Dynamic hedging of copper options

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Helsinki School of Economics **Abstract**
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DYNAMIC HEDGING OF COPPER OPTIONS

OBJECTIVES OF THE STUDY
The objective of this thesis is to study the pricing of copper options by comparing the premiums of traded options and delta hedging cost. Furthermore, this thesis analyzes the profits of using options in the copper price risk management of an industrial company that uses copper as a raw material. Delta hedging of options and option returns have been widely studied in equity markets but relatively few studies have been conducted in the commodity markets. This paper thus extends on the literature concerning commodity options.

DATA AND METHODOLOGY
The data set consists of prices of traded copper forwards and options of 6 different maturities in London Metal Exchange in 1998-2008. Delta hedging is conducted with both the basic Black (1976) model and with a GARCH application to account for stochastic volatility in the underlying instrument. In addition, the delta hedging returns are regressed on simultaneous copper returns to account for systematic bias in the hedge ratios. The returns are also analyzed against the performance of the S&P 500 Composite Index, the US Government Bond Index, the GSCI, as well as the SMB and HML factors.

The profitability of using traded options in risk management is assessed by comparing the returns of long positions in copper forwards and options. Furthermore, in a second strategy the term structure of copper is used to select a position in a forward or an option.

RESULTS
This study finds evidence on the cost of delta hedging being lower than the premiums traded options in the copper market. The application of GARCH volatility does not influence profits of delta hedging further, but reduces the deviation of returns slightly. Overall, the returns on delta hedging are not related to copper returns or systematic market risk factors.

Due to their high price, traded options are not optimal in risk management over the observance period. Furthermore, using copper term structure to choose between a forward or an option does not affect the cost of risk management.

KEYWORDS: Commodity options, delta hedging, risk management, GARCH volatility

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DYNAMIC HEDGING OF COPPER OPTIONS

TUTKIMUKSEN TAVOITTEET
Tutkielma käsittelee kuparioptioiden hinoittelua vertailemalla kuparioptioiden hintoja ja deltasuojauksen kustannuksia. Lisäksi tutkielma analysoi optioiden käytön hyötyä kuparin hinnan riskienhallinnassa yrityksessä, joka käyttää kuparia raaka-aineena tuotteissaan. Optioiden deltasuojausta ja optioiden tuottoja on tutkittu laajalti osakeindeksimarkkinoilla, mutta hyödykemarkkinoiden osalta tutkimusta on niukasti. Tämä tutkielma laajentaa hyödykeoptiokirjallisuutta.

TUTKIMUSAINEISTO JA –MENETELMÄT
Tutkimusaineisto koostuu Lontoon metallipörssissä (London Metal Exchange) kaupattujen kuparitermiinien ja kuparioptioiden hinnoista vuosina 1998-2008. Deltasuojauksen tuottoja tutkitaan sekä Black:n (1976) mallin avulla että GARCH-mallilla, joka huomioi option pohjana toimivan kuparin stokastisen volatiliteetin. Lisäksi deltasuojaustuotot regressoidaan samanaikaista kuparin hinnan muutosta vastaan, deltan mahdollisen systemaattisen riskin analysoimiseksi. Deltasuojaustuottoja verrataan myös S&P 500 osakeindeksin, USA:n valtion velkakirjaindeksin, GS hyödykeindeksin sekä SMB- ja HML-faktorien muutoksiin.

Osakkeiden käyttöä riskienhallinnassa arvioidaan vertaamalla tuottoja kuparitermiini ja –optiopositioista. Lisäksi arvioidaan toista strategiaa, jossa optio- tai termiinipositio valitaan termiinipisteiden mukaan.

TULOKSET
Tutkimus löytää näyttöä siitä, että deltasuojauksen kustannus on alempi kuin optioiden hinta. GARCH-volatiliteetin käyttö ei vaikuta deltasuojauksen tuottoon, mutta vähentää tuottojen hajontaa hieman. Tuotot eivät korreloi kuparin hinnan tai systemaattisten markkinariskifaktorien kanssa.

Optioiden korkeasta hinnasta johtuen niiden käyttö riskienhallinnassa ei tarkasteluajanjaksolla ole optimaalista. Termiinipisteiden käyttö optio- tai termiiniposition valinnassa ei vaikuta tuloksiin merkittävästi.

ASIASANAT: Hyödykeoptiot, deltasuojaus, riskienhallinta, GARCH-volatiliteetti

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1. Introduction

Despite its simplifying assumptions, the no-arbitrage framework of the Black and Scholes option-pricing model has preponderantly dominated the way market participants have viewed option pricing for the last decades. Ever since the 1987 stock market crash, however, the violations of the Black and Scholes prices have become more and more evident. It is now understood that the underlying asset is subject to stochastic volatility and jumps, which generate market incompleteness and lead to rejection of the model. Furthermore, the incompleteness brought on by this stochasticity seems to be reflected in the option premiums. Many authors such as Bakshi and Kapadia (2003) and Coval and Shumway (2001) show that returns generated by delta hedging or delta-neutral option positions are significant – in fact, the returns have been argued to be completely anomalous even in the presence of risk premiums.

It has been argued that options appear overpriced because the demand for options is high due to risk management purposes. For example, large institutions may buy substantial amounts of out-of-the-money put options to protect their equity portfolios. Although much of the previous literature has focused on the equity markets and the risk management of banks and financial institutions, in times of high turmoil in the market, the right risk management strategies can suddenly become crucial in the prosperity or survival of industrial companies as well. One major risk for many manufacturing companies is the exposure to volatile commodity markets.

1.1. Objective and contribution of the thesis

The objective of this thesis is to extend the growing literature on the anomalous returns on options thus far concentrated on the equity markets to commodity options, and copper options in particular. The returns on equity index options are substantial, as Bakshi and Kapadia (2003) find that buying an at-the-money index call option and delta hedging it with the underlying instrument on average loses 8 percent of the option value. Coval and Shumway (2001) further corroborate above findings by showing that selling even crash-protected puts is significantly profitable. Furthermore, in a portfolio optimization setting, Driessen and Maenhout (2004) find that even extremely risk-averse investors optimally hold short positions in put options and beta-neutral straddles. Against this setting, it can be argued that the commodity markets, consisting of industrial hedgers and speculators, are even more likely to exhibit significant “inefficiency” in option pricing.

The efficiency of copper options is studied in this thesis by comparing market prices to the cost of creating synthetic options by delta hedging with the underlying asset – the copper future. The foundation of the dynamic hedging scenario is on Black and Scholes option pricing model (1974) and particularly on Black’s (1976) modification of the model for futures options. The returns obtained from the basic Black and Scholes are further compared to delta hedging when the underlying futures are modeled to exhibit stochastic GARCH volatility.

This thesis extends the existing literature also by viewing the pricing of call options from the point of view of an industrial company hedging its exposure to copper price risk. Specifically, the thesis studies whether the benefits of the flexibility of options outweigh their premiums. This analysis is loosely related to Coval and Shumway’s (2001) test of returns on equity call options, which, according to capital asset pricing model (CAPM), should be positive if the underlying asset has a positive beta.

Finally, this thesis contributes to the literature on momentum strategies in the commodity markets. The term structure strategies present in many empirical papers (for example Miffre and Rallis, 2006 and Rusi, 2006) are modified to an option strategy. In essence, the term structure is used to distinguish between the purchase of an option and entering into a forward contract. The returns on this term structure strategy are then compared to the case where no distinguishment is made. This comparison allows to observe whether copper returns can be predicted by the term structure.

1.2. Presentation of the risk management setting

This thesis examines the commodity, namely copper, price risk management strategies of a global manufacturing company. The company buys copper as raw material and sells the copper products it manufactures to its customers for a price that includes a separate copper price component. For the most part, the company fixes the price for which it buys copper from producers to the market forward price. The copper price component in the selling price is also fixed beforehand so that customers know in advance the price they will have to pay. However, the price-fixed purchases and price-fixed sales do not automatically even each other out since sales and purchases occur at different times and at different quantities. Historically, the company's price-fixed sales have always been somewhat higher than price-fixed purchases at each moment. This creates a physical net short position on copper, which is hedged in the forward market.

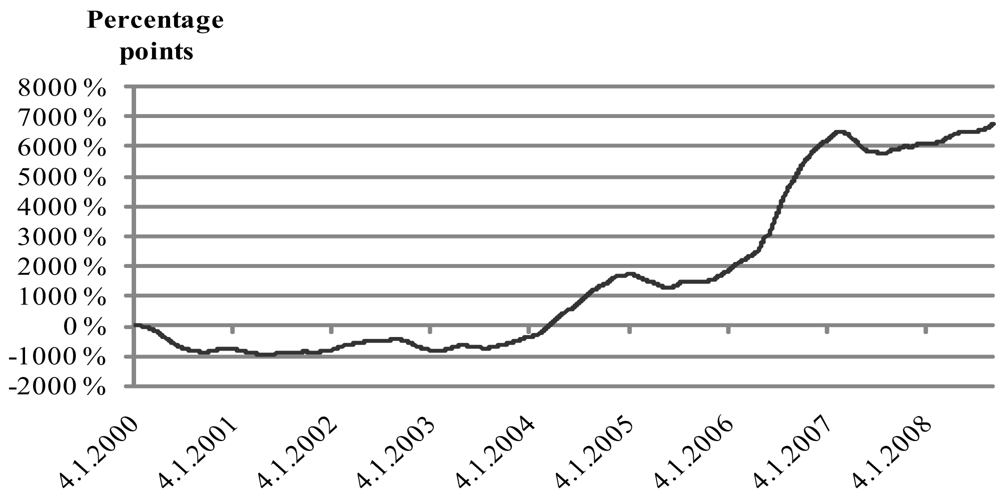
Since the company's current hedging strategy uses only forwards to fix the price of copper, it is not able to gain from a beneficial move in the copper price. If the price falls, they may have to pay a price substantially above current cash price at the expiration of their forward contracts, implying that their customers must pay a high price for their product as well. Price fixations can extend to very long maturities, which, while adding to the predictability of earnings, can harm the company's competitive position during a period of falling prices. Competitors may have more flexible hedging strategies and thus be able to bring their prices closer to market price.

An additional risk arises when large positions at brokers, especially if maturities are extended far, can have an impact on liquidity. Commodity brokers usually require a margin for the position held under their account. This margin is not significant when price changes are moderate, since the fair value of the forward contracts is fairly close to zero. However, if prices move considerably, the margin requirement increases rapidly for two reasons. First, the fair value, assuming a negative price change, falls, prompting an incremental deposit. Second, amid large price fluctuations the increased volatility triggers an augment in the so-called initial margin. The initial margin is calculated as a fixed portion of the position in tons, but the portion is determined based on the underlying asset's volatility and can change at any point.

The use of options decreases these risks by adding flexibility in the pricing of copper and by reducing margin payments. Especially, it should be noted that brokers bear no credit risk when options are used since the premium required is paid up-front. The premiums can, however, generate significant cost to the hedger whereas for forwards no premium is required. Therefore, a strategy where part of the forwards are replaced by options seems reasonable, since premium payments are decreased by lower volumes but additional flexibility is incorporated in the pricing strategy.

To obtain options, the company can purchase a traded option or create the option synthetically by entering into the forward market. Both alternatives create a cost to the hedger, that should be of equal magnitude. Figure 10 shows the cumulative difference between implied volatility of three-month copper options and the realized volatility over the options' maturity. The interpretation of the figure is somewhat ambiguous in the sense that the differences in percentage points are added up to create a total difference over the observed period. However, it is clear that on average the difference between impled and realized volatility should be zero for the implied volatility to be an unbiased estimate of expected volatility. From Figure 10 it can be observed that this has not been the case, but the realized volatility has been substantially lower than the implied volatility from 2004 onwards. This can be seen as clear motivation to study the costs of dynamic hedging, since the cost is determined by the realized volatility.

Figure 1 Cumulative Difference of Implied and Realized Volatility for 3-Month Options



1.3. Structure of the study

The following paper is organized as follows. In Chapter 2 I will provide a review on the existing literature in the field of pricing and hedging of options, as well as special characteristics of commodity options and commodity return disrtibutions. Chapter 3 presents the hypotheses and research questions of the study. Chapter 4 contains the presentation of the data set used in the empirical part as well as the methodology and theoretical foundation of the analysis. Chapter 5 discusses the results from the empirical tests regarding dynamic hedging of options. In Chapter 6, risk management strategies using options are examined from the point of view of the case company. Chapter 7 concludes.

2. Related research and theories

The following chapter provides insight into the empirical findings and theoretical frameworks in the field of option pricing and hedging. The chapter also discusses the characteristics of the copper market. The first part presents research concerning option pricing and the assumptions within, as well as literature on dynamic hedging and its performance. In the second part, research concerning the special characteristics copper return distribution is introduced.

2.1. Option pricing assumptions and dynamic hedging

The option pricing equation by Black and Scholes (1976) is based on the assumption of no arbitrage. In this framework, an option can be synthetically replicated by entering into the market of the underlying instrument in the amount of the so-called option delta[1], and adjusting for the position continuously to create a delta-neutral and, in this setting, risk-free hedge. The continuous adjusting creates a cost to the hedger since more of the underlying is bought every time the price of underlying goes up and sold as the price goes down to maintain a delta-neutral position. The value of the option, the option premium, equals in this framework the expected cost of adjusting to the hedger. If the cost of this dynamic hedging is less than the option premium, arbitrage is possible.

Ever since the Black and Scholes equation was made public, the so-called synthetic options formed using only the underlying security have been under much review. The first section of this part elaborates on the literature that analyzes the violations of the basic Black and Scholes model. The second section first presents empirical findings pointing towards rejection of the Black and Scholes model. Alternative option pricing models are then presented.

[1] Option delta is the first derivative of option price with respect to the price of the underlying asset. It shows how much the price of the option changes as the price of the underlying changes. A delta-neutral position is thus insensitive to small changes in the price of the underlying.

2.1.1. The Black and Scholes Model

Created in the early 1970s, the Black and Scholes model has had an enormous effect on how traders price and hedge options. The model involves setting up a riskless portfolio consisting of a position in the derivative and the underlying instrument. This portfolio is assumed riskless since the price of the underlying and that of the derivative are affected by the same source of uncertainty: the volatility of the underlying. In order for no arrbitrage opportunities to exist, the portfolio must earn risk-free interest. This leads to the Black-Scholes-Merton differential equation, from which the option pricing formula can be derived.

The Black and Scholes option pricing formula works in the presence of several simplifying assumptions, some of which are known not to hold in reality. With regard to the effectiveness of the Black and Scholes formula when these assumptions are relaxed, three of the assumptions are of particular interest. These are:

1) Transaction costs. The Black and Scholes model assumes no transaction costs or taxes, but in reality transaction costs are present in all markets.
2) Continuous rebalancing. For dynamic hedging to work perfectly, adjusting should be continuous. In reality this would lead to infinite transaction costs. Discrete rebalacing, however, creates an imperfect hedge.
3) The volatility of the underlying. The Black and Scholes model assumes constant, known volatility. In reality, however, volatility fluctuates over time and is never known in advance.

To name a few, Cox at al. (1979), Leland & al. (1985), and Boyle and Vorst (1992) are among those studying the effect of transaction costs on hedging and pricing of options. In the presence of transaction costs, the traditional Black and Scholes has two problems. First, hedging errors from discrete rebalancing will not be small unless the portfolio revision is frequent. Therefore, transaction costs will add significantly to the cost of hedging. Second, transaction costs themselves are random and will add to the error of the replicating strategy. To account for this, Leland & al. produce an adjustment to the BS to include transaction costs. In the model they assume that transaction costs are roughly proportional to price and inversely proportional to the revision period. They argue that this provides and intuitive explanation for slightly higher observed implied volatilities in the option market. They suggest that the difference could be due to transaction costs since the bid-ask spread makes one sell for a slightly lower and buy for a slightly higher price. This can be modelled as if the volatility were a bit higher than the actual expected volatility.

Boyle and Vorst (1992) also address the impact of transaction costs on dynamic hedging. Following Leland and the Cox-Ross-Rubinstein model (Cox et al. 1979) they derive a closed-form expression for the long call price inclusive of transaction costs. They show that when there is a large number of revisions, the price can be approximated with the ordinary BS formula but with an adjusted, slightly higher variance. Accuracy naturally increases with trading frequency. The influence of transaction costs increases with trading and also with proximity of strike to the current discounted value of the underlying. This is natural, since if the option is deep in-the-money or deep out-of-the-money, there will be hardly any transactions as the delta is not sensitive to minor movements in the underlying asset.

