays relatively low during the whole time that the market is active, while figure 8 for ZI-C markets shows that as time progresses also the number of bids and asks needed for a transaction is increases. This example shows well the idea of the progressive narrowing of the opportunity sets of ZI-C traders: as time progresses, the number of traders in the market decreases, which means that also the number of different valuations in the trader population decreases. Especially, the number of traders with intramarginal valuations has to decrease, because bids and asks tend towards the equilibrium price. In effect, this means that the range of different valuations for both buyers and sellers becomes narrower as the rounds increase. In ZI-C market this affects the trading, because traders cannot trade with a price that does not satisfy their valuation: a ZI-C seller cannot sell at a price that is lower than her valuation and a ZI-C buyer cannot buy at a price that is higher than her valuation.

There are two empirical arguments presented in figure 8 that support the idea that the traders with intramarginal valuations trade with a high probability during the beginning rounds of the ZI-C market. First, in the middle panel of figure 8, the best quotes are lined so that after 50 rounds the highest bids are just below the equilibrium price and the lowest asks are just above it. Now, because there are no transactions taking place after the 50 rounds with a price far from the equilibrium price and the best quotes are next to the equilibrium price, it has to be that there are no traders left with intramarginal valuations. This claim is based on the counter example: because we do not see any trades with transaction prices far from the equilibrium value, there are no traders with intramarginal valuations left to trade. If there was a trader left with an extreme valuation, then she would trade with a large probability at a price that is far from the equilibrium value.

Thus, it has to be that traders with extreme valuations have already left the market, because in the beginning they were in the market by the design of the experiment and each trader participating in the market is given on every round a possibility to trade if no trade occurs during that particular round. However, this is only an empirical result from the simulations now presented and corresponds only to the figure 8 presented.

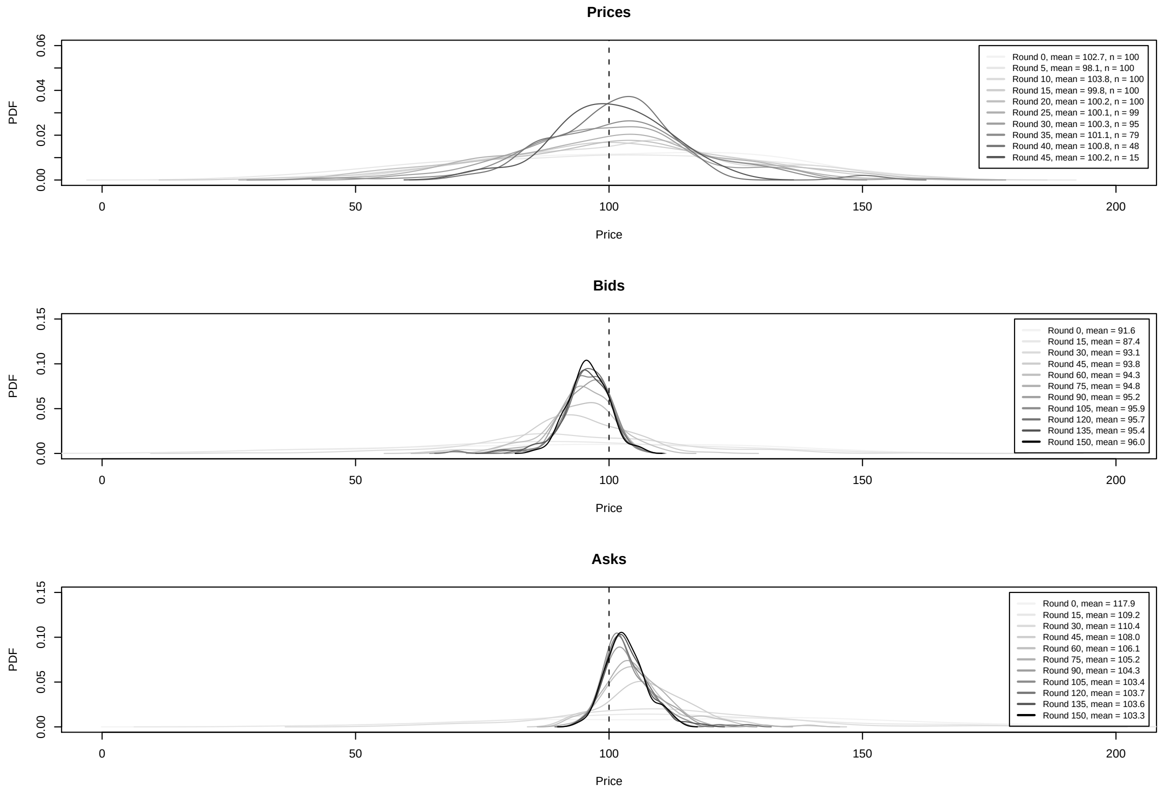
Also the second argument is based on the characteristics of ZI-C traders. The bottom panel of figure 8 shows that as the rounds increase, the number of quotes in the limit order book also increases. This should be thought of with the ZI-C traders logic in mind: the ZI-C buyers place bids uniformly on the range from minimum price p_{min} to trader specific valuation v and the ZI-C sellers place asks uniformly on the range from the trader specific valuation v to maximum price p_{max}. Thus, if the traders need a lot of bids and asks to trade, it has to mean that there is a very low probability to see a bid that is higher than the lowest ask in the limit order book and that there is a very low probability to see an ask that is lower than the highest bid in the limit order book. If the probabilities just mentioned are very low, it has to be that the maximum of all buyer's valuations b_{max} has to be very close to the minimum of seller valuations a_{min} in the population of traders.

The closeness can be also defined more rigorously. Before a trade[33], the closeness can be defined for buyers by looking at the difference between the $b_{max} - a_{min}$ and by determining how large a probability there is, for example, to see a bid that is situated in the interval from a_{min} to b_{max} when drawing uniformly a buyer from the population of buyers participating in the market. A similar measure could be determined for sellers by looking at the probability of seeing an ask in the same interval when drawing a seller uniformly from the population of sellers participating in the market. Thus, because the number of quotes needed to trade increase with every round and the transaction prices are close to the equilibrium price, it seems that the traders with the intramarginal valuations actually do trade in the beginning of the continuous double auction, while the traders with the valuations close to the equilibrium value trade in the end of the continuous double auction.

The first of the two claims presented above can also be looked at more extensively by using the computational methods. By simulating a ZI-C market model for multiple times, one can create a large number of observations about the best bids and asks at each round of the CDA. The results from such an experiment are summarized in figure 10 by plotting the estimated largest bid and lowest ask densities at different rounds in the two

[33] If a trade takes place, then we know in a ZI-C market that before the trade took place it had to be that the maximum of all buyer's valuations was greater than then minimum of all sellers valuations or otherwise no trade could have taken place.

Figure 10: Estimated transaction price PDFs, best bid PDFs and best ask PFDs at eleven different rounds for a ZI-C market with 150 traders and 150 rounds. The ZI-C market was run 100 times. The demand and supply schedules used are depicted in figure 8, while the statistics are presented in table 2. For the transaction price densities, also the number of observations, i.e. the number of transaction prices, accompanied with the their mean are also described for each estimated density in the legend. For the best bid and ask densities, the number of observations was 100 in each round, so it was omitted from the graphs, but the means are presented in the legend. The light gray lines in the picture correspond to densities estimated from the beginning rounds, while the darker lines corresponds to densities from ending rounds.



bottom panels[34].

[34] The PDFs presented henceforth in this thesis were estimated using the standard library density-routine of the statistical package R. The advantage of using density functions instead of histograms comes from the number of needed parameters. For a histogram, one has to select the number of subintervals dealing the data, the size of the intervals and the locations of the intervals (Tarter and Kronmal, 1976). As a result, one obtains a discontinuous description of the data. In contrast, when using a non-parametric kernel density estimate, one needs to only select the size of the intervals, while the endpoints are not needed. The endpoints can be forgotten, because the kernel function is situated at each observation instead of grouping observations (Tarter and Kronmal, 1976). As a result one obtains a continuous density estimate of the data. In kernel density estimation, the size of the intervals corresponds to the standard deviation of the kernel density and is referred as bandwidth. When estimating density functions the essential parameters that have to be chosen are the bandwidth and the smoothing kernel. To avoid making biased judgments, I used the default values of the R-function. This means that the smoothing kernel was normal and the bandwidth was selected using the Silverman's rule of thumb. Refer to the documentation of R for more elaborate descriptions: `http://cran.r-project.org/`. The chosen parameters should be sufficient for making comparisons on the level presented in this thesis although especially the bandwidth could be selected using more advanced methods (Jones et al., 1996).

As the figure shows, the lighter densities, i.e. the densities estimated from the quotes from the beginning rounds, are clearly flatter and more spread out on the interval from 1 to 200, while the darker densities, i.e. the densities estimated from the quotes from the ending rounds, are more peaked close to the value 100, which the equilibrium price for the demand-supply schedules used in the experiments. Thus, as the rounds increase, it seems that the bid and asks densities tend to become more peaked and their means tend towards the equilibrium price. Thus, it appears that the number of intramarginal bids and asks decreases as the rounds increase in all of the 100 runs of the ZI-C market exactly the same way as was argued above for best bids and asks given in figure 8 for a single ZI-C market.

This is a quantification of the argument of Gode and Sunder (1993a), who claimed that the opportunity sets of ZI-C traders progressively narrows as the rounds increase. This result also seems to suggest that the arguments presented above about the traders with intramarginal valuations have been satisfied in the 100 simulations for which the results are presented in figure 10. However, it is important to note that the results presented now only cover the symmetric demand-supply schedule presented in figure 8. A more extensive inquiry would have to be taken to really find out which market types also support empirically these claims.

It is also worth noting that the transaction price densities presented in the top panel of figure 10 seem to suggest a tend towards the equilibrium price: the transaction price density becomes more peaked close to the equilibrium price as the rounds increase. This essentially means that the two properties defined by Brewer et al. (2002) for the convergence are satisfied: initial transaction prices are further from the equilibrium prices than final prices, because the variance of transaction prices decreases as indicated by the more peaked transaction price densities. However, the problem with drawing conclusions using the densities for transaction prices is that their number in each round decreases as the rounds increase beyond a certain limit. Best quotes do not have the problem with the number of observations, because in each round the bids and asks are delivered at least by a single seller or a single buyer.

As a conclusion for the first model, it appears that instead of looking at the transaction price density of all trades as suggested by Cliff and Bruten (1997), it seems to be more interesting to look at how the transaction price density evolves round by round during the continuous double auction experiment. It appears that as the group of traders participating in the market changes, also the transaction price density evolves dramatically. This result is intuitive and was also suggested already by Gode and Sunder (1993a) when they described the progressive narrowing of traders opportunity sets. The following section will evaluate and use the methods first proposed by Cliff and Bruten (1997) to draw more precise conclusions about the convergence of transaction prices towards the equilibrium price and especially about the reasons why the ZI-C markets seem to exhibit such convergence.

5.2 The progress of the CDA affects bids, asks and transaction prices

Cliff and Bruten (1997) claim that the PDF of transaction prices is given by the intersection of PDFs of bids and asks on the market level. The background of this issue was analyzed in the literature review, and the critique of Othman (2008) was also presented. However, there appears to be a simpler way to assess the plausibility of the results by Cliff and Bruten than the one presented by Othman (2008). Now proposed method is based on the idea that the theoretical PDFs of transaction prices, bids and asks as defined by Cliff and Bruten (1997) can be compared to the empirical ones from the simulations by plotting the different PDFs. Thus, this section will first look at how the theoretical PDFs of bids and asks can be characterized properly. The results will then used to evaluate the evolution of bids, asks and transaction prices in the SCDA as time progresses.

5.2.1 Theoretical PDFs of bids and asks

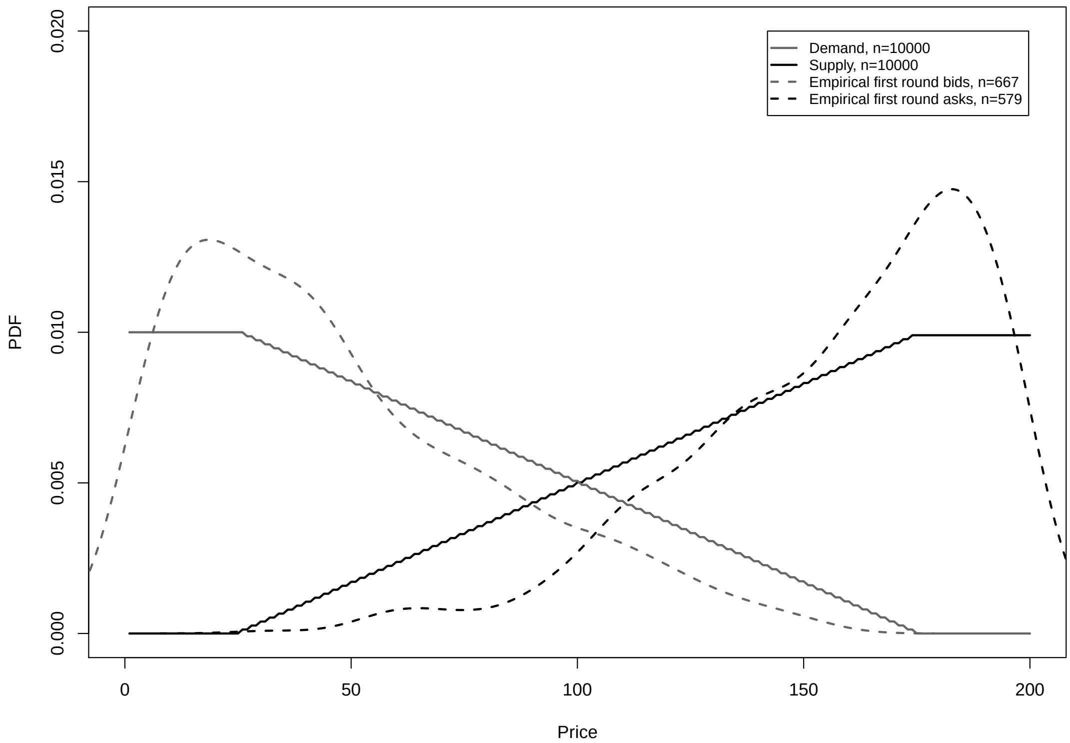
According to Cliff and Bruten (1997), the empirical PDFs of bids and asks should be the same as the theoretical PDFs of bids and asks. An example comparing the theoretical bids and ask to the empirical ones is presented in figure 11. It plots the theoretical PDFs of bids and asks as suggested by Cliff and Bruten (1997) against the empirical PDFs of bids and asks obtained in the first round from 100 runs of the ZI-C market. However, as figure 11 suggests, it seems that the qualitative description of the method of creating market wide densities for bids and asks by Cliff and Bruten (1997) does not create the correct PDFs of bids and asks submitted by the traders in the simulations.

Although Cliff and Bruten (1997) do not provide an exact description of how they create the theoretical PDFs, I assume that as their PDFs have constant slopes for a market with symmetric demand and supply curves, the theoretical pdfs presented in figure 11 correspond to their description. This assumption is strengthened by the fact that Cliff and Bruten (1997) use an assumption that the intersection of the probability density functions is shaped as a triangle in their analytic calculations. Using such an assumption requires the market wide densities for bids and asks to have constant slopes in the price range from minimum valuation (26) to maximum valuation (174)[35], so it

[35] Minimum valuation is 26 and maximum valuation is 174 for the market given in figure 11, because the valuations are from the arithmetic sequence of integers $26, 28, ..., 172, 174$.

seems quite appropriate to claim that the PDFs depicted in figure 11 are exactly the ones proposed by Cliff and Bruten (1997).

Figure 11: The theoretical PDFs of bids and asks as suggested by Cliff and Bruten (1997) and the empirical PDFs of bids and asks submitted by the traders in the first round in 100 runs of ZI-C market. The demand and supply schedules used are depicted in figure 8, while the statistics for the simulations are presented in table 2. The theoretical PDFs of bids and asks were created using the demand and supply functions, and depict the ideas presented by Cliff and Bruten (1997) as the PDFs have constant slopes. The theoretical PDF of bids is depicted in light gray and the theoretical PDF of asks is depicted in black. In addition, the black dashed line depicts the empirical pdf for asks during the first rounds of the 100 runs, and the light gray dashed line depicts the empirical pdf for all bids during first rounds of the 100 runs.



The characterizing idea in creating the theoretical PDFs of Cliff and Bruten (1997) is to count the number of traders ready to quote at a certain valuation, and weight all valuations equally[36]. The R code for creating the theoretical PDFs using these assumptions is included in Appendix B. The difference with empirical results can be qualitatively seen from figure 11, because the empirical density functions for bids and asks seem to have an exponential nature in contrast to the theoretical ones presented by Cliff and Bruten (1997). In general, figure 11 suggest that the theoretical PDFs of bids and asks by Cliff

[36] According to Cliff and Bruten (1997), this idea is derived from the fact how the theoretical demand and supply are derived in the double auction market.

and Bruten (1997) suggest too much probability mass for bids and asks in the price range from minimum valuation (26) to maximum valuation (174)[37], and too little probability mass for bids in the price range from minimum price (1) to minimum valuation (26) and asks in the price range from maximum valuation (174) to maximum price (200). Thus, the problem is that the theoretical PDFs by Cliff and Bruten (1997) are weighting the different prices differently than what simulations seem to indicate.

It appears that a different method has to be used to characterize the real density functions for bids and asks. Theoretically speaking, in the single-unit continuous double auction, the market wide PDFs of bids and asks have to take into account precisely the way how the traders are chosen to trade during each round. Thus, to create the theoretical market wide PDFs of bids and asks, one has to take into account that each single trader is selected with an equal probability to submit a quote from the group of traders, who are still participating in the market and have not submitted a quote during the ongoing round. This means that in the first round the probability to get a particular buyer or seller is 1/75, and this probability is the one that is used when weighting the probability density functions of individual traders to create the market wide probability density functions for bids and asks. Thus, in a market with valuations described by equation 3, the market wide density functions for asks have actually increasing slopes, because each trader is selected with an equal probability, but the trader's valuations are restricted to the price range from 26 to 174.

The increase in slope is derived from the fact that each seller draws her ask from a uniform distribution on interval her valuation v to maximum valuation (200). Thus, when the valuation is increased, the probability mass of the uniform distribution is divided equally on a shorter interval as the upper end is fixed. Take, for example, a seller, with a valuation 100: her probability to get selected from a population of n sellers is $1/n = 1/75$, while her probability to ask a particular value in the range from 100 to 200 is 1/100. Thus, this trader contributes to the probability to see an ask in the range from 100 to 200 $1/75 \times 1/100$. Similarly, a seller with a valuation v equal to 150, contributes to the probability to see an ask in the range from 150 to 200 $1/75 \times 1/50$. The increase in the slope is derived from the fact that the contribution of the seller with valuation 150 to the probability to see an ask in the range from 150 to 200, i.e. $1/75 \times 1/50$, is larger than the contribution of the seller with valuation 100, i.e. $1/75 \times 1/100$.

I will now generally define the PDF $f_S(p) : \mathbb{R} \to [0,1] \subset \mathbb{R}$ for asks at each price $p \in [1,200] \subset \mathbb{R}$. First, define the population of sellers to contain N_S agents. Second, define the set of valuations for sellers as set $S \subset (1,200)$, and assume that all valuations s_i in S are indexed uniquely by $i = 1, 2, ..., N_S$. Third, assume that a single seller is selected

[37] These numbers are derived from the fact that the valuations in the symmetric market in figure 11 are from the arithmetic sequence of integers $26, 28, ..., 172, 174$.

from the population of sellers with equal probability $1/N_S$, which is the case in the model of Gode and Sunder (1993a). To calculate the PDF $f_S(p)$ for asks, the next step is to sum over all the possibilities that a single seller is selected to trade at a certain price p. Now, because all the sellers are ZI-C agents, the probability that they ask a certain price equals the value of their PDF of uniform distribution in the range from s_i to 200. The next step is to multiply the latter with the probability to select a single seller from the group of sellers $1/N_S$, because all the sellers have equal probability to get chosen. Finally, one has to sum the probabilities so that all the valuations that are smaller or equal to price p are taken into account to the probability to see an asks at price p:

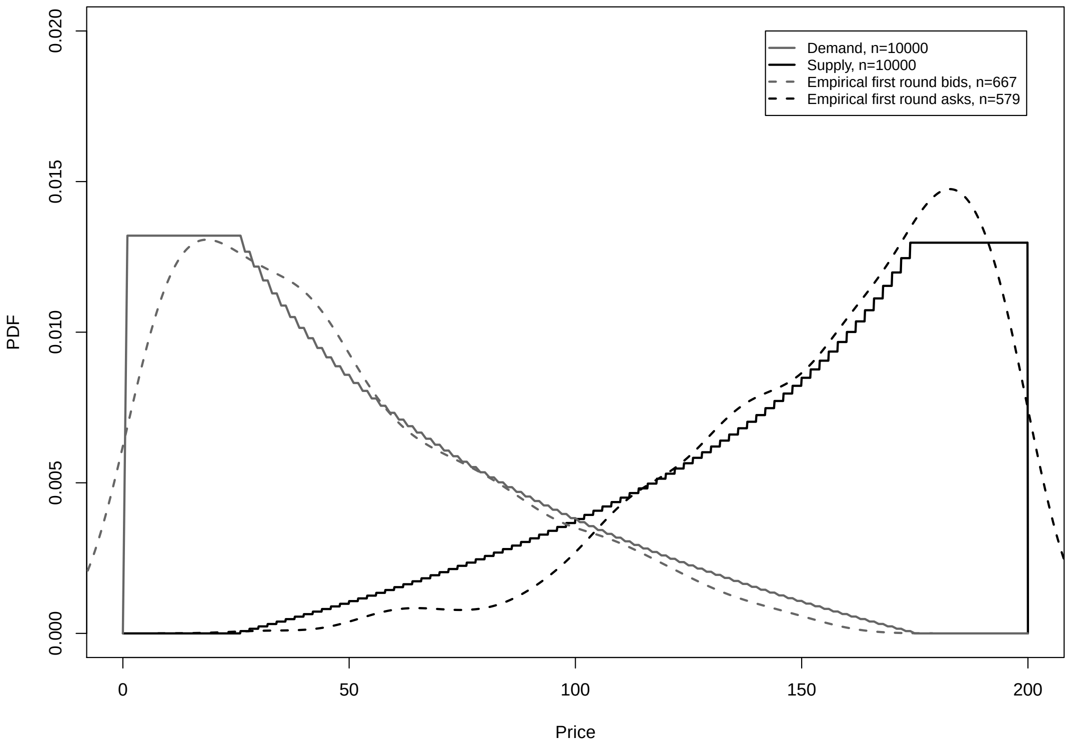
$$f_S(p) = \sum_{s_i \in S, p \geq s_i} \frac{1}{N_S(200 - s_i)}. \tag{6}$$

With bids, the idea is otherwise similar to the case with asks, but this time one has to sum the other way around. First, define the PDF $f_B(p) : \mathbb{R} \to [0,1] \subset \mathbb{R}$ for bids at each price $p \in [1,200] \subset \mathbb{R}$. Second, define that the population of buyers contains N_B agents. Third, define the set of valuations for buyers as set $B \subset (1,200)$, and assume that all valuations b_i in B are indexed uniquely by $i = 1, 2, ..., N_B$. Third, assume that a single buyer is selected from the population of buyers with equal probability $1/N_B$. To calculate the PDF $f_B(p)$ for bids, the next step is to sum over all the possibilities that a single buyer is selected to trade at a certain price p. Now, because all the buyers are ZI-C agents, the probability that they asks a certain price equals to the value of their PDF of uniform distribution in the range from 1 to b_i. The next step is to multiply the latter with the probability to select a single buyer from the group of buyers $1/N_B$, because all the buyers have equal probability to get chosen. Last, one has to sum the probabilities so that all the valuations that are larger or equal to price p are taken into account to the probability to see a bid at price p:

$$f_B(p) = \sum_{b_i \in B, p \leq b_i} \frac{1}{N_B(b_i - 1)}. \tag{7}$$

The functions presented in equations 6 and 7 corresponds to PDFs, because their integrals over the interval from 1 to 200 equal to one. The intuitive reason for this property, for example, with sellers is that the PDF of each seller integrate to unity by definition. Thus, by weighting all of them equally and summing will produce a function that also integrates to unity. The calculations to create the PDF for bids, which are similarly decreasing in price, are similar. The PDFs presented in figure 12 for the market wide asks were created in this manner by summing the probabilities for each possible valuation. The R code used to do these calculations and to draw the figures is included in the Appendix.

Figure 12: Suggestions as the theoretical PDFs of bids and asks and the empirical PDFs of bids and asks submitted by the traders in the first round in 100 runs of ZI-C market. The demand and supply schedules used are depicted in figure 8, while the statistics for the simulations are presented in table 2. The theoretical PDFs of bids and asks were created using the demand and supply functions, and the idea was to weight the PDFs of individual traders equally. The theoretical PDF of bids is depicted in light gray and the theoretical PDF of asks is depicted in black. In addition, the black dashed line depicts the empirical pdf for asks during the first rounds of the 100 runs, and the light gray dashed line depicts the empirical pdf for all bids during first rounds of the 100 runs.

