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Construction of an Economic Model of Resource Use: Research and Development of Substitutes for Non-renewable Resources and Waste

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Abstract

The research questions for the thesis were: How the existing economic models based on Romer's (1990) endogenous research and development (R&D) growth model could be extended to be able to research the optimal allocation of R&D resources? What are the most significant factors affecting on the R&D sector investment allocation?

A literature review on the existing resource economics and backstop substitution models was introduced. In order to put this thesis in context a closer look at the historical development of the substitution technologies, theoretical concepts and resource economics theory was taken from the early 1900 century to this day. In focus was especially recent literature with economic models with similar characteristics as in the model constructed here, most notably models derived from Romer (1990). Some of the literature was also used as a reference for the results part of the thesis.

The purpose of this thesis was to construct a framework for economic model with four sectors. The model is a discrete time model that is constructed so that the primary resources (non-renewable and renewable) are competitors for the secondary recycled resources as the input for the final production. The use of each resource is determined by the marginal cost of production. As the non-renewable resource becomes scarcer and their price increases, the investments in R&D sector increases as the possibilities for substitution increases as the increasing capital base increases the resource utilization level which increases the possibilities for substitution for backstop resources. Meanwhile the increasing technological level decreases efficiency of technological improvement.

We have four resource stocks that can be utilized at different steps of the production process as raw materials for the production. These include the non-renewable resources that can be substituted with renewable resources as raw materials to the system. There are the waste and resources-in-use stocks that are dependent on the production volumes. The model is structured on the basis of four sectors, two industrial sectors and two technology sectors. This approach is different from commonly used three sectors model with two industries and one technology. The non-renewable resources can be substituted by renewable resources through substitution technology R&D. On the other hand the waste associated with depreciation of resources-in-use, which also causes harm to renewable resources, can be substituted by recycled resources through recycling technology R&D. The recycled resources are secondary resource base that can replace a part of primary natural virgin resources consumption. The model has intermediate sector producing intermediate goods from natural resources. The intermediate sector is using technology licenses bought from the substitution technology sector to replace a part of non-renewable resource use. The final product sector is producing the final goods for consumption and capital by utilizing labor, capital and intermediate sector production and recycled resources for which the recycling technology sector sells licenses. The basic inputs for the system are capital, resources and labor.

The questions of sustainable steady state growth path and optimal resource allocation are only discussed but not solved in this thesis. The most remarkable results from the model are that the R&D resource allocation in equilibrium is mainly dependent on the volumes and the price changes in different sectors.

Tiivistelmä

Tämän työn tutkimuskysymyksiä olivat: Kuinka olemassa olevia Romerin (1990) taloustieteelliseen endogeeniseen kasvu malliin perustuvia tuotekehitys tutkimuksia voitaisiin laajentaa siten, että voitaisiin tutkia tuotekehitysresurssien optimaalista allokaatiota eri tuotekehityssektorien välillä? Mitkä ovat tärkeimpiä tekijöitä, jotka vaikuttavat T&K sektorin resurssien allokaatioon?

Taustoituksena näihin kysymyksiin suoritettiin kirjallisuuskatsaus olemassa oleviin resurssien taloustieteeseen kuuluviin substituutio malleihin ja muuhun oleelliseen kirjallisuuteen. Työn asettamiseksi historialliseen kontekstiinsa oli tarpeen tutustua substituutioteorioiden historialliseen kehitykseen, teoreettisiin konsepteihin ja resurssien taloustieteen teoriaan alkaen 1800-luvun kirjallisuudesta ja päättyen viimeisimpään tutkimustietoon. Erityisesti keskityttiin työn kannalta oleellisimpiin tuoreisiin tutkimuksiin, joka ovat pääasiassa Romerin (1990) mallista kehitettyjä teorioita.

Työn tarkoituksena oli rakentaa taloustieteellisen mallin kehikko, joka perustuu neljään taloudelliseen sektoriin perinteisesti käytetyn kolmen sektorin mallin sijaan. Malli on diskreetti aikamalli joka rakentuu siten, että primääriset resurssit, eli uusiutumattomat ja uusiutuvat resurssit, ovat kilpailijoita sekundäärisille resursseille lopputuotantosektorin tuotannossa. Tämän työn kontekstissa on käytössä vain yksi sekundäärinen resurssi, joka on kierrätettävät resurssit. Kunkin resurssin käyttö riippuu resurssin rajakustannuksista. Uusiutumattomien resurssien kulutuksen kasvaessa ja toisaalta resurssien harvinaistuessa, ja tuotantopääoman kasvaessa, luonnon resurssien hinnat nousevat nopeasti, jolloin investoinnit uusiutumattomia resursseja korvaavalle tuotekehityssektorille kasvavat substituutiomahdollisuuksien tullessa kannattavimmiksi. Toisaalta kasvava teknologisen tason kasvu vaikeuttaa uusien innovaatioiden tekemistä tuotekehityssektorilla.

Työssä esitellyssä mallissa on neljä mahdollista resurssia, joita voidaan hyödyntää tuotantoprosessin erivaiheessa. Näihin sisältyvät uusiutumattomat resurssit ja näitä korvaavat uusiutuvat resurssit, sekä jäte ja jätettä korvaavat kierrätettävät resurssit, jotka riippuvat tuotannon volyymistä. Malli rakentuu neljästä sektorista, joista kaksi on teollisia sektoreita ja kaksi tuotekehityssektoreita tavanomaisen kahden teollisen sektorin ja yhden tuotekehityssektorin sijaan.

Mallissa uusiutumattomat resurssit on mahdollista korvata uusiutuvilla resursseilla panostamalla näiden kahden resurssin välisen substituution mahdollistavaan tuotekehitykseen. Toisaalta käytössä olevien resurssien kulumisesta aiheutuvaa jätettä, joka tuottaa vahinkoa uusiutuville resursseille, on mahdollista muuttaa kierrätettäviksi resurssiksi myös tuotekehitystyön kautta. Kierrätettävien resurssien käytöllä on mahdollista alentaa primääristen neitseellisten raaka-aineiden käyttöä ja siten myös hidastaa uusiutumattomien resurssien hinnan nousua.

Mallissa on kaksi tuotantosektoria, joista toinen on jalostussektori, joka tuottaa puolivalmisteita neitseellisistä raaka-aineista lopputuotesektorille. Jalostussektori voi halutessaan ostaa uusiutumattomia resursseja korvaavaa teknologiaa substituutioteknologiaa kehittävältä sektorilta. Lopputuote sektori puolestaan käyttää jalostussektorin tuotannon omassa tuotannossaan, jonka lisäksi lopputuote sektori ostaa teknologiaa, jolla se voi halutessaan korvata osan jalostussektorin tuotannosta kierrättämällä käytöstä poistuvia resursseja takaisin tuotantoon. Mallissa hyödynnettäviä tuotantoresursseja luonnon resurssien lisäksi on pääoma ja työvoima.

Kestävään tasapainotettuun kasvuun ja optimaaliseen resurssien allokaatioon liittyviä kysymyksiä ei tässä mallissa ratkaista, mutta asiaan liittyen käydään keskusteluja. Mallin kannalta merkittävimpiä tuloksia on se, että tasapainossa tuotekehitysresurssien allokaatio riippuu vain substituoitavan resurssin volyymistä ja toisaalta hinnan kasvu nopeudesta eri sektoreilla.

Acknowledgements

I would like to thank for this Thesis one of those moments when I got to understand the fundamental threat that the humankind is facing in form of resource scarcity. It was a moment few years ago as I was standing at the peer at my family cottage in Espoo, in a natural silence, where the only voice disturbing the peace was the sound of an aero plane that struck like a thunder. Then it hit me that the surroundings we live in are in a threat, the human influence is everywhere and expanding at tremendous speed to the most deserted places on earth. Now this wasn't really news for me, but I realized that everything we are used to are in serious threat. The threat came from the human itself and its greed to expand to the areas never explored before. A greed that was created by the lust for new resources was the main motive for this expansion. Not too much later the deep sea oil rig BP Horizon exploded creating the biggest oil leak ever experienced on sea with unaccountable harm to the sea life. Thousands of fishers lost their income. The lust for non-renewable resources had excluded renewable resource from being used. With this in mind I started to think of the role of the research and development and the possibilities it could offer for solving these problems. So I thought to build an economic model.

Of course I must thank my dearest family for the support over the years. The education I was given and the interesting friends I've made during my studies. Some discussions with my friends have been most useful for this thesis. I would thank the Pauli Murto for patience and for guidance to the light, when I was trembling in the darkness.

Sincerely,

Karri Lehtonen

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Nomenclature

A	factor productivity of final sector production capital and labor
C	consumption
C_x	cost function of the intermediate sector
D	factor productivity of intermediate sector capital and resources
d	extraction rate of non-renewable resources
α	quality depletion
g_d	growth of extraction rate of non-renewable resources
g_{R_E}	growth of renewable resources
H	maximum sustainable yield
I	investments
I_K	investment in industrial capital
$I_{R\&D}$	investment in research and development
K	capital
K_D	capital used in intermediate production sector
K_E	carrying capacity of renewable resources
K_P	capital used in final sector production
$K_{R\&D}$	capital allocated on R&D sector
L	labor
L_P	labor allocated to final sector production
L_{Re}	labor allocated to recycling research
$L_{R\&D}$	labor allocated to R&D sectors
L_S	labor allocated to substitution research
$\boldsymbol{l_{Re}}$	technical substitution licenses for recycling, recycled resources
l_S	technical substitution licenses for renewable resources, used renewable resources
N_{Re}	technical recycling rate of resources-in-use
$N_{R\&D}$	technical rate in R&D sector
N_S	technical substitution rate of renewable resources
P	price
P_0	initial price
P_D	price of intermediate sector virgin raw material inputs
P_N	price of non-renewable resources
P_{Re}	price of recycled resources
P_x	price of the intermediate sector goods
R_C	resources from production allocated to consumption
$R_{C,agg}$	aggregate consumption resources
R_D	extracted resources, resources used by intermediate sector
R_E	renewable resource stock
R_K	resources from production allocated to capital
$R_{K,agg}$	aggregate resources allocated for capital
R_N	non-renewable resource stock
R_P	resources used by final production sector
R_R	recycled resources flow to the final sector
R_{use}	resources-in-use
R_x	resources from intermediate sector

r	interest rate (price of capital)
r_E	intrinsic growth rate of renewable resources
r_N	intrinsic growth rate of technological rate
$r_{R\&D}$	intrinsic growth rate of technology
$r_{N_{Re}}$	intrinsic growth rate of recycling rate
r_S	intrinsic growth rate of substitution rate
s	saving rate
s_D	savings rate for intermediate sector capital
s_K	saving rate for capital
$s_{K,P}$	saving rate for final sector capital
$s_{R\&D}$	saving rate for research and development
$s_{R\&D,Re}$	savings rate for recycling rate R&D
$s_{R\&D,S}$	savings rate for substitution rate R&D
u	instant utility
U	welfare
v_P	final sector labor share
v_{Re}	recycling R&D sector labor share
v_S	substitution R&D sector labor share
W	waste stock
W_P	waste from final sector production
W_N	waste stock of non-renewable resources
$\mathrm{W}_{\mathrm{Use}}$	waste from depreciation of resources-in-use
w	general wage level
w_p	wage level on the final sector
$\boldsymbol{w_{Re}}$	wage level on recycling sector
$\boldsymbol{w_S}$	wage level on substitution sector
x	output of the intermediate sector (intermediate sector production)
X	total resource demand in final sector
Y	output

α	output elasticity of final sector labor
β	output elasticity of final sector capital
γ	growth parameter for the price
γ_{1t}	natural renewable resources growth parameter
γ_{2t}	consumption rate of renewable resources
ε	demand elasticity for final sector
δ	output elasticity of intermediate sector capital
η	elasticity of marginal utility with respect to resources-in-use
λ	depreciation rate of capital
π_D	intermediate sector profits
π_{Re}	recycling sector profits
π_S	substitution R&D firm profit
$\boldsymbol{\pi_y}$	final sector profit
σ_W	waste parameter for non-renewable resource extraction
σ_F	final sector production waste parameter
ρ	waste harmfulness factor
φ	efficiency of R&D effort
φ_{Re}	efficiency of R&D for recycling rate
φ_S	efficiency of R&D for substitution rate
ω	natural depreciation rate of waste

Subscripts

D	extraction/extractive sector
E	renewable resources
K	capital
N	non-renewable resources
P	final sector production
Re	Recycling
R&D	research & development
S	Substitution
t	time
Use	resources-in-use
x	intermediate sector
Y	Final sector output

1 Introduction

There has been increasing alert on the depletion of resources for the past decades as more and more of the world countries have increased their industrialization levels. This has increased the standards of living all over the world but at certain unavoidable costs. As the production capital and resource use have increased significantly due to industrialization the resource scarcity as well as the waste generation that is degrading the renewable resources has become a great concern. The serious threat to nature and to human survival has raised questions. The debate begun with Medows et. al. (1972) book "Limits to Growth" with the fundamental questions:

Do we have enough resources to fulfill our and future generations increasing needs?

What do we need to do to have the necessary resources to fulfill these needs and how can we avoid the resource related Malthusian catastrophe?

The current economic theories are trying to find some answers to these questions. Commonly used argument claims that these questions can be answered through technological innovation and R&D (research and development) investments on substitution for backstop technologies. A less researched topic is how these R & D investments should be allocated, which brings us to the research question:

What kind of an economic model is needed to find an answer to the question of how the R&D investments should be allocated for optimal and sustainable resource use?

With the endogenous discrete time model constructed in the thesis we could be able to analyze R&D investments allocation on substitution between non-renewable resources and renewable resources[1] and substitution between waste and recycling[2] which are important factors for sustainable resource use. For the time being the model is constructed and a meta-analysis of the model is being done. For more specific mathematical analysis the optimized steady state growth path of the model would need to be solved.

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[1] generally referred as "substitution technology" or "substitution" in this thesis unless the context makes it clear that we are referring to recycling technology or backstop technology more generally.

[2] generally referred as "recycling technology" or "recycling".

The final answer is complex and there are many factors that need to be discussed (economical, political etc.). This thesis will approach these questions by including the technological development processes through R&D investments into the model together with resource use. The focus of this Thesis is in the technological processes that are needed to archive a sustainable solution from resource use point-of-view. Specifically we are most interested in the substitution of non-renewable resources with renewable resources as well as the substitution of generating waste from the end-of-life products by generating recycled resources. The theoretical background for the model is given by most importantly Solow (1974) and Romer (1990) while features from recent studies (such as Tahvonen and Salo, 1991, Tahvonen and Kuuluvainen, 2001, Tsur and Zemel, 2001 and 2006, Zon and Yetkiner,2003, Di Vita, 2006 and Acemoglu, 2011) are used for their similarities with the constructed model.

1.1 Model description

An endogenous economical model needs to be constructed where the main focus is in the effects of technological progress on resource use in economy when considering a closed system with limited resources of which part is renewable (resources that will be regenerated in short time period) and another part is depleting resources (resources that have very long or infinite regeneration rate). These resources can be seen as the basis for the primary resource pool that will be available for the humanity. This resource pool represented in this thesis can be either wasted or recycled after it has been altered so that it can be utilized by the final sector once more. Alternatively we can turn from using non-renewable resources into using renewable resources by substituting a part of our depleting non-renewable resources. In order to benefit from the substitution technological development is required. This technological development is driven by the non-renewable resource price change. There is one factor that limits the possibilities of substitution which is the renewable resource regeneration rate which shouldn't be exceeded in order to avoid the sudden extinction of renewable resources (such as human beings) (Bolden and Robinson, 1999).

Some general assumptions[3] are required for being able to construct the model that could be defined here.

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[3] All the assumptions made in this Thesis can be found in the Appendix C

The Earth can be considered to be a closed system that is in equilibrium with its surroundings (the space). This notion is made so that for example sun light is considered as characteristics of the system, and the source for all renewable processes on earth. It is thus justified to model the earth resource use as simplified closed economy without the government consumption since it's indifferent who consumes and invests in this context.

An assumption is made that the depleting resources are plentiful in the beginning as compared to the capital base but as capital is building up and non-renewable resources are being utilized they become scarce and the price of these resources will become eventually higher as lower grade deposits will be needed to replace high grade resource deposits. This development has occurred for example with copper where the ore grade has been falling significantly while the technological progress has made it possible to recover even lower grades. While the production technology has improved significantly the costs related to the extraction has grown enormously (Ayres et al., 2002 pp. 14-15). Another example could be oil where the easily accessible crude oil reserves have been consumed in most parts of the world while more difficult and unconventional production such as shale oil has become a costly alternative[4] (N.A. Owen et al. 2010). To slow down the depletion of non-renewable resources we need technological development. One possibility for slowing down the natural resource depletion is to recycle more and more of the existing storage of processed resources to new products as well as to possibly extract the resources from waste streams. These two sources of resources are comparable with each other in the end. This is because waste from production could be considered as a stock of resources that has not yet been recycled which creates a clear analogy. We will treat recyclable resources as one in a simplified manner and the possibility to convert waste to products is not included within the model.

This scarcity as well as recycling comes at a cost in production as it is a fact of physical world that is related to the second law of thermodynamics and the related concept of exergy (or available energy as it is commonly referred, which will be defined later) that the higher recycling rate we want to archive the more energy and materials will be needed to recover the resources from recycling streams causing a significant loss of exergy. Besides this the more processing steps there are the more it will consume resources and create waste. At some point we'll end up producing more waste (in form of wasted energy or materials) than we will be able to recover through recycling. Thus in thermodynamic and material means the process itself eventually becomes

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[4] This could be interpreted as an example of technological advancement as well.

inefficient[5]. Whilst the improvements in recycling rate as well as in substitution rate do have an economic logic:

"If it were possible to accurately correlate the exergy content of any resource with that resource's economic value, then arguments for avoiding stock exergy depletion could be based on purely economic arguments. Any industrial system that met consumer needs with a reduced level of resource depletion could be considered a more economically efficient system." (Connelly and Coshland, 2000a, pp.157)

Where the exergy can be defined as follows:

"An exergy is a thermodynamic state property which describes the available work of the system or more precisely the exergy of a system can be defined as the theoretical maximum amount of work which can be extracted from physical system when it is brought to thermodynamic equilibrium with the environment by means of reversible processes." (Lehtonen, 2013).

We could make a simplifying assumption that takes Connelly and Coshland (2000a) notion into account by introducing a hypothesis that exergy content of the product and production function are interrelated as in the case of traditional Cobb-Douglas production function we have labor and capital that do physical work on the system to improve the exergy content of the resources which then improves the value of the resource in economic terms.

This definition states that if some property of the initial system is altered (ie. the chemical composition or the physical properties of the input resources) it requires exergy in some form or another. For practical limitations not all of the generated exergy can be fully captured, which leads to waste of exergy in the form of chemical and/or physical exergy. The more the waste has exergy, the more it has potential to do harm to the nature (to renewable resources in this context) giving a motivation to recapture as much of the exergy as possible and recycle it back into the system with certain limitations. From thermodynamic point of view the recycling makes sense only when the exergy conserved through reprocessing saves more exergy than is required for recapturing the resource. Thus there is a practical limit for how much can be thermodynamically effectively recycled considering the fact that the higher recovery rate we want the more exergy will likely be wasted in the process. (Connelly and Coshland, 2000a and 2000b)

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[5] See H.E. Daly's (1992) comments on J.T. Young's (1991) article on entropy as a constraint of economy. For more discussion on the exergy as a constraint for recycling and reuse see L. Connelly and C.P. Coshland, 2000a and 2000b.