In practice, however, they observe that the implied volatilities are actually higher for deep in-the-money or deep out-of the-oney options, which should incur less transaction costs to the hedger. This can be due to the demand of these options exceeding their supply as options are used as hedges against extreme movements in the market. The overdemand requires the market-makers to hedge these options by replication, since they cannot purchase offsetting options.

Gregoriu et al. (2006) contribute to the concurrent literature by including transaction costs in the hedging equation by using the correct bid or ask price of the underlying, instead of the mid-price. They find that this procedure leads to superior pricing performance. Both bid and ask prices follow slightly different stochastic processes, which will give rise to alternative option valuation than their linear combination – the mid-price.

The effect of discrete rebalancing has been studied by for example Asay and Edelsburg (1986) and Mello and Neuhaus (1998). The latter use the variance of accumulated hedging errors up to maturity as a proxy for the risk arising from discrete hedging. Their main critique towards previous studies is that the hedging error of one rebalancing period does not represent the total risk in discrete rebalancing since it does not account for heteroskedasticity in hedging errors. Mello and Neuhaus's main result states that this risk can be significantly reduced by combining options into portfolios.

Asay and Edelsburg (1986) assess all three problems of transaction costs, discrete rebalancing and nonconstant volatility associated with dynamic strategies in the interest rate option market. In their results, ex post option prices closely approximate their ex ante theoretical value. The simulation results are largely insensitive to adjustments in rebalancing frequency. The authors tackle the problem of transaction costs, which take the form of commissions in the case of interest rate futures, by concluding that such costs are not relevant, at least in the market examined. The transaction costs arising from the synthetic option rarely exceed two times an outright fully covered with futures, and commissions on options are often twice that of the futures.

Empirical findings suggest that out of the three violations, the effect of misspecification of volatility swamps the other two. For example Boyle and Emanuel (1980), Asay and Edelsburg (1986), and more recently, Jiang and Oomen (2001) and Dotsis (2007) have studied and emphasized the effect of mis-estimated volatility. Boyle and Emanuel assert that the unknown variance of the underlying tends to exacerbate the skewed nature of hedge returns, but in the case of more profound misestimation, it will inundate all other components affecting the return. Asay and Edelsburg, on the other hand, state that inputting a wrong volatility estimate into the hedge ratio seems to have a minimal effect. Instead, they assert that anticipating changes in the volatility is the key consideration in the success of the dynamic hedging strategy. Jiang and Oomen and Dotsis also reach the same conclusion through extensive simulations.

Following the previous studies addressing the assumptions of the Black and Scholes formula, I will now focus on the effect of the volatility parameter on hedging results. In the next section, before delving further into questions regarding commodity markets, research addressing the alternative ways to price and hedge options is presented.

2.1.2. Redundancy of options – delta hedging and option returns

Early tests of Rubinstein (1985) supported the notion of Brownian motion for the option market and, thus, the Black and Scholes model[2]. After the 1987 stock market crash, however, evidence has been strikingly different with findings implying large violations of the Black and Scholes model (see, for example, Rubinstein 1994). In particular, the well-documented volatility smile or smirk and the volatility term structure have puzzled many authors.

The volatility smile refers to a circumstance where the implied volatility of the underlying asset, inferred from the market price of the option using the Black and Scholes model, exhibits a convex relationship to the exercise price of the option. Often the relationship is tilted towards the out-of-the-money call options, thus forming a so-called volatility smirk. Another well-documented finding is the volatility term structure, which describes a situation where the implied volatilities for different maturities on a given underlying asset, are different. In a Brownian motion, Black and Scholes world, the implied volatilities should be the same.

In practice, an ad-hoc approach is often used to account for these two forms of predictable biases. Specifically, the past Black and Scholes implied volatility is first related to its strike price and maturity by, for example, a regression. This estimated relationship then serves as the basis for calibrating the future Black and Scholes implied volatility with a different strike price and/or maturity. However, it has been under heavy scrutiny to solve the puzzle of the volatility smile.

In addition to the volatility smile and term structure, the observed returns on options have raised doubts about their redundancy as indicated by the Black and Scholes framework. Option risk consists of two separable components: the leverage effect and the curvature of option payoffs. The leverage effect stems from the fact that an option allows an investor to assume much of the risk of the option's underlying asset with a relatively small investment. Therefore, options have characteristics similar to a levered position in the underlying. Thus,

[2] Brownian motion refers to a stochastic process where the change in a variable during each short period of time Δt has a normal distribution with mean equal to zero and a variance equal to Δt. The Black and Scholes model has been derived under the assumption of this sort of process in the return of the underlying. See for example Hull (2005) for further reference.

call options written on securities with expected returns above the risk-free rate should earn expected returns above that of the underlying, since in essence, they have a higher beta than the underlying asset. Following the same logic, put options should earn less than the underlying since their beta is negative.

The other component of option returns relates to the fact that option values are nonlinear functions of the underlying asset. Therefore, option returns are sensitive to the higher moments of the underlying asset's returns. Inherent in the Black and Scholes model is that any risks associated with the higher moments are not priced, since asset returns follow geometric Brownian motion, which is a two-parameter process. Therefore, options are redundant assets. The implication of this is that option positions that are delta-neutral, meaning that they have a market beta of zero, should earn the risk free rate on average.

Coval and Shumway (2001) and Jones (2008) study the redundancy of options as an asset class by examining returns on straddles, which are delta-neutral combinations of put and call options. As noted earlier, over short time intervals, straddle positions are largely invariant to movements in the underlying asset, and thus, their CAPM betas are near zero. Both studies find evidence that the straddles still earn returns that are significantly different from the risk free rate. Moreover, the returns are negative. Coval and Shumway also test whether calls have positive betas and puts negative betas. Their results suggest that in equity, bond and foreign exchange markets call options do actually earn significantly higher returns than the underlying asset. Furthermore, put options earn significantly negative returns.

Goyal and Saretto (2008) use both straddle returns and delta-hedged gains as indicators of volatility risk premium in the market. They study returns on individual stock options by sorting stocks into decile portfolios based on the difference between their implied and historical volatility. Taking a long position in a portfolio of options with a large positive difference between historical and implied volatility, and a short position in the one with a large negative difference, they then study the returns of straddles and delta-hedged calls and puts on stocks in each portfolio. The returns for both delta-hedged portfolios and straddles are significantly positive. The authors also take into account transaction costs and liquidity effects and find that while they reduce the returns of both strategies, the returns remain significantly positive. As for the perseverence of the difference between historical and implied volatility, Goyal and Saretto find that while the implied volatility does predict future volatility as deviations between implied and historical volatility are transitory, the change in realized volatility is on average smaller in magnitude than implied. This suggests that investors may in fact over-react to current events in their estimation for future volatility, which causes the straddle and delta-hedging returns.

Balyeat (2002), Bakshi and Kapadia (2003), Szakmary et al. (2003) and Doran and Ronn (2008) have observed the delta-hedged gains and the predictive power of the implied volatility on future realized volatility. The papers by Balyeat as well as Bakshi and Kapadia both test equity index option markets and find that delta-hedged gains are nonzero. Where Balyeat analyzes the volatility difference of implied and realized volatility of traded options Bakshi and Kapadia use a strategy where equity index options are sold and then hedged using the Black-Scholes delta at the volatility implied by the no-premium base case. The strategy produces significant profits.

Szakmary et al. (2003) complement previous literature, which has been largely concentrated on equity markets, by studying a large number of futures including commodities, currencies interest rates and equity indices. In their study, efficacy is improved as all options trade in the same exchange as the underlying future and hence have the same closing time. Further, all options are based on futures contracts with less trading frictions, such as trading costs, than the commonly examined S&P cash equity index. They examine the predictive power of implied volatility of futures options relative to historical, 30-day moving average, and GARCH volatility estimates. Authors run several regressions and find results indicating that implied volatility is not an unbiased estimate of future realized volatility. However, its explanatory power relative to historical volatility is higher in most futures. In addition, in regressions where both implied and historical volatility are used as explanatory variables, the additional information from historical volatility does not appear economically meaningful. Therefore, Szakmary et al. conclude that futures options markets do seem to be weak form efficient.

Doran and Ronn (2008) further extend the research into energy commodity markets. By using Monte Carlo simulation, they demonstrate that negative market risk premium is the key factor in explaining the fact that implied volatilities in energy markets are consistently higher than realized volatilities. Thus, option traders tend to be short options: they write options at the higher implied volatilities and gather the risk premium by delta hedging the exposures.

Finally, Driessen and Maenhout (2003) study the redundancy of options through portfolio optimization. If options are redundant, they need not be included in the optimization equation. However, the authors obtain sriking results when index options are added to the portfolio of an investor with constant relative risk aversion. Every investor would optimally hold large short positions in both out-of-the-money puts and at-the-money straddles. This is especially puzling in the sense that even extremely risk-averse investors hold short positions in the put, rather than the protective-put strategies implied by portfolio insurance that one might expect for extreme risk aversion. The negative weights for straddles and puts persist after accounting for transaction costs in the form of bid-ask spread and margin requirements.

2.1.3. Alternative option pricing models

As discussed, several authors find evidence that the Black and Scholes model does not apply in the option markets. The three most popular natural explanations can be summarized as below (see, for example, Bondarenko, 2003):

1) Risk premium. Option returns are affected by risk factors not screened by returns in the underlying, towards which investors are strongly risk-averse. Investors are thus willing to pay premiums for holding options. In addition, a "true" model, which can explain the data exists.
2) The Peso problem. The presence of a rare, influential event that could have reasonably happened but did not happen in the sample, results in unexplained option returns appearing unreasonable.
3) Biased beliefs. Investors' subjective beliefs are mistaken.

In addition, Bates (2000) points out the possible reasons for the change in option returns after the 1987 market crash as 1) a change in investors' assessment of the stochastic process of the underlying, 2) a change in investors' risk aversion, or 3) mispricing of post-crash options due to e.g. market frictions. It is noteworthy that the second explanation could be justified by crash-related relative wealth redistribution between less and more risk-averse investors.

The Peso hypothesis has been largely rejected by several empirical studies. Bondarenko (2003) finds that market crashes of the magnitude experienced in 1987 would have to occur

1.3 times per year for ATM put option returns to break even. This would point towards rejection of the Peso hypothesis explaining the option returns, at least by itself. Results by Coval and Shumway (2001) and Driessen and Maenhout (2004) also find evidence supporting rejection of the Peso hypothesis. They “crash-neutralize” their option positions by simultaneously going short 0.96 moneyness out-of-the-money puts and long 0.92 moneyness “deep” out-of-the-money puts. At-the-money straddles are also crash-neutralized. Even with crash insurance, short put and straddle returns remain significantly positive.

The next step would thus be to try to explain option prices and option returns by a model that relaxes the assumptions of the Black and Scholes and fits the observed option market better, possibly by incorporating a risk premium in the prices. Bakshi et al. (1998) provide a summary of different pricing models and assess their empirical performance. They use three ways to assess the goodness-of-fit of a model, namely, 1) the level of misspecification in the model, 2) level of pricing errors when prices gotten from the model are compared to those obtained in the market, and finally, 3) the hedging performance from using the model. Whereas the in-sample and out-of-sample pricing errors reflect the static performance of the model, hedging errors reflect its dynamic performance. This notion is important since the authors find that while some models fit options data well based on their static performance, the dynamic performance of the Black and Scholes is just as good as that of the extended models. Yung and Zhang (2003) later note that the unimproved dynamic performance could be explained by the fact that hedging performance depends on both the price and delta estimates, and while more complicated models may produce more accurate price estimates, their delta estimates may be less precise.

The extensions of the Black and Scholes can broadly be divided into two groups based on whether they are deterministic or stochastic (Buraschi and Jakwerth, 2001). Deterministic models have many attractive features. In this setting, markets are dynamically complete and thus it is possible to hedge options without the need to estimate risk premia or equilibrium models. With deterministic models, time-varying volatility and volatility clustering can be captured, as can the relationship between the price of the underlying and the level of volatility. Deterministic models also provide the ability to fit the observed volatility smile by calibrating the volatility surface. However, the improved static pricing performance often does not result in improved dynamic performance, as noted above. Dumas et al. (1998) assess the functionality of the so-called deterministic volatility function, where volatility is deterministic of asset price and/or time, by fitting a cross-section of option prices and then testing the results both in- and out-of-sample. In their results, however, the hedge ratios determined from Black & Scholes appear more reliable than those obtained from the deterministic function. The more parsimonious models seem to dominate the ones with more parameters. In addition, the deterministic models seem to vary over time, resulting in poor out-of-sample results. In fact, deterministic models often suffer from overfitting the data.

Other deterministic models include the constant elasticity of variance model of Cox and Ross, the implied binomial tree by Rubinstein, the deterministic volatility models of Dupire (1994) and Derman and Kani (1994) and the kernel approach by Aït-Sahalia and Lo (1998). Buraschi and Jackwerth (2001), in their study, reject all deterministic models based on empirical data.

The stochastic models can be further grouped into three main strands: 1) the stochastic volatility models of Heston, Hull and White, Scott, Wiggins, Melino and Turnbull, and Stein and Stein, 2) the jump-diffusion models of Merton, Bates (1991) and Madan and Chang (1996), and 3) the stochastic interest rate models of Merton (1973) and Amin and Jarrow (1992). In addition, Amin and Ng (1993) and Scott (1997) have developed models that combine stochastic volatility and stochastic interest rates and Bates (1996) and Scott (1997) among others have combined stochastic volatility with jump diffusion. Furthermore, the stochastic processes can take on several forms, as can the jumps. The stochastic interest rate has been found not to be of great importance in studies by Bakshi et al. (1998) and Buraschi and Jackwerth (2001).

The stochastic models imply that option returns are affected by three distinctively different risk factors: diffusive price shocks, price jumps and diffusive volatility shocks. In a model in which these risks are not priced (such as the Black and Scholes model), strong inconsistency arises between the level of volatility observed in the spot market and in that implied by the model. The stochastic volatility factor (as in Heston, 1993) accounts for the volatility risk premium. Adding jumps in the returns of the underlying accounts for the jump-risk premium (Merton). Finally, for example Eraker et al. (2003) have added a jump component in the volatility process to account for the third risk factor.

The results from the plethora of papers studying the perfomance of stochastic option pricing models are encouraging, but not fully exhaustive. The results of Bakshi et al., Bates, Buraschi and Jackwerth, Liu and Pan, along with many others, have all come to similar conclusions. First, the deterministic models are rejected in favour of the stochastic ones that incorporate risk premia not screened by the underlying assets into option prices. It should be noted that especially jumps, either in the returns of the underlying or in option volatility, typically cannot be hedged away and thus investors may demand a large premium to carry these risks. Furthermore, when stochastic volatility is incorporated, the volatility risk premium is significant, as is the improvement to the goodness-of-fit. Fitting the jump into the equation improves the fit further. Moreover, the jump-risk premium in Pan (2002) and Eraker at al (2003) dominates the volatility risk premium. For pricing and internal consistency, the stochastic volatility and jump components are important. For hedging, however, the improvement is less pronounced, as the simple Black and Scholes performs not significantly worse in the dynamic sense.

It has also been argued that even the stochastic models and jumps cannot fully explain the returns on options discovered by for example Coval and Shumway (2001), Bondarenko (2003) and Bakshi and Kapadia (2003). Bondarenko, for instance, considers a broad class of models none of which can explain the highly negative returns on put options. Jones (2008) finds that the two- or three-factor models such as above are the most successful in explaining both expected and realized option returns. Furthermore, volatility and jump risk are priced in the cross-section of equity index options. More importantly, however, he asserts that these systematic risks are insufficient to explain average option returns as the risk-neutral parameters take on unrealistic values compared to the observed dynamics of the underlying index.