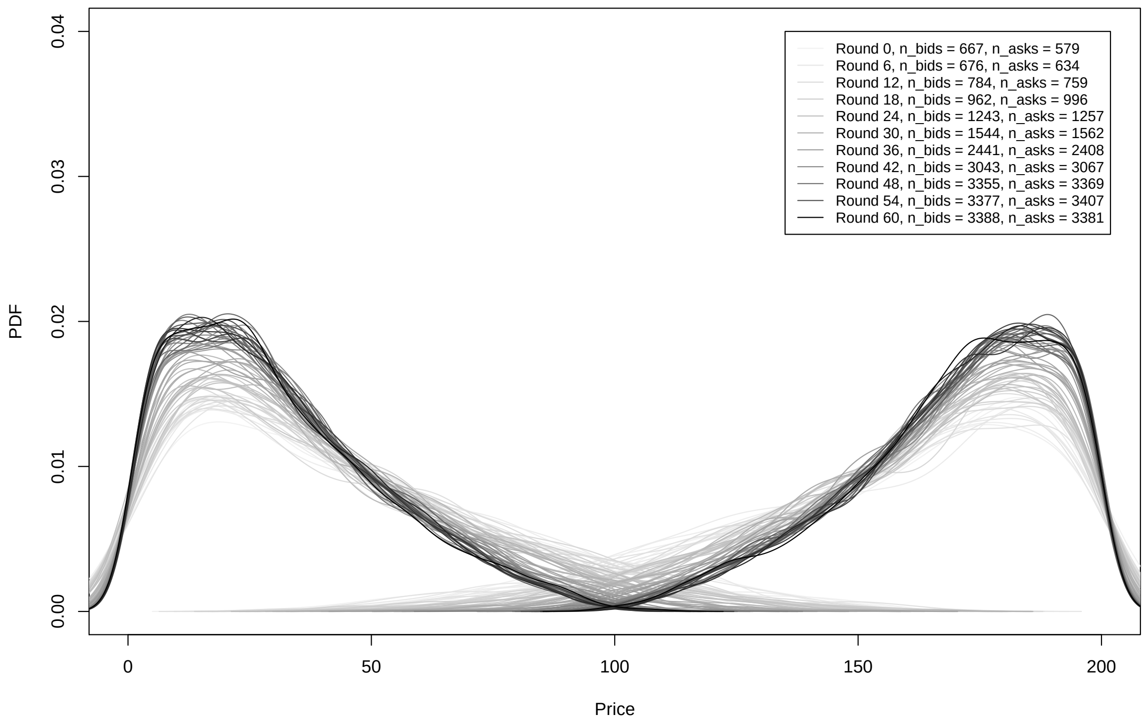


The suggested theoretical PDFs of bids and asks are depicted in the figure 12. It shows that these theoretical PDFs for bids and asks seem to be in line with the empirical PDFs estimated from the simulation results, because the modes and slopes of the PDFs seem to be similar when eye-balling the picture. Thus, it appears that the proposed theoretical PDFs fit the simulated data better than the theoretical PDFs of Cliff and Bruten (1997). Next step is to look how the changes in the trader population participating in the market change the theoretical and empirical PDFs of bids and asks.

5.2.2 How the progress of the SCDA affects bids and asks

Cliff and Bruten (1997) claim that the shifting of the PDFs of bids and asks as a result from trades taking place and traders leaving the market can be ignored. According to them, because their empirical results support their theoretical arguments, the shifting can be ignored. However, the empirical results of Cliff and Bruten (1997) consider only the expected transaction price. Considering only expected transaction prices seems inadequate, because Othman (2008) showed that by choosing the valuations of the traders in a certain way, the results of Cliff and Bruten (1997) can be shown to be false at least in one situation. Thus, it seems that actually the assumptions of Cliff and Bruten (1997) about the market types fit the predictions of their method. I will now first look at how the PDFs of bids and asks evolve in symmetric ZI-C markets, and use the results from this subsection when looking at the PDFs of transaction prices in the next subsection.

Figure 13: PDFs from 100 runs of the ZI-C market for bids and asks for all rounds from 0 to 60. The demand and supply schedules used in the simulations are depicted in figure 8, while the statistics are presented in table 2. The lines presented on the left side of the figure correspond to the PDFs of bids, while the lines presented on the right side of the figure correspond to the PDFs of asks. The light gray lines depict the PDFs for both bids and asks from the beginning rounds of the simulations, while as the color of the line changes to darker, the number of round increases.

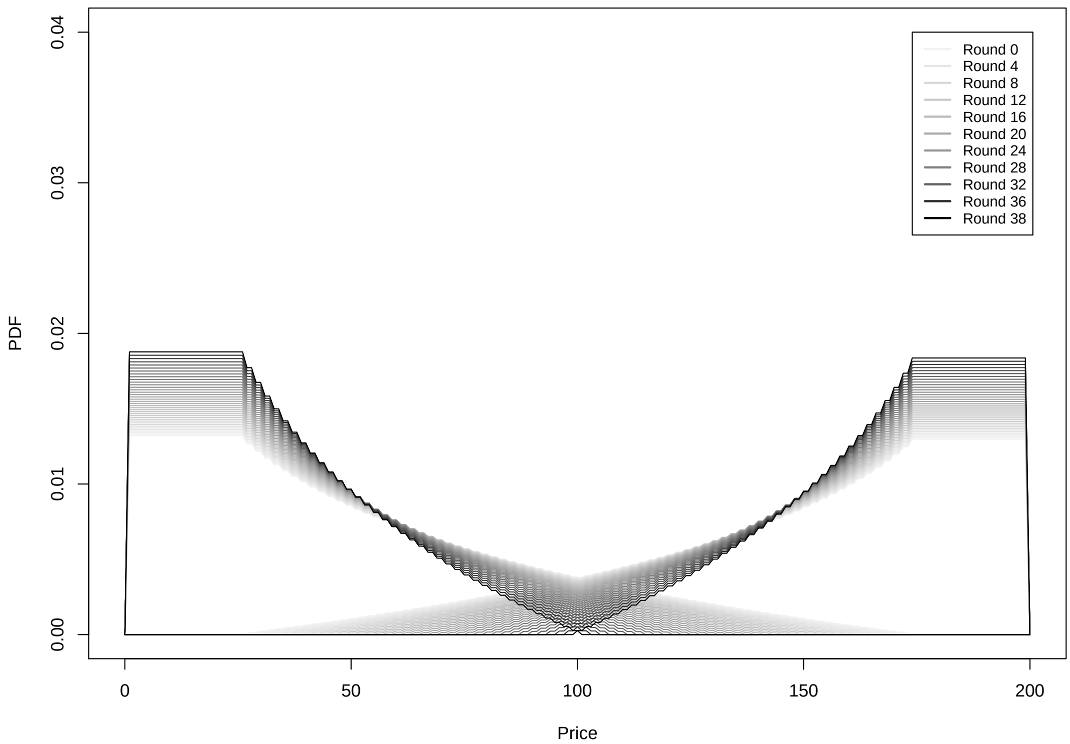


To investigate the importance of changes in the group of traders participating in the market, the PDFs of bids and asks submitted in all of the 150 rounds in the 100 simulations were estimated using R. The PDFs are depicted in figure 13, which shows that the PDFs of bids and asks change dramatically as time progresses. Figure 13 indicates that the change is towards a certain direction: for the PDFs of asks, the probability mass concentrates around the prices from 150 to 200, while for the PDFs of bids, the probability mass concentrates around the prices from 1 to 50. This concentration also seems to mean that in the end of the auction, the probability to see asks with a price lower than 100 is close to zero, while the probability to see bids with a price higher than 100 is also close to zero. In practice, this means that seeing a trade far from the theoretical equilibrium price 100 seems to be impossible, i.e. it has a zero probability, because neither buyers nor sellers can quote such prices. Thus, it seems that in all of the 100 simulations now evaluated, the population of traders participating in the market changed and this change caused the transaction prices to tend towards the equilibrium price.

Figure 13 seems to support the fact that the intramarginal traders are the most probable traders to trade at the beginning of the SCDA. In addition, comparison of figure 13 to figure 14 suggests that the trading mimics close the theoretical Marshallian path as predicted by Brewer et al. (2002). Thus, the intramarginal traders with the most intramarginal valuations appear to trade before the traders with less intramarginal valuations, which is exactly the case with the Marshallian path. In practice, this means that the buyer with the highest valuation is the most probable trader in the group of buyers to trade in the beginning of the SCDA. Similarly, the seller with the lowest valuation is the most probable trader in the group of sellers to trade in the beginning of the SCDA. As the SCDA progresses, the group of traders participating in the market decreases so that the most probable traders to transact leave the market.

Thus, the probability for an intramarginal trader to trade before an extramarginal trader in a SCDA seems to be a very important factor contributing to the converge of transaction prices towards the equilibrium price. Especially, the interesting issue is how the intramarginal traders are able to displace the extramarginal traders in the beginning of the SCDA. The displacing is an issue that has already discussed by Gode and Sunder (1993b, 1997), who were in both articles interested in the overall efficiency of the continuous double auction and did not take the price discovery process into account. The results now presented show that the price discovery process in ZI-C markets appears to be governed by the intramarginal traders ability to trade in the beginning of the SCDA.

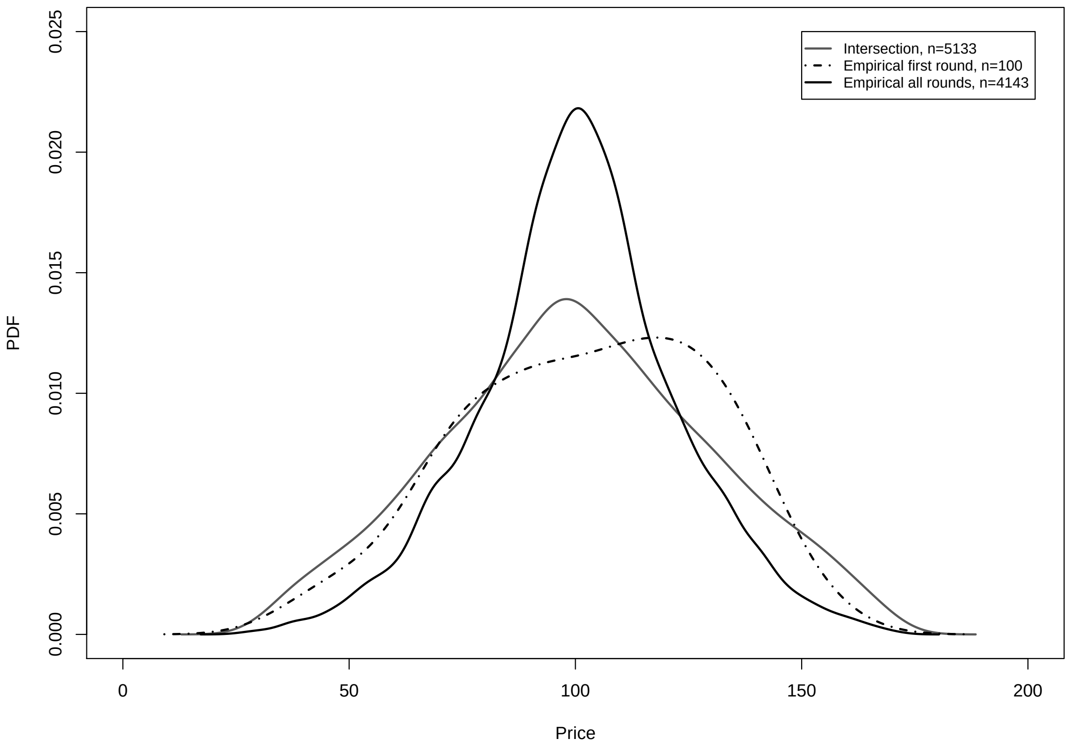
Figure 14: Theoretical PDFs for the ZI-C market for bids and asks when trading takes place exactly according to the Marshallian path. The initial demand and supply used to produce the theoretical probability density functions are depicted in figure 8. The lines presented on the left side of the figure correspond to the probability density functions of bids, while the lines presented on the right side of the figure correspond to the probability density functions of asks. The light gray depicts the PDFs when only first trades on the Marshallian path have taken place, while as the color of the line becomes darker, the number of trades already taken on the Marshallian path increases. The theoretical densities are depicted after each transaction, i.e. every time one seller and one buyer exit the market, on the Marshallian path takes place.



5.2.3 How the progress of the SCDA affects transaction prices

Cliff and Bruten (1997) essentially claimed that the intersection of the PDFs of bids and asks can be used to characterize the PDF of transaction prices in the SCDA. In contrast to this, the results presented now show that even with symmetric demand and supply schedules, the intersection as suggested by Cliff and Bruten (1997) can be only used to describe the PDF of transaction prices for the first round, while the PDF of transaction prices for all rounds should instead probably be a result of weighting equally the intersection densities of bids and asks from all of the rounds. In essence, it appears that the changes in the group of traders participating in the market, are the main reasons for the results shown next.

Figure 15: The PDF of the intersection of theoretical PDFs of bids and asks (IPDF), an empirical PDF of the transaction prices in the first round and an empirical PDF of transaction prices in all rounds in 100 runs of ZI-C market. The demand and supply schedules used are depicted in figure 8, while the statistics for the 100 runs of the ZI-C market are presented in table 2. The IPDF is depicted in light gray line and was created by sampling the theoretical PDFs of bids and asks using the accept-reject algorithm for which the code is included in appendix B. First the theoretical PDFs of bids and asks were sampled for 10000 observations, and then the points from the intersection were chosen as the theoretical transaction prices. The IPDF was estimated using these sampled points. The dashed black line is the PDF estimated using the empirical transaction prices only from the first round, while the solid black line is the PDF estimated using the transaction prices from all of the rounds.



The previous sections have shown that the proposed PDFs of bids and asks seem to characterize the empirical PDFs of bids and asks more accurately than the PDFs of bids and asks by Cliff and Bruten (1997). Thus, it is interesting to look at how the theoretical transaction prices from the intersection of the PDFs of bids and asks compare to the empirical transaction prices. Figure 15 shows the PDF of the intersection of theoretical PDFs of bids and asks (IPDF), an empirical PDF for the transaction prices in the first round and an empirical PDF for transaction prices in all rounds in 100 runs of ZI-C market. The IPDF is depicted in light gray line and was created by sampling the theoretical PDFs of bids and asks using the accept-reject algorithm[38] for which the code is included in appendix B.

Figure 16: The PDF of the intersection of theoretical PDFs of bids and asks (IPDF) on the Marshallian path and an empirical PDF of the transaction prices in all rounds in 100 runs of ZI-C market. The demand and supply schedules used are depicted in figure 8, while the statistics for the 100 runs of the ZI-C market are presented in table 2. The IPDF is depicted in light gray line and was created by sampling the theoretical PDFs of bids and asks on the Marshallian path using the accept-reject algorithm for which the code is included in appendix B. Each of the theoretical PDFs of bids and asks on the Marshallian path were sampled for 800 observations, and then the points from the intersection of the two were chosen as the theoretical transaction prices. The IPDF was estimated using these sampled points. The dashed black line is the PDF estimated using the empirical transaction prices from all of the rounds.

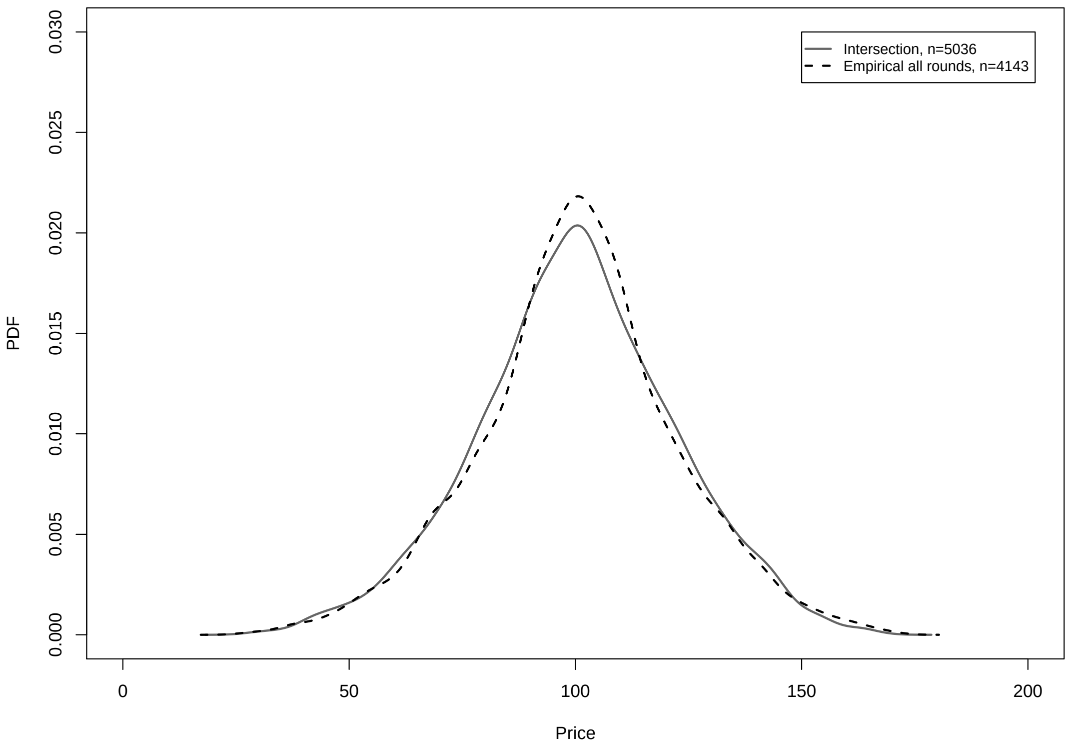


Figure 15 suggests that the claim of Cliff and Bruten (1997) about the PDF of transaction prices of all rounds being characterized by the intersection of the first round PDF is false. Although both PDFs have single modes approximately at price 100, the PDF of transaction prices in all rounds is more peaked around price 100 than the IPDF. However, the IPDF seems to at least somehow characterize the PDF of empirical transaction prices

[38] The purpose of accept-reject method is to simulate a certain known PDF f. The accept-reject method can be used when one does not know how to simulate f, but there exists a majorizing PDF g such that for a constant $M > 0$ we have $f \leq Mg$ in the support of f and one knows how to simulate PDF g. Refer to the book of Robert and Casella (2005) for more a elaborate description.

from the first round. This latter results also seems more intuitive, because IPDF is created using the PDFs of bids and asks that were created from the initial demand-supply schedules. All in all, with this amount of simulations one can only reject the results of Cliff and Bruten (1997), while it is not possible to confirm the results that the IPDF would be able to characterize the probability density function of transaction prices during the first round.

One can also compare the transaction prices on the Marshallian path to the empirical PDF for transaction prices in all rounds in 100 runs of ZI-C market. As figure 16 shows, it appears that the theoretical transaction prices from the Marshallian path seem to characterize the empirical transaction prices from the ZI-C market closely. Thus, it appears that the heuristic ideas of Cliff and Bruten (1997) seem to work correctly when they are refined in the way presented above. I will next use the methods presented in this section to look at the different market types initially presented in this context by Cliff and Bruten (1997).

5.3 Different market types

This section is devoted to the analysis of different market types. As the previous section showed explicitly that the model created exhibits similar characteristics as the original model of Gode and Sunder (1993a), the created model can be utilized also to analyze different market types. The way to analyze the ZI-C markets with symmetric demand and supply schedules, henceforth referred to as the symmetric case, can be also utilized to analyze markets with asymmetric demand and supply schedules.

There are some earlier results from different market types that are worth noticing. Most prominent results were presented by Cliff and Bruten (1997), who found that the transaction prices from simulations deviated clearly from the theoretical equilibrium price in markets where supply was fixed. According to their results the mean daily transaction price in markets with fixed supply was clearly above the equilibrium price, which was supposed to indicate that the tend towards the theoretical equilibrium did not take place. I will in the following look at this particular example, but in addition I will also review the other characterizing examples that are derived from the symmetric demand and supply schedule by making either demand and/or supply fixed and by limiting the number of traders participating in the market.

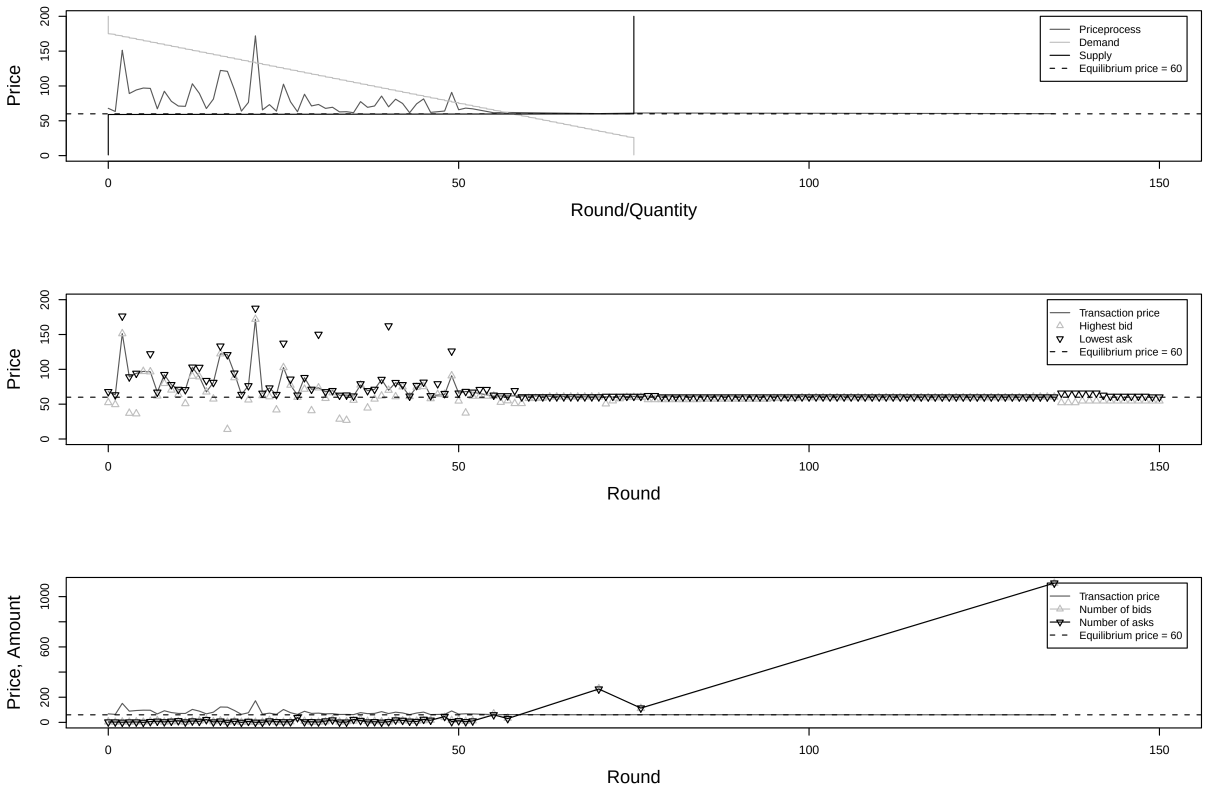
In general, it appears that the analysis presented in the previous subsection applies well also in these different cases. Actually, when comparing the cases presented here to the symmetric base case, it seems that the analysis is actually even simpler in these cases, because either the population of buyers and/or sellers is permitted to only consist of agents with same valuations. Thus, essentially this section shows that the analysis presented above applies also to all of the different market types presented by Cliff and Bruten (1997).

Table 5: Descriptive statistics from 100 runs of the model for ZI-C markets with non-symmetric demand-supply schedules. The results for the 100 runs are presented in the six columns by using averages and standard deviations. In the markets with fixed supply, all the sellers were given equal valuations at price 60, while the valuations of the buyers were defined as in equation 3. In the markets with fixed demand, all the buyers were given equal valuations at price 140, while the valuations of the sellers were defined as in equation 3. In the markets with excess supply, there were 50 buyers with equal valuations at price 140 and 100 sellers with equal valuations at price 60.

	Fixed supply		Fixed demand		Excess supply	
	Average	St.dev.	Average	St.dev.	Average	St.dev.
Equilibrium price	60.0	0.00	140.0	0.00	60.0	0.00
Efficiency (%)	99.9	0.0002	0.99	0.0002	1.0	0.00
Number of transactions	56.9	0.26	56.9	0.29	50.0	0.00
Mean of prices	80.9	2.24	119.7	2.19	99.5	3.28
Median of prices	74.5	2.56	126.2	2.27	99.1	5.44
Maximum price	146.2	12.80	139.8	0.17	138.4	1.55
Minimum price	60.2	0.14	54.5	13.41	61.2	1.25
Standard deviation of prices	20.3	2.33	20.2	2.58	23.4	1.50
Kurtosis of prices	4.4	1.43	4.5	1.25	1.83	0.16
Skewness of prices	1.3	0.33	-1.3	0.29	0.01	0.19
25 percentile	65.1	1.30	110.3	4.57	80.1	4.91
75 percentile	90.7	4.97	135.0	1.24	119.3	4.33
Coefficient of Convergence	48.39	5.01	20.4	2.28	76.4	4.71

Table 5 reports the different market types and the results for each type from 100 runs of the model with ZI-C traders. It seems to support the results presented by Cliff and Bruten (1997) as in all of the markets the theoretical equilibrium price does not equal the mean of transaction prices reported from simulations. Although, at first sight, this could be seen as a proof of the fact that the ZI-C markets do not converge in these markets, the analysis presented next will carefully consider the different markets and show that actually there are good reasons to expect exactly the behavior now witnessed in the results even when transaction prices do exhibit convergence. Again, it seems that actually it is not that interesting to speak about the convergence of transaction prices, but instead look at the evolution of the trader population participating in the single-unit continuous double auction market.

