

A Survey into Private Equity: With a Case Study of Private Equity Allocation in Stanford Endowment

Economics

Master's thesis

Jussi Liimatainen

2012

Tiivistelmä:

Tämän tutkielman tarkoitus on tutkia yksityistä pääomaa sijoituskohteena. Tutkielmassa perehdytetään lukija yksityisen pääoman perusominaisuuksiin kirjallisuuskatsauksen muodossa. Empiirisessä osassa tutkitaan yksityisen pääoman allokaatiota Stanfordin yliopiston lahjoitusrahastossa. Kirjallisuus yksityisestä pääomasta on melko niukkaa ja vain muutamia tutkimuksia on tehty yksityisen pääoman roolista strategisessa portfolioallokaatiossa. Yksityinen pääoma on melko vaikeaselkoinen arvopaperi. Tämän ja suuren pääomavaateen johdosta se onkin varattu melkein kokonaisuudessaan suurille institutionaalisille sijoittajille. Kirjallisuudessa esiintyy paljon debattia yksityisen pääoman todellisesta tuotosta verrattuna julkisiin osakkeisiin. Osa tutkimuksista näyttää, että yksityinen pääoma olisi tuottanut vähemmän ja osa tutkimuksista näyttää, että yksityinen pääoma tuottaa enemmän kuin julkisesti vaihdetut osakkeet. Väärin hinnoittelusta, sijoittamaan oppimisesta, muista hyödyistä ja eräänlaisesta salaliitosta agentin ja päämiehen välisissä sijoitussopimuksissa on tarjottu selitykseksi siihen miksi ihmiset sijoittavat yksityiseen pääomaan vaikka tutkitusti se on korkeariskisempi eikä näytä tarjoavan merkittävää preemiota julkisiin markkinoihin nähden.

Empiirisessä osiossa havaitaan, että historiallisen tiedon valossa, 12 % allokaatio yksityiseen pääomaan Stanfordin lahjoitusrahastossa on ymmärrettävää vain jos kyseistä rahastoa voidaan pitää korkeana riskin kantajana jolla on suuri painotus pääomiin (yksityiseen ja julkiseen) portfolioissaan.

Abstracts:

The purpose of this thesis is to study private equity as an asset class. The thesis familiarizes the reader with the basic aspects of private equity in the form of literature review. In the empirical section I will study the private equity allocation in Stanford endowment fund. The literature regarding private equity is scarce and only a few studies have been conducted about private equity in the framework of strategic asset allocation. Private equity is a rather obscure asset and because of this and large capital commitments required it is usually reserved to large institutional investors. There is a lot of debate in academic literature regarding the actual returns of private equity. Some studies show excess returns over common stocks and some studies show underperformance or similar returns. Learning, mispricing, side benefits and even conspiracy between the limited partner and general partner in private equity investing has been offer as an explanation why investors choose private equity in their portfolios (mixed views of returns and known high volatility). In the empirical section of the study I found that the 12% policy portfolio allocation in Stanford endowment can only be justified if we consider the fund a high risk bearer with high emphasis on equities in its portfolio.

Contents

Contents	3
List of Tables and Figures.....	5
1 Introduction.....	7
1.2 Personal motivation.....	9
1.3 Thesis Scope	9
1.4 Structure of study.....	10
2 Literature Review.....	10
2.1 Overview.....	10
2.2 Difference Between Public- and Private Companies.....	11
2.3 Structure of Private Equity Investing.....	13
2.4 Investment Strategies	15
2.4.1 Venture Capital	16
2.4.2 Leveraged Buyout.....	16
2.5 Private Equity Returns	16
2.6 Risks and Correlations of Private Equity.....	20
2.7 Explaining the Returns and Risks	21
2.8 Endowment Funds.....	22
2.9 Strategic Asset Allocation.....	23
2.10 What is an Efficient Allocation to Private Equity.....	24
3 Research Methodology	25
3.1 Modern Portfolio Theory	25
3.2 The Markowitz Model and Mean Variance Optimization	26
3.2.1 Matrix Formulation.....	28
3.2.2 Issues with Using Modern Portfolio Theory to Evaluate Private Equity.....	29
3.3 Efficient Frontier.....	30

4 Data.....	31
4.1 Different Asset Classes in the Stanford Endowment	32
4.3 Historical Data on Stanford Endowment	32
4.4 Comparative Data from Other University Endowments.....	35
5 Analysis.....	38
5.1 Implementation by MATLAB	40
5.2 Analysis by Using Historical Risk Component	41
5.3 Analysis by Using Correlation Estimates from Literature.....	46
5.4 Optimization with Asset Constraints	48
5.5 Speculating the Results	50
6 Conclusion	52
References.....	54
Appendix A.....	57

List of Tables and Figures

Figure 1: PE Asset Class	11
Figure 2: J-Curve (JPMorgan, 2008)	14
Figure 3: PE Investment Structure	14
Figure 4: Efficient frontier With 1000 Random Assets	30
Figure 5: Stanford Policy Portfolio Allocations	33
Figure 6: Mean, Standard Deviation, Minimum and Maximum Values of Asset Return During the Observation Period.....	34
Figure 7: PE Returns by Endowment 2011 - 2009	37
Figure 8: The Capital Market Line	38
Figure 9: Efficient Frontier with Purely Historical Data	42
Figure 10: Efficient Portfolios with Purely Historical Data (case 1a)	43
Figure 11: Efficient Portfolios with Historical Risk Component and Adjusted Returns (case 1b) ...	44
Figure 12: Efficient Portfolios with Historical Data, 50% Equity Oriented Portfolio (case 3a)	45
Figure 13: Efficient Portfolios with Adjusted Returns, 50% Equity Oriented Portfolio (case 3b)....	46
Figure 14: Efficient Portfolios with Historical Return and Risk Component from Literature (case 2a).....	47
Figure 15: Efficient Portfolios with Historical Returns, Risk Component from Literature and 50% equity constraint (case 4a).....	48
Figure 16: Efficient Portfolios with Purely Historical Data, Asset Bounds 5-20% (case 5)	49
Figure 17: Efficient Portfolios with Purely Historical Data, Bounded Asset Allocations, 50% Equity Constraint (case 6)	50
Figure 18: Historical returns, standard deviations and mean returns.....	57
Figure 19: Esimated Correlations and standard deviations (Bekkers, Doeswijk, Lam 2009)	57
Figure 20: Efficient frontier allocations with purely historical data, PE expected return 27%	58
Figure 21: Efficient frontier with Different Assets with Purely Historical Returns, Risk Component from Literature	59
Figure 22: Efficient Portfolios with Adjusted Return and Risk Component from Literature (case 2b)	59
Figure 23: Efficient Portfolios with Adjusted Returns, Risk Component from Literature and 50% Equity Constraint (case 4b).....	60
Table 1: Differences Between Public- and Private Equity Backed Firms (source ipeit.com)	12

Table 2: Returns per Asset Class in Stanford Endowment 34

Table 3: Average Return by Asset Class (NACUBO) 36

Table 4: Summary of Cases Analyzed 40

Table 5: MATLAB Return Series Column and Row Composition 40

Table 6: Purely historical correlations and standard deviations 41

Table 7: Historical Expected Returns 41

Table 8: Adjusted Expected Asset Returns 44

1 Introduction

Private equity provides capital to corporations not listed on any public exchange. It can be used in the same operations as public equity like strengthen the balance sheet or develop new products.

Private equity (PE) is a broader concept with many subsets. For example venture capital (VC) is a form of private equity investment.

PE is an asset class among many others and it can be used as a tool by investors to diversify their holdings. Most of the private equity raised for investments comes from institutional investors such as banks, pension funds and insurance companies (EVCA).

PE funds that directly invest in private companies are usually arranged as limited partner and general partner setting, where limited partner provides the general partner (fund manager) capital which the general partner then invests.

There exists a lot of academic debate about the performance of PE and there are still some aspects of it that are not understood profoundly.

The aim of this thesis is to familiarize the reader with the very basis of PE. I will concentrate specifically on the aspects of PE as an asset class, returns, risks and typical investments schemes. In the second part I will conduct a simple mean variance optimization (MVO) by using data from Stanford University Endowment to find out if the policy portfolio (passive) allocation of 12% to PE in Stanford endowment (SE) can be justified in the light of historical data and current literature.

The data is gathered from annual reports published by Stanford Management Company (SMC) that governs the endowment fund. Finding good data for PE research is notoriously difficult due to lack of transparency and efficient market pricing. The freely available data for PE is voluntarily given and as such could suffer from selection bias (i.e. we only observe funds that do not fail and thus the data does not give the true risks and returns, failed funds don't report statistics). Most data used in current research is gathered through Thomson Venture Economics system (or other commercial portals like Bloomberg), but that portal is not publicly available to all users and as such is out of my reach. SMC has chosen PE as an asset in its portfolio, which they have invested as a limited partner (the actual investment to the private companies are made by the general partner), mainly to VC and buyout funds (LBO).

In the empirical section of this thesis I find that the 12% allocation to PE could be justified only if we consider Stanford endowment as a high risk bearer with a strong emphasis on equity assets. In unconstrained optimization with historical returns and risks, PE did not get any allocation to the efficient portfolios. Constraining the optimization so that 50% of all assets should be allocated to equity assets (both public and private) we find that PE gets allocated to the riskiest portfolios. The findings in my empirical study are similar to that of Ibbotson (2007) and Ennis & Sebastian (2005) which both find that PE should only get allocation to portfolios with high risk bearing capabilities and strong emphasis on equity assets.

1.2 Personal motivation

PE encompasses a lot of problems and opportunities from economics point of view. The scarcity of data, illiquidity and other multidimensional aspects makes it challenging and interesting choice, but the driving force in selecting this subject was the fact that the literature concerning strategic allocation of PE is minimal if non-existing.

PE can also be considered as a “rising star”. More and more players in the financial markets are viewing it as a lucrative opportunity to hedge. PE, especially VC, is also one of the main contributors to research and development. As VC investments are made to small growing firms they also act as a catalyst for new innovations, for example investments in PE has enabled the emergence of many modern day giants like Google and Baidu (Metrick & Yasuda, 2010)

Over the past decade, the funds raised by PE funds have been increasing steadily. European PE funds invested a record sum of 36.9 billion € in about 7,000 portfolio companies in 2004. This represents a 27% increase from the 29.1 billion € invested in 2003 and a 52% increase since 2001 (EVCA) and worldwide PE funds manage approximately 1 trillion worth of assets (Phalippou, 2007)

There is also a danger to the rise of PE. Public companies are very transparent. They have annual shareholder meetings, release regular earnings releases and annual reports. When such a company is taken over with private equity, all of those things vanish from the public. This would lead to information blackout and loss of transparency on the markets (Cheffins & Armour, 2007). While no doubt, a subject of great interest, it is out of the scope of this thesis.

1.3 Thesis Scope

My aim is to give broad insight in the form of literature review to the world of PE. I will concentrate specifically to PE investment strategies, structure of PE investing (limited partner-general partner setting), risks and returns and how PE can and is used in strategic asset allocation. I will also go through the basis of asset allocation and its significance on portfolio performance.

I will omit such topics as agency problems in limited partner – general partner deals, taxation issues and business ethics.

In the empirical section I am trying to answer whether the 12% allocation to PE in SE can be justified in the light of historical data and current literature. I will limit my analysis on mean variance optimization (MVO) because it is the most widely recognized concept of portfolio valuations methods (CAPM, and constrained value at risk methods could also be considered even though their inputs are a bit different) and also one of the easiest to implement in practical analysis.