To address the shortcomings of previous models uncapable of capturing or explaining the empirical properties of option prices, Gârleanu et al. (2007) develop a model that departs fundamentally from the earlier framework. They recognize that option market makers cannot fully hedge their inventories, and, that option prices are thus impacted by their demand. The authors state that since options are traded, they cannot be redundant for all investors. Furthermore, they divide the option market into end users, who have fundamental need for option exposure, and intermediaries, who provide liquidity to end-users by taking the opposite side of the net demand. If intermediaries can hedge their exposure perfectly, then option prices are determined by no-arbitrage and the demand pressure has no effect on prices. In reality, however, markets are incomplete due to discrete hedging, stochasticity, jumps and transaction costs. Therefore, a marginal change in net demand of an option contract increases its price by an amount proportional to the non-hedgeable part. Also, this demand pressure increases the price of any other option on the same underlying asset by an amount proportional to the covariance of their unhedgeable parts. Empirically, the authors find evidence supporting the demand pressure theory in the equity index option market. Overall, options are more expensive when there is more end-user demand, and the expensiveness skew across moneyness is positively related to the skew in end-user demand across moneyness. For individual options, results are similar with the added feature that the relationship becomes stronger when there is less option activity, which points towards less intermediary capacity for risk-taking.

In a related study, Han (2008) observes the relationship between investor sentiment and option prices. Bollen and Whaley (2004) find that the slope of the volatility smile is not constant, but changing. Further, Han finds that market sentiment is significantly related to the shape of the smile. Specifically, the risk-neutral density for the equity index return is more negatively skewed when the market turns more bearish, and less negatively skewed in bull markets.

2.1.4. Relaxing the assumption of constant volatility: GARCH-models

As stated earlier, the assumption of constant volatility inherent in the Black and Scholes formula, is widely seen as its most significant shortcoming. The volatility of the underlying instrument is unknown and fluctuating, although possibly around a long-term mean. The different models developed to trace this unknown are various, as are the attempts to extend the widely used option pricing formula to incorporate a changing volatility. Furthermore, there is a consensus among researchers that asset return distributions are skewed and leptokurtic. Consequently, the ARCH family of models has in recent years been established as the strongest contender for the asset return generating process, at least in equity markets. Also, the available evidence suggests that GARCH option-pricing models are capable of describing option prices well (Amin and Ng, 1994, Duan, 1996 & 2001 and Heston and Nandi, 2000).

ARCH stands for autoregressive conditional heteroskedasticity. This implies that volatility has a long-run mean, but in the short run the asset price experiences periods of higher and lower volatility. Volatility is thus divided into a conditional (stochastic) and an unconditional (constant) component. The most popular of the ARCH-class models is the GARCH (1,1) model, in which the most recent realization of volatility and of the error term as well as the long-term variance are taken into account. The ARCH-class however includes various modifications of the model taking into account for example the asymmetry of the effect of negative and positive shocks to the market empirically witnessed in many markets.

Lehar et al. (2002) compare the performance of constant, GARCH and Heston's (1993) stochastic volatility in option pricing and find that GARCH dominates the other two with regard to minimizing pricing errors. However, they also find that in all three cases the performance of value-at-risk estimates for hypothetical derivatives positions are weak as the fit to realized profits and losses is poor.

Heston and Nandi (2000) present a closed-form option pricing formula for stochastic volatility model with GARCH. They show in their empirical analysis of the S&P 500 index options that the out-of sample valuation errors of the GARCH pricing formula are much lower than those from other models. They see that most of the benefit from the GARCH option pricing results from its recognition of path dependence in volatility and the negative correlation of volatility with index returns. They show that if the spot asset has a positive return premium and variance is negatively correlated with spot returns, then the risk-neutral drift of variance will be higher than the true drift of variance. This means that implied volatility will typically be higher than expected future volatility. Therefore, a short option position with a constant Black and Scholes hedge would actually be bullish on the market.

Dotsis and Markellos (2007) later study the GARCH option pricing model developed by Heston and Nandi and find that the parameters estimated by maximum likelihood in the Heston-Nandi model may be biased even in samples as large as 3000 observations. However, the option prices obtained by using these parameters do not contain the bias in as large amount, except for short-term, out-of-the-money options, and even then the errors are marginal in monetary terms.

Duan and Zhang (2001) study the performance of the GARCH option pricing in times of financial turmoil, and specifically its performance in equity option markets during the Asian financial crisis. They note that as the GARCH-models in fact contain the basic Black and Scholes model as a special case, this class of models must outperform the Black and Scholes when both are calibrated directly against market prices. However, the superior performance may be due to over-fitting, as is the case in many deterministic models. Using out-of-sample analysis is therefore a more credible way of determining whether the GARCH- model is indeed superior. The authors find, using out-of-sample analysis, that the GARCH model clearly dominates the benchmark Black and Scholes. Furthermore, it outperforms its practical application of "living-with-the-smile", where the implied volatilities are adjusted according to strike price and maturity. The GARCH model also conducts well in market turmoil, as a substantial increase in the market volatility (due to the financial crisis) in the second half of the sample left its pricing performance unaltered.

The final remark concerns delta hedging when the underlying asset exhibits non-constant volatility. As noted above, the dynamic performance of the stochastic and deterministic models alike has not been notably better than that of Black and Scholes, possibly due to the incorrect estimation of delta. The use of the Black and Scholes hedge ratio when the volatility of the underlying exhibits stochastic volatility is theoretically incorrect. Still, Dumas et al. (1998) find that the Black and Scholes hedge ratios outperform those obtained from the deterministic model. In addition, Bakshi and Kapadia use a GARCH model to depict volatility but find the bias from using Black and Scholes hedge negligible. This is noted by Barone-Adesi et al. (2004) as well, who suggest that unless a very strong departure from "local homogeneity" occurs in asset price during the maturity of the option, hedge ratios calculated from Black and Scholes at the implied level of volatility prove unbeatable. The GARCH models presented above come to the same conclusion – dynamic hedging is just as efficient with basic Black and Scholes, as it is with GARCH deltas.

In conclusion, the consensus of the research done in the field of option pricing states that in large, the Black and Scholes model is violated. Furthermore, the most important breach is that the volatility of financial assets is non-constant. Most of the research has concentrated on equity markets and equity index options in particular. In the next section I will elaborate on the research done in the field of commodity markets and the copper market in particular, and analyze further the market's features.

2.2. Characteristics of copper markets

As the scope of this study is to examine dynamic hedging of options in the copper markets, it is vital to understand the characteristics of this market. Literature concerning copper options, or commodity options in general, is very limited. To name a few exceptions, Miltersen and Scwartz (1998) and Bellalah (1999), following Black (1976), who modified the Black and Scholes option pricing formula to be suitable for futures, extend the formula to take into account the stochastic term structure of commodities and the information costs associated with investing in commodities. The special characteristics of the price process of commodities, and copper in particular, have, however, been studied extensively. In the following section I will go through findings on copper volatility estimation crucial for the pricing and hedging of options.

It should be noted that in the risk-neutral Black an Scholes setting, the expected return on the underlying asset does not affect the pricing and hedging of options. However, for a company trying to effectively hedge copper price risk while benefitting from beneficial price moevements, the ability to predict returns on copper would be highly advantageous. In the second part of this section I will thus also discuss recent findings on the profitability of momentum and term strategies in the commodity markets.

2.2.1. Estimating volatility in copper markets

Copper prices are determined in several metal exchanges. A large bulk of trade (approximately 95 %) occurs through the London Metal Exchange. The prices of metals are affected by both 1) financial considerations, such as information effects, speculative pressures and hedging activity, which typically lead to short-run volatility effects, and 2) market fundamentals, such as inventory levels, macroeconomic uncertainty, and strength of demand.

Metals are storable commodities and are not subject to seasonal production. However, some evidence indicates that price cycles in metal markets do exist. This is driven by the fact that supply is inelastic in the short-run whereas demand responds rapidly to changes in the industrial production, which is closely related to the business cycle. In periods of excess supply, consumption variability is attenuated through stock. However, in periods of excess demand, prices must adjust. Gilbert (1989) suggests that suchs an asymmetry of price responses leads to increased volatility in times of excess demand. Fama and French (1988) state that futures prices are less variable than spot prices when inventory is low, but when inventory is high, they have roughly the same variablity since the marginal convenience yield on inventory declines at higher levels.

A number of papers have examined the volatility of copper futures markets. Bracker and Smith (1999) in their approach, try to detect the sub-periods of changing variance in copper markets using an application of an ICSS algorithm and try to find the best model to capture the volatility in these markets. In their sample data ranging from 1974 to 1996, they find that overall, copper futures returns were characterized by heavy tails and negative skewness. Not surprisingly, given their findings, the random walk model performed poorly compared to GARCH –models. However, the symmetric GARCH performed equally well compared to the ones designed to model asymmetry.

Watkins and McAleer (2008), on the other hand, address the question of changing volatility in a longer time horizon. The metals trade has, in recent years, seen increased activity of investment funds and speculators in the exchanges. In addition to the positive effect of increased liquidity, some argue that this type of activity also increases market volatility. The paper examines the change in volatility over the years using a rolling AR-GARCH-model of 1000 observations (around four years of data). Results indicate that the models do vary over time, although at various times the estimates exhibit a high degree of stability. However, contrary to general belief, copper markets seem to have become more stable compared with previous periods of large price increases: returns are less volatile particularly in terms of less extreme events happening, and distributions are less fat-tailed than before. It should be noted that at the time of writing of the article, the ongoing economic expansion and interest rate volatility reducing monetary policy quite likely contributed to the more stable markets. Metal futures can be seen as having a close relationship with global macroeconomic trends, particularly with industrial production levels. Consequently, if the tests were re-run to include the market turmoil of 2008, results might look very different.

In addition to detecting changes in the models, Watkins and McAleer also assess the forecast errors of the models. Their results indicate that the GARCH-model seems to over-predict volatility. In general, large negative errors tend to be associated with the model under-predicting when a substantial shock hits the market (see 1996 Sumimoto LME scandal or 1987 stock market crash), and any large positive forecast errors are associated with the model over-predicting the persistence of the shocks.

McMillan and Speight (2001) contribute to the literature by dividing the copper volatility into short-run and long-run components. The model is called Component-GARCH (CGARCH, Engle, Lee 1999), in which the constant unconditional variance is replaced with a time-varying long-run volatility component. The component model suggests that the half-life of shocks to market-driven short-run volatility typically extends to no more than eight days, while that of shocks to the fundamentals-driven long-run volatility decays very slowly. This implies that metals price volatility is very slowly mean reverting.

Szakmary et al. (2003) and Schwartz and Trolle (2007) have studied the metal option markets. Szakmary et al. study the predictive power of implied volatilities in copper markets along with many other futures markets. They use implied volatility and historical volatility as explanatory variables for realized volatility, and find that the additional information from historical volatility is not economically meaningful. However, when a GARCH parameter is added to the regression, the coefficient becomes significant.

Following the research done in equity options markets, Schwartz and Trolle find evidence of components in copper volatility not spanned by the forward market. They first regress returns on option straddles and changes in implied volatilities on futures returns. For this regression the R-squared values are low, indicating that most volatility risk cannot be hedged by trading in the futures contracts. Moreover, they assess the importance of unspanned volatility when hedging an option portfolio, and find that hedging with only futures contracts causes only a small reduction in the variation of the portfolio's profit or loss. The conclusion of this article points towards copper options not being redundant assets that could be created synthetically with futures. However, it should be noted that the portfolios examined consist of both puts and calls and are relatively close to delta-neutrality to begin with.

2.2.2. Risk management considerations arising from price process of copper

For a company aiming to hedge copper price risk effectively while gaining from advantageous price movements, the ability to predict price movements provides a lucrative opportunity. To evaluate the existence of this sort of predictability, this section briefly looks into the literature concerning commodity pricing theories and momentum strategies in commodity markets.

2.2.2.1.Theories on price behaviour of commodities

The theories created to depict the characteristics and price behaviour of commodities can be broadly summarized into four classes. The theory of normal backwardation, the hedging pressure hypothesis and the theory of storage are the three most widely used theories. In addition, the returns have been considered through the CAPM.

The theory of normal backwardation, which was first introduced by Keynes in 1930, states that hedgers must compensate speculators for assuming the risk of holding futures contracts and therefore, futures prices are downward biased estimates of future spot prices. Inherent in the theory is the assumption that most hedgers are producers of the commodity in question, and thus hold a short position in futures. In subsequent decades the theory was extended to the hedging pressure hypothesis (Cootner, 1960, Deaves and Krinsky, 1965), which recognizes that since hedgers may in fact be net short or long futures depending on their position in the physical commodity, a risk premia will exist in the futures price regardless of the commodity showing a contango or backwardation[3]. The theory of normal backwardation has been widely tested (see, for example, Cootner, 1960 and Dusak, 1973) but it has been difficult to find evidence of the phenomenon. The practical weakness of the hedging pressure hypothesis, on the other hand, lies in the inability to test it, since no reliable measure of this hedging presure exists.

[3] Contango and backwardation are terms widely used in commodity markets when referring to forward premium or forward discount, respectively. Thus, if the forward price of a commodity is higher than the respective spot price, the term structure of the commodity is said to be showing a contango. If the forward price is lower than the spot price, the term structure is in backwardation.

The theory of storage, as presented by Kaldor in 1939, states that the relationship between cash and forward prices is determined by the net cost of carrying stock.The net cost depends not only on the opportunity cost of tying up funds in the inventory and the cost of carrying inventory but also on the convenience yield referring to the advantages from the security of supply. The convenience yield therefore reflects the market's expectations of future supply and demand of the commodity and is closely connected to the inventory levels of the commodity. (Erb & Harvey, 2006) However, the inability to accurately estimate the convenience yield is the shortfall of this theory and of the studies which have tried to assess its performance.

Finally, it has been argued by for example Lummer and Siegel (1993) and Kaplan and Lummer (1998) that according to the CAPM, commodities low correlation in relation to stocks and bonds, implies that they should have a long-term return approximately equal to the risk-free rate of interest. However, it has been argued that commodities are in fact not part of the market portfolio since they are not capital assets (Fama & French 1988, Dusak 1973). The characteristic of commodity prices being mean-reverting, however, has gathered support.

2.2.2.2.Momentum in commodities

Contrary to prices of stocks and other capital assets, driven by their long-term prospectives, commodity prices are mainly driven by current economic activity directly related to the scarcity of the commodity. This cyclicality and possible mean-reversion in commodity prices has led to studies of momentum strategies in commodity futures.

Many authors, such as Kat and Oomen (2006), Erb and Harvey (2006), Gorton and Rouwenhorst (2006), and Miffre and Rallis (2007) find evidence on the profitability of the so-called term structure strategy whereby backwardated contracts are bought and contangoed contracts are sold. The rationale behind the strategy lies in both the hedging pressure hypothesis and the theory of storage. Inherent in the strategy is that in many commodity markets, producers are more vulnerable to price changes than consumers. Therefore, producers are willing to pay a premium to hedge their position. Furthermore, backwardation may imply scarcity in the market of a commodity. Therefore, investors should be long commodities in the presence of backwardation, since odds are that the price of the commodity will rise as maturity approaches, or at the very least, cash price at maturity will not have fallen to net out the backwardation on trade date. During periods of contango, the situation is reversed and the investor should be short commodities, since contango implies possible oversupply of the commodity in the market.

Above simple strategy has created profits for investors even after accounting for time-varying risks in the market (Rusi, 2006). However, the risks in the commodity markets should not be neglected. Due to the cyclicality and inelasticity of the market, the effect of severe market turmoils is often large and unpredictable in commodity markets.

Finally, Miffre and Rallis (2007) complement earlier literature by looking into contrarian strategies in commodity markets. The contrarian strategies imply that since commodity prices are assumed mean-reverting in the long term, shorting the distant past well-performed commodities and buying the underperformed should eventually create profit to the investor. However, in their observance period Miffre and Rallis find no evidence of such profits.

In conclusion, the theories of and hedging pressure aiming to explain price behaviour in commodities, are intuitive, but their actual performance is difficult to test empirically. However, it has been found that in the past it has been possible to somewhat predict returns in commodity markets and thus gather momentum profits. In the next section I will move on to the research questions and hypotheses of this paper.

3. Hypotheses and research questions

This chapter presents the hypotheses and research questions of the study. The three principal hypotheses address the efficiency and characteristics of copper option markets and Black and Scholes hedge ratios. Two additional research questions concern the profitability of using options in hedging copper price risk in the case company, a large multinational copper products manufacturer.

The first hypothesis is associated with the pricing of options in the market. If copper options are efficiently priced and are redundant securities, creating synthetic options by means of dynamic delta hedging in the underlying forward market should create a cost approximately equal to the premiums paid for the purchased option.