In the model we will assume that all production in final sector will cause some form of pollution or waste that will degrade the quality of renewable resources. This should be understood so that any production directly or indirectly will cause some form of damage to the nature. It is necessary to give one example that should make this point clear: if we produce wooden table we will need to cut a tree for the materials and this will result in decrease in the nature's capacity to absorb some potentially harmful substances (that the tree that was cut would have otherwise absorbed). Besides this we'll have to have tools to be able to produce the table and the production of these tools will cause some form of waste as well etc. not only this but the energy for tools, and in this case the damages caused by the energy production, could be understood as an indirect damage to nature as well. The generated waste will lower the absolute amount of renewable resources by decreasing the regeneration rate of the renewable resources. This far what is described the approach is closely related to the life cycle assessment and the thesis resource use can be understood from one point of view as such. Through technological progress the waste levels could be altered so that there will be decreasing relative share of pollution/unit per produced good as the share of renewable resources increases through substitution.

Though this possibility is not considered in the model it is worth mentioning that through technological innovation it might become possible even to recover resources from the waste possibly later at increasing cost of recovery as the exergy required recovering the increasingly mixed and low concentrate waste increases substantially. The recovery process would decrease the environmental damage for renewable resources that is caused by waste. This leads to a conclusion that the recycling and substitution technology problems are interrelated through waste generation. There is one factor that should be reflected in the model construction, which is the fact that the more we lack in the research efforts now the more we have cumulated waste that is destroying the stock of renewable resources in the future. The total waste cumulated is also assumed to be the actual cause affecting the quality depreciation of renewable resources.

We will allow the harmfulness of the waste for natural resources to depreciate at some certain level so that at the infinity the harmfulness of any damage from waste for renewable resources approaches to zero as the exergy of the waste (similarly for the wasted resource) is approaching zero as the concentration is approaching that of the system ie. the nature. For control reasons there is a limit for the maximum amount of renewable resources described as carrying capacity.

There will be still one important factor where the technological progress will have important role that is the substitution of resources from non-renewable to renewable resources. Through investments in substitution technology it becomes possible to replace the non-renewable resources in production. As the price of non-renewable resources grow the incentives becomes significant to lower the increase through substitution.

Eventually the goal is to find an optimal substitution level, where the resource use is in balance and there exists a stabile flow of resources into the economy. The challenge would then be in finding a path that would not step into a trap of depleting the renewable resources which would result in a significant destruction of capital and labor. Similarly optimal recycling technology development allows replacing part of the natural resource demand. This would decelerate the price growth of natural resources. These questions of optimal resource use will not be examined further in this research.

1.2 Contents

We will start by going through the literature on resource use starting from the 18th century literature and we will end up to latest literature available on the issue. We'll try to bind the literature together so that it will become evident why this literature was chosen in this context. After the literature review we will introduce a general description of the model that is based partly on this literature and partly on the independent work of the author that are of interest for the equilibrium model. This way we should be able to form endogenous model on technological substitution of resources that includes non-renewable and renewable resources as well as waste and recyclable resources. The distribution of the resources will be determined. In the end we will analyze some of the results from the model.

The structure of the Thesis will be as follows:

This Chapter describes the purpose of this thesis and discusses about the contents.

In the second Chapter we will take a closer look in to the literature which is required for understanding the model and its purpose. We will discuss about the historical development in resource economics literature that is related to this thesis as well as some thermodynamic and process technology concepts that are closely related to the resource use are introduced. We will also go through most of the modern resource economics literature that is somehow relevant for the model structuring.

In the third Chapter the general model construction starts by introducing some basis for model construction. Based on the purpose of this thesis a general structure of the model is introduced.

In the fourth Chapter the model is being formulated. We begin by introducing the state variables, which includes the capital, technology improvements in R&D sectors, non-renewable resources, renewable resources, waste and resources-in-use. The production functions for intermediate, final, recycling R&D and substitution R&D sectors are introduced. The utility functions are shortly discussed. Also the profit functions for each sector are formulated. The natural resource prices and intermediate sector prices as well as the demand are being discussed. The wages and labor allocation as well as the interest rate and capital investment allocation are solved in equilibrium. In the end a summary of the model is made.

In the fifth Chapter the most important findings concentrating on the R&D labor division are introduced and the expected model behavior is shortly discussed. The next steps that are required for solving the equilibrium and optimal growth are shortly discussed.

In the sixth Chapter the model is once more discussed, suggestions for experimentation of the model are made. Some propositions for model improvement and development are discussed. Also the suggestions for further research are made.

2 Literature Review

An integral part of resource use that is presented in this thesis is the non-renewable resources and their substitution with renewable resources as well the introduction of recycling for reducing waste accumulation and the harm caused by it to the renewable resources. This is why it is logical to begin with a review of recycling and waste literature after which we'll go deeper into the resource and backstop literature and contemporary studies in resource economics literature. The recycling and closed loop literature is treated here separately as the origins of such literature are based more on industrial sciences rather than economics while resources literature is strongly economics orientated.

The literature introduced in this Chapter is used as a basis when constructing the model in Chapters 3 and 4 while the biases and recommendations are discussed in the discussion part of the research in Chapter 7.

2.1 Recycling and Closed Loops

Probably one of the first authors to address the possibilities of waste reuse and waste use in other manufactured goods production was Simmonds (1862) in his book "Waste and Undeveloped Substances: a Synopsis of Progress Made in Their Economic Utilisation During the Last Quarter of a Century at Home and Abroad" where he made notices about the huge economical potential and real world examples in the utilization of waste in production. The initial inspiration for this Thesis comes from the following sentences of Simmonds (1862):

"…It may be truly said that there is scarcely any manufacture in which there does not remain, in the form of residue or waste, something which, though not suited for that special manufacture, has still a considerable economic value…This is one of the characteristics and salient points of modern enterprise, not only to allow nothing to be wasted but to recover and utilize with profit the residues from former workings. The diminution in price which results from utilizing matters otherwise wasted, may easily be conceived… extensive works and factories are in better position than small ones, in concequence of the larger quantity of residues at their command, and which necessitate special machinery for working up or utilizing…" (Simmonds,1862, pp. 4).

He was also probably first one to address the possibilities of innovation in waste utilization or as he puts it *"…since every day furnishes new instances of what has become one of the most striking features of modern industry – to let nothing be lost, and to re-work with profit and advantage the residues of former manufacturers - …"* (Simmonds,1862, pp. 477).

The question of recycling and waste recovery was again forgotten in academic research for more than century as the prices of resources declined and it made no sense to recycle in economical means in many industries. Not much before 1972 that is, when Smith (1972) constructed modern approach to waste recycling where he defined the utilitarian economic conditions for recycling as well as he included the harmful effects of pollution in a model for the first time.

Ayres (1999) shows in his research pragmatic approach towards the resource use in a closed system and the limits of resource recyclability where he argues that despite of the second law of thermodynamics that is related to the natural tendency of a system to increase entropy (chaos) the system can still be sustainable and no limit for recycling might exists if the flux of energy (in case of earth: solar energy or thermal energy) is on adequate levels that enables the recycling processes. For simplicity the flux of exergy is assumed to be fixed in the model, which creates an actual constraint for the renewable resource use in the system.

The questions of waste recovery and closed loops in material use have gained some momentum in industrial sciences lately. One interesting concept towards which the real world industries might be developing in the future, which is partially supported also in the economic model presented in this thesis by including recycling and its return loop into the production, is the concept called "Industrial ecology" (Frosch and Gallopoulos, 1989, Ehrenfeld, 1997).

Connelly and Coshland (2000a, 2000b) suggests that in order to improve the industrial system based on industrial ecology it is of importance to reduce the exergetic losses of the system. The reduction of exergetic losses has various beneficial implications to the system. As exergy changes are always present wherever resources are transformed from one form to another *"Avoiding the depletion of non-renewed stock resources through the establishment of closed resource cycles driven by flow or renewed stock exergy sources would eliminate a significant driver of environmental change… to avoid future environmental crises caused by current resource depletion in immature industrial ecosystems…"* (Connelly and Coshland, 2000a, pp. 159). Connelly and Coshland (2000b) has shown a possible way to express the depletion of resources by dimensionless "depletion number" that is expressed through recycling, efficiency and use of renewable resources, which all are a part of this thesis.

2.2 Resource Use, Substitution and Growth

The concerns related to growth and limited resources were first addressed by Thomas Malthus when he wrote his long sighted "Essay on Principles of Population as It Affects the Future Improvement of Society" in 1798. There Malthus proved that the population growth will be limited by food resources at some point as population growth is exponential while the food production is linear, leading to a situation that is known (introduced by later authors) as "Malthusian catastrophe"[6]

. This is analogous to the current situation where growing economy is demanding more and more of the natural resources while the resource stock available is practically limited to some finite point (that is, if the space is not utilized). It took a while before this side of resource use was approached first by Harold Hotelling (1928) in his fundamental article "The Economics of Exhaustible Resources". Hotelling's model was first to show the optimal depletion rate of resources as well as the price path that this depletion rate would cause, a principle that is now known as the "Hotelling's rule" as introduced in equation 1.

$$P = P_0 e^{\gamma t} \qquad 1$$

Where $P =$ price, $P_0 =$ initial price, $\gamma =$ growth rate, $t =$ time.

This rule could be implemented in discrete form as:

$$P_{t+1} = (1 + \gamma) P_t \qquad 2$$

von Ciriacy-Wantrup (1952) was probably one of the first ones to describe and define the conditions for sustainability as well as to divide exhaustible resources into renewable and non-renewable resources in the modern sense, though the difference might have been acknowledged previously by some authors. He also connected the depletion of non-renewable resources into the possibility of technological change in his insightful book "Resource conservation: economics and policies". He suggested that technological substitution might occur from non-renewable resources

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[6] A situation where food production/person is at its minimal level that is still sustainable but the resource use/person is virtually zero. Thanks to the "green revolution" in the 1960's and 1970's we have been able to avoid this logic for now.

into renewable resources. Ayres (2007) discusses generally the weak and strong sustainability that is the possibilities to substitute the natural capital with man-made capital. According to weak sustainability point-of-view all of the natural capital can be replaced by man-made capital (a position taken by Solow and many of his followers) while strong sustainability point-of-view opposes this view. A point taken in this thesis is that of strong sustainability. It is observed that eventually the non-renewable resources will be depleted or the share of those resources is so small in the production in the end that it makes only a fraction of total input and resources available after that will be practically renewable resources and recycled resources.

Some of the findings made in the von Ciriacy-Wantrup (1952) book might have been one of the initial motives for The Club of Rome publication “Limits to Growth” in 1972 (Meadows et. al, 1972). This book concentrated on issues concerning the human population growth, industrialization, poverty, dependency on non-renewable resources and loss of environmental quality. The approach was a scenery analysis. The results of the analysis clearly showed that the economic growth relying heavily on non-renewable resources can’t be sustained due to the limited resources and that the humankind is on a way to economic, environmental and social collapse in the mid 21st century. It was concluded that through technological improvement new substitute technologies could decrease the depletion rate, the pollution levels could be lowered and in the long run the production (or resource use) could be halted to some sustainable level either through predetermined policies or through collapse. These scenarios will be experimented in the analysis part of the thesis. The book emphasizes the possibility to unlock the connection between resource use and growth as way to have possibly unlimited growth, which case will be not discussed in this context. (Meadows et. al, 1972)

The issue of resource exhaustibility was forgotten in economics mainstream debate for decades because the resources seemed to be limitless until it became more relevant during the 1970’s when the World experienced two oil crises, first in 1973 and latter in 1978. This also significantly increased interest on The Meadows et. al. (1972) work. Probably due to the first crisis there were two extraordinarily influential papers published in 1974 in “The Review of Economical Studies”, Vol. 41. The other paper was Solow’s (1974) “Intergenerational Equity and Exhaustible Resources” where the depletion of resources was considered as a question of justice between each generation and the just rate of depletion was defined for the first time to find out how the resources should be used in order to have the following generations needs fulfilled. The interest for defining the intergenerational justice was possibly motivated by Rawls (1971) book "Theory of justice". In his paper Solow was assuming exogenous growth of technology together with non-renewable resources use. He was able to prove a connection between raw materials prices ("Hotellings rule") and cost of capital (interest rate), which was a significant finding. The other paper was one by Dasgupta and Heal's (1974) article titled "The Optimal Depletion of Exhaustible Resources" where they established ground for modern resource models in economics. Their exogenic model was one of the first ones to actually include the substitution of exhaustible resources by technological change into backstop resources which can be interpreted as renewable resources in this context. Many later scientific articles related to substitution technology are based on their work.

When approaching the subject from resource use sustainability point-of-view, the Dasgupta and Heal (1974) model had certain evident flaws. The main argument against their model is related to an assumption that there will always be some substitute for the input. Problem with this assumption comes clear when huge and widely used input resource is not easily accessible anymore putting pressure on the price. This would lead to development of new technologies and backstop resources use as Dasgupta and Heal (1974) suggests. The new backstop resource stock needed, as in this case renewable resources, would likely be very large to replace the original resource. But the larger the old resource utilization was the larger the new deposits would need to be and the faster it would be consumed leading to possible exhaustion of renewable resources in this case. Thus the solution should not be addressed completely by traditional backstop technologies but instead we should admit the resource constraints to resource use in the growth theory when the non-renewable resources substitution with renewable resources is considered. This is why the recycling technologies as well as the renewable resources become significant part of this problem offering alternative interpretation for backstop technology development.

Some of the assumption made in Dasgupta and Heal (1974) model has been questioned later for example by Smith and Krutilla (1984) but none the less many of later researchers rely on their findings and their research shouldn't be ignored for its limitations. The essential backstop technology development we will be focusing is the change and transition of depleting resource into renewable resources and waste into recyclable resources that will happen during the transition between this day and the future date when the non-renewable resources are consumed and substituted with renewable resources. Stigliz (1974) has proved that given sufficiently fast technological progress it is possible to archive consumption per capital that does not decrease.

After the initial steps taken by Solow (1974), Dasgupta and Heal (1974) and Stigliz (1974) the resource research on economics started to gain some momentum that lasted until the mid 80’s while the academic research on resource economics took a rest until the mid-90’s when the environmental questions started to raise concern once again as the discussions about the global warming, that led to the Kyoto agreement in 1997, started to increase. Ever since, the research in environmental and resource economics has increased and it has become one of the mainstream research topics lately.

An interesting addition to Solow’s intergenerational equity was done by Riley (1977) where considerations on substitution of non-renewable resources by renewable resources were combined with intergenerational equity. The intergenerational justice constraint should lead to a situation where existing generation should have lower consumption than it otherwise could have thus a part of resources should be saved for the following generations. Riley’s (1977) third proposition states in practice that the initial level of stock lengthens the time until the alternative source is utilized as well as it lengthens the time the natural resource is fully depleted. This feature should have effect within the model of this thesis as the proposition should have affect through the recycling rate development and substitution, both of which increases the level of resources available and thus lowers the extraction rate of non-renewable resources. On the other hand Riley’s (1977) seventh proposition, which suggests that initially lower utilization of non-renewable resources, would lead to delayed adaptation of substitution technology.

Kamien and Scwartz (1978), as well as Dasgupta, Heal and Majumdar (1976), introduced endogenous technological change into exhaustible resource depletion models where research effort had effect on the probability of an invention. Kamien and Scwarz (1978) introduced the possibility that the output could be used for consumption, research and development investment or capital accumulation. In their model natural resource extraction is dependent on capital and resource use. Kamien and Scwarz (1978) noticed that the insertion of extraction costs into the model did not alter the basic resource use path for which reason the extraction costs can be left out of the model. Kamien and Scwarz (1978) were concerned on the possibility that if a R&D

development was started too late it could become too expensive to continue the development compared to the actual output.

Around the same time Modiano and Shapiro (1980) concentrated on dynamic optimization of depleting resource leaving out the renewable resources. Their work gives insights on how the actual resource prices are sensitive to substitute technologies development and capital investments that were confirmed in later researchers such as Bretchger and Smulders (2006).

De La Grandville (1980) based his research on Solow and Samuelson's work (1978) but it addressed more the optimal substitution of non-renewable resources with renewable resources through technical development. Most interesting of his results might be that the community should be indifferent of using some resource at given time or another which contradicts Kamien and Scwarz (1978) notice that delayed R&D efforts could halter the efficient development of resource use and thus lower the resources available at any given time which is more in line with Riley's (1977) notices on the meaning of initial stock size. Besides this La Grandville (1980) noticed that the interest theory and optimal growth theory are intimately related and this has effects on the optimal allocation of different types of resources. His research also suggests that generalization of discrete time models provide similar results as dynamical models in similar questions, which possibility is utilized in this thesis.

Lewis (1981) took renewable resource research one step further by taking into account the uncertainty concerning the renewable resources stock by using tuna fisheries as an example and Markov decision process (as introduced in Howard 1961 and 1970) in his analysis. Since in my model we've expected the resource stocks to be known for practical reasons the uncertainty factor can be excluded but Lewis uses some dynamic models that can be easily generalized for our renewable resource model so his research becomes useful. The most significant finding from Lewis (1981) research is that the maximum sustainable yield (MSY) of renewable resource might not be optimal under uncertainty because the risk of depleting the resource is large in case of significant variation in the renewable resource yield. In this thesis the possibility of uncertainty is not considered and equations describing the MSY can be used as such as constraints in the model.

Similarly M. Eswaran, T.R. Lewis and T. Heaps (1983) continued Lewis's (1981) work by examining the competitive market equilibrium in markets where there are decreasing costs to find out that equilibrium didn't actually exists. Their main concern was that scale economies together with U-shaped average cost curve would create non-convexity that could destroy the equilibrium conditions and at the same time it could make socially sustainable solution for resource exploitation impossible. We won't discuss about their findings in this thesis further but it's important to take the findings of Esweran et. al. (1983) into account when considering policy controls for finding optimal resource use

Simon (1981) disagreed with the common belief that the resources are finite. He argued that the resources are in fact infinite in relative terms. That is, from the same amount of resources it will be possible to get a higher utility as technologies develop. In this thesis no stance is taken in relative terms where as the model represented here is mainly relevant in absolute terms of resource use. When considering the resource use in absolute terms (which is important when considering the negative externalities of production or recycling), the limits to growth due to physical world limitations becomes relevant.

Interesting but controversial research that concentrates on the uncertainty of non-renewable resource use and exploration comes from Arrow and Chang (1982) using Poisson process for the distribution of the resources contributing to the resource research. Their approach was established on the basis that "Hotelling's rule" (Hotelling, 1928) was not performing well for the time being probably for the reason (speculation) that the exact resource stock is not known at certain time and the market prices are likely to adjust for the new findings affecting the initial price. Their analysis shows that the resource prices are changing at random due to new explorations but when new explorations are not made their view supports the use of "Hotelling's rule in the model. Their findings support the use of Hotelling's rule for the non-renewable resource prices.

Another controversial research was done by Farzin (1984) as he found exceptions to the common belief that has it's basis in the Hotelling's (1931) findings according to which the higher discount rate leads faster depletion of exhaustible resources as well as lower discount leads to slower depletion. An important notice that could have some effects in some discrete model was made by Farzin (1984). According to him a breakthrough in the substitute technology will increase the current extraction rate of the resource to be substituted lowering its price and increasing the demand. There is an analogy from this to my model as an increase in renewable resource use or recycling rate could alter the price of extracted resources (understood as equal price of renewable and non-renewable resources in this context) and their demand. Farzin's (proposition 1, 1984) observation that is also worth noticing is that the capital intensity of the competing technologies will have significant counter intuitive effects on depletion rates in case if we adjust the discount rate. This effect is due to two unrealistic assumptions in Hotelling's model: no technological substitution and no capital involved with extraction. Farzin (1984) found out that there exist resource stock sizes that could have faster depletion with lower discount rate and the other way round. For practical reasons we might need to rule this kinds of scenarios out of the model as they might complicate the analysis but none the less this is good to understand as one possible weakness of my model.