1.4 Structure of study

The structure of the study is as following. The second chapter is a literature review in to the world of private equity. Third chapter presents the research method. Fourth chapter describes and discusses the data used. In the fifth chapter I will conduct my analysis and chapter 6 concludes.

2 Literature Review

In this literature review I will go through the basic organizational structure and aspects of PE firms and funds (the main structure of PE investing). Then I will move into the returns and risks associated with PE. Then I will discuss endowments; what they are and how they work. Lastly, strategic asset allocation is discussed. The aim of this literature review is also to make the reader better understand the empirical part of this thesis.

2.1 Overview

Unlike public equity, PE is not traded in any stock exchange. With public equity investments an investor constantly knows what the market value of his/her investment is; he/she can buy more or sell the investment at almost any time. That is not the case with private equity as there are no markets to set the price and provide or offer a trading platform.

PE is usually perceived as an asset class reserved for large institutional investors mainly due to large capital commitments required. “Average Joe” can however still invest in PE by buying stocks

from firms that invest in PE. This kind of PE is called listed private equity (PE firms invest to private companies, but the PE firm itself is listed and thus its shares have a market price). Hence by this indirect investment, most private investors can attain exposure to PE.

PE is usually *perceived* to have high risk and above average returns (which has been challenged by many recent studies), illiquidity, long investment horizon and inconsistent valuations. Investment professionals usually describe PE as difficult asset class to invest in and the success in PE investment is heavy dependent on the skill of individual investor/manager (Brown, Garlappi, Tiu (2010))

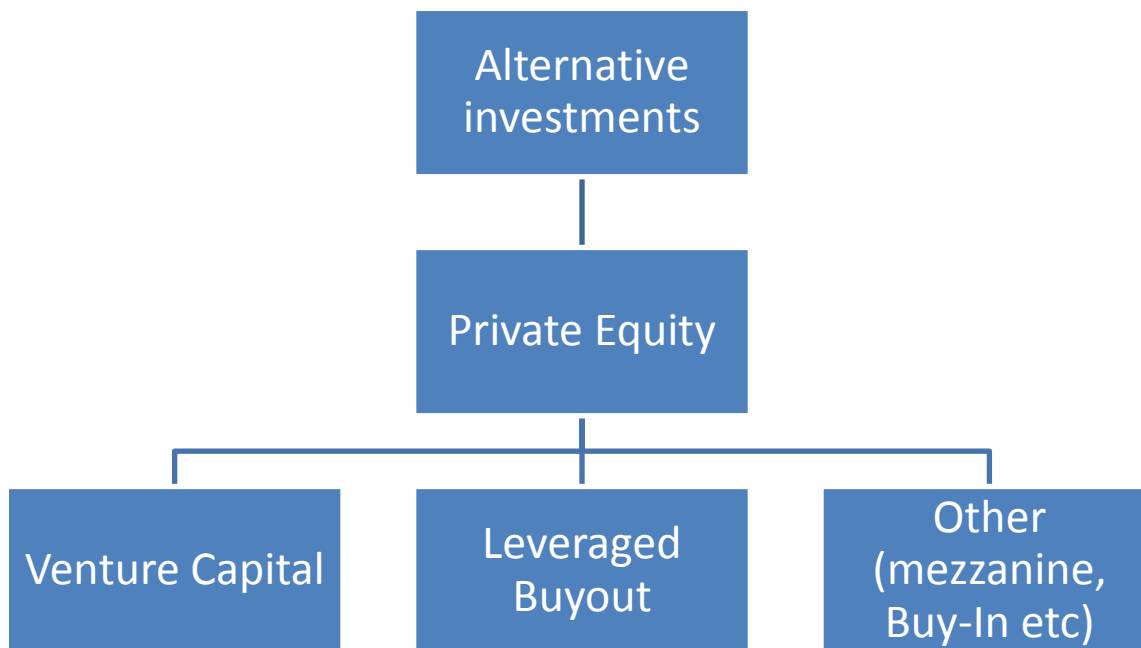


Figure 1: PE Asset Class

In figure one is listed the main structure of PE and its subsets. PE is thought to belong to the broader category of alternative investments with VC, LBO and mezzanine capital as its subsets. Mezzanine capital is a form of capital that has characteristics of both growth equity and debt (Metrick & Yasuda, 2009). I will omit the study of mezzanine capital from this study and concentrate on the two main subsets, VC and LBOs.

2.2 Difference Between Public- and Private Companies

The differences between public and companies backed by PE can be summarized as follows.

source ipeit.com	Public	Private
Number of shareholders	Large number of small shareholders	Low number of large shareholders
Owners involvement in operations	Most shareholders have little or no operational input	Private Equity investors often on the board and involved operationally
Management commitment	Difficult for management to have a meaningful economic interest in the company	Management normally very highly incentivized
Decision making	Public companies often concentrate on short-term earnings figures - can make it hard to take tough decisions if it hits earnings	Shareholders not concerned about taking tough decisions if that is the optimal strategy
Decision making speed	Need to seek shareholder approval for large transactions - costly,slow and time consuming	Very quick decision making process means companies can move swiftly and keep costs down
Leverage	Tend not to use much leverage - suboptimal WACC?	Happy to employ large amounts of leverage - probably closer to optimal capital structure
Organizational changes	Difficult for shareholders to change management	Changes to management are easy
Legislation	Increasing regulation and disclosure requirements	Relaxed regulation and disclosure requirements

Table 1: Differences Between Public- and Private Equity Backed Firms (source ipeit.com)

Basically PE backed companies are much more dynamic than their private counterparts and they are much less limited by regulations that apply to public companies.

2.3 Structure of Private Equity Investing

PE funds are usually arranged as general partner - limited partner contract, where the limited partner (LP) supplies capital for the general partner (GP) to invest in its funds. It is customary for the GP to invest at least 1 % of the total capital required to the fund (Kaplan & Strömberg, 2008). LPs are usually institutional investors (or wealthy individuals) and the general partner is the firm that governs the fund.

In typical agreement the LP commits to provide a certain amount of capital which the GP then “calls” from the LP during the investment phase. It is important to note that the GP might not call all of the capital in the beginning but rather calls it over the year the fund is active (Phalippou, 2007). In this “investment phase” the GP looks for lucrative investment opportunities. Once such opportunity is found, the GP calls the committed capital from the LPs to make the necessary investments. After the investment the GP has a limited time in which to return the investment and the cost of capital to the LP.

The PE firm has usually around five years to invest the capital committed to the fund. Ljungqvist & Richardson (2003) state that it takes around 6 years for 90% of the capital to be invested. The fund then has an additional five to eight years to return the capital to its investors after which the fund dissolves (Kaplan & Strömberg, 2008). Typical life of a fund is around 10 years but can be extended (for up to 3 (Kaplan & Strömberg, 2008) to 4 (Phalippou, 2007) years).

PE firms usually raise funds every three years. According to Kaplan & Schoar (2005) better performing funds have easier time to raise capital for follow-up funds, which would imply that knowledge of previous success plays a pivotal role when investors decide which fund to invest in.

This cyclical life of a PE fund can be depicted with a so called “j-curve”.

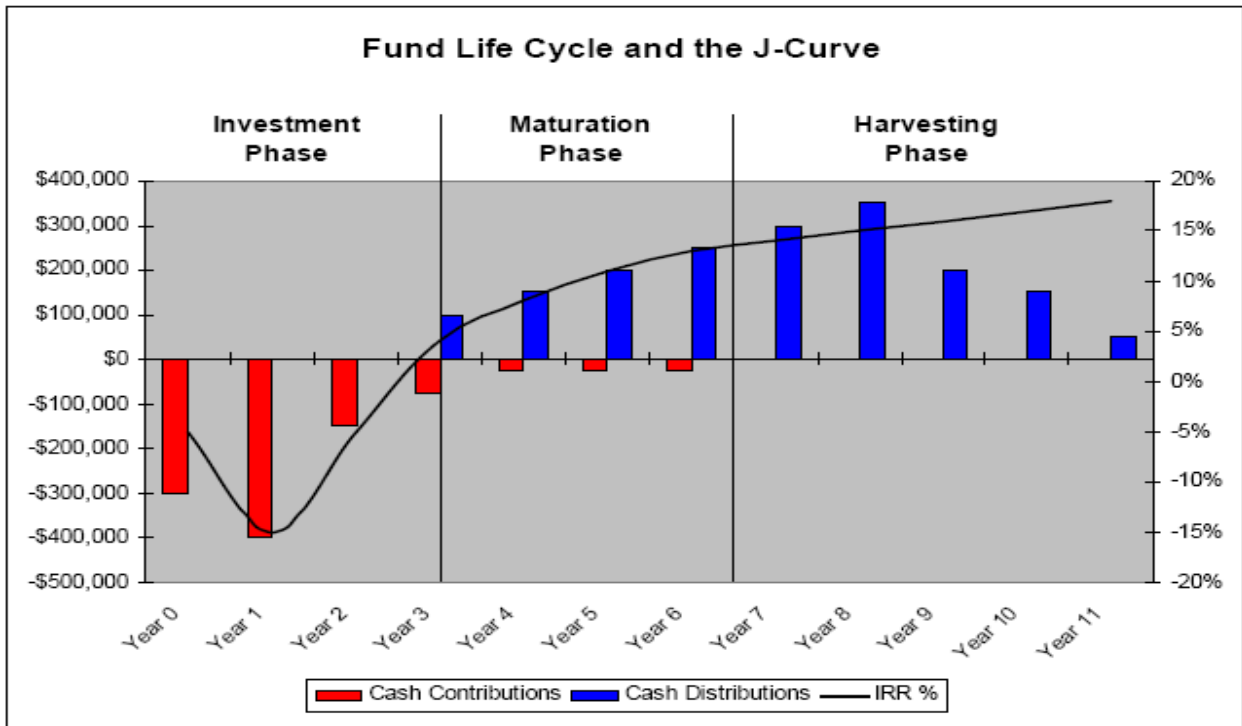


Figure 2: J-Curve (JPMorgan, 2008)

A PE firm can govern multiple funds at the same time, choosing either VC or LBO investment by fund (Phalippou, 2007). Detailed structure of PE firms, funds and investors are depicted in following figure 3.

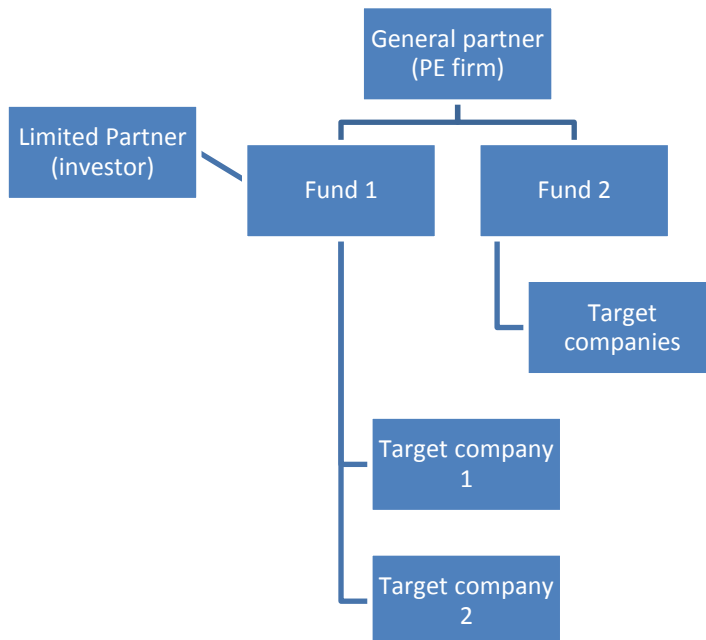


Figure 3: PE Investment Structure

The investments made in the target companies are not just passive investments. PE managers actively manage their portfolio companies by offering their own expertise to the company with the aim of increasing the value of their investment. In practice this means a seat on the board, which they in reality control (Kaplan & Strömberg, 2008).