H1. *Copper option market is efficient and creating synthetic options with delta hedging will generate a cost approximately equal to the option premium.*

In order to test the hypothesis, dynamic hedging scenarios are simulated. At the initiation, instead of purchasing an option, forwards of the same maturity are purchased in proportion to the option delta. This position in forwards is then adjusted daily or weekly based on changes in the delta. Dynamic hedging will generate a cost to the hedger, namely because more forwards are added to the portfolio when the price goes up and sold when the price goes down. If options are efficiently priced and redundant, the cost should roughly equal the premium of the traded option, with minor differentials due to the discrete adjusting interval,. If the costs differ significantly from the market premium, a possibility for arbitrage in the copper option market may exist. Alternatively, the option prices include a risk premium for something not screened by the underlying future or are not efficintly priced.

The second hypothesis relates to the scenario, where the first hypothesis is rejected and delta hedging does produce returns. If these returns from shorting options and creating a reciprocal long position by dynamic hedging are significantly different from zero, they should represent a compensation for risk.

H2. *Systematic market risks explain the returns of dynamic hedging.*

The second hypothesis is studied by analyzing the correlation of the difference between traded option premium and hedging cost and returns on stock, bond and commodity market risk premiums.

The final hypothesis relates to the efficacy of the Black and Scholes hedge ratio, the delta. Many authors including Heston and Nandi (2000) state that implied volatilities are often higher than expected future volatilities and Black and Scholes hedge ratios are in fact bullish on the market. This takes place if the underlying instrument exhibits stochastic volatility which is negatively correlated with the market and the asset has a positive expected return. Copper is a commodity and its price can be expected to be mean reverting. In addition, its volatility is more likely to be positively correlated with price. Therefore, to test for some systematic error in the Black and Scholes hedge ratio, the hedging results from upward trending and downward trending markets are compared. If Black and Scholes hedge ratios are systematically bearish due to the positive correlation of the volatility and return, delta hedging in downward trending markets should create higher returns to the hedger.

H2. *Black and Scholes hedge ratios exhibit no systematic error, and therefore, the return on the underlying asset does not affect hedging return.*

In addition to the three hypotheses presented above, two additional research questions are discussed relating to the profitability of using copper options for hedging copper price risk in the case company. Currently, only forwards are used to hedge copper price risk. Hence favorable changes in copper price cannot benefitted from. Using options in addition to forwards may provide advantages because of the unlimited upside potential in such an arrangement. However, the premiums paid for options reduce the potential gain, and on average the gains and costs should outweigh each other. The question is tested by constructing a model in which a long position in copper forwards is replaced by at-the-money call options spread evenly for maturities of 1 through 3 months, 6 months, 9 months and 12 months. This structure resembles the maturity structure of company's historical forward positions. The costs or gains from this strategy are compared to the cost or gain of using forwards of the same maturities for each time period.

The situation in which the company has a constant long position in call options relates loosely to the call strategy studied by Coval and Shumway (2001). If the underlying asset has a positive beta, the call option should have an even higher beta due to the leverage effect implicit in options. However, according to the CAPM, it can be argued that commodities are not part of the market portfolio, or that at least they have very low betas, and thus earn the risk-free rate. In other words, it is not evident that copper call options should have positive betas either.

The final research question relates to a situation where the visible term structure is used to define whether the company should enter into a forward agreement or purchase a call option. This is a modification of the term structure momentum strategies in commodity markets summarized earlier. To test the question, an option is bought when copper term structure shows a contango (that is, forward prices are higher than the spot price on trade date). In the case of backwardation a forward contract is entered into and no option is purchased. The maturity structure remains the same as in the previous strategy.

4. Data and methodology

This chapter first presents the data set used in the empirical part of the study. In essence, there are two data sets used in the study. The first consists of the implied volatilities of traded copper options and the second comprises the forward prices of copper used in all calculations. In addition to the data description, this chapter presents the methodology of the dynamic hedging simulations.

4.1. Description of the base data set

The calculations are based on London Metal Exchange copper forwards and traded European options based on these forwards. Observation period covers the 10-year period from September 1998 to December 2008. The data set for forward prices consists of daily official settlement prices for cash copper and 3-month, 15-month as well as 27-month forwards for which the official prices are quoted. Forward prices for additional maturities, namely 1-2 months, 6 months, 9 months and 12 months are needed in the empirical part of the analysis as well. To calculate the price for these maturities, I have performed the following adjustments:

(1) for 1- and 2-month forwards, I have multiplied the difference between cash and 3-month price by 1/3 and 2/3, respectively

(2) for other forwards of maturity x (F_x), I have used an average of the differences of cash and 3-months prices, and cash and 15-months prices so that $F_x = 0{,}5 * \frac{x}{3} * (F_3 - C) + 0{,}5 * \frac{x}{15} * (F_{15} - C)$ and so forth, where C denotes the cash (or spot) price of copper

This approach includes a slight inaccuracy since the forward curves for copper are not linear. However, since the time horizon is fairly long, it can be assumed that on average, the prices for other maturities besides 3-months and 15-months are close to correct as well. The source of the data for forward prices is Reuters and Datastream. Altogether there are 2610 observations for each contract.

To calculate option prices, I use LME traded option implied volatilities for years 2000-2008. The maturities are rounded to the closest month so that for example an option with maturity of 35 days and an option with maturity of 27 days both are grouped as 1-month options. When calculating the price, the correct maturity in days is used. All options are at-the-money, meaning that the strike of for example a 3-month option is the 3-month forward price of the trade date rounded to the nearest hundred dollars per metric ton. There are on average 2000 observations for each maturity. The volatilities are obtained from Reuters and RBS Sempra. The option prices are calculated from volatilities using Black (1976) formula for futures options. The risk-free interest rate needed for the formula is obtained using US Treasury Bill rates of matching maturities since all prices are quoted in US dollars.

Finally, for the multifactor model I have collected data on Goldman Sachs Commodity Index and the US Government Bond Index from Datastream. In addition I have obtained the SMB (small-minus-big i.e. small company risk premium) and HMB (high-minus-low i.e. risk premium on high market-to-book ratio companies) factors from the database maintained by Mr. French. The data for the multifactor models ranges from 2000 to 2008.

Option prices are calculated from the implied volatilities using the Black (1976) formula for futures options. Futures differ from other securities in the sense that the value of a futures contract is zero at the end of each trading day. Therefore, the traditional Black & Scholes formula is modified in relation to the interest terms. Note that this pricing equation assumes that changes in futures prices are distributed log-normally and that variance is known and constant, as is the expected return on market and the short-term interest rate. The pricing formula is as follows:

$$w(F,t) = e^{r(t-t^*)}\left[FN(d_1) - SN(d_2)\right] \tag{1}$$

$$d_1 = \left[\ln\frac{F}{S} + \frac{\sigma^2}{2}(t^* - t)\right] \Big/ \sigma\sqrt{(t^* - t)} \tag{2}$$

$$d_2 = \left[\ln\frac{F}{S} - \frac{\sigma^2}{2}(t^* - t)\right] \Big/ \sigma\sqrt{(t^* - t)} \tag{3}$$

where

w = *Premium of option*

F = *Forward price at trade date*

S = *Strike price*

$t\text{-}t^*$ = *Time until maturity*

r = *Risk-free interest*

σ = *Volatility*

Strictly speaking, the LME copper contracts used in the calculations are forwards, not futures. However, the LME forwards have most of the properties of futures. They are standardized with regards to size, metal purity and delivery location. In addition, there are arrangements for initial margins and margin calls and there is an organized clearing house for the contracts. Therefore, the forwards can be called futures in the sense the term is used in literature. As for the Black formula, their usage is also correct with respect to the interest adjustment since the positions are indeed settled daily with regards to the margin, as are futures contracts.

4.2. Calculating returns on dynamic hedging

The first hypothesis of the study states that the copper options market is efficient and options are redundant securities, and therefore no gains can be made through dynamic hedging. In the case of this study, this means that if the company chooses to replace part of its long forward position by call options, it will incur an equal cost if they do this synthetically or by purchasing options on the market.

To test the efficiency of the option market, simulations of dynamic hedging are performed for all option maturities and the cost is compared to the premiums of respective traded options. The adjustment period is set to be one day or one week, and the results are evaluated with regard to the cost incurred.

Option delta is defined as the first partial derivative of the option value with regard to the price of the underlying instrument or:

$$\Delta = \frac{\partial w}{\partial F} = N(d_1) \qquad (4)$$

It shoud be noted that in the case of futures options, the underlying instrument is the future, and delta is the partial derivative of the option value with regard to changes in the value of the future, not the cash price. As the time to maturity decreases, so must the maturity of the underlying forward. All options are at-the-money, so the initial delta should be relatively close to 0.5, due to the it being equally likely for the option to be in-the-money or out-of-the-money at maturity. It is not, however, exactly 0.5 since the forward price includes the contango or backwardation with respect to the cash price. The delta dictates the amount of forwards bought at the intitiation of the option simulation. In other words, assuming a delta 0.5 and that the call option is wanted for x tons of copper, the forwards bought must equal 0.5x.

At each adjustment period, a new delta is calculated and the forward position is adjusted accordingly. At maturity, delta is either 0 or 1. If the option is in-the-money, delta is one, forward position is x and the option has been exercised. Profit thus becomes market price less strike price, less the cumulative cost incurred. If the option is out-of-the-money at maturity, forward position is 0. All that remains is the cumulative cost to the hedger. After the profit or loss has been recorded, it is compared to the premium of the respective traded option of same maturity and trade date.

The return on dynamic hedging of an option of maturity *i* is calculated in percentage terms as:

$$R_{i,T} = \left\lfloor w_i - (\Delta_0 F_{i,0} + \sum_{t=1}^{T} (\Delta_t - \Delta_{t-1}) F_{i,1}) \right\rfloor \Big/ w_i \qquad (5)$$

where *t* denotes the redjustment interval of one day or one week. The return for a portfolio of options of maturities 1 through 12 months thus becomes:

$$R_T = \sum_{i=1}^{12} \left\lfloor w_i - (\Delta_0 F_{i,0} + \sum_{t=1}^{T} (\Delta_t - \Delta_{t-1}) F_{i,1}) \right\rfloor \Big/ \sum_{i=1}^{12} w_i \qquad (6)$$

The gains are also presented in nominal terms, as dollars per ton, where the return on option of maturity *i* is:

$$R_{i,T} = w_i - (\Delta_0 F_{i,0} + \sum_{t=1}^{T} (\Delta_t - \Delta_{t-1}) F_{i,1}) \qquad (7)$$

and the return on the portfolio of options is:

$$R_T = R_{1,T} + R_{2,T} + R_{3,T} + R_{6,T} + R_{9,T} + R_{12,T} \qquad (8)$$

In all cases the average return on dynamic hedging over the observance period is calculated as:

$$\pi = \frac{\sum_{T=1}^{N} R}{N} \qquad (9)$$

In order to calculate deltas for the simulations, the volatility of the underlying forward must be defined. I use a rolling historical average volatility for the historical volatility, and calculate the standard deviation of one year's (252 observations) overlapping monthly logarithmic returns for each trade date. The volatility is calculated as $\hat{\sigma}_T \sqrt{12}$, where

$$\hat{\sigma}_T^{\ 2} = \sum_{t=T-252}^{t=T-1} r_t^{\ 2} \Big/ 252 \qquad (10)$$
$$r_t = \ln(F_t - F_{t-22})$$

To assess the possible improvement on delta hedging from the incorporation of stochastic volatility, I also calculate hedging cost for 1-month options when the volatility of the underlying forward is depicted by a GARCH model. The GARCH (1,1) model depends on the long-term volatility, one lagged squared error term and the previous period's volatility measure. To estimate the parameters I use the observations from the previous year so that returns from 1999 are used to estimate the model used in 2000, returns from 2000 for 2001 and so on. This enables comparing the stability of the parameters over time and also testing the model out of sample, which more relevant from a practical point of view than the in-sample results. Also, as mentioned by Duan and Zhang (2001), the GARCH model in fact includes the Black and Scholes as a special case and thus is necessarily a better fit in-sample. Thus, the out-of-sample testing also avoids the problem of overfitting.

The Black formula assumes constant volatility for the maturity of the option. I therefore resort to Monte Carlo simulation to create the hedge ratios used. Following Barone-Adesi et al. (2004):

$$y_t = \mu + \varepsilon_t = \ln\left(F_t \big/ F_{t-1}\right), \qquad (11)$$
$$\sigma^2{}_t = \omega + \alpha \varepsilon^2{}_{t-1} + \beta \sigma^2{}_{t-1}$$

where μ determines the constant returns on a daily basis (set to the risk free rate), $\varepsilon_t = \sigma_t z_t$, and $z_t \sim i.i.d.(0,1)$. σ_t^2 is the conditional volatility at time t, and ω measures the long-term level of volatility. At time t = 0 the dollar price of a European at-the-money call option with time to maturity of one month is calculated by simulating log-returns under the risk neutral parameters so that $\mu = r - \sigma_t^2/2$ where r is the risk-free rate. Specifically, T independent random variables z are drawn to compute (y_i, σ_i^2) and set $F_T = F_0 \exp(\sum_{i=1}^{T} y_i)$. The call option payoff is then calculated as $C = \exp(-rT)\max(0, S_T - K)$. Iterating the payoff 10,000 times gives the estimate of option value at time t. To reduce the variance of the Monte Carlo estimates for option value, I set $C = (C_a + C_b)/2$ where (for each z) C_a is computed using z_i and

C_b using $-z_i$. This procedure is repeated each day during the option's maturity, and the starting price is adjusted according to current market price. The delta for each day is then calculated as the change in option price relative to the change in the underlying future, and delta hedging cost is computed as before.

As I use daily data to calculate option premiums and hedging costs, the resulting returns for delta hedging will be highly autocorrelated. In addition, as copper price and its volatility have experienced large-scale changes during the observance period, the error terms are likely to experience heteroskedasticity. Therefore, to assess the statistical significance of the returns, I calculate the t-values using both non-overlapping data and Newey-West standard errors. As the non-overlapping data is used, most of the observations are lost but to assess the total profitability of the strategy I also report the mean returns from daily observations.

Finally, to test whether the Black and Scholes hedge ratios are systematically biased due to the positive correlation of the volatility and return, the returns on delta hedging are regressed on the return on copper during the options maturity.

$$R_{p,T} = \alpha + \beta \ln\left({}^{C_T}\!\big/_{C_{T-t}} \right) + \varepsilon_{p,T} \qquad (12)$$

$R_{p,T}$ denotes the percentage gain of delta hedging compared to the option premium (see Equation 5). C_T denotes cash copper price at maturity and C_{T-t} the cash price on trade date. If any kind of systematic error exists in the hedge ratios, the return should explain positive or negative returns. Moreover, the alpha in the regression, which accounts for the abnormal gains, should become insignificant. It should be noted that if it is assumed that volatility and return of copper price are positivey correlated, delta hedging in downward trending markets should create relatively higher returns to the hedger due to downward-biased deltas. This implies that the beta in the above regression should be negative.

4.3. Multifactor models

To assess the profitability of the dynamic hedging strategy after accounting for risk, the Fama and French (1993) multifactor model is applied. In addition to the original model first created for the stock and bond market, I also test the multifactor model modified for commodity markets as presented by Miffre and Rallis (2005).

The basic variables in the multifactor model by Fama and French are return on the stock market, the SMB-variable implying the return of small companies over the large and the HML implying the return of companies with high market-to-book ratio over those with low.

$$R_{p,t} = \alpha + \beta_M (R_{M,t} - R_{f,t}) + \beta_{S,t} R_{SMB,t} + \beta_{H,t} R_{HML,t} + \varepsilon_{p,t} \qquad (13)$$

In the equation, $R_{p,t}$ is the difference of the hedging cost and the option premium in relation to the premium price (see Equation 5), an $R_{M,t}$, $R_{SMB,t}$, and $R_{HML,t}$ are the logarithmic returns on the stock market, the SMB and HML (as defined above) for the hedging period. It should be noted that the risk-free interest rate has not been subtracted from the return. This is due to the fact that when entering into a forward contract, no up-front payment is needed, but when an option is bought, the premium is paid immediately. However, in the dynamic hedging scheme the interest is settled in the margin account and the two terms cancel each other out.

$$R_{p,t} = \alpha + \beta_M (R_{M,t} - R_{f,t}) + \beta_{B,t} (R_{B,t} - R_{f,t}) + \beta_{C,t} (R_{C,t} - R_{f,t}) + \varepsilon_{p,t} \qquad (14)$$

In the modified factor model above, $R_{p,t}$ and $R_{M,t}$ are as before, $R_{B,t}$, and $R_{C,t}$ are the logarithmic returns on the US government bond index and the Goldman Sachs commodity index, respectively.

