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The minimum biofuel requirement and the oil monopolist

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Abstract

The aim of the thesis is to describe how the minimum biofuel requirement affects the monopolistic oil markets. This is an increasingly important questions, because the governments are smoothing the biofuel's way into the fuel markets for both the economical and environmental reasons. The focus in this thesis is mainly on the minimum biofuel requirement, but at times that policy is compared also to a subsidizing policy.

To answer the research question of how the oil markets are affected by the minimum biofuel requirement, the interaction between the oil, fuel and biofuel markets was defined. From that the residual oil demand was derived, which was applied into two different cost structures, a static and a dynamic one. Solving these two cases both analytically and numerically reveals their price and quantity paths for oil, fuel and biofuel as well as the oil stock's path. The final step of the analysis was to derive how changing the minimum biofuel requirement affects these paths.

The resulting contributions from this thesis are theoretical and methodological. Numerical results are not given as no real, empirical demands, supplies and costs were used. The first contribution relates to defining the residual oil demand in case of the minimum biofuel requirement and taking time aspect into account, which forces the minimum biofuel requirement break at a certain production levels at a certain time instant. The second contribution relates to handling the production path when a monopolist faces a substitute, and that affects the price and quantity paths. Solving these paths with the static and the dynamic cost structures and analyzing the changing biofuel requirements on them is the third contribution. The final contribution of this thesis relates to presenting a subsidized biofuel case, and showing that contrary to some results in literature, the biofuel subsidy does not neccessarily postpone the oil exhaustion in linear demands.

Keywords: biofuel, monopoly, oil, resource

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1 Introduction

The fuel market is under a change. Because of both the environmental and the economical reasons governments for example in US and Europe have been pushing biofuel to the market. The governments have greatly subsidized the biofuel in order to make the oil substitute cheaper and to let it take over part of the fuel demand. Using subsidies in US already 30% of the fuel demand is served with biofuel. On the other hand, in Europe also a mandatory minimum biofuel requirement has been applied. The European Union Biofuels Directive from 2003 dictates that the nations have to require 20% of biofuel in transportation by year 2020. As biofuel has entered the fuel market and it shall be pushed there even more strongly, the biofuel is a relevant topic. I take my main focus on the European approach, the minimum biofuel requirement.

The main research question is, *how the minimum biofuel requirement affects the oil market, and furthermore the environment.* To answer this I will set up a framework, in which I assume that the fuel and biofuel markets are competitive, but the oil stock is owned by the monopolist. The monopolist has market power as she freely decides the production quantities. However, the minimum biofuel requirement affects the oil price and therefore also monopolist's optimum quantity path. As OPEC has at least some market power, and as I consider the biofuel production and the fuel distribution operating on more competitive markets, I claim that the presented framework describes aspects of the current oil and fuel markets and carries therefore relevancy with it. Using the framework I will derive various effects that the minimum biofuel requirement causes. In that I try to keep most of the mathematics in appendixes and dig into the economical issues in the main text.

While reaching the deeps of the research question, I noticed problems with the previous studies in entering substitutes. I describe some of that already here in the introduction as it affected heavily my research path, even if I have not described few of the concepts yet. To start with, Hoel [1978] and Hoel [1983] described well the basic case of a perfect substitute entering monopoly markets and taking over the market fully at some market price. However, a substitute entering gradually into the markets and finally taking over was not satisfactory described in the found references in the biofuel field. A reference of Grafton, Kompas, and Long [2010] for example studies if and how the biofuel subsidies cause a Green Paradox[1]. They miss the crucial fact that the price elasticity of the oil residual demand is not continuous. Applying the Hotelling rule as a standard procedure leads their price path to a wrong direction. The Hotelling rule is just piecewise correct,

[1] A Green Paradox relates to environmental policies meant to improve the state of the environment, which actually worsen the situation. I will discuss them more later on.

and as it turns out that the production path rests a considerable amount of time at the discontinuity. Realization of some problems in the previous studies caused me to be careful with my derivations and statements.

Throughout the thesis I assume full information. Hence, each party considered in the analysis knows the other's best choice. There are the oil monopolist, fuel distributors, the government, fuel consumers and biofuel producers that know the game of each and every of the others. No question asked that it would be interesting to widen the analysis to the direction of less information and even to disinformation, but that I have to leave to the future and keep this thesis compact.

I further assume a closed economy and a partial equilibrium. The whole globe is naturally a closed economy, but the closed economy assumption leaves few interesting questions like the distribution of wealth aside. However, I decided to concentrate on the biofuel requirement from the environmentalist's perspective and in that path the closed economy felt like the best first step. Further, with the partial equilibrium assumption I mean that the whole economy is at equilibrium at all times, but not as a result of this framework.

Results of the thesis are derived using the above assumptions. The first result is the framework for analyzing the minimum biofuel requirement and especially the residual oil demand. I consider mainly the oil monopoly, but the competitive case can be handled in a similar manner. The second result is more general and relates to how the exhaustible resource monopolist handles a substitute. I derived conditions for the time, how long the monopolist stays in the quantity of a discontinuous marginal profit, as well as the effects on the production path. Third, I solve the production and price paths for oil, fuel and biofuel in cases of the static as well as the dynamic cost structures. The solutions are given in analytical form, where possible and also evaluated numerically for illustrations. Finally, as mentioned above I point out the problems in the analysis of Grafton, Kompas, and Long [2010] and present a case where their analysis fails.

The thesis is organized as follows. Chapter 2 describes background and the most important concepts that are referred later in the analysis. Next in chapter 3, I present the fuel, biofuel and oil markets and describe the concept of the residual oil demand. In addition to the demand, the assumptions on the cost structure affect heavily on the results. I took there two approaches. First, in chapter 4 the costs are fixed and depend only on the production quantity. That means that there are both low and high cost oil till the exhaustion. Second, in chapter 5 the costs depend on the remaining oil stock and the costs increase when the stock is produced. This reflects the assumption that the low cost oil is produced first. Finally, in chapters 6 and 7 I give discussion on few remaining issues and draw conclusions, respectively.

2 Background

2.1 Oil and transportation

Initially, the focus of this thesis was on fuel in transportation. I adopted my argumentation mostly from the work of Kverndokk and Rosendahl [2010]. They give as the main reasoning for their concentration on the transportation sector that out of the produced oil the transportation consumes more than half and the trend is increasing. Also, for most of the other energy consumption needs there are clear substitutes that are available already in great quantities, but in transportation sector any substitutes are not available in quantities close to the current oil consumption. Further, Kverndokk and Rosendahl [2010] claim that policy makers prefer policies that are targeted to the transportation sectors; fuel taxes, minimum biofuel requirements and emission standards for the vehicles. I considered the reasoning valid, but do not want to press the argument to the point. Even if governments may consider it easiest to apply the policies on the transportation, their main target is to cut the total oil production. Therefore, in following text I will not refer to transportation, but to the oil production and assume that that can be considered in the similar manner as the transportation parts.

2.2 Fossil fuels, environment and exhaustible resources

The Earth's atmosphere has a stock of carbon dioxide, CO_2. Burning fossil fuels, e.g. oil, affects the CO_2 stock as that releases energy via oxidization process. In that the carbon of the fuel reacts with the oxygen from the air or from the fuel and forms CO_2 as one of the end results. Therefore, using fossil fuels does not only diminish their stock in the ground, but at the same time the CO_2 stock in the atmosphere increases. Sinn [2008] calls this the double stock-adjustment problem. Affecting for example the oil price might be in the ambitions of policy makers, just in order to increase consumers welfare. If oil would not cause any harm to the environment and it would be inexhaustible, the lower the price the better that would be. However, the increasing stock of CO_2 in atmosphere is a real problem increasing Earth's average temperature. Stern et al. [2006] call this as the *greatest and widest-ranging market failure ever seen*. As releasing CO_2 has been free, there has been no incentive to take the pollution into account. Even if the total cost of the increasing CO_2 stock is unsure, affecting the atmosphere and Earth's living conditions are reasons, why governments have nationally and internationally applied policies to decrease green house gases. Green house gases are all gases in atmosphere that affect the Earth's temperature, and CO_2 is one of them.

Which are then good environmental policies? Environmental policies might affect supply or demand of the environmental unfriendly product, or on the other hand on its environmental friendly substitute. The policy might be a technical requirement, a tax, a subsidy or any form of a rule that affects the environmentally unfriendly product usage. And it is not only today's usage that matters, but the whole production path. According to Sinn [2008] flatter production paths should be favored. That makes sense as that means that releasing of the harmful CO_2 is more evenly distributed in time. If that is enough for not reaching harmful levels of atmospheric CO_2 is however not sure.

According to Sinn [2008] *the fossil fuels are available at a given quantity in the Earth's crust as a gift of nature apart from the extraction costs.* It is great that nature has provided us a gift, is't it. On contrast to the usual fear of the oil exhaustion, Gerlagh [2011] points out that actually there is too much oil available – oil is so cheap and its stock that great the atmosphere may not be able to handle all CO_2 from oil consumption. Even if the CO_2 level decreases naturally in time due to its absorption to the Earth's biomass and oceans, Sinn [2008] argues that the decrease is very slow compared to the pace of its current increase. Adjusting the fossil fuel usage in near terms may be crucial to keep the atmospheric CO_2 stock in tolerable balance. In this there are many uncertain issues related, for example the acceptable atmospheric CO_2 level and how that affects the Earth's average temperature, but I leave those out of the scope in this context.

All fossil fuels are exhaustible resources. They are exhaustible as, cost aside, they can be depleted till there is no left and new resources are not created in any sensible time scale. And the exhaustible resources are of a special kind. Their owner must think all the time the whole production period till the exhaustion. The owner's crucial question is the timing the production. Producing a part of the resource today leaves less of her resource for tomorrow. Hotelling [1931] gave the major contribution on exhaustible resources. I go through some of his main arguments and results later on in 2.4, but there are also further development in the field since.

One development that is important to this analysis is the concept of the Green Paradox. That is a fail in applying a policy, which is meant to be environmentally friendly. The concept was as such introduced by Sinn [2008]. I, however, use the definitions given by Gerlagh [2011], as he distinguishes between the weak and strong Green Paradoxes. The weak Green Paradox arises when the applied policy increases the current environmental damage. In the context of this thesis that means that the current oil consumption would increase as the damage here is just the CO_2 absorption to the atmosphere. The strong Green Paradox is created by a policy that increases the aggregate cumulative damages. Here I want to be quite careful. I will explain later on that changing the minimum biofuel requirement does not change the total amount of the oil stock that shall be produced over the whole production period. Hence, in any case equal amount of CO_2 is released to the atmosphere. However, the timing may change. One possibility is then assume that all of the atmospheric CO_2 remains there. In that case the measure of the cumulative damage would be the cumulatively consumed stock of oil. There are two difficulties in this approach. The first one is that CO_2 is absorbed naturally to the biomass and oceans. In this study I can not take a strong position if the absorption is remarkable or not compared to the oil production time scale. The second one is that in some cases the exhaustion does not occur, but the production lasts for ever. In those cases comparing the cumulative effects is even more difficult, and the CO_2 absorption from the atmosphere to the Earth's crust occurs for sure at least at some level.

The environmental aspect is not the main focus in this thesis even if it is one of the main motivations. That aspect just arrives through the assumption that the governments want to apply policies on the oil or fuel consumption due to environmental reasons.

2.3 Subsidy or biofuel requirement

Rajagopal and Zilberman [2007] states that the main two methods especially in US and Europe to boost biofuel usage have been subsidies and minimum biofuel requirements. Subsidies to the biofuel are given often in a form of reducing the fuel taxes for biofuels. They have been in use in US and in many European countries. On the other hand, there are also efforts to control the fuel contents directly. In the European Union Biofuels Directive from 2003 dictates that the nations have to require certain percentages of biofuel compared to their fuel usage in transportation. This is a proportional minimum biofuel requirement that is in the main focus in this thesis. There are also other kinds of biofuel requirements, for example a basic quantity requirement, but I touch them only in few places in this study. Therefore, when referring to the proportional biofuel requirement, I leave the proportional usually out just to shorten the discussion.

There are no proper studies combining the minimum proportional biofuel requirement and the dynamics of the oil monopolist. Grafton, Kompas, and Long [2010] study subsidies and in that their results should relate to mine, but there are crucial differences and also true problems. They study at which conditions biofuel subsidies will create a Green Paradox. To start with the merits of their framework, the residual oil demand approach is quite similar to that I decided to choose. The dynamics of the residual demands are naturally different due to different biofuel policies in our works, and I will explain that in details in chapter 3.5. In their article, Grafton, Kompas, and Long [2010] blame the biofuel subsidies and list two reasons. First, subsidies are not first-best choice environmentally as a carbon (or generally greenhouse-gas) tax or a cap-and-trade scheme should give a preferred result. The tax would be the Pigouvian solution. Second, the subsidies increase food prices. These are important issues, but I have to leave them aside.

The downside in the work of Grafton, Kompas, and Long [2010] is that their approach to the monopoly or cartel case lacks one of the most crucial ingredients; the discontinuous marginal revenue. The residual demand functional form is not differentiable at the oil quantity when the substitute turns competitive. The reason is that the residual demand has different derivatives when arriving from greater or lesser quantity to the appropriate policy limit. This causes a mandatory price limiting phase, during which the quantity and price do not change. Grafton, Kompas, and Long [2010] does not handle this properly and that shadows their work throughout. That is also a reason why I try to be very careful with the phase changing, when a policy turns to binding or non-binding. In any case, the discontinuous revenue has a real influence on price and quantity paths.

2.4 Hotelling's heritage

The sole groundwork on the theory of nonrenewable resources was introduced by Hotelling [1931]. Most of the analysis in this theses leans heavily on his work (and to some additions later on by for example Krautkraemer [1998]). Therefore, I consider it valuable to replicate here some of his thinking in order to be able to reflect that to the current case. The starting point is the resource monopolist's optimization problem. She wants to maximize her discounted profit over the whole production period.

Consider the maximization problem without substitutes. The profit of the monopolist is the revenue minus the costs, where she may choose the quantity as she pleases. The whole profit will not be available now, when she defines the production quantities for the whole production period, but she anticipates that they arrive for sure when the production occurs. That is I do not include uncertainty in this respect into the analysis. Furthermore, because there is a time difference between the decision phase and the receiving of the profit, I introduce the discounting factor into the maximization function. I consider here only a constant rate of the required interest and the exponential discounting. Hence, the problem is

$$\max_{x(t)} \int_0^T \left(p\left(x\right)x - C\left(x\right)\right)e^{-rt}dt. \tag{1}$$

Here $x\left(t\right)$ is the oil quantity path to be selected, $p(x)$ its price at the given quantity and $C(x)$ are the quantity dependent costs. The production lasts is T, and the discount factor is e^{-rt}, where the discount interest rate is r. I assume here that the profit does not depend on the stock of the oil, but only on the quantity that is produced, and later on loosen this assumption. To solve the optimum price and quantity paths let us turn to the calculus of variation. Hence, the optimum quantity path must satisfy the Euler–Lagrange equation. Let me use a notation f for the integrand, and write its

Euler–Lagrange equation as

$$\frac{\partial f}{\partial x} - \frac{d}{dt}\frac{\partial f}{\partial \dot{x}} = 0. \tag{2}$$

As function f does not depend explicitly on x's first time derivative, the second term in equation vanishes. Hence, the equation reduces to

$$\frac{\partial f\left(x,t\right)}{\partial x} = 0. \tag{3}$$

This simple condition states that the optimal path passes through optima – the discounted marginal profit vanishes along the path. Note that f is a function the quantity and time, and that each point on the optimum path must satisfy this condition separately. Hence, to get the optimum path, at each time instant the maximum of f in respect to the quantity has to be solved. This simplifies the calculations. The path that connects these optima is the optimum path. If there would be several maxima at some time instants, the process of selection the optimal path should be considered further as the quantity path might jump between maxima paths during its course.

To help the calculations, consider of adding a constant to the original maximization problem. Adding a constant does not affect the optimization problem. I choose this constant here in a specific way. Namely, it is a constant times the stock of the oil, which can be presented as a time integral over the production period - even if I do not know the quantity path at the moment! With this selection I nail down the final stock as I explicitly claim that the whole oil stock shall be used till the last drop. In addition, I choose to multiply the stock with a constant $\bar{\lambda}$ is such a way that solving the problem becomes easier. The procedure is formally

$$\begin{aligned}
&\max\left[\int_0^T \left(p\left(x\right)x - C\left(x\right)\right)e^{-rt}dt - \bar{\lambda}S_0\right] \\
&= \max\left[\int_0^T \left(p\left(x\right)x - C\left(x\right)\right)e^{-rt}dt - \bar{\lambda}\int_0^T x dt\right] \\
&= \max\int_0^T \left[\left(p\left(x\right)x - C\left(x\right)\right)e^{-rt} - \bar{\lambda}x\right]dt
\end{aligned} \tag{4}$$

Adding the constant could not affect the result. The last line is hence equally valid maximization problem. This new problem also has to respect the Euler-Lagrange equation. Applying that again from (3) gives

$$\lambda\left(x,t\right)e^{-rt} = \bar{\lambda} \tag{5}$$

This is one form of Hotelling's rule. Here I marked unconventinally the marginal profit as $\lambda(x,t)$. Hence, the discounted marginal profit at a given time on the optimal path is constant as the Lagrange multiplier was defined in this case as a constant. This discounted marginal profit at any time instant on the optimal path is called also as the in situ value or profit. It is just the difference of the marginal revenue and the marginal cost curves at the optimum production quantity.

In case the costs are stock dependent similar analysis applies with few new features. The multiplier, λ has to be time dependent and the added constant of the form $\int_0^T \lambda(t)xdt$. Also that results the Hotelling's rule. The added integral is constant when the condition applies. I will continue on this topic in chapter 5 and select there also a special cost function.

2.5 Static solution

Kverndokk and Rosendahl [2010] analyzed as one of their subjects the biofuel requirement in static context. As in the static world time is irrelevant, they do not face the problematic discontinuities met in continuous time. Even if their work was as one of the starting points to my study, lacking the time is a huge downside concerning the biofuel requirement. Because of that the biofuel requirement they applied is not breakable. That is, they considered only such levels of demands, at which the biofuel requirement is binding. They did not consider the *minimum* biofuel requirement, but just the biofuel requirement. Contrary, I decided to take in this study also into account that at some demand levels the minimum biofuel requirement in not binding.

2.6 Efficiency requirements and taxes

In order to focus the research question, I have not introduced even some very obvious aspects of the oil market. One of these is the fact that some countries produce oil and some others do not. Reflecting this the governments take different positions for example in fuel taxes and tariffs based on their oil production, consumption and importation. Hochman, Rajagopal, and Zilberman [2010] wrote a comprehensive study on these subjects of the case, when biofuel was introduced to the markets dominated by a cartel of nations. As nations may regulate their own taxation and in practice they also do that based on their position on oil, Hochman, Rajagopal, and Zilberman [2010] argue that the cartels in oil production should be considered be formed out of nations rather than firms optimizing their profit. Recognizing the merits of the approach, I considered that the path of taking open economy aspects into consideration would widen the research too much, and left them out of this thesis.

A further analysis in the an open market was presented by Hertel, Tyner, and Birur [2010]. They analyze using a partial equilibrium model how especially US and EU biofuel mandates affect globally. Their model goes down to detail of trading and using of land, but lacks again time aspect.

3 Framework of demand and supply

In this chapter I will go through the ways and means I have modeled both the oil and fuel industry and their operators. As biofuel is the main topic of this thesis, it is added to the concept.

I assume that there is one closed economy, and hence there are no trading between countries or any taxes related. Further, the operators that need to be taken into account are consumers, fuel distributors, biofuel producers and the single oil monopoly. I consider this as the minimum setting for modeling. I shall go through all of these aspects in this chapter and finally reveal, how the oil price is affected by these assumptions.

3.1 Definitions and assumptions of fuel, oil and biofuel

Let me start from the beginning, and define here the fuel related terms that I will use.

- *Fuel* I will use fuel as term for the product that the consumers buy for commuting. It may contain both oil and biofuel in any proportions.

- *Oil* Oil refers to every oil based fuel for simplicity. This is not technically accurate and does not reveal the chemical composition of the oil based fuel, but simplifies discussion.

- *Biofuel* Biofuel is a liquid fuel of any renewable source that can be mixed with oil. It may be e.g. sugar cane based ethanol or palm oil based biodiesel or any other fuel as long as it is produced of renewable resource. This means in practice that the biofuel is based on plants harvested or animals' biomass. Hence, as plants and animals get their energy out of solar power, biofuel is solar energy reserved into them.

In order to follow the text I find it important to make the notation clear, and therefore I will use following notation.

- x refers to fuel quantity.

- q refers to oil quantity.

- s refers to biofuel quantity.

- a is the minimum proportion of biofuel that has to be sold with oil. Note that the proportion may be also higher, when the minimum biofuel requirement is not binding, and $s \geq aq$.

As the sold fuel consist purely out of oil and biofuel, and as I assume that none of them can be stocked for more that a relatively short period, the sold fuel quantity is equal to the sum of the produced oil and biofuel at any time instant, and (6) follows.

$$x(t) = q(t) + s(t) \tag{6}$$

I stated earlier that there is a minimum biofuel requirement, but that is not plugged in into (6). The reason is that the requirement sets the minimum. Hence, there are some market conditions and some part of the production path that the biofuel requirement holds as equality. Otherwise, the market parties have found better to sell more biofuel than the minimum requirement. I will come back to this time and again.

The nature of the biofuel and oil are different from the resource point of view. The only stock that I assume is the unproduced, unrefined oil stock in the ground. It is owned by the monopolist and the main question of this thesis is how shall she take it into production. On the other hand, is there a stock of biofuel? As said, I assume no greater produced biofuel stock than needed for the logistics and related to timely differences in harvesting, producing and consuming. But there is the land, in which the ingredient plants for the biofuel grows. Is that related to stocks? Land can be organized to produce material for biofuel. If the biofuel production is at a sustainable, long-run level, the production may last forever. Hence, the biofuel stock related even to the smallest piece of land is infinite even if there are some sustainability, growth and production limits related to the particular piece of land, as well as a cost curve. Further, if the long-run biofuel production level increases either through adding more land or by increasing productivity, the biofuel stock increases to a greater infinity. Such boost might arrive for example through better fertilizers. Nevertheless, I consider discussing of biofuel stock problematic, and as it does not add anything into the analysis, I consider only the current production and its limits.

3.2 Overview of markets involved

This thesis leans to the results derived from a fuel market model. I shall explain the stucture of this model throughout this chapter. The model describes how the oil, fuel and biofuel markets interact with each others mainly in cases the government dictates a mandatory biofuel requirement. To help gasping the model I give here first a short overview to the three different markets that are involved and which are referred later on.

1. *The fuel market* provides fuel for the consumers. The market parties are the consumers and the distributors, who exchange fuel to be used in commuting. The fuel market is assumed to be competitive, because there are both numerous consumers as well as distributors.

   The consumers are price takers and they buy fuel till its price meets their marginal utility. Of the aggregate marginal utilities of all consumers I assume a fuel demand, $D_{fuel}(p_{fuel})$. In derivations I assume a linear demand.