After the limited partner has made the investment, it has very little to say how the PE firm invests the capital, as long as the basic covenants of the fund agreement are followed. Common covenants include restrictions on how much fund capital can be invested in a single company, the types of securities a fund can invest in and restrictions on debt at the fund level (Kaplan & Strömberg, 2008).

2.4 Investment Strategies

PE is an asset class with two main “sub-strategies” VC and LBO. Academic study is usually conducted in the broader category PE, because both VC and LBO share the same characteristics (like illiquidity, fee structure and so on) (Phalippou, 2007). However there are differences in risks (standard deviations) and returns (see for example Cochrane (2005) and Kaplan & Schoar (2005)). This gap between asset class and investment strategy can be confusing for investors. PE investment is never a passive investment to companies that form the PE asset class. It is an allocation to a skill-based strategy in which two primary “sub-strategies” are chosen, leveraged buyout or venture capital (Ibbotson, 2007).

Of all the invested PE capital, about 2/3 is invested in LBO funds and 1/3 to VC funds (Metrick & Yasuda, 2009).

Because of the fragmented nature of PE markets, the investor cannot fully diversify the corporate specific risk. Hence all investments private companies are a mixture of systematic risk related to the company in question and the asset class itself (Ibbotson, 2007). This would imply that PE investments are always riskier than investments made in public equity.

2.4.1 Venture Capital

VC is basically an investment to a small, typically young growth firms. The aim is to create value by injecting capital into the young business and helping the business to mature and generate value by capitalizing on the expertise of the manager (Kaplan & Strömberg, 2008).

VC is sometimes used as a synonym for PE, after all, all the capital that “business angels” or venture capitalists inject into these firms is private. These business angles usually have some knowledge of a certain industry or aspect of the company they are investing in. They inject capital into the firm and offer their expertise in developing the company, usually however in an exchange for a big share of the company.

These VC investments can be characterized as hectic and “hands-on”.

2.4.2 Leveraged Buyout

In a LBO the company is acquired by a specialized investment firm using a relatively small portion of equity and a relatively large portion of outside debt financing (leverage). According to Kaplan & Strömberg (2008) the amount of debt is usually around 60% up to 90% (hence the name *leveraged* buyout).

In a typical leveraged buyout transaction, the private equity firm buys majority control of an existing, usually mature firm. This is distinct from VC firms that typically invest in young or emerging companies, and typically do not obtain majority control (Kaplan & Strömberg, 2008).

2.5 Private Equity Returns

Capital commitments to PE have increased from 5 billion USD in 1980 to 300 billion USD in 2004. This increase is at least to some part attributable to the perceived high returns (Phalippou & Gottschal, 2009). There however exists a wide discrepancy between different academic studies regarding PE returns.

Before we can discuss about returns we must first understand how PE is evaluated since there are no markets to set the price (exception being listed private equity which some studies have used as proxy like Brown, Garlappi & Tiu (2010)).

Usually the valuation is carried out by using internal rate of return (IRR) method of the investment from its inception (or as PE industry calls it, “vintage”) to dissolution. This is usually how GPs report their performance to LPs. This method is also widely used in academic research to analyze performance. There has also been criticism about using IRR as a performance measure, like upward biasness (Phalippou & Gottschal, 2009) and there exists other performance methods like the “multiples” method (which is mainly used in academic research only). The pros and cons of these different methods of analyzing the performance are however beyond the scope of this thesis.

Since there are no enforcing guidelines for PE valuation it’s always to some degree at the discretion of the GP how it reports its performance, as can also be deducted from SMC statement on its 2011 annual report (p.51) “The NAV [net asset value] is reported by the external investment managers, including general partners, *in accordance with their policies* as described in their respective financial statements and offering memoranda”.

It is also important to mention that there exists a big heterogeneity amongst the funds when it comes to return. Bigger and more mature funds tend to make much larger profits (Kaplan & Schoar, 2005).

Kaplan & Schoar (2005) found out that the fund return, net of fees, was not much different from S&P 500 (big heterogeneity among the funds). The returns strongly correlated with the funds raised by the fund. Better performing funds are more likely to raise more funds on the follow-up period (a period after a fund has dissolved and new funds being introduced) and generally raise larger funds than funds that are performing poorly. They also found out that market entry in the PE industry is cyclical. Funds started in boom times are less likely to raise follow-up funds, suggesting that these kinds of funds perform worse. This could be interpreted that in boom times people with less skill managing PE will start funds, and the market mechanics eventually separate the sheep from the goats.

Ljungqvist & Richardson (2003) found out that PE has actually outperformed public markets by around 6%. Unlike Kaplan & Schoar (2005) who extracted their data from the Thomson Venture Economics (TVE) service which data relies on self-reported statistics (which might lead to selection bias) and also observe both realized and unrealized investments which could distort the data set, Ljungqvist & Richardson (2003) used a unique data set that depicts actual cash flows on fund level

(TVE observes data on aggregate level) to different investment. Similar finding that of Ljungqvist & Richardson (2003) are reported by Higson & Stucke (2012) who also find returns over S&P 500 with buyout funds.

Phalippou & Gottschal (2009) found that PE funds underperform public markets 3% (net-of-fees) and up to 6% risk adjusted but outperforms S&P 500 gross-of-fees by 3%. It is worth mentioning that they use largely the same dataset that of Kaplan & Schoar (2005) extracted from TVE.

When it comes to data from TVE a serious problem was raised by Stucke (2011). According to this study it seems that many funds stopped reporting data in 2001 but contrary of removing them from the database, their net asset values were always rolled to the next observation period. These kinds of funds might be “living deads” as described by (Phalippou, 2007) i.e. they are failed investments that haunt the database. But because their net asset value is constantly rolled forward it follows that the fund level IRRs drop over time (which is used as a proxy for evaluating returns). This would lead to under estimation on PE returns when using TVE databases. This finding was also confirmed by Harris, Jenkinson & Kaplan (2012) who by studying multiple different databases found out that average US. Buyout fund has outperformed the S&P 500 more than 3% per year. They also found out that even though VC funds have outperformed public markets in the 1990’s they have underperformed in the 2000’s.

There is also some evidence that the low PE returns are the results of excessive fees charged by the GPs. There are usually two types of fees that the GP collects from LPs. First there are management fees which are typically around 2% of the committed capital of the LP. Secondly is a fee called “carried interest” which is usually charged upon successful exit (succeeded fund). These “carried interests” are usually around 20% but the calculation of it can be quite complex and thus I won’t go deeper into it. Combined, the fees usually add up around 7% p.a. (Phalippou, 2007). Big fees also imply that PE managers must add some serious value to the investment, why else pay these fees?

An interesting finding was made by Lerner, Schoar & Wong (2007). They measured the performance of PE in the point of view of different LPs. They found out, that particularly private endowments seem to generate much higher IRR than other LPs. They beat the other LPs by whopping margin of 14%. I will get back to this “best pickers” argument result in chapter 2.7.

Alternative method of evaluating returns of PE is by using private equity indices. This kind of approach was taken by Ibbotson (2007) and Bekkers, Doeswijk & Lam (2009), findings of the latter I will be using in my own analysis. They used different PE indices to estimate returns and risks of

PE investment and also constructed efficient allocations. However the use of indices as a measure of performance and risk are not yet fully understood in the literature, but as they will more or less give PE a market mechanism they at least give some tool to study PE in mean-variance setting and also neutralize the appraisal bias that haunts studies that use datasets like TVE as its data source. As previous academic literature that concentrate on cash flows (either at aggregate GP level or fund level) to calculate PE returns gave pretty grim picture about PE, the Ibbotson study of 2007 showed quite the contrary. PE has returned significantly more than public markets when listed PE is used as a proxy.

Even if the returns of PE have been debated a lot in the academic literature, it does not however reflect to the expectations of LPs, at least when it comes to endowments. A survey conducted in 2009 by Preqin shows that even though there exists contradicting evidence in the literature about the true performance of PE, 97% of endowments expects their PE investments to beat the public market.

Since the expectation of returns and volatilities are pivotal when determining asset allocation to a specific asset, this issue also concerns my own analysis, because as we see later, the historical data I have at my disposal gives quite mixed results when determining whether or not the PE allocation in SE has been justifiable.

2.6 Risks and Correlations of Private Equity

On the contrasts to fees, there is no academic debate whether or not PE is riskier than public markets, it certainly is. According to Bekkers, Doeswijk & Lam (2009) PE was estimated to have a standard deviation of 30% with public markets having a standard deviation of 20% (measured by using listed PE). Finding of Phalippou (2007) also support the fact that PE markets are much more volatile than public markets. There are some private investment groups that also publish their research of PE, but since these studies seems in general to show a tendency to report PE in a very positive light (low risk, high return) it might be because these research papers are used as material to possible investors when raising capital to new funds, so I will mostly omit these findings (see for example Partners Group (2012)).

Like stated above, there are two main perspectives to analyze the performance of PE, using appraisal based data from sources such as TVE or by using listed private equity as a proxy to depict the behavior of PE. Because the illiquid nature of investments in PE and the fact that returns are not fully realized until the end of the fund life, measuring risk using standard time-series correlations with market and other assets is difficult (Metrick & Yasuda, 2009). This problem however can be overcome by using listed private equity as a proxy. How well listed private equity actually measure the performance and risk has not yet been determined in academic literature. PE indices like LPX America, follows different listed private equity companies across U.S. and from the performance of those underlying companies, form the index.

In figure 19 in appendix A is summary of those parameters estimated by Bekkers, Doeswijk & Lam (2009). It shows a correlation of PE and public markets to be 0,8 which is similar to that of Ibbotson (2007) which shows a correlation of 0,7. Phillips, Hager & North (2008) estimated a correlation as low as 0,5 but this study is again a investment management firm research paper. My historical data set shows a correlation of 0,9.

The correlations and risk are highly dependent on 3 things. First what method was used to construct the correlations (indices or appraisal based estimations like TVE), what was the data source (which index, which database) and thirdly what kind of weights were used to approximate the values.

However there is no academic consensus on the correlations of PE (not studies much because of the lack of good benchmark). As the asset mix in the study of Bekkers, Doeswijk & Lam (2009) is quite similar to that of SE; I will be using parameters of that study in my own analysis.

2.7 Explaining the Returns and Risks

What we have learned from the previous two sections is that there is no consensus on PE returns and most of the academic studies show mixed results. On the average it seems that PE has no substantial premium on public equity. Since there however is evidence on the higher risk involved in PE investing, the question arises, why would anyone want to invest in an asset that offers lower returns than the less riskier public markets?