Romer (1990) presents endogenous model that takes into account technological change. He's working paper is very fundamental for structuring a model for R&D activities on monopolistic markets and introduces the human capital as a separate input into the R&D equations. He's constructs a simplified two sector model with innovation and final production sectors. In Romer (1990) model the production function can be expressed by human capital, capital, knowledge and labor. Romer's (1990) approach on endogenous innovation has been widely used ever since in academic discussion. In Romer (1990) the growth of knowledge is introduced in general terms and no limits for growth exist as there are no limiting inputs such as resources included to the model. The Romer's model is described in figure 1. Orange colors indicates R&D sector, blue represents the industrial sector and inputs are indicated by light green.

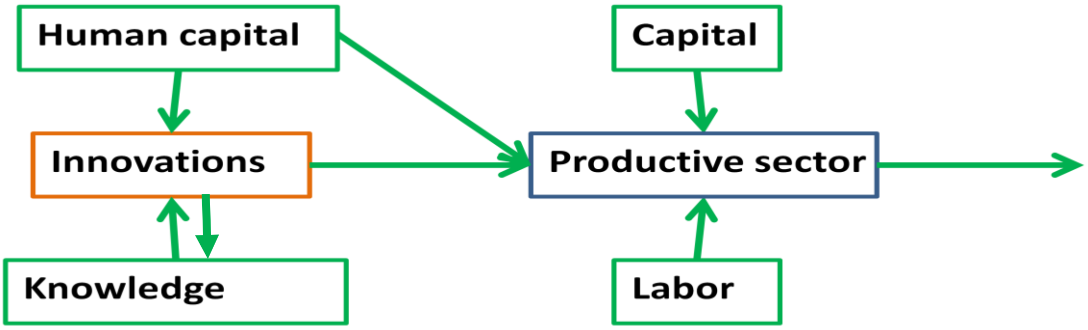

Figure 1, Romer (1990) Endogenous Technical Change model

In the later research some additions, limiting assumptions and restrictions have been implemented to the model. Most importantly the resource economics has benefitted remarkably from Romer's (1990) approach when studying the transition to backstop technologies such as the substitution from non-renewable resources into renewable resources. Romer's approach can also be applied to the substitution from waste into recycling. In this thesis some upper limits for technological knowledge are introduced for substitution and recycling rates. These model limits can be understood as thermodynamic limitations for efficiency.

The idea of net national income was introduced by Samuelson in (1961). Hartwick (1990) expanded the concept by introducing the pollution and its harmful effects on total output of the nation. He suggested that any harm caused by the pollution to the renewable resources should be deducted from the overall Gross National Product (GNP) calculations. An interesting calculation example that is slightly related to Hartwick (1990) proposal was done by Constanza (1997) where he calculated the value for the ecosystem services. The idea of Hartwick (1990) is carried out in the resource part of our model through introduction of waste as a stock variable that is dependent on the part of resources-in-use depreciation rate and volume as well as the non-renewable resource extraction and final sector production volumes.

An important step in sustainability research was taken by Barbier and Markandaya (1990) where they determined the conditions for sustainable development as well as they found the necessary conditions for non-sustainable development. Their findings supported the points of views of Malthus in case where the natural resources are low as compared to population when the survival (or the high utility discount rate) becomes a significant limiting factor for the sustainability. According to their research, for as long as the utility discount rate is adequately and the resource stock is high enough a sustainable resource use path can be archived. (Barbier and Markandaya, 1990)

Tahvonen and Kuuluvainen (1991) introduced the pollution into an endogenous growth model where the pollution is regarded as a part of the production function that has adverse effects on regeneration rate of renewable resources. The model of Tahvonen and Kuuluvainen(1991) has three inputs: capital, renewable resources and emissions. The idea of the waste behavior in the model is much similar to our model, while in our model the waste is left out of the production function by treating the waste stream as a separate loop causing harm to renewable resources through depreciation of produced goods and capital rather directly through production function. Besides this any social problems caused by pollution as suggested by Tahvonen and Kuuluvainen

(1991) are left out of our model[7]. This choice is made in order to concentrate on the phenomenon we are most interested, that is the R&D in our case.

In Tsur and Zemel's (2001) article a model on optimal substitution of non-renewable technologies with backstop technologies which assumes smoothly improving technology curve is introduced. Smooth improvement is an approach that can be considered as characteristic development for recycling rate technology as well as for backstop substitution rate technology where large leaps in technology are not expected to happen. The Tsur and Zemel (2001) model assumes that technology can become obsolete due to ageing which reduces the efficiency of technological innovations. This assumption does not necessarily hold true for recycling rate or substitution rate as it can be expected that there won't be transition to less efficient technologies. On the other hand in our model the efficiency of technology improvement is expected to decrease as the technology approaches its theoretical maximum rate since it becomes more and more difficult to make improvements to the system.

Tahvonen and Salo (2001) have also made research on the transition from non-renewable resources into a backstop resource. They have concentrated on the dynamics of the system in time and in their model the approach is very general. Their models main problem is that it takes no stance on the allocation of resources such as capital or labor as the main focus is on the resource consumption as a whole. On the other hand the results are of such a general type that our model might have similar time paths for natural resource use as represented in their model.

The model represented by Zon and Yetkiner (2003) introduces an intermediate sector to the model which is utilizing the innovations that are created by the R&D sector in order to produce products for the final sector. The intermediate sector is utilizing energy and capital as inputs, while the innovations improve the productivity of the intermediate sector. The innovations that are created on R&D sector require knowledge and labor as inputs. On the other hand the final sector utilizes the intermediate sector products and labor to produce the final goods. Their model had many common features with our model. Zon and Yetkiner (2003) model is shown in figure 2 for its importance in model construction. The dark green lines represent the flow of natural resources.

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[7] Although the harm to renewable resources does have indirect effects on utility since the possibilities to utilize the renewable resources are decreased.

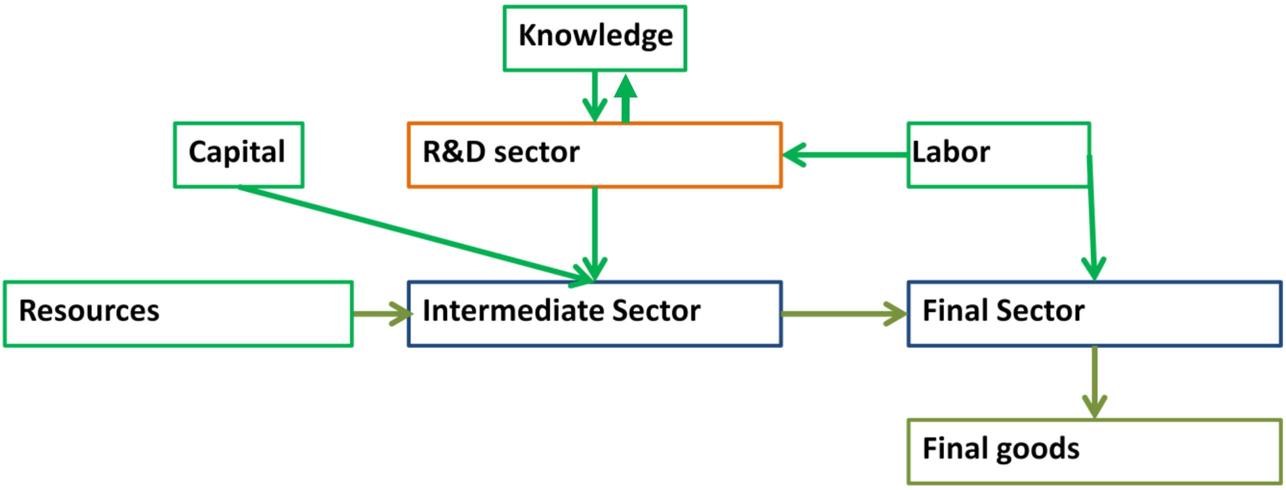

Figure 2, Three sector model by Zon and Yetkiner (2003)

Bretschger and Smulders (2006) introduced a five sector model where technology and production are direct competitors for input resources, whereas their model has only one R&D sector and it doesn’t have renewable resources nor pollution, their model focus more on the R&D sector development under different conditions. Bretschger and Smulders (2006) model concentrates most importantly on the differences between sectoral substitution opportunities which affects in labor division between the sectors, which is also one interest in our model. They find that increasing resource scarcity price makes the sector with least innovation opportunities relatively expensive to other sectors. This phenomenon is represented in our model R&D sectors through decreasing efficiency of innovative sectors.

Tsur and Zemel (2006) constructed an endogenous three sector model with smooth backstop technology process through R&D sector activities, intermediate sector that utilizes the R&D and a final sector production. In their model the resource stock is expected to be limited while the backstop technology is not, as the focus is in the optimal capital allocation and R&D process for substitution. Their model is expected to have similarities with our model when it comes to the prices effects of R&D, competition for resources and scarcity effects on R&D efforts.

De Vita (2006) experimented on changes in rate of technological substitution in a three sector model where the waste and its negative externalities were included into the model. The main difference between our model and that of De Vita’s (2006) is in the additional focus on recycling of resources-in-use, which requires the inclusion of additional recycling technology R&D sector into the model which would have significant effects on the waste generation. The latter is crucial since in the real world exergy losses can be significantly reduced if the processed material is not wasted even if the material itself could renew in the nature and thus it makes sense (at least from thermodynamic point of view) to take into account the waste generated by the processing of renewable resources at some stage. The most significant exergetic losses of the system are caused by the depletion in the quality of the resource when it is discarded as waste as the work done to the system is practically lost. None the less the production processes where the resource is modeled from one form to another can cause significant exergetic losses as well. Even though this “waste” might not be direct, the process itself consumes energy and creates pollution in exergetic means and affects the renewable resources growth in a way or another, which needs to be introduced into our model.

The main difference between my model and that of Acemoglu et. al. (2010) is that I assume that there doesn’t exists two different kind of production technologies where the other is polluting and the other isn’t. Generally we will assume that all production causes some form of depletion of renewable resource. Additionally in our model there is the intermediate sector where the pollution is assumed to be dependent on the utilization of non-renewable resources in the production and through substitution the pollution on intermediate sector can be decreased, which brings some similarities with the Acemoglu et al. (2010) model. Our assumption is well defined since in order to produce anything (product or capital) one must extract the raw-materials from the nature and in practice all extraction processes are indeed causing some form of environmental degradation at some point of the process. This degradation of resources can be seen as either direct (materials are extracted directly from nature such as iron ore or timber) or indirect (where raw-materials are extracted to produce the capital that generates for example virtually non-polluting energy). The degradation when utilizing the renewable resources in the production is accounted for when the resource is used by the final production sector.

3 Overview of the Model

One of the main focuses of this thesis was the resources and their use. The real resources (or the resources that can be used to produce products or capital) are extracted in low grade from the natural world and they are consumed in production to fulfill the materialistic needs of humans by producing products that the human needs (such as housing, food, energy, mobility etc.) and their use is commonly related to the economic activity. Bagliani, Bravo and Dalmazzone (2006) research paper provides evidence that there exists high correlation (and causality) between the GDP and ecological footprint that describes the overall resource consumption in economy. This relation justifies the assumption made in this thesis that the resource use and economic wellbeing are highly connected to each other and the latter can be expressed in terms of the first. In this thesis the "resources-in-use"[8] actually includes all forms of capital and consumption goods that create utility to humankind. The production function in this thesis can be understood to produce exergy in that sense that the more the resource is processed the more exergy is contained into the actual product which requires use of resources and thus adds to the economic value of the product. This might not be the actual case as this is more of a conceptual interpretation, while the real world is much more complex than that.

Besides resources our focus was in the technological processes that lead to improvements in recycling rate as well as in the substitution of non-renewable resources with renewable resources. The model should have much similar dynamical behavior as in Kuuluvainen and Tahvonen (2001) general transition model. Theoretically our approach should have similarities with Tsur and Zemel (2001, 2006) where they are trying to establish a connection between the scarcity and R&D growth. Otherwise the model we are about to construct should have much similar economy structure as was represented by Romer (1990) or Zon and Yetkiner (2003) but instead of two or three sectors (R&D, intermediate production and final production) the model represented in this Thesis should have one additional R&D sector (recycling) as suggested Bretschger and Smulders (2006) as we are interested on how the resource allocation between different R&D sectors affects the actual resource use and scarcity of natural resources. Additionally the model would require resource loops for recycling and waste effects similar to the industrial ecology based efficiency models (Connelly and Coshland 2000a and 2000b).

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[8] The concept of "resources-in-use" is introduced later in Chapter 4.6

We will try to construct the model in such a way that we can answer some of the points[9] represented by Simon (1981, p. 61): *"…that supply of a service will depend upon (a) which raw materials can supply that service with the existing technology, (b) the availabilities of these materials at various qualities, (c) the costs of extracting and processing them, (d) the amounts needed at the present level of technology to supply the services that we want, (e) the extent to which the previously extracted materials can be recycled, (f) the cost of recycling, (g) the cost of transporting the raw materials and services, and (h) the social and institutional arrangements in force..."*

The model which we are about to discuss here consists of four sectors. There is the extractive sector that uses labor and capital in its production to extract renewable and non-renewable resources from the earth and its surface as intermediate products for the final sector. The substitution R & D sector then uses labor in its production to create patents and directs its research on the substitution of non-renewable resources with renewable resources and offers licenses of this technology for the intermediate extractive sector through which the intermediate sector can use more of the renewable resource in its production. The resources allocated on substitution technology are in practice directed by the price change of non-renewable resources (which on the other hand must be equal to renewable resource price changes) and affected by the share of renewable resources used in production of the intermediate sector. Additionally there is the final sector that uses shares of labor and capital to create final products from the resources extracted by the intermediate sector and from the resources that are being recycled using the technology provided by the recycling technology firm. The recycling R&D sector uses labor in its production to create technology patents for final sector where the extracted resources are substituted with recycled resources which decreases the pollution caused by depleting capital and consumption. The licenses for the patents are then sold for the final sector which can utilize the technologies in its production. The price increase of extracted resources is the driving force to create more efficient technologies. Thus the R&D on recycling has two way impacts on resource use: through increased recycling and through decreased harm to the nature. Eventually the harm to nature (waste) is decreasing as recycling approaches 100% and as non-renewable resources becomes substituted by renewable resources. In such case the only source of pollution becomes from the final sector production.

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[9] Points a,b,c,d,e and f are relevant for the model

We can describe this R&D model as in figure 3, which can be seen as a combination of the presented models with addition of waste loop. The harmful effect of waste for renewable resources is shown in red.

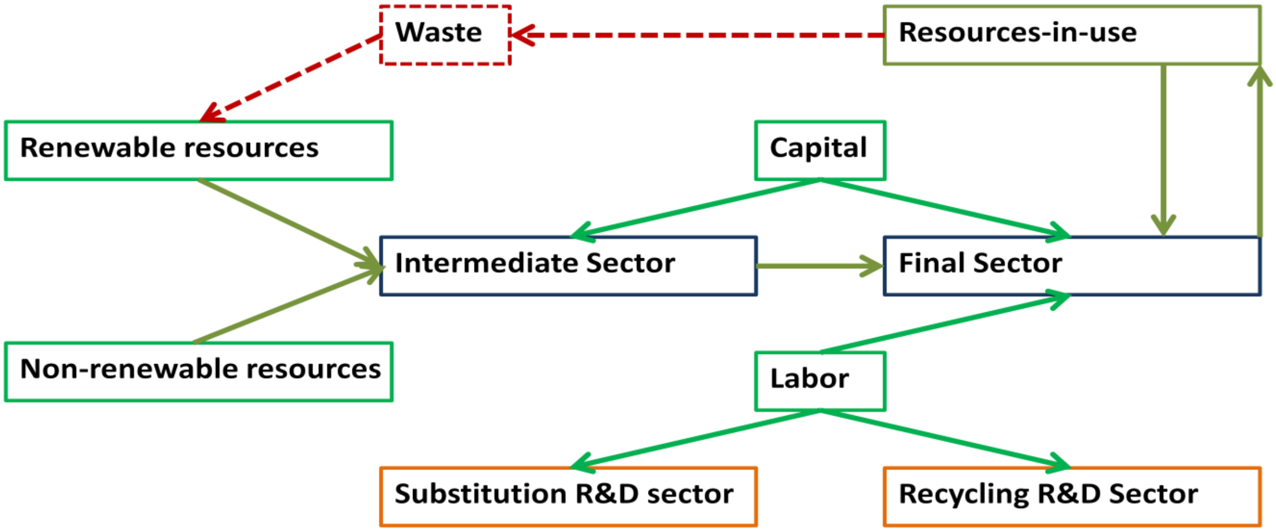

Figure 3, General description of the model

In this model labor is assumed to be a constant and thus there is no population growth. This is done to reduce the amount of state variables in order to reduce the complexity of the model. On the other hand the labor can be divided between three sectors: recycling research, substitution research and final sector production. The division of labor is one question that needs to be solved for the model.

4 The Model

In this Chapter the discrete model is being formulated. The model construction starts by introducing the state variables after which the utility, production and profit functions are described. The interest rates, wages and prices are introduced after which the labor and capital allocations in equilibrium are discussed.

4.1 State Variables

State variable are variables that can be described as stocks in a way or another. These stocks can be reduced or added as resources flow in or out of the stock according to the economic structures and system characteristics. There are seven state variables in this model, which includes capital, recycling rate, substitution rate, non-renewable resources and renewable resources, waste and resources-in-use.

The model has capital as one state variable which can be divided into two main subcategories: the intermediate and final sector capital. It is noticeable that the investments in R&D are similar to capital investments when it comes to the value of the technology and investment decisions. The division of investments allocated on industrial capital (intermediate and final sectors) is determined by the production in each sector and output elasticities of capital.

The resources are initially either non-renewable resources that can be extracted only once or they can be renewable resources that can regenerate. Basically the model assumes that non-renewable resources can be replaced by renewable resources. Beside these characters there are some loops that have effects on resource use. The renewable resource regeneration rate can be affected by the waste generated in final sector production. Meanwhile a part of the depreciating resources-in-use stock can be regenerated back into raw materials through recycling processes.

4.1.1 Capital

Capital accumulation is allowed through investments on either industrial capital or R&D and as such capital is one of the state variables. There are some differences between these capital types in our model. The savings rate determines the investments (Ramsey, 1928) as:

$$sY = I \qquad 3$$

Where $s =$ savings rate, $Y =$ output and $I =$ investments. The total savings rate is regarded as exogenous constant.

The special characteristics of industrial capital is that a certain part of the industrial capital becomes obsolete and is reduced from the capital stock. In general form the industrial capital accumulation can be described by equation:

$$\boldsymbol{K_{K,t+1} = f(I_{K,t}, K_{K,t}; \lambda) = (1-\lambda)\, K_{K,t} + I_{K,t} = K_{K,t} - \lambda K_{K,t} + s_{K,t} Y_t} \tag{4}$$

Where $K_K =$ industrial capital, $\lambda =$ depreciation rate of capital (constant) and $I_K =$ investments in industrial capital, $s_K =$ savings rate for industrial capital and the subscript t refers to time[10]. Depreciation is regarded as an exogenic property of the model. The initial level of capital would need to be determined while the capital is endogenously developing.