The distributors face a full price competition, and set the price at the same price as they themselves buy the fuel ingredients. Fuel may contain both oil and biofuel at various proportions. Therefore, the fuel price is the weighted average of its ingredients' prices.

2. *The biofuel market* provides biofuel to the fuel market. Market parties are the fuel distributors and the biofuel producers. The biofuel market is assumed also to be competitive as there are plenty of both distributors as well as biofuel producers. Also, each producers' product is assumed to be a substitute for the others'.

   Distributors buy the required amount of biofuel from the markets, and meet the aggregate supply curve, $S_{bio}(p_{bio})$. The other party, the biofuel producers face a demand of quantity s. As there are plenty of producers and full competition, they are willing to sell that quantity without extra profit.

   It becomes later on inevident that the biofuel demand varies in time. The reason for this is that the biofuel demand bases on the oil production level that is dictated by the oil monopolist. I assume that the biofuel producers are able and willing to fully cope with the demand. This assumption is probably not valid in real life as sovereign nations might want to protect their own production, and I discuss this later on, but keep the assumption for simplicity.

3. *The oil market* provides oil to the fuel markets. Market parties are the oil monopolist and the fuel distributors. The oil market is governed by the oil monopolist, who may choose the oil production quantity and hence, also the fuel and biofuel quantities if their proportions are fixed.

   The monopolist maximizes her profit over time. That dictates her quantity and price paths. At any time, she sells oil at such a price $p_{oil}(t)$ that fuel market just manages to clear. In this fuel market clearing the mandatory biofuel requirement and higher price of the biofuel has to be taken into account. Hence, monopolist must consider all other parties - consumers, distributors and biofuel manufacturers - when deciding the quantity and price paths.

   Distributors are price takers. Furthermore, as the monopolist releases the just quantity that sums up to fuel market clearing price and quantity, she milks all the profit and leaves none to the distributors.

3.2.1 Markets visualized

Even if the interaction between the markets involved is very simple there are few issues that need to be clarified. To make the points clearly, I will use figure 1 for representing the overall markets interaction. It has the fuel demand $D_{fuel}(p_{fuel})$ and biofuel supply $S_{bio}(p_{bio})$ plotted. They are here linear and their parameters selected only for the representation. However, the following discussion applies also to any monotonically decreasing fuel demand and increasing biofuel supply. Figure 1 shows also an oil price and

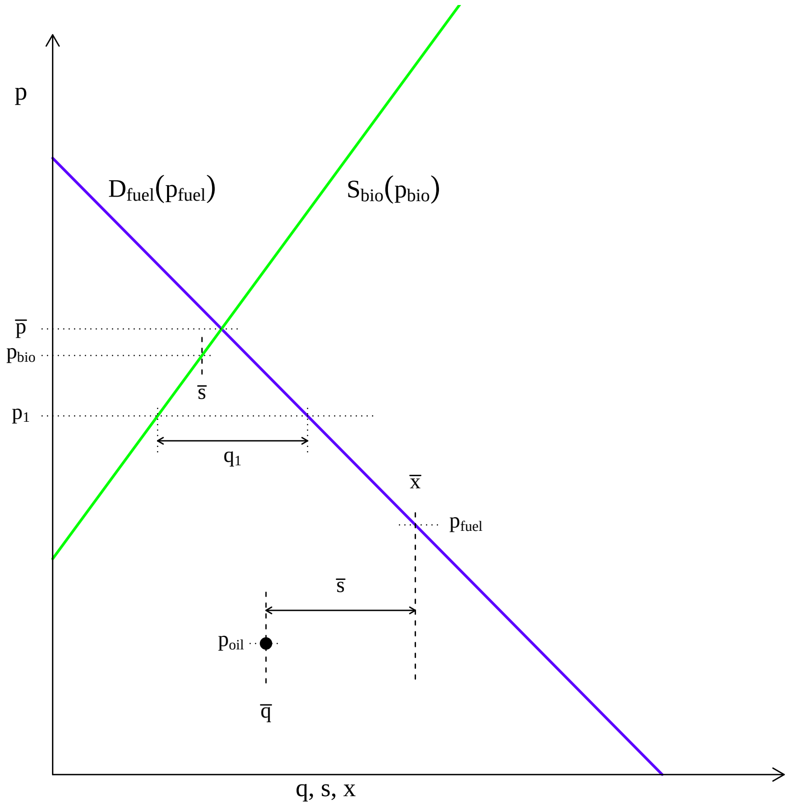


Figure 1: The markets summarized; the demand for oil and the biofuel supply

quantity pair $(p_{oil}, \bar{q})$ that clears all the markets. That results in that the whole monopoly profit arrives to the monopolist and no excess profit to the distributors or biofuel producers. At each oil price level there is one such a quantity. These make up to the residual oil demand. I will discuss that next, and define how that can be derived.

The monopolist knows that the distributors and biofuel suppliers face competition and neither of them manages to gain excess profits. Therefore, the biofuel supply remains even if the minimum biofuel requirement wears off. The distributors mediate the biofuel price through to the consumers in the proportion the sold fuel contains biofuel. If the fuel distributors need to buy a biofuel aggregate an amount of $\bar{s}$, the market price will settle to p_{bio} as shown in the figure 1.

The price of the biofuel is not necessarily the same as either the oil or the fuel prices. The government has dictated the minimum biofuel requirement just for the reason – to force biofuel to the market even if its price is higher than the oil price. Currently no or only too small amount of biofuel would be sold within fuel if no minimum requirement would be stated. To affect this, to increase biofuel proportion and most of all to decrease oil consumption, the government has given a minimum biofuel requirement. It is crucial to understand for the following analysis that this minimum biofuel requirement holds with any sold fuel quantity level, but it is not necessarily binding.

The fuel contains both oil and biofuel. I have marked in figure 1 random quantities of oil, fuel and biofuel as $\bar{q}$, $\bar{x}$ and $\bar{s}$. Even if these quantities are random, they hold always $\bar{q} + \bar{s} = \bar{x}$. That is they represent quantities that could clear the markets. With the competition that means also that the distributors aggregate profits vanish and

$$\bar{x}p_{fuel} - \bar{q}p_{oil} - \bar{s}p_{bio} = 0. \tag{7}$$

However that is not yet enough. The given minimum biofuel requirement adds a twist. If that requirement is binding or not depends on the prices. Hence, there are additional conditions depending on if the minimum biofuel requirement is binding or not. I go next these two cases separately through.

1. *The binding minimum biofuel requirement* binds the oil and biofuel prices together, but their price levels are not equal. The prices arrive through (7), which can be solved using the binding condition in quantities, $q = as$. That is

$$q \geq q_1 \tag{8a}$$

$$s = aq \tag{8b}$$

$$x = (1 + a)q \tag{8c}$$

$$\bar{x}p_{fuel} = \bar{q}p_{oil} + \bar{s}p_{bio}. \tag{8d}$$

   Here q_1 marks the quantity level, when the minimum biofuel requirement breaks. At a higher production level than q_1 the above equations hold.

2. *The non-binding minimum biofuel requirement* ties all prices equal. When there is no binding proportions, distributors would buy oil or biofuel, whichever they get at the lowest price. Distributors are indifferent between biofuel and oil in their equilibrium only if oil and biofuel have equal price. This occurs when the prices are relatively high. That is between the break-up price (p_1) and the choke price ($\bar{p}$), which are

shown in figure 1. The conditions for non-binding biofuel requirement are

$$q < q_1 \tag{9a}$$

$$x = q + s \tag{9b}$$

$$p_1 \leq p_{fuel} = p_{oil} = p_{bio} \leq \bar{p}. \tag{9c}$$

The breaking point of the minimum biofuel requirement occurs at a quantity q_1 and a price p_1. There both of the conditions (8b) and (9c) hold. By using these, q_1 and p_1 can be solved. The minimum biofuel requirement is binding with prices that are lower than p_1. The breaking point, or p_1 and q_1 are also shown in figure 1.

The oil monopolist limits her oil production, but she realizes that at each oil quantity level there are two or three market conditions that define the oil price if markets are in partial equilibrium. If she one day wants to operate at oil quantities, at which the minimum biofuel requirement is binding, selecting an oil quantity dictates both biofuel and fuel quantities. To be able to sell all produced fuel and to cover the expensive biofuel costs the distributors have to take a specific price level, and these define the maximum price the monopolist may take for the oil at that quantity level. This as the same as the conditions in (8). On the other hand, if the monopolist wants to operate at oil quantity levels at which the minimum biofuel requirement is not binding, she competes directly with biofuel. The oil and biofuel prices are therefore equal. Monopolist may select the produced oil quantity, but that defines both the prices as biofuel producers are able to meet that. These were the conditions in (9).

3.3 Fuel market and demand

The fuel market is in this context a market where the consumers buy fuel. In real life the market realizes itself e.g. at gas stations. The market is in fact geographically very distributed, but considered here as a market as I assume that the full information and a dense enough distribution network exists. The fuel market is considered as a very competitive market. Consumers can buy fuel from any distributors' gas station and fuel is distributed to the consumers in relatively small amounts compared to the worldwide consumption.

There are order of few billions decision makers making the fuel buying decision quite often, adding up as the fuel demand. Here, I take that the consumers' aggregated demand is known by all parties as full information is assumed. This demand is not assumed to change over time.

On supply side there are many fuel distributors available, out of which the consumers may choose. Even if there are naturally efforts to lock the customers to certain distributors, I ignore these. In addition, the fuel distributors may have other interest to meet the customers. For example selling other profitable products might be the approach of some gas stations, and this might affect the fuel pricing decisions. I ignore also this argument and claim for the sake of the analysis that fuel market is fully competitive.

In order to make the framework manageable, I simplify further the fuel market radically. First, I simplify fuel market by not going into different fuel varieties like gasoline, petrol, etc. I settle to discuss all oil based liquid fuels as one market, as I consider them as more or less substitutes in the long run. In this I recognize possibility for difficulties if further, especially numerical analysis shall follow, but leave that subject out of this thesis. Second, fuel is assumed to be used in small portions. Hence, as consumers buy fuel often and by small amounts they are not able to affect the current or future price.

Adding biofuel into the oil based fuel does not affect much the fuel characteristics by Kverndokk and Rosendahl [2010]. As in the beginning of 2011 the E10 fuel variety became widely available in Finland, customers' reactions revealed that some of them do expect contrary. Regardless of all this, I assume that the consumers do not care of the differences in fuel characteristics when biofuel is added at any proportion.

A very simplified representation of the assumed fuel market is given in figure 2a. I will next explain the fuel demand and supply in more details.

3.3.1 Fuel demand

It is important to realize that the consumers demand in this model for fuel, not oil. I have borrowed this point of view from Kverndokk and Rosendahl [2010], who argue that if consumers use fuel only for commuting they demand for mileage with their vehicles.

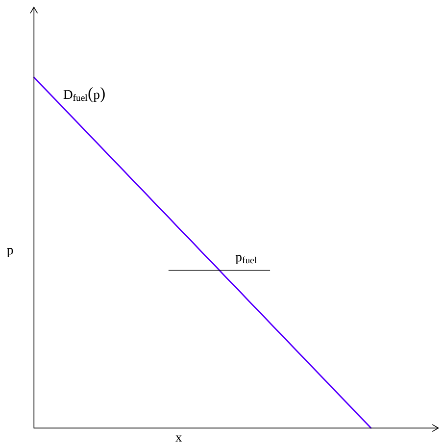
I keep the fuel demand fixed in these studies. Hence, I have to assume for example that any of the changes in any of the markets does not change the fleet of vehicles. In real life for example in U.S. there are signs that the high fuel prices are finally diminishing the demand for the oversized and high consuming vehicles.

As the demand is for fuel, introducing biofuel does not directly affect the fuel demand. The change in consumers' behavior arrives however through the fuel price. Consumers are not interested how much there is biofuel in the fuel they buy, but they are very interested of its price. Introducing the biofuel to the fuel market affects the consumers' behavior and therefore the utility only through the fuel pricing. Consumers buy more fuel till their marginal utilities are equal to the fuel price.

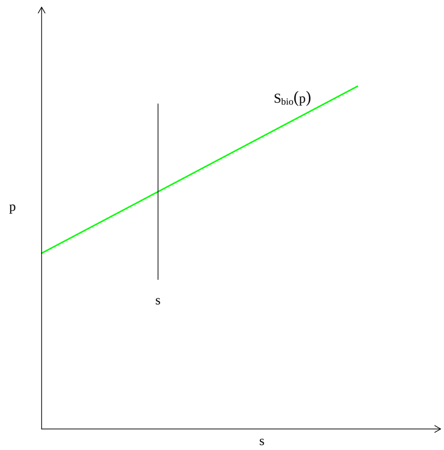
Let me take consumers' demand as an aggregated demand of all consumers within the closed economy. This assumption reveals that there is only one price as no countries are assumed, and the economy is not even divided into the oil producing and the oil consuming parts of the World. This is naturally a limited view of the World's economics. It leaves questions for example of wealth distribution effects aside, but the subject was considered too large to be included within this thesis.

There is a number of consumers. Their aggregate behavior is revealed in theory by setting their needs into order in price wise. I do not let the consumers affect the price. Hence, the consumers problem is presented in (10).

$$\max_x \left(U\left(x\right) - px\right) \tag{10}$$



(a) The fuel demand and supply



(b) The biofuel demand and supply

Figure 2: The fuel and biofuel markets

3.3.2 Fuel supply

Fuel distributors are in pressure. To see this let me consider, how their operation is affected by others. As discussed earlier, I assume here that there is a monopolist, who owns all oil based fuel. If the fuel sold to the consumers is pure oil-based, the monopolist sets the prices and quantities to maximize her profits, and the distributors mediate the price to the fuel markets. It is the monopolist who takes all profit as she sells oil to the distributor at the same price as she knows that the distributor has to sell it forward in order to get all sold. And there is nothing more to sell, so there is no reason why the price could be lower. (I ignore here all other services and products that the distributor might offer to the customers as well as I ignore the small marginal, the rent that the distributor must be able to bill to run the business, which might be the actual business.) However, I assume that distributors have incentive to sell as much fuel as she can and price competition drives her into that. This is important assumption when the constant proportion biofuel condition does not hold. If the distributor would not face the full competition, she had freedom to increase the fuel price over the oil and biofuel prices by lowering the biofuel proportion.

The entering biofuel affects the distributors, as they have to sell a different product and operate also on biofuel market. To sell an amount of fuel they have to buy certain quantities of biofuel and oil based fuel. Distributors do not themselves decide the fuel quantity even when biofuel is available assuming that biofuel is more costly than oil. It is the monopolist who decides all quantities by deciding the oil production quantity. Distributors just provide that to the market. Also the prices are just passed through the distributors. The prices of the both fuel components, biofuel and oil are non-negotiable and as fuel market is competitive, the fuel price goes down to distributors' zero profit level. Hence, the price that the distributors pay for the fuel and mediate to the consumers is the weighted average of the different fuel prices.

3.4 Biofuel market

The biofuel supply is quite different from the fuel supply. First, biofuel is not a single product. Contrary, biofuel is produced through different procedures out of different biomaterials. However, I smooth the analysis considering all biofuels totally equal in all their characteristics. Second, unlike oil producing, there are many biofuel producers and no one has ability to own the resources. Hence, biofuel market is considered here as a competitive market. Figure 2b shows a simplified presentation of that assuming a linear biofuel supply for convenience.

The biofuel demand arises here solely from the fuel markets needs. The minimum biofuel requirement states that the quantity of biofuel has to be greater that a fixed proportion of the sold oil. I assume further that it is the distributor, who buys the required biofuel, but that the oil monopolist is aware of all prices and quantities. The biofuel is assumed to be available in a stable market, which clears at all prices. The clearing price is assumed here to be affected only by the biofuel demand, and the biofuel supply is fixed. This is a heavily simplifying assumption for the sake of the analysis. In the real life there are demands for biofuel, the needed biomaterial and the land also elsewhere. Hence, the biofuel supply and demand in fact may be volatile as they depend on plethora of factors, which are not analyzed in this thesis.

3.5 Oil market

The monopolist dominates the oil market. She defines the quantities that are sold on the market and, as usual, limits the quantity and receives monopoly profit for that.

There are many uses for oil. The products and the by-products that are received in its refining process are numerous and almost all of them are valuable. In this sense concentrating on oil's contribution on the fuel seems to be very limiting. And more so, when considering that many of these oil products and by-products have various uses themselves and they may be sold within different markets. However, when considering substitutes for the liquid oil based fuel in commuting there seems to be not that many good candidates. Biofuel is naturally one, but even that is hard or impossible to produce in high quantities.

There are two major features that oil monopolist has to take into account throughout these analysis. First, there is the time horizon involved as the monopolist owns that great stock of oil that it can not be or it is not maximizing her profits to produce it all at once. She wants to maximize her profits over the whole production period. Second, the introduction of biofuel affects the demand she experiences. For this reason, I have to introduce the residual demand for oil. I describe that next and come back to the timing problem in next chapters.

3.5.1 Residual demand for oil

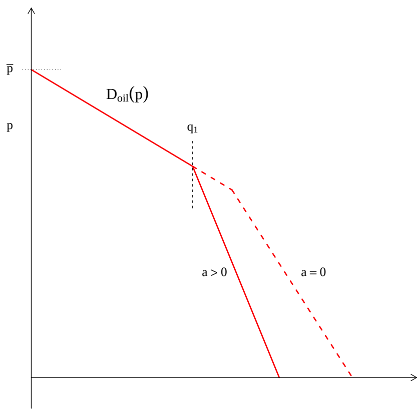
Next I will discuss the model for the oil demand and form the residual demand for oil modifying the approach of Grafton et al. [2010]. Their case was to study, how the biofuel subsidies affect on the length of the oil production. However, note that the subsidizing policy influences the lowest demands of oil. I will show later that this low demand will occur at the end of the production. Contrary to that, the minimum biofuel requirements affects only the highest demands – or at the beginning of the production. I discuss these differences of the policies and also a bit of their combined effects later on.

The key into understanding the oil demand is to realize the fact that when there is a biofuel requirement the fuel demand is fulfilled by oil and biofuel

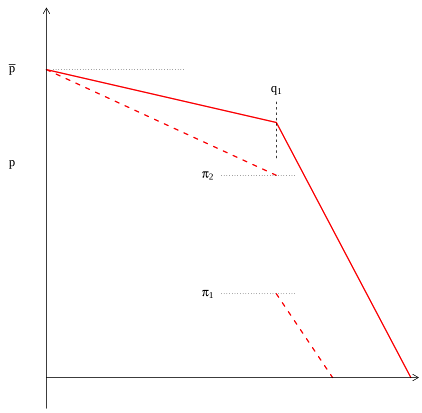
$$\begin{aligned} D_{oil}\left(p_{oil}\right) &= D_{fuel}\left(p_{fuel}\right) - D_{bio}\left(p_{bio}\right) \\ &= D_{fuel}\left(p_{fuel}\right) - S_{bio}\left(p_{bio}\right). \end{aligned} \tag{11}$$

Here the biofuel market clearing is assumed at price p_{bio}, and S_{bio} is the biofuel market supply that meets the demand. What is here to note, is that the demand for the oil is derived from both the fuel demand and the biofuel supply. It is the fuel demand that arrives through the customers demand, and part of that is served by the biofuel supply. The rest is the residual oil demand. Also notable is that the prices of oil, biofuel and fuel are not equal. I will discuss that next.

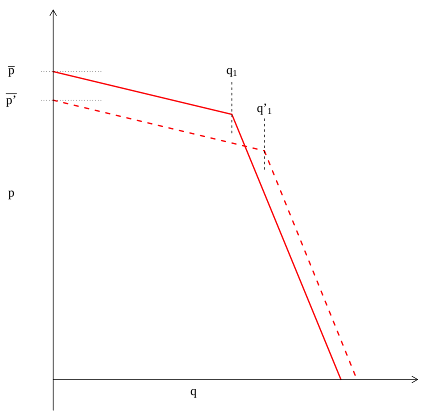
Figure 3 represent the residual oil demands in four cases to clear the concept. In 3a there is the residual oil demand with and without the minimum biofuel requirement. The marginal revenue is introduced in subfigure 3b.



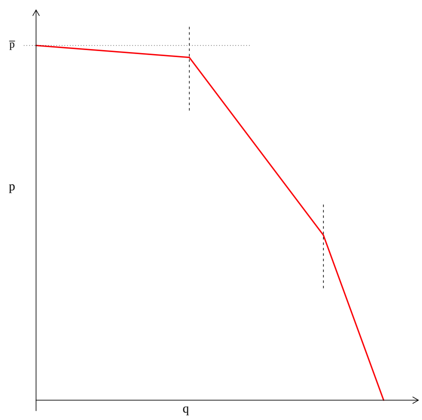
(a) The oil residual demand with (solid) and without (dashed) a minimum biofuel requirement.



(b) The oil residual demand (solid) and the marginal revenue(dashed) when a minimum biofuel requirement applies.



(c) The oil residual demand without (solid) and with a subsidy (dashed) when a minimum biofuel requirement applies.



(d) The oil residual demand with a fixed minimum biofuel quantity in addition to the minimum proportional requirement.

Figure 3: The oil demand in changing biofuel requirements and in changing subsidizing

Subfigure 3c shows the residual oil demand with and without a biofuel subsidy. Finally, subfigure 3d represent the residual oil demand when there is a fixed biofuel quantity requirement in addition to the minimum proportional biofuel requirement.

3.5.2 Price of oil

How does the monopolist set the oil price? When the biofuel is introduced, the monopolist knows that her product, oil is only one ingredient of the final product that is sold to the consumers. Hence, the oil price's connection to the consumers demand has loosened a bit.

Let me start from the related fuel market. The fuel price is the competitive market solution. Hence, each of the distributors make their own decisions of the proportions of fuel constitutes and gains zero profit by assumption. This condition ties the oil price to the fuel market equilibrium. If in this equilibrium the fuel distributor does not make profit, the revenue she gains from the selling of fuel is equal to the costs of buying the oil and biofuel,

$$xp_{fuel} = qp_{oil} + sp_{bio}. \tag{12}$$

Using (11) and (12) the oil price and oil demand are

$$p_{oil}\left(q\right) = \frac{x}{q}p_{fuel}\left(x\right) - \frac{s}{q}p_{bio}\left(s\right) \tag{13a}$$

$$D_{oil}\left(p_{oil}\right) = D_{fuel}\left(\frac{q}{x}p_{oil} + \frac{s}{x}p_{bio}\right) - S_{bio}\left(p_{bio}\right). \tag{13b}$$

As stated earlier, there is a minimum proportion of biofuel that has to be sold with a quantity of oil. When the requirement is binding, the equations reduce to

$$x = (1+a)q \tag{14a}$$

$$p_{oil} = (1+a)p_{fuel} - ap_{bio}, \tag{14b}$$

where the functional forms of the prices are from the relevant inverse demand and supplies. Selecting one of the quantities, the rest of them are given by the first equation and the second equation states the oil price. On the other hand, when the minimum biofuel requirement does not hold similar equations read

$$x = q + s \tag{15a}$$

$$p_{oil} = p_{bio}. \tag{15b}$$

Selecting a quantity of fuel or biofuel defines the other's quantity by the latter equation. It also states the oil price, and the first equation gives the oil quantity.