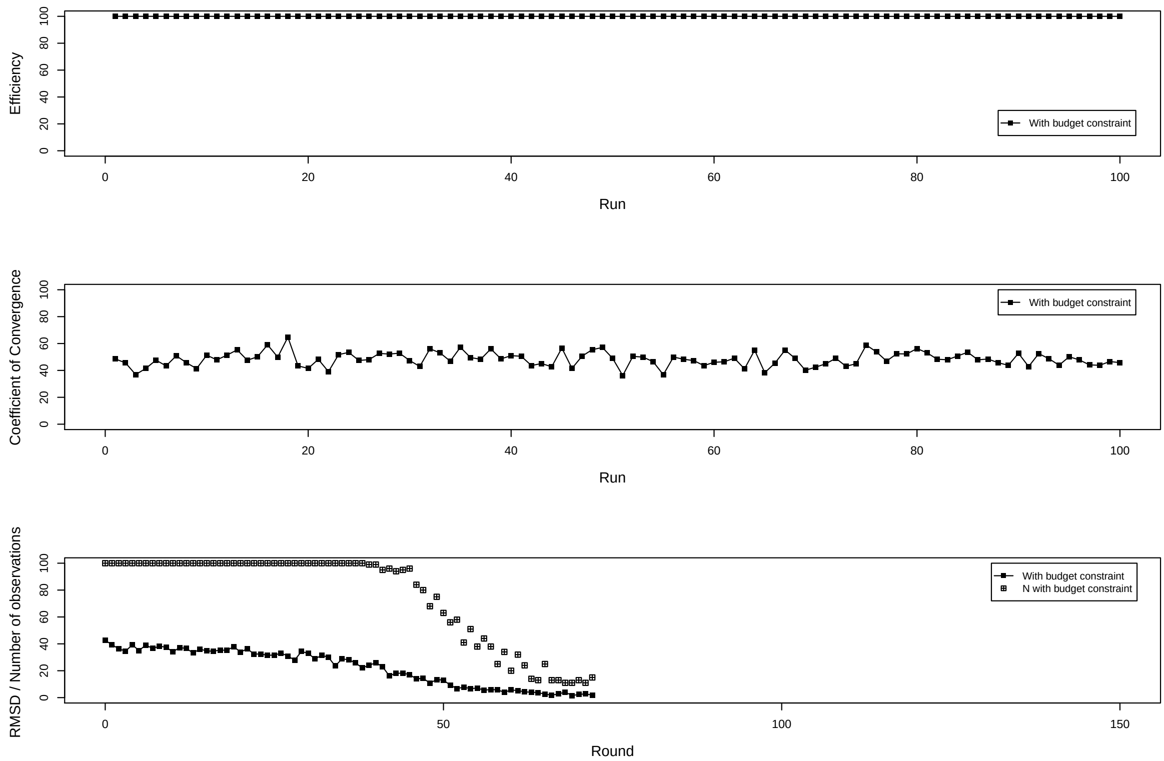
Figure 17: Transaction price time series, demand-supply schedules, best quotes and the amount of bids and ask in the limit order book for a single run of ZI-C market with 150 traders and 150 rounds when supply is fixed. The traders were divided into buyers and sellers equally, and the valuations of the buyers are specified in equation 3, while for all sellers the valuations were set at price 60. In all of the three panels, the transaction price times series is presented by a dark gray solid line as a function of rounds, and the theoretical equilibrium price is presented by a black dashed line. In the top panel, demand as a function of quantity is presented in light gray, and supply as a function of quantity is presented in black. Demand and supply functions were counted using the valuations of individual traders, and the equilibrium price was determined by the intersection point of demand and supply functions. In the middle panel, the "best" quotes in each round are presented; the best quotes are defined as the highest bid and the lowest ask in each round. Highest bids are reported by light gray triangles and the lowest asks are reported by black triangles. In the bottom panel, the number of bids is depicted by light gray line with triangles, while the number of asks is presented by black line with triangles. The number of both bids and asks are reported for each transaction that took place during the single run of the model.



5.3.1 Fixed supply

The results from a single run of the market with fixed supply are reported in figure 17. It shows exactly the same results as found already by Smith (1962) with human subjects, as the transaction prices seem to tend towards the equilibrium price from above the supply schedule, i.e. the transaction price is in all transactions above the price 60. In practice, this is a very natural results, because there exists no ZI-C seller with a valuation lower than price 60 in the population of traders during the simulations. Although the average of the coefficients of convergence reported in table 5 is rather high when compared to the symmetric case, at least qualitatively it seems that the transaction prices tend towards the equilibrium price in this one simulation experiment. The quotes seem to cluster around the equilibrium price and also the distance between the transaction prices and the equilibrium price seems to decrease as time progresses.

Figure 18: Efficiency, coefficient of convergence and root mean squared deviation of transaction prices from the equilibrium price for 100 runs of ZI-C and ZI-U markets with fixed supply, 150 traders and 150 rounds. The traders were divided into buyers and sellers equally, and the valuations of the buyers are specified in equation 3, while for all sellers the valuations were set at price 60. The efficiency of the SCDA markets is presented in the top panel. It is determined as the ratio of the total profit the traders actually earned in the market and the total profit the traders could have earned in the market. In the bottom panel, the root mean square deviation (RMSD) of transaction prices from the equilibrium price in ZI-C markets as a function of rounds is presented in black. In addition to RMSDs, the bottom panel also depicts the number of observations, i.e. number of transaction prices from 100 simulations, on each round. The plot shows RMSDs only in rounds, which had more than 10 observations. The number of observations from ZI-C markets are depicted in black squares with crosses. In the middle panel, the coefficient of convergence is presented for each run of the model.

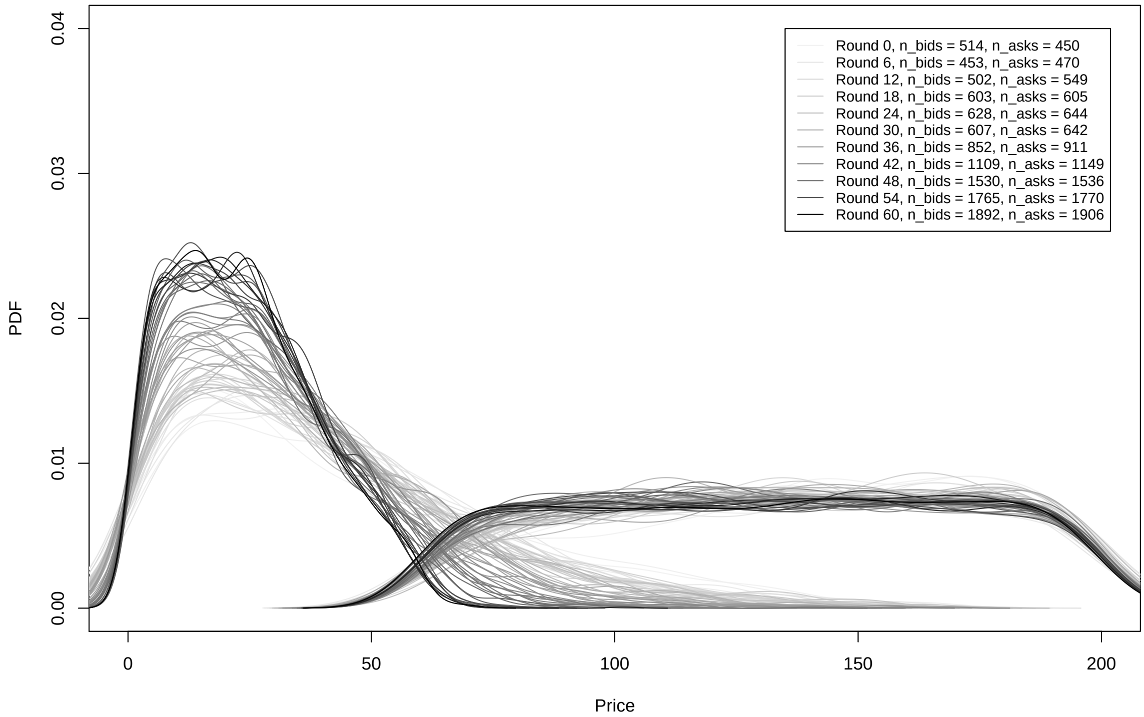


In essence, the results suggest that it is not possible to rule out the convergence argument given by Gode and Sunder (1993a) by comparing only the average of the transaction prices to the theoretical equilibrium price as suggested by Cliff and Bruten (1997). At least in this particular case, it seems that the prices do converge towards the equilibrium price although the mean of the transaction prices is definitely above the equilibrium price. Thus, it seems that the quantitative consideration by Cliff and Bruten (1997) does not take into account the fact that the average transaction price does not need to equal the equilibrium price to make the transaction prices to converge towards the equilibrium price during the simulations. Actually, the behavior suggested by Cliff and Bruten (1997) would only be expected if the transaction prices approached the equilibrium price from both above and below the equilibrium price.

A first more quantitative look at the convergence can be made using the root mean squared deviation (RMSD) of transaction prices from the equilibrium price. Although Cliff (1997) claims that the ZI-C markets with fixed supply do not converge, he reports that the RMSD of transaction prices from the equilibrium price seems to decay as the time progresses. This results is also confirmed in the simulations presented now, as shown by the RMSD of transaction prices from the equilibrium price depicted in figure 18 for all of the 100 simulations. This suggests that the convergence is definitely present in the ZI-C markets with fixed supply schedule. However, also in this case, it seems that the number of observations clearly decreases as the RMSD starts to decay towards the equilibrium price. This is natural, because there are less intramarginal traders participating in the market, which makes the trading to demand more quotes for a single trade to take place. However, to make the concrete judgement about this issue using the RMSD of transaction prices from the equilibrium price, one would certainly have to increase the number of simulations.

However, the convergence of transaction prices towards the equilibrium price in ZI-C markets with fixed supply schedule can also be reviewed using the PDFs of bids and asks estimated from the simulation data as was done with in the symmetric case. Figure 19 shows the PDFs of bids and asks in all of the 150 rounds in the 100 runs of the model. It clearly shows that as the sellers form a homogeneous population, there seems to be practically no change in the probability density function of asks during the 150 rounds. This suggests that the seller population does not seem to change in any meaningful way during the simulations. However, as the valuations of buyers are still exactly the same as in the symmetric case, the population of buyers is heterogeneous. This heterogeneity is also shown in figure 19, because when time progresses, the probability density function for bids becomes more peaked. This is exactly the same result that was found in the markets with symmetric demand and supply schedules. Thus, the convergence can be explained by the fact that the intramarginal buyers leave the market in the beginning of

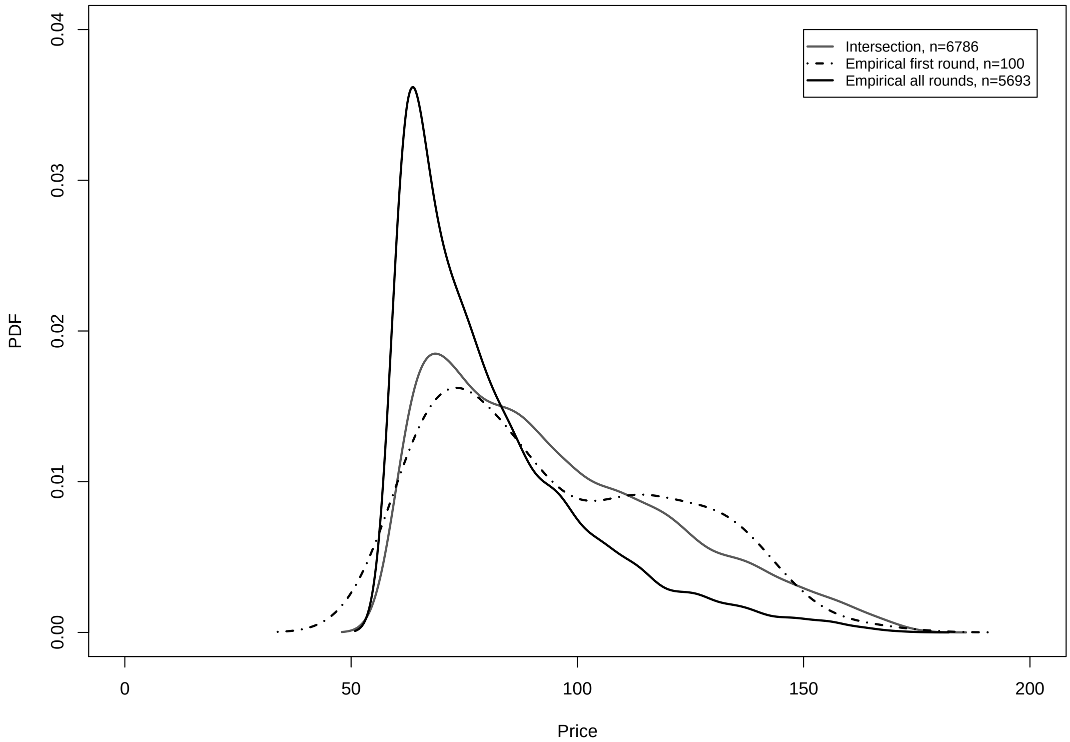
Figure 19: PDFs from 100 runs of the ZI-C market with fixed supply for bids and asks for all rounds from 0 to 150. The traders were divided into buyers and sellers equally, and the valuations of the buyers are specified in equation 3, while for all sellers the valuations were set at price 60. The lines presented on the left side of the figure correspond to the PDFs of bids, while the lines presented on the right side of the figure correspond to the PDFs of asks. The light gray lines depict the PDFs for both bids and asks from the beginning rounds of the simulations, while as the color of the line becomes darker, the number of rounds increases.



the simulation after they have traded, and the trading ceases as none of the intramarginal traders participate in the market any more. The distinction between this case and the symmetric case is that with fixed supply only the characteristics, i.e. the PDF of bids, of the population of the buyers change as time progresses, while the population of sellers does not change in any way that would affect the price discovery process.

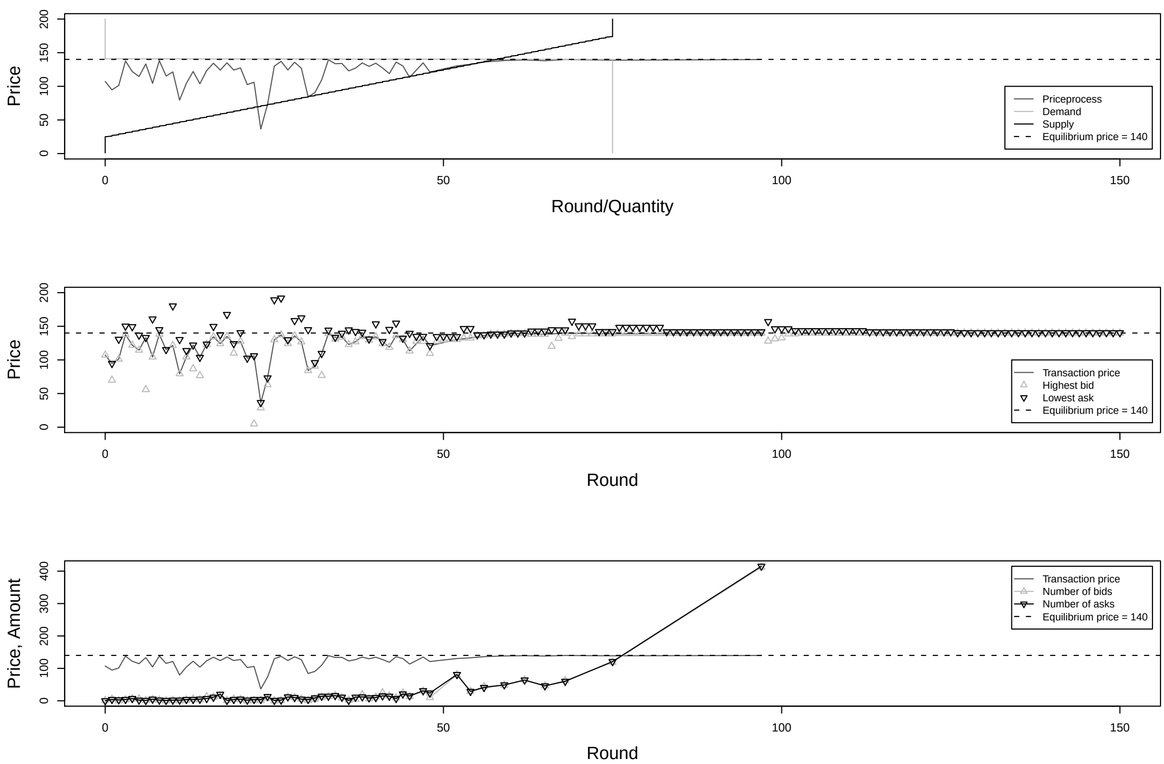
To strengthen the ideas presented for the ZI-C markets in the symmetric case, it is also interesting to look at how the theoretical framework presented for the symmetric case fits this slightly altered situation. Figure 20 shows the theoretical PDF of the intersection of the PDFs of bids and asks (IPDF) and empirical PDFs of transaction prices for the first round and all rounds. In general, figure 20 supports the fact that the results from the 100 simulations are in line with the theoretical ideas presented for the symmetric demand and supply schedules. It also appears that the transaction prices from the first round are well in line with the intersection of the theoretical bid and asks densities, because

Figure 20: The PDF of the intersection of theoretical PDFs of bids and asks (IPDF), an empirical PDF of the transaction prices in the first round and an empirical PDF of transaction prices in all rounds in 100 runs of ZI-C market when supply is fixed. The demand and supply schedules used are depicted in the figure 17, while the statistics are presented in the table 5. The IPDF is depicted in light gray line and was created by sampling the theoretical PDFs of bids and asks using the accept-reject algorithm. First the theoretical PDFs of bids and asks were sampled for 10000 observations, and then the points from the intersection were chosen as the theoretical transaction prices. The IPDF was estimated using these sampled points. The dashed black line is the PDF estimated using the empirical transaction prices only from the first round, while the solid black line is the PDF estimated using the transaction prices from all of the rounds.



the transaction prices seem to have a very similar distribution as the intersection of the theoretical bid and asks densities. The slight differences in the distributions could be expected to decay as the number of simulations is increased. However, the claim of Cliff and Bruten (1997) that the transaction prices from all rounds would be characterized by the distribution of the intersection of the theoretical bid and ask densities, is rejected in this case. The density of the transaction prices from all rounds is clearly more peaked than the density of the intersection.

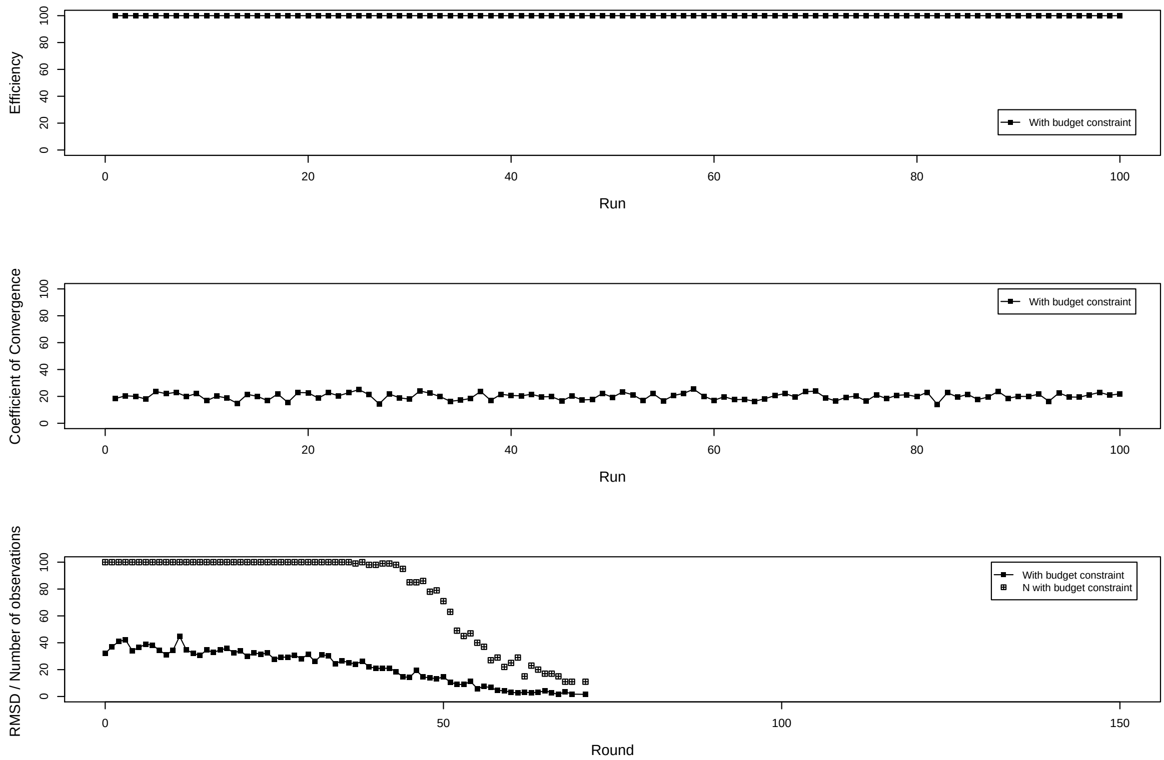
Figure 21: Transaction price time series, demand-supply schedules, best quotes and the amount of bids and ask in the limit order book for a single run of ZI-C market with 150 traders and 150 rounds when demand is fixed. The traders were divided into buyers and sellers equally, and the valuations of the sellers are specified in equation 3, while for all buyers the valuations were set at price 140. In all of the three panels, the transaction price times series is presented by a dark gray solid line as a function of rounds, and the theoretical equilibrium price is presented by a black dashed line. In the top panel, demand as a function of quantity is presented in light gray, and supply as a function of quantity is presented in black. Demand and supply functions were counted using the valuations of individual traders, and the equilibrium price was determined by the intersection point of demand and supply functions. In the middle panel, the "best" quotes in each round are presented; the best quotes are defined as the highest bid and the lowest ask in each round. Highest bids are reported by light gray triangles and the lowest asks are reported by black triangles. In the bottom panel, the number of bids is depicted by light gray line with triangles, while the number of asks is presented by black line with triangles. The number of both bids and asks are reported for each transaction that took place during the single run of the model.



5.3.2 Fixed demand

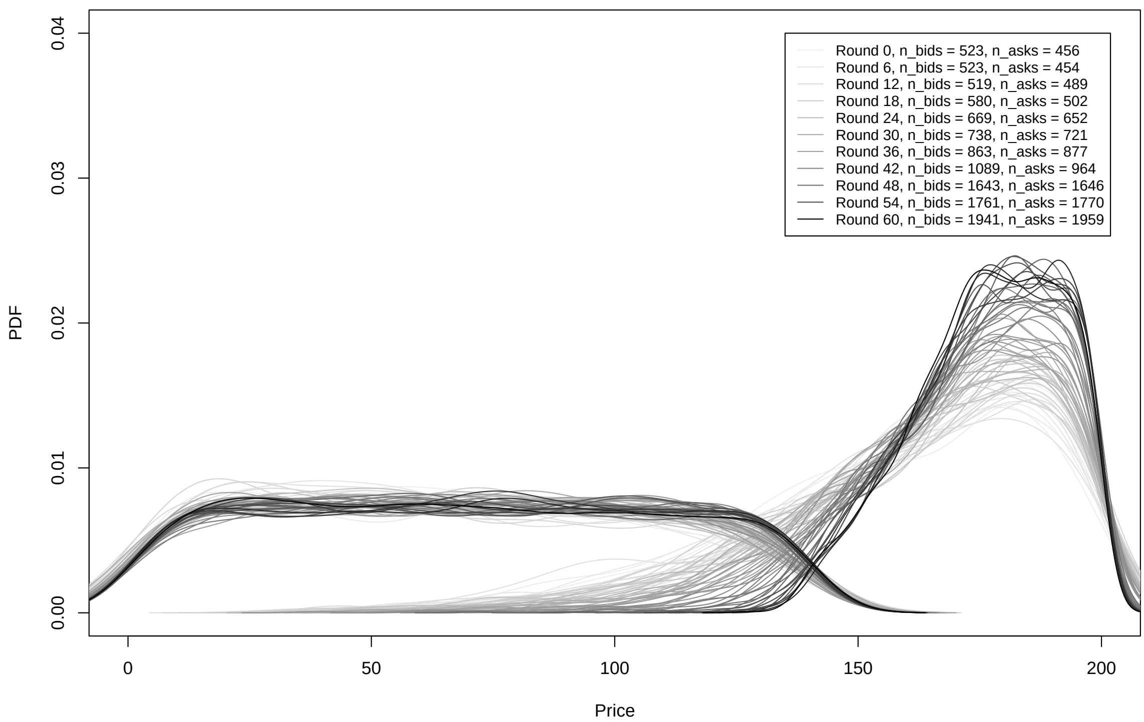
The results from a single run of the market with fixed demand are reported in figure 21. It shows similar results as the previous subsection showed for the ZI-C markets with fixed supply, but this time the interest is directed at the changes in the population of buyers participating in the market. This time the transaction prices seem to tend towards the equilibrium price from below the demand schedule, i.e. the transaction price is in all transactions below the price 140. In practice, this is a very natural result, because there exists no ZI-C buyer with a valuation higher than 140 in the population of traders during the simulations.