The investments on R&D are known to be equal to the labor costs on each sector. It is also assumed that there is no depreciation of R&D. Thus we can easily express the capital allocated to R&D as:

$$\boldsymbol{K_{R\&D,t+1} = f(K_{R\&D,t}, L_{R\&D}; w) = K_{R\&D,t} + wL_{R\&D} = K_{R\&D,t} + s_{R\&D,t} Y_t} \tag{5}$$

Where $K_{R\&D} =$ capital allocated for R&D, $L_{R\&D} =$ labor allocated for R&D, $w =$ general wage level, $s_{R\&D} =$ savings rate for R&D activities. The savings rates in equations 4 and 5 add together:

$$\boldsymbol{s_t = s_{K,t} + s_{R\&D,t}} \tag{6}$$

4.1.2 Research and Development of Technological Rates

This Chapter represents the general form for R&D efforts. It should be noted that in this model we have two R&D sectors which have basically same equations for technology development. For practical reasons the specific equations for substitution and recycling technologies are represented with the production functions in Chapters 4.2.3 and 4.2.4.

In this model investments are not restricted to traditional capital as the investment can be allocated on technologies that improve the efficiency of resource use. Such technologies in this case are recycling technology and substitution technology. An investment in technologies can be understood as capital investment. A common feature between each type of capital is that the marginal cost of capital should be equal.

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[10] for the rest of the thesis, t+1 refers to next period and so on.

Now according to Romer (1990) and van Zon and Yetkiner (2003) the increase in technological rate is determined by general equation for R&D efforts:

$$\frac{dN_t}{dt} = f(\varphi_t, N_t; L_t) = \varphi_t N_t L_{R\&D,t} \qquad 7$$

Where $\varphi =$ efficiency of research effort and $N =$ technological rate. Equation 6 can be regarded as the production function of R&D sector[11]. Thus the state variable for the technological rates becomes:

$$N_{t+1} = f(\varphi_t, N_t, L_t) = (1 + \varphi_t L_t) N_t \qquad 8$$

The efficiency factor is assumed to follow an inverse of logistic curve thus making an improvement more difficult the higher technological level has been archived. This is partially reflected by the R. Davidsons (1978) findings for the probability of technological improvement in time. This interpretation also follows the findings of B. Achilladelis et. al. (1988, p. 12), where technology development can be seen to be following a sigmoid shaped curve. Eventually the efficiency of technological development could be described by equation:

$$\varphi_t = f(N_t; r_N) = 1 - N_t(1 + r_N(1 - N_t)) \qquad 9$$

Where $r_N =$ intrinsic growth rate of technology.

Now we can now express the technology $\boldsymbol{N_{t+1}}$ as state variable:

$$N_{t+1} = \mathrm{f}(N_t, L_t; r_N) = \left(1 + \left(1 - N_t\left(1 + r_N(1 - N_t)\right)\right) L_{R\&D,t}\right) N_t \qquad 10$$

4.1.3 Non-renewable Resources

The non-renewable resource stock is understood as concentrated natural resource such as metal ore deposit or crude oil that can be extracted from the ground and utilized in one form or another in production to be transformed into utility providing product (eg. car, mobile phone, house, production machine) or service (eg. energy) . The extraction of non-renewable resource reduces the stock of the non-renewable resources and makes it scarcer affecting the price of the resource. This creates another state variable. Generally the non-renewable resources can be described by[12]:

$$R_{N,t+1} = f(R_{N,t}, d_t) = (1 - d_t) R_{Nt} \qquad 11$$

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[11] Chapters 4.2.3 and 4.2.4 for production functions.

[12] similar equation has been used by most of the authors from Dasgupta and Heal (1974) to Tsur and Zemel (2006) or Bentchenkroun and Withagen (2011)

Where $R_N =$ non-renewable resource and $d =$ the extraction rate which is proportional to the natural resource demand, non-renewable resource stock and substitution rate according to:

$$\boldsymbol{d_t = f\left(R_{D,t}, N_{S,t}, R_{N,t}\right) = \left(1 - N_{S,t}\right)\frac{R_{D,t}}{R_{N,t}}} \qquad \mathbf{12}$$

Where $N_S =$ technical substitution rate of non-renewable resources and $R_D =$ extracted resources or natural resources. The aggregate extracted resources can be expressed as:

$$\boldsymbol{R_D = f\left(R_{N,t}, R_{E,t}, d_t, \gamma_{2t}\right) = \gamma_{2t}R_{Et} + d_tR_{Nt}} \qquad \mathbf{13}$$

Where $\boldsymbol{\gamma_{2t}} =$ extraction rate of renewable resources and $R_E =$ renewable resources. The renewable resource extraction rate can be expressed by the renewable resource use $N_SR_{D,t}$and renewable resources stock:

$$\boldsymbol{\gamma_{2t} = \frac{N_SR_{D,t}}{R_{Et}}} \qquad \mathbf{14}$$

The state variable $R_{N,t+1}$ can now be rewritten by utilizing these equations as:

$$\boldsymbol{R_{N,t+1} = f\left(N_S, R_{D,t}, R_{Nt}\right) = R_{Nt} - \left(1 - N_{S,t}\right)R_{D,t}} \qquad \mathbf{15}$$

4.1.4 Renewable resources

The renewable resources are considered similar resource by their qualities as non-renewable resource with a clear distinction: the stock of renewable resources (such as plants and animals) may reproduce by them self through some natural processes (autogenesis, multiplication and other). The reproductive processes are sensitive for pollution and waste which can decrease the regeneration rate of the renewable resources[13]. The renewable resources can be regarded as another state variable. Unlike the non-renewable resources the renewable resources have some upper limit of use that is restricted by the regeneration rate of the renewable resource beyond which the renewable resource could be exhausted, while the use of non-renewable resources is restricted only by their availability.

There are some limits for the maximum renewable resource stock. The relative size of the renewable resources as compared to their maximum value has an effect on the regeneration rate of the renewable resources as well (Bolden and Robinson, 1999). In this thesis the natural regeneration rate of renewable resources $\boldsymbol{\gamma_1}$ is assumed to have a logistic relation to the stock size

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[13] It's not necessary to go deeper in the mechanics of such processes.

in relation to the carrying capacity[14] (Lewis, 1981). This can be expressed using renewable resource stock $\boldsymbol{R_E}$, intrinsic growth rate of renewable resources $\boldsymbol{r_E}$ and carrying capacity of the population $\boldsymbol{K_E}$ which can be assumed as constant[15]:

$$\gamma_{1t} = f(R_{E_t}; r_E, K_E) = R_{E_t} r_E \left(1 - \frac{R_{Et}}{K_E}\right) \underset{K_E=1}{\Longleftrightarrow} \gamma_{1t} = R_{E_t} r_E \left(1 - R_{E_t}\right) \tag{16}$$

Solving for maximum yield (Bolden and Robinson, 1999) gives:

$$\frac{d}{dR_{Et}}\left(R_{E_t} r_E - \frac{{R_{Et}}^2}{K_{Et}}\right) = 0 \leftrightarrow R_{E_t} = \frac{r_E K_{Et}}{2} \tag{17}$$

Thus the maximum yield is archived when the stock of renewable resources is equal to intrinsic growth rate multiplied by half of the carrying capacity.

We can use equation represented by Bolden and Robinson (1999) to solve the maximum sustainable yield (MSY), beyond which the renewable resource utilization can’t exceed without depleting the renewable resources.

$$H = \frac{K_E\, \gamma_{1\mathrm{t}}}{4} \tag{18}$$

Where H=maximum sustainable yield[16].

In equilibrium this growth rate must be equal to the depletion rate of the renewable resources.

$$\gamma_{1t} = \gamma_{2t} + \frac{\alpha W_t}{R_{E_t}} \tag{19}$$

Where α = quality depletion parameter and W = waste.

The equilibrium condition presented in equation 19 could be substituted into equation 18 to solve utilization of renewable resources in final equilibrium:

$$\mathrm{H} = \frac{K_E\left(\gamma_{2t} + \frac{\alpha W_t}{R_{E_t}}\right)}{4} \tag{20}$$

Generally the next period renewable resource stock is determined by the regeneration rate, renewable resource use and quality losses caused by waste:

$$R_{E_{t+1}} = (1 + \gamma_{1t} - \gamma_{2t}) R_{E_t} - \alpha W_t \tag{21}$$

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[14] Carrying capacity is the maximum size of the total population.

[15] The carrying capacity of population is assumed to be a constant with value 1 in the equation.

[16] This is the situation, where $\boldsymbol{R_{E_{t+1}} = R_{E_0}}$

We can ignore the waste generation from extraction of renewable resources since no renewable resources are wasted at extraction as this "renewable waste" will dissolve through natural processes and does not necessarily create wasted exergy[17]. The conditions for parameters are $\gamma_{1t} \geq 0, 1 \geq \gamma_{2t} \geq 0, 0 < \alpha \leq 1$.

By replacing $\boldsymbol{\gamma_{1t}}$ and $\boldsymbol{\gamma_{2t}}$ into the equation we get:

$$\boldsymbol{R_{E_{t+1}} = f\left(R_{E_t}, K_{D,t}, W_t; \alpha\right) = R_{E_t} + r_E\left(1 - R_{E_t}\right) R_{E_t} - N_S R_{D,t} - \alpha W_t} \qquad 22$$

Where

$$\boldsymbol{\alpha = f\left(W_t, R_{E_t}; \rho\right) = 1 - \rho^{\frac{W_t}{R_{E,t}}}} \qquad 23$$

Where $\rho =$ exogenous waste harmfulness constant, $0 < \rho < 1$.

Thus we can notice that the renewable resource stock is endogenously determined state variable. The natural regeneration rate is dependent on the existing stock and intrinsic growth rate. The consumption parameter is dependent on the substitution rate of non-renewable resources and natural resource demand. The quality depletion parameter is related to the waste stock.

4.1.5 Waste

In production and recycling processes a great share of resources are actually wasted in some form or another creating waste of exergy. We will consider the waste as a stock pollutant in this model but it should be considered also as a possible resource. As a resource, waste fulfills the description by Dasgupta and Heal (1974) for backstop resource, though it doesn't have similar characteristics as the backstop represented by them as waste is dependent on previous production and it does also have negative side-effects on other resources. In this thesis we will pay some attention to the waste accumulation, waste depletion and recycling processes because they have interesting characteristics on resource use in general.

Waste is defined here as a resource that is dumped in one form or another to the nature (through depreciation of the resources-in-use or pollution from the extraction of non-renewable resources or as a side stream of production) because the resource might not have economic value (or it might have even a negative value) currently or the treatment costs exceeds their economic value. Waste is something that is a byproduct of production in general. The waste generation is closely

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[17] This is dependent on the actual definition of the system.

connected to the output of the economy in a way or another. The waste is one of the state variables. The waste differs from other state variables as it can cause harm to the regeneration rate of the renewable resources reducing the productivity of the renewable resources and lowering renewable resource amount that can be extracted periodically. (Smith, 1972)

The waste could also decrease the utility of a consumer according to some authors (Tahvonen and Kuuluvainen, 2002, Acemoglu et al., 2010), which case is not considered as such in this thesis, though there are indirect impacts on the consumer utility as discussed previously in Chapter 2.2. If we would restrict the possibility of recycling we would end up with the result represented by d'Arge and Kogiku (1973)[18] $\boldsymbol{R_D} = \boldsymbol{W}$.

In this model the waste stock accumulation can be regarded to consist of various sources of pollution:

$$\boldsymbol{W_{t+2} = f\left(W_{D,t+1}, W_{P,t+1}, \mathrm{W}_{\mathrm{Use,t+1}}, W_{t+1}; \omega\right) = W_{D,t+1} + W_{P,t+1} + \mathrm{W}_{\mathrm{Use,,t+1}} + (1-\omega)W_{t+1}} \quad \mathbf{24}$$

Where W = waste stock, W_D = waste from non-renewable resource extraction, W_P = waste from final sector production, $\mathrm{W}_{\mathrm{Use}}$ = waste from depreciation of resources-in-use, ω = natural depreciation rate of waste. Depreciation is regarded as an exogenic property of our model.

The use of three period waste accumulation model is considered for one reason only. The consumption goods use (and recycling) is taking place in one period while the waste is generated in the second period so that the use of consumption goods can be shown in the resource balance.

The produced resources are expressed in terms of intermediate sector production rather than virgin natural resources since in the intermediate sector a considerable amount of work is already allocated on the extraction processes. From thermodynamic point of view it is this kind of raw material from intermediate sector and recycling of waste that can be utilized in the final sector. Thus when considering the material balance for resources-in-use it makes sense to express the amounts in terms of intermediate sector goods. In such case the input resources for final sector can be expressed as:

$$\boldsymbol{R_{P,t+1} = x_{t+1} + \left(\lambda R_{K.agg,t+1} + (1-s)R_{P,t}\right)N_{Re,t+1}} \qquad \mathbf{25}$$

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[18] Using the notations from this Thesis.

Where R_P = resources for production, $R_{K.agg}$ = aggregate capital stock (expressed as resources) and N_{Re} = recycling rate.

The recycling rate here defines how great a share of the recycled materials can be economically recycled and how much is eventually released to the nature as waste. Through technological progress the recycling rate can be increased.

From this we can make a conclusion that the waste generated $\mathrm{W}_{\mathrm{Use},t+1}$, when the resources-in-use stock (See Chapter 4.1.6) is depreciating, becomes:

$$\mathbf{W}_{\mathbf{Use},t+1} = (1 - N_{Re,t+1})(\lambda R_{K.agg,t+1} + (1-s)R_{P,t}) \qquad 26$$

The final sector waste is on the other hand proportional to the inputs of final sector, thus we can write this as:

$$W_{P,t+1} = \sigma_P(x_{t+1} + (\lambda R_{K.agg,t+1} + (1-s)R_{P,t})N_{Re,t+1}) \qquad 27$$

Where σ_P =waste parameter for final sector production (constant).

Finally the intermediate sector waste is proportional only to the use of non-renewable resources as it can be assumed that renewable resources will regenerate naturally and these resources will become a part of the natural circulation without causing any waste. Thus the extraction related waste term is:

$$W_{D,t+1} = d_t R_{N,t}\sigma_w = (1 - N_{S,t})R_{D,t}\sigma_w \qquad 28$$

Where σ_w = waste parameter for non-renewable resource extraction

By replacing the waste terms from equations 26,27 and 28 into equation 24 we get:

$$W_{t+2} = f(W_{t+1}, R_{D,t+1}, R_{Nt+1}, R_{K.agg,t+1}, R_{P,t}, N_{Re,t+1}, N_{S,t+1}; \omega, \sigma_{P,}, \sigma_w, \lambda, s) \qquad 29$$

Or more conveniently:

$$W_{t+2} = (1-\omega)W_{t+1} + \sigma_P x_{t+1} + (1 + (\sigma_P - 1)N_{Re,t+1})\lambda R_{K.agg,t+1} + \Big((1-s)(1 + (\sigma_P - 1)NRe,t+1RP,t+1-NS,tRD,t+1\sigma w \qquad 30$$

4.1.6 Resources-in-use

The resources-in-use are resources that can be found in products or capital. As long as they are used they produce utility. The resources-in-use wear when they are used as capital or as consumption goods and a constant share of the resources-in-use are reduced to waste in during each time period. It is possible to create innovative processes to recover used resources (waste) or alternative uses for once used resources thus creating recycling loops to the system which lowers the need of virgin raw materials. In this thesis technological improvement enables part of this waste stream to be recycled back as input for the final sector. This development is of interest in our model. The introduction of resources-in-use is an un-traditional approach and it is done since the inclusion of recycled resources into the model requires a real resource base that can be recycled instead of some abstract output. This also allows an approach where the process can be examined through exergetic efficiency.

The resources-in-use stock is regarded as state variable which is dependent on the production in final sector, depreciation of capital, savings rate and the recycling technology. It is proper to begin the construction of the resources-in-use equation by introducing an equation for recycled resources $\boldsymbol{R_{R,t}}$ which can be defined as:

$$\boldsymbol{R_{R,t+1} = (\lambda R_{K.agg,t+1} + (1-s)R_{P,t})N_{Re,t+1}} \qquad \textbf{31}$$

Where R_R =recycled resources. The first part on RHS represents the part of depreciating of capital that is recycled while the second part on RHS represents the consumption goods that are recycled. The obvious assumption here is that the capital and consumption resource use is homogenous, thus both type of use consumes the same proportion of resources.

For the recycling rate we have $0 \leq N_{Re} < 1$, thus on every recycling cycle at least some small fraction of materials[19] is wasted (Reuter, 2011). The reprocessing will require significant amounts of resources and the closer to 100% we are approaching in recycling rate the more resources and processing will be required in order to keep the recycling rate up. This condition is binding from thermodynamic point of view and this satisfies also the second law of thermodynamics for entropy as defined by Georgescu-Roegen (1971) and others. This is also related to exergy generation in such a way that the exergy required for recycling shouldn't exceed the exergy content of the recycled material or otherwise the recycling would lead to a situation where more natural resources are wasted than recovered (Connelly and Coshland, 2001a). This is indicated indirectly in the model by the decreasing efficiency of R&D efforts which highly increases the use of resources

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[19] Or exergy in broader context.

(in form of labor) required to perform the recycling process. The upper limit for recycling rate could be then understood as exergy loss constraint.

The resources for production can be expressed as a sum of recycled resources and intermediate sector production thus we can write:

$$R_{P,t+1} = x_{t+1} + R_{R,t+1} = x_{t+1} + \left(\lambda R_{K.agg,t+1} + (1-s)R_{P,t}\right)N_{Re,t+1} \qquad 32$$

The produced resources could be divided in two categories: capital and consumption goods. Periodic resources allocated to capital are:

$$R_{K,t+1} = sR_{P,t} = s\left(\left(\lambda R_{K.agg,t+1} + (1-s)R_{P,t}\right)N_{Re,t+1}\right) \qquad 33$$

Where R_K =produced resources allocated for capital.

While the accumulation of resources allocated to capital can be expressed with the following equation[20]:

$$R_{K,agg,t+1} = R_{K.agg,t} + R_{K,t} - \lambda R_{K,agg,t} \qquad 34$$

Which can be interpreted as parameterized version of equation 4. Similarly the resources allocated to consumption can be expressed as:

$$R_{C,t+1} = (1-s)R_P = (1-s)\left(x_t + \left(\lambda R_{K.agg,t+1} + (1-s)R_{P,t}\right)N_{Re,t+1}\right) \qquad 35$$

Where R_C = resources allocated for consumption.