The factors in the above models are designed to reflect systematic market risk. If the return from dynamic hedging is correlated with one or several of the factors, it can be concluded that the return is merely compensation of the risk embedded. Furthermore, the alpha should remain significant for there to exist abnormal profit in dynamic hedging of copper options.

5. Analysis and empirical results

This chapter presents the findings from the empirical analysis regarding the data set and gains on dynamic hedging. The first part discusses the general characteristics of copper price development over the examination period and its implications on the results. The second, third and fourth part delve into the effectiveness of the replicating strategy. The fifth part summarizes the main findings.

5.1. Copper price development and volatility term structure

The development of copper price over the observation period may have a profound effect on the results obtained in this study. As evident from Figure 2, copper price has been reasonably stable until summer 2003, after which it has quadrupled to the highs of almost 9,000 dollars per metric ton. During 2008 and specifically towards the end of the observation period, the global financial crisis and upcoming recession have forced copper price to a free-fall. By the end of the period the price has fallen back to barely over 3,000 dollars per metric ton as demand for the industrial metals has been forecasted to drop dramatically. This dramatic behaviour in the price evolution, while making the generalization of the analysis difficult, further motivates the notion of enhancing risk management strategies.

Figure 2 Copper 3-Month Forward Price 1998-2008

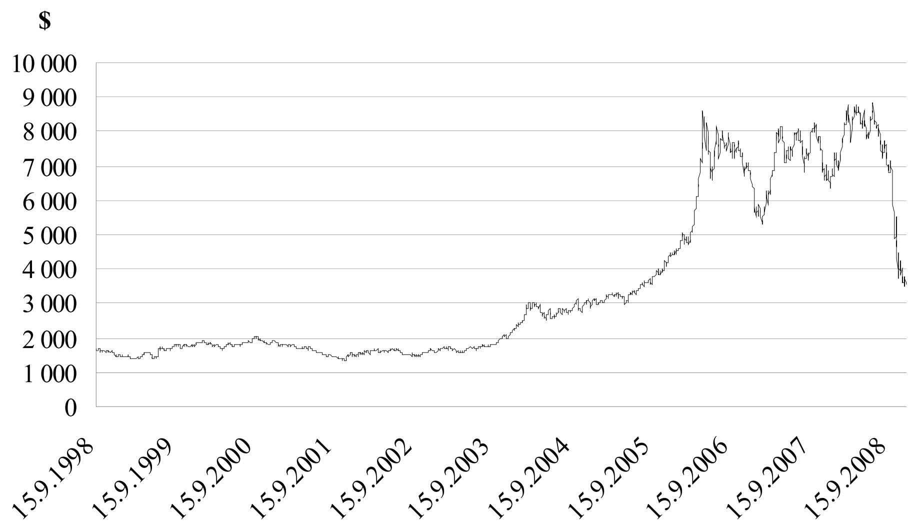


Table 1 Price Statistics of LME Copper

Table 1 presents the price range and average and mean prices of cash and 3-month forward prices of copper 1998-2008. In addition, the range, average and median of backwardation calculated as cash price less 3-month forward price is presented.

	Cash price	3-month price	Backwardation
Mean	3 510	3 481	29
Median	1 987	2 001	-14
Min	1 319	1 340	-125
Max	8 983	8 812	297
Count	2 666	2 666	2 666

Table 2 Return on LME Copper 1998-2008

Table 2 presents the summary statistics of annualized daily logarithmic returns of copper 1998-2008. The statistics are presented for the whole period as well as for four 2.5-year subperiods. The significance level of the annualized return is also shown for all periods.

Annualized figures		Daily return	Standard deviation	Skewness	Kurtosis
Whole period	Cash	0,074***	0,267	-0,112	4,109
	3M	0,074***	0,258	-0,208	5,033
1998-2001	Cash	0,0036	0,195	0,369	2,110
	3M	0,0086	0,188	0,312	2,350
2001-2003	Cash	0,0884	0,176	0,433	1,073
	3M	0,0860	0,168	0,447	1,128
2003-2005	Cash	0,5660	0,264	-0,121	1,611
	3M	0,5488	0,257	-0,237	1,903
2005-2008	Cash	-0,3502	0,381	-0,105	2,246
	3M	-0,3274	0,373	-0,139	2,780

*/**/*** denote significance levels of 0.05, 0.01 and 0.001, respectively

Tables 1 and 2 present the price and return statistics of copper price in more detail. As noted earlier, copper price has moved in large scale, from a minimum value of 2,000 dollars per ton to a maximum of almost 9,000, whereas average yearly return for the whole period is only 7.4 percent. The average annual return is still statististically siginficantly different from zero, but compared to the returns of more than 50 percent in years 2003-2005 followed by an abrupt fall, it is evident that copper market has been mercurial in recent years.

Standard deviation, skewness and kurtosis have also varied substantially in the period. In 1998 - 2008, volatility is in the range of 26 percent, negative skewness of -0.112 and kurtosis of 4.109. Looking at the four subperiods, however, the skewness has been both positive and negative and kurtosis not as significant.

As the Black and Scholes assumes constant volatility and lognormal returns the distribution of daily returns is important for the effectiveness of option pricing and hedging. Figure 3 presents the distribution of daily returns as compared to a normal distribution with the same mean and variance. Overall, the distribution is relatively close to normal. Skewness is undistinguishable but fat tails are clearly visible in the figure. Excess kurtosis implies that the probability of extreme price movements in the underlying instrument is higher than suggested in Black and Scholes valuation. Figure 4, which shows the squared monthly returns for the period from 1998 to 2008 also supports this observation. The clustering of higher than average squared returns indicates that historically there have been periods of high volatility leading to extreme movements in price.

Figure 3 Probability Distribution of Logarithmic Daily Returns on Copper

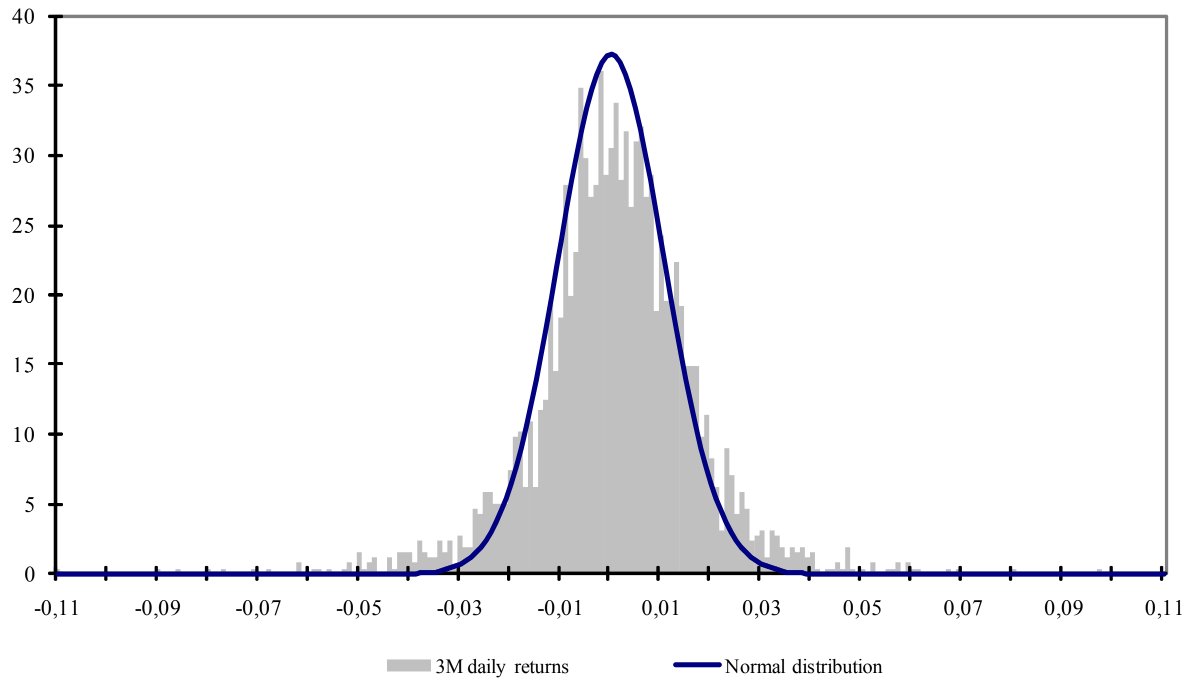


Figure 4 Squared Monthly Logarithmic Returns on 3-Month Copper Forwards

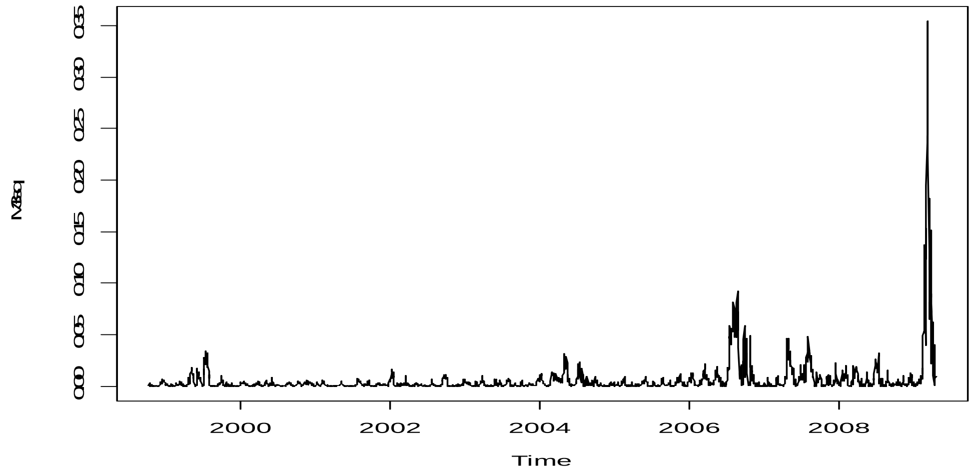


Figure 5 Autocorrelation and Partial Autocorrelation Functions of Squared Copper Returns

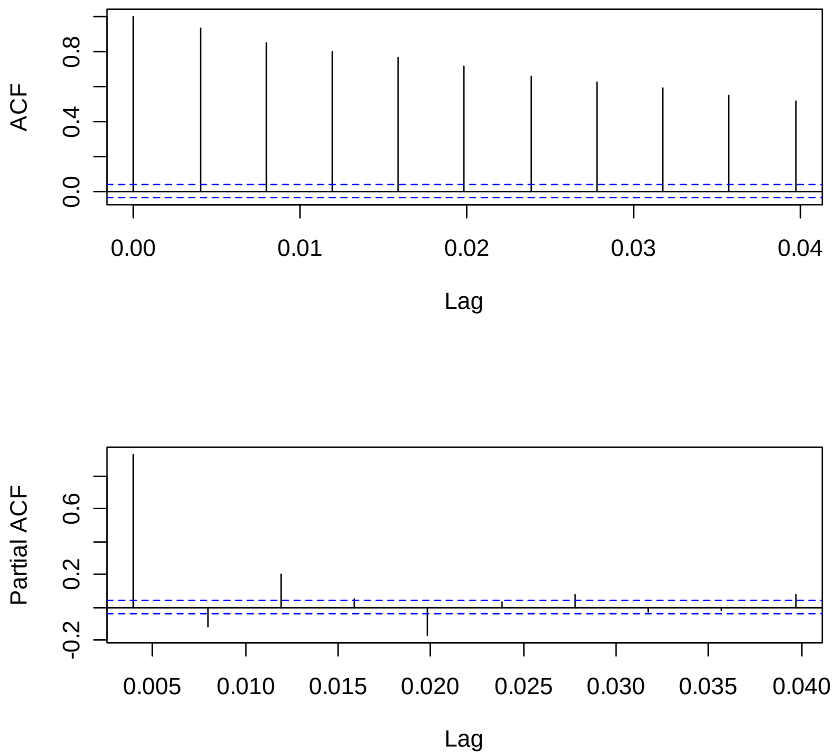


Figure 5 shows the autocorrelation and partial autocorrelation functions of squared returns. The autocorrelation function, which shows the correlation of returns up to 10 lags indicates that recent realizations of the volume of change in copper price are significant in explaining future returns. Furthermore, the partial autocorrelation function shows that the first lag makes up most of the relation. Hence, it can be concluded that the return characteristics point towards a GARCH model being well-suited to depict copper price volatility, as it relates the predicted volatility to the recent realizations as well as a long term mean level of volatility.

Finally, Figure 6 presents the volatility term structure relating the implied volatility to maturity. It is clear that the market's expectation of volatility is not constant. Rather, the volatility first rises and then starts to decline as time-to-maturity increases. Furthermore, the effect has become more profound in the latter part of the sample period.

Figure 6 Volatility Term Structure of Copper At-The-Money Options

5.2. Performance of dynamic hedging

This section presents the results from dynamic hedging scenario, referring to the case where instead of purchasing traded call options, synthetic ones are created. First, I summarize the results from the basic strategy using historical volatility. Next, I compare the performance of a more sophisticated GARCH-volatility model in dynamic hedging, as compared to arithmetic historical estimates of volatility.

5.2.1. Historical rolling average volatility

Table 3 shows the premiums for traded options and compares them to the cost of creating the same option synthetically. The premiums and hedging costs are shown both as dollars per ton and as percentages of copper price. The returns on dynamic hedging, defined as the difference between cost synthetic and traded options are sown as dollars per ton and as percentage of traded option premium. The statistics in Table 6 are calculated from daily observations and thus include overlapping data, but the T-values are calculated using Newey-West standard errors to account for autocorrelation and heteroskedasticity.

The returns to dynamic hedging are positive for all option maturities, with readjustment periods of both one day and one week. The average return ranges from 7.8 % for 1-month options and with daily readjustment to 17.6 % for 9-month options and weekly readjustment. The positive returns from dynamic hedging is statistically significant. Only the options with the longest maturities, namely, 9 and 12 months have low t-values, but even the returns from these maturities are significant with the weekly readjustment. The returns from weekly readjustment are in fact higher for all maturities. However, in line with theory, the standard deviation of the hedging cost is also higher for weekly readjustment period. The standard deviations of option premiums and hedging costs, which range from 1.3 % to 3.2 % of copper price, are higher for option premiums than for hedging costs. The only exceptions are the 1-month option where the standard deviation of the traded options is lower, and the 2-month option where the standard deviation of the cost of daily dynamic hedging is the lowest. In 2-month options the standard deviation of traded option premiums comes next and the highest is the standard deviation of the cost of weekly readjusted dynamic hedging.

Table 3 Premiums, Hedging Costs and Returns on Dynamic Hedging

Table 6 shows the descriptive statistics of traded at-the-money option premiums for maturities of 1-3, 6, 9 and 12 months in 2000-2008 as well as the cost of dynamic hedging for the same maturities and time period. All figures are shown as dollars per ton as well as percentages of copper price on trade date. Hedging cost is shown for readjustment periods of one day and one week. Table also shows the mean returns on hedging calculated as percentages of traded option premiums as well as dollars per ton. The statistics are calculated from daily, partly overlapping periods and thus the standard error of mean differences and t-values are calculated with the Newey-West modification to account for the resulting autocorrelation and heteroskedasticity.