Phalippou (2007) offered four explanations to this bizarre phenomenon. First he claimed that GPs learn to invest as time passes, a finding which is also supported by Kaplan & Schoar (2005) and Lerner, Schoar & Wong (2007). Funds owned by mature GPs that tend to make superior profits are usually oversubscribed and those LPs that have previously been investing in the funds of that GP usually take precedence over new investors. This means that those better performing funds might not be available to all investors. This could lead to a situation where LPs might want to commit to funds governed by inexperienced and possibly poorly performing GPs just to get the precedence in the follow-up funds if the early investments are successful. Thus LPs are constantly screening for new GPs in the business to pick out the better performing ones. This learning exists on both sides of the equation, GPs learn to be better in investing the funds committed, and LPs learn to screen GPs more efficiently and pick those GPs that will perform the best (also supported by big difference in expected returns between difference LP's Lerner, Schoar & Wong (2007))

Secondly it is quite possible that investors have simply mispriced PE or do not fully understand the fees involved in PE investing (usually GPs report performance gross-of-fees when they seek new LPs). Both of these are due to the lack of skill and understanding of PE as an asset class and investment process. As stated above the current information about the profitability of PE is mixed and the lack of benchmarks makes asset allocation to PE very difficult.

Third, there might also be some side benefits engaging in PE investment with some GP in the sense that maximizing the return of the investment is not actually the main goal of the LP. Ljungqvist & Richardson (2003) found out that the firms in their dataset mainly invest in PE because they want to engage in other business ventures with the GP (these other businesses might be for example advisory work on underwriting securities). Also since most PE investors are institutional investors like pension funds and endowments, there might be some other broader economic incentives to invest in them like boosting the local economy.

Lastly Phalippou (2007) argued that there might be some kind of conspiracy going on and that the GPs overstate their performance in their reports to the LPs. The main arguments are that the managers actually handling the PE portfolio of the LP are happy to get overstated performance measures from the GPs since their incentive bonuses and salaries are usually tied to the performance of the asset class under their management. The GPs is also happy with this arrangement since he can charge higher fees. If it actually is the case that there is a lot of “air” in the reported figures of the GPs, it might threaten the existence of the whole industry as we know it. Of course allegations like this also increase the pressure to standardize and monitor the industry more rigidly.

2.8 Endowment Funds

Since my main data source comes from an endowment it is good to go through what endowments really are.

Endowment funds, like the name suggests, are funds that are primarily operated by contributions and donations (financial endowment). They are mainly used by non-profit institutions such as universities, hospital, churches etc. Endowments get their funding from philanthropist, corporations, governments etc. In some countries, donations to endowments are tax deductible (NACUBO). In PE investment they are LPs who provide capital to GPs.

Academic institutions particularly use endowment funds to finance a portion of their operating expenses. When talking about university endowments, they usually have two major objectives. Firstly they must preserve their long-term value by beating the inflation. Secondly, they must provide a steady income for the university to help it run its operations.

University endowment funds typically invest in various assets like public equity, private equity, natural resources, real estate and fixed income to protect itself against a slump in a single asset category. This of course is the very basis of diversification.

The endowment funds give the universities, especially the richest ones, a big advantage. A successful endowment fund means more income for the university to use on its operations. This means, higher quality teaching by lowering the student-faculty ratios and getting the best professors.

On the average, the bigger endowments yield greater returns. This could be because the bigger the fund, the more it has possibilities to invest in more riskier and possibly much more lucrative assets (NACUBO). This has led to speculations that in the future we might see even more of these “super-elite” schools that have the financing to operate a very effective schooling system, with lower teacher/student ratio and with the best professors around the world.

2.9 Strategic Asset Allocation

Strategic asset allocation decision is widely considered the most important decision an investor has to make. Brinson, Hood and Beebower found out in their 1995 study that over 90% of fund performance variation can be explained through their policy portfolio selection. This explanatory power of a single decision is huge. Combined with the market timing decision the explanatory power rose up to 95%. Ibbotson & Kaplan (2000) found out that asset allocation explained 40% of cross-fund performance but nearly 100% of the performance at the fund level.

Current literature usually divides the asset allocation decision in to three steps. First, there is the passive allocation decision or policy portfolio. This policy portfolio identifies the long-term asset allocation and the assets in the portfolio selected to control the overall risk and meet funds objectives. These decisions in the case of endowment fund or institutions in general are made by the board that governs the entity (in the case of Stanford the decision is done by SMC)

Benchmark return is a consequence of this policy portfolio. The benchmark return is calculated by using the asset allocation in the policy portfolio and the benchmark return expected from that asset class. Funds try to outperform the benchmark return, by actively managing the fund.

The second decision is the active investment decisions made that deviate from the policy portfolio. This is usually called market timing decision. In reality the weights in the fund do not stay static over the course of the year, but instead they fluctuate. These active allocation decisions (market timing decisions) are not made by the board, but the managers that are in charge of the day to day operations of the fund. This is called active management.

Lastly there is the security or asset selection. This is simply choosing the right assets in to the fund to meet with the risk and funds objectives.

In my analysis I will study the allocation to PE based on the passive policy portfolio of SE, since I don't have the data on the actual market timing allocations.

2.10 What is an Efficient Allocation to Private Equity

The topic of efficient asset allocation to PE has not been studied a lot in literature. The only studies I was able to find was that of Ennis & Sebastian (2005) and Ibbotson (2007). The Ibbotson study is a corporate research paper by Ibbotson Associates and thus could be subject to some amount of bias, but since Ibbotson associates do not manage any funds (only provide consultation) and in the absence of academic literature I chose this one as the second paper on this subject. Ennis & Sebastian (2005) found out first of all, that largest diversification benefits gained from PE investment are experienced by equity portfolios rather than balanced ones. They conducted their research in quite similar way than I will do in my analysis i.e. they used mean variance analysis. They argue that only moderate size investors with equity balanced portfolios with substantial skill in PE investing should allocate more than 10%.

The findings of Ibbotson (2007) and Ennis & Sebastian (2005) were much alike, they too found out that PE investment most benefit equity oriented portfolio with high risk bearing capabilities and preferably only those institutional investors should invest large proportions that have access to the best GPs. They however also concluded that the role of PE in well diversified asset allocation is not yet fully understood. They concluded in similar way as Ennis & Sebastian (2005) that only aggressive investors with a very high skill and knowledge of PE should commit more than 10% of their portfolio to the asset.

Since SE is in the top 5 of largest endowments in the U.S. we can conclude that it is of adequate size to invest more than 10% to PE. They have also had a long policy of investing in PE and have maintained allocations between 10-14% in their policy portfolio. This long and quite big exposure to it would suggest that they have received some benefits from it, other vice they would have not kept exposure of this magnitude in their policy portfolio for so long. This could imply that they are better at picking up good GPs and have significant knowledge of PE and the skill to invest in it. As stated previously, private endowments seem to get the most performance out of their PE investments compared to other LPs. Taking all this in consideration it would seem that 12% to PE is perfectly justifiable as an allocation to PE in the case of SE.

In the following chapters I will test this hypothesis in practice by using purely historical data from SE from 10 year period, after which I adjust my analysis by taking correlations from other study because the biggest drawback of my own analysis are probably correlations since 10 years of yearly data is not very good dataset to form covariance matrix.

3 Research Methodology

As stated earlier, the main difficulty in analyzing private equity is the lack of available data. I have chosen Stanford University Endowment fund as my main source of data. It depicts yearly returns and policy portfolio allocations in their different asset classes from the past ten years.

In the following chapters I will first discuss the method I will be using to conduct my analysis and describe the underlying data.

3.1 Modern Portfolio Theory

The modern portfolio theory is largely based on the seminal work of Harry Markowitz and the 1952 paper “Portfolio Selection”. In his paper Markowitz divides portfolio selection in two stages. The part is where we observe historical returns and variances and the second part is to use that information to generate expectations of the future.

The theory itself is very simple and assumes among other things that markets are efficient and that agents in the market are risk averse; from the bundle of two identical portfolios they will choose the one with less risk. Variance is used as a proxy for risk and the mean of historical value can be used as expected returns.

The idea behind the model is to find a set of diversified portfolios that maximizes the return at a given level of risk measured by the standard deviation of that portfolio. This concept is more generally known as mean variance optimization.

Next I will present the model in more general terms.

3.2 The Markowitz Model and Mean Variance Optimization

In this sub-chapter I will loosely follow the notation and presentation by Harry Markowitz in his 1952 classic paper which formed the basis for using mean variance optimization for portfolio optimization.

Consider the simple case with N assets and let r_{it} be the expected return for asset i at time t . Let d_{it} be the discount rate at which the returns of the i^{th} asset at time t is discounted back to the present. ω_i is the allocation to asset i . There is no short selling so $\omega_i \geq 1$ for all assets.

Hence the discounted expected value of portfolio r_p is:

$$\begin{aligned} r_p &= \sum_{t=1}^{\infty} \sum_{i=1}^N d_{it} r_{it} \omega_i \\ &= \sum_{i=1}^N \omega_i \left(\sum_{t=1}^{\infty} d_{it} r_{it} \right) \\ r_i &= \sum_{t=1}^{\infty} d_{it} r_{it} \end{aligned}$$

Where r_i is the discounted return of the i^{th} security. Hence we can conclude that the expected return for portfolio r_p is simply

$$r_p = \sum \omega_i r_i$$

In other words, the expected return on portfolio is the sum of weights times the discounted expected return of that asset. To maximize r_p we simply set $\omega_i = 1$ for the security with maximum return r_i .

As we can clearly see in no case in this setting, does a diversified portfolio be preferred over all non-diversified portfolios.

The golden idea of Markowitz was to introduce risk in to this setting. The main assumption with risk is that there exists a portfolio which will maximize the expected return with a given level of risk (or minimize the risk with a given level of return).

The portfolio with the maximum return is not necessarily the one with the lowest risk (variance), indeed there is a trade-off between return and variance and the investor must choose between a higher return and higher variance or lower return with lower variance.

Variance can be defined as the mean of the square minus the square of the mean so that the variance of an asset X_i is

$$\sigma_{X_i}^2 = E[X_i^2] - E[X_i]^2$$

The variance at the portfolio level is defined as

$$\sigma_p^2 = \sum_{i=1}^n \omega_i^2 \sigma_i^2 + 2 \sum_{i=1}^n \sum_{j=1+i}^n \omega_i \omega_j \sigma_{ij}$$

Where ω_i is the asset weight to that specific asset i and σ_{ij} is the covariance between assets i and j .

The covariance can also be expressed through correlation coefficient ρ .

$$\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j$$

The return on portfolio level is

$$E(r_p) = \sum_{i=1}^n \omega_i E(r_i)$$

Where $E(R_i)$ is the return of asset i . So the portfolio return is defined by the expected returns of the assets and the weights allocated to them.

So now the investor faces a decision based on his believes on the future expected returns and the future risk related to that asset. The expected return can be formed from historical mean and the covariance matrix captures the risk effect between different asset classes. There of course might be a situation where one asset class is so lucrative that it might be optimal to allocate all assets to this one class (the case with so called maximum earnings portfolio), but in general the best risk/return ration can be achieved by diversifying the assets.

The covariance aspect captures both risk and the behavior of the assets with respect to other assets. The model suggests that negative covariance (movement in the opposite direction) between assets reduces the total portfolio risk (benefits of diversifying).

By using this model, we can construct a set of efficient portfolios on expected return expected variance universe and compute an efficient frontier. The efficient frontier represents combinations of diversified portfolios on expected return/variance plane. Each of these portfolios are efficient in the sense that they will maximize the expected return with that given level of risk, i.e. there is no other such portfolio that would offer more profit with that level of variance.

This model of course simplifies a lot of real life issues and it has been recently criticized from its assumptions, mainly that markets are efficient and that all agents are rational and risk averse.

3.2.1 Matrix Formulation

Matrices are preferred when conducting MVO for practical reasons. The matrix form of the above presented problem can be constructed as follows.