3.5.3 Breaking the minimum biofuel requirement

The biofuel requirements may break off from the minimum when the oil quantity drops. I shall show now how this occurs.

Hotelling [1931] showed in case of the exhaustible resource monopoly that the price is increasing and the quantity is decreasing on the optimal path. If the biofuel quantity is fixed in proportion to the oil quantity, decreasing oil quantity requires decreasing biofuel quantity. But the competitive biofuel market sells the lesser amount of biofuel at a lower price. Hence, the price of biofuel is decreasing in time.

If the increasing oil and the decreasing biofuel price meet at some fuel demand, the fuel price is also equal to the oil and biofuel prices. And further, that common price level is the last time instant at which the minimum biofuel requirement binds. If it held further, the biofuel price would decrease under the increasing oil price as both quantities would still decrease. As distributors face price competition they would cut prices and buy more biofuel leading to the breaking of the minimum biofuel requirement. This leads to price equality in the all markets. The prices of fuel, biofuel and oil are the same, when the biofuel requirement does not hold at minimum level.

After the minimum biofuel requirement breaks, the prices keep on increasing. The reason lies in that the monopolist still has market power and she wants to continue in the path of decreasing quantities. But as the supply of biofuel is increasing in price, the competitive fuel and biofuel markets ensure that the biofuel quantity starts to increase after the break. The fuel supply decreases in time with or without the break due to the monopolist's actions, and hence, the oil quantity decreases now at greater pace. The proportion of biofuel in fuel market increases rapidly until it fills alone the market at the choke price.

What happens to the monopolist market power? As discussed, the monopolist has total control of the market in the first phase. She selects the oil quantities so that the distributors mix it with a fixed proportion of biofuel, and just manage to sell it all with no profit at the fuel market. On the other hand, after the break there is a substitute or competing product in the fuel market, the biofuel. However, biofuel is sold at a competitive biofuel market, and the biofuel producers cannot control its price. Therefore the oil monopolist manages to hold market power. She just takes into account that the biofuel affects the fuel market in a different, but still in a predictable way. She realizes also that at some price, namely at choke price, biofuel served all fuel demand. However, there is a fuel demand range in which she still may operate. Biofuel serves there an increasing proportion of the fuel demand, but it may be a very appealing range if the monopolist costs are low with small quantities.

What if there is dominant players also in other markets? Execution of market power leads to limiting supply. In fuel market there may be temporal or local monopoly power, but I assume that the competition and government's efforts in removing cohesion does not allow that grow into any remarkable scale. On the other hand, in biofuel markets some oligopoly with competitive fringe might be plausible. The efficient scale of the biofuel production may be that great and require so large areas of land, production units and logistics that some monopoly power might arise that affect the biofuel market. In that case, there would be a game between the oil monopolist and the biofuel monopolist from the start. When a binding biofuel requirement, the biofuel monopolist would not be able directly to affect the fuel quantities, but through increasing biofuel price, which in turn affects the optimal paths of the oil monopolist. After the requirements wears off, the biofuel monopolist would face either a fierce game against the oil monopolist or a collusion with her. A game against the biofuel monopolist would hurt the oil monopolist's profits, and hence she tries to prevent this by increasing the oil production in first phase. This subject seems to me very interesting, but within this thesis I do not go further in this, but assume competitive biofuel market.

3.5.4 Change in requirements and subsidies

Governments have implement policies to boost the biofuel usage. I shall discuss here how two of them, the minimum biofuel requirement alone and the biofuel subsidy under the minimum biofuel requirement, affect the residual oil demand.

Implying a minimum biofuel requirement affects differently the low and high demand phases of the oil production. To help gasping the effect see figure 3a, which presents the case in linear demands. As the requirement does not affect the prices directly in low demand where all prices are equal, the demand remains unchanged as in (15). In high oil demand, the biofuel demand is proportional to the oil quantity. At a fixed fuel price the fuel demand remains, but the proportion of biofuel increases. Hence, the price of biofuel increases. To keep the fuel market in balance the oil price must decrease. Therefore, the oil demand moves. As both price and quantity decrease, the demand moves down and left. In the special case given in figure 3a, the slope of the demand steepens. This is not yet the whole story. The breaking point changes also. An increased minimum biofuel requirement induces a smaller fuel demand at the breaking point, and a smaller oil quantity. Continuity of the demand requires that the breaking point is on the unchanged low demand curve.

How does the increase in the biofuel requirement affect the monopolist? As we saw, the requirement increase influences only the high demand. It decreases the demand starting from the new breaking point. From the monopolist point of view this can not be good, as her profit can not in any case increase. If she operates before and after the requirement change in the unchanged residual demand range, the profits are unchanged. In all other cases she will lose. What about timing then? The decreased demand curve does not encourage her to sell her oil at least in a quicker pace, but the contrary. I shall go into details of this shortly, but for that I have to make assumptions of the cost function.

As decision makers have various views and as the policies do affect differently, using multiple environmental policies at the same time might serve best in cases. Let me discuss the effects of adding a subsidy when already a minimum biofuel requirement applies. For simplicity, I go through only the case of *ad valorem* subsidies - giving a certain sum of money for each biofuel produced. According to Rajagopal and Zilberman [2007] this applies for example in U.S. and in countries in Europe. Now, in low demand the only thing changing by the subsidy in the biofuel supply. It increases as the government is paying part of total biofuel price. The low demand of oil decreases by the amount of the subsidy as shown for the linear demands in figure 3c. In high demand, at a fuel demand level in the subsidized world the biofuel is cheaper and oil more expensive than in the unsubsidized world. This is because consumers are willing to pay the same price for fuel in both cases, but the biofuel producers in competitive biofuel market are willing to sell for less as government pays as a subsidy part of the price. Hence, the subsidy flows through to the oil price and the oil price increase. Increasing subsidies without increasing biofuel requirements would increase oil demand in high demand and lower it in low. That would not be an environmental friendly solution.

3.5.5 Competitive oil market

What if also the oil market would be competitive? Even if this is not in the main scope of this thesis, it is a natural case to compare the monopolist's behavior with. The difference between the competitive and the monopolist behavior is that the monopolist holds back the production knowing that she can affect the prices. On the other hand, the competitive producers will sell at the given price.

I continue in assuming that the demands and supplies are given. In a similar manner as in the monopolist case that results in the same residual oil demand. That was not dependent on the monopoly power. Now, it is worth noting that the residual demand is continuous. That is the competitors in oil production choose the quantity paths knowing that all possible small decreases in production do not cause discrete jumps in prices. Therefore, the entering of the biofuel into the fuel markets marks just the demand change or more competition. The competitive oil producers follow the smooth price curve according to Hotelling [1931]. Hence the competitive producers do not have to wait to equalize the present values of the profits as will be the case for the monopolist and will be discussed in details later on.

The biofuel requirement changes however the demand curve also in the competitive case. If the requirement is given the competitive oil producers' quantity and price paths are straightforward to evaluate. The price path of competitive oil producers is given by the appropriate Hotelling rule. The final price is the price at the backstop. At any time instant before that the price is given by the rule. The appropriate quantity can be checked from the residual oil demand curve. That is after evaluating the appropriate price level of the date (on the y-axis), the quantity is on the x-axis. I will show next that this is not that simple for the monopolist, as she experiences a discontinuity in the marginal revenue, an important factor in her decision making.

4 Static cost structure

Having gone through explaining the framework in the previous chapter I shall now show what that implies. What have still missed from the framework are the costs. Therefore, first in this chapter I will go through the fixed or static cost model. Thereafter, in the next chapter I turn to a case of dynamic costs, namely to one that depends on the stock of remaining oil.

In the case of static costs the oil monopolist experiences the same marginal cost function throughout the whole production period. At any time instant, she has the same decision to make: how much to produce today? Does she then choose the same production quantities and get the same price at every time instant? No, she does not. Thanks to the work by Hotelling [1931], we understand that she wants an increasing marginal profit. Let me start with the maximization problem, where all is about the monopolist choosing the quantities of oil.

To make the calculation simpler let me modify the residual demand. Evaluations are easier to follow, when the marginal cost is zero by defining prices differently. (In appendixes, where the derivations are given, this presentation is turned back to the original one when necessary.) Nothing changes in the case but notation. In short, the profit in the maximization problem is unchanged, and with old (left) and new notation (right) it is

$$\int_0^T \left[qP\left(q\right) - C\left(q\right)\right] e^{-rt} dt = \int_0^T \left[qP^{\dagger}\left(q\right)\right] e^{-rt} dt. \tag{16}$$

Here $P\left(q\right)$ is the inverse demand function and $C\left(g\right)$ the costs. When the inverse demand incorporates also the costs I call it as the modified inverse demand function, $P^{\dagger}\left(q\right)$. In literature this form of notation is called sometimes net pricing. Now, $qP^{\dagger}\left(q\right)$ gives directly the net priced revenue or the profit at the production level q, and its marginal profit is $\pi^{\dagger} = \frac{\partial qP^{\dagger}(q)}{\partial q}$. This notation is used in following to simplify presentations and calculations. The value of the discounted profit with the new modified inverse demand function is just the same as before the modification, the levels and slopes of the resulting curves are just different.

4.1 Very expensive biofuel - no backstop

I start with the simplest case, where biofuel is very expensive. Let biofuel be so expensive that the fuel distributors would not add biofuel into the fuel they sell without a mandatory requirement at any given, reachable oil price level. The demand for fuel would stop if no oil would be available. The only reason for adding biofuel to the fuel is because the government had made it mandatory; no one may sell fuel without selling a given proportion of biofuel. As distributors will never sell more than the mandatory proportion of biofuel and as the government does not let anyone sell less, the governmental ruling of the minimum biofuel requirement is binding. Biofuel is sold at the same proportion starting from the day one till the exhaustion of the oil. No biofuel or nothing is sold afterwards. It is typical to say that there is no backstop as no substitute is stopping the oil consumption even if the biofuel regulation is limiting it a bit, as I will show in few moments.

4.1.1 Dividing production over time

With the given assumptions the monopolist has quite the same problem as Hotelling solved in his days. The problem is to find the quantity path that maximizes monopolist's discounted profits. Contrary to the case of Hotelling [1931], the price is here determined from the competitive conditions in the fuel markets. And especially the biofuel regulations affect the solution.

There are few details that needs to be clarified. How does the time affect the quantities or prices? Why does the monopolist not choose the same quantity at each time instants if nothing changes? How long does the production last? At which price and quantity levels to start with?

The first question to answer is how to handle the time. Time may be handled using different methods. First, most optimization problems can be solved by the calculus of variations. It hands out the Euler–Lagrange equation of motion, which is a differential equation that is valid on the optimal path. Second, it turns out that this case is solvable also using basic Lagrange multiplier method. That is at each time instant optimization is performed separately. The path is the trajectory combining these optima at varying time instants. This was the method Hotelling used in his famous paper in 1931. Third, in post-Hotelling era researchers tend to first write down the Hotelling's rule. In this case, writing down the current marginal profit and forcing that equal to the one tomorrow discounted to today gives the rule,

$$\frac{\dot{\pi}^{\dagger}}{\pi^{\dagger}} = r. \tag{17}$$

As I use net pricing also the marginal profits are net priced.

The marginal profit is monopolist's choice parameter at each time instant. If the discounted marginal profit would differ at some time instant from the others, at that time instant the quantity should be changed. The last drop of oil on any day should not be produced if the marginal profit was less than that of discounted from any other day, but saved. At the optimum path none of the quantities should be changed at any time instant. That is when the Hotelling's rule applies. Finally, optimal control is a modern method that can be spotted in research during last few decades. It is a more formal method and bases on Hamiltonian mechanics.

The second question 'Why does the monopolist not choose the same quantity at each time instants if nothing changes' is wrong from the start. The analysis of the first question already stated how the quantity must change over the time. But there is also something else in this as there is something that changes in time, namely the stock. It is the remaining stock that dictates the current production quantity. Let me clarify this a bit. Producing something today lets you less for tomorrow. I state that even stronger; there is no memory, but the stock. What decision the monopolist did yesterday does not affect today's decision, but through the smaller stock. Keeping that in mind and starting from the end, what would monopolist do if she had just one infinitesimal unit of oil? She will maximize her profit and hence sell that at the price of just vanishing demand (I take here that demand vanishes at some price.). Monopolist knows that that is the highest marginal profit she can ever reach. If she reaches a situation where she has that infinitesimal stock, she sells it at this profit. If she had a bit greater stock, would she sell that at once if possible or divide it over time? From the discussion of the first question we already know that the monopolist wants to equalize the discounted marginal profits leading to a smooth quantity path in our continuous case. Hence, the monopolist divides the production over time even if she owns just a small stock. She also wants to cash the highest marginal profit and to follow the increasing marginal profit as stated by Hotelling's rule. And quantity produced today is such that the marginal profit at exhaustion is at its maximum and that the stock today is just exhausted at the end. That is the optimum path. This answers also the rest of questions, as they seem to be the same question. The price and quantities on day one are determined so that the optimum path can be followed given that the whole stock will be exhausted in the end.

What if the monopolist does not care about time? In other words let me assume that the interest rate is zero. She does the basis school book solution to maximize the profit today. She produces a quantity, at which the marginal profit vanishes. I will call this solution as the zero-interest solution. It is discussed and given in appendix A.

4.1.2 Solution

I shall go next through the solution for the static cost function case with no backstop. Even if the previous discussion was independent of the forms of the demands and supplies, in order to get concrete results I use in this part linear forms, given and the evaluations done in appendix A. As the derivations take lots of space and may distract from the actual matter I give them in appendix C.

The quantity path and net price paths for the optimal solution are

$$q\left(t\right)=\hat{q}\left(1-e^{-r(T-t)}\right) \tag{18a}$$

$$p\left(t\right)=\hat{p}+\left(\bar{p}-\hat{p}\right)e^{-r(T-t)} \tag{18b}$$

Here $\hat{q}$ and $\hat{p}$ refer to the quantity and the net price in the static monopoly solution, where no discounting occurs. I call this as the zero-interest monopoly solution, see appendix A. $\bar{p}$ is the exhaustion net price. The quantity path is decreasing and the quantity vanishes at exhaustion as required. This solution is familiar already from Hotellings work in 1931, as is the net price path, which is increasing and end to the net price where the demand stops. The relationship with the zero-interest solution is very close one.

Let me write down the initial and final values for quantities and net prices

$$
\begin{aligned}
q_0 &= \hat{q}\left(1 - e^{-rT}\right) = r\left(\hat{q}T - S_0\right) < \hat{q} \\
q_T &= 0 \\
p_0 &= \hat{p} + \left(\bar{p} - \hat{p}\right) e^{-rT} \\
p_T &= \bar{p}.
\end{aligned}
$$

Here the final values are not that interesting as they were given into the model by assumptions. But the initial quantity and net price reveal some interesting information. First, the quantity is never greater than $\hat{q}$, the optimal solution in the zero-interest case, at which quantity the net marginal profit goes to zero (remembering that the cost function was modified to zero). This is natural, but I underline here for the later analysis that there is no reason for the quantity exceed the zero-interest solution of given linear demand function. There is also a second piece of information. The further the exhaustion time is, the closer the initial price and quantity are to the zero-interest solution. For very far away exhaustion times their initial values reach the zero-interest solution without a limit. This has an interesting consequence I discuss next.

Any finite stock of can be exhausted optimally with any linear demand. This is a results of the fact that the quantity limit reaches without limit to the zero-interest solution. The form or parameters of the demand just define the time till exhaustion. The exhaustion time is defined as the time it takes to follow the optimal path given the initial stock. I have evaluated its condition in appendix C.2. The condition is

$$
\frac{1}{r}\left(1 - e^{-rT}\right) = -\frac{S_0}{\hat{q}} + T. \tag{19}
$$

Figure 4 presents the condition. Note that all the information of the demand and costs is in the single factor, $\hat{q}$. There is no analytical solution for the condition, but the exhaustion time can be evaluated numerically. And there is always one and single solution.

4.1.3 Discussion

No backstop case is not a very good model to explain the fuel markets. There are clearly biomaterial sources from which biofuel can be produced at

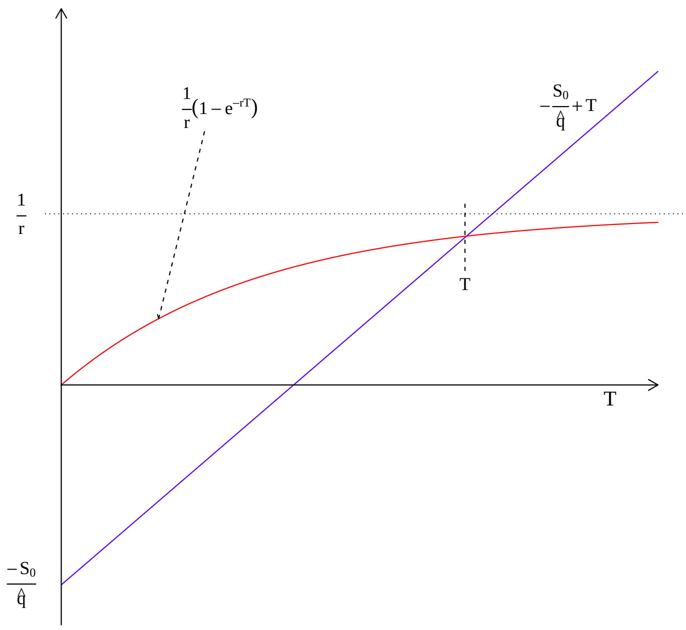


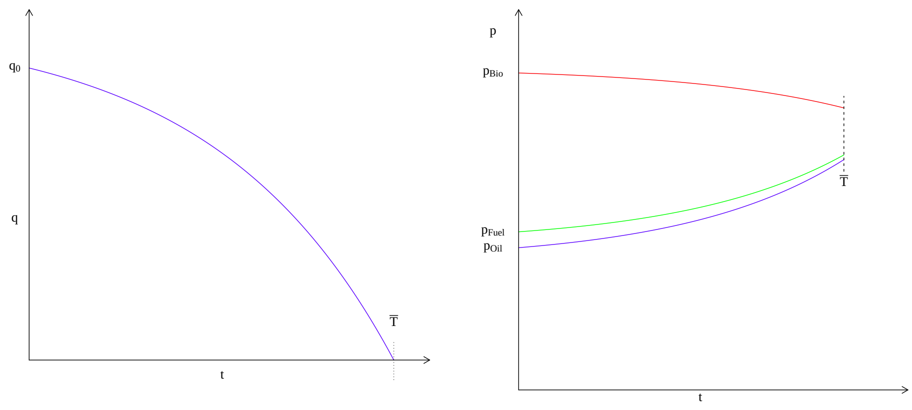
Figure 4: The condition for the exhaution time in case of static costs and no backstop.

reasonable prices. One example of these is the Brazilian ethanol production. It is competitive in parts of the fuel market and hence can not be considered as very expensive. Also, consider leftover oils or other biomass coming e.g. from food industry. They are raw materials for biofuels. In order to get rid of leftovers, and even get paid of it, the food industry should sell this raw material. It is naturally a question at what prices they sell, but I leave that aside.

4.2 Biofuel as backstop

Let me turn now to another case, in which the biofuel can serve the whole fuel demand at some price level. This will occur after some finite time period in this case. Thereafter the fuel market is fully competitive and biofuel serves the whole fuel demand. Biofuel acts there as a backstop.

At this moment of changing to pure biofuel, oil exhausts. It exhausts because the monopolist wants to sell all of her oil as it is profitable till the last drop. And the biofuel producers are not pushing her out. They are small in this framework and operate on a competitive biofuel market and hence, they do not have any means for that. On the other hand, costs are neither



(a) Quantity path (b) Price paths of oil, fuel and biofuel.

Figure 5: The quantity and price paths in case of static costs with backstop.

driving her out, as the cost curve is fixed and the marginal profit is actually increasing. The monopolist just realizes that she does not want to operate on competitive fuel market, where she has no market power. Therefore, the monopolist has willingly planned from the day one to exit the fuel market at that day at that price by exhausting her oil stock.

4.2.1 Timing when discontinuous marginal profit

As there is now a backstop, the marginal demand the monopolist faces is continuous but its derivative is not. That means that the marginal profit is discontinuous. Does that affect somehow the case? Yes, it does a great deal. Remembering Hotelling's rule, the discounted marginal profit should be equal at all times. And the rule is strong as we will see.

Due to the discontinuous marginal profit the oil production faces three phases, which I shall discuss in detail shortly. To phase down the jump in marginal profit the monopolist executes limit pricing for some time during which she sells a constant quantity. Discontinuity causes need for a further analysis also because the profits of the later phases have to be taken into account as salvage values in prior decisions. That affects the results in those prior phases' decisions. Let me show how this goes. I note here three production phases, where the final phase is the one where oil is exhausted, in the second phase the price limiting is executed and the initial phase opens the production.

To start with I write down the maximization problem of the profit explicitly for the three phases of the demand,

$$\Pi = \max_{q(t)} \left[\int_0^{t_1} P_1^{\dagger}(q)\, q e^{-rt} dt + \int_{t_1}^{t_2} p_1^{\dagger} q_1 e^{-rt} dt + \int_{t_2}^{T} P_2^{\dagger}(q) q e^{-rt} dt \right]. \quad (20)$$

Here, the first, second and third terms are the total profits from the respective phases. The time instants t_1, t_2 and t_3 refer to the ends of the initial, price limiting and final phases. The profits of the last two phases are discounted to the initial, decision time. I use here net pricing, and prior the discontinuity $P_1^{\dagger}$ (for quantities greater than q_1 in the initial phase) and after that $P_2^{\dagger}$ (for $q < q_1$). In the initial and in the price limiting phases the minimum biofuel requirement is binding and in the final phase it is not.

4.2.2 Maximization problem and timing

As typical for such problems, let me start from the final phase. There is no memory in the system, and the past path does not affect the decisions but through current values. Hence, whenever reaching a definite set of conditions the same optimum path shall be executed regardless of the history. And here that means that when a certain oil stock is reached, the final phase begins. When the final phase is entered, all the prices are equal as the requirement is not binding. And at the exhaustion they all reach the same choke price. At the same time the oil quantity vanishes and oil is exhausted. Going further with the exhaustion argument there is some maximum stock, S_3^{max} that can be produced within the optimum path in the last phase. That is the total stock produced on the path from the initial quantity of the phase, q_1 and price $p_1^{\dagger}$, till exhaustion. If the original stock is no more than S_3^{max}, the monopolist chooses to operate only in the final phase. That is the minimum biofuel requirement is never binding. If the stock is greater than S_3^{max} the final phase's monetary value enters the monopolist decision in the prior phases.

The monopolist takes the value of the last phase into account when choosing her path in the first two phases. If the initial stock is not greater than the sum of the maximal consumable stocks in the final and price limiting phases, $S_2^{max} + S_3^{max}$, the initial phase is never executed. I continue however here with the assumption that all phases are entered as limited cases can be derived from the full solution.