Maturity			Average	Median	Standard deviation	Min	Max	# of Contracts	Return	N-W Standard error	T-value
1 Month	Premium of traded option		134,49	82,03	126,19	8,20	704,27	2069			
			3,01 %	3,03 %	1,33 %	0,56 %	10,57 %				
	Hedging cost	Daily	119,56	70,89	110,30	-2,69	689,72		14,93	4,38	3,411***
			2,77 %	2,65 %	1,39 %	-0,17 %	17,34 %		7,80 %	4,58 %	1,702*
		Weekly	109,57	61,36	112,83	-8,78	839,16		19,66	6,03	3,259**
			2,60 %	2,42 %	1,50 %	-0,57 %	11,89 %		11,18 %	5,35 %	2,091*
2 Months	Premium of traded option		188,57	116,02	173,56	19,18	850,47	1988			
			4,19 %	4,16 %	1,56 %	1,34 %	10,03 %				
	Hedging cost	Daily	161,53	96,24	144,94	10,14	754,56		27,04	5,01	5,396***
			3,69 %	3,57 %	1,44 %	0,61 %	9,77 %		11,75 %	4,83 %	2,433*
		Weekly	148,64	88,26	139,86	-0,56	945,07		33,53	8,07	4,156***
			3,57 %	3,38 %	1,63 %	-0,04 %	13,41 %		13,75 %	6,76 %	2,032*
3 Months	Premium of traded option		223,33	133,64	209,66	25,23	1015,29	2070			
			5,04 %	4,90 %	1,85 %	1,74 %	12,05 %				
	Hedging cost	Daily	184,72	109,97	166,18	0,00	782,81		38,61	8,33	4,637***
			4,36 %	4,19 %	1,56 %	0,00 %	10,83 %		13,52 %	4,27 %	3,169**
		Weekly	179,00	105,94	165,21	7,64	943,45		44,33	8,00	5,541***
			4,26 %	3,93 %	1,75 %	0,49 %	12,29 %		15,50 %	6,20 %	2,500*
6 Months	Premium of traded option		300,20	172,69	288,25	46,36	1265,40	1976			
			7,07 %	6,51 %	2,51 %	3,19 %	15,68 %				
	Hedging cost	Daily	241,96	149,82	220,86	41,65	754,97		58,24	15,08	3,863***
			5,95 %	5,52 %	1,94 %	2,37 %	12,36 %		15,77 %	6,50 %	2,426*
		Weekly	233,68	138,07	213,48	25,45	1013,15		66,52	9,35	7,118***
			5,81 %	5,39 %	2,00 %	1,44 %	14,00 %		17,81 %	6,21 %	2,870**
9 Months	Premium of traded option		332,96	167,57	322,50	65,37	1345,02	485			
			8,32 %	7,45 %	2,88 %	4,46 %	18,18 %				
	Hedging cost	Daily	274,73	146,30	254,31	61,84	922,84		58,23	20,43	2,850**
			7,10 %	6,38 %	2,24 %	3,65 %	13,96 %		14,74 %	11,16 %	1,321
		Weekly	262,15	147,73	240,69	41,51	1014,52		70,82	13,20	5,363***
			6,86 %	6,35 %	2,28 %	2,35 %	16,05 %		17,58 %	8,76 %	2,008*
12 Months	Premium of traded option		353,44	163,25	351,48	72,80	1440,40	462			
			9,32 %	8,30 %	3,28 %	5,02 %	20,57 %				
	Hedging cost	Daily	287,84	152,80	257,92	77,65	980,73		65,60	30,10	2,179*
			8,01 %	7,60 %	2,17 %	4,44 %	15,31 %		14,05 %	12,28 %	1,144
		Weekly	280,68	156,47	248,71	66,16	1127,13		72,75	14,77	4,925***
			7,87 %	7,51 %	2,22 %	3,76 %	16,54 %		15,57 %	8,49 %	1,834'

*/**/*** denote significance levels of 0.05, 0.01 and 0.001, respectively

Table 4 lays out the returns from delta hedging using non-overlapping data. The results are significantly positive in this case as well, with the exception of the 12-month options. The returns are in the same range than with overlapping data, but somewhat higher for 2-month and 9-month options and lower for 12-month and 6-month options. For 1-month options the results are very similar, and for 3-month options the daily readjustment provides higher return with the non-overlapping data. For the weekly readjustment, however, the situation is reversed. It can be concluded that the returns on dynamic hedging are robust in the sense that they are similar and significant with both overlapping and non-overlapping data.

Table 4 Return on Dynamic Hedging Using Non-overlapping Data

Table 5 shows the mean returns on delta hedging with non-overlapping data. The statistics show no autocorrelation. All returns and standard errors are shown as dollars per ton as well as percentages of traded option premium, and for readjustment periods of one day and one week.

Maturity	Readjustment period	Mean Return	Standard error	T-value	# of Contracts
1 Month	Daily	14,29	4,89	2,921**	101
		3,55 %	3,68 %	0,963	
	Weekly	16,20	6,71	2,413*	
		9,27 %	4,36 %	2,413**	
2 Months	Daily	31,60	10,49	3,012**	48
		14,25 %	2,88 %	4,945***	
	Weekly	43,64	13,68	3,189**	
		16,75 %	3,58 %	4,677***	
3 Months	Daily	41,15	17,48	2,353*	35
		12,66 %	3,34 %	3,790**	
	Weekly	37,52	16,25	2,308*	
		13,27 %	4,30 %	3,087**	
6 Months	Daily	46,65	24,58	1,897'	17
		12,00 %	3,39 %	3,542**	
	Weekly	50,14	26,93	1,862'	
		16,07 %	4,33 %	3,714**	
9 Months	Daily	75,49	30,27	2,493*	13
		15,89 %	5,19 %	3,060**	
	Weekly	90,02	37,69	3,137**	
		19,58 %	4,80 %	4,075***	
12 Months	Daily	54,55	40,11	1,36	8
		6,14 %	8,28 %	0,742	
	Weekly	65,83	47,28	1,392	
		8,56 %	8,92 %	0,960	

*/**/*** denote significance levels of 0.05, 0.01 and 0.001, respectively

The cumulative return from dynamic hedging over the observance period is depicted in Figure 7 as dollars per ton, and the percentage return for individual, non-overlapping periods in Figure 8. Figures are shown for 3-month options but the graphs look similar for all maturities. From 2000 through 2008, the dynamic hedging strategy would have produced cumulative returns of approximately 80,000 dollars per ton for 3-month options. Clearly, most of the return derives from the period post to 2004, although looking at Figure 8, percentage returns of over 30 % have been collected even in 2001. Observations where the returns is negative can be detected, but the positive returns easily absorb the negative. Thus, returns from dynamic hedging are positive.

Figure 7 Cumulative Return on Dynamic Hedging of 3-Month Options ($/ton)

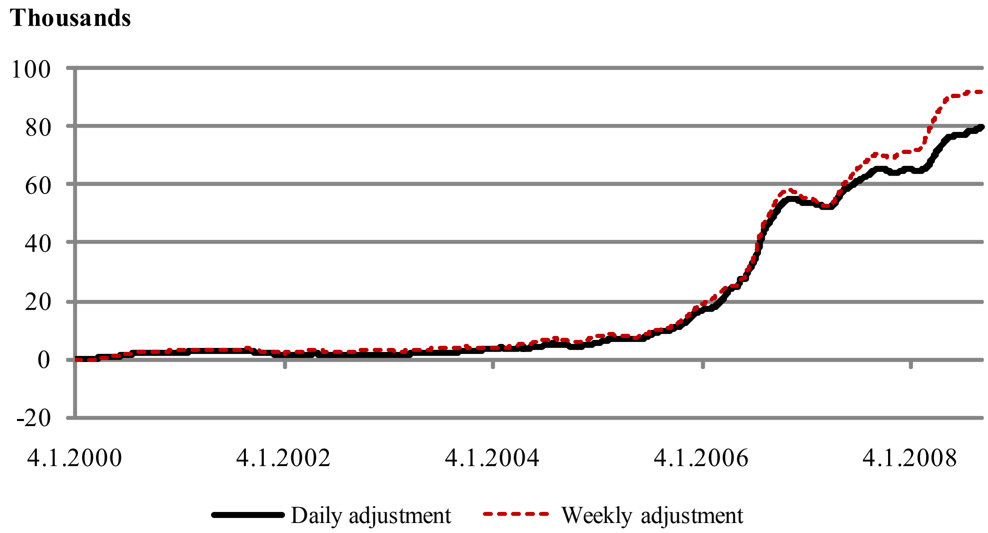
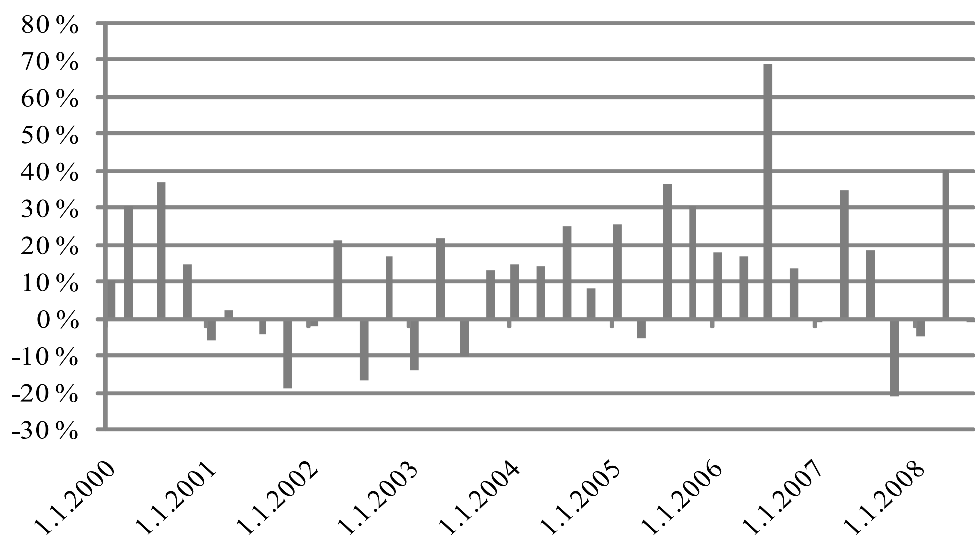


Figure 8 Returns on Dynamic Hedging of 3-Month Options



As the returns on dynamic hedging seem positive and statistically significant, the first hypothesis must be rejected. As found in various studies concerning the equity markets (see, for example, Bakshi and Kapadia, 2003), delta hedging of copper options produces returns not compatible with the Black and Scholes model. To account for the important violation of non-constant volatility, in the next section I study the effect of GARCH volatility modelling on the efficiency of hedging, before moving on to the multifactor models measuring the returns against risk factors.

5.2.2. GARCH volatility

This section presents the effect to the returns of dynamic hedging when GARCH (1,1)-volatility model is used instead of simple historical rolling average. If the returns on delta hedging in the previous section are explained by a volatility risk premium, and the GARCH model spans the dynamics of copper futures well, dynamic hedging with GARCH deltas should reduce the risk of hedging as measured by the standard deviation of the costs.

Table 6 shows the simulated GARCH parameters for each year as presented in Equation 11. As the model was only tested out-of-sample to provide practical results in terms of hedging and to avoid the problem of overfitting, each year's parameters are estimated from the previous year's price data. The ϖ-term signifies the annualized long-term volatility and ranges from 17 to 42 percent. Somewhat surprisingly, the estimate for long-term volatility in 2000 is as high as 30 percent whereas that of 2006 is only 24 percent. In all, however, the level of volatility seems reasonable. α-coefficient determines the weight of the previous error term and β-coefficient the weight of previous local volatility. The alphas are all highly significant whereas the betas are not in any of the observation years. This suggests that the model could possibly be enhanced further.

Table 5 Estimated GARCH Parameters

Table shows the estimated GARCH parameters used in simulations for years 2000-2008. Note that as the parameters are tested out-of-sample, previous year's data has been used for each year. ϖ depicts the level of (annual) long-term volatility, α is the coefficient related to previous error term and β is the coefficient for previous estimate of volatility. T-statistics of all coefficients are shown below in parenthesis.

	ω	α	β
2000	0,300	13,975	0,066
	(5,276)***	(4,376)***	(0,004)
2001	0,173	9,289	6,050
	(4,023)***	(3,519)***	(1,037)
2002	0,227	14,115	5,124
	(3,477)***	(5,382)***	(0,836)
2003	0,234	14,489	2,261
	(3,022)**	(5,673)***	(0,6266)
2004	0,264	14,489	0,000
	(3,396)***	(4,042)***	(0,001)
2005	0,282	13,854	6,691
	(4,050)***	(5,582)***	(1,844)'
2006	0,237	14,013	6,520
	(3,745)***	(5,363)***	(1,267)
2007	0,398	13,584	6,413
	(4,071)***	(5,498)***	(0,795)
2008	0,428	14,302	5,320
	(2,748)***	(5,594)***	(0,4718)

*/**/*** denote significance levels of 0,05; 0,01 and 0,001, respectively

Table 6 and Table 7 present the return on GARCH delta hedging and the difference in the hedging cost to the constant-volatility Black and Scholes. The returns remain positive and significant for the 1-month options observed. With daily rebalancing the return is on average 9.25 percent, and with weekly it is in the same range at 9.71 percent. As for the improvement to the constant, historical volatility, the results are not so explicit. The difference in mean return is not significant. In fact, for daily and weekly readjustment intervals the return difference is of opposite sign. This is in line with previous studies where it has been found that more sophisticated models, while improving the pricing performance, do not significantly improve the dynamic performance of Black and Scholes (Duan and Zhang, 2001). As for the reduction in hedging risk as measured by the standard error of return, the results are slightly more promising. Amid the introduction of the GARCH-volatility, a reduction of 1 percent point is observed for both readjustment frequencies, which implies a relative reduction of approximately 20 percent. Thus, it can be concluded that while the GARCH model does not improve returns on hedging, it may reduce their deviation.

Table 6 Differences to the Black and Scholes Hedging Cost

Table 6 presents the differences between returns on Black and Scholes delta hedging and GARCH delta hedging. Mean differences along with differences in standard errors are shown in percentage points of the respective hedging returns and standard errors for both daily and weekly readjustment intervals. T-statistics for the mean differences are shown below in parenthesis.

Readjustment frequency	Mean Difference	Difference in Standard Error	# of Contracts
Daily	1,45 %	-0,99 %	104
	(0,803)		
Weekly	-1,48 %	-0,99 %	
	(1,005)		

*/**/*** denote significance levels of 0.05, 0.01 and 0.001, respectively

Table 7 Returns on Delta Hedging using GARCH Volatility

Table 7 presents the returns on delta hedging with simulated GARCH deltas. Returns are shown as percentages of traded option premium for maturity of one month, for hedging readjusting frequencies of one day and one week.

Readjustment frequency	Return	Standard error	T-value
Daily	9,25 %	3,59 %	2,58*
Weekly	9,71 %	4,36 %	2,23*

*/**/*** denote significance levels of 0.05, 0.01 and 0.001, respectively

5.3. Performance of Black and Scholes in upward- and downward trending markets

Table 8 shows the results from regressing the delta hedging returns on copper price movement during the option's maturity (Equation 12). This is done to see if the Black and Scholes delta is biased due to the formula's assumption of constant volatility. As noted earlier, copper volatility is stochastic and positively correlated with price due to the scarcity effect. Therefore, Black and Scholes hedge ratios may be systematically bearish and hedging returns in downward trending markets should create higher returns to the hedger. In the regressions, this would be indicated by a negative coefficient relating copper returns and hedging returns.

The empirical results do not imply a significant bias in the Black and Scholes hedge ratio. Out of the coefficients in the twelve regressions five are positive and seven negative. The only significant negative coefficient is that of the two month option with daily readjustment period. However, as the coefficient for the same maturity with weekly readjustment period is insignificant and in fact positive, it can be concluded that the bias is negligible.

The alphas in the regressions indicate the abnormal return on delta hedging. The returns are not materially affected by the inclusion of copper return in the equation. Similarly to the results in the previous section, returns are positive for all maturities and readjustment periods, and significant for all maturities with the exception of the one-month options and daily readjustment period and the twelve-month options. However, there are only eight observations in the regression for the twelve-month options, which most likely explains the lack of significance of the alpha.

In conclusion, the third hyopthesis cannot be rejected. The Black and Scholes hedge ratio does not appear to exhibit any systematic bias relating to copper price movements. Furthermore, the abnormal returns in general remain positive and significant even as the copper returns are included in the return equation.

Table 8 Regression of hedging returns on copper price returns

This table presents the results from regressing the returns on delta hedging on simultaneous returns on the price of copper. The statistics are calculated from non-overlapping periods and show no autocorrelation. Results are shown for all maturities and for readjustment periods of one day and one week.