First we can define the expected asset return vector as:

$$R = \begin{bmatrix} E(r_1) \\ \vdots \\ E(r_n) \end{bmatrix}$$

The covariance matrix between the assets as:

$$\theta = \begin{bmatrix} (\sigma_{11} & \cdots & \sigma_{n1}) \\ \vdots & \ddots & \vdots \\ (\sigma_{n1} & \cdots & \sigma_{nn}) \end{bmatrix}$$

And the weights of each asset as a vector:

$$\omega = \begin{bmatrix} \omega_1 \\ \vdots \\ \omega_n \end{bmatrix}$$

Where $\sum_{i=1}^n \omega_i = 1$. The expected return of the portfolio can then be written as:

$$E(r_p) = \omega^T R = \sum_{i=1}^n \omega_i E(r_i)$$

And the variance of the portfolio is:

$$\sigma_p^2 = \omega^T \theta \omega = \sum_{i=1}^n \omega_i^2 \sigma_i^2 + 2 \sum_{i=1}^n \sum_{j=1+i}^n \omega_i \omega_j \sigma_{ij}$$

Where σ_{ij} is the covariance between assets i and j.

3.2.2 Issues with Using Modern Portfolio Theory to Evaluate Private Equity

If we look at the implicit assumptions in the previous chapter, we can clearly see that PE does not necessarily fit in to those frames very well. For example it assumes that markets are efficient. All the data is readily available for all participants. Clearly PE markets are nowhere near efficient, since only the investors who invest in private equity funds receive data of its performance. PE markets are inefficient; valuations are infrequent and unreliable, investments are illiquid and there is much less investors on the market compared to public equity.

In my case correlation will probably be my biggest drawback. Correlations are the very basis of the model since they depict the risk-return tradeoff between the assets. This is not a problem with public equity since we can get daily data from the stock exchange but with PE all the data is at best quarterly reported figures, which are not ideal when forming correlation between the assets.

So when we are dealing with PE the ideal dataset would show quarterly data. Unfortunately I only have access to yearly data, which was also heavily twisted by the market turmoil of early and late 2000.

However I try to overcome this issue by using alternative data source for the risk component in section 5.3.

3.3 Efficient Frontier

In my analysis I will concentrate on my analysis on the efficient frontier (also known as mean-variance frontier or “Markowitz bullet”). This kind of analysis assumes that there is no risk free asset, otherwise the location of the *optimal* portfolio is somewhere on the efficient frontier, I will get back to this in chapter 5. Since my goal is to find all possible efficient portfolios that SE might choose from I opted to only analyze the efficient frontier portfolios, not individual *optimal* portfolios.

If we look the previous sections, in MVO one minimizes the variance or maximize the returns of the portfolio (with possible constraints). MATLAB forms efficient frontier in a way that it always calculates the minimum variance portfolio and the maximum return portfolio and distribute a set number of portfolios (in my case 50) between these two maximum points on the frontier.

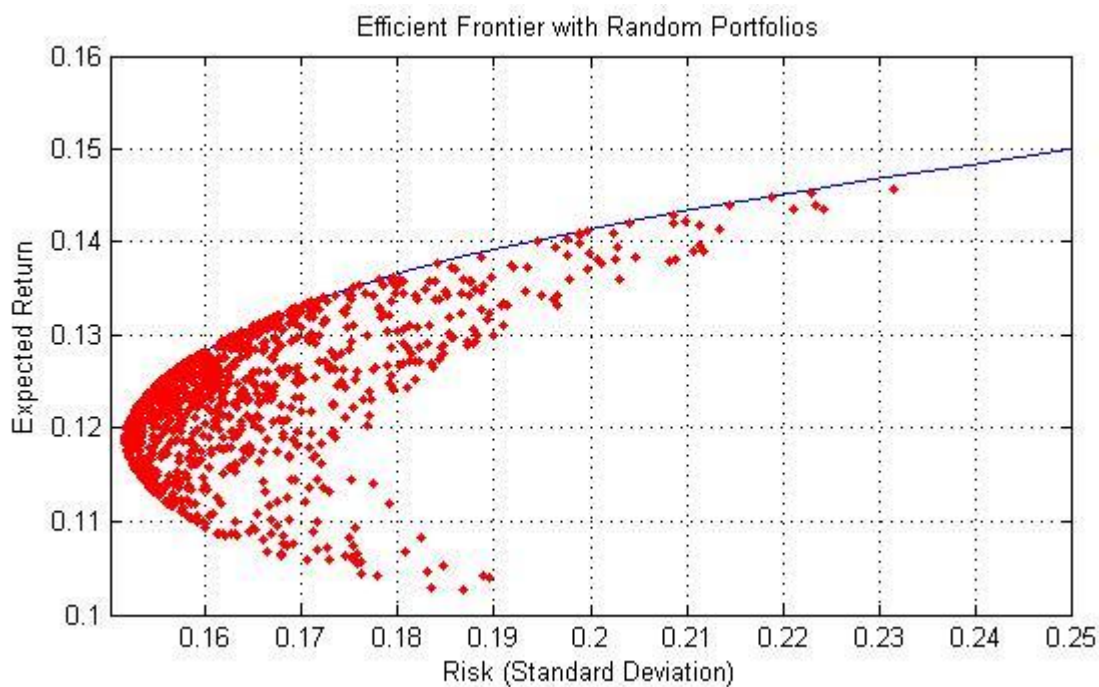


Figure 4: Efficient frontier With 1000 Random Assets

To illustrate this I have formed an efficient frontier from 1000 randomly generated assets shown in figure 4. The red dots in the pictures are the asset (their expected return and standard deviation) and the blue frontier is called the efficient frontier. It means that any point on that frontier is efficient in a sense that the combination of the assets that form that point on the frontier is the best possible; with that level of risk you cannot receive better returns. MATLAB then given you 3 output

matrices, expected return matrix, expected risk matrix and finally the weights matrix which depicts the weights of that given efficient portfolio.

In my analysis I will study these weights on the efficient frontier to check what kind of efficient portfolios containing allocation to PE exist with the data and parameters at my disposal.

4 Data

Stanford University endowment is the fifth largest endowment in the U.S (NACUBO). The Stanford endowment is managed by the Stanford Management Company (SMC). For the past ten years it has diversified its portfolio to 6 different assets, public equity, private equity, absolute return, natural resources, real estate and fixed income.

According to SMC yearly report of 2010, the actively managed financial and real estate assets were valued at approximately \$15.9 bn. as of June 30, 2010. The majority of the endowments assets are invested through merged pool which is an actively managed, diversified portfolio. Over the past 10 years it has achieved an annualized return of 6.9 % and growth from \$7.9bn. to \$15.9bn.

SMC also states that the basis of the portfolio construction is modern portfolio theory and strategic asset allocation. The portfolio is designed to optimize long-term returns, to create consistent annual payouts to the University's operating budget and preserve purchasing power for future generations of Stanford faculty and students.

SE invests its PE assets through limited partner agreements mainly to VC and LBO funds (SMC, 2011).

4.1 Different Asset Classes in the Stanford Endowment

According to SMC their different asset classes can be characterized as following (SMC, 2011).

- **Public Equity:** Include investments that are directly held as well as commingled funds which invest in publicly traded equities
- **Private Equity:** Discussed in literature review. Investments mainly to VC and LBO funds
- **Absolute return:** Out of all assets, I think this one deserves a bit more explanation. As an investment vehicle, an absolute return seeks to make positive returns by using investment management techniques that differ from the traditional ones. Absolute return investment techniques include short selling, futures, options, derivatives, leverage and other unconventional assets. In SE investments in this asset class are typically commingled funds that employ multiple strategies to produce positive returns, regardless of the direction of the financial markets.
- **Natural resources:** Mostly held in commodities and energy related investments
- **Real estate:** Investments represents directly owned real estate and other real estate interests held through limited partnerships (LPs)
- **Fixed income:** Include investments that are actively traded fixed income securities or mutual funds.

SMC has been investing in these assets for the past 10 years. As diversification is one of the cornerstones of SMCs investment strategy these assets give them the broad exposure to different market risks.

4.3 Historical Data on Stanford Endowment

I will next present some historical data from the endowment that I will be using as data source in my analysis

Stanford Policy Portfolio Asset Allocations 2011-2001

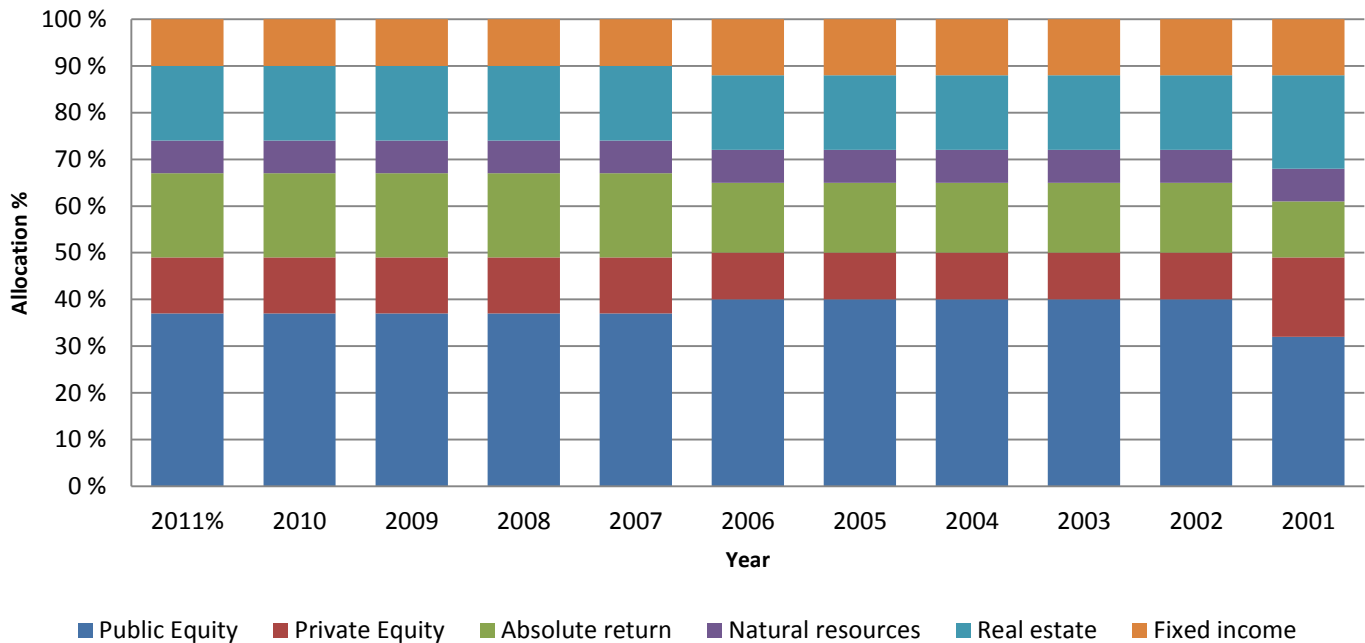


Figure 5: Stanford Policy Portfolio Allocations

In Figure 5, we can see the allocations to different assets that SMC has been using from 2001 to 2011. As we can see from the figure public equity has had the biggest emphasis in their endowment with 37-40% allocations. Private equity has had a steady allocation around 10-12% per year, with 2001 being exemption with 17% allocation. Investments to absolute return have been climbing from the 12% in 2001 to 18% to where it has been since the year 2007. Natural resources have been allocated 7% the entire observation period. Real estate allocations have been 16% with the exemption of 2001 when the allocation was 20%. Fixed income has been allocated 10-12%. What is worth mentioning that both equities (PE and public equity) has received around 50% of the total portfolio allocation indicating heavy emphasis on equity in the portfolio. It might be the case that the large proportion of stocks in the portfolio offers liquidity and PE is meant to generate excess profits.