Figure 22: Efficiency, coefficient of convergence and root mean squared deviation of transaction prices from the equilibrium price for 100 runs of ZI-C and ZI-U markets with fixed demand, 150 traders and 150 rounds. The traders were divided into buyers and sellers equally, and the valuations of the sellers are specified in equation 3, while for all buyers the valuations were set at price 140. The efficiency of the CDA markets is presented in the top panel. It is determined as the ratio of the total profit the traders actually earned in the market and the total profit the traders could have earned in the market. In the bottom panel, the root mean square deviation of transaction prices from the equilibrium price (RMSD) in ZI-C markets as a function of rounds is presented in black. In addition to RMSDs, the bottom panel also depicts the number of observations, i.e. number of transaction prices from 100 simulations, on each round. The plot shows RMSDs only in rounds, which had more than 10 observations. The number of observations from ZI-C markets are depicted in black squares with crosses. In the middle panel, the coefficient of convergence is presented for each run of the model.



The average of the coefficients of convergence reported in table 5 is slightly, i.e. approximately 5 percent, lower than in the symmetric case. This indicates that the transaction prices in the ZI-C market with fixed demand are rather close to the equilibrium price when compared to the other market types already considered in this thesis. It also seems that the transaction prices tend towards the equilibrium price in this one simulation experiment: the quotes cluster around the equilibrium price and also the distance between the transaction prices and the equilibrium price decreases as time progresses.

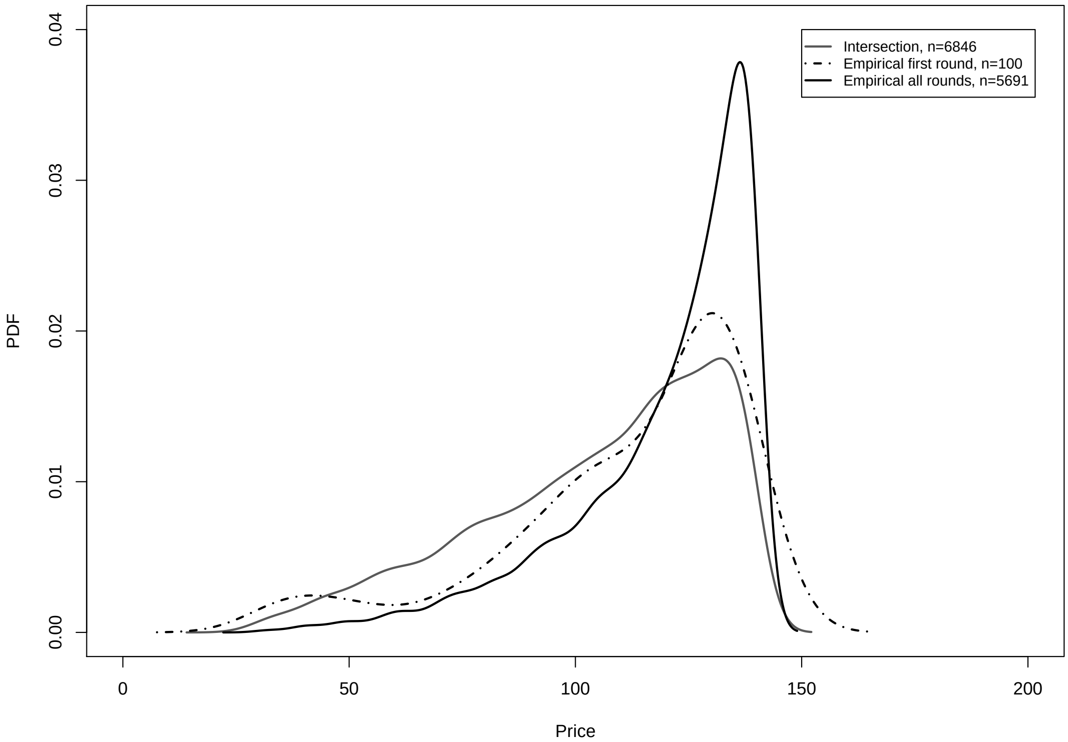
Figure 23: PDFs from 100 runs of the ZI-C market with fixed demand for bids and asks for all rounds from 0 to 150. The traders were divided into buyers and sellers equally, and the valuations of the sellers are specified in equation 3, while for all buyers the valuations were set at price 140. The lines presented on the left side of the figure correspond to the PDFs of bids, while the lines presented on the right side of the figure correspond to the PDFs of asks. The light gray lines depict the PDFs for both bids and asks from the beginning rounds of the simulations, while as the color of the line becomes darker, the number of rounds increases.



Although Cliff and Bruten (1997) did not report the results from the ZI-C markets with fixed demand, it seems in general that the results seem to be quite similar to those reported in this thesis already for the ZI-C markets with fixed supply. It seems that the transaction prices converge towards the equilibrium price as time progresses. Note also that in this case the average of the transaction prices gives a false indication about the convergence, because all of the transaction prices are below the equilibrium price. Thus, the mean of the transaction prices is below the equilibrium price unless all the transaction take place exactly with the equilibrium price.

Evaluating the convergence of ZI-C markets with fixed demand yields the same result as in the ZI-C markets with fixed supply. The RMSD of transaction prices from the

Figure 24: The PDF of the intersection of theoretical PDFs of bids and asks (IPDF), an empirical PDF of the transaction prices in the first round and an empirical PDF of transaction prices in all rounds in 100 runs of ZI-C market when demand is fixed. The traders were divided into buyers and sellers equally, and the valuations of the sellers are specified in equation 3, while for all buyers the valuations were set at price 140. The IPDF is depicted in light gray line and was created by sampling the theoretical PDFs of bids and asks using the accept-reject algorithm. First the theoretical PDFs of bids and asks were sampled for 10000 observations, and then the points from the intersection were chosen as the theoretical transaction prices. The IPDF was estimated using these sampled points. The dashed black line is the PDF estimated using the empirical transaction prices only from the first round, while the solid black line is the PDF estimated using the transaction prices from all of the rounds.



equilibrium price seems to decay as time progresses. To make the result visually concrete, the RMSD of transaction prices from the equilibrium price is plotted for each round in the bottom panel of figure 23. Again the same result is repeated as the number of observations starts to decrease at the same time as the RMSD of transaction prices from the equilibrium price starts to decay. More concrete results about the convergence can be obtained by looking at the probability density functions of bids and asks estimated from the simulations and by examining how the densities change over time presented in figure 23. It appears again that the homogeneous population, i.e. the population of buyers, stays this time exactly the same, while the heterogeneous population, i.e. the population of sellers, seems to change so that the intramarginal traders leave the market and after there are none of them left the trading ceases.

The final look at the theoretical framework given in figure 24 shows that also in this case the theoretical characterizations for the first round seem to be similar as the results from the simulations. However, again, the probability density function of all transaction prices is clearly different from the probability density function of the intersection of the probability density functions of market wide bids and asks. Thus, the claim presented by Cliff and Bruten (1997) is rejected also in this case.

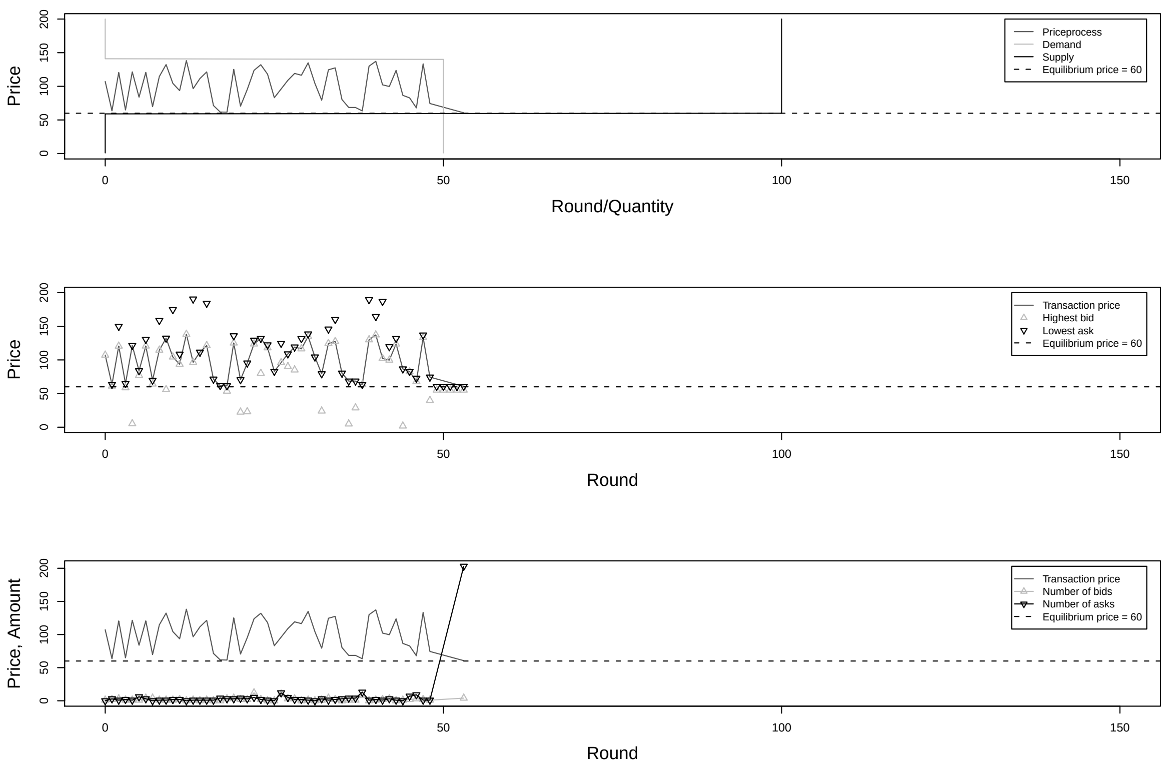
5.3.3 Fixed demand and supply - excess supply

This last case is especially interesting, because Cliff and Bruten (1997) claim that with fixed demand and supply transaction prices in the ZI-C markets do not converge towards the equilibrium price. As the previous presentation has used different arguments than Cliff and Bruten (1997), it is interesting to see how the methods presented in the earlier section fits this particular problem. As the first note, one should understand that a market with fixed demand and supply schedules is the simplest case to analyze using the methods presented, because the populations of buyers and sellers are homogeneous. Thus, one can expect that with fixed demand and supply, the changes in the population of traders should not affect the transaction price in any way. In essence, the expected transaction price should be random, which in this case means that it should have a uniform distribution on a certain interval.

The market parameters were chosen so that the market exhibits excess supply. I chose the parameters so that there were 50 buyers with a common valuation at price 140 and 100 sellers with a common valuation at price 60. This implies that the distribution of the transaction prices should be a uniform distribution from price 60 to price 140, because in that interval any seller will accept a bid and any buyer will accept an ask. From the statistics presented in table 5, one should notice that in each of the runs presented here, 50 transactions took place in each one of the 100 runs of the model. The market was closed after that, because there were no buyers left in the market, which meant that it was impossible for the sellers to find counterparts to trade with. In addition, the efficiency of the ZI-C market with excess supply was in all of the runs 1.0 with a standard deviation 0.00.

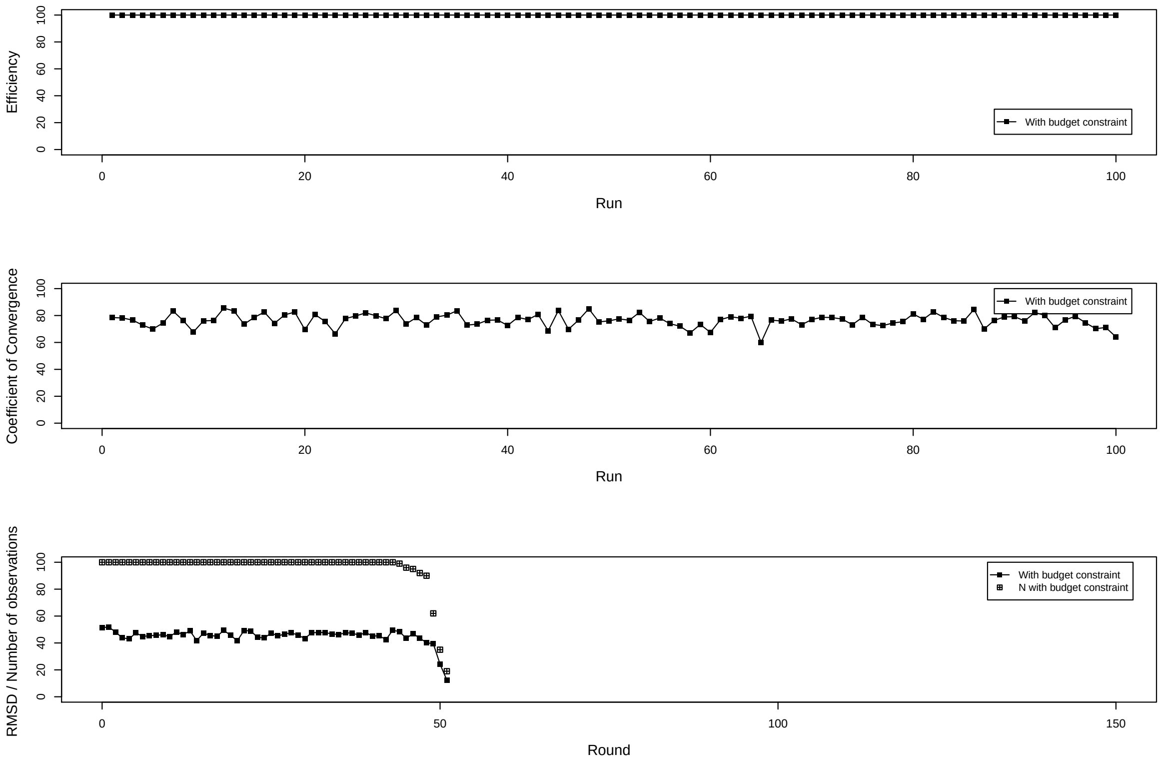
The top panel of figure 25 shows the transaction prices for a single run of the model, and it seems that the hypothesis about the randomly fluctuating transaction prices in the price interval from 60 to 140 seems to have been well satisfied in this single simulation. Actually even a more concrete result about this issue can be presented, because table 5 shows that the mean of the maximum prices was 138.4. Although the fact that the

Figure 25: Transaction price time series, fixed demand supply schedules, best quotes and the amount of bids and ask in the limit order book for a single run of ZI-C markets with 150 traders and 150 rounds. The traders were not divided into buyers and sellers equally: there were 50 buyers with a common valuation at price 140 and 100 sellers with a common valuation at price 60. In all of the three panels, the transaction price times series is presented by a dark gray solid line as a function of rounds, and the theoretical equilibrium price is presented by a black dashed line. In the top panel, demand as a function of quantity is presented in light gray, and supply as a function of quantity is presented in black. Demand and supply functions were counted using the valuations of individual traders, and the equilibrium price was determined by the intersection point of demand and supply functions. In the middle panel, the "best" quotes in each round are presented; the best quotes are defined as the highest bid and the lowest ask in each round. Highest bids are reported by light gray triangles and the lowest asks are reported by black triangles. In the bottom panel, the number of bids is depicted by light gray line with triangles, while the number of asks is presented by black line with triangles. The number of both bids and asks are reported for each transaction that took place during the single run of the model.



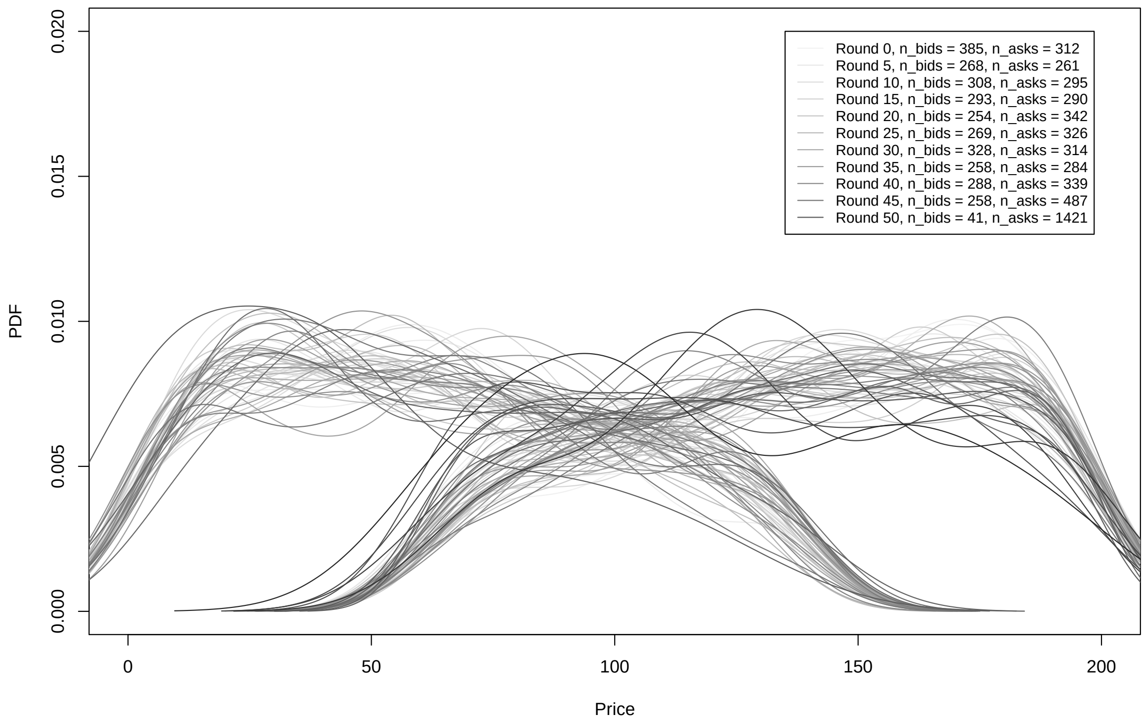
mean of the maximums is lower than 140 does not show that there could not have been a transaction price higher than 140, in this case, it still shows that there were transaction prices close to 140. Similarly, as the mean of the minimums of transaction prices is 61.9, it is clear that there were transaction prices close to the lower bound at price 60. Thus, it seems that in all of the 100 runs of the model, the transaction prices have varied in the price range from 60 to 140 with a mean around a price of 100. As also the skewness has been very close to zero on average, it seems that the distribution of transaction prices has been symmetric around its mean.

Figure 26: Efficiency, coefficient of convergence and root mean squared deviation of transaction prices from the equilibrium price for 100 runs of ZI-C market with excess supply and fixed demand-supply schedule, 150 traders and 150 rounds. The traders were not divided into buyers and sellers equally: there were 50 buyers with a common valuation at price 140 and 100 sellers with a common valuation at price 60. The efficiency of the CDA markets is presented in the top panel. It is determined as the ratio of the total profit the traders actually earned in the market and the total profit the traders could have earned in the market. In the bottom panel, the root mean square deviation of transaction prices from the equilibrium price (RMSD) in ZI-C markets as a function of rounds is presented in black. The measures were counted for each round using the data of transaction prices of either ZI-C markets. In addition to RMSDs, the bottom panel also depicts the number of observations, i.e. number of transaction prices from 100 simulations, on each round. The plot shows RMSDs only in rounds, which had more than 10 observations. The number of observations from ZI-C markets are depicted in black squares with crosses. In the middle panel, the coefficient of convergence is presented for each run of the model.



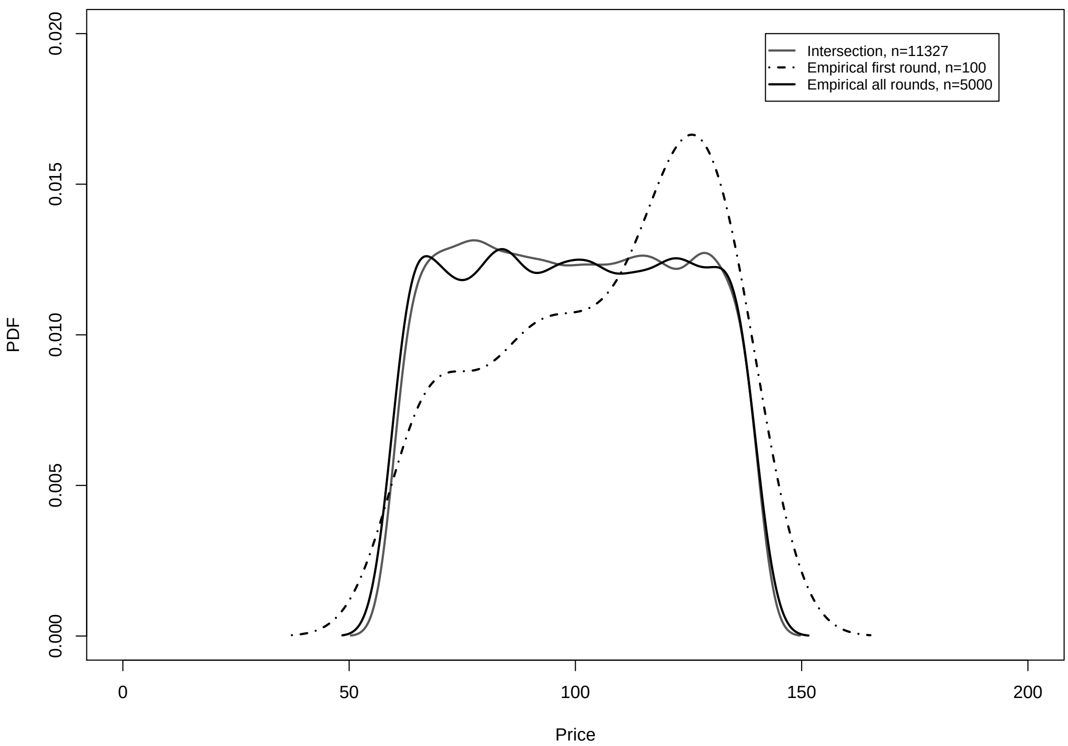
The transaction prices shown in figure 25 seem to suggest that there would not be a convergence towards the equilibrium price. The best bids and asks shown in the middle panel of the figure 25 also support this story: neither the best bids nor asks seem to concentrate around the equilibrium more tightly as time progresses. In addition, the bottom panel of figure 25 reports the number of bids and asks in the limit order book for each transaction, and shows that the number of bids and asks needed for a trade seems to stay on the same level all the time. Thus, it seems as already suggested by the qualitative analysis that at least in the single simulation presented in figure 25 no convergence of transaction prices towards the equilibrium price took place.

Figure 27: PDFs from 100 runs of the ZI-C market with fixed supply for bids and asks for all rounds from 0 to 150. The demand and supply schedules used in the simulations are depicted in the figure 7, while the statistics are presented in table 2. The lines presented on the left side of the figure correspond to the PDFs of bids, while the lines presented on the right side of the figure correspond to the PDFs of asks. The light gray lines depict the PDFs for both bids and asks from the beginning rounds of the simulations, while as the color of the line becomes darker, the number of rounds increases.



A more detailed analysis of the convergence towards the equilibrium price can be done using the coefficients of convergence and the RMSD of transaction prices from the equilibrium price. Figure 26 depicts in the middle panel the coefficients of convergence for all of the 100 runs of the model. The first impression from the middle panel of figure 26 is that the coefficients of convergence are relatively high in all of the runs when compared to, for example, the symmetric case. However, the more concrete proof for the lack of convergence is the fact that the RMSD of transaction prices from the equilibrium price does not seem to exhibit any convergence. In the round close to the round 50, the RMSD of transaction prices from the equilibrium price seems to decay, but it is only a result of the fact that the number of observations decreases dramatically at the same time. Thus, both the RMSD and the coefficient of convergence support the fact that there seems to be no convergence of transaction prices towards the equilibrium price.

Figure 28: The PDF of the intersection of theoretical PDFs of bids and asks (IPDF), an empirical PDF of the transaction prices in the first round and an empirical PDF of transaction prices in all rounds in 100 runs of ZI-C market when there is excess supply and both demand and supply are fixed. The demand and supply schedules used are depicted in the figure 25 while the statistics are presented in table 5. The IPDF is depicted in light gray line and was created by sampling the theoretical PDFs of bids and asks using the accept-reject algorithm. First the theoretical PDFs of bids and asks were sampled for 10000 observations, and then the points from the intersection were chosen as the theoretical transaction prices. The IPDF was estimated using these sampled points. The dashed black line is the PDF estimated using the empirical transaction prices only from the first round, while the solid black line is the PDF estimated using the transaction prices from all of the rounds of the model.