Maturity	**Readjustment frequency**	**α**	β_{Ret}	R^2	**# of Contracts**
1 Month	Daily	0,0241	0,8002	0,0253	101
		(0,6466)	(1,6015)		
	Weekly	0,0962	-0,2477	0,0017	101
		(2,159)*	(0,4143)		
2 Month	Daily	0,6325	-8,0009	0,2377	49
		(2,045)*	(3,828)***		
	Weekly	0,1639	0,0106	0,0000	49
		(4,568)***	(0,044)		
3 Month	Daily	0,1236	0,1795	0,0302	35
		(3,690)***	(1,013)		
	Weekly	0,1310	0,1024	0,0059	35
		(2,996)**	(0,443)		
6 Month	Daily	0,1326	-0,1762	0,0591	17
		(3,649)**	(0,970)		
	Weekly	0,1796	-0,2644	0,0817	17
		(3,918)**	(1,155)		
9 Month	Daily	0,1594	-0,0072	0,0001	13
		(2,847)*	(0,037)		
	Weekly	0,1955	0,0034	0,0000	13
		(3,775)**	(0,019)		
12 Month	Daily	0,0701	-0,0525	0,0020	8
		(0,587)	(0,110)		
	Weekly	0,1242	-0,2315	0,0338	8
		(0,981)	(0,458)		

*/**/*** denote significance levels of 0.05, 0.01 and 0.001, respectively

5.4. Multifactor models

This section presents the results from the multifactor model. The multifactor models are used to test whether the returns from dynamic hedging are compensation for systematic market risk. First, I test the basic multifactor model by Fama and French (1993). Second, the multifactor model modified for commodity markets is examined.

5.4.1. Basic multifactor model

Table 10 presents the results from the basic multifactor model for all option maturities (Equation 13). The alpha, which measures the abnormal return is positive for all option maturities and significant for maturities 2, 3, 6 and 9 months. With the exception of the 3-month option which is significantly correlated at 5 % level with the SMB and HML factors, the returns do not seem to bear significant relation to the market risk factors. In addition, the betas vary in sign over different maturities, which seems counterintuitive considering the underlying instrument is the same for all contracts.

Furthermore, the model has very low explanatory power over the returns from hedging. The 12-month option has the highest R squared (0.396). However, considering that the model has three explanatory variables and that the 12-month option has the least observations, only eight, this observation is not likely to be caused by the model truly fitting the returns better.

In conclusion, the basic multifactor model provides does not support the notion of dynamic hedging returns being compensation for market risks. The alphas remain positive and significant in most cases and the risk factors bear little relation to the returns.

Table 9 Results from the Basic Fama-French Multifactor Model

Table presents the results from the original multifactor model for dynamic hedging of copper options of different maturities. The abnormal return is α, β_M, β_{SMB} and β_{HML} measure the correlation with the returns of the stock market, small company premium and high book-to-market premium. T-statistics are presented below in parenthesis.

	Maturity					
	1 Month	**2 Month**	**3 Month**	**6 Month**	**9 Month**	**12 Month**
α	0,044	0,135	0,137	0,107	0,16	0,056
	(1,112)	(4,123)***	(3,889)***	(2,522)*	(1,966)'	(0,460)
β_M	-0,614	0,582	1,235	0,023	-0,492	0,697
	(-0,606)	(0,946)	(2,69)*	(0,051)	(-1,109)	(1,136)
β_{SMB}	-1,001	0,16	-1,651	-0,246	0,129	-1,266
	(-0,947)	(0,300)	(-2,541)*	(-0,471)	(0,127)	(-1,508)
β_{HML}	-0,502	0,312	0,681	0,365	-0,205	0,561
	(-0,382)	(0,480	(0,554)	(0,964)	(-0,304)	(0,798)
R^2	0,015	0,024	0,254	0,095	0,134	0,396
# of observations	101	48	35	17	13	8

'/*/**/*** denote significance levels of 0,1; 0,05; 0,01 and 0,001, respectively

5.4.2. Commodity multifactor model

As seen in Table 11, the results from the multifactor model modified for the commodity markets (Equation 14) shows results similar to the basic multifactor model. The alphas are positive and significant for options with maturities 2, 3, 6 and 9 months. The alpha of the 12-month option is slightly negative, but not significant and as the degrees of freedom for this maturity is very low, cannot be considered a reliable estimate of the true alpha.

The betas vary in sign for different maturities, and are not significantly related. The explanatory power of the model is very low, ranging from 0.024 to 0.326. Furthermore, the model, although modified to better represent the risk in commodity markets, does not seem to provide better explanation for the dynamic hedging return than does the basic Fama-French multifactor model.

Table 10 Results from the Commodity Multifactor Model

Table presents the results from the multifactor model (modified for the commodity market) for dynamic hedging of copper options of different maturities. The abnormal return is α, β_S, β_B and β_S measure the correlation with the returns of the stock market, the bond market and the commodity market. T-statistics are presented below in parenthesis.

	Maturity					
	1 Month	**2 Month**	**3 Month**	**6 Month**	**9 Month**	**12 Month**
α	0,041	0,132	0,126	0,103	0,145	-0,02
	(1,081)	(4,300)***	(3,495)**	(2,279)*	(1,903)'	(-0,190)
β_S	-0,725	0,657	0,386	-0,145	-0,483	1,237
	(-0,807)	(1,062)	(0,862)	(-0,329)	(-0,839)	(1,243)
β_B	-3,551	0,114	-1,424	0,019	-0,225	6,298
	(-1,348)	(0,077)	(-0,933)	(0,016)	(-0,072)	(1,239)
β_C	-0,088	0,391	0,175	0,197	0,073	-0,185
	(-0,147)	(1,073)	(0,512)	(0,677)	(0,145)	(-0,370)
R^2	0,024	0,044	0,083	0,048	0,127	0,326
# of observations	101	48	35	17	13	8

'/*/**/*** denote significance levels of 0,1; 0,05; 0,01 and 0,001, respectively

Since the returns cannot be explained by the common risk factors, and options with maturities of 2, 3, 6 and 9 months all provide significant abnormal returns, the second hypothesis can be rejected. As mentioned by Doran and Ronn (2008), options are often purchased as hedges against significant market movements and thus, the buyers are willing to pay a higher price for this "insurance premium". The fact that dynamic hedging return is positive supports this notion, especially if the buyers of the option deem the extreme movement risk insufficiently incorporated in any of the risk factors of the mulitifactor models.

5.5. Main findings

The findings of the empirical part regarding dynamic hedging of copper options provide evidence of large violations of the Black and Scholes option pricing model in copper the option market. The first part, which assesses the redundancy of copper options and the efficiency of their pricing by comparing dynamic hedging cost to the premium paid on the traded option, produces significant results. The return on dynamic hedging is positive and statistically significant. Although most of the cumulative profit over the observance period between 2000-2008 is obtained during the latter part of the sample, the net gains are positive in the first part as well.

Since copper return distribution have in the past been found to experience high kurtosis (see for example Bracker and Smith, 1999) and all assets are widely recognized to exhibit stochastic volatility, in the second section a GARCH volatility model is compared to the historical volatility in hedging efficiency. The returns on hedging with GARCH deltas are positive and significantly different from zero but they do not significantly differ from the returns obtained using constant-volatility Black and Scholes-deltas. The deviation of returns is, however, reduced somewhat.

To account for the possibility of some form of systematic bias in the hedge ratios of the Black and Scholes caused by the assumption of constant volatility, in the third section the hedging returns are also regressed on copper price returns during the option's maturity. The results do not indicate a significant bias as correlations between hedging returns and copper returns are mixed in sign and in general not siginificant. Furthermore, the abnormal returns as indicated by alphas in the regressions remain positive and significant.

Finally, the returns on dynamic hedging are compared with risk measures in the multifactor model by Fama and French (1993). The returns on delta hedging remain statistically significant for most maturities and bear little relation to the risk factors. Thus, the empirical findings provide support to the profitability of creating synthetic options in the copper market even after accounting for market risk.

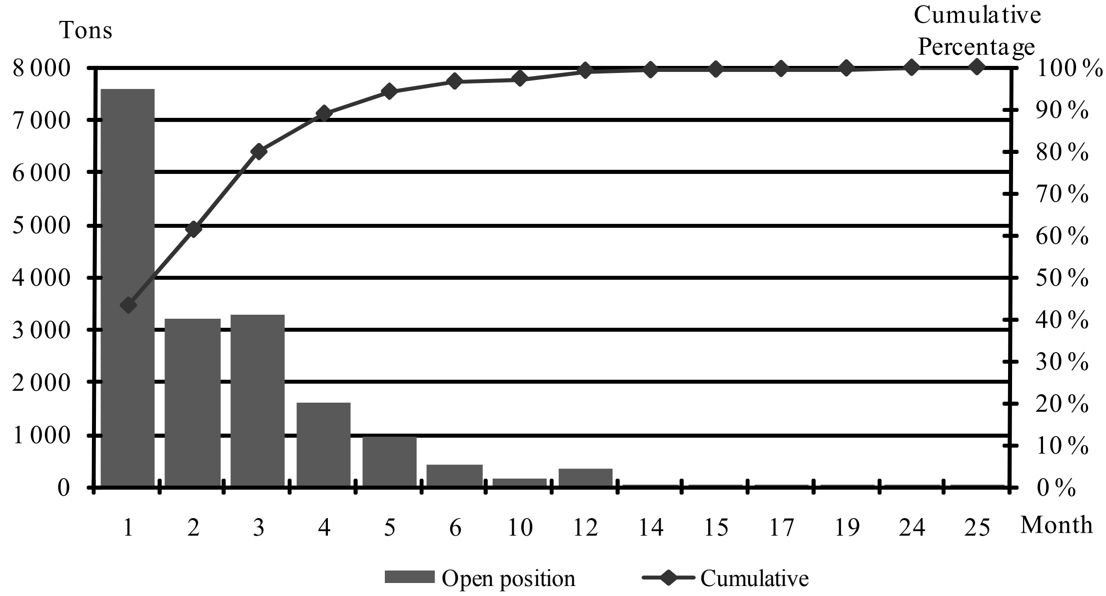
6. Risk management with options

The previous chapters have provided evidence of substancial gains to dynamic hedging of copper options. However, the question of gains to be made from using option strategies in price risk management has thus far been left unanswered. As noted, options allow the hedger to gain from beneficial price movements while protecting it from harmful developments in the price. The hedger must naturally pay a price for this protection in the form of the option premium. To assess the benefits of options to the case company, the first part of the following chapter examines the profitability of replacing a fixed part of the case company's long position in copper forwards with more flexible options. The second part furher examines a strategy in which forwards are replaced by options only when the term structure of copper forwards shows a contango. This so called momentum option strategy thus tries to exploit predictability of returns on copper as indicated by previous research.

6.1. Basic option strategy

The historical maturity structure of the case company's forward position is depicted below in Figure 9. A large part, approximately 40 percent of all contracts are for maturity of 1 month or less. Maturities from 2 to 6 months represent another 40 percent of contracts, and the last 20 percent are for maturitites from 7 to 12 months.

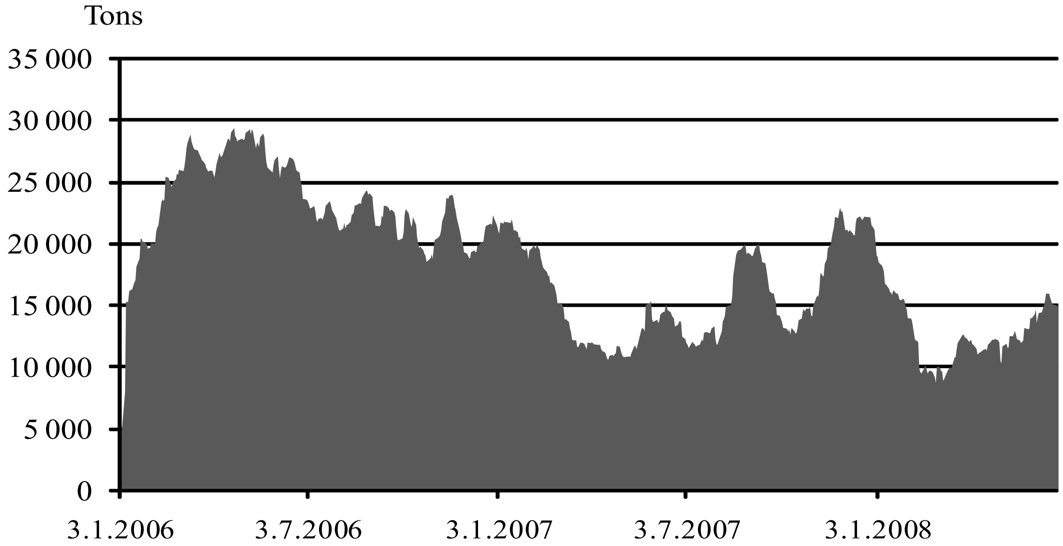
Figure 9 Maturity Structure of Case Company's Open Derivative Positions



Option allocation of the basic option strategy should represent the maturity structure of the case company's forward positions as in Figure 10. However, as it is more unlikely for the price movements to be large in a short period of one month or less, the forwards with a short maturity do not create as large a risk as longer maturities in either respect, for liquidity or high pricing. On the other hand, the forwards with a maturity of over 6 months can be seen as the most risky. Therefore, the option portfolio can be contsructed so that it includes options of each maturity (1 through 3 months, 6 months, 9 months and 12 months) equally.

The total amount of options in this strategy is stable through time. The net long forward position of the case company, as seen in Figure 11, has varied substantially over time. As the objective is to hedge only part of the long position with options, it is reasonable to choose the minimum long position of 10,000 tons to be the option tonnage. This 10,000 tons is then divided equally for all maturities chosen.

Figure 10 Case Company's Net Long Copper Position 2006-2009



The gain or loss from the basic option strategy is calculated as compared to entering into corresponding forward contracts at the initiation. It is not compared to a no-hedge scenario since restraining from hedging price risk altogether would present too grave an uncertainty of cash flows for the company. The gain or loss is defined as follows:

$$\pi_i = MAX(F_i - S_i - w_i ; F_i - C - w_i) \qquad (15)$$

$$\pi = \pi_1 + \pi_2 + \pi_3 + \pi_6 + \pi_9 + \pi_{12} \qquad (16)$$

where

π_i =*Profit from option of maturity i vs. forward*

F_i = *Forward price for maturity i on trade date*

S_i = *Strike price for at-the-money option of maturity i*

w_i = *Premium for option of maturity i*

C = Cash price of copper at maturity

With regards to the proft formula, possible scenarios are several:

(1) The cash price of copper at maturity is higher than the forward price, in which case forward strategy is more profitable than the option strategy unless the difference between strike price and forward price favours option strategy (strike price is lower than forward price) and the difference is higher than the option premium.

(2) The cash price at maturity is lower than the forward price, but the difference between forward and cash price is smaller than the option premium and hence the forward strategy is more profitable.

(3) The cash price at maturity is lower than the forward (and strike) price and the difference between the prices is greater than the option premium, making the option strategy is more profitable.

Table 12 includes the mean and median returns, standard deviations as well as the ranges for returns from buying at-the-money call options instead of entering into forward contracts (Equations 15 and 16). The statistics are presented for all maturities separately, as well as for the whole portfolio. Returns are reported as dollars per ton as well as percentages of copper spot price on trade date. This results in slightly different results due to the effect of the substantially higher prices of both copper forwards and options in the latter part of the sample swamping the earlier observations in dollar terms. The nominal returns are still reported in order to easily obtain cumulative measures of the performance. The observation period ranges from 2000 through 2008 and daily observations have been used to calculate the returns. However, to calculate the significance level of the mean returns, non-overlapping data has been utilized to ensure that autocorrelation does not violate the validity of the results.

Table 11 Return on Basic Option Strategy

Total return is calculated as dollars per ton and as percentage of copper cash price on trade date. Returns are calculated assuming even distribution for all maturities, and returns are also shown for individual option maturities. Table shows statistics for daily trades, but the significance levels are calculated from data using non-overlapping observations.

Maturity		Mean	Median	Standard deviation	Min	Max	# of Contracts
1 Month	$/ton	-6,76	-38,71	298,34	-538,73	2961,40	2069
	%	-0,07	-0,14	6,40	-10,07	79,60	
2 Month	$/ton	-8,47	-59,78	454,03	-744,21	3255,21	1988
	%	0,18	-3,00	10,31	-9,46	88,32	
3 Month	$/ton	-13,8'	-68,84	544,76	-933,65	3681,71	2076
	%	0,35	-3,44	12,89	-11,81	104,29	
6 Month	$/ton	-86,11	-101,29	581,70	-1031,49	4100,04	1981
	%	-0,72	-4,60	14,18	-14,86	116,14	
9 Month	$/ton	-143,1*	-119,02	585,25	-1298,65	3995,74	1821
	%	-1,53	-4,98	14,94	-16,89	113,19	
12 Month	$/ton	-204,16'	-133,88	532,52	-1514,07	2611,47	1742
	%	-2,39	-5,10	12,84	-19,60	70,57	
TOTAL	$/ton	-31,37'	-68,38	418,10	-868,92	2975,98	2205
	%	-1,22'	-3,78	7,37	-11,45	53,58	

'/*/**/*** denote significance levels of 0.1, 0.05, 0.01, and 0.001, respectively

Average returns to the option strategy are negative in all cases, with the exception of average return in percentage terms for 2- and 3-month options (0.2 % and 0.4 %, respectively). However, even for those maturities the median return is negative. The 12-month options have the lowest relative return of -2.4 %. This is not suprising in the sense that the premiums of options with longer maturities are very high due to time value of options. The returns in general are not statistically significant as the standard deviation of the returns is very high – in percentage terms the standard deviation ranges from 6.4 percent of copper price in 1-month options to 14.9 percent in 9-month options.