The consumption goods accumulation can be expressed by:

$$R_{C,agg,t+2} = R_{C,t+1} - R_{C,t} \qquad 36$$

The resources-in-use $R_{use,t}$ is the stock of resources that generally is the source of utility for the consumers and thus these resources could be used in the utility function instead of production (see Chapter 4.3). We will need to determine the resources that are available for the final product sector. This means that we must take into account the depletion rate of capital λ and recycling rate N_{Re}. We must also take into account the resources flow from the extractive sector to production sector in order to define the resources-in-use equation properly. One should observe that the consumption good resources are counted only for one period after which they are

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[20] This is similar to capital accumulation equation

reduced from the resources-in-use stock for the simple reason that this way the accumulation of consumption goods in the resources-in-use equation is properly accounted:

$$R_{use,t+2} = R_{K,agg,t+2} + R_{C,agg,t+2} = (1-\lambda)R_{K.agg,t+1} + R_{P,t+1} + R_{C,t+1} - R_{C,t} \qquad 37$$

Replacing equations 32, 33, 34 and 35 into equation 37 we can rewrite the resources-in-use as:

$$R_{use,t+2} = f\left(R_{K.agg,t+1}, R_{P,t}, N_{Re,t+1}, R_{C,t}, x_{t+1}; s, \lambda\right)\left(1-\lambda+\lambda N_{Re,t+1}\right)R_{K.agg,t+1} + (1-s)R_{P,t}N_{Re,t+1} - R_{C,t} + x_{t+1} \qquad 38$$

Resource use is generally affected by the price which determines the amount demanded. Renewable resources are affected by the intermediate sector substitution technology where better substitution technology increases the renewable resource use in the production. Meanwhile the aggregate demand affects the waste level and the regeneration rate of the renewable resources and the recycling rate development decreases the negative effects of waste. Similarly non-renewable resources are mainly affected by the intermediate sector demand and technological level. The resources-in-use on the other hand are affected by all of these factors directly or indirectly (as is the case for waste).

4.2 Production functions

The production functions are important sector specific functions that determine the output flows. The production functions can be used to determine the use and distribution of input resources between each sector. In the model we had four sectors which give us four production functions as well. The sectored division follows that of Yon and Yetkiner (2003), Di Vita (2006) and Tsur and Zemel (2006) and many others, while the addition of R&D sector on recycling is new approach. The production functions included the intermediate sector which extracts the natural resources and buys technologies (or renewable resources) from the R&D sector which is improving the substitution between non-renewable and renewable resources; the final sector then consumes the intermediate goods and buys technologies (or recycled resources) from R&D sector which is improving the recycling rate.

Direct capital investments can be allocated either to intermediate or final sector. These allocations should be in balance in such way that there is arbitrage condition so that the expected return for investment is the same on each sector, no capital is wasted and all of the production in the intermediate sector is consumed in the final sector. In order to define how the capital is allocated between each sector we'll need to represent the production functions for intermediate and final production sectors as well as for the R&D sectors.

We have two technology developing firms that concentrate on improving recycling rate and substitution rate as traditionally the concentration has been on substitution in general[21]. These technologies can be bought by the intermediate or final sector in order to alleviate the price increase of the non-renewable resources. The recycling technology R&D sector develops technologies that enable to recycle the resources-in-use and these recycled resources can be transformed back into inputs for final sector. The substitution technology R&D sector is trying to substitute non-renewable resources with renewable alternative which allows the intermediate sector to use renewable resources as inputs for production. This has simultaneous effect on the renewable resources consumption possibilities as the waste, which directly affects the renewable resource regeneration level, decreases through the R&D efforts. These technological advances are inclusive in that sense that it is likely that both of these processes are going on simultaneously and exclusive in that sense that these development firms are competing for the same limited resources. The technological development firms have indirect competitive situation where both companies are trying to sell as much of their technology as possible guaranteeing effective markets for innovations. We will make here an assumption that the only cost of technology for the producing sectors comes from the use of licenses. This requires that the technology firm practically owns the resources it is selling and thus no costs are related to the processing of these resources other than the technology development costs.

In this thesis slightly different approach is taken to the production. It is assumed that production is closely related to the exergy generation and the economic value for the product is the higher the more exergy is contained within the product and the more exergy losses are related to the production. This assumption is logical in that sense that in the intermediate sector the raw material is processed to concentrate or purified raw material by utilizing resources and the existing capital which increases the exergy contained with the product. In the final sector labor (work) and capital (another form of work) is used together to produce even higher exergy contained products that costs more than the intermediate product, while the reutilization of the final products resources is actually exergy consuming[22]. It is worth mentioning that in this sense

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[21] Since Romer (1990) to most recent research

[22] See discussion on recycling in Chapter 4.1.6

the waste associated with extraction and production contains unavoidable exergy losses which are expected to have linear relation with the resource use volumes. It would be possible to include technologies to improve these efficiencies, but for practical purpose (for not expanding the model beyond the limits) this is left for later researchers.

4.2.1 Intermediate Sector Production

The intermediate sector utilizes capital and extracted resources in order to produce the products for the final sector. The production function x_t for intermediate goods sector has two inputs: capital $K_{D,t}$ and extracted resources $R_{D,t}$. The production function can be written as:

$$\boldsymbol{x_t = f\left(K_{D,t}, R_{D,t}; \delta, D\right) = DK_D^{\delta}R_D^{1-\delta}} \qquad 39$$

Where $\delta =$ output elasticity of extraction capital $0 \leq \delta \leq 1$, $1 - \delta =$ partial elasticity of resources, $K_D =$ capital used at intermediate sector, $R_D =$ resources used by the intermediate sector, $D =$ factor productivity of intermediate capital and resources, $x_t =$ intermediate sector production function. Subscript D refers to intermediate sector from here on. We could take partial derivatives of capital and resources:

$$\boldsymbol{\frac{\partial x_t}{\partial K_D} = D\delta\left(\frac{R_D}{K_D}\right)^{1-\delta}} \qquad 40$$

$$\boldsymbol{\frac{\partial x_t}{\partial R_D} = D(1-\delta)\left(\frac{K_D}{R_D}\right)^{\delta}} \qquad 41$$

The extracted resources consists of non-renewable resources and the licenses bought from the substitution technology R&D sector which allows to replace a part of the non-renewable resources with renewable resources.

$$\boldsymbol{R_D = f\left(\gamma_{2t}, R_{E,t}, d_t, R_{N,t}\right) = l_D + d_t R_{N,t}} \qquad 42$$

Where $l_D =$ licenses sold for intermediate sector $=$ extracted renewable resources.

$$\boldsymbol{l_D = \gamma_{2,t}R_{E,t} = N_{s,t}R_{D,t}} \qquad 43$$

This allows us to rewrite the production function as:

$$\boldsymbol{x_t = f\left(K_{D,t}, R_{E,t}, R_{N,t}, \gamma_{2,t}, d_t; \delta, D\right) = DK_{D,t}^{\delta}(\gamma_{2,t}R_{E,t} + d_t R_{N,t})^{1-\delta}} \qquad 44$$

4.2.2 Final Sector Production

The final goods sector consumes eventually all the resources of the economy in its production and produces products for the final consumers. The final goods sector buys resources from the intermediate goods sector and alternatively it can use recycling technology licenses bought from waste recovery and recycling technology R&D firm to expand its resource base and to alleviate the pressure on natural resource prices. This means that the R&D firms are competing against each other indirectly as both of their technologies solve a part of resource scarcity problem caused by non-renewable resources.

The final sector production function is assumed to follow the Cobb-Douglas function described in Chang (2010, p. 7) consisting of the labor and capital allocated to the final production sector as well as the intermediate sector output $x_t(K_D, R_D)$ and as an additional feature the recycling technology licenses:

$$\boldsymbol{Y_t = f(X_t, K_{P,t}L_{P,t}; \alpha, \beta, A) = AL^{\alpha}_{P,t}K^{\beta}_{P,t}{X_t}^{1-\alpha-\beta}} \tag{45}$$

Where L_P = Labor used in production, K_P = Capital used in final sector production, A =factor productivity of production capital and labor, α = partial elasticity of labor, β = partial elasticity of production capital, $1-\alpha-\beta$ = partial elasticity of resources, X_t = resource input for final sector. The subscript P refers to the final sector here and later on. By taking the partial derivatives of labor, capital and resource input we get the following first order conditions:

$$\boldsymbol{\frac{\partial Y_t}{\partial L_{P,t}} = A\alpha L^{\alpha-1}_{P,t}K^{\beta}_{P,t}{X_t}^{1-\alpha-\beta}} \tag{46}$$

$$\boldsymbol{\frac{\partial Y_t}{\partial K_{P,t}} = A\beta L^{\alpha}_{P,t}K^{\beta-1}_{P,t}{X_t}^{1-\alpha-\beta}} \tag{47}$$

$$\boldsymbol{\frac{\partial Y_t}{\partial X_t} = A(1-\alpha-\beta)L^{\alpha}_{P,t}K^{\beta}_{P,t}{X_t}^{1-\alpha-\beta}} \tag{48}$$

The input resources in the production function can be expressed as a sum of intermediate sector production and recycling technology R&D sector license sales:

$$\boldsymbol{X_t = x_t + l_{Re}} \tag{49}$$

Where $\boldsymbol{l_{Re,t}}$ = licenses for recycling technology. The license sale amount l_{Re} for the final sector on the other hand is determined by the savings rate, depreciation rate of capital, recycling rate and resources-in-use stock:

$$\boldsymbol{l_{Re,t+1} = f\left(N_{Re,t}, R_{use,t}; s, \lambda\right) = R_{R,t+1} = \left(\lambda R_{K.agg,t+1} + (1-s)R_{P,t}\right)N_{Re,t+1}} \tag{50}$$

Replacing equations 39 and 50 into the final sector production function in equation 45 we get:

$$Y = AL_{P,t}^{\alpha}K_{P,t}^{\beta}\left[DK_{D,t}^{\delta}R_{D,t}^{1-\delta} + \left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)N_{Re,t}\right]^{1-\alpha-\beta} \tag{51}$$

4.2.3 Recycling R&D Sector Production Function

The development of recycling rate will be such that in the beginning the development might be slow as the benefits of recycling are rather minimal as stock of resources-in-use are small and the raw resources seem to be virtually unlimited and so the benefits from recycling are minimal. As the resources are consumed, the limit for non-renewable resource use becomes evident. The resource prices can be expected to follow some form of Hotelling’s (1928) formulation. It can be similarly expected that there is an increased interest in developing a technology that will cancel out these price changes (see for example Dasgupta and Heal, 1974). One such technology is recycling. Increased prices and volumes in resources will lead to increased investments into R&D that has the goal in increasing the recycling rate. Investments in recycling rate R&D will increase the R&D sectors production as is shown in this Chapter. The recycling rate development and recycling technology development costs are assumed to have a logistic function relation to each other. The logic behind this can be described as follows. The benefits from R&D increases as the price of the raw materials increases. The higher the recycling rate gets the more difficult (costly) it becomes to make innovations that will eventually increase the recycling rate even more and the higher the investments and production in R&D sector will become in absolute terms as the technologies that can have an effect on recycling rate becomes more and more sophisticated and requires more labor and knowledge. Eventually the efficiency of R&D investments is decreasing and approaching zero[23].

The only input in R&D sector is the labor, which can be interpreted more broadly as human capital. The recycling rate growth process is similar to the technology development formulation by Romer (1990):

$$\frac{dN_{Re}}{dt} = \varphi_{Re}N_{Re}L_{Re} = \varphi_{Re}N_{Re}(L - L_P - L_S) \tag{52}$$

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[23] For more detailed discussion on the limits of recycling I can advice to go through the book “Sustainable Metals Management: Securing our Future – Steps towards a Closed Loop Economy” Chapter 10 by G. Rombach (edited by A. Von Arnim, R.U. Ayres and S. Gößling-Reisemann, 2006) and M.A.Reuter (2011) “Limits of Design for Recycling” `Sustainability`: A Review”, Waste Biomass Valor, Vol. 2 pp. 183-208. These articles emphasize the real limits to recycling.

Where φ_{Re} = productivity of R&D for recycling rate, N_{Re} = recycling rate, $1 > N_{Re} \geq 0$, L_{Re} = labor allocated to recycling research. Thus one unit of labor will produce $\varphi_{Re}N_{Re}$ units of improvement in the technology.

The recycling technology development efficiency is assumed to follow an inverse of logistic function as the efficiency of R&D is declining as higher recycling rate is approached. Formally this can be written as:

$$\boldsymbol{\varphi_{Re} = 1 - N_{Re}\left(1 + r_{Re}\left(1 - \frac{N_{Re}}{K_{Re}}\right)\right)} \qquad \textbf{53}$$

Where N_{Re} = recycling rate, r_{Re} = intrinsic growth parameter for recycling R&D and K_{Re} = theoretical maximum recycling rate. Standardizing $K_{Re} = 1$ gives us $0 \leq N_S \leq 1$ and $\lim_{N_S \to 1} \varphi_S = 0$. Replacing the equation 53 into the substitution technology R&D production function allows us to rewrite equation 52 as:

$$\boldsymbol{\frac{dN_{Re}}{dt} = \left(1 - N_{Re}\left(1 + r_{Re}\left(1 - \frac{N_{Re}}{K_{Re}}\right)\right)\right)N_{Re}L_{Re}} \qquad \textbf{54}$$

4.2.4 Substitution R&D Sector Production Function

Substitution technology is defined here as a technology that directly substitutes non-renewable resources with renewable resources. The logic behind substitution is that as non-renewable resources are consumed they become more expensive while the relative price of backstop resource development decrease and relatively cheaper substitutes will become available (Dasgupta & Heal, 1974), in this case renewable resources. Another logic for substitution is the sustainability argument related to resource use and production. If the non-renewable (or renewable resources) were consumed too fast we could find ourselves in a situation where the intergenerational equality (Solow, 1974, Riley, 1977,) could not be met and we could end up in resource use situation described by Malthus (1798)[24] where the resource consumption would approach asymptotically zero.

In this model the use of renewable resources becomes available through research effort and thus it makes sense for intermediate sector to buy services from technology sector that can provide

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[24] By intergenerational equality we refer to Solow's concept where each present and future generation is allowed to consume the same amount in relative terms.

technologies that can substitute the non-renewable resources by renewable resources and thus slow down the price increase in the natural resources.

This R&D process is expected to have similar characteristics as the recycling rate R&D process, thus the substitution growth process can be described by using similar equation as we had earlier (Romer, 1990, van Zon and Yetkiner, 2003, Chang, 2010):

$$\frac{dN_S}{dt} = \varphi_S N_S L_S = \varphi_S N_S (L - L_P - L_{Re}) \qquad 55$$

Where φ_S = productivity of R&D process of substitution and L_S =labor allocated to substitution research.

The substitution technology development efficiency is assumed to follow an inverse of logistic function as we had for the recycling technology[25]. There are limits for substitution $1 > N_S \geq 0$ and φ_S is assumed to have a control function that follows an inverse of a logistic curve:

$$\varphi_S = 1 - N_S\left(1 + r_S\left(1 - \frac{N_S}{K_N}\right)\right) \qquad 56$$

Where N_S = Substitution rate, r_S = intrinsic growth parameter for substitution R&D and K_N = maximum substitution rate. Standardizing $K_N = 1$ gives $0 \leq N_S \leq 1$ and $\lim_{N_S \to 1} \varphi_S = 0$. When setting the maximum $K_N = 1$ it should be noticed that the value represents the theoretical maximum substitution rate rather than absolute value. Replacing the efficiency equation into the substitution technology R&D production function gives:

$$\frac{dN_S}{dt} = \left(1 - N_S\left(1 + r_S\left(1 - \frac{N_S}{K_N}\right)\right)\right) N_S L_S \qquad 57$$

4.3 Utility functions

The capital inflows and out flows are determined by two factors: the investments in capital and destruction speed of the capital. The investments in capital are assumed to be a constant share of the total production as well as the depreciation of capital is assumed to be a constant. The investments in capital are determined by the marginal utility of capital investments as suggested already by R. Ramsey (1928). This marginal utility of investments is assumed to be equal to the utility from instant consumption:

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[25] See Chapters 4.1.2 and 4.2.3

$$\frac{\partial u(K,C)}{\partial K} = \frac{\partial u(K,C)}{\partial C} \qquad 58$$

Generally we could measure consumer utility by using the consumption as a variable for utility. In this thesis however the consumption can be expressed in terms of resources-in-use. In practice this is similar to Solow (1974), Dasgupta and Heal (1974), Kamien and Scwarz (1978) and many other author's concept on capital accumulation but there exists recycling of produced goods and capital (resources-in-use) in the model which changes the structure and dynamics by some degree. The modified[26] instant utility function can be expressed using resources-in-use:

$$u(R_{use,t}) = \begin{cases} \frac{R_{use,t}^{1-\eta}-1}{1-\eta}, \eta \neq 1 \\ \log(R_{use,t}), \eta = 1 \end{cases} \qquad 59$$

Where η = elasticity of marginal utility with respect to resources-in-use.

The optimal policy would maximize the welfare function that takes into account the sum of periodic utilities which are then discounted by utility discount rate. Continuous welfare function (see for example Tsur and Zemel, 2006, p. 487) can be translated in discrete form as:

$$U = \sum_{t=0}^{\infty} \frac{u(R_{use,t})}{(1+\vartheta)^t} \qquad 60$$

Where U = welfare function, ϑ = utility discount rate.

The welfare function could be used to solve the optimal consumption of resources as well as the capital and technology development paths. Similarly the final sector price can be solved as shadow price of the Hamiltonian.

4.4 Profit Functions

The profit functions determine the economic profit of the sectors. In our case we have four sectors that need to be discussed. In this Chapter we will introduce the profit functions for final sector, intermediate sector, recycling sector and substitution sector. For the two technology sectors the wage costs are discussed in detail in order to be able to solve the labor demand on these secors.

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[26] Eg. Bentchekroun and Withagen (2011) has used similar formulation but their variable is consumption as is the case in most of the articles. From resource use point-of-view it makes more sense to treat the resources-in-use stock as the source of utility although the emphasizing of consumption could still be considered as a fair approximation.

4.4.1 Final Sector Profit

The final sector utilizes capital, labor and resources in its production. Thus the final sector profit function is dependent on the total production, the general wage level and amount of labor required to produce the output, the general interest rate and amount of capital required to produce the output as well as the intermediate sector prices and amount of input resources, which can be linked to the intermediate sector production and recycling rate licenses sold for the final sector. Given these factors the final sector production function can be described as:

$$\pi_Y = Y - w_p L_p - rK_p - X_t P_{x_t} = AL_P^{\alpha} K_P^{\beta} {X_t}^{1-\alpha-\beta} - w_p L_p - rK_p - (x_t + l_{Re}) P_{x_t} \qquad 61$$

Where $\pi_Y =$ final sector profit function, $\boldsymbol{w_p}$ = final sector wage, $\boldsymbol{r} =$ rent, $\boldsymbol{P_{x_t}}$ = price level of the intermediate sector, subscript **P** refers to the final sector generally.

4.4.2 Intermediate Sector Profit

The intermediate sector utilizes capital and resources in its production[27]. Thus the intermediate sector profit function depends on the price and amount of resources produced for final sector, cost and amount of capital required for the production and price and amount of the extracted resources, which can be linked to the substitution rate development as follows:

$$\pi_D = P_{x_t} x_t - r_D K_D - P_D R_D = P_{x_t} D K_D^{\delta} R_D^{1-\delta} - r_D K_D - P_D R_{D,t} \qquad 62$$

Where $\pi_D =$ intermediate sector profit.

4.4.3 Recycling Sector Profit

The recycling sector utilizes only labor in its production. The recycling sector profits come from the license sales, which is equal to the amount of recycled resources (equation 31) multiplied by the price of recycled resources, which is equal to intermediate sector price, while the costs are direct wage costs that are paid to the employees. Thus the periodic profit function in recycling sector becomes:

$$\pi_{Re,t} = \left(\lambda R_{K.agg,t} + (1-s) R_{P,t-1}\right) N_{Re,t} P_{x_t} - w_{Re,t} L_{Re,t} \qquad 63$$

Where $\pi_{Re} =$ recycling sector profit.