In the beginning of this subsection, the claim was made that he characteristics of the ZI-C trader populations, i.e. the buyer and seller populations, do not change during the auction in any meaningful way from the price discovery perspective in ZI-C markets. Figure 27 shows that this is indeed the case in the 100 simulations reported now. It shows that the probability density functions of bids and asks do not essentially change in any way as time progresses. This can be seen as the main reason for the lack of convergence exhibited in the model, because in the earlier markets considered in this thesis the evolution of the PDFs of bids and asks has been the driving force of the price discovery in the ZI-C markets. All in all, as the ZI-C traders trade in exactly the same manner throughout the SCDA, the trading exhibits a similar pattern as the trading of ZI-U traders, but this time only in the price range from 60 to 140. Thus, by changing the demand and supply schedules to this extreme case, Cliff and Bruten (1997) have chosen a particular situation, which is not even expected to exhibit convergence of transaction prices towards the equilibrium price.

Final inspection of the model is done using the theoretical PDFs. Figure 28 shows the theoretical PDF of the intersection of the PDFs of bids and asks (IPDF) for the first round. The most interesting result is that the PDF estimated from the simulated transaction prices from all rounds seems to be very close to the uniform distribution in the price range from 60 to 140. This is also exactly what is suggested by the theoretical IPDF. Thus, as the populations of both buyers and sellers are homogeneous, the transaction price density stays constant as the time progresses, which makes it possible to use the IPDFs from the first round to approximate the PDFs of all transaction prices. In essence, this example shows that the method proposed by Cliff and Bruten (1997) works correctly when the buyer and seller populations are homogeneous throughout the SCDA. However, as the previous examples with different market types have shown, this method seems to work correctly only in this particular example.

6 Conclusions

Agent-based models have been motivated by the possibility to analyze real markets more accurately when compared to the traditional analytic models. However, at the moment agent-based modeling in general lacks synthesis about the modeling principles used. One of the primary objectives of this thesis was to argue how a single step toward the synthesis could be taken when the interest is in price discovery in double auction markets. The extensive literature and heterogeneous models led me to choose a model that is as simple as possible. By constraining to a simple model, it has been possible to account carefully for the different issues that contribute in this model to the price discovery process.

This thesis analyzed single-unit continuous double auction markets and especially the ZI-trader paradigm that was introduced by Gode and Sunder (1993a). The results of Gode and Sunder have been widely recognized and also criticized. The most prominent critique for the ZI-trader approach has been given by Cliff and Bruten (1997) and Brewer et al. (2002). These two studies were used as the starting point of this thesis, and the method proposed is a refinement of the ideas of Cliff and Bruten (1997). The new method proposed to analyze the ZI-C trader markets seems to describe the PDFs of bids, asks and transaction prices in the SCDA more accurately in different types of markets than the method proposed by Cliff and Bruten (1997). In addition, it appears that the ideas by Cliff and Bruten (1997) corresponds only to situations, where there exists no distinction between intra- and extramarginal traders.

The results presented in this thesis support the results presented by Brewer et al. (2002). Essentially, the results suggest that by using the ideas presented by Cliff and Bruten (1997) more carefully, it appears to be possible to explain the price discovery process in the ZI-C trader markets. In addition, when analyzing the behavior of ZI-C markets, it is important to look at how the population of traders changes over time and how the changes contribute to the characteristics of the market. Generally, it seems that the earlier literature has overlooked the importance of the evolution in the trader population participating in the ZI-C market. In addition, I find that the trading in ZI-C markets seems to approximate the trading that takes place exactly on the Marshallian path as was heuristically suggested also by Brewer et al. (2002).

The results are especially interesting in the light of recent research. Although the ZI-trader framework abstract quite far from the real markets, it is interesting that a recent study by Ladley and Schenk-Hoppé (2009) found that a modified ZI-C trader market was able to produce many of the characteristics of real-life order book markets. Thus, it is important to understand how the ZI-C trader markets actually function, and this thesis has presented quantitative results that show how the trading in ZI-C markets seems to approximate the trading along the Marshallian path.

There appears to be two main areas of further research. First, the results presented in this thesis should be confirmed using a larger amount of simulations to be able to compare the results to the ones presented by Othman (2008). However, such an experiment would require a new and more efficient implementation of the ZI-model than the one presented in this thesis. Second, it could also be interesting to look at the bid and ask densities from real markets and, for example, compare the evolution of the two to the different market regimes appearing in the real markets at a certain moment. This could be done for example by somehow estimating the demand and supply in the market at a certain moment, and looking how the bid and ask densities develop after that.