The optimal paths in the initial and final phases must obey the Hotelling rule. Therefore their marginal profits are increasing in time. The total profit is now simple to calculate as there is no costs in the modified residual inverse demand function. In addition, in the second phase the price limiting strategy is followed. The profits for the initial, price limiting and final phases are noted as V_1, V_2 and V_3, respectively and they are

$$\begin{aligned} V_1 &= \int_0^{t_1} P_1^{\dagger}(q)\, q e^{-rt} dt \\ &= -\frac{1}{r} p^{\dagger} q e^{-rt}\Big|_0^{t_1} + \frac{1}{r}\int_0^{t_1} \frac{\partial P_1^{\dagger} q}{\partial q}\frac{dq}{dt} e^{-rt} dt \\ &= \frac{1}{r}\left[p_0^{\dagger} q_0 - p_1^{\dagger} q_1 e^{-rt_1^\star} + \pi_1^{\dagger} q_1 e^{-rt_1^\star} - \pi_0^{\dagger} q_0\right] \end{aligned} \tag{21a}$$

$$V_2 = \frac{p_1^{\dagger} q_1}{r}\left(1 - e^{-rt_2^\star}\right) \tag{21b}$$

$$V_3 = \frac{q_1}{r}\left(p_1^{\dagger} - \pi_1^{\dagger}\right). \tag{21c}$$

Here I note the lengths of the phases as $t_1^\star$, $t_2^\star$ and $t_3^\star$. The used notation for the marginal profits is; $\pi_1^{\dagger}$ is the net marginal profit when arriving the discontinuity from above (in quantity), $\pi_2^{\dagger}$ when arriving below and $\pi_0^{\dagger}$ is the initial net marginal profit. In solving the profits I have used the optimum path as used there the Hotelling rule.

Now I am ready to tackle the case. To proceed let me write down the total profit of the whole production,

$$\begin{aligned} \Pi &= \max_{q(t)}\left[V_1 + V_2 e^{-rt_1^\star} + V_3 e^{-r\left(t_1^\star + t_2^\star\right)}\right] \\ &= \frac{q_0}{r}\left(p_0^{\dagger} - \pi_0^{\dagger}\right) + e^{-rt_1^\star}\frac{q_1}{r}\left(\pi_1^{\dagger} - \pi_2^{\dagger} e^{-rt_2^\star}\right). \end{aligned} \tag{22}$$

The last line results from plugging the profits in from (21a), (21b) and (21c). In this case I do not yet want to use any heavier mathematical machinery that is needed in the dynamic cost case, but use simple reasoning.

Assume now that the optimum path was solved for the problem. The profit is given by (22). It has however a term that depends on the initial period's duration, $e^{-rt_1^\star}$. Nevertheless, in the optimal path changing the timing should cost in profits. Delaying costs due to $e^{-rt_1^\star}$, but starting before $t_1^\star$ would not. Therefore, I claim that the second term, related to the timing has to vanish at the optimal phase timing. Profit's derivative in respect of the initial period's duration vanishes and reveals a condition for the price limiting phase

$$\frac{\partial \Pi}{\partial t_1^\star} = 0 \tag{23a}$$

$$\pi_2^{\dagger} = \pi_1^{\dagger} e^{rt_2^\star}. \tag{23b}$$

Here the discounted marginal profit follows the Hotelling rule also in the price limiting phase, or at least over it. Hence, the whole production period runs optimally when at each time period the discounted marginal profit is constant. If there is a discontinuity the production should wait till time equalizes the marginal profits.

The total profit simplifies still. When taking into account the optimal condition for the length of the price limiting phase (23b) and the total profit (22), reduces to

$$\Pi = \frac{q_0}{r} \left(p_0 - \pi_0 \right). \tag{24}$$

Even if there is a radical change in the rules of the game when the minimum biofuel requirement wears off, and even if there is the price limiting phase, the profit depends on the initial quantity, price and marginal profit in addition to the interest rate. The first three of them can be evaluated by the Hotelling rule of the discounted marginal profit.

4.2.3 Maximizing solution

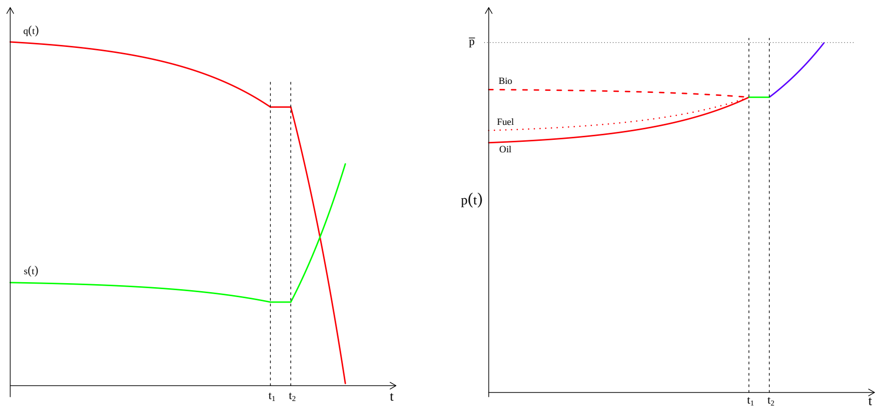
Did I found the maximizing solution? Hotelling states in his article (1931) that if the path found is the maximizing solution the forms

$$\frac{\partial F}{\partial q}, \qquad F - q\frac{\partial F}{\partial q} \tag{25}$$

must be continuous. These are also called Weierstrass-Erdmann sufficiency conditions. Neither of them is continuous in the present case, but with the help of the price limiting phase. If price limiting is put there in between, both of the Weierstrass-Erdmann conditions fulfill. Is there something wrong here? No. There is no freedom in the final phase. The monopolist starts executing the final phase from quantity of q_1 regardless of history, when the remaining stock is at certain level. Also the initial price and all final values in that phase are given by the theory. There is nothing to manipulate in that path unless changing exogenous parameters. It must be at discontinuity if anywhere to make these conditions work. Further, the first form in (25) states that the marginal profit should be continuous. As the marginal profit at any quantity in the final phase is greater than the marginal profit in the initial phase at any quantity, this continuity condition can never be satisfied, but adding a phase in between. The price limiting phase helps to dilute the difference of the marginal profits. The delay is just that long that it takes for the initial discounted marginal profit of the final phase to reach the value the initial phase ended. Their discounted marginal profits are equal.

4.2.4 Solution

The solution of the optimal path consists of the optimal paths in all three phases. They are calculated in appendix D.1 with the Euler– Lagrange equations and by cluing the solutions together with the initial and final



(a) The quantity path (b) The price paths of oil, fuel and biofuel

Figure 6: The oil quantity path and the price paths for oil, fuel and biofuel

conditions. The quantity path in case of biofuel as the backstop in the initial, price limiting and the final phases are, respectively,

$$q\left(t\right)=\left(q_1-\hat{q}_1\right)e^{-r(T-t)}+\hat{q}_1 \qquad 0<t<t_1 \tag{26a}$$

$$q\left(t\right)=q_1 \qquad t_1<t<t_2 \tag{26b}$$

$$q\left(t\right)=\hat{q}_2\left(1-e^{-r(T-t)}\right) \qquad t_2<t<T. \tag{26c}$$

The final quantity value in the initial phase shall be q_1, and the gap between q_1 and the zero-interest solution $\hat{q}_1$ is increasing exponentially in time. In the later analysis for the existence of a Green Paradox I will need also the initial value, and therefore write that down here

$$q_0=\left(q_1-\hat{q}_1\right)e^{-rT}+\hat{q}_1. \tag{27}$$

However, there is a condition for entering this initial phase altogether. If the initial phase is executed depends on whether the zero-interest monopoly solution is greater or smaller than the final quantity. If the initial quantity is greater than than the quantity at the discontinuity of the marginal revenue (or $q_1>\hat{q}$), the quantity path should be increasing. That is not optimal by Hotelling [1931]. The marginal profit would be negative in that case. The monopolist should in that case start directly from the price limiting phase. Nevertheless, I assume that there is the initial phase and that the price limiting phase follows just to smooth the discontinuity in marginal profits. And in the final phase the total stock shall be exhausted.

In my opinion, even more interesting issue are the lengths of the phases. As discussed already there are continuity requirements both for the stock, but also for the marginal profit. The lengths of the initial, price limiting and final phases are, respectively

$$\frac{1}{r}\left(1 - e^{-rt_1^\star}\right) = -\frac{S_1}{\hat{q}_1 - q_1} + \frac{\hat{q}_1}{\hat{q}_1 - q_1} t_1^\star \tag{28a}$$

$$t_2^\star = \frac{1}{r} \ln \frac{\pi_2^\dagger}{\pi_1^\dagger} \tag{28b}$$

$$t_3^\star = \frac{1}{r} \ln \frac{\hat{q}_2}{\hat{q}_2 - q_1} \tag{28c}$$

The lengths of the initial and final phases depend on the quantity and stock levels. These levels can be gathered from the continuous parts of the residual demand related to the initial and final phases. Contrary, the price limiting phase's length depend on the discontinuity of the residual demand. That was already discovered by Hoel [1983]. He got the same length of the price limiting in the case of a perfect substitute and a full backstop.

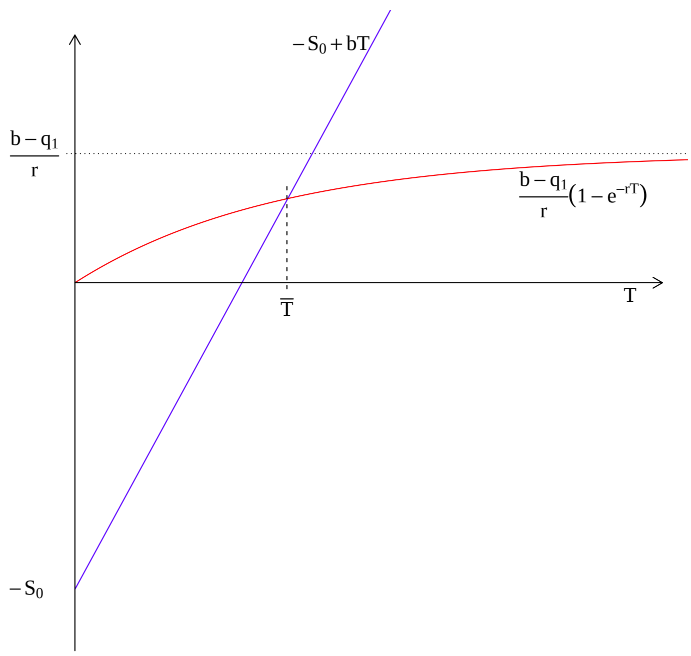


Figure 7: The exhaustion time when biofuel acts as a backstop

4.2.5 No Green Paradox

Increasing the minimum biofuel requirement shortens the final and the price limiting phases, but increases the length of the final phase. It does however not cause a Green Paradox. I show this next.

The analysis of biofuel goes hand in hand with the analysis of the residual demand of oil. Let me remind results is 3.5.4 that increasing the biofuel requirement lowers the quantity where the discontinuity of the marginal revenue occurs and also lowers the demand at low oil prices. When there are the fixed costs involved, these results do not change if the cost curve is monotonically and smoothly increasing, or constant.

Increasing the biofuel requirement leaves the marginal profit unaffected in the final phase, but changes its length. According to Hotelling's rule the paths do not change close to the exhaustion. This is because the final marginal profit is unchanged and the Hotelling's rule does not change. The Hotelling's rule dictates how the marginal profit changes in time, and nothing changes there close the exhaustion. With the unchanged marginal profit, the price and quantity paths do not change, either. But altogether the final phase lasts for a shorter period as the marginal profit has not as much room to increase.

The price limiting phase is shorter for the linear forms, because the relation of the marginal profits, $\frac{\pi_2^\dagger}{\pi_1^\dagger}$ is decreases with an increasing biofuel requirement. This results is derived in details in appendix B.2 for the linear forms of demand and costs. That is

$$\frac{\partial}{\partial a} \frac{\pi_2}{\pi_1}\bigg|_{q_1} < 0. \tag{29}$$

Hence, by (28b) the length of the price limiting phase is shorter with the increasing biofuel requirement. There is now a smaller gap in the marginal profits and it takes a shorter time to equalize that. As the second, price limiting phase is shorter, and the produced oil quantity q_1 is lower with increased biofuel requirement, also the stock produced in the second phase, S_2 decreases.

As both the second and final phases are shorter, and less oil is produced during them, how does this affect the oil usage in general? To start with, the proportion of the whole stock produced in the initial phase must increase. This does not anyhow cause Green Paradox.

To analyze this in details, let me write down the stock produced in the final phase in two ways

$$S_1 = \hat{q}t_1^\star + \frac{1}{r}(q_1 - \hat{q})\left(1 - e^{-rt_1^\star}\right) \tag{30a}$$

$$S_1 = \hat{q}t_1^\star + \frac{1}{r}(q_1 - q_0). \tag{30b}$$

In the second equation I used the initial quantity relation, (27). Now, take their partial derivative in respect of the minimum biofuel requirement

$$\frac{\partial S_1}{\partial a} = \hat{q}\frac{\partial t_1^\star}{\partial a} + t_1^\star \frac{\partial \hat{q}}{\partial a} + (q_1 - \hat{q})\, e^{-rt_1^\star} \frac{\partial t_1^\star}{\partial a} + \frac{1}{r}\left(1 - e^{-rt_1^\star}\right) \frac{\partial\,(q_1 - \hat{q})}{\partial a} \tag{31a}$$

$$\frac{\partial S_1}{\partial a} = t_1^\star \frac{\partial \hat{q}}{\partial a} + \hat{q}\frac{\partial t_1^\star}{\partial a} + \frac{1}{r}\frac{\partial q_1}{\partial a} - \frac{1}{r}\frac{\partial q_0}{\partial a} \tag{31b}$$

Next I solve from the latter one the initial quantity's change in respect of the minimum biofuel requirement, $\frac{\partial q_0}{\partial a}$. In that I need the initial phase's time change in changing requirement $\frac{\partial t_1^\star}{\partial a}$, which can be solved from the first equation. Putting those together I gather that the initial quantity decreases in the increasing biofuel requirement,

$$\begin{aligned} \frac{1}{r}\frac{\partial q_0}{\partial a} = & \frac{\frac{\partial S_1}{\partial a} - t_1^\star \frac{\partial \hat{q}}{\partial a} - \frac{1}{r}\left(1 - e^{-rt_1^\star}\right)\frac{\partial (q_1 - \hat{q})}{\partial a}}{1 + \left(1 - \frac{q_1}{\hat{q}}\right) e^{-rt_1^\star}} \\ & - \frac{\partial S_1}{\partial a} + t_1^\star \frac{\partial \hat{q}}{\partial a} + \frac{1}{r}\frac{\partial q_1}{\partial a} < 0. \end{aligned} \tag{32}$$

Here $\frac{\partial S_1}{\partial a} > 0$, $\frac{\partial q_1}{\partial a} < 0$ and $\frac{\partial \hat{q}}{\partial a} < 0$. In the first term the denominator is greater than one. Therefore its nominator's terms are out powered by the lasts three straight terms, but the partial derivative of $\hat{q}$. It turns out to be negative and hence it does not need a balancing term.

The minimum biofuel requirement is a good environmental policy, at least in case of fixed costs and linear supply and demand. It decreases the current demand and decreases the initial quantity. The monopolist serves the market for a longer time, but with smaller quantities, if a more strict biofuel requirement applied.

4.3 Fixed minimum biofuel quantity

The basic solution just presented is problematic for the biofuel producers. The biofuel quantity decreases in the initial production phase. Then it stays put in the price limiting phase and finally increases in the final phase, till oil exhausts. This is probably not a path that will occur as the regulator has opportunity to affect the path in many ways. I will discuss next shortly one additional feature that the regulator may use to protect the biofuel producers from such a heavy volatility. She may set a minimum biofuel quantity. Such a minimum biofuel quantity requirement is already in use in US according to Rajagopal and Zilberman [2007].

4.3.1 Solution

I give here a solution only to the extreme case, in which the consumption of the biofuel is initially above the long-run equilibrium. That is the relative low priced oil compensates the demand for the high priced biofuel that much that its price rises above the choke price. This case is related to the case just presented in chapter 4.2 if the oil quantities in the beginning are very high. Now, assume that the minimum biofuel quantity is set to the long-run equilibrium. The derivations for this case are given in appendix E.

Now there is only one phase as from the start the aim is at the choke price. The quantity path is

$$q = q_s \left(1 - e^{-r(T-t)}\right). \tag{33}$$

In this the steady state quantity q_s and the initial quantity q_0 are

$$q_s = \frac{\alpha - \gamma - 2\beta s_s}{2\beta + \sigma}. \tag{34a}$$

$$q_0 = q_s \left(1 - e^{-rT}\right). \tag{34b}$$

There is just the equation for the exhaustion time to complete the case, and that is

$$-\frac{S_0}{q_s} + T = \frac{1}{r}\left(1 - e^{-rT}\right). \tag{35}$$

Hence, if there is a minimum biofuel requirement as great as the long-run biofuel equilibrium, the oil monopolist behaves like in case of no backstop. I discuss that a bit more next and widen to the case of lower minimum requirements.

4.3.2 Five phases

The oil monopolist faces the residual oil demand. This was discussed already in chapter 3.5. In case there is both a minimum biofuel requirement (that is a proportional requirement) and a fixed minimum quantity requirement. Hence, the residual demand has two discontinuities. When a fixed minimum quantity requirement is applied, the residual oil demand decreases at the phase when the fixed minimum biofuel quantity requirement is binding, but is unchanged in any other parts.

On the base of the previous analysis, there are now five phases. That is because there are two discontinuities in the marginal profit and in those the monopolist executes price limiting separately. Between them there is the phase in which the quantity of biofuel is fixed and hence, the minimum biofuel requirement breaks. In addition, there is the initial phase during which the minimum biofuel requirement is binding and the final exhaustion phase.

Also based on the previous results, the Hotelling rule is strong. It applies also over the discontinuities and it has to apply also to the middle phase, when the biofuel quantity is fixed. I call this on purpose as a middle phase to emphasize that the policy of the fixed quantity applies here in the middle of the demand. Contrary to that the minimum biofuel requirement applied to high demand and the subsidies to the low demand. Remembering that the production starts with high quantities and the a policy affects also all quantity decisions before it is binding. Therefore, subsidies affect whole production from the beginning till the end, but the proportional biofuel requirement only the initial phase. The fixed biofuel quantity requirement affects all but the final period. The fixed biofuel quantity requirement lowers the production quantities in the middle of the production, and increase the length of that phase. This is resulting just by the Hotelling rule. In the example solution given above, only the last phase was needed and the minimum proportional biofuel requirement was not assumed to apply at any time. Therefore, the solution was so simple.

5 Dynamic cost structure

When utilizing a resource of any kind, some parts of it are easier available than the others. The production cost varies based on which part of the resource is currently produced. It is quite obvious that the monopolist would not start from the hardest piece of her stock if others are easily available. Nevertheless, it is not that clear that she would start from the easiest one either. She could slice off from different parts of the resource to meet the optimal production. This might lead to the case in the previous chapter, in which the costs of production at any time is a fixed function. Now, I turn to a different case, in which the stock is taken into use based on the costs.

When the least costly parts of the stock are produced first, the cost curve that the monopolist faces depends on the stock. This is partly the case in the oil market. The oil rush started from the easily accessed Texas and currently oil is drilled already from deep seas and refined from the Canadian and Venezuelan tar sands. In future, as the oil price rises even the hardest reserves might come into production. But this is not the whole story as in fact there are quite many oil producing countries and firms owning oil resources of different qualities. There is no question about that the Saudi oil is less costly than that pumped out of deep sees. And both are produced currently. Nevertheless, I set that aside for the moment, and assume that the oil monopolist owns all these oil resources and takes them into production based on the costs.

5.1 Profit maximization and timing

I take in this part of the study a bit different route in the analysis than in the previous one. More mathematics is needed and I mix the Euler–Lagrange, the bounded optimal control and the Hotelling's rule methods. Following the derivations in the text is becoming more of a burden than helping in this case, and therefore I give that in appendix F.

As nothing is changed in the demand side, the residual oil demand is the same as previously. Based on the previous analysis, I assume that there is again a price limiting phase between the initial and final phases. (However, in appendix F.4.2 I show with the bounded optimal control theory that the price limiting phase is mandatory.) Taking that into account, let me first write down the total profit

$$\begin{aligned}\Pi = \max_{q(t)} \int_0^{t_1} \left(P_i^\dagger(q) + \delta S\right) q e^{-rt} dt + \int_{t_1}^{t_2} \left(p_1^\dagger + \delta S\right) q_1 e^{-rt} dt \\ + \int_{t_2}^{T} \left(P_f^\dagger(q) + \delta S\right) q e^{-rt} dt, \qquad (36)\end{aligned}$$

where $P_i^\dagger$, $p_1^\dagger$ and $P_f^\dagger$ are the modified inverse demands for the initial, the price limiting and the final phases, respectively,

$$P_i^{\dagger}(q) = b_1^i - b_2^i q, \qquad t \leq t_1 \tag{37a}$$

$$p_1^{\dagger} = p_1 - \gamma - \delta S_0, \qquad t_1 \leq t \leq t_2 \tag{37b}$$

$$P_f^{\dagger}(q) = b_1^f - b_2^f q, \qquad t_2 \leq t \tag{37c}$$

and the coefficient b_1^i and b_2^i for the initial phase and b_1^f and b_2^f for the final phase are

$$b_1^i = \alpha(1+a) - \gamma - a\mu - \delta S_0 \tag{38a}$$

$$b_2^i = \beta(1+a)^2 + \eta a^2 \tag{38b}$$

$$b_1^f = \bar{p} - \gamma - \delta S_0 \tag{38c}$$

$$b_2^f = \frac{\bar{p} - p_1}{q_1} \tag{38d}$$

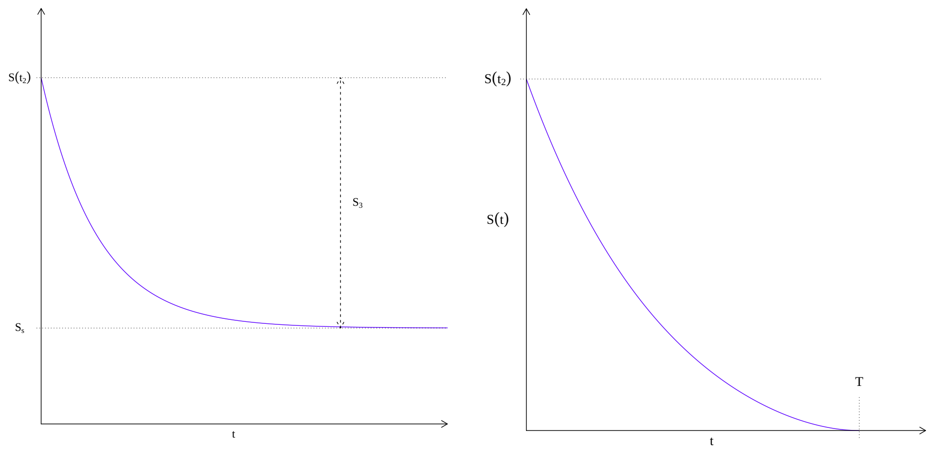
I have simplified the notation here by using net pricing. I continue noting the marginal revenue by $\pi^{\dagger}$ and in addition note the marginal profit by $\lambda^{\dagger}$. Also, there are now the stock (S) and a term that depends on that in the profit functionals. At first it may seem cumbersome just to add the stock term to the inverse demand. This is just for making the analysis more readable, but some explanation is needed. First note also that the modified inverse residual demand hides all terms that are constant or dependent just on the quantity. Especially, it includes the initial stock S_0 and its term is highly negative. Second, when time passes, the stock gets shorter. Therefore, the stock related term is a cost and that increases in time. See appendix F for more details.