The median returns are all lower than the average returns, indicating positive skewness in the return distribution. This observation is further strengthened by looking at the maximum and minimum returns for options. The lowest return in relative terms is 19.6 % of copper price whereas the highest is 116.1 %. Clearly, this is due to the nature of options - in relatively stable times the premiums paid may outweigh the benefit of obtaining optionality in fixing the price. However, as an unexpected jump in the market occurs, the optionality can be of very high value. Futhermore, the downside is bounded since the holder of the option can at most lose the premium paid.

The return for the portfolio consisting of all maturities produces an average return of -1.2 % of copper price. It is statistically significant from zero at 10-percent level. The range of return varies from -11 % to 53 % of copper price. In dollar terms the highest return is a massive 2975 dollars per ton whereas the lowest return amounts to a substantial -869 dollars per ton. Figures 11 and 12 showing the return in percentage terms and the cumulative return as dollars per ton further clarify the characteristics of the option strategy. For a large part of the observance period, the return is slightly negative, making the cumulative return fall to very low values. However, when a shock hits the market as seen in the last year of the observation period, the profit from options is materialized in substantial significance. As seen in Figure 12, the profit is nevertheless quickly reversed as option premiums rise as a countereffect to turmoil in the market. The reversal is not observable in the cumulative returns of Figure 11 as the weight of returns of the last few observations is too small to be reflected in the cumulative profits of the entire period.

It should be noted that the level of the price decline in 2008 is quite abnormal, and without this “crash” the long call option position would have produced highly negative returns. Thus, as it can be presumed that copper does not have a negative beta, if not positive, the call options have been somewhat expensive – perhaps incorporating a jump risk premium that has materialized in the end of the sample.

Figure 11 Cumulative Return from Option Strategy ($/Ton)

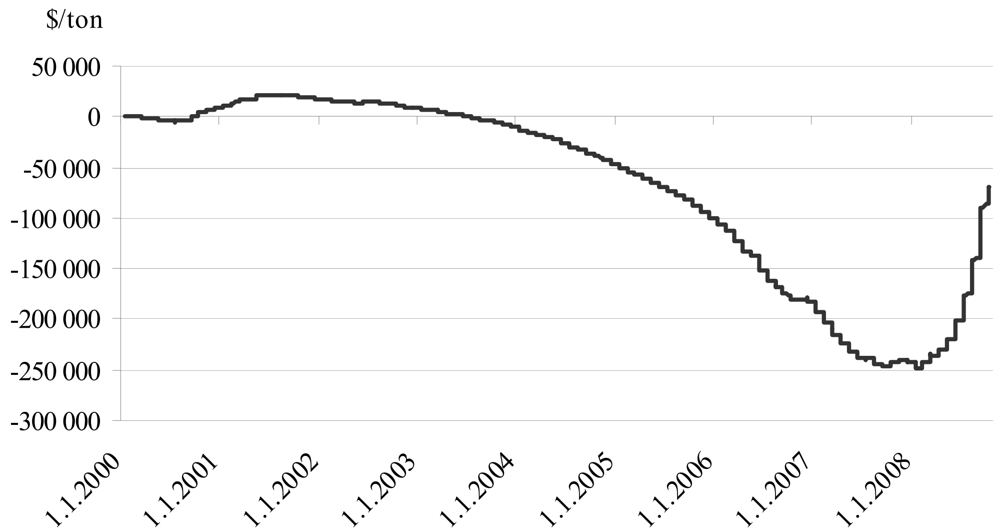
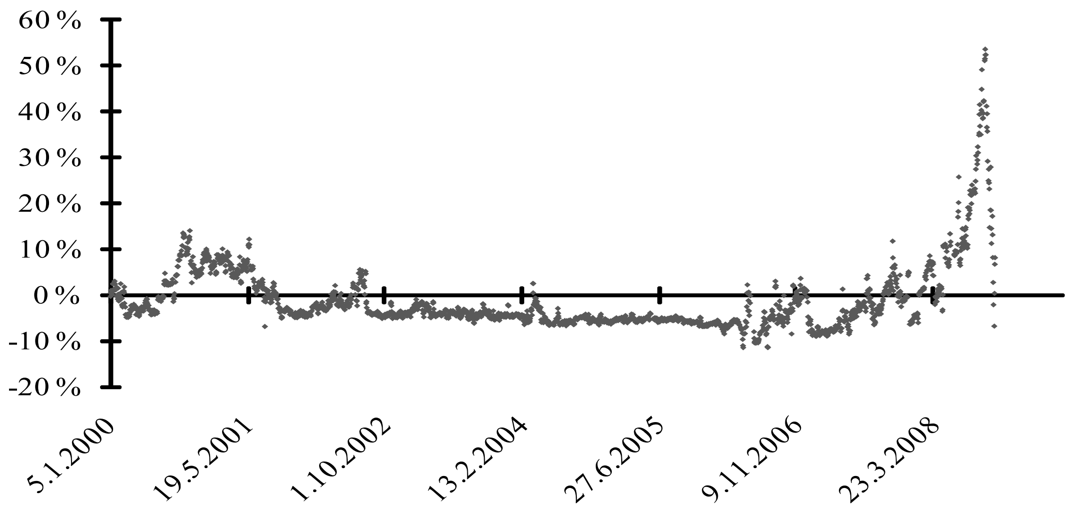


Figure 12 Return on Option Strategy (% of Copper Price on Trade Date)



The results show that while the profit from using options has been slightly negative in general, it is not statistically significant from zero. On average, the option premiums seem to outweigh the benefit obtained. I next go on to momentum options strategy to test whether the profitability can be improved with timing the option purchases according to term structure. Following many authors such as Miffre and Rallis (2006) and Rusi (2006), who have studied the returns of momentum strategies based on the term structure of commodities, I will study the improvement to the returns if options are only purchased when the term structure shows contango.

6.2. Momentum option strategy

In commodity literature, the basic term structure strategy implies that the investor enters into a long position if the commodity shows backwardation and into a short position if it shows contango. The modified momentum option strategy as presented here thus entails that in the case of contango, rather than shorting the contract (or entering into a long forward contract as the company's current forwards-only strategy suggests), an option is be bought so as not to be obliged to pay an above market price at maturity. In case of backwardation, the bet is on copper price increasing (or not decreasing) and a forward contract is entered into. This strategy is compared to the options-only strategy to account for predictability of the term structure. Return on the momentum option strategy is calculated as below:

$$\pi_i = \begin{cases} MAX(F_{i,T-t} - S_{i,T-t} - w_{i,T-t}; F_{i,T-t} - C_T - w_{i,T-t}) & , F_{i,T-t} > C_{T-t} \\ 0 & , F_{i,T-t} \leq C_{T-t} \end{cases} \qquad (17)$$

$$\pi = \pi_1 + \pi_2 + \pi_3 + \pi_6 + \pi_9 + \pi_{12} \qquad (18)$$

Table 13 presents the returns from the momentum option strategy. Results are shown for all maturities separately as well as for the entire portfolio, as dollars per ton and as percentage of copper price on trade date. In some cases, the term structure can be such that for example the one-month price is lower than the cash price but the 6-month price is higher. In such case only the options whose underlying shows a contango are included in the portfolio. The statistics are calculated from daily, partly overlapping observations but the significance levels are from non-overlapping samples.

The returns for the momentum option strategy are slightly positive (3,90 and 17,57 dollars per ton but 0,00 % of price, respectively) for one- and two-month options, but negative (from -0,00 % to -1,55 %) for other maturities as well as for the entire portfolio (-0,50 %). The median returns are all negative, but none of the returns are statistically significantly different from zero, however.

The number of observations in the momentum strategy analysis is approximately half of that of the basic option strategy, indicating that the term structure has shown contango approximately half of the time. This is also evident from Figure 13 showing the cumulative return on the momentum strategy. The longest periods of constant backwardation can be detected from the flat part of the curve as being years 2004-2006, part of 2007 and 2008. The curve shows no indication of the profitability of the momentum strategy. The cumulative return per ton for the whole period is approximately -17,000 dollars. This is less negative than the cumulative return from the basic strategy (approximately –50,000 dollars per ton) but it is more likely caused by the fact that less options are purchased in this strategy than by the profitability of the momentum strategy itself.

Table 14 shows the return difference between the momentum option strategy and the basic option strategy in more detail. The differences are shown for daily observations as well as for non-overlapping periods, but t-statistics are presented only for the latter. In nominal terms the differences all favor the momentum strategy, as the mean differences range from 3.5 dollars per ton to 155.5 dollars per ton. As percentages of copper price the mean differences vary in sign, however, ranging from –0.5 % to 0.6 %. As the standard deviation is rather high for all option maturities, the differences are not statistically significant. For the portfolio consisting of all maturities the mean difference is 0.7 % of copper price, favoring the momentum strategy. However, the mean difference for the entire portfolio is not statistically significantly different from zero. It seems that no material gains can be achieved by looking at the term structure of prices.

Table 12 Return on Momentum Option Strategy

Total return is calculated as dollars per ton and as percentage of copper cash price on trade date assuming even distribution for all maturities. However, only options where the term structure shows contango are bought. Returns are also shown for individual option maturities. Table shows statistics for daily trades, but the significance level is calculated from data using non-overlapping observations.

Maturity		Mean	Median	Standard deviation	Min	Max	# of Contracts
1 Month	Nominal ($)	3,90	-18,62	185,29	-376,04	2780,93	1084
	%	0,00	-1,08	0,04	-10,05	69,78	
2 Month	Nominal ($)	17,57	-33,34	218,83	-514,56	1431,56	918
	%	0,00	-2,05	0,05	-6,89	21,55	
3 Month	Nominal ($)	-10,26	-42,08	247,19	-547,69	1565,05	1054
	%	0,00	-2,61	0,06	-8,17	22,37	
6 Month	Nominal ($)	-32,87	-63,48	166,14	-729,78	375,09	924
	%	-0,82	-3,93	0,07	-11,40	23,50	
9 Month	Nominal ($)	-50,81	-80,40	212,18	-867,02	383,15	934
	%	-1,55	-4,92	0,08	-14,91	22,08	
12 Month	Nominal ($)	-48,65	-88,75	223,94	-985,86	462,78	912
	%	-1,75	-5,65	0,09	-14,69	25,44	
TOTAL	Nominal ($)	-14,94	-41,03	226,79	-712,72	2780,93	1112
	%	-0,50	-2,51	0,05	-10,18	50,06	

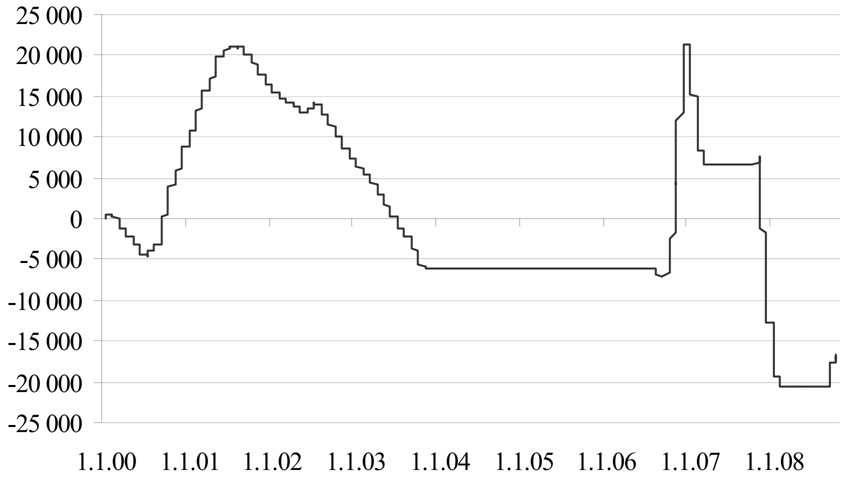
Table 13 Return Differences of Momentum and Basic Option Strategy

Table 5 shows the mean difference of returns from the two option strategies as dollars per ton and percentage points of copper price on trade date. Differences are calculated both for all daily observations and non-overlapping observations, but t-statistics (shown in parentheses) are shown only for the non-overlapping sample due to autocorrelation in the daily observations.

Maturity	**Nominal difference ($/ton)**		**Difference in %-points**	
	All	Non-overlapping	All	Non-overlapping
1 Month	10,66	-0,73	0,21	-0,16
		(-0,016)		(-0,1786)
2 Month	26,04	34,86	0,07	0,58
		(0,3467)		(0,2659)
3 Month	3,51	42,48	-0,03	1,16
		(0,5457)		(0,69)
6 Month	53,24	106,63	-0,10	2,06
		(0,7408)		(0,6811)
9 Month	92,29	161,63	-0,02	2,31
		(1,2298)		(0,5061)
12 Month	155,51	221,19	0,64	4,81
		(1,2340)		(0,7793)
ALL Maturities	16,43	83,12	0,72	2,20
		(0,8111)		(0,7672)

*/**/*** denote significance levels of 0.05, 0.01 and 0.001, respectively

Figure 13 Cumulative Return from Momentum Option Strategy ($/ton)



6.3. Main findings

This chapter has examined the profitability of using copper options instead of forwards in risk management of copper price risk. Specifically, the results relate to a copper product manufacturer with a long position in copper forwards due to customer price fixations. The empirical results are not encouraging towards favouring options in risk management. On average the strategy in which forwards are replaced by portfolio of at-the-money call options divided equally to different maturities produces slightly negative returns. The returns are not statistically significantly different from zero. A so-called momentum option strategy whereby options are only purchased when the term structure shows contango, does not significantly improve the returns, which remain negative and not significantly different from zero.

7. Conclusion

This thesis extends the growing literature on option pricing, hedging and option returns to commodity markets. Many authors have found that options strategies involving shorting of options and delta hedging produce returns unexplained by a variety of pricing models in the equity markets. Similar results are obtained in this study with respect to delta hedging returns on copper options. Two delta hedging scenarios are considered using 1) the basic Black and Scholes model with constant, historical volatility, and 2) a GARCH-model accounting for stochastic volatility. The results show no significant differences in the hedging performance of the two models, besides a slight reduction in return deviation when the GARCH-volatility is applied. This supports previous literature finding that various stochastic option pricing models rarely outperform the Black and Scholes in dynamic hedging scenarios.

As the returns on delta hedging are regressed against market betas of the Fama-French factor model, no significant relation of returns with the systematic market risks is found. In addition, the returns are not related to the returns on copper, which implies that the Black and Scholes hedge ratios exhibit no systematic bias that might contort the results.

A possible explanation for the returns lies in some risks simply unspanned by the underlying copper futures. It is left for further study to test the explanatory power of other stochastic or jump-diffusion models on the dynamic hedging returns of the copper option market. Alterrnatively, the returns may be explained by an imbalance in the demand and supply of options in the market and the incapability of perfectly hedging options. With regard to future studies, there are numerous unexplored scopes of research, not least because commodity markets have been largely neglected thus far in option literature. Especially issues such as demand pressure on commodity option markets and returns on straddles and other option strategies are yet unknown, but certainly of genuine interest to market participants.

As the scope of this paper is in the risk management of an industrial company hedging its exposure to copper price risk, it is also examined whether the flexibility of options would have outweighed their price in the past. In the equity markets it has been found that even the most risk-averse investors would optimally hold short positions in options as they are so expensive (Driessen and Maenhout, 2003). The returns on long call positions in this paper support the results by Driessen and Maenhout, being on average negative, although not statistically different from zero. This topic thus presents room for further study as well.

In conclusion, this paper provides genuinely interesting results both in terms of commodity price risk management and the pricing of commodity options, as represented by copper in the study. In particular, the large unexplained returns on dynamic hedging of copper options calls for further investigation, as the soaring option prices make the use of options in risk management at present unoptimal.

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