Table 2 depicts yearly returns by asset class. We can see that natural resources have been the most lucrative asset with some astounding returns during the 10 year observation period. Other asset classes have usually returned less than 20%, but as you can see in figure 6, it has also had the second highest volatility.

	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
Public Equity	24 %	16,7 %	-27,8 %	-0,3 %	24,4 %	22,5 %	13 %	23,8 %	1,7 %	-7,6 %	-25,8 %
Private Equity	36 %	16,7 %	-20,8 %	6,5 %	36,6 %	19,8 %	48 %	11,6 %	-6,6 %	-30,5 %	-43,2 %
Absolute return	15 %	22,8 %	-16,7 %	2,0 %	13,1 %	12,5 %	17 %	17,9 %	14,2 %	10,0 %	13,5 %
Natural resources	27 %	6,0 %	-31,2 %	43,8 %	29,2 %	61,2 %	71 %	19,7 %	9,1 %	17,1 %	2,5 %
Real estate	20 %	-8,1 %	-41,1 %	1,6 %	26,8 %	19,2 %	29 %	18,9 %	19,6 %	6,5 %	13,4 %
Fixed income	6 %	13,8 %	15,1 %	13,3 %	12,3 %	0,5 %	8 %	1,1 %	11,5 %	4,5 %	15,2 %

Table 2: Returns per Asset Class in Stanford Endowment

The next figure shows the mean and standard deviation of asset returns during the 10 year observation period for each asset class. It also includes the minimum and maximum values during that period.

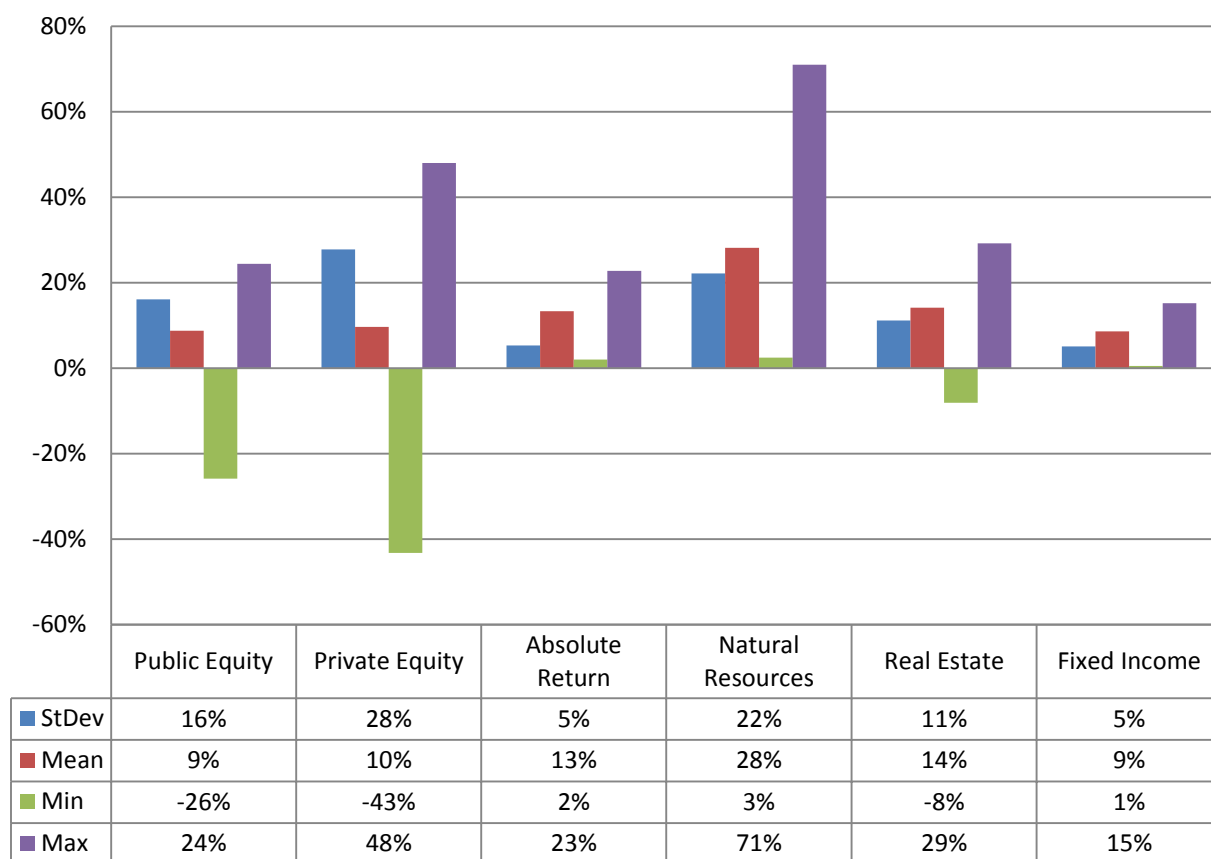


Figure 6: Mean, Standard Deviation, Minimum and Maximum Values of Asset Return During the Observation Period

Natural resources has the highest mean but also high standard deviation. Private and public equity both have relatively low mean and high standard deviation. It is worth mentioning that private

equity has the biggest downside but also the second highest upside of all the assets, while natural resources is the most lucrative out of all asset classes. However, if we look at figure 5, we can also see that natural resources have the lowest allocation in that portfolio.

From this data the following can be concluded with respect to PE. It has a relatively low mean and high volatility with respect to other asset classes in Stanford endowment. It also has a big gap between the maximum and minimum returns from the past 10 years. If we think about the setup of mean-variance optimization PE does not look very lucrative asset class.

4.4 Comparative Data from Other University Endowments

This sub-chapter provides some background information on other universities. According endowment surveys conducted by the National Association of College and University Business Officers (NACUBO), the average return to PE asset classes (excluding venture capital) was 14,1% in the fiscal year 2010. Venture capital yielded the return of 9,6%. In this light, Stanford endowment performed averagely with its PE assets, having return of 16,7% in 2010 (even though they report all PE assets as one asset class).

Average Return by Asset Class for Fiscal Year 2010*

<i>numbers in percent (%)</i>	Total Institutions
Average FY2010 total return	11.9
Domestic equities	15.6
Fixed income	12.2
International equities	11.6
Alternative strategies	7.5
Private equity (LBOs, mezzanine, M&A funds and international private equity)	14.1
Marketable alternative strategies (hedge funds, absolute return, market neutral, long/short, 130/30, event-driven and derivatives)	9.9
Venture capital	9.6
Private equity real estate (non-campus)	-15.8
Energy and natural resources, commodities and managed futures	13.2
Energy and natural resources	11.0
Commodities and managed futures	9.5
Distressed debt	24.6
Short-term securities/cash/other	2.7
Short-term securities/cash	0.9
Other	8.4

*614 responding institutions

Table 3: Average Return by Asset Class (NACUBO)

In the next figure I have taken the returns of private equity from three other endowments from 2011-2009. These three, Harvard, Princeton and Stanford are all in the top 5 endowments in the U.S. The sources of the data in the chart are the yearly endowment reports by each university (Harvard, Princeton, Stanford 2011-2009)

PE Returns by Endowment 2011 - 2009

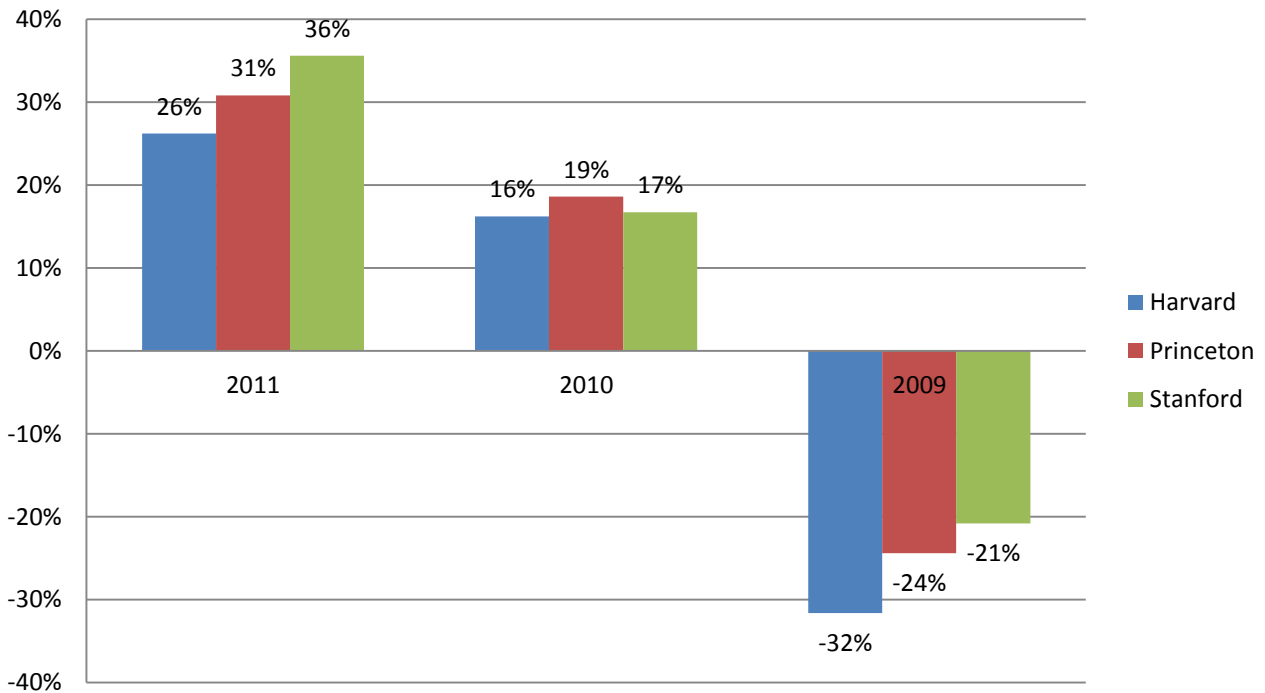


Figure 7: PE Returns by Endowment 2011 - 2009

The allocation in the policy portfolio for PE was 12% for Harvard, 23% for Princeton and 12% for Stanford.

If we look at the mean return from the time period it seems that Stanford has performed the best with a mean return of 11%, followed by Princeton with 8%. Harvard mean was 4%.

Of course we cannot conclude from this data how well does Stanford data mirror the behaviour of other universities because there is no historical data from the other universities before 2009 and since we do not know what valuation has been used. This is the main reason why I chose Stanford's data, since it makes no sense to form a covariance matrix with just three years of historical data.

However it does give an insight to the performance of other universities and how Stanford has performed against them in the last three years even though there's always the problem with valuation of PE assets and how they might not always be comparable.

5 Analysis

In the analysis part I will try to explain if the current SE policy portfolio allocation of 12% to PE can be justified by the historical performance of the asset. Since my data is limited I will conduct my research in three parts. In the first part I will use purely historical data and in the second part I will conduct the same analysis as in part one but use the risk factors from the research of Bekkers, Doerswijk & Lam (2009). In the third part I introduce additional constraints to the optimization. Since the analysis of the endowment as a whole is beyond the scope of this thesis I will not comment more about the overall allocation they have chosen, but rather, concentrate on the allocation of PE.