5.2 Solution

Let me again start from the end and work out the solution backwards. I did this already in the previous chapter, but now the case is quite different. The reason is that costs are now increasing in time. In the fixed cost case the marginal costs were at their minimum, now in the current case they are at their maximum at the exhaustion. Next, I will go through the solution phase by phase starting from the last phase.

5.2.1 Final phase

In the final phase the stock exhausts, but it may exhaust in two ways. Either it exhausts physically or economically. The physical exhaustion means that the final stock vanishes as it is profitable to produce all of it. In following this is noted as setting the final stock, S_T to zero. The other option is that at some positive level of the stock it is not more profitable to produce more of it, and the final stock left to the ground is the stationary solution, S_s. In



(a) The economical exhaustion (b) The physical exhaustion

Figure 8: A economical and a physical exhaustion when dynamic costs

this case, it is $S_T = S_s$. Therefore, the production of oil stops when it is not profitable anymore.

In the present case there is a given definite and finite stock. At least some of that can be produced profitably. Hence, the initial stock defines the form of the exhaustion. The stock dynamics over time are evaluated in Appendix F.1. The solution is reached just by defining the maximization problem, writing down its Euler – Lagrange equation and solving that with the basic tools of differential equations (see Martio and Sarvas [1993] for examples). Let me write down the result of that procedure, the generic solution for the oil stock is

$$S(t) = C_1 e^{w_1 t} + C_2 e^{w_2 t} + S_s. \tag{39}$$

To illustrate this result, figures 8a and 8b show the time paths of the oil stock for the physical and economical exhaustions, respectively. The figures reveal also further that the exhaustion time is infinite in case of the economical exhaustion, and finite in case of the physical exhaustion. To understand this further, I write down the solutions in more details and go through the economical and physical exhaustions separately.

Both of the exhaustion cases have the same boundary conditions. At the beginning of the final phase the produced oil quantity and the oil stock are given q_1 and $S(t_2)$, respectively. The final stock S_T is also given, but it is zero in case of a physical exhaustion and greater than zero in an economical exhaustion case. The general conditions for the final phase's solution are derived in Appendix F.1 (see equations 115). I apply them here for the both exhaustion cases separately to solve the coefficient for the stock and quantity paths.

1. **Economical exhaustion** Using the initial and final conditions the coefficient for the oil quantity and stock paths reduce to

$$T = \infty \tag{40a}$$

$$C_1 = 0 \tag{40b}$$

$$C_2 = -\frac{q_1}{w_2}. \tag{40c}$$

   From this the stock and quantity paths for the economical exhaustible case are

$$S(t) = S_s + \frac{q_1}{|w_2|} e^{w_2 t} \tag{41a}$$

$$q(t) = q_1 e^{w_2 t} \tag{41b}$$

2. **Physical exhaustion** In case of the physical exhaustion the coefficients for the generic solution do not reduce that much and they have to be solved numerically from the conditions

$$\frac{-S_s}{q_1} = \frac{e^{rT}}{e^{w_1 T} - e^{w_2 T}} \left[\frac{1}{w_1} - \frac{1}{w_2} \right] \tag{42a}$$

$$C_1 = \frac{q_1 e^{w_2 T}}{w_1 \left(e^{w_1 T} - e^{w_2 T}\right)} \tag{42b}$$

$$C_2 = -\frac{q_1 e^{w_1 T}}{w_2 \left(e^{w_1 T} - e^{w_2 T}\right)}. \tag{42c}$$

   However, now the exhaustion time is finite.

5.2.2 Price limiting phase

The price limiting phase is still similar to the fixed cost case presented earlier. If there is a discontinuity in the marginal profit, the monopolist stays put at the fixed price p_1 and fixed quantity q_1 for some time $t_2^\star$ and lets the time fix the discontinuity. However, the present values of the marginal profits will not be equal anymore over the price limiting phase in this case. And there is an additional handle to take care of, namely the stock dependent costs.

In the price limiting phase the quantity is q_1 and the price p_1. There is only the length of the phase that is unknown. I will use two methods to tackle that, namely the direct evaluation and the bounded optimal control. Let me first discuss how that can be directly evaluated. A more detailed discussion and derivations are given in F.4.1. First, the marginal profits in the beginning and at the end of the price limiting phase are equal to the ones at the end of the initial and at the beginning of the final phase, respectively. If they were not the profit could be improved by modifying the timing. Second, the stock is decreasing at a constant pace during the price limiting phase because the quantify is fixed to q_1. From that I get the relation between the initial and final stocks binded by the length of the phase. Now, the stock has to be continuous over the phases. The final stock in the price limiting phase is equal to the one at the beginning of the final phase which was defined by the optimality of the final phase. Hence, taking these two facts together the initial stock in the price limiting phase is just a function of its length and the given final stock. Finally, the dynamics of the marginal profit is given by the optimality condition. That condition arrives formally via the Hamiltonian. By solving that and plugging in the findings, I reach the condition for the price limiting phase. This is done in F.4.1 in details. As a result, the condition for the length of the price limiting phase is

$$e^{-rt_2^\star} = \frac{\pi_1 + \delta\left(S\left(t_2\right) - \frac{q_1}{r}\right)}{\pi_2 + \delta\left(S\left(t_2\right) - \frac{q_1}{r}\right)} + \frac{\delta q_1}{\pi_2 + \delta\left(S\left(t_2\right) - \frac{q_1}{r}\right)} t_2^\star. \tag{43}$$

Here t_2 refers to the time instant, when the price limiting phase ends, and $t_2^\star$ to the length of the phase. The length can be solved numerically in order to solve all phases fully, and to plot price, quantity and stock paths.

Is there a price limiting phase? I will show next that there is always a solution for the condition indicating that the price limiting phase exists. The form of the condition is $e^{-rt_2} = k_1 + k_2 t_2$, where k_1 and k_2 are constant coefficient. The exponential function is decreasing in the phase's length, $t_2^\star$ and vanishing at infinity. Its initial value at zero time is 1. The right hand side is not that obvious, and that is discussed in more details in Appendix F.4. In result, it turns out that $k_1 > 1$ and $k_2 < 0$. Therefore, the curves drawn of the left hand and right hand sides of the equation 43 cross each others at exactly one length of the price limiting phase, $t_2^\star$ and it is finite. I come back to the subject again in discussing the effect of changing the minimum biofuel requirement later on, and draw figure 10 of the case.

To get a different view, the price limiting phase can be thought with the help of the bounded optimal control theory. Here, the monopolist controls the production quantity. As seen in 5.2.1 there is the final phase that begins, when the certain stock level has been reached, namely $S(t_2)$. This leaves the rest of the stock to the previous phase. For that phase the monopolist has defined initial stock, final stock and the final quantity. That is enough to solve the general solution, which is similar to (39). However, there is a further constraint; in the initial phase the minimum biofuel requirements holds and therefore, the whole quantity path must be above the breakup quantity level, q_1. That is $q \geq q_1$ during the phase. Taking this into account there is always a price limiting phase, during which the price and quantity are fixed at p_1 and q_1, respectively. Derivations with the bounded optimal control theory are given in F.4.2. They result into the same condition for the length of the price limiting phase as the direct derivation in (43).

5.2.3 Initial phase

The first phase is more complex than it at first glance might seem. However, when understanding that there is the price limiting phase, and solving the final stock at the end of the initial phase with that, the generic solution gives the solution for the initial phase.

The solution also in the first phase must fulfill the Euler – Lagrange equation. Therefore, apart from the fact that the coefficients and the steady state are different than in the final state, the form of the solution is the same

$$S(t) = C_1 e^{w_1 t} + C_2 e^{w_2 t} + S_s. \tag{44}$$

Furthermore, the solution has natural boundary conditions;

1. The initial stock is given
$$S(t_0) = S_0 = C_1 + C_2 + S_s. \tag{45}$$
2. The final stock can be evaluated from the last two phases
$$S(t_1) = S_1 = C_1 e^{w_1 t_1} + C_2 e^{w_2 t_1} + S_s. \tag{46}$$
3. The final quantity is q_1
$$q(t_1) = q_1 = -C_1 w_1 e^{w_1 t_1} - C_2 w_2 e^{w_2 t_1}. \tag{47}$$

The boundary conditions and the form of the solution is everything that is needed for the full solution.

Again, I discuss the solution also with the help of the bounded control variable. The quantity is the monopolist's control variable, and its path is bounded below. This requirement arrives just because the residual demand with the minimum biofuel requirement binding is valid only with greater quantities than q_1. If unbounded, the quantity path could be U-shaped – first decreasing and then increasing. However, the final value of the phase is q_1, and that is the lowest quantity allowed for the first phase. The reason that the shape of the unbounded solution arrived U-shaped lies in the form of the solution in (44), remembering that $|w_1| > |w_2|$. Hence, as time goes by the term with w_1 weights more and more, and it pushes the quantity path finally to increase. And if the quantity at some time instant would start to increase, it would increase thereafter at all time instants. The bound in quantity helps to find the optimal and monotonically decreasing quantity path.

The solution to the bounded quantity case is presented in F.4.2. To evaluate the time instant at which the quantity path reaches the bound q_1, an additional transversality condition has to be taken into account. That condition states that at the end of the initial period the average profit equals the marginal profit plus required rent of the remaining production's profits. This condition helps in defining the unknown length of the initial phase when assuming the quantity bound is binding at some values.

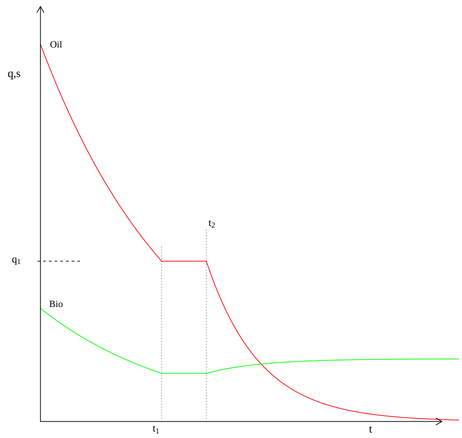
5.2.4 Stock, quantity and price paths

I turn now to discuss the resulting paths of quantities, prices and the oil stock. As a base for that an illustration in one specific case is given in figure 9. It has three subfigures; the first representing the quantity paths of oil and biofuel, the second showing the price paths for oil, fuel and biofuel and the third plotting the oil stock in time.

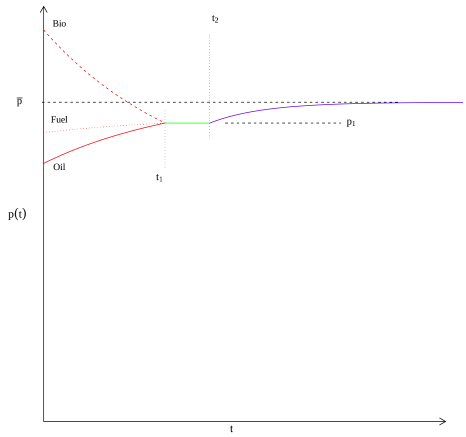
To be able to plot these figure I solved all given equations numerically backwards by writing appropriate scripts in R. The solving process starts from the final phase and its conditions. First solving the timing, and when knowing the timing the coefficient. When timing and coefficient of the final phase were known, that phase was solved, and I knew the stock at the end of the price limiting phase. The process went through phase by phase revealing the solution. Now, all figures share the time scale. The vertical, dotted lines mark few important time instants, t_1 and t_2. They indicate the changes from the initial phase to the price limiting phase and from the price limiting phase to the final phase, respectively.

The quantity paths are shown in figure 9a. The oil quantity is plotted with red and biofuel with green. In the initial phase both of the quantities decrease till t_1 and enter the price limiting phase. Finally, in the exhaustion phase the minimum biofuel requirement is broken, and the oil quantity decreases and the one of the biofuel increases rapidly. I have few notes on this. First, having the price limiting phase means that the monopolist may not execute the unbounded solution for the Euler – Lagrange equation in oil production, but has to take the lower bound into account. The oil quantity is therefore fixed staring from t_1 till t_2. Second, the quantity paths are decreasing quickly in the first phase. Compared to the fixed cost case the quantity paths have quite different shapes. This is however a matter of chosen parameters. If the cost dynamics were very limited, almost similarly shaped paths as in the fixed cost case would result. A limited cost dynamics with a short initial oil stock causes the physical exhaustion. Finally, when considering biofuel, again the biofuel quantity is tied to the oil quantity by the minimum biofuel requirement till the end of second phase.

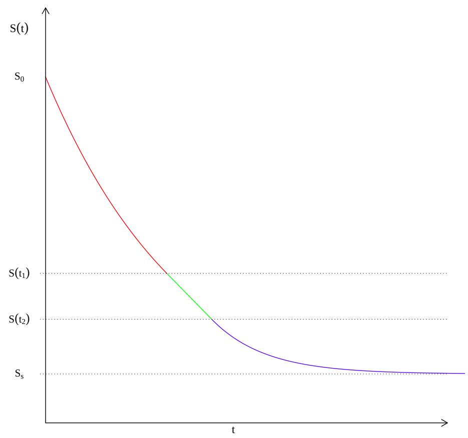
Figure 9b shows the oil, fuel and biofuel price paths. The choke price ($\bar{p}$) and the price limiting price (p_1) are marked in it. As was in the fixed cost case the biofuel price starts at a higher level and then decreases to the price limiting price and the oil price level experiences simultaneously an increase. If the fuel price path is increasing or decreasing in the first



(a) The quantity paths



(b) The price paths



(c) The oil stock path

Figure 9: Paths of; the produced oil and biofuel quantities in (a); the oil, fuel and biofuel prices in (b); and the remaining oil stock in (c) in case of the economical exhaustion

phase depends especially on the minimum biofuel requirement, but also on the other parameters. The effect of having a bounded solution is seen also on the price paths as the price limiting price is reached at t_1, when all the prices meet. And again, as the quantity stayed put till the end of the price limiting phase, so does the price. In the last phase the prices increases till the choke price.

Finally, figure 9c represent the oil stock path. The stock levels at t_1 and at t_2 are given in the figure, as well as the stationary stock level S_s.

As the absolute value of the oil stock's time derivative is the value of the oil quantity, the quantity path analysis should be combined to the stock analysis. In the first phase the stock is originally S_0. As the produced oil quantity decreased over time, the oil stock path is first decreasing quickly, but the pace is smoothing till it reaches t_1. After that the stock path decreases linearly till the end of the price limiting phase at t_2. This is because the produced oil quantity is constant, meaning that the stock path is linear. In the last phase the stock decreases slowly as the given case was of the economical exhaustion.

5.3 Hotelling rule

To complete the solution I will write down the Hotelling rules, when the substitute product enters gradually the monopolist's market. The 'internalized' Hotelling rule (the term used by Grafton, Kompas, and Long [2010]) for the monopolist applies at all times, and it is

$$\dot{\lambda} = r\lambda - \frac{\partial f}{\partial S}. \tag{48}$$

I continue using λ, unconventionally, as the marginal profit. Let me derive this a bit further. With the given cost function for the case, $\frac{\partial f}{\partial S} = \delta q = -\delta \dot{S}$ and the rule reduces to one of the following,

$$\frac{\dot{\pi}}{\lambda} = r \tag{49a}$$

$$\frac{\dot{\lambda} + \delta q_1}{\lambda} = r. \tag{49b}$$

The first rule relates to the continuous marginal profit, and the second to the point of its discontinuity, the price limiting phase. In the first rule, the marginal revenue's change in time relates to the marginal profit by the interest rate. However, note that the rule is not defined at the point of the discontinuity of the marginal revenue, and there the second rule applies.

The final Hotelling rule for the current case, in which the residual demand was linear in parts is one of the following,

$$\frac{\dot{q}}{\lambda} = -\frac{r}{2d_i} \tag{50a}$$

$$\frac{\dot{\lambda} + \delta q_1}{\lambda} = r. \tag{50b}$$

The first rule refers again to the continuous marginal revenue parts. In that, the relation of the oil production quantity's change in time to the marginal profit is a constant. That constant depends on the interest rate and the slope of the appropriate residual demand, d_i. The rule for the price limiting phase does not change.

5.4 Effects of the biofuel requirement

The biofuel requirement affects the case of the stock dependent costs more deeply than in the fixed costs case. I go through next phase by phase, how the changing minimum biofuel requirement affects the oil monopolist's decisions.

5.4.1 Final phase

The final phase is not itself affected if the minimum biofuel requirement is changed, because its residual oil demand does not change. The form and the roots (w_1 and w_2) of the solution are unaffected. The reason is that the biofuel is competitive in the last phase and the minimum requirement is not binding. Biofuel is sold at greater proportions than the minimum requirement states.

Reaching the exhaustion is not either affected by the changing biofuel requirements. For example the final stock is not changing. In case of the economical exhaustion it is still the same condition $S_T = S_s$ that applies, and the stationary solution of the last phase does not depend on the biofuel requirement. On the other hand, in the case of the physical exhaustion, all of the stock is still produced.

The initial condition of the final phase however changes. The discontinuity of the residual demand arrives at a lower oil quantity when the minimum biofuel requirement is increased as derived in B.1, $\frac{\partial q_1}{\partial a} < 0$. As the form and especially the roots (w_1 and w_2) are unaffected, the new price limiting quantity with the increased biofuel requirement is on the optimal quantity path of the old one. Therefore, the length of the final phase is shorter. The initial stock of the final phase $S(t_2)$ is also lower and the stock produced in the final phase is less than it was with a lower biofuel requirement.

5.4.2 Length of price limiting

The price limiting phase occurs just at the discontinuity of the marginal revenue, and depends on its magnitude. The change of the minimum biofuel requirement affects both the timing and the magnitude of the discontinuity.

The price limiting phase is longer if the minimum biofuel requirement increases. To show this let me study how the price limiting phase's length was determined. It is determined by (43), and I rewrite it here

$$e^{-rt_2^\star} = \frac{\pi_1^\dagger + \delta\left(S\left(t_2\right) - \frac{q_1}{r}\right)}{\pi_2^\dagger + \delta\left(S\left(t_2\right) - \frac{q_1}{r}\right)} + \frac{\delta q_1}{\pi_2^\dagger + \delta\left(S\left(t_2\right) - \frac{q_1}{r}\right)} t_2^\star. \tag{51}$$

On the left there is just the decreasing exponential function, which is unaffected by any changes in biofuel. On the contrary, in the right hand side's linear function almost everything depends on the biofuel requirements. I

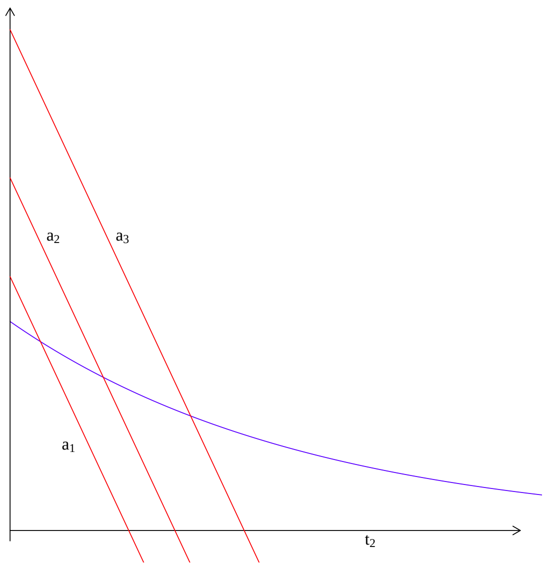


Figure 10: The lengths of the price limiting phase in cases of various biofuel requirement; in a_3 the biofuel requirement is highest and no requirement is implemented in a_1.

rewrite the equation as $e^{-rt_2^\star} = k_1 + k_2 t_2^\star$. The coefficient k_1 increases in increasing minimum biofuel requirement, but at the same time the coefficient k_2 is unaffected. See appendixes F.4 and F.7 for details. In total, as the exponential curve stays put and the linear function moves upwards without changing its slope, the curves coincidence later as shown in figure 10. The price limiting phase becomes longer. It is to note for the later analysis that k_1 increases a certain, constant amount with a constant increase of biofuel requirement.

The price limiting phase's duration increases and the production quantity decreases, but what about the produced stock. The produced stock in the price limiting phase changes with increasing biofuel requirement as

$$\begin{aligned}\frac{1}{q_1}\frac{d}{da}q_1 t_2^\star &= \frac{dt_2^\star}{da} + \frac{t_2^\star}{q_1}\frac{dq_1}{da} \\ &= \frac{dt_2^\star}{da} - \frac{t_2^\star}{1+a}. \end{aligned} \tag{52}$$

Here I have just used linear demands in the last line, and the definition for q_1 (and set $\eta = 0$ and $\beta = 1$ for simplicity). This equation should be considered as a difference equation rather than a continuous one as the changes in the requirements are discrete jumps. Nevertheless, the equation serves as a approximation. Now, the increase in the biofuel requirement increases k_1 at a constant pace. The price limiting phase's duration increases quick for the small increases in the biofuel requirement, but the response stabilize in the higher increases (see figure 10). The latter term is negative, and depends on the initial duration, and it is a question, which of these terms effects count more. The produced stock increases everywhere in increasing biofuel requirement, if initially, at no requirement the first term is greater than the latter. If the increase of the requirement were greater, the first term would be greater and the latter smaller, resulting in favor of the increased stock consumption in the price limiting phase. I gathered results of decreasing stock in the numerical analysis, but only for small biofuel requirements. That means that if the biofuel requirement is originally set to a very low standard, the produced stock in the price limiting phase may decrease slightly. If the standard is increased more, the produced stock however increases.

5.4.3 Initial phase

I start handling the initial phase from the Hotelling's rule. As given earlier, for this case it is

$$\frac{\dot{q}}{\lambda} = -\frac{r}{2d}. \tag{53}$$

There the slope of the residual demand steepens, when the biofuel requirement is implemented. Hence, at any given marginal profit level the dynamics gets slower. The drive in the change of the production quantity slows down, and the stock is produced slower. This was to be expected as the demand was decreased. The production path becomes more flat, and according to Sinn [2008] that was a sign of a good environmental policy.