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[27] See equation 39

It is assumed that under arbitrage condition the value of patent, which can be seen as equivalent for capital asset, must be equal to the net capital income. Net income of capital is defined as the capital rent r minus the capital depreciation δ_t. Under arbitrage condition it should be totally indifferent for the consumer whether he owns capital or a patent. As in Romer (1990) the income from improving the technology at t gives an improvement for income at t+1, which is equal to the capital income from the license sales. The value of patent should fulfill the condition[28]:

$$(r \quad - \lambda)K_{A,t} + K_{A,t} = r_{A,t} + K_{A,t+1} \qquad 64$$

Where $K_{A,t}$ = capital value of recycling, $r_{A,t}$ = rent for patents sold for the final sector.

Given that technology development is occurring at rate:

$$\frac{dN_{Re}}{dt} = \varphi_{Re} N_{Re} L_{Re} = N_{Re,t+1} - N_{Re,t} \qquad 65$$

It becomes possible to express the capital value of current technology and next period technology. We can assume a semi-rational decision maker that takes into account the changes in the technology and prices while the changes in resources-in-use stock are ignored[29]. It is assumed that the labor costs are one-time costs while the revenue from the invention is practically infinite[30]. With these pre-assumptions the capital values becomes:

$$K_{At} = \frac{\left(\theta R_{K.agg,t} + (1-s) R_{P,t-1}\right) N_{Re} P_{x_t}}{r-\lambda} \qquad 66$$

$$K_{At+1} = \frac{\left(\theta R_{K.agg,t} + (1-s) R_{P,t-1}\right)(1+\varphi_{Re} L_{Re}) N_{Re,t} P_{x_{t+1}}}{r-\lambda} \qquad 67$$

The wage costs and the discounted net value of new innovations should be equal, thus we have:

$$w_{Re} L_{Re} = K_{At+1} - K_{At} = \frac{\left(\lambda R_{K.agg,t} + (1-s) R_{P,t-1}\right)(1+\varphi_{Re} L_{Re}) N_{Re,t} P_{x_{t+1}} - \left(\lambda R_{K.agg,t} + (1-s) R_{P,t-1}\right) N_{Re,t} P_{x_t}}{r-\lambda} \qquad 68$$

This value is equally the capital investment into the recycling technology.

In order to take into account the most rapid approach (MRAP) assumption for technology development (Tsur and Zemel, 2001) we can make an assumption of competitive markets rather than monopolistic markets on R&D sector. We could write the profit function of recycling

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[28] As described in SØrensen and Whitta-Jacobsen (2005, p.289).

[29] Taking into account the changes to the resources-in-use would complicate the analysis for which reason this change is ignored here. It could be released in later studies.

[30] This might not be exactly true in real world and the effects of shorter patent times on the value of patents could be examined. It is clear that shorter patent viability times would make innovation relatively more costly by reducing the future profits.

technology firm as zero and replace the wage costs into the profit function utilizing equation 68. The net profit function for firm turns out to be, when taking into account arbitrage condition for wage $w_{Re} = w$, equal to:

$$\boldsymbol{\pi_{Re} = \frac{\left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)(1+\varphi_{Re}L_{Re})N_{Re,t}P_{x_{t+1}} - \left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)N_{Re,t}P_{x_t}}{r-\lambda} - wL_{Re} = 0} \qquad \textbf{69}$$

Equation 69 could be used for solving $\boldsymbol{L_{Re}}$:

$$\boldsymbol{L_{Re} = \frac{\left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)N_{Re,t}\left(P_{x_{t+1}} - P_{x_t}\right)}{\left((r-\lambda)w - \left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)N_{Re,t}P_{x_{t+1}}\varphi_{Re}\right)}} \qquad \textbf{70}$$

The labor demand cannot be negative, since negative labor doesn't exist in any practical sense, we must have: $\boldsymbol{(r-\lambda)w} > \left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)\varphi_{Re}N_{Re,t}P_{x_{t+1}}$.

4.4.4 Substitution Sector Profit

The substitution sector profits come from the license sales while the costs are direct wage cost paid to the employees. Thus the typical profit function in substitution sector could be described as:

$$\boldsymbol{\pi_S = R_D N_S P_{x_t} - w_S L_S = \gamma_{2t} R_{E,t} P_{x_t} - wL_S} \qquad \textbf{71}$$

Where π_S = profit function of substitution sector.

As for recycling technology the value of patent for substitution technology must be equal to the net capital income. Net income of capital is defined as the capital rent r minus the capital depreciation δ_t. Thus it is totally indifferent for the consumer whether he owns capital or a patent. As in Romer (1990) the income from improving the technology at t gives an improvement for income at t+1 that is equal to the capital income from the capital equivalent to the amount of the licenses sold. The value of patent should fulfill the definition[31]:

$$\boldsymbol{(r \quad -\lambda)K_{AS,t} + K_{AS,t} = r_{AS,t} + K_{AS,t+1}} \qquad \textbf{72}$$

Where K_{AS} = capital value of substitution technology, r_{AS} = rent paid by intermediate sector for the patent for substitution.

As the amount of licenses required in extraction is equal to amount $\boldsymbol{N_S R_{D,t}}$ and the price of the patent is equal to the price of renewable resources $P_{E,t}$ we can write the rent as:

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[31] As was described in SØrensen and Whitta-Jacobsen (2005, p.289).

$$r_{AS,t} = P_{E,t} N_S R_{D,t} \tag{73}$$

Given the arbitrage condition for the resource prices we have:

$$P_{E,t} = P_{N,t} = P_{D,t} \tag{74}$$

The technology development process was similar to Romer (1990) technology development giving us a substitution technology development rate:

$$\frac{dN_S}{dt} = \varphi_S N_S L_S \tag{75}$$

Given these equations it becomes possible to express the capital value of current substitution technology and next period substitution technology. We can assume a semi-rational decision maker[32] that takes into account the price changes and the technology changes while the stock changes are ignored. The capital values then becomes:

$$K_{AS,t} = \frac{P_{D,t} N_S R_{D,t}}{r-\lambda} \tag{76}$$

$$K_{AS,t+1} = \frac{P_{D,t+1}(1+\varphi_S L_S) N_S R_{D,t}}{r-\lambda} \tag{77}$$

The cost of performing substitution R&D should be equal to the capital value increase from the R&D effort, thus we have equality:

$$w_S L_S = K_{AS,t+1} - K_{AS,t} = \frac{P_{D,t+1}(1+\varphi_S L_S) N_S R_{D,t} - N_S R_{D,t} P_{D,t}}{r-\lambda} \tag{78}$$

In order to take into account the most rapid approach path (MRAP) assumption for technology development (Tsur and Zemel, 2001) we would need to assume perfectly competitive markets rather than monopolistic markets on substitution R&D sector. We assume that the substitution technology sector will not generate economic profit in perfect competition. In such case we can set the profit function to zero. Utilizing equation 78 and an arbitrage condition for wages $w_S = w$ we can write the actual profit function of the substitution technology firm as:

$$\pi_S = \frac{P_{D,t+1}(1+\varphi_S L_S) N_S R_{D,t} - N_S R_{D,t} P_{D,t}}{r-\lambda} - w L_S = 0 \tag{79}$$

We could use this function to solve the substitution R&D sector labor demand L_S, which gives us.

$$L_S = \frac{P_{D,t+1} N_S R_{D,t} - N_S R_{D,t} P_{D,t}}{w(r-\lambda) - P_{D,t+1} N_S R_{D,t} \varphi_S} \tag{80}$$

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[32] This assumption is made in order to simplify the analysis. For more complex analysis the stock changes of non-renewable should be taken into account as well as the intermediate sector capital in next period. This would complicate the analysis significantly.

Where we must have $w(r-\lambda) \geq P_{D,t+1} N_s R_{D,t} \varphi_S$.

By replacing this into the actual profit function we could now rewrite:

$$\boldsymbol{\pi_S = \frac{P_{D,t+1}(1+\varphi_S L_S) N_s R_{D,t} - N_S R_{D,t} P_{D,t}}{r-\lambda} - \frac{P_{D,t+1} N_s R_{D,t} - N_S R_{D,t} P_{D,t}}{(r-\lambda) - P_{D,t+1} N_s R_{D,t} \varphi_S} = 0} \qquad \mathbf{81}$$

4.5 Resource Prices and Demand

4.5.1 Natural Resource Prices

We will assume that the price of raw materials is a function of the resources and the demand in the intermediate sector, thus $\boldsymbol{P_D(R_D, x_t)} = \boldsymbol{P_D}(\boldsymbol{R_{Nt}}, R_{Et}, x_t)$. The price of raw materials is assumed to follow the Hotelling's rule (Hotelling, 1931) in such a way that the increase in the share of renewable resources decreases the price growth rate of the non-renewable resources. This will be an extension to the Hotelling's rule stating that $\boldsymbol{P_{N,t+1} = (1+\gamma) P_{N,t}}$.

We want to take a closer look into price growth parameter γ and see how the substitution technology development could have effects on the price growth of non-renewable resources. We can define that γ is dependent of the scarcity of the resource as compared to the initial level of the resource and the current resource use as compared to the current resource stock, as well as the substitution rate, which transfers the price growth rate closer to the utilization rate of the renewable resources and further away from the scarcity, thus we can express γ as:

$$\boldsymbol{\gamma = \frac{R_{N,t,0}}{R_{N,t}} \frac{d_t R_{Nt}}{R_{Nt}} + N_{S,t}\gamma_{2t} = \frac{R_{N,t,0}}{R_{N,t}} d_t + N_{S,t}\gamma_{2t} = \left(1 - N_{S,t}\right) \frac{R_{N,t,0}}{R_{N,t}} \frac{R_{D,t}}{R_{N,t}} + N_{S,t}\gamma_{2t}} \qquad \mathbf{82}$$

Proposition 1: in the absence of technological substitution from non-renewable resources into renewable resources the discrete form of Hotelling's rule can be expressed in modified form, where the scarcity increases the price whereas the increase in substitution rate decreases the price growth rate while the substitution moves the price growth closer to the utilization rate of the renewable resources:

$$\boldsymbol{P_{N,t+1} = \left(1 + \left(1 - N_{S,t}\right) \frac{R_{N,t,0}}{R_{N,t}} \frac{R_{D,t}}{R_{N,t}} + N_{S,t}\gamma_{2t}\right) P_{N,t}} \qquad \mathbf{83}$$

This follows the logic that as the substitution effect takes place the pressure on the non-renewable resource price is lowered. On the other hand, when the non-renewable resources are fully substituted and we reach equilibrium we see an interesting phenomenon: the price increase will eventually stop (or becomes infinitely small) as the growth of extracted resources use becomes to a halt and the renewable resources will be consumed at a constant MSY rate since the renewable resources becomes practically the only available input. This decrease in the price growth rate is derived from the original Hotelling rule's definition that it is indifferent for the owner of a resource if he sells the resource now or keeps it and sells it in consecutive period with a price added with an interest rate. The renewable resources growth rate decreases if less than MSY of renewable resource is consumed while by Hotelling's rule the price is growing at certain rate, but by not consuming MSY the owner of the resource earns less than he would earn by consuming MSY. This leads to a conclusion that when all non-renewable resources have been consumed the price level growth of renewable resources must stabilize as it is optimal to consume constant MSY each period and it is non-rational option to postpone the consumption to the next period due to decreased income on resources sales. The limiting factor for the price growth is eventually the net growth rate of the renewable resources. This leads to conclusion that $\lim_{t\to\infty} r = \gamma_{2t}$.

4.5.2 Intermediate Sector Prices and Demand

The demand of the final sector is not as straight forward to solve as the price. We can start by solving the price level of the intermediate sector products by taking a partial derivative of the profit function in terms of X_t. By arbitrage condition the price of the intermediate products and recycled resources must be equal $P_r = P_{X_t}$, which allows us to write:

$$\frac{d\prod_Y}{dX_t} = (1-\alpha-\beta)AL_p^{\alpha}K_p^{\beta}X_t^{-\alpha-\beta} - P_{X_t} = 0 \Leftrightarrow P_{X_t} = (1-\alpha-\beta)AL_p^{\alpha}K_p^{\beta}X_t^{-\alpha-\beta} \qquad 84$$

Taking a partial derivative of the intermediate sector profit function in terms of input resources R_D:

$$\frac{\partial \prod_D}{\partial R_D} = (1-\delta)DK_D^{\delta}R_D^{-\delta}P_{X_t} - P_D = 0 \qquad 85$$

This could be solved for natural resource price:

$$P_D = (1-\delta)D\left(\frac{K_D}{R_D}\right)^{\delta}P_{X_t} \qquad 86$$

None the less a monopolist producer on intermediate sector solves cost minimization function:

$$\min C_{x_t} = rK_D + P_D R_D \qquad 87$$

$$st.\, DK_D^{\delta}R_D^{1-\delta} \qquad 88$$

Where C_{x_t} = cost function of the intermediate sector and r = rent for capital.

The solution for this problem gives following results[33]:

$$R_D = K_D\left(\frac{r}{P_D}\right)\left(\frac{1-\delta}{\delta}\right) \qquad 89$$

$$K_D = R_D\left(\frac{P_D}{r}\right)\left(\frac{\delta}{1-\delta}\right) \qquad 90$$

When these are replaced into the equation 87 we get:

$$C_{x_t} = \frac{1}{D}\left(\frac{r}{\delta}\right)^{\delta}\left(\frac{P_D}{1-\delta}\right)^{1-\delta} R_{x,t} \qquad 91$$

Replacing costs from equation 91 into the profit function (equation 62) gives:

$$\prod_D = DK_D^{\delta}R_D^{1-\delta}P_{X_t} - \frac{1}{D}\left(\frac{r}{\delta}\right)^{\delta}\left(\frac{P_D}{1-\delta}\right)^{1-\delta} R_{x,t} = 0 \qquad 92$$

Replacing the capital and resource equations 89 and 90 into the profit function (equation 92) allows us to solve the price of the intermediate sector goods (which turns out to be equal to the final sector input resource price as should be expected):

$$P_{X_t} = \frac{1}{1-\alpha-\beta}\left[\frac{1}{D}\left(\frac{r}{\delta}\right)^{\delta}\left(\frac{P_D}{1-\delta}\right)^{1-\delta}\right] = \frac{d\prod_Y}{dX_t} = (1-\alpha-\beta)AL_p^{\alpha}K_p^{\beta}X_t^{-\alpha-\beta} \qquad 93$$

The total amount of resources demanded by the final sector X_t can be solved as:

$$X_t = \left[\frac{(1-\alpha-\beta)^2 AL_P^{\alpha}K_P^{\beta}}{\frac{1}{D}\left(\frac{r}{\delta}\right)^{\delta}\left(\frac{P_D}{1-\delta}\right)^{1-\delta}}\right]^{\frac{1}{\alpha+\beta}} = DK_D^{\delta}R_D^{1-\delta} + \left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)N_{Re,t} \qquad 94$$

If the demand function was donated by $X_t(P_{xt})$ then the demand elasticity ε for final sector becomes:

$$\varepsilon = \frac{-X_t{}'(P_{xt})P_{xt}}{X_t(P_{xt})} = \frac{1}{\alpha+\beta} \qquad 95$$

It is evident that the intermediate sector production x_t must be equal to the final sector demand minus the recycled resources, thus we can write the intermediate sector production as:

$$x_t = X_t - R_R = \left[\frac{(1-\alpha-\beta)^2 AL_P^{\alpha}K_P^{\beta}}{\frac{1}{D}\left(\frac{r}{\delta}\right)^{\delta}\left(\frac{P_D}{1-\delta}\right)^{1-\delta}}\right]^{\frac{1}{\alpha+\beta}} - \left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)N_{Re,t} \qquad 96$$

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[33] More detailed solution can be found in Chang (2010, pp. 8-9)

4.6 Wages and Labor Allocation

In order to solve the general wage level we will need to take a partial derivative of profit function of the final sector (equation 61) in terms of labor to get the first order condition.

$$\frac{d\Pi_Y}{dL} = w_p - \frac{\alpha Y}{L_p} = 0 \Leftrightarrow w_p = \frac{\alpha Y}{L_p} \Leftrightarrow L_p = \frac{\alpha Y}{w} \qquad 97$$

Taking a partial derivative of profit function for recycling sector (equation 69) in terms of labor gives us an equation that can be used for solving the wage level and labor demand on final sector:

$$\frac{\partial \pi_{Re}}{\partial L_{Re}} = \frac{\left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)\varphi_{Re} N_{Re,t} P_{x_{t+1}}}{r-\lambda} - w_{Re} = 0 \Leftrightarrow w_{Re} = \frac{\left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)\varphi_{Re} N_{Re,t} P_{x_{t+1}}}{r-\lambda} \qquad 98$$

By utilizing the arbitrage condition $w_{Re} = \boldsymbol{w}$ and by setting the first order condition for final sector production function in terms of labor $w = \frac{\alpha Y}{L_p}$ (equation 97) and the final sector production function $Y_t = AL_{P,t}^{\alpha} K_{P,t}^{\beta} {X_t}^{1-\alpha-\beta}$ (equation 45) we can solve the final sector labor demand from equation 98:

$$L_{P,t}^{1} = \left[\frac{\left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)\varphi_{Re} N_{Re,t} P_{x_{t+1}}}{(r-\lambda)\alpha A K_{P,t}^{\beta} {X_t}^{1-\alpha-\beta}}\right]^{\frac{1}{\alpha-1}} \qquad 99$$

Taking a partial derivative of substitution technology R&D firms profits (equation 79) in terms of labor gives us a possibility to solve the wage in substitution sector:

$$\frac{\partial \pi_S}{\partial L_S} = w_S - \frac{\varphi_S P_{D,t+1} N_s R_{D,t}}{r-\lambda} = 0 \Leftrightarrow w_S = \frac{\left(1 + (1-N_{s,t})\gamma\right)\varphi_S P_{D,t+1} N_s R_{D,t}}{r-\lambda} \qquad 100$$

By utilizing the arbitrage condition $w_S = w$ and replacing $w = \frac{\alpha Y}{L_p}$ and the final sector production function $Y_t = AL_{P,t}^{\alpha} K_{P,t}^{\beta} {X_t}^{1-\alpha-\beta}$ into equation 100 we can express an alternative form for the final sector labor demand as:

$$L_{P,t}^{2} = \left[\frac{\varphi_S P_{D,t+1} N_s R_{D,t}}{\alpha A (r-\lambda) K_{P,t}^{\beta} {X_t}^{1-\alpha-\beta}}\right]^{\frac{1}{\alpha-1}} \qquad 101$$

From the profit functions of R&D sector we had already solved the labor demanded on these sectors (equations 70 and 81). Using these results it is possible to solve the labor demand, which can be normalized as has been done in equations 103-106:

$$\boldsymbol{L = L_{Re} + L_S + L_P = \frac{(\lambda R_{K.agg,t}+(1-s)R_{P,t-1})N_{Re,t}P_{x_{t+1}}-(\lambda R_{K.agg,t}+(1-s)R_{P,t-1})N_{Re,t}P_{x_t}}{(r-\lambda)\alpha A L_{P,t}^{\alpha-1}K_{P,t}^{\beta}{X_t}^{1-\alpha-\beta}-(\lambda R_{K.agg,t}+(1-s)R_{P,t-1})\varphi_{Re}N_{Re,t}P_{x_{t+1}}} + \frac{P_{D,t+1}N_sR_{D,t}-N_SR_{D,t}P_{D,t}}{w(r-\lambda)-P_{D,t+1}N_sR_{D,t}\varphi_S} + \left[\frac{\varphi_S P_{D,t+1}N_sR_{D,t}}{\alpha A(r-\lambda)K_{P,t}^{\beta}{X_t}^{1-\alpha-\beta}}\right]^{\frac{1}{\alpha-1}}} \qquad 102$$

Given that the labor supply is fixed we can solve the relative shares of labor allocated on each sector as:

$$\boldsymbol{v_{Re} = \frac{\frac{(\lambda R_{K.agg,t}+(1-s)R_{P,t-1})N_{Re,t}P_{x_{t+1}}-(\lambda R_{K.agg,t}+(1-s)R_{P,t-1})N_{Re,t}P_{x_t}}{(r-\lambda)\alpha A L_{P,t}^{\alpha-1}K_{P,t}^{\beta}{X_t}^{1-\alpha-\beta}-(\lambda R_{K.agg,t}+(1-s)R_{P,t-1})\varphi_{Re}N_{Re,t}P_{x_{t+1}}}}{L}} \qquad \boldsymbol{0 \le v_{Re} < 1} \qquad 103$$

$$\boldsymbol{v_S = \frac{P_{D,t+1}N_sR_{D,t}-N_SR_{D,t}P_{D,t}}{w(r-\lambda)-P_{D,t+1}N_sR_{D,t}\varphi_S}} \qquad \boldsymbol{0 \le v_S < 1} \qquad 104$$

$$\boldsymbol{v_p = \frac{\left[\frac{\varphi_S P_{D,t+1}N_sR_{D,t}}{\alpha A(r-\lambda)K_{P,t}^{\beta}{X_t}^{1-\alpha-\beta}}\right]^{\frac{1}{\alpha-1}}}{L}} \qquad \boldsymbol{0 \le v_p < 1} \qquad 105$$

Where v_{Re} =relative share of recycling R&D labor, v_S = relative share of substitution R&D labor and v_p = relative share of final sector production labor.