In portfolio theory, the optimal portfolio on the efficient frontier is usually decided by comparing the risk free rate, investors risk aversion and possible borrowing which form the capital market line (CML). However my goal is not to find the optimal portfolio for SE, but rather, to study all the possible efficient allocations that the historical data indicates. The analysis of these efficient portfolios should give me some insight to what kind of portfolios would justify the 12% allocation and does the SE policy portfolio match these portfolios. This is why I have opted not to use risk free rate and different risk aversions to find the one and only optimal portfolio in the whole efficient frontier. The capital market line theory is depicted in figure 8.

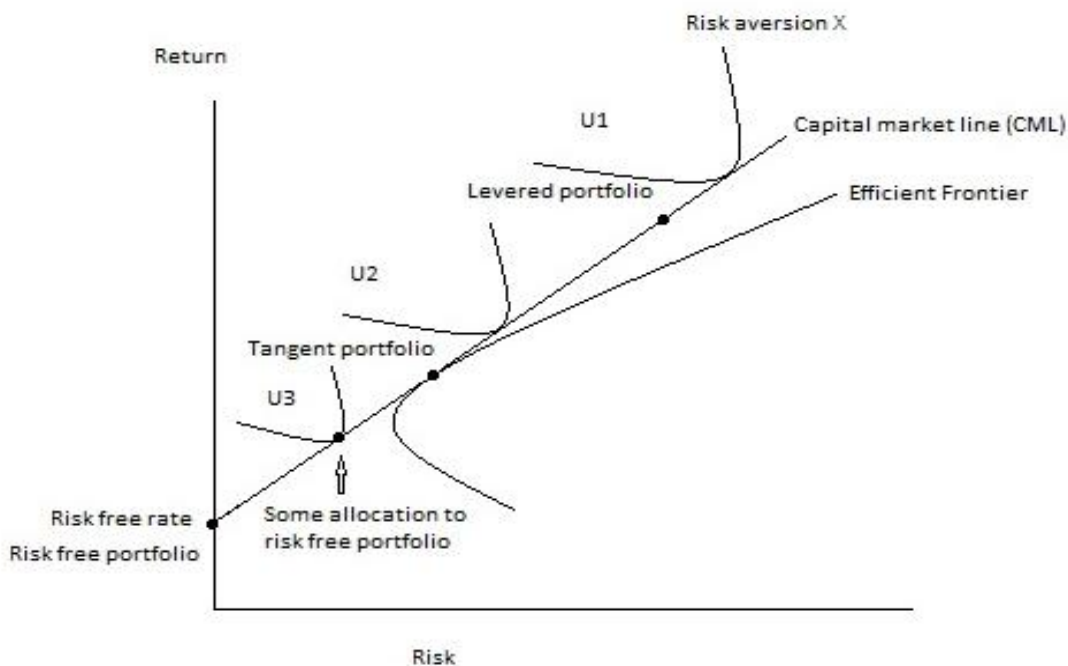


Figure 8: The Capital Market Line

The efficient frontier is the set of all efficient portfolios given the expected return of the assets and their covariance. However the *optimal* portfolio on the efficient frontier is determined by the risk free asset, investors risk aversion and possible borrowing rate. The degree of risk aversion, which is usually in the form of utility function (U1,U2,U3 in figure 8), moves the investor up and down the capital allocation line. The higher the risk aversion, the closer the investor to the risk free asset and the less risk averse investor moves up, and away from the risk free asset. Notice that the case where there is no borrowing rate, then the risk aversion after the tangent portfolio just moves the investor to more risky portfolio choices.

My goal in this analysis, however, is not to form a combined optimal portfolio out of all assets, but rather, to find out the set of all possible efficient portfolios on the frontier and see if there exist portfolios where the 12% allocation is justifiable and what are the risk characteristics of that portfolio.

I will analyze the data in 6 different cases (with cases 1-4 each containing 2 “sub” cases), which differ from each other by the expected return, risk component used and optimizations constraints. Chapter 5.2 analyses the allocation with the historical risk component. Chapter 5.3 analyses the allocation with the risk component taken from the study of Bekkers, Doerswijk & Lam (2009). Chapter 5.4 adds individual asset constraints in to the optimization

The first sub case uses only the purely historical returns. Since my data has been affected heavily by the market turmoil of early 2000 and also by the financial crisis, the second sub case is to exclude those returns from the analysis and see what would have been the expected return without this market turmoil and use that return to run the simulation. This means that I remove the “bad” years affected by the market turmoil and see if the simulation changes.

Finally, I will analyze the case where 50% must be allocated to the equity assets (public and private equity), since they have had a combined allocation in SE policy portfolio of around 50% in the past 10 years suggesting that SMC might have a stronger emphasis on equity based portfolio.

In chapter 5.4 I will further constrain the analysis in a way that no asset can have 100% allocation (diversification) and every asset must have at least 5% allocation but less than 20% allocation in the portfolio.

Table 4 summarizes the cases analyzed.

Chapter	Case	Risk component	Expected returns	Grouped	Asset Constraints
5.2	1a	Historical	Historical	No	None
5.2	1b	Historical	Adjusted	No	None
5.3	2a	From literature	Historical	No	None
5.3	2b	From literature	Adjusted	No	None
5.2	3a	Historical	Historical	50% to equity assets	None
5.2	3b	Historical	Adjusted	50% to equity assets	None
5.3	4a	From literature	Historical	50% to equity assets	None
5.3	4b	From literature	Adjusted	50% to equity assets	None
5.4	5	Historical	Historical	No	5-20%
5.4	6	Historical	Historical	50% to equity assets	5-20%

Table 4: Summary of Cases Analyzed

5.1 Implementation by MATLAB

MATLAB is a powerful tool for matrix calculus. It contains many toolboxes that are suited for example to portfolio optimization. The use of the functions in my analysis requires MATLAB Financial Toolbox (2011 or newer).

With MATLAB the portfolio optimization starts by determining the return series matrix. The return series is formed so that each asset has its own column and each row is an observation. The return observations are sorted from oldest to newest. The order of columns and rows is thus the following.

Public Equity	Private Equity	Absolute Return	Natural Resources	Real Estate	Fixed Income
Oldest return observation	Oldest return observation	Oldest return observation	Oldest return observation	Oldest return observation	Oldest return observation
.....
Newest return observation	Newest return observation	Newest return observation	Newest return observation	Newest return observation	Newest return observation

Table 5: MATLAB Return Series Column and Row Composition

After this matrix has been done, the user can then use functions like *ewstats* to get the expected returns and covariance matrix. I used the function *frontcon* to form my different efficient frontiers

and asset allocations. *Frontcon* function creates efficient portfolios between the minimum variance and maximum return portfolio and evenly distributes the rest of the portfolios between these two points. The advantage of this is that there is no need to run the simulation twice, e.g. once for minimum variance portfolio and once for maximum earnings portfolio. The user can then specify different constraints as he pleases. MATLAB website has a broad library of different commands with examples.

5.2 Analysis by Using Historical Risk Component

In this section I will go through cases 1 and 3 in table 4. The historical covariance of different assets and expected returns are depicted in the following tables.

	Public Equity	Private Equity	Absolute Return	Natural Resources	Real Estate	Fixed Income	StDev
Public Equity	1,0	0,9	0,4	0,4	0,3	-0,5	0,16
Private Equity	0,9	1,0	0,3	0,6	0,4	-0,2	0,28
Absolute Return	0,4	0,3	1,0	-0,2	0,1	-0,1	0,05
Natural Resources	0,4	0,6	-0,2	1,0	0,4	-0,4	0,22
Real Estate	0,3	0,4	0,1	0,4	1,0	-0,3	0,11
Fixed Income	-0,5	-0,2	-0,1	-0,4	-0,3	1,0	0,05

Table 6: Purely historical correlations and standard deviations

Public Equity	Private Equity	Absolute Return	Natural Resources	Real Estate	Fixed Income
8,8 %	9,7 %	13,3 %	28,1 %	14,1 %	8,6 %

Table 7: Historical Expected Returns

MATLAB uses weighted mean returns to calculate the expected return. In this case however, I opted not to include any decay factor to the calculation so the weighted mean returns in this case are the same as arithmetic means. With decay factor I could emphasize the newer observations over the old ones, but since my data set is as limited as it is, I decided to ignore it.

As we can see from table 6, PE has a strong positive historic correlation with public equity and has quite a high negative correlation with fixed income (which is the case with all assets). Since mean variance optimization suggests that an investor can reduce the portfolio risk by investing in assets that are not perfectly positively correlated (i.e. correlation coefficient is not one), we can see that public equity and PE do not necessarily work very well together in MVO setting. Both have relatively high standard deviation and low mean compared to other assets in the portfolio and they have a strong positive correlation between one another. Even though the analysis of public equity and PE is beyond the scope of this thesis, it is important for the reader to know the mechanics of mean variance optimization.

Another important aspect to notice from tables 6 and figure 7 is that natural resources have a high mean, high standard deviation and relatively low correlations with other assets. This would imply that it will take the role of the high yield asset in this analysis.

Plugging the expected returns and covariance I can now construct the efficient frontier. The first frontier is constructed by using pure historical data from tables 6 and 7.

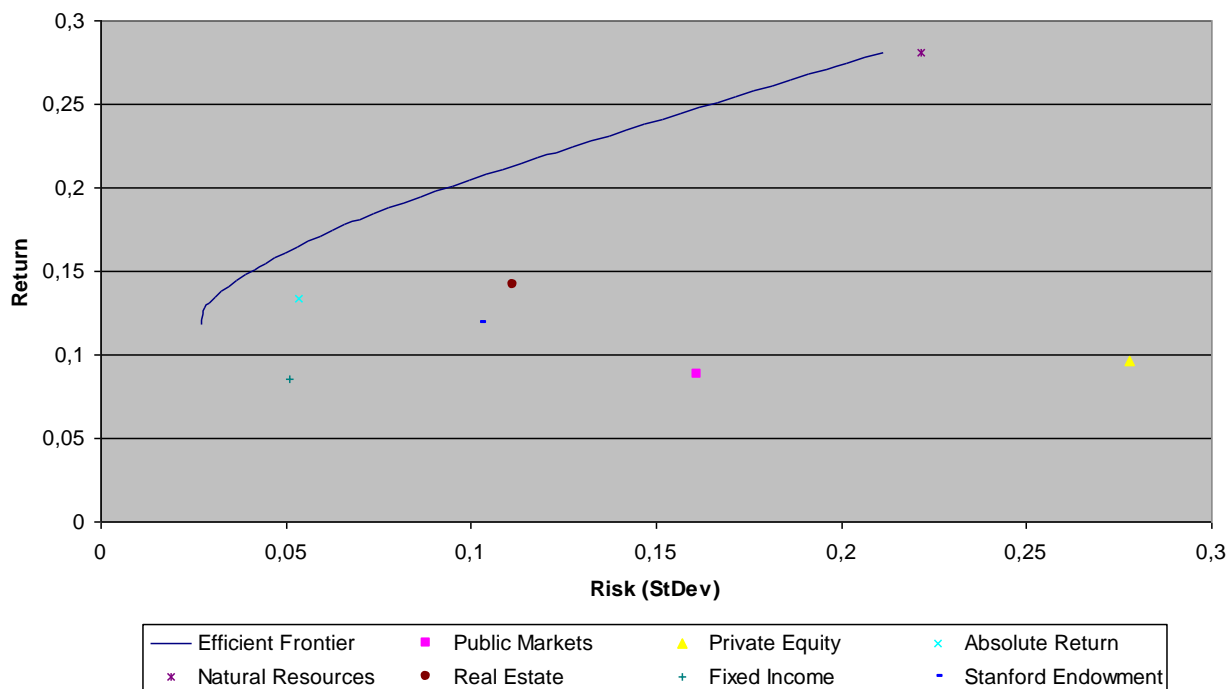


Figure 9: Efficient Frontier with Purely Historical Data

Figure 9 represents all the 50 efficient, diversified portfolios on expected return/variance universe, distributed evenly between the minimum variance - and maximum return portfolio. Individual assets as well as SEs current policy portfolio is also depicted. Clearly by using only historical data

and no restrictions to the optimization process, SE policy portfolio is not even close to the efficient frontier and PE is a very bad asset with high risk and low expected return. Two other assets also stand out; natural resources (NR) and absolute return (AR). They are both relatively close to the efficient frontier at opposite ends, so it is quite possible that the two assets get a relatively big role in the portfolios.