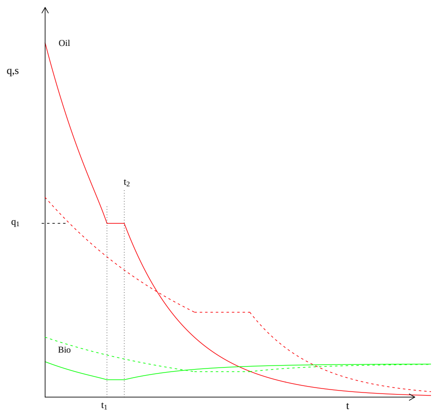
The initial quantity decreases. Let me first take a look at the linear case. In that the total profit in linear demands was evaluated in F.5, and it is

$$\Pi = \frac{q_0}{r}\left(p_0^\dagger - \pi_0^\dagger\right). \tag{54}$$

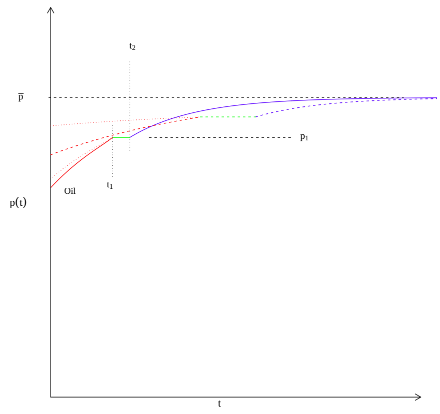
Increasing the biofuel requirement must not increase the present value of the total profit, as otherwise the monopolist would start her own fuel distribution network and apply herself a greater biofuel requirement. Therefore,

$$\begin{aligned} r\frac{\partial \Pi}{\partial a} &= \frac{\partial q_0}{\partial a}\left(p_0^\dagger - \pi_0^\dagger\right) + q_0\frac{\partial (p_0^\dagger - \pi_0^\dagger)}{\partial a} \\ &= \frac{\partial q_0}{\partial a}\left(p_0^\dagger - \pi_0^\dagger + q_0 d_2\right) \leq 0. \end{aligned} \tag{55}$$

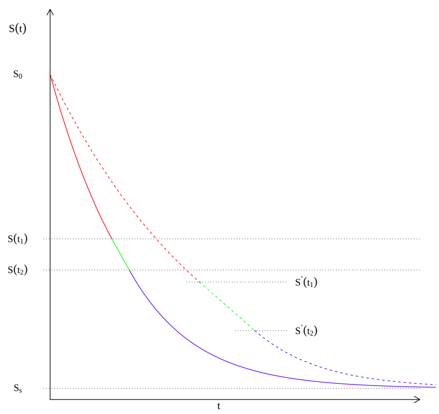
As in the last line the term in parenthesis is positive, the initial quantity must not increase in increasing biofuel requirement. Here $d_2 > 0$ is the slope of the residual demand in the initial phase. In general, consider now the ending of the price limiting phase with the increased biofuel requirement. After that time instant the stock path is the same with the original requirement. Before that the price limiting phase has just ended, and that was longer and at lower



(a) The quantity paths; the increased requirement with a dashed line



(b) The price paths; the increased requirement with a dashed line



(c) The oil stock path; the increased requirement with a dashed line

Figure 11: Effects of changing the biofuel requirement; on quantities in (a); on price in (b); and on the remaining oil stock in (c)

quantity than with the original requirement. Also in the initial phase the stock was produced slower, as the quantity dynamics was slower and that ended at a lower quantity. Hence, with the increased biofuel requirement the stock was produced at a lower quantity path, and the initial stock decreased.

The problematic part arrives in the stock behavior. I spent a considerable amount of efforts in solving how the increase in the biofuel requirement affect the oil stock. Having solved all the needed conditions, the influence should be just a direct derivation. But the case has more in it.

The understanding of the stock behavior in the initial phase arrives through the last two ones. In the final phase, a more strict requirement decreases the amount of stock produced quickly. And the greater the change the less the additional effect. On the other hand, in the price limiting phase the opposite is true. A small increase in the requirement may decrease the produced stock during the phase. That turns around quickly, and the produced stock increases.

In total, an initially small biofuel requirement will increase the production in the initial phase. This is because both of the later phases are producing less. A stricker requirement will decrease the production less in the final, but starts to increase that in the price limiting phase. The production in the initial phase turns to decreasing when the second phase's increasing effect becomes larger than that of decreasing in the final. The timing accommodates respecting the given conditions.

The effects of increasing the biofuel requirement are shown in figure 11. The changing quantity paths are given in subfigure 11a, the price paths in 11b and the oil stock in 11c. In all of these figures the dashed lines represent the increased biofuel requirement.

The effect of increasing the biofuel requirement on the quantity paths is shown subfigure 11a. The initial oil quantity has decreased and the biofuel quantity increased. The initial phase last longer and the price limiting quantity has decreased.

Subfigure 11b represents the influence of the increasing biofuel requirement on the oil price. The initial price as well as the price limiting price increases with the increasing biofuel requirement. The final, choke price is unchanged.

The oil stock path is shown in subfigure 11c. The biofuel requirement is in this case environmental friendly. Even if the the same amount of oil is produced with both levels of the biofuel requirement, the increased on results in more conservative oil usage.

6 Discussion

At this point, I have gone through how the minimum biofuel requirement affects the fuel, oil and biofuel markets. Few related issues have been left out, which rose under my studies as they had to be dealt with and which bothered me for a quite long time. I will discuss these in this chapter. First, I will go through how the biofuel subsidy affect monopolist. In that I have opposite results than Grafton, Kompas, and Long [2010] gives. Second, I will discuss shortly also on uncertainty. As the monopolist has a great value in her oil resource asset, which she want to secure, short period quantity path might be a result of coping to the changing expectations of the biofuel policies.

6.1 Biofuel requirement and subsidy

The minimum biofuel requirement is one option for an environmental friendly governmental policy. It is not however the only one. Is it better than a direct subsidy to the biofuel producers? I go first through how the biofuel subsidy affects the oil production.

6.1.1 Effects of subsidy

The biofuel subsidy was not in the main focus in this thesis. Nevertheless, as it could be used instead of the minimum biofuel requirement or in addition to it, I want to discuss that also even if I am not giving the full path of derivations. In subsidizing the biofuel production is supported and I consider here just one case. In this case the government supports all biofuel production by an equal amount of money per a produced quantity. Each gallon of biofuel receives an equal subsidy. And further, the subsidy influences fully and hence the market price of the biofuel decreases by the amount of the per quantity subsidy.

Figure 12 presents the effects of the subsidy in one example of the dynamic costs case with linear demands and supplies. The dashed lines refer to the subsidized case. In addition to the subsidy, there is no minimum biofuel requirement present. Also, note that this case is physically exhausted. The economically exhausted case would not be a fair comparison as when the subsidy present the exhaustion occurs with a greater stock. In that case also removing the subsidy path should be considered.

The quantity paths of the subsidized (dashed) and the non-subsidized (solid) biofuel are shown in subfigure 12a. Increasing the subsidy decreases the initial oil quantity at least in this case. (If this occurs in every case was not studied as that was not in the main scope.) The initial phase is shorter and the final phase starts earlier. The exhaustion occurs also earlier. In this case there is clearly a sort of Green Paradox even if the quantity decreases in the beginning.

The oil price increases in the beginning, but flips below the original after a while during the price limiting phase as subfigure 12b states. The initial price is greater and backstop price lower in subsidized case than the one without.

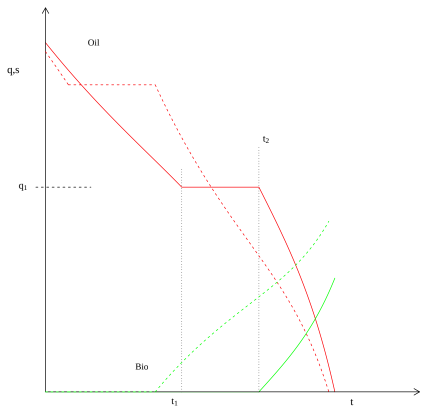
Subfigure 12c shows that the oil stock is depleted quicker in the subsidized case than in the one without, but just in the beginning. Contrary, in the economical exhaustion case the stock would exhaust at a greater stock when subsidized. However, this has its price as the subsidy would have to be paid forever if that level is wanted to hold. Therefore, this gain in the unproduced oil stock should be thought as a temporary advantage. Subsidizing affects the residual demand straightforwardly, as was shown in 3.5.4. The discontinuity occurs at a greater quantity, but the slopes of the residual demand do not change if no minimum biofuel requirement. Therefore, the solutions of the initial and final phases do not change, but in their timing and end conditions. The initial phase last for a shorter, the final phase for a longer time and the price limiting phase arrives earlier and at a greater quantity than in the unsubsidized case. In the given example, the price limiting phase also lasts for a longer period, but that might not be the case always.

This description contradicts the one given by Grafton, Kompas, and Long [2010]. They explicitly claim that increasing the subsidy will delay the exhaustion time in the linear demand and supply case with dynamic costs. However, in their presentation the discontinuity of the marginal revenue is not taken into account. That arrives in their presentation in the discontinuity of the elasticity. One of their main claim is the condition for delaying the exhaustion date, and it does not take the discontinuity – meaning the price limiting phase – into account. Piecewise it applies still to the initial and final phases, but does not have a meaning without the price limiting phase.

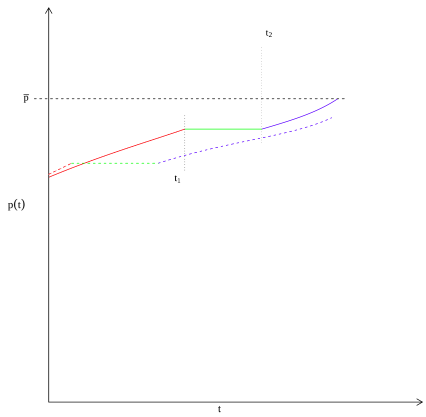
6.1.2 Is biofuel requirement better than subsidy?

The biofuel requirement has some advantages over the subsidy. It is cheap. It acts on the present more than the future. It acts also in a way that is easy to comprehend. However, there are some issues also. In the long run operating with a subsidy on an economically exhausting oil stock leaves some of that into the ground if biofuel subsidized forever. Also, with the biofuel requirement biofuel production is in troubles. I shall discuss next these subjects.

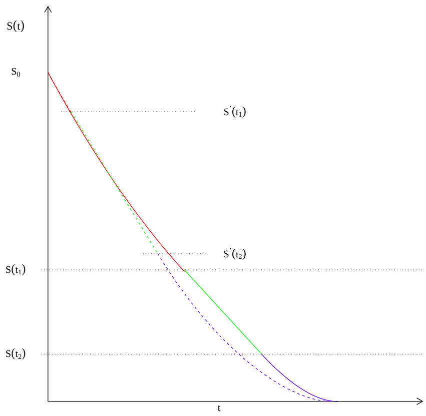
The costs of the minimum biofuel requirement are low. The government does not have to pay more than the control of the requirement takes. On the other hand subsidizing would require cash payments for a long time and committing to that might be problematic for politicians. Looking this from a different view, the biofuel requirement is not free at all. The costs are paid in the oil or actually in the fuel prices. As the requirement forces to buy expensive biofuel with the cheaper oil, the consumers pay for this



(a) The quantity paths; the subsidized biofuel case with a dashed line



(b) The price paths; the subsidized biofuel case with a dashed line



(c) The oil stock path; the subsidized biofuel case with a dashed line

Figure 12: Effects of subsidizing biofuel; on quantities in (a); on price in (b); and on the remaining oil stock in (c)

environmental act.

The biofuel requirement is a policy of an immediate impact if markets are ideal. Setting a requirement today affect already today as the oil production drops immediately. The reason to this is that the biofuel requirement decreases the demand for the lowest oil prices. Therefore, the greatest percentual drop in the oil production is today compared to the no-policy case. To be a realistic, there is natural hinges in this. The biofuel production set a limit for the biofuel requirement and also for the timing. Set that aside, and assume competitive biofuel production, the biofuel requirement in a good environmental policy for an immediate act.

In the long run the effect of the minimum biofuel requirement wears off. In the final phase, when the biofuel is a competitive substitute to oil, the minimum biofuel has no effect on the quantities, prices or stock. Anyhow, it had one on the timing. if something is left to the ground that is of an equal amount than it would have been without the requirement. Would the biofuel subsidy work better as its solution states that more oil is left unproduced into the ground? That requires that the subsidies are paid forever in order to keep the oil production non-profitable. In reality, the subsidy will finally vanish. Knowing or anticipating this affects the monopolist's action. The decisions and paths in the exhaustion phase are therefore questionable when using subsidies in the economically exhaustible case.

A biofuel requirement decreases the current oil production, increases prices and causes the oil stock to last longer. That is easy to comprehend. On the contrary, acting with a subsidy can cause a Green Paradox as was seen in previous example in chapter 6.1.1.

The biofuel producers will be troubled by a new biofuel requirement. They experience first a sudden increase in the demand of biofuel. Before there is enough biofuel production capacity and producers are able to react and meet the demand, it is far fetched to claim that the competitive biofuel market exists. Nevertheless, I assume that the increased biofuel demand meets the supply after some time. The demand turns to decreasing, because it dictated to a certain proportion of the oil production, and the production is decreasing. In this phase the biofuel producers are indeed competitive as some of them run out of the business. There is a distress till the prices of the biofuel and oil are equal. In conclusion, applying a minimum biofuel requirement without carefully thinking of a backup production for the biofuel producers is morally questionable policy.

The biofuel is subsidized at the moment and it is sold even in US, where the fuel prices are cheaper than in EU. This reveals that the markets should be in the price limiting phase already with the help of subsidizing. Further, let me still consider just one big closed market, even in this that assumption is clearly stretching the facts. If already in price limiting phase, the forecast is that the quantities and prices stay the same for a quite long time. Increasing the subsidy would send us deeper into the price limiting phase or end it, decrease the price level and increase the current oil consumption as more competition enters, and vice versa. On the other hand, increasing the biofuel requirement would probably increase or delay the price limiting phase, increase the oil price and decrease the quantity.

6.2 Critics on certainty

Having gone through the analysis with the fixed and known demand, it is time to take a few steps back and see the pitfalls. World is changing, whatever we might do to stop that. And in addition, if there is something uncertain at the moment, the level of uncertainness increases in time.

The uncertainty has dramatical effects on the framework. Remember that solving the quantity and price paths started from the end, the exhaustion conditions and took bit by bit every detail into account till today. Decisions of the whole production path were made on the first day. And there was no uncertainty. Decide once, sit back and collect profits till the exhaustion. On the contrary, uncertainty forces the monopolist to reconsider the decisions during the production. It forces also her to take into account the possibilities beforehand, and possibly execute actions or take different paths already before the realization of the uncertain situation. The effects of uncertainty given may be of any size even if the changes are quite small.

It is the monopolist who decides the paths. What does she know? The most detailed knowledge she has on her own resources and the costs of production. It is the outside world that causes more the problems. I discuss two aspects, the demand and the governmental policy.

The monopolist knows the fuel demand at the moment. With the current oil, biofuel and fuel prices she knows the sold quantities. She could also be lured to assume that the state of the world has not been changing much from yesterday, but a bit more from the day before. Hence, the monopolist has at her hands today's and last few day's or year's demands (with estimates of how the world's state has changed). She is also able to say something about the demand in past, but she has to estimate the changing conditions, which becomes more and more challenging the further in past. How about the future? Even if the state of the world would not change, there is no way in this framework to predict the demand at higher prices, toward which the monopolist should be heading according to Hotelling [1931]. That is the territory, of which there is no empirical knowledge. And that affects the current decisions. The monopolist must form expectations. Every time the expectations change the paths change. Let us hope that the monopolist is not changing the expectations too often.

The governmental biofuel policies affect greatly the optimal production and price paths. What kind of policy is declared and when it becomes effective affects the monopolist. The effect of different policies was discussed throughout this thesis and I put that aside now, and assume that the monopolist assumes that the government is tightening the biofuel policy. For simplicity assume that the date when the policy becomes affective is known, but the level of the tightening is not. At that date, the value of the monopolist's remaining oil stock will drop, because the more strict policy will be applied. Therefore, she want to stick will the shorter oil stock the more strict policy she expects. And to reach a lower stock, she will try to push the more oil to the market now, the greater tightening she is afraid. That could cause a green paradox. Even if the government did nothing, but gave a reason for an anticipation.

The governmental policy anticipation is a powerful tool. The monopolist maximized her profit and considers the unproduced oil as an asset. The uncertainty regarding the rules or policies could affect her to change the current production. Also, the monopolist understands that all the policies are not that quick to apply. Giving out a subsidy could be effective tomorrow. On the other hand, forcing a minimum biofuel requirement needs a long smoothening period, and it can be totally effective only in years' time. Anticipating the policy tools, how they are used and how they become fully effective, are aspect the monopolist should consider every day. This kind of uncertainty was not discussed more in this thesis.

7 Conclusions

In this thesis I have discussed the biofuel from the mandatory proportional biofuel requirement point of view. I set up the framework to analyze the dynamics of the oil and biofuel production in time. I consider the framework as my first merit. However, handling the dynamics I adopted mostly from Hotelling [1931].

The basis of the framework was the discovery that the mandatory biofuel requirement affects the residual oil demand in two ways. First, it affects the oil quantity level, at which the minimum biofuel requirement breaks up. Second, it affects also the oil demand at the higher oil quantities, because the mandatory, minimum biofuel requirement answers to a part of the fuel demand. Finding the ways the biofuel affects the demand I consider as my second merit.

I studied the framework with two different cost structures. In the first one the oil production cost structure was fixed in time and linear in production quantities. In the second case the costs were dependent on the remaining oil stock. Solving both of these cases independently I consider as the third merit. I did not found any previous solution in case of the minimum biofuel requirement. To illustrate the solutions I solved the cases also numerically.

Entering of a substitute affects the monopolist's choices. The substitute causes a mandatory price limiting phase. I solved the condition for the length of the phase in two ways, using a direct derivation and also by using a bounded optimal control, and consider that as my further merit.

As final words I would like to conclude that the minimum biofuel requirement is at least in this context a very efficient environmental policy. On the contrary to the subsidizing policy, which caused a Green Paradox at least in some cases, the minimum biofuel requirement did not create a Green Paradox in any of the studied cases.

A Zero interest monopoly solution

To begin this more mathematical part of the thesis, I go back to the principles. I will derive in this chapter the zero interest solution for the oil monopolist. This means that the monopolist values the future's earnings as much as she values today's earnings and hence no discounting needs to be considered. Further, the monopolist does not take into account the yet unproduced oil stock she owns. However, the biofuel is introduced into the framework and the main contribution of the zero interest solution arrives through understanding this dynamics.

There are deeper reasons in this than just to fill requirements of a comprehensive study. In fact, the basic monopolist's results of the zero interest case appear in the more advanced scenarios. It is like a measuring stick against which the monopolist scales the quantities in those scenarios.

Also knowing that the derivations in the following chapters in appendix might feel sometimes cumbersome, understanding the basic zero interest solution might help in skipping some parts of the derivation if this reasoning convince also in more mathematical cases.

A.1 Linear demands and supplies

I will use in most of the evaluations linear version for supplies and demands. Here, (56a) gives the form of the inverse fuel demand, (56b) the inverse biofuel supply and (56c) the fixed form of the marginal cost of oil. Further, (56d) applies when the biofuel requirement is binding.

$$D_{fuel}^{-1} = \alpha - \beta x \tag{56a}$$

$$S_{bio}^{-1} = \mu + \eta s \tag{56b}$$

$$\frac{\partial C_{oil}}{\partial q} = \gamma + \sigma q \tag{56c}$$

$$x = (1 + a)\, q \tag{56d}$$

A.2 Maximization in the zero interest model

The monopolist maximizes her profit. As there is no future to consider, the profit is just the revenue reduced by the costs without discounting. Further, as the monopolist operates on the oil market the revenue is the product of the oil price and the oil quantity. The costs are her total costs. The optimization problem is written down in (57a). Without discounting that is the problem, which the monopolist faces every day. As was discussed in chapter 3, the monopolist controls also the fuel markets in the given framework, and the oil price is connected to the fuel price. In the optimization problem this is taken into account in (57b), where the quantities are still oil quantities, but the prices arrive through the fuel and biofuel markets. Finally, in (57c) the fuel price is replaced by the inverse demand in fuel market. I shall use this version of the optimization problem to evaluate the optimal quantity for the monopolist.

$$\max \left[p_{oil} q - \left(\gamma + \frac{1}{2} \sigma q \right) q - C(q) \right] \tag{57a}$$

$$= \max \left[(1+a) q p_{fuel} - a q p_{bio} - \gamma q - \frac{1}{2} \sigma q^2 \right] \tag{57b}$$

$$= \max \left[(\alpha - \beta (1+a) q)(1+a) q - a (\mu + \eta a q) q - \gamma q - \frac{1}{2} \sigma q^2 \right] \tag{57c}$$

A.3 Zero interest solution

Let me find next the profit maximizing quantity for the monopolist. That is found in (58) simply by studying where the partial differential of the profit in respect of the quantity vanishes. It is where the profit is in its maximum.

$$\frac{\partial \Pi}{\partial q} = (1+a)\alpha - 2\beta (1+a)^2 q - a\mu + 2a^2 \eta q - \gamma - \sigma q = 0 \tag{58}$$

Finally, given the optimal condition, the optimal quantity is

$$\hat{q} = \frac{\alpha (1+a) - \gamma - a\mu}{2\beta (1+a)^2 + \sigma + 2a^2 \eta} = \frac{(\alpha - \gamma) - a(\mu - \alpha)}{2\beta (1+a)^2 + \sigma + 2a^2 \eta}. \tag{59}$$

B Marginal revenue at discontinuity

As the marginal revenue or profit has important role in the study, I will next discuss that subject a bit further. The reason for that the marginal revenue is important comes again from Hotelling's rule for the monopolist.