The normalized labor constraint can be expressed as:

$$\boldsymbol{v_{Re} + v_S + v_p = 1} \qquad 106$$

4.6.1 Labor Allocation Identity

By utilizing the labor demand equations 99 and 101 we can write an identity, which can be reduced significantly to more convenient form:

$$\boldsymbol{L_{P,t}^1 = L_{P,t}^2 \Leftrightarrow (\lambda R_{K.agg,t} + (1-s)R_{P,t-1})N_{Re,t}\varphi_{Re}P_{x,t+1} = \varphi_S N_s R_{D,t} P_{D,t+1}} \qquad 107$$

$$\boldsymbol{\Leftrightarrow R_R \varphi_{Re} P_{x,t+1} = l_S \varphi_S P_{D,t+1}} \qquad 108$$

This is an important result which suggests that the next period value of recycled resources weighted with the efficiency of recycling R&D should be equal to the next period value of substituted resources weighted with the efficiency of substitution technology R&D. The volume of recycled resources as compared to substituted resources can be easily solved as:

$$\boldsymbol{R_R = l_S \frac{\varphi_S P_{D,t+1}}{\varphi_{Re} P_{x,t+1}}} \qquad 109$$

This result will be discussed later in Chapter 5.1.1 as part of the results of the model.

4.7 Interest Rate and Capital Investment Allocation

In this Chapter the allocation of investments are discussed. The shares of investments can be interpreted as savings rates allocated on each sector by generalizing the equation $I = sY$, which will be utilized in this Chapter.

4.7.1 R&D Sectors

The investments on R&D are dependent on the labor allocated on R&D. The R&D investments are competing against the capital demand from intermediate and final sectors. Thus the (capital) investments allocated on R&D takes a share of the total production. These investments are mainly dependent on the labor unit costs (wage), current level of technology and the efficiency of innovation. The total labor costs on R&D sector correspond to the value of new innovations. Thus we can express the investments on R&D as shares of total production to get following shares for investments:

$$\boldsymbol{I_S = wL_{S,t} = s_S Y_t \Leftrightarrow s_S = \frac{wL_{S,t}}{Y_t}} \tag{110}$$

Where $\boldsymbol{I_S} =$ investments on substitution technology development, $s_S =$ savings rate for substitution technology development.

$$\boldsymbol{I_{Re} = wL_{Re} = s_{Re} Y_t \Leftrightarrow s_{Re} = \frac{wL_{Re}}{Y_t}} \tag{111}$$

Where $I_{Re} =$ investments on recycling technology development, $s_{Re} =$ savings rate for recycling technology development.

4.7.2 Industrial Sectors

In order to solve the capital allocation between industrial sectors (intermediate and final sectors) we will need to solve the first order conditions for capital on each sector.

Taking a derivative of the final sector profit function in respect to final sector capital and solving for the zero-point gives us the interest rate for capital in the final sector:

$$\boldsymbol{\frac{d\prod_Y}{dK_P} = \frac{\beta Y}{K_P} - r_P = 0 \Leftrightarrow r_P = \frac{\beta Y}{K_P}} \tag{112}$$

Taking a derivative of the intermediate sector profit function in respect to intermediate sector capital and solving the zero-point allows us to solve the interest rate for capital in the intermediate sector:

$$\frac{\partial \Pi_D}{\partial K_D} = \frac{\delta x_t}{K_{D,t}} - r_D = 0 \Leftrightarrow r_D = \frac{\delta x_t}{K_D} \qquad 113$$

We can assume an arbitrage condition for the interest rate which allows us to solve the relative share of production capital in terms of one another.

$$r = r_D = r_P = \frac{\delta x_t}{K_D} = \frac{\beta Y}{K_P} \Leftrightarrow K_D = \frac{\delta x_t}{\beta Y} K_P \qquad 114$$

4.7.3 Capital Investment Allocation

We were assuming a constant savings rate s for which the capital investments and R&D investments are competing. The capital investments can be expressed as:

$$I_{K,t} = I_{K,D,t} + I_{K,P,t} \Leftrightarrow s_{K,t}Y = s_{K,D,t}Y + s_{K,P,t}Y \qquad 115$$

Where $I_{K,t}$ = investments in industrial capital, $I_{K,D,t}$ = investments in intermediate sector capital, $I_{K,P,t}$ = investments in final sector capital.

Given the results from interest rate equality (equation 114) it becomes possible to write:

$$s_{K,D,t} = \frac{\delta x_t}{\beta Y} s_{K,P,t} \qquad 116$$

By introducing equation 116 into equation 115 and reducing the outputs gives:

$$s_{K,t} = \left(1 + \frac{\delta x_t}{\beta Y}\right) s_{K,P,t} \qquad 117$$

In order to solve the capital allocation on R&D sector we will need to express the R&D sector investments more conveniently in terms of savings rates. R&D savings rate can be expressed as sum of investments on substitution and recycling:

$$I_{R\&D} = I_S + I_{Re} \Leftrightarrow s_{R\&D}Y = s_S Y + s_{Re} Y \qquad 118$$

Where $I_{R\&D}$ =investments allocated on R&D sector, I_S = investments allocated on substitution technology development, I_{Re} = investments allocated on recycling technology development.

By using arbitrage condition for wage rate and identity equations 110 and 111 to get the relative savings rates:

$$Y_t = \frac{wL_{S,t}}{s_S} = \frac{wL_{Re}}{s_{Re}} \Leftrightarrow s_{Re} = s_S \frac{L_{Re}}{L_{S,t}} \qquad 119$$

By introducing equation 119 into equation 118 we can write:

$$s_{R\&D} = \left(1+\frac{L_{Re}}{L_{S,t}}\right) s_S \tag{120}$$

The savings rate can then be expressed as the sum of these:

$$s = s_{K,t} + s_{R\&D} = \left(1+\frac{\delta x_t}{\beta Y_t}\right) s_{K,P,t} + \left(1+\frac{L_{Re,t}}{L_{S,t}}\right) s_S \tag{121}$$

By utilizing the total production and savings rate equation 121 we can rewrite the investments as:

$$I = sY = \left(\left(1+\frac{\delta x_t}{\beta Y_t}\right) s_{K,P,t} + \left(1+\frac{L_{Re,t}}{L_{S,t}}\right) s_S\right) Y = \left(L_{S,t}+L_{Re}\right) w + \left(1+\frac{\delta x_t}{\beta Y_t}\right) s_{K,P,t} \tag{122}$$

The RHS equation 122 can be used for solving the investment allocated on final sector capital.

$$s_{K,P,t} = \frac{sY - w\left(L_{S,t}+L_{Re}\right)}{1+\frac{\delta x_t}{\beta Y_t}} \tag{123}$$

By using equations 116, 119, 120, 121 and 123 we can solve the investment allocation for the rest of the sectors.

$$s_S = \frac{s - s_{K,P,t}}{\left(1+\frac{L_{Re}}{L_{S,t}}\right)} \tag{124}$$

$$s_{Re} = \frac{s - s_{K,P,t}}{\left(1+\frac{L_{Re}}{L_{S,t}}\right)} \times \frac{L_{Re}}{L_{S,t}} \tag{125}$$

$$s_{K,D,t} = \frac{sY - w\left(L_{S,t}+L_{Re}\right)}{1+\frac{\delta x_t}{\beta Y_t}} \times \frac{\delta x_t}{\beta Y} \tag{126}$$

4.8 Summary of the Model

The model can be described using the functions that were represented in Chapters 4.1 - 4.7. The state variables can be divided in the following sub categories: resources, technology and capital. The other important equations included production functions, profit functions, prices and equations related to other input variables.

4.8.1 Instant Utility and Welfare Functions

Instant utility function in this contex can be derived from the resources-in-use.

$$u\left(R_{use,t}\right) = \begin{cases} \frac{R_{use,t}^{1-\eta}-1}{1-\eta}, \eta \neq 1 \\ \log\left(R_{use,t}\right), \eta = 1 \end{cases}$$

Whereas the target function for the model is the welfare function that would need to be maximized that is the discounted sum of instant utilities from this period to infinity:

$$\boldsymbol{U} = \sum_{\boldsymbol{t=0}}^{\infty} \frac{\boldsymbol{u(R_{use,t})}}{\boldsymbol{(1+\vartheta)^{t}}}$$

The behavior of the model is described by the state variables, production functions profit functions and other significant functions .

4.8.2 State Variables

Resources use

$$\boldsymbol{R_{N,t+1} = f(N_S, R_{D,t}, R_{Nt}) = R_{Nt} - (1 - N_{S,t})R_{D,t}}$$

$$\boldsymbol{R_{E_{t+1}} = f(R_{E_t}, K_{D,t}, W_t; \alpha) = R_{E_t} + r_E(1 - R_{E_t})\, R_{E_t} - N_S R_{D,t} - \alpha W_t}$$

$$R_{use,t+2} = (1 - \lambda + \lambda N_{Re,t+1})R_{K.agg,t+1} + (1 - s)R_{P,t}N_{Re,t+1} - R_{C,t} + x_{t+1}$$

$$W_{t+2} = (1 - \omega)W_{t+1} + \sigma_P x_{t+1} + (1 + (\sigma_P - 1)N_{Re,t+1})\lambda R_{K.agg,t+1} + \big((1 - s)(1 + (\sigma_P -$$

$$1)NRe,t+1RP,t+1-NS,tRD,t+1\sigma w$$

It is useful to define also the additional resource functions for $\boldsymbol{R_{R,t}}$, $R_{P,t}$**,** $R_{C,t}$ and $R_{K.agg,t+1}$:

$$\boldsymbol{R_{R,t}} = (\lambda R_{K.agg,t} + (\boldsymbol{1} - \boldsymbol{s})\boldsymbol{R_{P,t-1}})N_{Re,t}$$

$$R_{P,t+1} = x_{t+1} + R_{R,t+1} = x_{t+1} + (\lambda R_{K.agg,t+1} + (1 - s)R_{P,t})N_{Re,t+1}$$

$$R_{C,t} = (1 - s)R_{P,t}$$

$$R_{K,agg,t+1} = R_{K.agg,t} + R_{K,t} - \lambda R_{K,agg,t}$$

Technology development

$$\boldsymbol{N_{Re,t+1} = \left[1 + \left(1 - N_{Re,t}\left(1 + r_N(1 - N_{Re,t})\right)\right) L_{Re,t}\right] N_{Re,t}} \tag{127}$$

$$\boldsymbol{N_{S,t+1} = \left[1 + \left(1 - N_{S,t}\left(1 + r_N(1 - N_{S,t})\right)\right) L_{S,t}\right] N_{S,t}} \tag{128}$$

Capital and R&D Capital accumulation

$$K_{t+1} = \boldsymbol{K_t - \lambda K_t + s_t Y_t}$$

$$K_{R\&D,t+1} = K_{R\&D,t} + \boldsymbol{s}_{\boldsymbol{R\&D,t}}\boldsymbol{Y}_t$$

4.8.3 Other Functions

Price Function

$$P_{N,t+1} = \left(1 + (1 - N_{S,t})\frac{R_{N,t,0}}{R_{N,t}}\frac{R_{D,t}}{R_{N,t}} + N_{S,t}\gamma_{2t}\right)P_{N,t}$$

Production Functions

$$x_t = \boldsymbol{DK_{D,t}^{\delta}}(\gamma_{2,t}R_{E,t} + d_t R_{N,t})^{\boldsymbol{1-\delta}}$$

$$Y = AL_P^{\alpha}K_P^{\beta}\left[\boldsymbol{DK_D^{\delta}R_D^{1-\delta}} + \left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)N_{Re,t}\right]^{1-\alpha-\beta}$$

$$\frac{dN_{Re}}{dt} = \left(1 - N_{Re}\left(1 + r_{Re}\left(1 - \frac{N_{Re}}{K_{Re}}\right)\right)\right)N_{Re}L_{Re}$$

$$\frac{dN_S}{dt} = \left(1 - N_S\left(1 + r_S\left(1 - \frac{N_S}{K_N}\right)\right)\right)N_S L_S$$

Profit functions

$$\pi_Y = AL_P^{\alpha}K_P^{\beta}\left[\boldsymbol{DK_D^{\delta}R_D^{1-\delta}} + \left(\lambda R_{K.agg,t} + (\boldsymbol{1} - \boldsymbol{s})\boldsymbol{R_{P,t-1}}\right)N_{Re,t}\right]^{1-\alpha-\beta} - w_p L_p - r_p K_p - (x_t + l_{Re})P_{x_t}$$

$$\pi_D = P_{x_t}\boldsymbol{x_t} - rK_D - P_D R_D = P_{x_t}DK_D^{\delta}R_D^{1-\delta} - rK_D - P_D(dR_{Nt} + l_D)$$

$$\pi_{Re} = \frac{\left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)(1+\varphi_{Re}L_{Re})N_{Re,t}P_{x_{t+1}} - \left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)N_{Re,t}P_{x_t}}{r-\lambda} - wL_{Re} = 0$$

$$\pi_S = \frac{P_{D,t+1}(1+\varphi_S L_S)N_S R_{D,t} - N_S R_{D,t}P_{D,t}}{r-\lambda} - wL_S = 0$$

Intermediate sector price

$$P_{x_t} = \frac{1}{1-\alpha-\beta}\left[\frac{1}{D}\left(\frac{r}{\delta}\right)^{\delta}\left(\frac{P_D}{1-\delta}\right)^{1-\delta}\right]$$

Final sector input

$$X_t = \left[\frac{(1-\alpha-\beta)^2 AL_P^{\alpha}K_P^{\beta}}{\frac{1}{D}\left(\frac{r}{\delta}\right)^{\delta}\left(\frac{P_D}{1-\delta}\right)^{1-\delta}}\right]^{\frac{1}{\alpha+\beta}} = \boldsymbol{DK_D^{\delta}R_D^{1-\delta}} + \left(\lambda R_{K.agg,t} + (\boldsymbol{1} - \boldsymbol{s})\boldsymbol{R_{P,t-1}}\right)N_{Re,t}$$

Labor Demand on R&D Sectors

$$\boldsymbol{L = L_S + L_{Re} + L_P}$$

$$L_{Re} = \frac{\left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)N_{Re,t}\left(P_{x_{t+1}} - P_{x_t}\right)}{w(r-\lambda) - \left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)N_{Re,t}P_{x_{t+1}}\varphi_{Re}}$$

$$L_S = \frac{P_{D,t+1}N_s R_{D,t} - N_S R_{D,t} P_{D,t}}{w(r-\lambda) - P_{D,t+1}N_s R_{D,t}\varphi_S}$$

$$L_{P,t}^1 = L_{P,t}^2 \Leftrightarrow \left[\frac{\left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)\varphi_{Re}N_{Re,t}P_{x_{t+1}}}{\alpha A(r-\lambda)K_{P,t}^{\beta}{X_t}^{1-\alpha-\beta}}\right]^{\frac{1}{\alpha-1}} = \left[\frac{\varphi_S P_{D,t+1}N_s R_{D,t}}{\alpha A(r-\lambda)K_{P,t}^{\beta}{X_t}^{1-\alpha-\beta}}\right]^{\frac{1}{\alpha-1}}$$

Marginal Utility of capital and consumption

$$\frac{\partial u\left(R_{Use,t}\right)}{\partial R_{K,agg,t}} = \frac{\partial u(R_{Use,t})}{\partial R_{C,t}}$$

Wages

$$w = w_S = w_{Re} \underset{=}{\Leftrightarrow} \frac{\alpha Y}{L_p} = \frac{(1+\left(1-N_{s,t}\right)\gamma)\varphi_S P_{D,t+1}N_s R_{D,t}}{r-\lambda} = \frac{\left(\lambda R_{K.agg,t} + (1-s)R_{P,t-1}\right)\varphi_{Re}N_{Re,t}P_{x_{t+1}}}{r-\lambda}$$

Interest rates

$$r = \frac{\beta Y}{K_P} = \frac{\delta x_t}{K_D}$$

Savings rates

$$s_{K,P,t} = \frac{sY - w(L_{S,t} + L_{Re})}{1 + \frac{\delta x_t}{\beta Y_t}}$$

$$s_S = \frac{s - s_{K,P,t}}{\left(1 + \frac{L_{Re}}{L_{S,t}}\right)}$$

$$s_{Re} = \frac{s - s_{K,P,t}}{\left(1 + \frac{L_{Re}}{L_{S,t}}\right)} \times \frac{L_{Re}}{L_{S,t}}$$

$$s_{K,D,t} = \frac{sY - w(L_{S,t} + L_{Re})}{1 + \frac{\delta x_t}{\beta Y_t}} \times \frac{\delta x_t}{\beta Y}$$

5 Results

In this chapter the most important conclusions based on the analysis of the equilibrium of the model are introduced and discussed in detail. Most importantly we have focused on the R&D sector resource allocation and its possible behavior. On the other hand some discussion for solving the optimal steady state growth path of the model is also being introduced.

5.1 Meta Analysis and Model Behavior

The model presented here would require the solving of the optimal growth path to steady state. This is not done in this thesis. None the less based on the existing equations it is possible to make some conclusions on the equilibrium and behavior of the model. For practical reasons we will concentrate on the behavior of the model on topics that we are most interested, that is the allocation of R&D resources on substitution technology development and on the other hand on recycling rate development.

5.1.1 Labor Allocation on R&D Sector

According to traditional backstop theories such as Romer (1990), Tsur and Zemel (2006) and many others the backstop substitution possibilities are practically unlimited, whereas the model represented here had upper limits for technological rates. Besides this we had four sectors instead of traditionally used three sector model. For this reason the model should have some behavioral characteristics that can be expected to be different from the traditional models. In this context the special characteristic that is related to the R&D sector needs to be discussed, that is the allocation of R&D resources.