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A Source code for Python

```
"""
8.2.2011, Niklas Jahnsson

Zero-intelligence traders implemented for a master's thesis in Finance.
This file has got a model of both zero-Intelligence traders with and
without budget constraint. The model is implemented using a simulation
framework SimPy for Python. Essentially the model was created to replicate
the results of both Gode and Sunder (1993) and Cliff and Bruten (1997).
Thus, the market implemented is a continuous double auction without
replacement.

Uses
 -SimPy, Rpy
 -r.library('moments'), r.library('coin'), r.library('colorRamps')

It is important to note that if the model is used to review the convergence
using root mean squared deviation of transaction prices from the equilibrium
price, then the market parameters implemented in function initializeDemandAndSupply
should be kept the same in all of the runs.

commandline arguments available are

"debug"         to see debugging texts
"trades"        to see how trading takes place
"conv"          to force the using of same seed for random number generator in all
                of the runs
"singleSeed"    to force the using of single seed

Note that output parameters are set so that a run with 150 agents for 150 rounds
is outputted correctly. Set outPaths yourself to change the output directories.
"""

from SimPy.Simulation import *
from rpy import r
import random, time, math, sys

# Auxiliary variables
debug = False
trades = False
convergenceRun = False
singleSeed = False
if 'debug' in sys.argv:
 debug = True
if 'trades' in sys.argv:
 trades = True
if 'conv' in sys.argv:
 convergenceRun = True
if 'singleSeed' in sys.argv:
 singleSeed = True

OutPath = ""
outPathC = "/home/nikke/SimPyOut/ZI/C/080211/big/ZI_C_i_"
outPathU = "/home/nikke/SimPyOut/ZI/U/080211/big/ZI_U_i_"
outPathZI = "/home/nikke/SimPyOut/ZI/080211/big/ZI_n_"

# Experiment data -------------------------
numberOfRuns = 100
SEED = time.ctime()
equilibriumPrice = 100
numberOfRounds = 150
numberOfAgents= 150
simulationIndex = 1
maximumAssetValue = 200
budgetConstraint = True

# Model components ------------------------
# Variables
numberOfBuyers = 0
```

```
numberOfSellers = 0

# Queues for buyers and sellers
selectedBuyer = -1
selectedSeller = -1
buyerQueue = []
sellerQueue = []
selectedBuyers = []
selectedSellers = []

# Logging for output data
prices = []
bestBids = []
bestAsks = []
buyerValuations = []
sellerValuations = []
demand = []
supply = []
extractedProfit = 0
statistics = []
allBids = []
allAsks = []
allBids0 = []
allAsks0 = []

# For output using Rpy
rheight = rwidth = 500

# Auxiliary functions

def initializeDemandAndSupply():
 """
 Initialize buyer and seller valuations so that they are not stochastic, but
 instead sequences from 25 to 175 using equal intervals. The valuations are
 casted to integers, because integers are used later when determining demand,
 supply and equilibrium price.
 """
 global buyerValuations, sellerValuations, numberOfAgents, numberOfBuyers,\
     numberOfSellers
 numberOfBuyers = 0
 numberOfSellers = 0
 buyerValuations = []
 sellerValuations = []
 increment = round(1.0*(174-26)/(1.0*numberOfAgents/2))
 i = 26
 count = 0
 while count < 1.0*numberOfAgents/2:
  buyerValuations.append(int(i))
  sellerValuations.append(int(i))
  i += increment
  count += 1

def RMSD(priceSeries,eqPrice):
 """
 Parameters
  priceSeries: a list of prices
  eqPrice: equilibrium price

 Calculates the standard deviation of the prices
 in the priceSeries around the equilibrium price
 rather than around the mean of priceSeries prices.
 """
 if eqPrice < 0:
  raise Exception("RMSD: eqPrice < 0")
 if len(priceSeries) > 0:
  ret = 0
  for i in range(0,len(priceSeries)):
   ret += 1.0*pow((priceSeries[i]-eqPrice),2)
  return pow(1.0*ret/len(priceSeries),0.5)
 else:
```

```
  raise Exception("RMSD: length of priceSeries < 0")


def returnAgentNames(agentList):
 """
 Assumes that input is a list of agents, and returns all the names of the
 agents concatenated. Used for debugging purposes.
 """
 try:
  output = ""
  for agent in agentList:
   output += agent.name+","
  return output
 except Exception as inst:
  raise Exception("returnAgentNames: Exception: "+str(inst))

class Book(Resource):
 """
 Implements a generic limit order book as a common resource using the
 Resource class of SimPy. Both bids and asks need their own limit order
 books, because of this implementation. The book consists of a list, which
 can be allocated to a single trader at the time if they request it.
 """
 def __init__(self,sim):
  try:
   Resource.__init__(self,capacity=1,sim=sim)
   self.list = []
  except Exception as inst:
   raise Exception("Book: __init__(): Exception: "+str(inst))

class MarketMaker(Process):
 """
 Responsible for gathering both all buyer and seller agents
 and picking one of them uniformly to trade. Waits first for all
 agents to gather to the market, then chooses one and waits until
 the chosen one has acted. Stops market if the number of buyers AND
 sellers equals zero. Only one method, i.e. work(), implemented as
 it is the only thing market maker does.
 """
 def work(self):
  if debug:
   print "Market maker waiting for market to initialize"
  yield waituntil,self,self.sim.marketInitialized
  while True:
   global buyerQueue, sellerQueue, numberOfBuyers, numberOfSellers
   if debug:
    print "Market maker waiting for market to get ready. "+\
       "len(buyerQueue) = "+str(len(buyerQueue))+" and "+\
       "len(sellerQueue) = "+str(len(sellerQueue))+\
       " while numberOfBuyers = "+str(numberOfBuyers)+\
       " and numberOfSellers = "+str(numberOfSellers)
   if numberOfBuyers == 0 or numberOfSellers == 0:
    raise Exception("Buyers and sellers equal to zero: market done")
   yield waituntil,self,self.sim.marketReady
   try:
    global selectedSellers, selectedBuyers, returnAgentNames
    if debug:
     print "selectedSellers = "+returnAgentNames(selectedSellers)
     print "selectedBuyers = "+returnAgentNames(selectedBuyers)
    # Check if a new round should begin
    if (len(selectedSellers)+len(selectedBuyers)) == (numberOfBuyers+\
              numberOfSellers):
     if len(self.sim.bidB.list) > 0:
      bestBids.append([self.sim.now(),self.sim.bidB.list[-1][0]])
     if len(self.sim.askB.list) > 0:
      bestAsks.append([self.sim.now(),self.sim.askB.list[0][0]])
     selectedSellers = []
     selectedBuyers = []
     yield hold,self,1
     if trades:
```

```
     print "At "+str(self.sim.now())+": no trades in"+\
        " this round: beginning a new round"
    # See if only buyers or sellers left, select the other group if
    # other group is entirely consumed. If agents left in both groups,
    # then select either a buyer or a seller from their queues randomly.
    selectedGroup = -1
    # First check for the agents left
    allBuyers = set(buyerQueue)
    usedBuyers = set(selectedBuyers)
    buyersLeft = list(allBuyers-usedBuyers)
    allSellers = set(sellerQueue)
    usedSellers = set(selectedSellers)
    sellersLeft = list(allSellers-usedSellers)
    # Then see their amounts and decide about the group
    if len(buyersLeft) == 0:
     selectedGroup = 1
    elif len(sellersLeft) == 0:
     selectedGroup = 0
    else:
     selectedGroup = random.randint(0,1)
    if selectedGroup == 0:
     # Select buyer from unselected buyers left in this round
     global selectedBuyer
     if len(buyersLeft) == 1:
      selectedBuyer = buyersLeft[0]
     else:
      selectedBuyer = buyersLeft[random.randint(0,\
               len(buyersLeft)-1)]
     selectedBuyers.append(selectedBuyer)
     if debug:
      print "Market maker ready, chose buyer "+\
         selectedBuyer.name
    else:
     # Select seller from unselected sellers left in this round
     global selectedSeller
     if len(sellersLeft) == 1:
      selectedSeller = sellersLeft[0]
     else:
      selectedSeller = sellersLeft[random.randint(0,\
              len(sellersLeft)-1)]
     selectedSellers.append(selectedSeller)
     if debug:
      print "Market maker ready, chose seller "+\
         selectedSeller.name
    yield waituntil,self,self.sim.marketDone
   except ValueError as inst:
    print "len(buyersLeft) == "+str(len(buyersLeft))+\
       " and len(sellersLeft) == "+str(len(sellersLeft))
    raise Exception("Market maker: ValueError: "+str(inst))
   except Exception as inst:
    raise Exception("Market maker: Exception: "+str(inst))

class Agent(Process):
 """
 Responsible for agents work. The main method is work(), but also other
 auxiliary methods are used by the agents.

 Agents characteristics
  - self.type: 0 = buyer, 1 = seller
  - self.valuation: any integer in range (1,maximumAssetValue)
  - self.hasTrader: 0 = has not, 1 = has
  - selfname: agent + str(integer), identifies the each agents uniquely
 """
 def __init__(self,name,sim):
  """
  Initialize agents with their characteristics, name is initialized by the
  Generator() defined below. Note that if the number of agents is uneven, then
  this function initializes agents so that there is one buyer more than sellers.
  """
  global numberOfBuyers, numberOfSellers, maximumAssetValue, demand, supply,\
```

```
     buyerValuations, sellerValuations, numberOfAgents
 Process.__init__(self,name,sim)
 self.hasTraded = 0
 self.type = -1
 self.valuation = -1
 try:
  if numberOfBuyers < 1.0*numberOfAgents/2:
   self.type = 0
   self.valuation = buyerValuations[numberOfBuyers]
   numberOfBuyers += 1
   for i in range(0,self.valuation):
    demand[i][1] += 1
  else:
   self.type = 1
   self.valuation = sellerValuations[numberOfSellers]
   numberOfSellers += 1
   for i in range(self.valuation-1,len(supply)):
    supply[i][1] += 1
 except ValueError as inst:
  raise Exception("Agent: __init__: ValueError: "+str(inst))
 except Exception as inst:
   raise Exception("Agent: __init__: Exception: "+str(inst))

def appendBid(self,bid,amount):
 """
 Inserts a bid and sorts bids from smallest to largest
 """
 global allBids, allBids0
 allBids.append([self.sim.now(),bid])
 if (self.sim.now() == 0):
  allBids0.append(bid)
 self.sim.bidB.list.append([bid,amount,self])
 self.sim.bidB.list = sorted(self.sim.bidB.list, key = lambda bid: bid[0])

def appendAsk(self,ask,amount):
 """
 Inserts an ask and sorts asks from smallest to largest.
 """
 global allAsks, allAsks0
 allAsks.append([self.sim.now(),ask])
 if (self.sim.now() == 0):
  allAsks0.append(ask)
 self.sim.askB.list.append([ask,amount,self])
 self.sim.askB.list = sorted(self.sim.askB.list, key = lambda ask: ask[0])

def selectBid(self):
 """
 Returns the largest bid and remove it from the list. Largest bid is
 found from the last place in the list.
 """
 if(len(self.sim.bidB.list) == 0):
  return -1
 return self.sim.bidB.list.pop()

def selectAsk(self):
 """
 Return the smallest ask and remove it from the list. Smallest ask is
 found from the first place in the list.
 """
 if(len(self.sim.askB.list) == 0):
  return -1
 aux = self.sim.askB.list[0]
 self.sim.askB.list.remove(aux)
 return aux

def seeBid(self):
 """
 Returns the value of the largest bid, which is found from the last place
 in the list
 """
```

```
    if(len(self.sim.bidB.list) == 0):
     return -1
    return self.sim.bidB.list[-1][0] #bids[len(bids)-1]

   def seeAsk(self):
    """
    Returns the value of the smallest ask, which is found from the first
    place in the list
    """
    if(len(self.sim.askB.list) == 0):
     return -1
    return self.sim.askB.list[0][0]

   def cancelOldLimitBids(self):
    """
    Cancels all limit bids from the bid limit order book for the
    agent, who is given as a parameter.
    """
    try:
     # First find orders to be cancelled
     aux = []
     for order in self.sim.bidB.list:
      if order[2] == self:
       aux.append(order)
     # Then cancel them
     for order in aux:
      self.sim.bidB.list.remove(order)
    except ValueError as inst:
     print "cancelOldLimitBids(): ValueError: "+str(inst)
    except Exception:
     print "cancelOldLimitBids(): Exception: "+str(inst)

   def cancelOldLimitAsks(self):
    """
    Cancels all limit asks from the ask limit order book for the
    agent, who is given as a parameter.
    """
    try:
     # First find orders to be cancelled
     aux = []
     for order in self.sim.askB.list:
      if order[2] == self:
       #print "cancelling ask for agent %s"%agent.name
       aux.append(order)
     # Then cancel them
     for order in aux:
      self.sim.askB.list.remove(order)
    except ValueError as inst:
     print "cancelOldLimitAsks(): ValueError: "+str(inst)
    except Exception:
     print "cancelOldLimitAsks(): Exception: "+str(inst)

   def work(self):
    """
    The main routine for agent: agents push themselves into buyer or seller
    queues, and wait if they got selected. If a particular agent is selected,
    then she looks at either limit asks or bids, depending whether the agent
    is a buyer or a seller, and if any of them satisfy her, then she trades.
    If the agent doesn't wan't to trade at the quotes available, then she
    appends her bid or ask to the corresponding limit order book.

    Because agent is implemented in SimPy and the limit order books are derived
    from the resource class, the agents always request the particular limit
    order book they need when they trade. This is heavy structure for a simple
    ZI-market, but offers possibilities to create more complex market structures.
    """
    global maximumAssetValue, numberOfBuyers, numberOfSellers, selectedSeller,\
        selectedBuyer, buyerQueue, sellerQueue, extractedProfit,\
        selectedSellers,selectedBuyers, bestAsks, bestBids,\
        budgetConstraint
```

```
if debug:
 print self.name+" initialized"
while (not self.hasTraded):
 if self.type == 0:
  # Self a buyer
  buyerQueue.append(self)
  yield waituntil,self,self.sim.marketMakerDone
  # First check if a buyer of a seller was chosen
  if selectedBuyer != -1:
   if(self == selectedBuyer):
    if debug:
     print self.name+" came to market"

    # Trade
    yield request,self,self.sim.bidB
    if budgetConstraint:
     u = random.uniform(1,self.valuation)
    else:
     u = random.uniform(1,maximumAssetValue)
    askPrice = self.seeAsk()
    if askPrice != -1 and u >= askPrice:
     bestAsks.append([self.sim.now(),askPrice])
     if self.seeBid() > 0:
      bestBids.append([self.sim.now(),self.seeBid()])
     sellingAsk = self.selectAsk()
     prices.append([self.sim.now(),askPrice,\
           len(self.sim.bidB.list),\
           len(self.sim.askB.list)])
     sellingAsk[2].hasTraded = 1
     self.hasTraded = 1
     numberOfSellers -= 1
     numberOfBuyers -= 1
     selectedSellers = []
     selectedBuyers = []
     increaseInProfit = self.valuation-\
            sellingAsk[2].valuation
     #if increaseInProfit > 0:
     extractedProfit += increaseInProfit
     if trades:
      print self.name+" bought and "+sellingAsk[2].name+\
         " has sold and profit was "+\
         str(self.valuation-sellingAsk[2].valuation)
      print "price = "+str(askPrice)+" "+\
         str(self.valuation-sellingAsk[2].valuation)
    else:
     self.appendBid(u,1)

    # Try to clear books if traded
    if self.hasTraded:
     if debug:
      print self.name+" clearing the books"
     yield request,self,self.sim.askB
     self.sim.askB.list = []
     yield release,self,self.sim.askB
     self.sim.bidB.list = []
     yield hold,self,1
     if trades:
      print "At "+str(self.sim.now())+\
         ": trade took place: beginning new round"


    # Release resources
    yield release,self,self.sim.bidB
    buyerQueue = []
    sellerQueue = []
    selectedBuyer = -1
    if debug:
     print "Buyer "+self.name+" left market and"+\
        " selectedBuyer == -1 now"
     print "selectedSeller == "+str(selectedSeller)+\
```

```
      " and selectedBuyer == "+str(selectedBuyer)
  else:
   # Another buyer was chosen
   if debug:
    print "Buyer "+self.name+\
      " waiting for market to be done"
   yield waituntil,self,self.sim.marketDone
 else:
  # A seller was chosen
  if debug:
   print "Buyer "+self.name+\
      " waiting for market to be done"
  yield waituntil,self,self.sim.marketDone
else:
 # Self a seller
 sellerQueue.append(self)
 yield waituntil,self,self.sim.marketMakerDone
 # First check if a buyer of a seller was chosen
 if selectedSeller != -1:
  if(self == selectedSeller):
   if debug:
    print self.name+" came to market"

   # Trade
   yield request,self,self.sim.askB
   if budgetConstraint:
    u = random.uniform(self.valuation,maximumAssetValue)
   else:
    u = random.uniform(1,maximumAssetValue)
   bidPrice = self.seeBid()
   if bidPrice != -1 and u <= bidPrice:
    bestBids.append([self.sim.now(),bidPrice])
    if self.seeAsk() > 0:
     bestAsks.append([self.sim.now(),self.seeAsk()])
    buyingBid = self.selectBid()
    prices.append([self.sim.now(),bidPrice,\
         len(self.sim.bidB.list),\
         len(self.sim.askB.list)])
    buyingBid[2].hasTraded = 1
    self.hasTraded = 1
    numberOfSellers -= 1
    numberOfBuyers -= 1
    selectedSellers = []
    selectedBuyers = []
    increaseInProfit = buyingBid[2].valuation-\
           self.valuation
    #if increaseInProfit > 0:
    extractedProfit += increaseInProfit
    if trades:
     print self.name+" has sold and "+\
        buyingBid[2].name+" has bought and"+\
        " profit was "+str(buyingBid[2].valuation-\
        self.valuation)
     print "price = "+str(bidPrice)+" "+\
        str(buyingBid[2].valuation-self.valuation)
   else:
    self.appendAsk(u,1)

   # Try to clear books if traded
   if self.hasTraded:
    if debug:
     print self.name+" clearing the books"
    yield request,self,self.sim.bidB
    self.sim.bidB.list = []
    yield release,self,self.sim.bidB
    self.sim.askB.list = []
    yield hold,self,1
    if trades:
     print "At "+str(self.sim.now())+": trade took"+\
        " place: beginning new round"
```

```
      # Release resources
      yield release,self,self.sim.askB
      sellerQueue = []
      buyerQueue = []
      selectedSeller = -1
      if debug:
       print "Seller "+self.name+" left market and"+\
          " selectedSeller == -1 now"
       print "selectedSeller == "+str(selectedSeller)+\
          " and selectedBuyer == "+str(selectedBuyer)
     else:
      # Another seller was chosen
      if debug:
       print "Buyer "+self.name+\
          " waiting for market to be done"
      yield waituntil,self,self.sim.marketDone
    else:
     # A buyer was chosen
     if debug:
      print "Buyer "+self.name+\
         " waiting for market to be done"
     yield waituntil,self,self.sim.marketDone
  # Leave market
  if self.type == 0:
   self.cancelOldLimitBids()
  else:
   self.cancelOldLimitAsks()

class Generator(Process):
 """
 Responsible for generating all agents at the
 beginning of the simulation. Initializes the names
 of the agents and puts them acting in the market.
 """
 def execute(self,agentNumber):
  for i in range(agentNumber):
   a = Agent("Trader "+str(i), sim=self.sim)
   self.sim.activate(a,a.work())
   yield hold,self,0

### Model ----------------------------------
class NeedResourcesModel(Simulation):
 """
 Implements the simulation itself. The first four functions are
 used by the market maker and the agents to communicate between
 themselves. The main method is run: it initializes all the model
 variables, runs the model and outputs data.
 """
 def marketReady(self):
  """
  Market maker waits for this to return True before she starts
  to choose which trader gets to trade.
  """
  global numberOfSellers, numberOfBuyers, buyerQueue, sellerQueue
  return (len(buyerQueue) == numberOfBuyers) and\
      (len(sellerQueue) == numberOfSellers)

 def marketDone(self):
  """
  Both market maker and the agents who did not got chosen as the
  trading agent wait for this to become True. After the agent, who
  was selected to trade, is ready, she sets either the selectedBuyer
  or selectedSeller again to -1, and releases all the
  """
  global selectedBuyer, selectedSeller
  return (selectedBuyer == -1) and (selectedSeller == -1)

 def marketMakerDone(self):
  """
```

```
 Used by agents to check if the market maker has done his decision
 about the fact that which agents was chosen to trade.
 """
 global selectedBuyer, selectedSeller
 return (selectedBuyer != -1) or (selectedSeller != -1)

def marketInitialized(self):
 """
 Used by the market maker to check that the generator has initialized
 all agents. Needed, because otherwise it could be that the market maker
 would start to act, because the marketReady() may return True although all
 agents were not initalized.
 """
 global numberOfAgents, numberOfSellers, numberOfBuyers
 return ((numberOfBuyers + numberOfSellers) == numberOfAgents)

def run(self):
 """
 Runs the model. First initializes all needed variables for the model and
 output. Then runs the model, and after that outputs data from the single
 run.
 """
 global numberOfAgents, numberOfRounds, prices, numberOfBuyers,\
     numberOfSellers,selectedBuyer, selectedSeller, buyerQueue,\
     sellerQueue, maximumAssetValue, supply, demand, extractedProfit,\
     selectedBuyers, selectedSellers, buyerValuations,\
     sellerValuations, statistics, rheight, rwidth, bestBids,\
     bestAsks, RMSD, initializeDemandAndSupply, EquilibriumPrice
 # Initializations of variables
 initializeDemandAndSupply()
 selectedBuyer = -1
 selectedSeller = -1
 buyerQueue = []
 sellerQueue = []
 selectedBuyers = []
 selectedSellers = []

 # Initializations of logs
 # output data
 prices = []
 bestBids = []
 bestAsks = []
 demand = []
 supply = []
 statistics = []
 for i in range(1,maximumAssetValue+1):
  demand.append([i,0])
  supply.append([i,0])
 extractedProfit = 0

 # Initialization of simulation components
 self.initialize()
 self.bidB = Book(sim=self)
 self.askB = Book(sim=self)
 g = Generator(name='gen',sim=self)
 self.activate(g,g.execute(agentNumber=numberOfAgents))
 marketMaker = MarketMaker(sim=self)
 self.activate(marketMaker,marketMaker.work())

 # Simulation
 print "Simulation started at "+str(time.ctime())
 try:
  print self.simulate(until=numberOfRounds)
 except Exception as inst:
  print "\n*** Exception occured during Simulation ***\n"
  print inst
 print "Simulation ended at "+str(time.ctime())
 if debug:
  print "Bids in the end:"
  for i in self.bidB.list:
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  print "price = %f, amount = %f" %(i[0],i[1])
 print "Asks in the end:"
 for i in self.askB.list:
  print "price = %f, amount = %f" %(i[0],i[1])
 print "Prices:"
 for i in prices:
  print "price = "+str(i)

# Plotting
if(len(prices) > 1):
 global outPath
 # Separate data
 rets = []
 pricesOnly = []
 pricesTimes = []
 pricesBids = []
 pricesAsks = []
 outstr = outPath+str(simulationIndex)+"_n_"+\
   str(numberOfAgents)+"_r_"+str(numberOfRounds)
 f = open(outstr+".txt", 'w')
 fret = open(outstr+"_ret.txt", 'w')
 for i in range(0,len(prices)):
  if i != 0:
   ret = math.log(prices[i][1]/prices[i-1][1])
   rets.append(ret)
   fret.write(str(ret)+"\n")
  f.write(str(prices[i])+"\n")
  pricesAsks.append(prices[i][3])
  pricesBids.append(prices[i][2])
  pricesOnly.append(prices[i][1])
  pricesTimes.append(prices[i][0])
 f.close()
 fret.close()

 # Plots using Rpy

 # Draw density estimate for returns using R
 if len(rets) > 1:
  densityData = r.density(rets,kernel="gaussian")
  densityData = [densityData['x'],densityData['y']]
  r.postscript(outstr+"_density_rets.ps",width=rwidth,height=rheight)
  r.plot(densityData[0],densityData[1],xlab='Logreturn',
      ylab='Probability',type='l',col="blue4",lty=1,lwd=1)
  r.dev_off()

 # Draw histogram of returns using R
 r.postscript(outstr+"_hist_rets.ps",width=rwidth,height=rheight)
 r.hist(rets,breaks=10,xlab="Logreturn",ylab="Number of observations",\
     main="",col="blue4")
 r.dev_off()

 if len(pricesOnly) > 1:
  # Draw density estimate for prices using R
  densityData = r.density(pricesOnly,kernel="gaussian")
  densityData = [densityData['x'],densityData['y']]
  r.postscript(outstr+"_density_prices.ps",width=rwidth,\
      height=rheight)
  r.plot(densityData[0],densityData[1],xlab='Price',\
      ylab='Probability',type='l',col="blue4",lty=1,lwd=1)
  r.dev_off()

 # Draw histogram for prices using R
 r.postscript(outstr+"_hist_prices.ps",width=rwidth,height=rheight)
 r.hist(pricesOnly,breaks=10,xlab="Price",\
     ylab="Number of observations",main="",col="blue4")
 r.dev_off()

 # Index for printing statistics
 statIndex = 0
```

```
# Equilibrium price
statistics.append(equilibriumPrice)
print "Equilibrium price was "+str(statistics[statIndex])
statIndex += 1

# Draw pricesprocess + demand and sypply schedule using R
supplyQ = []
supplyP = []
demandQ = []
demandP = []
maxSupply = 0
for i,item in enumerate(supply):
 supplyQ.append(supply[i][1])
 supplyP.append(supply[i][0])
 demandQ.append(demand[i][1])
 demandP.append(demand[i][0])
 if supply[i][1] > demand[i][1] and supply[i][1] > maxSupply:
  maxSupply = supply[i][1]
 else:
  if demand[i][1] > maxSupply:
   maxSupply = demand[i][1]
r.postscript(outstr+"_price_demand_supply.ps",width=rwidth,\
    height=rheight)
r.par(mfrow=r.c(3,1))
r.plot([0,numberOfRounds],[0,maximumAssetValue],type="n",\
    xlab="Round/Quantity",ylab="Price",main="")
r.lines(pricesTimes,pricesOnly,col="blue4",lty=1,lwd=1)
r.lines(demandQ,demandP,col="darkorange",lty=1,lwd=1)
r.lines(supplyQ,supplyP,col="darkgreen",lty=1,lwd=1)
r.abline(equilibriumPrice,0,col="darkred",lwd=1,lty=2)
# legend
colors = r.c("blue4","darkorange","darkgreen","darkred")
linetype = r.c(1,1,1,2)
names = r.c("Priceprocess","Demand","Supply","Equilibrium price = "+\
   str(equilibriumPrice))
r.legend(numberOfRounds-15,maximumAssetValue,names, cex=0.8, col=colors,\
   lty=linetype, lwd=1)

# Draw priceprocess + best bids + best asks
bidsOnly = []
asksOnly = []
bidsTimes = []
asksTimes = []
for item in bestBids:
 bidsTimes.append(item[0])
 bidsOnly.append(item[1])
for item in bestAsks:
 asksTimes.append(item[0])
 asksOnly.append(item[1])
r.plot([0,numberOfRounds],[0,maximumAssetValue],type="n",\
    xlab="Round",ylab="Price",main="")
r.lines(pricesTimes,pricesOnly,col="blue4",lty=1,lwd=1)
r.points(bidsTimes,bidsOnly,col="darkorange",pch=2)
r.points(asksTimes,asksOnly,col="darkgreen",pch=6)
r.abline(equilibriumPrice,0,col="darkred",lwd=1,lty=2)
# legend
colors = r.c("blue4","darkorange","darkgreen","darkred")
names = r.c("Transaction price","Highest bid","Lowest ask",\
   "Equilibrium price = "+str(equilibriumPrice))
r.legend(numberOfRounds-14,maximumAssetValue,names, cex=0.8,\
   col=colors,lty=r.c(1,0,0,2),lwd=1,pch=r.c(-1,2,6,-1))

# Draw priceprocess + number of quotes, bids and asks, eq. price
r.plot([0,numberOfRounds],[0,max(maximumAssetValue,max(pricesBids),\
  max(pricesAsks))],type="n",xlab="Round",ylab="Price, Amount",\
  main="")
r.lines(pricesTimes,pricesOnly,col="blue4",lty=1,lwd=1)
r.lines(pricesTimes,pricesBids,col="darkorange",type="o",lty=1,\
  lwd=1,pch=2)
r.lines(pricesTimes,pricesAsks,col="darkgreen",type="o",lty=1,\
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  lwd=1,pch=6)
r.abline(equilibriumPrice,0,col="darkred",lwd=1,lty=2)
# legend
colors = r.c("blue4","darkorange","darkgreen","darkred")
names = r.c("Transaction price","Number of bids","Number of asks",\
   "Equilibrium price = "+str(equilibriumPrice))
r.legend(numberOfRounds-14,max(maximumAssetValue,max(pricesBids),\
   max(pricesAsks)),names,cex=0.8, col=colors,lty=r.c(1,1,1,2),\
   lwd=1,pch=r.c(-1,2,6,-1))
r.par(mfrow=r.c(1,1)) # need to be here to prevent a pop-up window
r.dev_off()

# Efficiency
possibleProfit = 0
for item in buyerValuations:
 if equilibriumPrice < item:
  possibleProfit += item-equilibriumPrice
for item in sellerValuations:
 if equilibriumPrice > item:
  possibleProfit += equilibriumPrice-item
statistics.append(1.0*extractedProfit/possibleProfit)
print "Efficiency was "+str(statistics[statIndex])+" composed of "+\
   str(extractedProfit)+" and "+str(possibleProfit)

# number of transactions
statistics.append(len(prices))
statIndex += 1
print "Number of transactions was "+str(statistics[statIndex])

# Mean of prices
statistics.append(r.mean(pricesOnly))
statIndex += 1
print "Mean was of prices "+str(statistics[statIndex])

# Median of prices
statistics.append(r.median(pricesOnly))
statIndex += 1
print "Median was of prices "+str(statistics[statIndex])

# Maximum of prices
statistics.append(max(pricesOnly))
statIndex += 1
print "Maximum of prices was "+str(statistics[statIndex])

# Minimum of prices
statistics.append(min(pricesOnly))
statIndex += 1
print "Minimum of prices was "+str(statistics[statIndex])

# stdev of prices
statistics.append(r.sd(pricesOnly))
statIndex += 1
print "Stdev of prices was "+str(statistics[statIndex])

# moments-package
r.library('moments')

# Kurtosis of prices
statistics.append(r.kurtosis(pricesOnly))
statIndex += 1
print "Kurtosis of prices was "+str(statistics[statIndex])

# Skewness prices
statistics.append(r.skewness(pricesOnly))
statIndex += 1
print "Skewness prices was "+str(statistics[statIndex])

aux = r.quantile(pricesOnly)
statistics.append(aux['25%'])
statIndex += 1
```

```
    print "25% quantile of prices was "+str(statistics[statIndex])

    statistics.append(aux['75%'])
    statIndex += 1
    print "75% quantile of prices was "+str(statistics[statIndex])


    # Mean of rets
    statistics.append(r.mean(rets))
    statIndex += 1
    print "Mean was of rets "+str(statistics[statIndex])

    # Median of rets
    statistics.append(r.median(rets))
    statIndex += 1
    print "Median was of rets "+str(statistics[statIndex])

    # Maximum of rets
    statistics.append(max(rets))
    statIndex += 1
    print "Maximum of rets was "+str(statistics[statIndex])

    # Minimum of rets
    statistics.append(min(rets))
    statIndex += 1
    print "Minimum of rets was "+str(statistics[statIndex])

    # stdev of rets
    statistics.append(r.sd(rets))
    statIndex += 1
    print "Stdev of rets was "+str(statistics[statIndex])

    # Kurtosis of returns
    statistics.append(r.kurtosis(rets))
    statIndex += 1
    print "Kurtosis of returns was "+str(statistics[statIndex])

    # Skewness returns
    statistics.append(r.skewness(rets))
    statIndex += 1
    print "Skewness returns was "+str(statistics[statIndex])

    # Coefficient of convergence
    # Taa pitaa laskea sd:na eq-hinnan ymparilla!
    statistics.append(100.0*RMSD(pricesOnly,\
          equilibriumPrice)/equilibriumPrice)
    statIndex += 1
    print "Coeff. of convergence (Smith) was "+str(statistics[statIndex])

    # Jarque-Bera test for normality of returns
    jarqueBera = r.jarque_test(rets)
    statistics.append(jarqueBera['p.value'])
    statIndex += 1
    print "p-value for jarque-bera test for the normality of"+\
       " returns was "+str(statistics[statIndex])
    statistics.append(jarqueBera['statistic']['JB'])
    statIndex += 1

   else:
    print "No trading occured"

# Experiment ------------------------------
seeds = range(1,numberOfRuns+1)
seedInd = 0
random.seed(SEED)
statsAll = []
statisticsC = []
statisticsU = []
pricesAll = []
pricesC = []
```

```
pricesU = []
bestBidsAll = []
bestAsksAll = []
bestBidsAllC = []
bestAsksAllC = []
bestBidsAllU = []
bestAsksAllU = []
outPath = outPathC
for ind in range(1,numberOfRuns*2+1):
 print "** Run number "+str(ind)+" **"
 if ind == numberOfRuns+1:
  budgetConstraint = False
  statisticsC = statsAll
  pricesC = pricesAll
  bestBidsAllC = bestBidsAll
  bestAsksAllC = bestAsksAll
  statsAll = []
  pricesAll = []
  bestAsksAll = []
  bestBidsAll = []
  fstats = open(outPathZI+"allBidsC.txt", 'w')
  for item in allBids:
   fstats.write(str(item)+";")
  fstats = open(outPathZI+"allAsksC.txt", 'w')
  for item in allAsks:
   fstats.write(str(item)+";")
  fstats = open(outPathZI+"allBids0C.txt", 'w')
  for item in allBids0:
   fstats.write(str(item)+",")
  fstats = open(outPathZI+"allAsks0C.txt", 'w')
  for item in allAsks0:
   fstats.write(str(item)+",")
  allBids = []
  allAsks = []
  allBids0 = []
  allAsks0 = []
  simulationIndex = 1
  fstats = open(outPath+str(simulationIndex)+"_n_"+\
       str(numberOfAgents)+"_r_"+\
       str(numberOfRounds)+"_stats.txt", 'w')
  for row in statisticsC:
   fstats.write(str(row)+"\n")
  fstats.close()
  seedInd = 0
  outPath = outPathU
  print "\n **Beginning runs with ZI-U agents** \n"
 if convergenceRun:
  print "Seed set back to 1"
  random.seed(1)
 elif singleSeed:
  # Do nothing
  print "No change in seed"
 else:
  random.seed(seeds[seedInd])
  print "Seed set to next from the seeds-array: "+str(seeds[seedInd])
  seedInd += 1
 NeedResourcesModel().run()
 simulationIndex += 1
 statsAll.append(statistics)
 pricesAll.append(prices)
 bestBidsAll.append(bestBids)
 bestAsksAll.append(bestAsks)

# for output
statisticsU = statsAll
pricesU = pricesAll
bestBidsAllU = bestBidsAll
bestAsksAllU = bestAsksAll

# statisticsU
```

```
fstats = open(outPath+str(simulationIndex)+"_n_"+str(numberOfAgents)+"_r_"+\
     str(numberOfRounds)+"_stats.txt", 'w')
for row in statisticsU:
 fstats.write(str(row)+"\n")
fstats.close()

# Statistics for the whole experiment
indexList = [1,2,3,4,5,6,7,8,9,10,11,19]
auxStats = []
fstats = open(outPathZI+str(simulationIndex)+"_n_"+str(numberOfAgents)+"_r_"+\
     str(numberOfRounds)+"_stats.txt", 'w')
for index in indexList:
 aux = []
 for row in statisticsC:
  aux.append(row[index])
 auxStats.append(r.mean(aux))
 auxStats.append(r.sd(aux))
 auxStats.append(len(aux))
fstats.write(str(auxStats)+"\n")
auxStats = []
for index in indexList:
 aux = []
 for row in statisticsU:
  aux.append(row[index])
 auxStats.append(r.mean(aux))
 auxStats.append(r.sd(aux))
 auxStats.append(len(aux))
fstats.write(str(auxStats)+"\n")
fstats.close()

# Tests for differences between constrained and unconstrained agents
efficienciesC = []
coefsConvergC = []
efficienciesU = []
coefsConvergU = []

for row in statisticsC:
 efficienciesC.append(100.0*row[1])
 coefsConvergC.append(row[19])

for row in statisticsU:
 efficienciesU.append(100.0*row[1])
 coefsConvergU.append(row[19])

r.library('coin')
wilcoxTestEff = r.wilcox_test(efficienciesC,efficienciesU,paired=1,\
         alternative="greater")
print "p-value for efficiencies using Wilcoxon test "+\
   str(wilcoxTestEff['p.value'])
wilcoxTestCoef = r.wilcox_test(coefsConvergC,coefsConvergU,paired=1,\
          alternative="less")
print "p-value of coefficients of convergence using Wilcoxon test "+\
   str(wilcoxTestCoef['p.value'])
fstats = open(outPathZI+str(numberOfRuns)+".txt", 'w')
fstats.write(str(wilcoxTestCoef)+"\n")
fstats.write(str(wilcoxTestEff)+"\n")
fstats.close()

# Print efficiencies and coefs of convergence and RMSD together
r.postscript(outPathZI+str(numberOfRuns)+"_eff_conv_RMSD.ps",\
    width=rwidth,height=rheight)
r.par(mfrow=r.c(3,1))

# Efficiencies
r.plot([0,numberOfRuns],[0,100],type="n",xlab="Run",\
    ylab="Efficiency",main="")
r.lines(efficienciesC,col="blue4",type="o",lty=1,lwd=1,pch=15)
r.lines(efficienciesU,col="darkorange",type="o",lty=1,lwd=1,pch=17)
colors = r.c("blue4","darkorange")
names = r.c("With budget constraint","Without budget constraint")
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r.legend(numberOfRuns-10,30,names,cex=0.8,col=colors,lty=r.c(1,1),\
   lwd=1,pch=r.c(15,17))

# Coefs of convergence
r.plot([0,numberOfRuns],[0,100],type="n",xlab="Run",\
    ylab="Coefficient of Convergence",main="")
r.lines(coefsConvergC,col="blue4",type="o",lty=1,lwd=1,pch=15)
r.lines(coefsConvergU,col="darkorange",type="o",lty=1,lwd=1,pch=17)
colors = r.c("blue4","darkorange")
names = r.c("With budget constraint","Without budget constraint")
r.legend(numberOfRuns-10,100,names,cex=0.8,col=colors,lty=r.c(1,1),\
   lwd=1,pch=r.c(15,17))

# Convergence using RMSD, results applicable only if same seed in all runs!
# Match prices buy their transaction rounds
# Only rounds with more than 10 observations accepted
pricesOnlyC = []
pricesOnlyU = []
for i in range(0,numberOfRounds+1):
 pricesOnlyC.append([])
 pricesOnlyU.append([])
for pricesItem in pricesC:
 for item in pricesItem:
  pricesOnlyC[item[0]].append(item[1])
for pricesItem in pricesU:
 for item in pricesItem:
  pricesOnlyU[item[0]].append(item[1])
convsC = []
convsCT = []
convsCN = []
convsU = []
convsUT = []
convsUN = []
for i,item in enumerate(pricesOnlyC):
 if len(item)>10:
  convsC.append(RMSD(item,statistics[0]))
  convsCT.append(i)
  convsCN.append(len(item))
for i,item in enumerate(pricesOnlyU):
 if len(item)>10:
  convsU.append(RMSD(item,statistics[0]))
  convsUT.append(i)
  convsUN.append(len(item))

r.plot([0,numberOfRounds],[0,100],type="n",xlab="Round",\
  ylab="RMSD / Number of observations ",main="")
r.lines(convsCT,convsC,col="blue4",type="o",lty=1,lwd=1,pch=15)
r.lines(convsUT,convsU,col="darkorange",type="o",lty=1,lwd=1,pch=17)
r.points(convsCT,convsCN,col="blue4",pch=12)
r.points(convsUT,convsUN,col="darkorange",pch=9)
colors = r.c("blue4","darkorange","blue4","darkorange")
names = r.c("With budget constraint","Without budget constraint",\
   "N with budget constraint","N without budget constraint")
r.legend(numberOfRounds-16,100,names,cex=0.8,col=colors,lty=r.c(1,1,0,0),\
   lwd=1,pch=r.c(15,17,12,9))
r.par(mfrow=r.c(1,1))
r.dev_off()

# Output to files
fstats = open(outPathZI+str(numberOfRuns)+"_pricesC_0.txt", 'w')
fstats.write(str(pricesOnlyC[0])+"\n")
fstats.close()

fstats = open(outPathZI+str(numberOfRuns)+"_pricesC.txt", 'w')
for item in pricesOnlyC:
 for price in item:
  fstats.write(str(price)+",")
fstats.close()

fstats = open(outPathZI+str(numberOfRuns)+"_convergence.txt", 'w')
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```
fstats.write(str(coefsConvergC)+"\n")
fstats.write(str(coefsConvergU)+"\n")
fstats.close()

fstats = open(outPathZI+str(numberOfRuns)+"_efficiencies.txt", 'w')
fstats.write(str(efficienciesC)+"\n")
fstats.write(str(efficienciesU)+"\n")
fstats.close()

fstats = open(outPathZI+str(numberOfRuns)+"_RMSD.txt", 'w')
fstats.write(str(convsC)+"\n")
fstats.write(str(convsCT)+"\n")
fstats.write(str(pricesOnlyC)+"\n")
fstats.write(str(convsU)+"\n")
fstats.write(str(convsUT)+"\n")
fstats.write(str(pricesOnlyU)+"\n")
fstats.close()

# Draw estimates for the ask, bid and price densities

# first separate prices, bids and asks from time data
bidsOnlyAllC = []
asksOnlyAllC = []
pricesOnlyAllC = []
bidsOnlyAllU = []
asksOnlyAllU = []
pricesOnlyAllU = []
for i in range(0,numberOfRounds+1):
 bidsOnlyAllC.append([])
 asksOnlyAllC.append([])
 pricesOnlyAllC.append([])
 bidsOnlyAllU.append([])
 asksOnlyAllU.append([])
 pricesOnlyAllU.append([])
for pricesItem in pricesC:
 for item in pricesItem:
  pricesOnlyAllC[item[0]].append(item[1])
for bidsItem in bestBidsAllC:
 for item in bidsItem:
  bidsOnlyAllC[item[0]].append(item[1])
for asksItem in bestAsksAllC:
 for item in asksItem:
  asksOnlyAllC[item[0]].append(item[1])
for pricesItem in pricesU:
 for item in pricesItem:
  pricesOnlyAllU[item[0]].append(item[1])
for bidsItem in bestBidsAllU:
 for item in bidsItem:
  bidsOnlyAllU[item[0]].append(item[1])
for asksItem in bestAsksAllU:
 for item in asksItem:
  asksOnlyAllU[item[0]].append(item[1])

fstats = open(outPathZI+str(numberOfRuns)+"_bidsOnlyAllC.txt", 'w')
fstats.write(str(bidsOnlyAllC))
fstats.close()
fstats = open(outPathZI+str(numberOfRuns)+"_asksOnlyAllC.txt", 'w')
fstats.write(str(asksOnlyAllC))
fstats.close()
fstats = open(outPathZI+str(numberOfRuns)+"_pricesOnlyAllC.txt", 'w')
fstats.write(str(pricesOnlyAllC))
fstats.close()
fstats = open(outPathZI+str(numberOfRuns)+"_bidsOnlyAllU.txt", 'w')
fstats.write(str(bidsOnlyAllU))
fstats.close()
fstats = open(outPathZI+str(numberOfRuns)+"_asksOnlyAllU.txt", 'w')
fstats.write(str(asksOnlyAllU))
fstats.close()
fstats = open(outPathZI+str(numberOfRuns)+"_pricesOnlyAllU.txt", 'w')
fstats.write(str(pricesOnlyAllU))
```

```
fstats.close()


# Then take only those prices and quotes, which are from
# Rounds 0,10,20,...,150
asksGroupedC = []
bidsGroupedC = []
pricesGroupedC = []
asksGroupedU = []
bidsGroupedU = []
pricesGroupedU = []

densityStepSize = 15
for i in range(0,numberOfRounds+1,densityStepSize):
 asksGroupedC.append([i,asksOnlyAllC[i]])
 bidsGroupedC.append([i,bidsOnlyAllC[i]])
 asksGroupedU.append([i,asksOnlyAllU[i]])
 bidsGroupedU.append([i,bidsOnlyAllU[i]])

densityStepSize = 5
for i in range(0,numberOfRounds+1,densityStepSize):
 pricesGroupedC.append([i,pricesOnlyAllC[i]])
 pricesGroupedU.append([i,pricesOnlyAllU[i]])

fstats = open(outPathZI+str(numberOfRuns)+"_bidsGroupedC.txt", 'w')
fstats.write(str(bidsGroupedC))
fstats.close()
fstats = open(outPathZI+str(numberOfRuns)+"_asksGroupedC.txt", 'w')
fstats.write(str(asksGroupedC))
fstats.close()
fstats = open(outPathZI+str(numberOfRuns)+"_pricesGroupedC.txt", 'w')
fstats.write(str(pricesGroupedC))
fstats.close()
fstats = open(outPathZI+str(numberOfRuns)+"_bidsGroupedU.txt", 'w')
fstats.write(str(bidsGroupedU))
fstats.close()
fstats = open(outPathZI+str(numberOfRuns)+"_asksGroupedU.txt", 'w')
fstats.write(str(asksGroupedU))
fstats.close()
fstats = open(outPathZI+str(numberOfRuns)+"_pricesGroupedU.txt", 'w')
fstats.write(str(pricesGroupedU))
fstats.close()

# Draw density estimate for prices, asks and bids at different times using R
# with budget constraint
r.postscript(outPathZI+str(numberOfRuns)+"C_densities.ps",width=rwidth,\
    height=rheight)
r.par(mfrow=r.c(3,1))
r.library('colorRamps')
numberOfColors = int(1.0*numberOfRounds/densityStepSize)+2
colorsA = r.matlab_like(numberOfColors*2)[numberOfColors:numberOfColors*2]

# Price
try:
 r.plot([0,200],[0,0.2],type="n", xlab="Price",ylab="PDF",main="Prices")
 namesA = []
 ltyA = []
 for i,item in enumerate(pricesGroupedC):
  if len(item[1]) > 10:
   densityData = r.density(item[1],kernel="gaussian")
   r.lines(densityData['x'],densityData['y'],type="l",\
     col=colorsA[i],lty=1,lwd=1)
   namesA.append("Round "+str(item[0])+", mean = "+\
        str(round(r.mean(item[1]),1))+", n = "+str(len(item[1])))
   ltyA.append(1)
 r.lines([equilibriumPrice,equilibriumPrice],[-1,10],col="darkred",lwd=1,lty=2)
 r.legend(173,0.1,namesA,cex=0.8,col=colorsA,lty=ltyA,lwd=1)
except Exception as inst:
 print inst
```

```
# bids
try:
 r.plot([0,200],[0,1.4],type="n", xlab="Price",ylab="PDF",main="Bids")
 namesA = []
 ltyA = []
 for i,item in enumerate(bidsGroupedC):
  if len(item[1]) > 10:
   densityData = r.density(item[1],kernel="gaussian")
   r.lines(densityData['x'],densityData['y'],type="l",\
     col=colorsA[i],lty=1,lwd=1)
   namesA.append("Round "+str(item[0])+", mean = "+\
        str(round(r.mean(item[1]),1)))
   ltyA.append(1)
 r.lines([equilibriumPrice,equilibriumPrice],[-1,10],col="darkred",lwd=1,lty=2)
 r.legend(180,1.4,namesA,cex=0.8,col=colorsA,lty=ltyA,lwd=2)
except Exception as inst:
 print inst

# Asks
try:
 r.plot([0,200],[0,1.4],type="n", xlab="Price",ylab="PDF",main="Asks")
 namesA = []
 ltyA = []
 for i,item in enumerate(asksGroupedC):
  if len(item[1]) > 10:
   densityData = r.density(item[1],kernel="gaussian")
   r.lines(densityData['x'],densityData['y'],type="l",\
     col=colorsA[i],lty=1,lwd=1)
   namesA.append("Round "+str(item[0])+", mean = "+\
        str(round(r.mean(item[1]),1)))
   ltyA.append(1)
 r.lines([equilibriumPrice,equilibriumPrice],[-1,10],col="darkred",lwd=1,lty=2)
 r.legend(180,1.4,namesA,cex=0.8,col=colorsA,lty=ltyA,lwd=2)
 r.par(mfrow=r.c(1,1))
 r.dev_off()
except Exception as inst:
 print inst

# Without budget constraint

r.postscript(outPathZI+str(numberOfRuns)+"U_densities.ps",\
    width=rwidth,height=rheight)
r.par(mfrow=r.c(3,1))

# Price
try:
 r.plot([0,200],[0,0.1],type="n", xlab="Price",ylab="PDF",main="")
 namesA = []
 ltyA = []
 for i,item in enumerate(pricesGroupedU):
  if len(item[1]) > 10:
   densityData = r.density(item[1],kernel="gaussian")
   r.lines(densityData['x'],densityData['y'],type="l",\
     col=colorsA[i],lty=1,lwd=1)
   namesA.append("Round "+str(item[0])+", mean = "+\
        str(round(r.mean(item[1]),1))+\
        ", n = "+str(len(item[1])))
   ltyA.append(1)
 r.legend(173,0.1,namesA,cex=0.8,col=colorsA,lty=ltyA,lwd=2)
except Exception as inst:
 print inst

# bids
try:
 r.plot([0,200],[0,0.1],type="n", xlab="Price",ylab="PDF",main="")
 namesA = []
 ltyA = []
 for i,item in enumerate(bidsGroupedU):
  if len(item[1]) > 10:
   densityData = r.density(item[1],kernel="gaussian")
```

```
   r.lines(densityData['x'],densityData['y'],type="l",\
     col=colorsA[i],lty=1,lwd=1)
   namesA.append("Round "+str(item[0])+", mean = "+\
        str(round(r.mean(item[1]),1)))
   ltyA.append(1)
 r.legend(180,0.1,namesA,cex=0.8,col=colorsA,lty=ltyA,lwd=2)
except Exception as inst:
 print inst

# Asks
try:
 r.plot([0,200],[0,0.1],type="n", xlab="Price",ylab="PDF",main="")
 namesA = []
 ltyA = []
 for i,item in enumerate(asksGroupedU):
  if len(item[1]) > 10:
   densityData = r.density(item[1],kernel="gaussian")
   r.lines(densityData['x'],densityData['y'],type="l",\
     col=colorsA[i],lty=1,lwd=1)
   namesA.append("Round "+str(item[0])+", mean = "+\
        str(round(r.mean(item[1]),1)))
   ltyA.append(1)
 r.legend(180,0.1,namesA,cex=0.8,col=colorsA,lty=ltyA,lwd=2)
 r.par(mfrow=r.c(1,1))
 r.dev_off()
except Exception as inst:
 print inst

# End of file
```