B.1 Marginal revenue at the end of the first phase

I start from the residual inverse demand and study the initial phase, or when the minimum biofuel requirement is binding. That is I take (56a) and (56d). Now, fuel price when the minimum biofuel requirement is binding, is

$$p_t = \alpha - (1+a) \beta q_t. \tag{60}$$

The discontinuity occurs when the prices of oil, fuel and biofuel meet. Therefore the fuel price is equal to the oil price at discontinuity, and it is

$$\begin{aligned} p_1 &= \alpha - (1+a) \beta q_1 \\ &= \alpha - \frac{(1+a) \beta (\alpha - \mu)}{\beta (1+a) + a\eta} \end{aligned} \tag{61}$$

Here I have used the quantity q_1 at the discontinuity, which can be evaluated by setting the fuel and biofuel prices equal and holding the minimum biofuel requirement binding, in (56a) and (56b). It is

$$q_1 = \frac{(\alpha - \mu)}{\beta\,(1+a) + a\eta}. \tag{62}$$

The reason of this part of analysis was to understand, how the the marginal profit at the end of the initial phase behaves. To proceed in that let me write the marginal profit explicitly down and evaluate it at the discontinuity with the help of (60),

$$\pi_1 = \left.\frac{\partial pq}{\partial q}\right|_{q_1} = p_1 + q_1 \left.\frac{\partial p}{\partial q}\right|_{q_1} = p_1 - (1+a)\,\beta q_1. \tag{63}$$

The marginal revenue depends on the price and quantity at the discontinuity, and also directly on the biofuel requirement. They all are affected when the biofuel requirement is changed. Next, I write down the change in the marginal revenue when changing the biofuel requirement,

$$\frac{\partial \pi_1}{\partial a} = \frac{\partial p_1}{\partial a} - \beta q_1 - (1+a)\,\beta \frac{\partial q_1}{\partial a}. \tag{64}$$

Using (61) and (62) the effect of changing biofuel requirement on the price p_1 and the quantity q_1 at discontinuity are

$$\frac{\partial p_1}{\partial a} = -\frac{[\beta\,(1+a) + a\eta]\,(\alpha-\mu) - (1+a)\,(\alpha-\mu)\,(\beta+\eta)}{[\beta\,(1+a) + a\eta]^2} \tag{65a}$$

$$\frac{\partial q_1}{\partial a} = -\frac{(\beta+\eta)\,(\alpha-\mu)}{[\beta\,(1+a) + a\eta]^2}. \tag{65b}$$

Putting these together and eliminating few term reveals the change in the marginal revenue π_1, and it is

$$\frac{\partial \pi_1}{\partial a} = \frac{\eta\,(\alpha-\mu)\,(1+\beta)}{[\beta\,(1+a) + a\eta]^2}. \tag{66}$$

B.2 The price limiting phase's length

The length of the price limiting phase depends on the ratio of the marginal revenues at discontinuity as was discussed for example in chapter 4.2.1. In order to understand how the price limiting phase's length changes when changing the minimum biofuel requirement, I have to understand how the ratio of the marginal revenues behaves in that change. Let me first write down the ratio of the marginal revenues, and spell it out

$$\left.\frac{\pi_2}{\pi_1}\right|_{q_1} = \frac{p_1 + q_1 \frac{\partial p_2}{\partial q}|_{q_1}}{p_1 + q_1 \frac{\partial p_1}{\partial q}|_{q_1}} = \frac{p_1 - d_2 q_1}{p_1 - (1+a)\,\beta q_1}. \tag{67}$$

Here I have used (61) for the first phase marginal revenue, where the minimum biofuel requirement is binding. Further, the case when the minimum requirement is broken in smaller quantities than q_1 was discussed already in chapter 3.5.1. Now, I have used d_2 as the slope of the residual inverse demand for that case. It is constant and especially independent of the minimum biofuel requirement, and explicitly

$$d_2 = \frac{\bar{p} - p_1}{q_1} = \frac{\beta\eta}{\beta + \eta}. \tag{68}$$

Let me now go to the actual question, how the change of the biofuel requirement affects the ratio of the marginal revenues. I write down the change,

$$\frac{\partial}{\partial a}\frac{\pi_2}{\pi_1}\bigg|_{q_1} = \left(\frac{\partial \pi_2}{\partial a} - \frac{\pi_2}{\pi_1}\frac{\partial \pi_1}{\partial a}\right)\frac{1}{\pi_1}. \tag{69}$$

In this only the change of π_2 is unknown. It can be solved in similar manner that it was solved for π_1. It is

$$\begin{aligned}\frac{\partial \pi_2}{\partial a} &= \frac{\partial p_1}{\partial a} - d_2\frac{\partial q_1}{\partial a} \\ &= \frac{(1+\beta)(\alpha-\mu)\eta}{[\beta(1+a)+a\eta]^2}\end{aligned} \tag{70}$$

Compare this with (66) to find that $\frac{\partial \pi_2}{\partial a} = \frac{\partial \pi_2}{\partial a}$. Now, it is a simple task to finalize the evaluation and get

$$\pi_1\frac{\partial}{\partial a}\frac{\pi_2}{\pi_1}\bigg|_{q_1} = \frac{(1+\beta)(\alpha-\mu)\eta}{[\beta(1+a)+a\eta]^2}\left(1-\frac{\pi_2}{\pi_1}\right) < 0. \tag{71}$$

Hence, the ratio of the marginal revenues decreases when the biofuel requirement is increased. As the length is related to the logarithm of the ratio, also the length of the price limiting second period decreases.

C Static costs with no backstop

C.1 Quantity and price paths

Let us go back to the optimization problem. As the monopolist is primarily interest in her oil, let us plug in the oil quantities and proper supply, demand and cost functions. She takes into account the future, understands the difference of profit at different time instants and also that her oil stock is limited.

Starting again from the monopolist who maximizes profit. I may add a constant to the problem as explained in 2.4,

$$\max\left[\left[(\alpha - \beta(1+a)q)(1+a)q - C_{oil}(q) - aqp_{bio}(aq)\right]e^{-rt} - \lambda q\right]$$

For this function, the first order condition (at each time instant t) is

$$\left[\alpha\left(1+a\right)-2\beta\left(1+a\right)^{2}q-\left(\gamma+\sigma q\right)-a\left(\mu+2\eta aq\right)\right]e^{-rt}-\lambda=0.$$

Further, the quantity path can be solved from that. Note that the equation contains still the Lagrange multiplier.

$$q=\frac{\alpha\left(1+a\right)-\gamma-a\mu-\lambda e^{rt}}{2\beta\left(1+a\right)^{2}+\sigma+2a^{2}\eta}$$

In order to get rid of the Lagrange multiplier I introduce a further condition. I require that the production is finished when the time of exhaustion (let me use notation T for the time of exhaustion). From this condition the Lagrange multiplier can be solved

$$\lambda=\left(\alpha\left(1+a\right)-\gamma-a\mu-\lambda\right)e^{-rT}.$$

Before taking the final steps towards solving the price and quantity paths let us study this result as we incorporated into the Lagrange multiplier some economical meaning. The Lagrange multiplier or in situ value is a present value measure of the marginal profits on the optimal path – the profit of selling the last unit of fuel at any time instant. Further, I stressed already previously that the solutions are valid throughout the optimal quantity and price paths. Finally, using the value of the Lagrange multiplier the oil quantity and price paths as functions of time are

$$\begin{aligned}q\left(t\right)&=\frac{\left(\alpha\left(1+a\right)-\gamma-a\mu\right)\left(1-e^{-r(T-t)}\right)}{2\beta\left(1+a\right)^{2}+\sigma+2a^{2}\eta}\\&=\hat{q}\left(1-e^{-r(T-t)}\right)\qquad(72)\\p\left(t\right)&=\alpha\left(1+a\right)-a\mu-\left(\beta\left(1+a\right)^{2}+a^{2}\eta\right)\frac{\left(\alpha\left(1+a\right)-\gamma-a\mu\right)\left(1-e^{-r(T-t)}\right)}{2\beta\left(1+a\right)^{2}+\sigma+2a^{2}\eta}\\&=\bar{p}+\hat{q}\left(\beta\left(1+a\right)^{2}+a^{2}\eta\right)\left(1-e^{-r(T-t)}\right)\qquad(73)\end{aligned}$$

C.2 Exhaustion time

There is a further requirement related to the stock. The whole stock shall be exhausted as there is nothing to stop the profitable oil production. The stock is just the sum of all production. Using the linear forms that is

$$\begin{aligned}S_{0}&=\int_{0}^{T}q\left(t\right)dt\\&=\int_{0}^{T}\frac{\left(\alpha\left(1+a\right)-\gamma-a\mu\right)\left(1-e^{-r(T-t)}\right)}{2\beta\left(1+a\right)^{2}+\sigma+2a^{2}\eta}dt\\&=\frac{\alpha\left(1+a\right)-\gamma-a\mu}{2\beta\left(1+a\right)^{2}+\sigma+2a^{2}\eta}T-\frac{1}{r}\frac{\left(\alpha\left(1+a\right)-\gamma-a\mu\right)\left(1-e^{-r(T-t)}\right)}{2\beta\left(1+a\right)^{2}+\sigma+2a^{2}\eta}.\end{aligned}$$

Now, using the zero interest solution given in (59) this reduces to

$$\frac{1}{r}\left(1-e^{-rT}\right)=-\frac{S_0}{\hat{q}}+T. \tag{74}$$

The exhaustion time is on both sides of the equation, and it is not analytically solvable. There is however always one single solution.

C.3 Biofuel affecting exhaustion time

The exhaustion time depends on the biofuel requirement. In (74) there was only one other coefficient namely the one step monopoly solution, $\hat{q}$ that depends on the biofuel requirement. Hence, to see the effect of the biofuel more clearly, let me write first the effect of the one step monopoly solution on the exhaustion time

$$\frac{dT}{d\hat{q}}=\frac{-S_0}{\hat{q}\left(1-e^{-rT}\right)}<0. \tag{75}$$

Increasing zero interest monopoly solution decreases the exhaustion time. Now, the change in the biofuel requirement affects the zero interest monopoly solution. Remembering that in linear no-backstop case $\alpha<\mu$, I obtain

$$\frac{d\hat{q}}{da}=\frac{-1}{2\beta\left(1+a\right)^2+\sigma+a^2\eta}\left((\mu-\alpha)+\frac{\beta\left(1+a\right)+2a\eta}{2\beta\left(1+a\right)^2+\sigma+a^2\eta}\right)<0. \tag{76}$$

Combining (75) and (76)

$$\frac{dT}{da}=\frac{\partial T}{\partial\hat{q}}\frac{d\hat{q}}{da}>0 \tag{77}$$

Hence, increasing the minimum biofuel requirement increases the exhaustion time.

D Biofuel as backstop in static costs

Consider that there was no oil in the market. The biofuel producers would respond to that by fulfilling the whole fuel demand, if capable. The monopolist knows this and realizes the price cap that limits her actions, namely the competitive biofuel price. This is called the choke price.

In fact there is another important price limit. As we saw earlier in case of expensive biofuel analysis, the produced oil and biofuel quantities decreased over time. At the same time the oil and fuel prices increased, but the biofuel price decreased. Now, if biofuel can act as a backstop, at some point the prices of oil, fuel and biofuel meet. This point is the smallest quantity when the biofuel requirement would still be binding. Biofuel would be provided to the fuel market according to the minimum requirement at that price level even without it. It turns out that the monopolist wants to stay at that price and quantity level for a while.

D.1 Quantity and price paths

Let me consider here of the monopolist's reaction to the backstop biofuel when there is a minimum biofuel requirement. First, there is the competition that limits the pricing when the fuel quantities are smaller than q_1, because there the competitive biofuel will eat up increasing portion of the demand. When the oil monopolist limits her production below q_1, distributors may add more biofuel to fuel than the minimum requirement. Second, at higher quantities part of the demand is responded automatically by biofuel due to the minimum biofuel requirement.

It is no surprise that at the point of exhaustion the monopolist has produced all of her stock. Production would be still profitable to produce leftover stock due to low marginal costs if some of it would be left. Therefore, monopolist does not leave anything into the ground.

I will derive next the optimal oil quantity and price paths for the monopolist and use the calculus of variations as my tool. The calculus of variations bases on the Euler–Lagrange equation, in (78). The optimum path must satisfy the equation at any time instant, but at discontinuities. Solution of the Euler–Lagrange equation applies therefore at any time instant, but when the quantity is q_1, which shall be discussed separately. Now, the Euler–Lagrange equation is

$$\frac{\partial f}{\partial S} - \frac{d}{dt}\frac{\partial f}{\partial \dot{S}} = \frac{\partial f}{\partial S} + \frac{d}{dt}\frac{\partial f}{\partial q} = 0. \tag{78}$$

In these static cost cases I have assumed that none of the factors in the optimization problem depends on the oil stock, and there are no other stocks of any kind available. Therefore, the Euler–Lagrange equation reduces to

$$\frac{d}{dt}\frac{\partial f}{\partial q} = 0. \tag{79}$$

The optimization problem that the monopolist faces is that of maximizing the discounted profits, when she may choose the oil quantities. With linear function forms it is formally

$$\max \int_0^T f dt \tag{80}$$

$$f = \left[(\alpha - \beta\,(1+a)\,q)\,(1+a)\,q - \left(\gamma + \frac{1}{2}\sigma q\right) q - a\,(\mu + \eta a q)\,q \right] e^{-rt}.$$

Now, before going to the Euler–Lagrange equation, let me write down the partial derivative of f and its time derivative,

$$\frac{\partial f}{\partial q} = \left[\alpha\left(1+a\right) - 2\beta\left(1+a\right)^2 q - \left(\gamma+\sigma q\right) - a\left(\mu + 2\eta a q\right)\right] e^{-rt} \quad (81)$$

$$\begin{aligned} e^{rt}\frac{d}{dt}\frac{\partial f}{\partial q} &= -2\beta\left(1+a\right)^2 \dot{q} - \sigma\dot{q} - 2a^2\eta\dot{q} \\ &- r\left[\alpha\left(1+a\right) - 2\beta\left(1+a\right)^2 q - \left(\gamma+\sigma q\right) - a\left(\mu + 2\eta a q\right)\right] = 0. \end{aligned} \quad (82)$$

Equation 82 is the Euler–Lagrange equation for this case.Its characteristics are easier to analyze, when it is organized in a typical form of a first order differential equation,

$$\dot{q} - rq + r\frac{\alpha\left(1+a\right) - \gamma - a\mu}{2\beta\left(1+a\right)^2 + \sigma + 2a^2\eta} = 0. \quad (83)$$

First step in solving this differential equation is to find the special, static solution. It is found by assuming that the quantity has found a static state, and hence its time derivative vanishes or $\dot{q} = 0$. The special, static solution in (84) is in fact the zero interest monopoly quantity

$$q_{static} = \hat{q} = \frac{\alpha\left(1+a\right) - \gamma - a\mu}{2\beta\left(1+a\right)^2 + \sigma + 2a^2\eta}. \quad (84)$$

The next step in solving the Euler–Lagrange equation is to find the general solution for the homogeneous equation 85, where the constant has been left out as the static solution shall fulfill that,

$$\dot{q} - rq = 0. \quad (85)$$

This homogeneous equation has solution (86a), and the general solution in (86b) is the sum of the specific and homogeneous solutions.

$$q = Ge^{rt} \quad (86a)$$

$$q = Ge^{rt} + \hat{q} \quad (86b)$$

To finalize the analysis, let me include the constraints. As there shall be two different phases, initial and final, where this same solution applies, I write this quite formally down. At the end, the oil quantity is $q\left(T\right) = q_T$. From that the constant G is

$$G = \left(q_T - \hat{q}\right) e^{-rT} \quad (87)$$

Finally, including the previous results the solution for quantity path is

$$q\left(t\right) = \left(q_T - \hat{q}\right) e^{-r(T-t)} + \hat{q}. \quad (88)$$

In case this is applied to the initial phase, where the minimum biofuel requirement is binding, the final quantity is q_1. On the other hand, if applied to the final period, the quantity vanishes at the end. Naturally, it has its own zero interest monopoly solution.

D.2 Exhaustion time

There is still the exhaustion time to solve. First, the monopolist shall produce again all of her oil stock (reserved for the particular phase), as oil production is profitable till the end and would be even further on. Therefore, the whole production shall sum up to the whole oil stock. Now, for the initial or final phase the produced stock during it is

$$\begin{aligned} S_i &= \int_0^T q(t)\,dt \\ &= \int_0^T \left((q_T - \hat{q})\, e^{-r(T-t)} + \hat{q} \right) dt \\ &= \hat{q}T + \frac{1}{r}(q_T - \hat{q})\left(1 - e^{-rT}\right) \end{aligned}$$

From this the equation the exhaustion time is

$$\frac{1}{r}\left(1 - e^{-rT}\right) = -\frac{S_i}{\hat{q} - q_T} + \frac{\hat{q}}{\hat{q} - q_T}T, \tag{89}$$

where $q_T = q_1$ in the initial phase, and $q_T = 0$ in the final phase.

E Case of fixed biofuel quantity in static costs

Previously I have discussed cases where the government has dictated a minimum proportional biofuel requirement. There is however a difficulty with the solutions related to those cases. The problem is that also the biofuel quantity decreases over the time when the biofuel requirement is binding. This is because as the oil monopolist has the market power, she optimizes her profits over time by reducing the oil quantity bit by bit and the biofuel production is binded to that quantity.

Decreasing biofuel quantity might be acceptable if that would lead to a stable solution, where the biofuel market would rest in an equilibrium. In case of very expensive biofuel this was the case. However, that biofuel market equilibrium would mean zero biofuel production. In case of the backstop biofuel the solution is much more cumbersome. In that case the biofuel production decreases as the oil production decreases, and after reaching a certain oil production level q_1 biofuel production stands still at constant production for a while and then increases gradually to the competitive biofuel level. At the same time oil vanishes altogether from fuel market.

Allowing the biofuel quantity to go first down and then up is not a solid governmental strategy. Therefore, I study here a very stable strategy in which the biofuel production is fixed to the competitive level, in which it would settle if no oil would be on the market anymore.

E.1 Quantity and price paths

To start with I evaluate the biofuel quantity level by setting the biofuel supply and fuel demand equal (see appendix A.1). The biofuel quantity level is

$$s_s = \frac{\alpha - \mu}{\beta + \eta}.$$

As before the fuel price is the weighted average of oil and biofuel prices,

$$p_{fuel} = \frac{s}{x} p_{bio} + \frac{q}{x} p_{oil}. \tag{90}$$

However, as the biofuel quantity is fixed there is not anymore a simple proportional dependence between oil and biofuel. Plugging in the required biofuel quantity level, the revenue from oil to the monopolist is

$$q p_{oil} = (q + s_s)\, p_{fuel} - s_s p_{bio}.$$

Again, monopolist maximized discounted profits, revenue minus costs

$$\int_0^T \left[q p_{oil} - C(q) \right] e^{-rt} dt$$

$$= \int_0^T \left[(\alpha - \beta (q + s_s))(q + s_s) - s_s p_{bio} - \left(\gamma + \frac{1}{2} \sigma q \right) q \right] e^{-rt} dt. \tag{91}$$

Proceeding here follows the same path as in earlier cases. I want to solve the Euler–Lagrange equation, and need the integrand's partial derivative in respect of the qunatity,

$$\frac{\partial f}{\partial q} = (\alpha - 2\beta (q + s_s) - \gamma - \sigma q)\, e^{-rt}.$$

The integrand of the optimization problem has no dependence on the stock and therefore, the Euler–Lagrange equation can be directly written down,

$$e^{rt} \frac{d}{dt} \frac{\partial F}{\partial q} = (-2\beta - \sigma)\, \dot{q} - r (\alpha - \gamma - 2\beta (q + s_s) - \sigma q) = 0. \tag{92}$$

To solve the equation, I write that in a typical differential equation form

$$\dot{q} - rq + r \frac{\alpha - \gamma - 2\beta s_s}{2\beta + \sigma} = 0. \tag{93}$$

Euler equation's special, static solution is found, when its dynamics is removed. The static, special solution q_s is

$$q_s = \frac{\alpha - \gamma - 2\beta s_s}{2\beta + \sigma}. \tag{94}$$

The next step is to find solution for the homogeneous equation, out of which the static solution is removed. The homogeneous equation is

$$\dot{q} - rq = 0. \tag{95}$$

This simple homogeneous equation has a familiar solution,

$$q = He^{rt}. \tag{96}$$

Now, combining the special solution from (94) and the solution of the homogeneous equation in (96) I get the the general solution for the Euler–Lagrange equation,

$$q = He^{rt} + q_s. \tag{97}$$

For finalizing the solution I need to consider the constraints. I require that the monopolist finalizes her oil production at the highest possible price level, at the backstop price. And hence, the final quantity reaches zero, and

$$q\left(T\right) = He^{rT} + q_s = 0 \tag{98a}$$

$$H = -q_s e^{-rT}. \tag{98b}$$

Including this into the general solution in (97), the quantity path finally is

$$q = q_s\left(1 - e^{-r(T-t)}\right). \tag{99}$$

E.2 Exhaustion time

Again, all of the oil stock shall be used as it is profitable to produce it till the end of the production. Let me therefore integrate the quantity path over the whole time period till the exhaustion time,

$$\begin{aligned} S_0 &= \int_0^T q\left(t\right) dt \\ &= \int_0^T q_s\left(1 - e^{-r(T-t)}\right) dt \\ &= q_s T + \frac{q_s}{r}\left(1 - e^{-rT}\right). \end{aligned}$$

Modifying that still, in (100) the condition for the exhaustion time is presented in a bit clearer way.

$$-\frac{S_0}{q_s} + T = \frac{1}{r}\left(1 - e^{-rT}\right) \tag{100}$$

The condition is of the familiar form that has been popping up in all of the cases presented till. For example, comparing that to the no backstop case in (74) or to the final phase in the backstop case, the only difference is the quantity measure that is here q_s and $\hat{q}$ in the previous cases.

F Derivations in dynamic oil costs

Contrary to the previous static oil supply cases, I shall now discuss and derive quantity paths for a case in which the costs of oil are dynamic. One might consider that the dynamic oil costs would be more realistic, and that it would fit the world's oil production better than the static models.

F.1 Stock dynamics

The starting point of the analysis is again the optimization problem of the oil monopolist. That problem is the same as previously in the static oil supply cases, but now the oil costs depend on the stock. I will solve the case first partially. That is for a part of the case in which there are no discontinuities in the demand, costs, marginal marginal cost or marginal demand. Later on I will clue the needed parts together.

I will derive the dynamics for the initial phase, but same dynamics apply to the final phase also just the coefficients change. The optimization problem for the initial phase is

$$\max \int_0^T \left[\left(\alpha - \beta \left(1 + a \right) q \right) \left(1 + a \right) q - \left(\gamma + \delta \left(S_0 - S\left(t\right) \right) \right) - aqp_{bio}\left(aq \right) \right] e^{-rt}. \tag{101}$$

The formal analysis follows similarly as the ones discussed when the oil supply was static and starts with the Euler–Lagrange equation. Hence, let me write down the partial derivative of the integrand f in respect of the stock and the quantity, and the time derivative of the latter,

$$\begin{aligned}
\frac{\partial F}{\partial S} &= \delta q e^{-rt} \\
\frac{\partial F}{\partial q} &= \Big((1+a)\,\alpha - 2\beta\,(1+a)^2 q - \left(\gamma + \delta\left(S_0 - S(t)\right)\right) - a\left(\mu + 2\eta a q\right) \Big) e^{-rt} \\
e^{rt}\frac{d}{dt}\frac{\partial F}{\partial q} &= -2\beta\,(1+a)^2 \dot{q} - \delta q - 2\eta a^2 \dot{q} \\
&\quad - r\Big((1+a)\,\alpha - 2\beta\,(1+a)^2 q - \left(\gamma + \delta\left(S_0 - S(t)\right)\right) - a\left(\mu + 2\eta a q\right) \Big).
\end{aligned}$$

Here the analysis start to differ from the previous cases, because now the partial derivative in respect of the stock does not vanish. Now, the Euler–Lagrange equation is in this case

$$\begin{aligned}
\left(\frac{\partial F}{\partial S} + \frac{d}{dt}\frac{\partial F}{\partial q} \right) e^{rt} &= \delta q - 2\beta\,(1+a)^2 \dot{q} - \delta q - 2\eta a^2 \dot{q} - r\,(1+a)\,\alpha \\
&\quad + 2r\beta\,(1+a)^2 q + r\left(\gamma\left(S_0 - S(t)\right)\right) + ar\left(\mu + 2\eta a q\right) = 0
\end{aligned} \tag{102}$$

This can be modified to a typical differential equation form

$$\ddot{S} - r\dot{S} - \frac{r\delta}{2\beta\left(1+a\right)^2 + 2a^2\eta}S = \frac{r}{2\beta\left(1+a\right)^2 + 2a^2\eta}\left(\left(1+a\right)\alpha - \gamma - a\mu - \delta S_0\right). \tag{103}$$

This is the second order differential equation of the oil stock. That and conditions at the starting and ending points are all I need in solving the equation. The first step in solving is to find the special solution. This steady state specific solution can be solved by finding a solution where all dynamics is removed by setting $\dot{S}$ and $\ddot{S}$ as zero. Hence, this static, specific solution carries also the right hand side's constant, and it is

$$S_s = S_0 - \frac{1}{\delta}\left(\left(1+a\right)\alpha - \gamma - a\mu\right). \tag{104}$$

The remainder of the equation is called homogeneous equation. The second step in solving is to find the solution for the homogeneous equation. Let me write down the homogeneous equation and to simplify the following steps introduce a constant c,

$$\ddot{S} - r\dot{S} - cS = 0, \qquad \text{where}$$
$$c = \frac{r\delta}{2\beta\left(1+a\right)^2 + 2a^2\eta}.$$

Solving the homogeneous second order differential equation starts with writing down the characteristic equation. I denote the roots of the characteristic equation by w. Hence, let me try out introducing $S = e^{wt}$, and derive its characteristic equation

$$w^2 - rw - c = 0.$$

There are two real roots, out of which one is positive and the second negative

$$w_1 = \frac{r + \sqrt{r^2+4c}}{2} = \frac{r}{2} + \sqrt{\left(\frac{r}{2}\right)^2 + \frac{r\delta}{2\beta\left(1+a\right)^2 + 2a^2\eta}} > 0 \tag{105}$$

$$w_2 = \frac{r - \sqrt{r^2+4c}}{2} = \frac{r}{2} - \sqrt{\left(\frac{r}{2}\right)^2 + \frac{r\delta}{2\beta\left(1+a\right)^2 + 2a^2\eta}} < 0. \tag{106}$$

Hence, putting together results in the solution for the oil stock is the sum of the general solution and the specific solution.

$$S\left(t\right) = C_1 e^{w_1 t} + C_2 e^{w_2 t} + S_s \tag{107}$$

It remains to define the constants involved using the conditions.