The equilibrium allocation of labor between the R&D sectors can be solved by replacing equations 46, 70, 80, 99, 101 and 107 into equation 119. This gives us an equation that solves the equilibrium labor division between R&D sectors:

$$s_{Re} = s_S \frac{R_R\left(P_{x_{t+1}} - P_{x_t}\right)}{l_s(P_{D,t+1} - P_{D,t})} \qquad \mathbf{129}$$

Based on equation 129 we can notice that the saving rate on each R&D sector (and the labor allocation between the R&D sectors) is based most importantly on the resource flow volumes. This is logical as the volume directly affects the income potential for the technology improvement. The effects of volumes and price are discussed in the following.

Volumes

The traditional literature based on Romer (1990) and others assume that the principal incentive for R&D comes from the prices only. In our model with two R&D sectors we have shown that in case where the non-renewable resources can be substituted with renewable resources as an alternative input for intermediate sector and where the waste can be substituted with recycling which produces alternative input for final sector also the resource stock volumes that can be substituted has significant impact on the allocation of resources between the R&D sectors. On the other hand this does not necessarily make Romer (1990) and others assumptions invalid as it can be expected that in general the driving force for aggregate R&D investments is in the price change.

If we examine the model in detail we can easily notice that the increase in capital stock increases the volumes of the non-renewable resource use (especially when the substitution level is low), while on the other hand this increases the speed at which the non-renewable resources are being depleted. The substitution technology development can be expected to grow initially rather slowly hand-in-hand with the increasing demand for resources[34].We can expect that the substitution technology development, when approaching the question from resource use volume perspective, will slow down as the non-renewable resources stock is being depleted due to the smaller amount of non-renewable resources that can be substituted.

The recycling technology development can be expected to increase to the point where the resources-in-use stock reaches its maximum at the level at a point where the production reaches its maximum. It could be concluded, that the R&D efforts in general can be expected to initially improve when the scarcity takes effect on the prices, while as the technological rate improves the decreasing efficiency makes R&D more and labor intensive, lower resource stocks makes the possibilities in R&D less attractive and high alternative costs in industrial sectors makes the R&D eventually really expensive.

If we take a look at the equation 109 $R_R = l_S \frac{\varphi_S P_{D,t+1}}{\varphi_{Re} P_{x,t+1}}$ we can notice that there exists a connection between the recycling volumes and the substitution volumes. The R&D efficiencies and price development on each sector can be seen as determining factors for the volumes. Based on this equation we can conclude that as the technological level on one sector increases (and the

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[34] We can notice that there are clear benefits from early investments on R&D in form of delayed depletion of the non-renewable resources, while the costs of doing so can be high as the volumes that can be substituted are low.

efficiency decreases) there can be expected decrease in the volumes of the other technology license sales given ceteribus paribus technological level and intrinsic growth rates are not equal. We have similar effect if say natural resource price is increasing faster than intermediate sector price, then recycling volume can be expected to increase.

Prices

Initially the price increases can be expected to be extremely slow as the capital level is low and the resource consumption is slow while there exists virtually no scarcity of the resource (if we compare to the initial level). When the capital builds up there is an accelerating price development due to relatively larger consumption of goods as compared to existing stock, but also due to increased scarcity. This accelerating price level makes it more and more profitable to invest in R&D on both sectors, while the limited labor supply and increasing labor cost will limit this progress. As we recall the price identity $P_{x_t} = \frac{1}{1-\alpha-\beta}\left[\frac{1}{D}\left(\frac{r}{\delta}\right)^{\delta}\left(\frac{P_D}{1-\delta}\right)^{1-\delta}\right]$ determines the intermediate sector prices as compared to natural resource prices. Given this equation we can predict that the price differential between the natural resources and intermediate resources is likely decrease and thus there is a small drift towards substitution technology R&D according to this equation.

5.1.2 Capital Allocation Between Sectors

The savings rate allocated on the final sector production capital is determined by current output of final and intermediate sectors and the elasticities, total savings rate (which was assumed to be a constant), and the wage costs on R&D sector giving an equation 123 indicating a relation between total savings, R&D efforts and intermediate and final sector outputs: $s_{K,P,t} = \frac{sY_t - w(L_{S,t}+L_{Re})}{\left(1+\frac{\delta x_t}{\beta Y_t}\right)}$.

We can see that the capital investments on final sector are negatively affected by the R&D sector consumption, which was expected. Similarly the output volume as well as the output elasticity of intermediate sector affects negatively on the final sector investments, which is similarly expected result as intermediate sector capital is competing for investments with final sector capital. It can be expected that as recycling technology advances the need for natural resources decreases affecting the intermediate sector output as equation 94 predicts. This would indicate that the relative share of final sector capital would increase as compared to intermediate sector capital according to equation 123. On the other hand the decreasing efficiency of R&D efforts would require higher shares of labor (as capital investment) to be allocated on R&D sectors thus lowering the total output of the system indirectly through the final sector production function.

5.2 Requirements for Solving the Model

It should be noted that in archiving the sustainable growth path for steady state there are basically two possible equilibriums: one where there is no consumption and one where the resource use is at stable (or marginally increasing) level. If the capital required (or the costs) for R&D sector development would become too high (resulting from lowering efficiency of R&D efforts) there is a significant risk that the capital investments on productive sector wouldn't be sufficient enough to cover the depreciation of capital, which would eventually lead to the possibility of destruction of all capital. On the other hand if the technology development is adequately fast, it should be possible to archive a sustainable level, where most (or virtually all) of the intermediate sector input resources has been substituted and all (or virtually all) of the waste can be recycled back into the production. In such case the intermediate sector production is just adequate to replace the depreciating resources-in-use and there is a stable use of resources. If the substitution and recycling developments are optimal there could be development curve for these technologies where the production levels would follow a path, where the recycling and substitution development would keep the resource use development path on sustainable level. The substitution of non-renewable resources for renewable resources in absolute terms should not exceed a level where the renewable resources would be consumed and the consumption would be larger than MSY (as described by Bolden and Robinson, 1999). The equilibrium with stable use of resources might not limit the growth possibilities of the real economy (unlimited backstop technologies as suggested by Romer, 1990 and many others), but the non-sustainable equilibrium path resulting to no input resources state surely sets limits for economy (Malthus, 1798).

Using the model it might become possible to answer the question of how big share in relative terms of the non-renewable resources will be needed to be replaced with renewable resources to maximize the overall resources-in-use (and thus the overall production/consumption levels. By using the conditions for sustainable resource use it would be possible to solve the optimal and sustainable steady state growth path for the model by utilizing the welfare function (equation 60) introduced in Chapter 4.3. This would require the maximization of welfare function in respect to the consumption of natural resources, accumulation of capital and technological level as well as the accumulation of resources-in-use and waste. The solution to the problem is likely to be rather complex, for which reason it is not represented in this thesis.

6 Discussion

This model has some common features with existing models while the novelty of the model lies in exclusion of additional R&D sector into the model, which makes it possible to research the R&D efforts and their effect on resource use dynamics. It should be noted that this model represents a thought where the resource use and economic growth are closely linked. The possibility to decouple these two activities from each others would weaken the arguments represented in this model (UNEP, 2011).

In our focus we had the R&D sector and the allocation of resources between each sector. It was found that the resource allocation between R&D sectors is mainly determined by the volume of the resource in question to be substituted (non-renewable resource or the depreciating resource) and the expected price changes of the resources in question. On the other hand the allocation of capital between R&D sectors and productive sectors is determined mainly by the marginal efficiencies of labor allocated on R&D sector and secondary by the interest rate and capital depreciation rate which both affect the profitability of the R&D efforts.

The model introduced in this thesis should be considered as a framework for later studies. First of all what needs to be done is to solve the steady state growth path for the model. When solving this problem the optimality conditions should include a sustainability argument, which restricts the depletion of renewable resources as we have discussed in the Chapter 6.2. The solution to the problem can be expected to be rather complex due to significant amount of state variables.

6.1 Research Suggestions

There are many interesting areas that would require further research. Most importantly the equilibrium growth path of the model needs to be solved. When this is done the optimal growth path would need to be solved as well. A continuous version of the model might be needed for solving the optimal growth path.

Alternatively numerical practice on the model could be performed to test some of the theories that have been suggested by previous authors. It would be interesting to test the model with similar assumptions for natural resources as did the Kamien and Scwarz (1978) where extraction costs are assumed to rise as the resource stock is decreasing while at the same time the unit extraction cost might be decreasing. Kamien and Scwarz (1978) were concerned of the possibility of starting R&D development too late, which could result in too expensive R&D efforts and make adaptation of the technology impossible. This could be experimented indirectly with the model by varying the intrinsic growth rates of the R&D sector, which affects the speed at which the R&D sector technological rates improves. With low intrinsic growth rate the technology improvement would become slower and similar effects as with delayed technology development would occur as this would increase the relative price of performing R&D.

Riley's (1977) third proposition which was supported by Barbier and Markandaya (1990) could be tested with numerical model by varying the initial level of stock to see how it affects the depletion of resources. Riley's (1977) seventh proposition which suggested that lower utilization in the beginning would delay the adaptation of substitution technologies could be experimented by varying the initial level of production capital. If the results for Riley could be confirmed it would mean the rejection of De La Grandville (1980) hypothesis.

It would be possible to test Tsur and Zemel's (2006) model against our model to see how prices effects of R&D, competition for resources and scarcity effects on R&D efforts perform in our model. When the optimal time path for the model is solved it would become possible to test how the model behaves as compared to Tahvonen and Salo (2001). The model could be used to see how the environmental Kuznets curve as presented by Bagliani, Bravo and Dalmazzone (2006) performs in the model.

6.2 Improvements to the Model

There are many features in the model that could be improved and experimented further. Most of the suggestions in this Chapter would require some additional features to be included into the model. For convenience we have listed these features.

- The inclusion of uncertainty to the model (see Lewis, 1981) could be useful for later research.
- Through technological innovation it might become possible to recover resources from the waste possibly later at increasing cost of recovery as the exergy required to recover the increasingly mixed and low concentrate waste increases substantially while relative cost of extraction per unit of waste could drop due to scale benefits in extraction.
- The implementation of depletion number introduced by Connelly and Coshland (2006b) into the model could offer interesting possibilities for analysis.

- Through technological progress (such as gene technologies) we could possibly have higher level of renewable resources available.
- Through technological progress we could also increase the overall productivity of the capital and through dematerialization less resources might be needed to produce the same product as before. This means that less of the resources are needed to produce the same amount of output as before and this will also contribute to the effects of technological progress. This case was not considered in the model.
- Efficiency was already mentioned as part of decreasing the waste streams but it will also mean that through technological progress the resources needed to produce one unit of product could be declining. This means that from the same amount of resources it could be possible to produce more than one unit of output through technological progress.
- One factor that could be improved by technological progress is the depreciation rate. The products could be designed to last longer (instead of current trend of "designed to brake") than previously so that fewer recycling loops are needed (and thus less waste generated) while the utility would remain unaltered. In practice this could mean that the production processes are designed to be flexible so that new parts could be added to the old modular constructions improving the efficiency this way instead of constructing totally new production plants or machines. This would lower the depreciation rate by some small fraction that could be significant in the long run for resource use and utility.
- The technological improvement of extraction and production waste parameters should be considered in future R&D models.
- Fully rational decision maker that takes into account the changes in stock levels when making an investment decision for R&D improvement.

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8 Appendixes

8.1 Appendix A: Proof for Price Equality on Final Sector

Solving the profit equation for the resource input of the final sector gives us:

$$\frac{d\prod_Y}{dR_D} = (1-\delta)(1-\alpha-\beta)AL_P^{\alpha}K_P^{\beta}DK_D^{\delta}R_D^{-\delta}\left(DK_D^{\delta}R_D^{1-\delta} \quad + (1+\theta s - s)R_{use,t}\boldsymbol{N_{Re,t}}\right)^{-\alpha-\beta} -$$

$$(1-\delta)DK_D^{\delta}R_D^{-\delta}P_{x_t} = 0$$

Solving P_{x_t} from the derivative gives:

$$P_{x_t} = (1-\alpha-\beta)AL_P^{\alpha}K_P^{\beta}\left(DK_D^{\delta}R_D^{1-\delta} \quad + \theta N_{Re}R_{use,t}\right)^{-\alpha-\beta}$$

Now derivating the same equation for resources in use gives us:

$$\frac{d\prod_Y}{dR_{use,t}} = (1-\alpha-\beta)A\theta N_{Re}L_P^{\alpha}K_P^{\beta}\left(DK_D^{\delta}R_D^{1-\delta} \quad + (1+\theta s - s)R_{use,t}\boldsymbol{N_{Re,t}}\right)^{-\alpha-\beta} - (1+\theta s -$$

sNRePxt=0

Solving P_{x_t} from the derivative gives:

$$P_{x_t} = (1-\alpha-\beta)AL_P^{\alpha}K_P^{\beta}\left(DK_D^{\delta}R_D^{1-\delta} \quad + \theta N_{Re}R_{use,t}\right)^{-\alpha-\beta}$$

As we can see the derivatives are equal, thus it is possible to use $R_{X,t}$ for the calculations.

8.2 Appendix B: Discussion for Proposition 1

We want to take a closer look into price growth parameter γ and see how the substitution technology development could have effects on the price growth of non-renewable resources. We can define that γ is dependent of the scarcity of the resource as compared to the initial level of the resource and the current resource use as compared to the current resource stock. The utilization rate of the renewable resources has an effect on the price growth rate as well, thus we can express γ as:

$$\boldsymbol{\gamma = \frac{R_{N,t,0}}{R_{N,t}} \frac{d_t R_{Nt}}{R_{Nt}} + N_{S,t}\gamma_{2t} = \frac{R_{N,t,0}}{R_{N,t}} d_t = \left(1 - N_{S,t}\right) \frac{R_{N,t,0}}{R_{N,t}} \frac{R_{D,t}}{R_{N,t}} + N_{S,t}\gamma_{2t}}$$

The first term on the RHS represents the effects of substitution on the price growth, the second term on RHS represents the scarcity of the resource as compared to the initial resource amount and the third term on RHS is connected to the resource consumption on intermediate sector. While the final term on the RHS indicates the utilization rate of the renewable resources.

Proposition 1: in the absence of technological substitution from non-renewable resources into renewable resources the discrete form of Hotelling's rule can be expressed in an alternative way, where the scarcity increases the price whereas the increase in substitution rate decreases the price growth rate while the substitution moves the price growth closer to the regeneration rate of the renewable resource:

$$\boldsymbol{P_{N,t+1} = (1 + \left(1 - N_{S,t}\right) \frac{R_{N,t,0}}{R_{N,t}} \frac{R_{D,t}}{R_{N,t}} + N_{S,t}\gamma_{2t}) P_{N,t}}$$

In stable equilibrium we have: $R_{E,t+1} = R_{E,t}$ and $K_{K,t+1} = K_{K,t} \Leftrightarrow s_{K,t}Y_t = \delta K_{K,t}$.

At MSY we have renewable resource stock size:

$$R_{E_t} = \frac{r_E K_{Et}}{2} \underset{K_{Et}=1}{\Longleftrightarrow} R_{E_t} = \frac{r_E}{2}$$

We have $R_D = \gamma_{2t} R_{E_t}$

$$\gamma_{2t} = r_E\left(1 - \frac{R_{E_t}}{K_E}\right) = r_E\left(1 - \frac{r_E}{2}\right)$$

Case 1:

Now if $\boldsymbol{\gamma_{2t} > r_E\left(1 - \frac{r_E}{2}\right)}$, which is equal to situation where more renewable resource is consumed than is the natural regeneration rate. This would result in situation where:

$$R_{E_{t+1}} < \frac{r_E}{2}$$

In equilibrium the renewable resources are just sufficient to compensate the capital depreciation, thus this would result in situation where:

$$K_{K,t+1} > K_{K,t}$$

Next period the consumption would be even greater than previously

$$R_{E_{t+2}} < R_{E_{t+1}}$$

Eventually this development would lead to destruction of all the capital once all the renewable resource was used. This scenario is not sustainable but plausible with very high utility discount rate.

Case 2:

Now if $\boldsymbol{\gamma_{2t} < r_E\left(1 - \frac{r_E}{2}\right)}$ there would be saving of resources. This would lead to a situation where:

$$R_{E_{t+1}} > \frac{r_E}{2}$$

In equilibrium this would lead to situation where:

$$K_{K,t+1} < K_{K,t}$$

Thus the next period capital and thus production would be eventually smaller than previously. Higher renewable resource amount would decrease the growth rate of renewable resource:

$$\gamma_{1t+1} < \gamma_{1t}$$

And thus the overall consumption despite of the fact that more resources could be utilized next period is lower than in case where no changes to the consumption pattern were made.

Thus the interest rate in equilibrium must be eventually equal to the utilization rate of the renewable resources:

$r = \gamma_{2t} = \gamma_{1t} - \frac{\alpha W_t}{R_{E_t}}$

MSY utilization rate of renewable resources should be equal to the depreciation rate of the capital:

$$\lim_{N_{S,t}\to 1} \boldsymbol{P_{N,t+1}} = \mathbf{lim}_{N_{S,t}\to 1}\left[\left((\mathbf{1} + \left(1 - N_{S,t}\right)\frac{\boldsymbol{R_{N,t,0}}}{\boldsymbol{R_{N,t}}}\frac{R_{D,t}}{R_{N,t}}\right) + N_{S,t}\gamma_{2t})\boldsymbol{P_{N,t}}\right] = (\mathbf{1} + \gamma_{2t})\boldsymbol{P_{N,t}} =$$
$$(\mathbf{1} + \boldsymbol{\lambda})\boldsymbol{P_{N,t}}$$

8.3 Appendix C: Assumptions

- closed system that is in equilibrium with its surroundings.
- no government consumption
- price of resource becomes higher as lower grade deposits will be needed to replace high grade resource deposits
- exergy content of the product and production function are interrelated
- all production in final sector will cause some form of pollution or waste that will degrade the quality of renewable resources
- cumulated waste affects the regeneration rate of the renewable resource
- constant labor force
- flux of exergy into the system is a constant
- endogenous R&D processes
- limited substitution possibilities (recycling and renewable resources)
- focus in transition
- smooth transition to new technologies
- no ageing of technology
- all production sectors pollute, the intermediate sector pollution is dependent on non-renewable resource use
- resource use and utility are correlated
- four sectors: intermediate and final production sectors, substitution R&D and recycling R&D sectors

- substitution of non-renewable resources with renewable resources is possible
- substitution of waste with recycling is possible
- the efficiency is assumed to follow an inverse of logistic curve, thus the higher technological level is reached the more difficult and costly it becomes to improve the system
- the regeneration rate of renewable resources follows a logistic function
- regeneration of renewable resources is a feature of the system
- carrying capacity of renewable resources is a constant
- capital and consumption resource use is homogenous
- cost of technology for the producing sectors comes from the use of licenses
- economic value and exergy consumption are correlated
- production functions are in Cobb-Douglas production function form
- constant savings rate
- depreciation of capital is a constant
- marginal utility of savings and consumption are equal
- value of patent is equal to the net capital income from the patent (PV of the patent)
- labor cost for improving technology as one time cost, while the income from the technology improvement is infinite
- Most Rapid Approach Path (MRAP) leads to practically competitive markets on R&D sector
- no economic profits for R&D sectors due to competitive markets
- price changes and technology changes are taken into account, while the resource stock changes and other R&D sectors developments are not included to the valuation of R&D investments
- price is a function that is dependent on the resource consumption (demand) and initial stock size as well as substitution rates
- price change is following a modified Hotelling’s rule
- arbitrage condition for interest rates
- arbitrage condition for wage rates
- arbitrage condition for price rates