B Source code for R

```
## 9.3.2011, Niklas Jahnsson
##
## .r-file
## to show how the generalized market wide bid and ask densities are created
##
## to generate pictures that show transaction price densities
##  -for theoretical bids and asks on the Marshallian path
##  -for theoretical bids, asks and transaction prices
##  -for theoretical transaction prices according to Cliff and Bruten 1997
##  -for transaction prices on first round
##  -for transaction prices on all rounds
##
## Three different demand-supply types are investigated: symmetric, fixed supply
## and fixed demand.
##
## Uses accept-reject method to generate a sample from the theoretical
## market bid and ask densities, and then selects those values that are
## in the intersection of the two densities.
##
## As a input needs
## sellerValuations, buyerValuations
## transactionPrices0
## transactionPricesAll
## bids
## asks
##

## An example of the general approach to calculate market wide
## bids and asks presented in equations 8 and 9.

sellerValuations = seq(26,174,2)
buyerValuations = seq(174,26,-2)

supplyPDF = function(p){
  ret = 0
  if(p<=200){
    for(i in 1:length(sellerValuations)){
      if(p>=sellerValuations[i]){
        ret = ret + 1/(length(sellerValuations)*(200-sellerValuations[i]))
      }
    }
  }
  ret
}

integrandS = function(x){
  y = numeric(length(x))
  for(i in 1:length(x)){
    y[i] = supplyPDF(x[i])
  }
  y
}
integrate(integrandS,0,200)
# 0.999957 with absolute error < 9.2e-05

demandPDF = function(p){
  ret = 0
  if(p>=1){
    for(i in 1:length(buyerValuations)){
      if(p<=buyerValuations[i]){
        ret = ret + 1/(length(buyerValuations)*(buyerValuations[i]-1))
      }
    }
  }
  ret
}

integrandD = function(x){
```

```
  y = numeric(length(x))
  for(i in 1:length(x)){
    y[i] = demandPDF(x[i])
  }
  y
}
integrate(integrandD,0,200,subdivisions=2000)
# 0.9999982 with absolute error < 1.1e-05

## Figures (14,16) for pdfs in the theoretical Marshallian path

# With colors
#library('colorRamps')
#colors = matlab.like(148)
#colors = colors[70:144]
#library('TeachingDemos')
#colors = col2gray(colors)

# With greyscale colors
library('grDevices')
colors = grey.colors(38,start=0.95,end=0.0)

# Output

postscript("/home/nikke/Kuvat/kehittyminen_intra_extra_symmetric.ps",\
   width=100,height=100)
plot(c(0,200),c(0,0.04),type="n",xlab="Price",ylab="PDF")
namesA = character(11)
colA = character(11)
ltyA = numeric(11)
nAR = 600
sampledPrices = 100
for(i in 1:38){
  sellerValuations = seq(24+2*i,174,2)
  buyerValuations = seq(26,176-2*i,2)#seq(174,26,-2)
  xsPDF = seq(0,200,1)
  ysPDF = numeric(201)
  for(j in 1:200){
    ysPDF[j] = supplyPDF(xsPDF[j])
  }
  if((i-1)%%1 == 0){
    lines(xsPDF,ysPDF,lwd=1,col=colors[i])
  }
  xdPDF = seq(0,200,1)
  ydPDF = numeric(201)
  for(j in 1:200){
    ydPDF[j] = demandPDF(xdPDF[j])
  }
  if((i-1)%%1 == 0){
    lines(xdPDF,ydPDF,lwd=1,col=colors[i])
  }
  if(((i-1)%%4) == 0){
    print(i)
    namesA[((i-1)/4)+1] = paste("Round ",(i-1),sep="")
    colA[((i-1)/4)+1] = colors[i]
    ltyA[((i-1)/4)+1] = 1
  }
  if(i == 38){
    print(i)
    namesA[11] = paste("Round ",(i),sep="")
    colA[11] = colors[i]
    ltyA[11] = 1
  }

  # ACCEPT-REJECT

  # Demand
  xd=numeric(nAR)
  yd=numeric(nAR)
  for(j in 1:nAR){
```

```
    while(1){
      z=runif(1,1,176-2*i)
      u=runif(1,0,1)
      if(u<(demandPDF(z)/(4/174))){
        print(j)
        xd[j]=z
        yd[j]=u*4/174
        break
      }
    }
   }
  #points(xd,yd,pch=".",col="gray40")
  # Supply
  xs=numeric(nAR)
  ys=numeric(nAR)
  for(j in 1:nAR){
    while(1){
      z=runif(1,24+2*i,200)
      u=runif(1,0,1)
      if(u<(supplyPDF(z)/(4/174))){
        print(j)
        xs[j]=z
        ys[j]=4/174*u
        break
      }
    }
  }
  #points(xs,ys,pch=".",col="black")

  accepted2 = -1
  accepted2y = -1
  index = 1
  for(j in 1:nAR){
    if(ys[j]<=demandPDF(xs[j])){
     accepted2[index] = xs[j]
     accepted2y[index] = ys[j]
     index = index + 1
    }
    if(yd[j]<=supplyPDF(xd[j])){
     accepted2[index] = xd[j]
     accepted2y[index] = yd[j]
     index = index + 1
    }
  }
  #points(accepted2,accepted2y,col="darkred",pch=".")
  sampledPrices = c(sampledPrices,accepted2)
}
legend(174,0.04,namesA,cex=0.8,col=colA,lty=ltyA,lwd=2)
dev.off()
dev.off()

# Compare prices on Marshallian path to all transaction prices
aux = read.csv("/home/nikke/SimPyOut/ZI/050211/big3/ZI_n_100_pricesC.txt",\
      sep=",",header=FALSE)
aux = data.matrix(aux)
transactionPricesAll = numeric(length(aux))
for(i in 1:length(aux)){
  transactionPricesAll[i] = aux[i][1]
}
postscript("/home/nikke/Kuvat/vertailu_prices_Marshall.ps",width=100,height=100)
sampledPrices = sampledPrices[sampledPrices>0]
plot(c(0,200),c(0,0.03),type="n",xlab="Price",ylab="PDF")
lines(density(sampledPrices),col="gray40",lty=1,lwd=2)
lines(density(transactionPricesAll),col="black",lty=2,lwd=2)
colors = c("grey40","black")
names = c(paste("Intersection, n=",length(sampledPrices),sep=""),\
    paste("Empirical all rounds, n=",length(transactionPricesAll),sep=""))
legend(150,0.03,names,cex=0.8,col=colors,lty=c(1,2),pch=c(-1,-1),lwd=2)
dev.off()
```

```
## fixed demand and supply: excess demand correct analysis

sellerValuations = array(60,dim=50)
buyerValuations = array(140,dim=100)
xsPDF = seq(0,200,0.1)
ysPDF = numeric(2001)
for(i in 1:2000){
  ysPDF[i] = supplyPDF(xsPDF[i])
}
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF)
area = 0
for(i in 1:200){
  area = area + supplyPDF(i)
}
area

xdPDF = seq(0,200,0.1)
ydPDF = numeric(2001)
for(i in 1:2000){
  ydPDF[i] = demandPDF(xdPDF[i])
}
lines(xdPDF,ydPDF,col="gray40")
area = 0
for(i in 1:200){
  area = area + demandPDF(i)
}
area

## Accept-reject sampling
n=10000

# Demand
xd=numeric(n)
yd=numeric(n)
for(i in 1:n){
  while(1){
    z=runif(1,1,174)
    u=runif(1,0,1)
    if(u<(demandPDF(z)/(2/174))){
      print(i)
      xd[i]=z
      yd[i]=u*2/174
      break
    }
  }
}
points(xd,yd,pch=".",col="gray40")

# Supply
xs=numeric(n)
ys=numeric(n)
for(i in 1:n){
  while(1){
    z=runif(1,26,200)
    u=runif(1,0,1)
    if(u<(supplyPDF(z)/(2/174))){
      print(i)
      xs[i]=z
      ys[i]=2/174*u
      break
    }
  }
}
points(xs,ys,pch=".",col="black")

accepted2 = -1
accepted2y = -1
index = 1
```

```
for(i in 1:n){
  if(ys[i]<=demandPDF(xs[i])){
      accepted2[index] = xs[i]
      accepted2y[index] = ys[i]
      index = index + 1
  }
  if(yd[i]<=supplyPDF(xd[i])){
      accepted2[index] = xd[i]
      accepted2y[index] = yd[i]
      index = index + 1
  }
}

## Output
points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=2,lwd=2)
transactionPrices0 = c(...)
bids0 = c(...)
asks0 = c(...)
lines(density(transactionPrices0),col="black",lty=3,lwd=2)
lines(density(asks0),col="black",lty=4,lwd=2)
lines(density(bids0),col="gray40",lty=4,lwd=2)
colors = c("grey35","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),\
    paste("Empirical first round, n=",length(transactionPrices0),sep=""),\
    "Demand, n=10000","Supply, n=10000")
legend(152,0.02,names,cex=0.8,col=colors,lty=c(1,1,1,1),pch=c(-1,-1,-1,-1),lwd=2)

postscript("/home/nikke/Kuvat/vertailu_intersection_fixedDS2_140211.ps",width=100,height=100)
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF,lwd=2)
lines(xdPDF,ydPDF,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=2,lwd=2)
lines(density(transactionPrices0),col="black",lty=3,lwd=2)
lines(density(asks0),col="black",lty=4,lwd=2)
lines(density(bids0),col="gray40",lty=4,lwd=2)
colors = c("grey35","black","gray40","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),\
    paste("Empirical first round, n=",length(transactionPrices0),sep=""),\
    "Demand, n=10000","Supply, n=10000",paste("Empirical first round bids, n=",\
    length(bids0),sep=""),paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.02,names,cex=0.8,col=colors,lty=c(2,3,1,1,4,4),pch=c(-1,-1,-1,-1,-1,-1),lwd=2)
dev.off()

# mukana kaikki transaction pricet
aux = read.csv("/home/nikke/SimPyOut/ZI/130211/big2/ZI_n_100_pricesC.txt",sep=",",header=FALSE)
aux = data.matrix(aux)
transactionPricesAll = numeric(length(aux))
for(i in 1:length(aux)){
  transactionPricesAll[i] = aux[i][1]
}
postscript("/home/nikke/Kuvat/vertailu_intersection_fixedDS22_140211.ps",width=100,height=100)
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF,lwd=2)
lines(xdPDF,ydPDF,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=2,lwd=2)
lines(density(transactionPrices0),col="black",lty=3,lwd=2)
lines(density(asks0),col="black",lty=4,lwd=2)
lines(density(bids0),col="gray40",lty=4,lwd=2)
lines(density(transactionPricesAll),col="black",lty=2,lwd=2)
colors = c("grey35","black","black","gray40","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),paste("Empirical first round, n=",\
    length(transactionPrices0),sep=""),paste("Empirical all rounds, n=",\
    length(transactionPricesAll),sep=""),"Demand, n=10000","Supply, n=10000",\
```

```
    paste("Empirical first round bids, n=",length(bids0),sep=""),\
    paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.02,names,cex=0.8,col=colors,lty=c(2,3,2,1,1,4,4),pch=c(-1,-1,-1,-1,-1,-1,-1),lwd=2)
dev.off()

## fixed demand and supply: excess supply correct analysis

sellerValuations = array(60,dim=100)
buyerValuations = array(140,dim=50)

xsPDF = seq(0,200,0.1)
ysPDF = numeric(2001)
for(i in 1:2000){
  ysPDF[i] = supplyPDF(xsPDF[i])
}
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF)
area = 0
for(i in 1:200){
  area = area + supplyPDF(i)
}
area

xdPDF = seq(0,200,0.1)
ydPDF = numeric(2001)
for(i in 1:2000){
  ydPDF[i] = demandPDF(xdPDF[i])
}
lines(xdPDF,ydPDF,col="gray40")
area = 0
for(i in 1:200){
  area = area + demandPDF(i)
}
area

# Accept-reject sampling
n=10000

# Demand
xd=numeric(n)
yd=numeric(n)
for(i in 1:n){
  while(1){
    z=runif(1,1,174)
    u=runif(1,0,1)
    if(u<(demandPDF(z)/(2/174))){
      print(i)
      xd[i]=z
      yd[i]=u*2/174
      break
    }
  }
}
points(xd,yd,pch=".",col="gray40")

# Supply
xs=numeric(n)
ys=numeric(n)
for(i in 1:n){
  while(1){
    z=runif(1,26,200)
    u=runif(1,0,1)
    if(u<(supplyPDF(z)/(2/174))){
      print(i)
      xs[i]=z
      ys[i]=2/174*u
      break
    }
  }
}
```

```
points(xs,ys,pch=".",col="black")

accepted2 = -1
accepted2y = -1
index = 1
for(i in 1:n){
  if(ys[i]<=demandPDF(xs[i])){
      accepted2[index] = xs[i]
      accepted2y[index] = ys[i]
      index = index + 1
  }
  if(yd[i]<=supplyPDF(xd[i])){
      accepted2[index] = xd[i]
      accepted2y[index] = yd[i]
      index = index + 1
  }
}

# Output
points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=2,lwd=2)
transactionPrices0 = c(...)
bids0 = c(...)
asks0 = c(...)
lines(density(transactionPrices0),col="black",lty=3,lwd=2)
lines(density(asks0),col="black",lty=4,lwd=2)
lines(density(bids0),col="gray40",lty=4,lwd=2)
colors = c("grey35","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),\
    paste("Empirical first round, n=",length(transactionPrices0),sep="")\
    ,"Demand, n=10000","Supply, n=10000")
legend(152,0.02,names,cex=0.8,col=colors,lty=c(1,1,1,1),pch=c(-1,-1,-1,-1),lwd=2)

postscript("/home/nikke/Kuvat/vertailu_intersection_fixedDS_140211.ps",width=100,height=100)
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF,lwd=2)
lines(xdPDF,ydPDF,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=2,lwd=2)
lines(density(transactionPrices0),col="black",lty=3,lwd=2)
lines(density(asks0),col="black",lty=4,lwd=2)
lines(density(bids0),col="gray40",lty=4,lwd=2)
colors = c("grey35","black","gray40","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),\
    paste("Empirical first round, n=",length(transactionPrices0),sep=""),\
    "Demand, n=10000","Supply, n=10000",paste("Empirical first round bids, n=",\
    length(bids0),sep=""),paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.02,names,cex=0.8,col=colors,lty=c(2,3,1,1,4,4),pch=c(-1,-1,-1,-1,-1,-1),lwd=2)
dev.off()

# including all transaction prices
aux = read.csv("/home/nikke/SimPyOut/ZI/130211/big/ZI_n_100_pricesC.txt",sep=",",header=FALSE)
aux = data.matrix(aux)
transactionPricesAll = numeric(length(aux))
for(i in 1:length(aux)){
  transactionPricesAll[i] = aux[i][1]
}
postscript("/home/nikke/Kuvat/vertailu_intersection_fixedDS2_140211.ps",width=100,height=100)
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF,lty=3,lwd=2)
lines(xdPDF,ydPDF,lty=3,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=1,lwd=2)
lines(density(transactionPrices0),col="black",lty=4,lwd=2)
lines(density(asks0),col="black",lty=2,lwd=2)
lines(density(bids0),col="gray40",lty=2,lwd=2)
```

```
lines(density(transactionPricesAll),col="black",lty=1,lwd=2)
colors = c("grey35","black","black","gray40","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),\
    paste("Empirical first round, n=",length(transactionPrices0),sep=""),\
    paste("Empirical all rounds, n=",length(transactionPricesAll),sep=""),\
    "Demand, n=10000","Supply, n=10000",paste("Empirical first round bids, n=",\
    length(bids0),sep=""),paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.02,names,cex=0.8,col=colors,lty=c(1,4,1,3,3,2,2),pch=c(-1,-1,-1,-1,-1,-1,-1),lwd=2)
dev.off()

# Only bids and asks
postscript("/home/nikke/Kuvat/vertailu_intersection_fixedDS2_quotes_090311.ps",width=100,height=100)
plot(c(0,200),c(0,0.015),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF,lty=1,lwd=2)
lines(xdPDF,ydPDF,lty=1,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
#lines(density(accepted2),col="grey35",lty=1,lwd=2)
#lines(density(transactionPrices0),col="black",lty=4,lwd=2)
lines(density(asks0),col="black",lty=2,lwd=2)
lines(density(bids0),col="gray40",lty=2,lwd=2)
#lines(density(transactionPricesAll),col="black",lty=1,lwd=2)
colors = c("gray40","black","gray40","black")
names = c("Demand, n=10000","Supply, n=10000",paste("Empirical first round bids, n=",\
   length(bids0),sep=""),paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.015,names,cex=0.8,col=colors,lty=c(1,1,2,2),pch=c(-1,-1,-1,-1),lwd=2)
dev.off()

postscript("/home/nikke/Kuvat/vertailu_intersection_fixedDS2_prices_090311.ps",width=100,height=100)
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
#lines(xsPDF,ysPDF,lty=3,lwd=2)
#lines(xdPDF,ydPDF,lty=3,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=1,lwd=2)
lines(density(transactionPrices0),col="black",lty=4,lwd=2)
#lines(density(asks0),col="black",lty=2,lwd=2)
#lines(density(bids0),col="gray40",lty=2,lwd=2)
lines(density(transactionPricesAll),col="black",lty=1,lwd=2)
colors = c("grey35","black","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),paste("Empirical first round, n=",\
  length(transactionPrices0),sep=""),paste("Empirical all rounds, n=",length(transactionPricesAll),sep=""))
legend(142,0.02,names,cex=0.8,col=colors,lty=c(1,4,1),pch=c(-1,-1,-1),lwd=2)
dev.off()

## Symmetric demand and supply: correct analysis

sellerValuations = seq(26,174,2)
buyerValuations = seq(26,174,2)
xsPDF = seq(0,200,0.1)
ysPDF = numeric(2001)
for(i in 1:2000){
  ysPDF[i] = supplyPDF(xsPDF[i])
}
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF)
area = 0
for(i in 1:200){
  area = area + supplyPDF(i)
}
area
xdPDF = seq(0,200,1)
ydPDF = numeric(201)
for(i in 1:200){
  ydPDF[i] = demandPDF(xdPDF[i])
}
lines(xdPDF,ydPDF,col="gray40")
area = 0
```

```
for(i in 1:200){
  area = area + demandPDF(i)
}
area

## Accept-reject sampling
n=10000

# Demand
xd=numeric(n)
yd=numeric(n)
for(i in 1:n){
  while(1){
    z=runif(1,1,174)
    u=runif(1,0,1)
    if(u<(demandPDF(z)/(3/174))){
      print(i)
      xd[i]=z
      yd[i]=u*3/174
      break
    }
  }
}
points(xd,yd,pch=".",col="gray40")

# Supply
xs=numeric(n)
ys=numeric(n)
for(i in 1:n){
  while(1){
    z=runif(1,26,200)
    u=runif(1,0,1)
    if(u<(supplyPDF(z)/(3/174))){
      print(i)
      xs[i]=z
      ys[i]=3/174*u
      break
    }
  }
}
points(xs,ys,pch=".",col="black")

accepted2 = -1
accepted2y = -1
index = 1
for(i in 1:n){
  if(ys[i]<=demandPDF(xs[i])){
      accepted2[index] = xs[i]
      accepted2y[index] = ys[i]
      index = index + 1
  }
  if(yd[i]<=supplyPDF(xd[i])){
      accepted2[index] = xd[i]
      accepted2y[index] = yd[i]
      index = index + 1
  }
}

# Output
points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=2,lwd=2)
transactionPrices0 = c(...)
bids0 = c(...)
asks0 = c(...)
lines(density(transactionPrices0),col="black",lty=3,lwd=2)
lines(density(asks0),col="black",lty=4,lwd=2)
lines(density(bids0),col="gray40",lty=4,lwd=2)
colors = c("grey35","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),\
    paste("Empirical first round, n=",length(transactionPrices0),sep=""),\
```

```
    "Demand, n=10000","Supply, n=10000")
legend(152,0.02,names,cex=0.8,col=colors,lty=c(1,1,1,1),pch=c(-1,-1,-1,-1),lwd=2)

postscript("/home/nikke/Kuvat/vertailu_intersection_symmetric_080211.ps",width=100,height=100)
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF,lwd=2)
lines(xdPDF,ydPDF,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=2,lwd=2)
lines(density(transactionPrices0),col="black",lty=3,lwd=2)
lines(density(asks0),col="black",lty=4,lwd=2)
lines(density(bids0),col="gray40",lty=4,lwd=2)
colors = c("grey35","black","gray40","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),\
    paste("Empirical first round, n=",length(transactionPrices0),sep=""),\
    "Demand, n=10000","Supply, n=10000",paste("Empirical first round bids, n=",length(bids0),sep=""),\
    paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.02,names,cex=0.8,col=colors,lty=c(2,3,1,1,4,4),pch=c(-1,-1,-1,-1,-1,-1),lwd=2)
dev.off()

# including all transaction prices
aux = read.csv("/home/nikke/SimPyOut/ZI/090311/big/ZI_n_100_pricesC.txt",sep=",",header=FALSE)
aux = data.matrix(aux)
transactionPricesAll = numeric(length(aux))
for(i in 1:length(aux)){
  transactionPricesAll[i] = aux[i][1]
}
postscript("/home/nikke/Kuvat/vertailu_intersection_symmetric2_080211.ps",width=100,height=100)
plot(c(0,200),c(0,0.025),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF,lty=3,lwd=2)
lines(xdPDF,ydPDF,lty=3,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=1,lwd=2)
lines(density(transactionPrices0),col="black",lty=4,lwd=2)
lines(density(asks0),col="black",lty=2,lwd=2)
lines(density(bids0),col="gray40",lty=2,lwd=2)
lines(density(transactionPricesAll),col="black",lty=1,lwd=2)
colors = c("grey35","black","black","gray40","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),paste("Empirical first round, n=",\
    length(transactionPrices0),sep=""),paste("Empirical all rounds, n=",length(transactionPricesAll),sep=""),\
    "Demand, n=10000","Supply, n=10000",paste("Empirical first round bids, n=",length(bids0),sep=""),\
    paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.025,names,cex=0.8,col=colors,lty=c(1,4,1,3,3,2,2),pch=c(-1,-1,-1,-1,-1,-1,-1),lwd=2)
dev.off()


# Only bids and asks
postscript("/home/nikke/Kuvat/vertailu_intersection_symmetric2_quotes_080211.ps",width=100,height=100)
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF,lty=1,lwd=2)
lines(xdPDF,ydPDF,lty=1,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
#lines(density(accepted2),col="grey35",lty=1,lwd=2)
#lines(density(transactionPrices0),col="black",lty=4,lwd=2)
lines(density(asks0),col="black",lty=2,lwd=2)
lines(density(bids0),col="gray40",lty=2,lwd=2)
#lines(density(transactionPricesAll),col="black",lty=1,lwd=2)
colors = c("gray40","black","gray40","black")
names = c("Demand, n=10000","Supply, n=10000",paste("Empirical first round bids, n=",\
  length(bids0),sep=""),paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.02,names,cex=0.8,col=colors,lty=c(1,1,2,2),pch=c(-1,-1,-1,-1),lwd=2)
dev.off()

postscript("/home/nikke/Kuvat/vertailu_intersection_symmetric2_prices_080211.ps",width=100,height=100)
```

```
plot(c(0,200),c(0,0.025),type="n",xlab="Price",ylab="PDF")
#lines(xsPDF,ysPDF,lty=3,lwd=2)
#lines(xdPDF,ydPDF,lty=3,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=1,lwd=2)
lines(density(transactionPrices0),col="black",lty=4,lwd=2)
#lines(density(asks0),col="black",lty=2,lwd=2)
#lines(density(bids0),col="gray40",lty=2,lwd=2)
lines(density(transactionPricesAll),col="black",lty=1,lwd=2)
colors = c("grey35","black","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),paste("Empirical first round, n=",\
   length(transactionPrices0),sep=""),paste("Empirical all rounds, n=",\
   length(transactionPricesAll),sep=""))
legend(150,0.025,names,cex=0.8,col=colors,lty=c(1,4,1),pch=c(-1,-1,-1),lwd=2)
dev.off()

## Symmetric demand and supply: Cliff & Bruten 1997

sellerValuations = seq(26,174,2)
buyerValuations = seq(26,174,2)

supplyF = function(p){
  ret = 0
  index = 1
  while (sellerValuations[index]<=p){
    index = index + 1
    ret = ret + 1
    if (index==76){break;}
  }
  if (p>200){ret = 0;}
  ret
}
supplyArea = 0
for(i in 1:200){
  supplyArea = supplyArea + supplyF(i)
}
supplyArea
supply = function(p){
  ret = 0
  index = 1
  while (sellerValuations[index]<=p){
    index = index + 1
    ret = ret + 1
    if (index==76){break;}
  }
  if (p>200){ret = 0;}
  ret/supplyArea
}
demandF = function(p){
  ret = 0
  index = 75
  while (buyerValuations[index]>=p){
    index = index - 1
    ret = ret + 1
    if (index ==0){break;}
  }
  if (p<1){ret = 0;}
  ret
}
demandArea = 0
for(i in 1:200){
  demandArea = demandArea + demandF(i)
}
demandArea
demand = function(p){
  ret = 0
  index = 75
  while (buyerValuations[index]>=p){
```

```
    index = index - 1
    ret = ret + 1
    if (index ==0){break;}
  }
  if (p<1){ret = 0;}
  ret/demandArea
}
s = -1
d = -1
x = seq(1,200,1)
for(i in 1:200){
  s[i] = supply(i)
  d[i] = demand(i)
}
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
lines(x,s,col="black",lwd=2)
lines(x,d,col="gray40",lwd=2)

## Accept-reject sampling
n=10000

# Demand
xd=numeric(n)
yd=numeric(n)
for(i in 1:n){
  while(1){
    z=runif(1,1,174)
    u=runif(1,0,1)
    if(u<(demand(z)/(2/173))){
      print(i)
      xd[i]=z
      yd[i]=u*2/173
      break
    }
  }
}
points(xd,yd,pch=".",col="gray40")

# Supply
xs=numeric(n)
ys=numeric(n)
for(i in 1:n){
  while(1){
    z=runif(1,26,200)
    u=runif(1,0,1)
    if(u<(supply(z)/(2/173))){
      print(i)
      xs[i]=z
      ys[i]=2/173*u
      break
    }
  }
}
points(xs,ys,pch=".",col="black")

accepted2 = -1
accepted2y = -1
index = 1
for(i in 1:n){
  if(ys[i]<=demand(xs[i])){
      accepted2[index] = xs[i]
      accepted2y[index] = ys[i]
      index = index + 1
  }
  if(yd[i]<=supply(xd[i])){
      accepted2[index] = xd[i]
      accepted2y[index] = yd[i]
      index = index + 1
  }
}
```

```
# Output
points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=2,lwd=2)
transactionPrices0 = c(...)
bids0 = c(...)
asks0 = c(...)
lines(density(transactionPrices0),col="black",lty=3,lwd=2)
lines(density(asks0),col="black",lty=4,lwd=2)
lines(density(bids0),col="gray40",lty=4,lwd=2)
colors = c("grey35","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),\
    paste("Empirical first round, n=",length(transactionPrices0),sep=""),"Demand, n=10000","Supply, n=10000")
legend(152,0.02,names,cex=0.8,col=colors,lty=c(1,1,1,1),pch=c(-1,-1,-1,-1),lwd=2)

postscript("/home/nikke/Kuvat/vertailu_Cliff_intersection_symmetric_080211.ps",width=100,height=100)
plot(c(0,200),c(0,0.025),type="n",xlab="Price",ylab="PDF")
lines(x,s,lwd=2)
lines(x,d,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=2,lwd=2)
lines(density(transactionPrices0),col="black",lty=3,lwd=2)
lines(density(asks0),col="black",lty=4,lwd=2)
lines(density(bids0),col="gray40",lty=4,lwd=2)
colors = c("grey35","black","gray40","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),\
    paste("Empirical first round, n=",length(transactionPrices0),sep=""),\
    "Demand, n=10000","Supply, n=10000",paste("Empirical first round bids, n=",length(bids0),sep=""),\
    paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.025,names,cex=0.8,col=colors,lty=c(2,3,1,1,4,4),pch=c(-1,-1,-1,-1,-1,-1),lwd=2)
dev.off()

# including all transaction prices
aux = read.csv("/home/nikke/SimPyOut/ZI/050211/big3/ZI_n_100_pricesC.txt",sep=",",header=FALSE)
aux = data.matrix(aux)
transactionPricesAll = numeric(length(aux))
for(i in 1:length(aux)){
  transactionPricesAll[i] = aux[i][1]
}
postscript("/home/nikke/Kuvat/vertailu_Cliff_intersection_symmetric2_080211.ps",width=100,height=100)
plot(c(0,200),c(0,0.025),type="n",xlab="Price",ylab="PDF")
lines(x,s,lty=3,lwd=2)
lines(x,d,lty=3,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=1,lwd=2)
lines(density(transactionPrices0),col="black",lty=4,lwd=2)
lines(density(asks0),col="black",lty=2,lwd=2)
lines(density(bids0),col="gray40",lty=2,lwd=2)
lines(density(transactionPricesAll),col="black",lty=1,lwd=2)
colors = c("grey35","black","black","gray40","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),paste("Empirical first round, n=",\
    length(transactionPrices0),sep=""),paste("Empirical all rounds, n=",length(transactionPricesAll),sep=""),\
    "Demand, n=10000","Supply, n=10000",paste("Empirical first round bids, n=",length(bids0),sep=""),\
    paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.025,names,cex=0.8,col=colors,lty=c(1,4,1,3,3,2,2),pch=c(-1,-1,-1,-1,-1,-1,-1),lwd=2)
dev.off()

# Only bids and asks
postscript("/home/nikke/Kuvat/vertailu_Cliff_intersection_symmetric2_quotes_080211.ps",width=100,height=100)
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
lines(x,s,lty=1,lwd=2)
lines(x,d,lty=1,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
#lines(density(accepted2),col="grey35",lty=1,lwd=2)
```

```
#lines(density(transactionPrices0),col="black",lty=4,lwd=2)
lines(density(asks0),col="black",lty=2,lwd=2)
lines(density(bids0),col="gray40",lty=2,lwd=2)
#lines(density(transactionPricesAll),col="black",lty=1,lwd=2)
colors = c("gray40","black","gray40","black")
names = c("Demand, n=10000","Supply, n=10000",paste("Empirical first round bids, n=",length(bids0),sep=""),\
    paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.02,names,cex=0.8,col=colors,lty=c(1,1,2,2),pch=c(-1,-1,-1,-1),lwd=2)
dev.off()

postscript("/home/nikke/Kuvat/vertailu_Cliff_intersection_symmetric2_prices_080211.ps",width=100,height=100)
plot(c(0,200),c(0,0.025),type="n",xlab="Price",ylab="PDF")
#lines(xsPDF,ysPDF,lty=3,lwd=2)
#lines(xdPDF,ydPDF,lty=3,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=1,lwd=2)
lines(density(transactionPrices0),col="black",lty=4,lwd=2)
#lines(density(asks0),col="black",lty=2,lwd=2)
#lines(density(bids0),col="gray40",lty=2,lwd=2)
lines(density(transactionPricesAll),col="black",lty=1,lwd=2)
colors = c("grey35","black","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),paste("Empirical first round, n=",\
    length(transactionPrices0),sep=""),paste("Empirical all rounds, n=",length(transactionPricesAll),sep=""))
legend(142,0.025,names,cex=0.8,col=colors,lty=c(1,4,1),pch=c(-1,-1,-1),lwd=2)
dev.off()

## Non symmetric demand and supply: fixed demand
## Correct analysis

buyerValuations = array(data=140,dim=75)
sellerValuations = seq(26,174,2)

xsPDF = seq(0,200,0.1)
ysPDF = numeric(2001)
for(i in 1:2000){
  ysPDF[i] = supplyPDF(xsPDF[i])
}
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF)
area = 0
for(i in 1:200){
  area = area + supplyPDF(i)
}
area

xdPDF = seq(0,200,0.1)
ydPDF = numeric(2001)
for(i in 1:2000){
  ydPDF[i] = demandPDF(xdPDF[i])
}
lines(xdPDF,ydPDF,col="gray40")
area = 0
for(i in 1:200){
  area = area + demandPDF(i)
}
area

## Accept-reject sampling
n=10000

# Demand
xd=numeric(n)
yd=numeric(n)
for(i in 1:n){
  while(1){
    z=runif(1,1,174)
    u=runif(1,0,1)
    if(u<(demandPDF(z)/(1/139))){
```

```
      print(i)
      xd[i]=z
      yd[i]=u*1/139
      break
    }
  }
}
points(xd,yd,pch=".",col="gray40")

# Supply
xs=numeric(n)
ys=numeric(n)
for(i in 1:n){
  while(1){
    z=runif(1,26,200)
    u=runif(1,0,1)
    if(u<(supplyPDF(z)/(3/174))){
      print(i)
      xs[i]=z
      ys[i]=3/174*u
      break
    }
  }
}
points(xs,ys,pch=".",col="black")

accepted2 = -1
accepted2y = -1
index = 1
for(i in 1:n){
  if(ys[i]<=demandPDF(xs[i])){
      accepted2[index] = xs[i]
      accepted2y[index] = ys[i]
      index = index + 1
  }
  if(yd[i]<=supplyPDF(xd[i])){
      accepted2[index] = xd[i]
      accepted2y[index] = yd[i]
      index = index + 1
  }
}

# Output
points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=2,lwd=2)
transactionPrices0 = c(...)
bids0 = c(...)
asks0 = c(...)
lines(density(transactionPrices0),col="black",lty=3,lwd=2)
lines(density(asks0),col="black",lty=4,lwd=2)
lines(density(bids0),col="gray40",lty=4,lwd=2)
colors = c("grey35","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),paste("Empirical first round, n=",\
    length(transactionPrices0),sep=""),"Demand, n=10000","Supply, n=10000")
legend(152,0.02,names,cex=0.8,col=colors,lty=c(1,1,1,1),pch=c(-1,-1,-1,-1),lwd=2)

postscript("/home/nikke/Kuvat/vertailu_intersection_fixed_demand_080211.ps",width=100,height=100)
plot(c(0,200),c(0,0.025),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF,lwd=2)
lines(xdPDF,ydPDF,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=2,lwd=2)
lines(density(transactionPrices0),col="black",lty=3,lwd=2)
lines(density(asks0),col="black",lty=4,lwd=2)
lines(density(bids0),col="gray40",lty=4,lwd=2)
colors = c("grey35","black","gray40","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),paste("Empirical first round, n=",\
    length(transactionPrices0),sep=""),"Demand, n=10000","Supply, n=10000",\
```

```
    paste("Empirical first round bids, n=",length(bids0),sep=""),\
    paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.025,names,cex=0.8,col=colors,lty=c(2,3,1,1,4,4),pch=c(-1,-1,-1,-1,-1,-1),lwd=2)
dev.off()

# including all transaction prices
aux = read.csv("/home/nikke/SimPyOut/ZI/050211/big/ZI_n_100_pricesC.txt",sep=",",header=FALSE)
aux = data.matrix(aux)
transactionPricesAll = numeric(length(aux))
for(i in 1:length(aux)){
  transactionPricesAll[i] = aux[i][1]
}
postscript("/home/nikke/Kuvat/vertailu_intersection_fixed_demand2_080211.ps",width=100,height=100)
plot(c(0,200),c(0,0.04),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF,lty=3,lwd=2)
lines(xdPDF,ydPDF,lty=3,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=1,lwd=2)
lines(density(transactionPrices0),col="black",lty=4,lwd=2)
lines(density(asks0),col="black",lty=2,lwd=2)
lines(density(bids0),col="gray40",lty=2,lwd=2)
lines(density(transactionPricesAll),col="black",lty=1,lwd=2)
colors = c("grey35","black","black","gray40","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),\
       paste("Empirical first round, n=",length(transactionPrices0),sep=""),\
    paste("Empirical all rounds, n=",length(transactionPricesAll),sep=""),
    "Demand, n=10000","Supply, n=10000",paste("Empirical first round bids, n=",\
    length(bids0),sep=""),paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.04,names,cex=0.8,col=colors,lty=c(1,4,1,3,3,2,2),pch=c(-1,-1,-1,-1,-1,-1,-1),lwd=2)
dev.off()

# Only bids and asks
postscript("/home/nikke/Kuvat/vertailu_intersection_fixed_demand_quotes_080211.ps",width=100,height=100)
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF,lty=1,lwd=2)
lines(xdPDF,ydPDF,lty=1,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
#lines(density(accepted2),col="grey35",lty=1,lwd=2)
#lines(density(transactionPrices0),col="black",lty=4,lwd=2)
lines(density(asks0),col="black",lty=2,lwd=2)
lines(density(bids0),col="gray40",lty=2,lwd=2)
#lines(density(transactionPricesAll),col="black",lty=1,lwd=2)
colors = c("gray40","black","gray40","black")
names = c("Demand, n=10000","Supply, n=10000",paste("Empirical first round bids, n=",\
    length(bids0),sep=""),paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.02,names,cex=0.8,col=colors,lty=c(1,1,2,2),pch=c(-1,-1,-1,-1),lwd=2)
dev.off()

postscript("/home/nikke/Kuvat/vertailu_intersection_fixed_demand_prices_080211.ps",width=100,height=100)
plot(c(0,200),c(0,0.04),type="n",xlab="Price",ylab="PDF")
#lines(xsPDF,ysPDF,lty=3,lwd=2)
#lines(xdPDF,ydPDF,lty=3,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=1,lwd=2)
lines(density(transactionPrices0),col="black",lty=4,lwd=2)
#lines(density(asks0),col="black",lty=2,lwd=2)
#lines(density(bids0),col="gray40",lty=2,lwd=2)
lines(density(transactionPricesAll),col="black",lty=1,lwd=2)
colors = c("grey35","black","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),paste("Empirical first round, n=",\
    length(transactionPrices0),sep=""),paste("Empirical all rounds, n=",length(transactionPricesAll),sep=""))
legend(150,0.04,names,cex=0.8,col=colors,lty=c(1,4,1),pch=c(-1,-1,-1),lwd=2)
dev.off()
```

```
## Non symmetric demand and supply: fixed supply
## Correct analysis

buyerValuations = seq(26,174,2)
sellerValuations = array(data=60,dim=75)

xsPDF = seq(0,200,0.1)
ysPDF = numeric(2001)
for(i in 1:2000){
  ysPDF[i] = supplyPDF(xsPDF[i])
}
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF)
area = 0
for(i in 1:200){
  area = area + supplyPDF(i)
}
area

xdPDF = seq(0,200,0.1)
ydPDF = numeric(2001)
for(i in 1:2000){
  ydPDF[i] = demandPDF(xdPDF[i])
}
lines(xdPDF,ydPDF,col="gray40")
area = 0
for(i in 1:200){
  area = area + demandPDF(i)
}
area

## Accept-reject sampling
n=10000

# Demand
xd=numeric(n)
yd=numeric(n)
for(i in 1:n){
  while(1){
    z=runif(1,1,174)
    u=runif(1,0,1)
    if(u<(demandPDF(z)/(2/139))){
      print(i)
      xd[i]=z
      yd[i]=u*2/139
      break
    }
  }
}
points(xd,yd,pch=".",col="gray40")

# Supply
xs=numeric(n)
ys=numeric(n)
for(i in 1:n){
  while(1){
    z=runif(1,60,200)
    u=runif(1,0,1)
    if(u<(supplyPDF(z)/(1/140))){
      print(i)
      xs[i]=z
      ys[i]=1/140*u
      break
    }
  }
}
points(xs,ys,pch=".",col="black")

accepted2 = -1
accepted2y = -1
```

```
index = 1
for(i in 1:n){
  if(ys[i]<=demandPDF(xs[i])){
      accepted2[index] = xs[i]
      accepted2y[index] = ys[i]
      index = index + 1
  }
  if(yd[i]<=supplyPDF(xd[i])){
      accepted2[index] = xd[i]
      accepted2y[index] = yd[i]
      index = index + 1
  }
}

# Output
points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=2,lwd=2)
transactionPrices0 = c(...)
asks0 = c(...)
bids0 = c(...)
lines(density(transactionPrices0),col="black",lty=3,lwd=2)
lines(density(asks0),col="black",lty=4,lwd=2)
lines(density(bids0),col="gray40",lty=4,lwd=2)
colors = c("grey35","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),\
    paste("Empirical first round, n=",length(transactionPrices0),sep=""),"Demand, n=10000","Supply, n=10000")
legend(152,0.02,names,cex=0.8,col=colors,lty=c(1,1,1,1),pch=c(-1,-1,-1,-1),lwd=2)

postscript("/home/nikke/Kuvat/vertailu_intersection_fixed_supply_080211.ps",width=100,height=100)
plot(c(0,200),c(0,0.025),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF,lwd=2)
lines(xdPDF,ydPDF,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=2,lwd=2)
lines(density(transactionPrices0),col="black",lty=3,lwd=2)
lines(density(asks0),col="black",lty=4,lwd=2)
lines(density(bids0),col="gray40",lty=4,lwd=2)
colors = c("grey35","black","gray40","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),paste("Empirical first round, n=",\
   length(transactionPrices0),sep=""),"Demand, n=10000","Supply, n=10000",\
   paste("Empirical first round bids, n=",length(bids0),sep=""),\
   paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.025,names,cex=0.8,col=colors,lty=c(2,3,1,1,4,4),pch=c(-1,-1,-1,-1,-1,-1),lwd=2)
dev.off()

# including all transaction prices
aux = read.csv("/home/nikke/SimPyOut/ZI/050211/big2/ZI_n_100_pricesC.txt",sep=",",header=FALSE)
aux = data.matrix(aux)
transactionPricesAll = numeric(length(aux))
for(i in 1:length(aux)){
  transactionPricesAll[i] = aux[i][1]
}
postscript("/home/nikke/Kuvat/vertailu_intersection_fixed_supply2_080211.ps",width=100,height=100)
plot(c(0,200),c(0,0.04),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF,lty=3,lwd=2)
lines(xdPDF,ydPDF,lty=3,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=1,lwd=2)
lines(density(transactionPrices0),col="black",lty=4,lwd=2)
lines(density(asks0),col="black",lty=2,lwd=2)
lines(density(bids0),col="gray40",lty=2,lwd=2)
lines(density(transactionPricesAll),col="black",lty=1,lwd=2)
colors = c("grey35","black","black","gray40","black","gray40","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),paste("Empirical first round, n=",\
    length(transactionPrices0),sep=""),paste("Empirical all rounds, n=",\
    length(transactionPricesAll),sep=""),"Demand, n=10000","Supply, n=10000",\
```

```
    paste("Empirical first round bids, n=",length(bids0),sep=""),\
    paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.04,names,cex=0.8,col=colors,lty=c(1,4,1,3,3,2,2),pch=c(-1,-1,-1,-1,-1,-1,-1),lwd=2)
dev.off()

# Only bids and asks
postscript("/home/nikke/Kuvat/vertailu_intersection_fixed_supply_quotes_080211.ps",width=100,height=100)
plot(c(0,200),c(0,0.02),type="n",xlab="Price",ylab="PDF")
lines(xsPDF,ysPDF,lty=1,lwd=2)
lines(xdPDF,ydPDF,lty=1,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
#lines(density(accepted2),col="grey35",lty=1,lwd=2)
#lines(density(transactionPrices0),col="black",lty=4,lwd=2)
lines(density(asks0),col="black",lty=2,lwd=2)
lines(density(bids0),col="gray40",lty=2,lwd=2)
#lines(density(transactionPricesAll),col="black",lty=1,lwd=2)
colors = c("gray40","black","gray40","black")
names = c("Demand, n=10000","Supply, n=10000",paste("Empirical first round bids, n=",\
    length(bids0),sep=""),paste("Empirical first round asks, n=",length(asks0),sep=""))
legend(142,0.02,names,cex=0.8,col=colors,lty=c(1,1,2,2),pch=c(-1,-1,-1,-1),lwd=2)
dev.off()

postscript("/home/nikke/Kuvat/vertailu_intersection_fixed_supply_prices_080211.ps",width=100,height=100)
plot(c(0,200),c(0,0.04),type="n",xlab="Price",ylab="PDF")
#lines(xsPDF,ysPDF,lty=3,lwd=2)
#lines(xdPDF,ydPDF,lty=3,col="gray40",lwd=2)
#points(xd,yd,pch=".",col="gray40")
#points(xs,ys,pch=".",col="black")
#points(accepted2,accepted2y,col="grey35",pch=".")
lines(density(accepted2),col="grey35",lty=1,lwd=2)
lines(density(transactionPrices0),col="black",lty=4,lwd=2)
#lines(density(asks0),col="black",lty=2,lwd=2)
#lines(density(bids0),col="gray40",lty=2,lwd=2)
lines(density(transactionPricesAll),col="black",lty=1,lwd=2)
colors = c("grey35","black","black")
names = c(paste("Intersection, n=",length(accepted2),sep=""),paste("Empirical first round, n=",\
    length(transactionPrices0),sep=""),paste("Empirical all rounds, n=",length(transactionPricesAll),sep=""))
legend(150,0.04,names,cex=0.8,col=colors,lty=c(1,4,1),pch=c(-1,-1,-1),lwd=2)
dev.off()
```