Same solution applies to the final phase. In that the coefficient has just a different value, $c = \frac{r\delta}{2d_2}$. Here, d_2 is the slope of the residual demand function. Note that not the coefficient nor the roots are dependent on the minimum biofuel requirement in the final phase.

F.2 Coefficients in initial phase

When considering the solution for the oil stock in (107), the first instinct might be to turn against the constraint of the exponential function with the positive root. There are no economical or technical reasons why the stock would increase at any point – no one is pumping oil back to the well! Hence, why would there be any such term in the oil stock time path that increases with time. And further, the absolute value of w_1 is greater than that of w_2 or $|w_1| > |w_2|$. Therefore, without a heavy weight on C_2 compared to C_1, it is the positive root w_1 term that affect the stock's time path the most. Nevertheless, I keep both terms still with me, and try not to jump into conclusions.

To fully solve the time path of the oil stock and define the coefficient and the exhaustion time T, I have to use the three conditions or constraints, because there are three unknown parameters: C_1, C_2 and T. The first of the constraints defines the amount of the original stock. The second constraint is the requirement that at the end of the phase the quantity must be q_1. The final constraint is the final stock. That arrives through the optimality requirements for the last two phases. In those, specific amounts of stock shall be produced leaving a defined stock for the final phase.

Conditions

1. The stock in the beginning is S_0.

$$\begin{aligned} S(0) = S_0 &= C_1 e^{w_1 0} + C_2 e^{w_2 0} + S_s = C_1 + C_2 + S_s \\ &= C_1 + C_2 + S_0 - \frac{1}{\delta}\left((1+a)\alpha - \gamma - a\mu\right) \end{aligned} \tag{108}$$

   From this I get the first equation for the constants

$$C_1 + C_2 = \frac{1}{\delta}\left((1+a)\alpha - \gamma - a\mu\right). \tag{109}$$

2. The final quantity shall be q_1. This gives the second equation

$$q(T) = q_1 = C_1 w_1 e^{w_1 T} + C_2 w_2 e^{w_2 T}. \tag{110}$$

3. The final stock is $S(t_1$. I do not want to fix the length of the period, but the final stock defines that. The final equation is

$$S(T) = S(t_1) = C_1 e^{w_1 T} + C_2 e^{w_2 T} + S_s. \tag{111}$$

F.3 Coefficients in final phase

Let me now go to the final phase's solution. In that the quantity vanishes in the end. This is the first condition. The production lasts either till there is nothing left or till it turn to unprofitable. In the first case oil is physically exhausted and in the latter case it is economically exhausted. In both cases that defines the stock at the exhaustion. Finally, let me assume that there is such a great stock of oil that the last phase is not enough for the whole production path. In that case also the initial quantity is given. It is q_1, and that is the last condition.

Conditions

1. The stock in the end is S_T. From this, the first equation for the constants is

$$S(T) = S_T = C_1 e^{w_1 T} + C_2 e^{w_2 T} + S_s. \tag{112}$$

2. The quantity shall vanish in the end, $q(T) = 0$. The second equation is

$$q(T) = C_1 w_1 e^{w_1 T} + C_2 w_2 e^{w_2 T} = 0 \tag{113a}$$

$$C_1 = -C_2 \frac{w_2}{w_1} e^{(w_2 - w_1)T}. \tag{113b}$$

3. The quantity in the beginning of the phase is q_1. Hence, the third equation is

$$q(0) = C_1 w_1 e^{w_1 0} + C_2 w_2 e^{w_2 0} = q_1 \tag{114a}$$

$$C_1 w_1 + C_2 w_2 = q_1. \tag{114b}$$

I write explicitly down the resulting equations for solving the constants as there are two possible out comes; an economical or a physical exhaustion,

$$\frac{S_T - S_s}{q_1} = \frac{e^{(w_1 + w_2)T}}{e^{w_1 T} - e^{w_2 T}} \left[\frac{1}{w_1} - \frac{1}{w_2} \right] \tag{115a}$$

$$C_1 = \frac{q_1 e^{w_2 T}}{w_1 \left(e^{w_1 T} - e^{w_2 T} \right)} \tag{115b}$$

$$C_2 = -\frac{q_1 e^{w_1 T}}{w_2 \left(e^{w_1 T} - e^{w_2 T} \right)} \tag{115c}$$

These can be solved starting from the first. There is an important economical difference related to the exhaustion. In case of physical exhaustion the final stock, S_T vanishes. In that case there is a finite solution for the exhaustion time. Nevertheless, in case of an economically exhausting oil the stock reaches its static solution, or $S_T = S_s$. This causes an infinite long exhaustion time.

F.4 Marginal profit dynamics in price limiting

F.4.1 Direct derivation

To evaluate the length of the second, price limiting phase I shall use optimal control theory. I have to stretch it in places, but claim that the final solution fulfills the requirements. In next chapter, F.4.2 I will go the same derivation using there the bounded optimal control theory and gain same result.

I know already the marginal revenue in the beginning and at the end of the price limiting phase. Also the stock dynamics during the phase is trivial as the quantity is constant. Now, the Hamiltonian and dynamic equation of the marginal profit for the price limiting phase are

$$H = \left(p^{\dagger} q + \delta S q\right) - \lambda_t q \tag{116a}$$

$$\frac{\partial p^{\dagger} q}{\partial q} + \delta S - \lambda_t = 0 \tag{116b}$$

$$\dot{\lambda}_t = r\lambda - \delta q. \tag{116c}$$

Here, λ meets the marginal profit in the beginning and at the end of the price limiting phase. And that is what I want to solve. Here I must be careful as the marginal revenue is not defined well between the beginning and at the end of the price limiting phase. In those end points I require that marginal profits are continuous and equal to their counterparts at the end of the initial and at the beginning of the final phases. The value of the actual marginal profit is not important during the price limiting phase and I give it there the value of λ. I use λ just to mediate the value between the time instants.

Let me write (116c) a bit differently

$$\frac{d\left(\lambda_t e^{-rt}\right)}{dt} = -\delta q_1 e^{-rt}. \tag{117}$$

In this form both sides of the equations are simple to integrate, giving

$$\int_0^{t_2} d\left(\lambda_t e^{-rt}\right) = -\delta q_1 \int_0^{t_2} e^{-rt} dt, \tag{118a}$$

$$\lambda_{t_2} e^{-rt_2} - \lambda_{t_1} = -\frac{\delta}{r} q_1 \left(1 - e^{-rt_2^{\star}}\right). \tag{118b}$$

This is the condition for the marginal profit over the second phase. To get the condition for the length of the period, I will use (118b), require continuous marginal profits over the phase transitions and obtain

$$\pi_1^{\dagger} + \delta S\left(t_1\right) - \frac{\delta}{r} q_1 = e^{-rt_2^{\star}} \left(\pi_2^{\dagger} + \delta S\left(t_2\right) - \frac{\delta}{r} q_1\right) \tag{119a}$$

$$e^{-rt_2^\star} = \frac{\pi_1^\dagger + \delta\left(S\left(t_2\right) - \frac{q_1}{r}\right)}{\pi_2^\dagger + \delta\left(S\left(t_2\right) - \frac{q_1}{r}\right)} + \frac{\delta q_1}{\pi_2^\dagger + \delta\left(S\left(t_2\right) - \frac{q_1}{r}\right)} t_2^\star. \tag{119b}$$

This is the rule for the second period, and here

$$\pi_1^\dagger = \pi_1 - \gamma - \delta S_0 \tag{120a}$$

$$\pi_2^\dagger = \pi_2 - \gamma - \delta S_0. \tag{120b}$$

In the rule for the price limiting period length, I have changed the notation and $t_2^\star$ marks the length of the price limiting phase and t_2 the final time instant of that period. I have also set in the stock at the end of the period, as that is fixed due to the final phase optimality requirement. That given, the length of the price limiting phase is now explicitly on both sides of the equation. On the left side it is in the discount factor, but it is also in the right hand side's linear function of the form $k_1 + k_2 t_2$. The coefficients k_1 and k_2 of the linear right hand side are constants, when the residual demand and the minimum biofuel requirement are given. They are determined at the phase change taking into account of the discontinuity of the marginal revenue. Now, taking into account that $\pi_2 + \delta\left(S(t_2) - \frac{q_1}{r}\right) > \pi_1 + \delta\left(S(t_2) - \frac{q_1}{r}\right)$ there is a important relation between the the coefficient k_1 and k_2. If the first coefficient, k_1 is less than one, the second coefficient is positive, and vice versa. The equation has therefore in any case exactly one solution.

I did not discuss here at all if second phase is in monopolist plan, and for example if it is profitable phase. And on purpose I left the conditions for the profitability out of this analysis. All I need to carry out out from this is that if the monopolist has decided to operate in the steeper part of oil's residual demand function, she faces the price limiting phase. And that phase is always of a finite length.

F.4.2 Optimal control with a bounded quantity

I will give here also a second approach to this case with the help of bounded optimal control theory. For reference see for example Kamien and Schwartz [1991].

In the first phase the quantity path is in fact bounded. There the minimum biofuel requirement is binding and that occurs only, when the produced oil quantity is greater than q_1. Without taking this explicitly into account the quantity path satisfying the boundary condition would in cases get values lower than q_1. Therefore, I take this limitation into account. However, as I have already solved the form of the solution with the help of Euler – Lagrange equation all I need from this are the values of the coefficients.

I write down the optimization problem in case of the bound,

$$\max \int f dt \tag{121a}$$

$$\dot{S} = -q, \qquad S(0) = S_0, \qquad q \geq q_1 \tag{121b}$$

$$f = \left(P_i^{\dagger} q + \delta S q\right) e^{-rt} \tag{121c}$$

$$P_i^{\dagger} = b_1^i - b_2^i q \tag{121d}$$

$$b_1^i = \alpha(1+a) - \gamma - a\mu - \delta S_0 \tag{121e}$$

$$b_2^i = \beta(1+a)^2 + \eta a^2 \tag{121f}$$

The Hamiltonian and Lagrangian are then

$$H = f - \lambda q \tag{122a}$$

$$L = f - \lambda q + z(q - q_1). \tag{122b}$$

In the Lagrangian there is the bound explicitly written down and the variable z ensures that the bound is respected. The conditions are

$$\frac{\partial L}{\partial q} = \frac{\partial f}{\partial q} - \lambda + z = 0 \tag{123a}$$

$$z(q - q_1) = 0, \qquad z \geq 0 \tag{123b}$$

$$\dot{\lambda} = -\frac{\partial H}{\partial S} = -\delta q e^{-rt} \tag{123c}$$

The quantity path is decreasing as was already discussed by Hotelling [1931]. Therefore, if the quantity ever reached the bound, the quantity remains at the bound till the end of the period. The period ends at the stock level S_2, which was defined as the optimal for the last period.

I need still one condition, because in this case the time instant when the quantity path reaches the bound q_1 is not given. Especially that time instant can not be evaluated using the initial stock at the beginning of the final phase – resulting quantity path would not respect the bound. Therefore, I use the transversality condition for the free final time instant according to results in part II, chapter 6 in Kamien and Schwartz [1991] and mark this final time as t_1. The condition states how the salvage values has to be taken into account. Applying that to the present case, the condition at the final time, when the quantity path reaches q_1 is

$$f - \lambda q - r\Pi(t_1) - re^{-rt_2^\star}\Pi_2(t_2) = 0\Big|_{t=t_1}. \tag{124}$$

Here, the second last and the last term are the time derivative of the salvage value of the remaining production till the start of the final phase, and from that till the exhaustion, respectively. The value of the price limiting phase is $\Pi_1(t_1)$ and delaying of that production infinitesimally costs $r\Pi(t_1)$ as an uncollected rent. Here,

$$\Pi_1(t_1) = \int_0^{t_2} \left[p_1^\dagger q + \delta q S\right] e^{-rt}$$
$$= \frac{q_1}{r}\left(p_1^\dagger + \delta S_1 - \frac{\delta q_1}{r}\right)\left(1 - e^{-rt_2^\star}\right) + \frac{\delta q_1^2}{r} t_2 e^{-rt_2^\star}, \tag{125}$$

where $p_1^\dagger = p_1 - \gamma - \delta S_0$. Now, let me evaluate the last phase profit. It is

$$\Pi_2(t_2) = \int_0^T \left[P_f^\dagger + \delta S\right] q e^{-rt} dt$$
$$= -\frac{1}{r}\left[\left(P_f^\dagger + \delta S\right) q e^{-rt}\right]\Big|_0^T + \frac{1}{r}\int_0^T \left[\dot{q}\left(\pi_f^\dagger + \delta S\right) - \delta q^2\right] e^{-rt} dt$$
$$= \frac{1}{r}\left(p_1^\dagger + \delta S(t_2)\right) q_1 + \frac{1}{r}\left[q\left(\pi_f^\dagger + \delta S\right) q e^{-rt}\right]\Big|_0^T$$
$$- \frac{1}{r}\int_0^T \frac{d}{dt}\left(\pi_f^\dagger + \delta S\right) q e^{-rt} dt - \frac{1}{r}\int_0^T \delta q^2 e^{-rt} dt$$
$$= \frac{1}{r}\left(p_1^\dagger - \pi_2^\dagger\right) q_1 \tag{126}$$

Here the integration by parts has been applied twice in the second and third phases. And the second last line I used a trick, and the two integrals vanish because those state the integral of the time path of λ, see (123c) and (123a) and apply those to the final phase. Hence, the integrand itself vanishes at every time instant on that path.

The variable z is nonzero only when the quantity path has reached the bound, q_1. Therefore, when arriving to the bound, z is zero. Taking that into account at t_1 and using 123a and 124, I get

$$f - q\frac{\partial f}{\partial q} - r\Pi_1(t_1) - re^{-rt_2}\Pi_2(t_2) = 0\Big|_{t=t_1}. \tag{127}$$

To proceed, note also that by using 121c

$$f - q\frac{\partial f}{\partial q}\Big|_{t=t_1} = -q_1^2\frac{\partial P_i^\dagger}{\partial q}\Big|_{t=t_1}. \tag{128}$$

Plugging that and both 125 and 126 into 127 I get

$$q_1\frac{\partial P_i^\dagger}{\partial q} + \left(p_1^\dagger + \delta S_1 - \frac{\delta q_1}{r}\right)\left(1 - e^{-rt_2^\star}\right) + \delta q_1 t_2^\star e^{-rt_2^\star} + \left(p_1^\dagger - \pi_2^\dagger\right) e^{-rt_2^\star} = 0. \tag{129}$$

Remembering that $\pi_1^\dagger = \frac{\partial(P_i^\dagger q)}{\partial q}|_{t_1}$ and removing counter balancing terms this evolves to

$$\pi_1^\dagger + \delta S(t_1) - \frac{\delta}{r}q_1 = \left(\pi_2^\dagger + \delta S_2 - \frac{\delta}{r}q_1\right) e^{-rt_2^\star} \tag{130}$$

Finally, as I want to write this in terms of the solvable $S(t_2)$ and after reorganization the condition reads

$$e^{-rt_2^\star} = \frac{\pi_1^\dagger + \delta\left(S(t_2) - \frac{q_1}{r}\right)}{\pi_2^\dagger + \delta\left(S(t_2) - \frac{q_1}{r}\right)} + \frac{\delta q_1}{\pi_2^\dagger + \delta\left(S(t_2) - \frac{q_1}{r}\right)} t_2^\star \tag{131}$$

The condition is the same derived in the previous chapter.

F.5 Total profit

In previous chapter I already evaluated the profits for the last two phases. I continue from there and evaluate now the total profit in net present value.

Only missing part for the total profit is the profit of the initial phase. That is

$$\begin{aligned}\Pi_0 &= \int_0^{t_1} \left[P_i^\dagger + \delta S\right] q e^{-rt} dt \\ &= -\frac{1}{r}\left[P_i^\dagger + \delta S\right] q e^{-rt}\Big|_0^{t_1} + \frac{1}{r}\left[\pi_i^\dagger + \delta S\right] q e^{-rt}\Big|_0^{t_1} \\ &= \frac{1}{r}\left(p_0^\dagger - \pi_0^\dagger\right) q_0 - \frac{1}{r}\left(p_1^\dagger - \pi_1^\dagger\right) q_1 e^{-rt_1} \end{aligned} \tag{132}$$

Here I used (126) in the second line to get rid of unnecessary terms.

The total profit is just the sum of the phase's profit at present value. Using (125), (126) and (132) I get

$$\begin{aligned}\Pi &= \Pi_0 + \Pi_1 e^{-rt_1} + \Pi_2 e^{-r(t_1+t_2^\star)} \\ &= \frac{q_1 e^{-rt_1}}{r}\left[\pi_1^\dagger + \delta S(t_1) - \frac{\delta q_1}{r} - \left(\pi_2 + S(t_2) - \frac{\delta q_1}{r}\right) e^{-rt_2^\star}\right] + \frac{1}{r}\left(p_0^\dagger - \pi_0^\dagger\right) q_0 \\ &= \frac{1}{r}\left(p_0^\dagger - \pi_0^\dagger\right) q_0. \end{aligned} \tag{133}$$

To reach the final result, I used (130).

The total profit is just the same as it was in the static cost case. It depends on the initial quantity, price and marginal revenue as well as the required interest rate.

F.6 Hotelling's rule and changing biofuel requirement

In linear demand the marginal revenue is a simple function of the produced quantity. For the initial phase it is

$$\pi(q) = \pi_1 - 2d_2(q - q_1), \tag{134}$$

where $d_2 = \beta(1+a)^2 + a^2\eta$ in this case, and the slope of the residual demand in general.

From the Hamiltonian and its optimal conditions, I get the conditions for the marginal profit, λ. They similar to those in (116), and

$$\begin{aligned} r\lambda &= \dot{\lambda} + \delta q = \frac{d}{dt}\left(\lambda - \delta S\right) \\ &= \frac{d}{dt}\pi^{\dagger} = \frac{\partial \pi^{\dagger}}{\partial q}\frac{dq}{dt} \\ &= -2d_2\dot{q}. \end{aligned} \tag{135}$$

This is condition for the marginal profit in the linear demand with dynamic costs. I differentiate that in respect of the minimum biofuel requirement and solve the change in $\dot{q}$,

$$\frac{\partial \dot{q}}{\partial a} = -\frac{r}{2d_2}\frac{\partial \lambda}{\partial a} - \frac{\dot{q}}{d_2}\frac{\partial d_2}{\partial a} > 0. \tag{136}$$

The quantity decreases at a lower pace in the start. The result follows from the facts that the slope becomes steeper and the initial marginal revenue decreases in increasing biofue requirement. The latter fact arrives as the initial costs are the same, and in order to produce the same stock with the decreased demand, the monopolist has to produce a longer period and to start with lower marginal revenue.

F.7 Changing biofuel requirement

To proceed the analysis of the changing biofuel requirement, few step are needed. I will show that the sign of $\pi_2^{\dagger} + \delta\left(S\left(t_2\right) - \frac{q_1}{r}\right)$ is negative and also that it is increasing with increasing biofuel requirement. By direct evaluation is get

$$\begin{aligned} \pi_2^{\dagger} + \delta\left(S\left(t_2\right) - \frac{q_1}{r}\right) &= \pi_2 + \delta S_s - \frac{\delta q_1}{w_2} - \frac{\delta q_1}{r} - \gamma - \delta S_0 \\ &= \bar{p}^{\dagger} - 2d_2 q_1 + \delta S_s - \frac{\delta q_1}{w_2} - \frac{\delta q_1}{r} \\ &= \bar{p}^{\dagger} + \delta S_s - q_1\left(\frac{\delta}{r} + 2d_2 + \frac{\delta}{w_2}\right) \\ &= -\frac{\lambda\left(t_1\right) - \lambda\left(t_2\right)}{1 - e^{-rt_2^{\star}}} < 0. \end{aligned} \tag{137}$$

Here in the last line I used another result, namely

$$\begin{aligned}
\frac{\delta}{r}+2d_2+\frac{\delta}{w_2} &= \frac{\delta}{r}+2\frac{\bar{p}-p_1}{q_1}+\frac{\delta}{w_2}\\
&= \frac{1}{q_1}\left(\delta q_1\left(\frac{1}{r}+\frac{1}{w_2}\right)+\bar{p}^{\dagger}-\pi_2^{\dagger}\right)\\
&= \frac{1}{q_1}\left(\frac{\delta q_1}{r}+\delta S_s-\delta S\left(t_2\right)+\bar{p}^{\dagger}-\pi_2^{\dagger}\right)\\
&= \frac{1}{q_1}\left(\frac{\lambda\left(t_1\right)-\lambda\left(t_2\right)e^{-rt_2^{\star}}}{1-e^{-rt_2^{\star}}}+\delta S_s+\bar{p}^{\dagger}-\lambda\left(t_2\right)\right)\\
&= \frac{1}{q_1}\left(\frac{\lambda\left(t_1\right)-\lambda\left(t_2\right)}{1-e^{-rt_2^{\star}}}+\delta S_s+\bar{p}^{\dagger}\right)>0. \qquad (138)
\end{aligned}$$

This result is useful also in evaluating the changing biofuel requirement, where direct derivation gives

$$\begin{aligned}
\frac{\partial}{\partial a}\left[\pi_2^{\dagger}+\delta\left(S\left(t_2\right)-\frac{q_1}{r}\right)\right] &= \frac{\partial\pi_2^{\dagger}}{\partial a}+\delta\frac{\partial S\left(t_2\right)}{\partial a}-\frac{\delta}{r}\frac{\partial q_1}{\partial a}\\
&= -\left(\frac{\delta}{r}+2d_2+\frac{\delta}{w_2}\right)\frac{\partial q_1}{\partial a}>0. \qquad (139)
\end{aligned}$$

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