Next we'll have a look at the allocations to different asset classes on those 50 efficient portfolios. They are presented in figure 10.

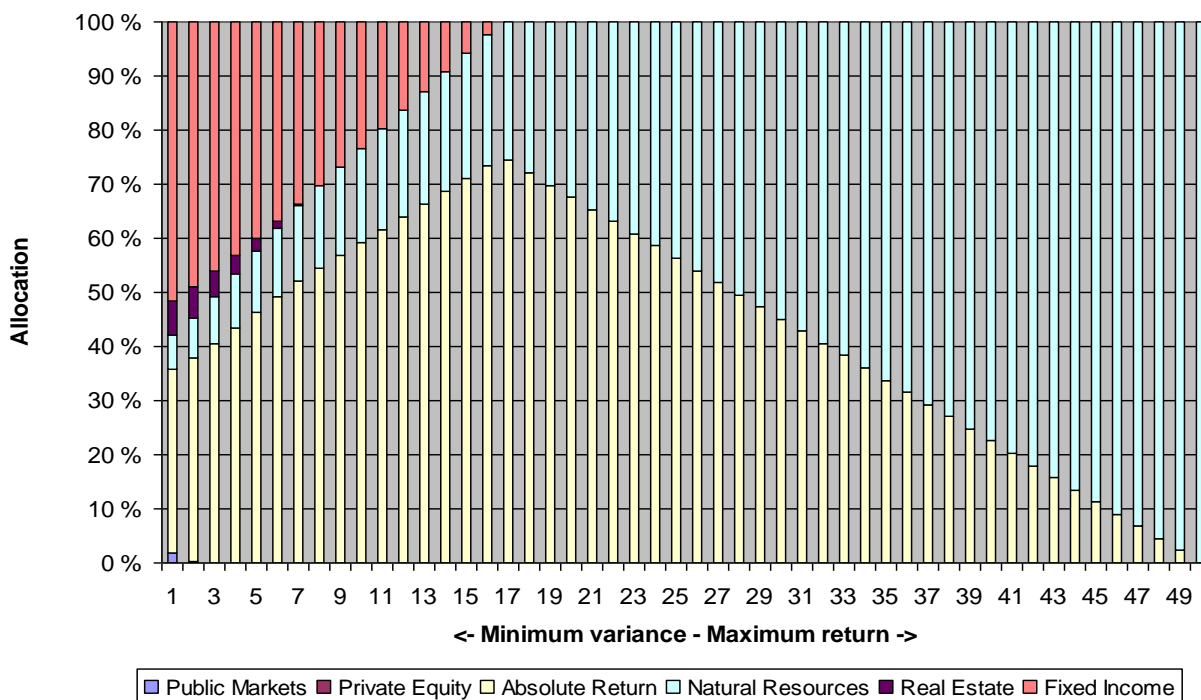


Figure 10: Efficient Portfolios with Purely Historical Data (case 1a)

These portfolios are again plotted between the minimum variance - and maximum return portfolio, so that the portfolio #1 is the minimum variance portfolio and portfolio #50 is the maximum return portfolio.

In this setting, PE gets no allocation at all in the efficient portfolios. In the minimum variance portfolio the carrying assets are fixed income due to its low correlation and low standard deviation and absolute return as the higher yield component. As we move closer to the maximum return portfolio the allocations are distributed to natural resources and absolute return portfolios.

The allocation to PE does not increase even if we use adjusted returns where all the volatile years of early 2000 and financial crisis have been eliminated. With the expected returns of table 18 (appendix A), the mean returns are as follow.

Public Equity	Private Equity	Absolute Return	Natural Resources	Real Estate	Fixed Income
18 %	26 %	15 %	34 %	16 %	7 %

Table 8: Adjusted Expected Asset Returns

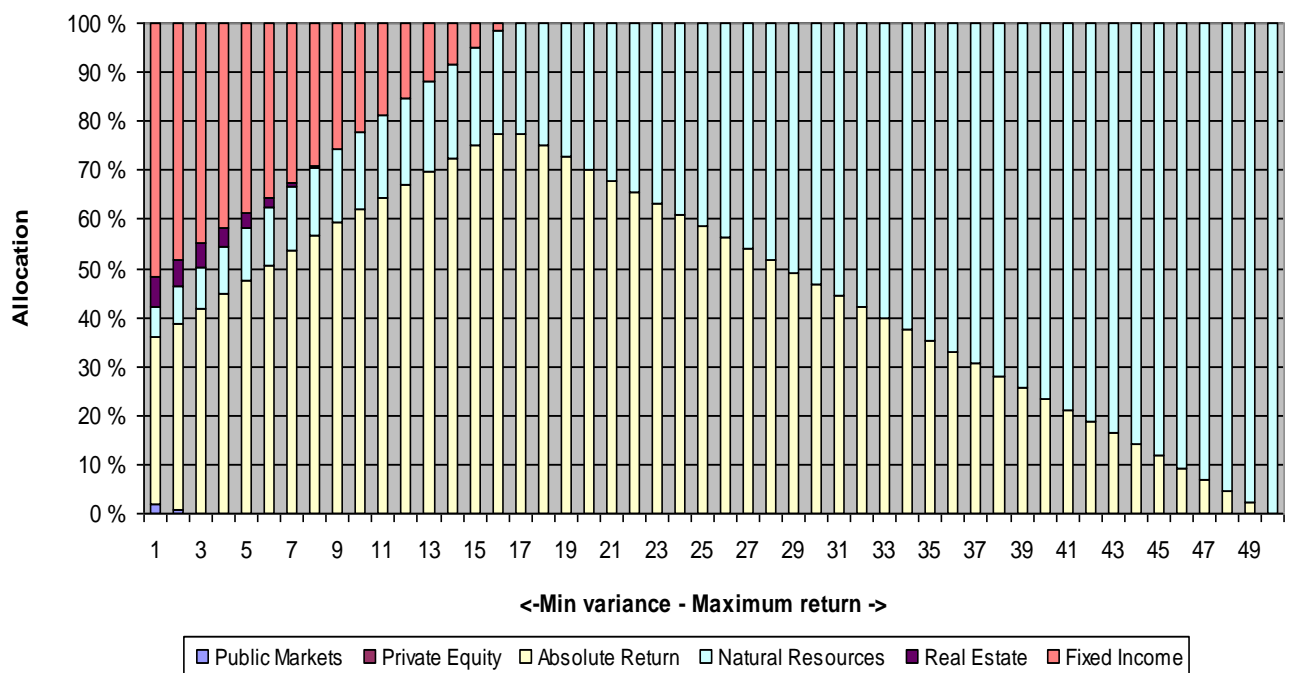


Figure 11: Efficient Portfolios with Historical Risk Component and Adjusted Returns (case 1b)

Even though the expected return of PE rose to 26% it still is not good enough asset in MVO to allocate it even to the riskiest, highest earning portfolios. This is purely because its risk is still greater and expected return smaller than that of NR (and worse correlation characteristics than NR)

What is the cut off expected return for PE in order for it to have allocation in one of the efficient portfolios? With 27% expected return there exists portfolios on the efficient frontier with PE allocation of 12%. Allocations are drawn in figure 20 (appendix A).

SMC has allocated roughly 50% of their assets in equities (PE and public equity) over the past 10 years. This might suggest that they emphasize their portfolio to be more equity oriented. For this

reason I group the asset in a way that PE and public equity (henceforth PM) must receive an allocation of 50% and rest of the assets distribute the 50% pot between them.

Again with purely historical and adjusted return data the allocation are as follows.

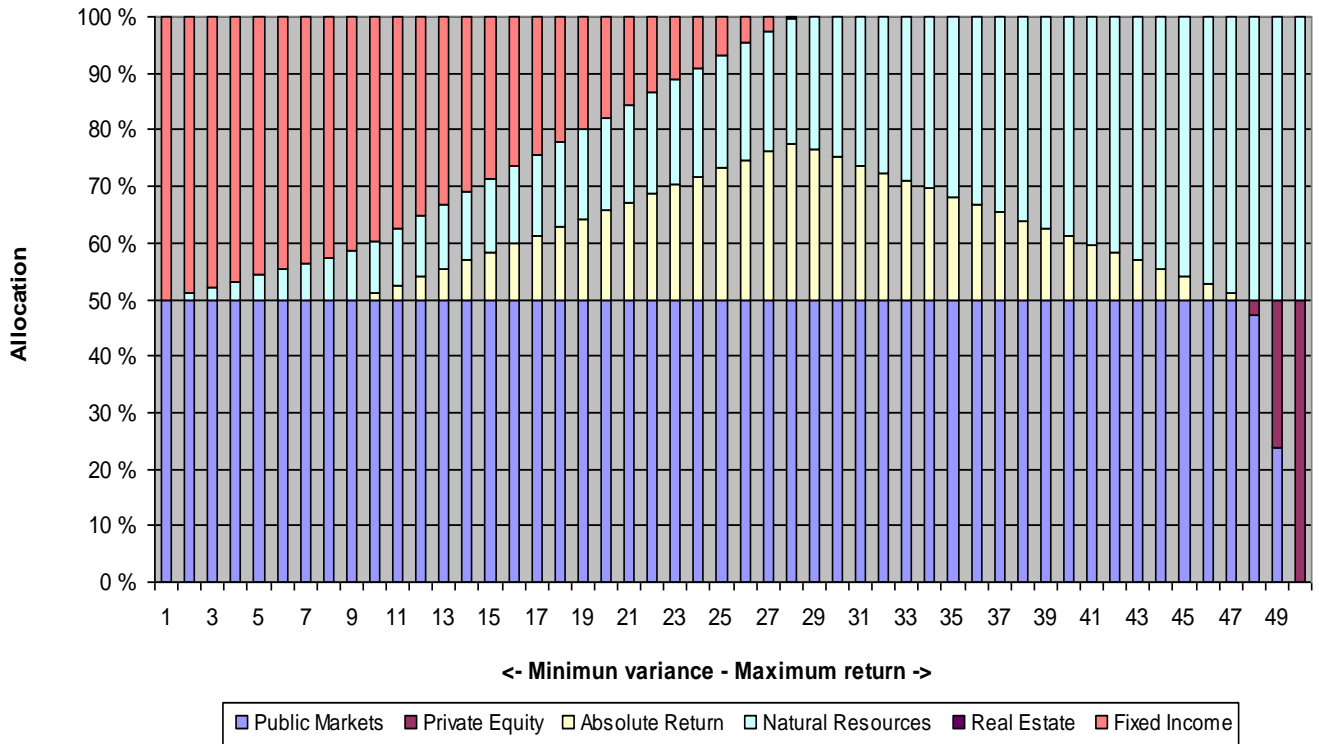


Figure 12: Efficient Portfolios with Historical Data, 50% Equity Oriented Portfolio (case 3a)

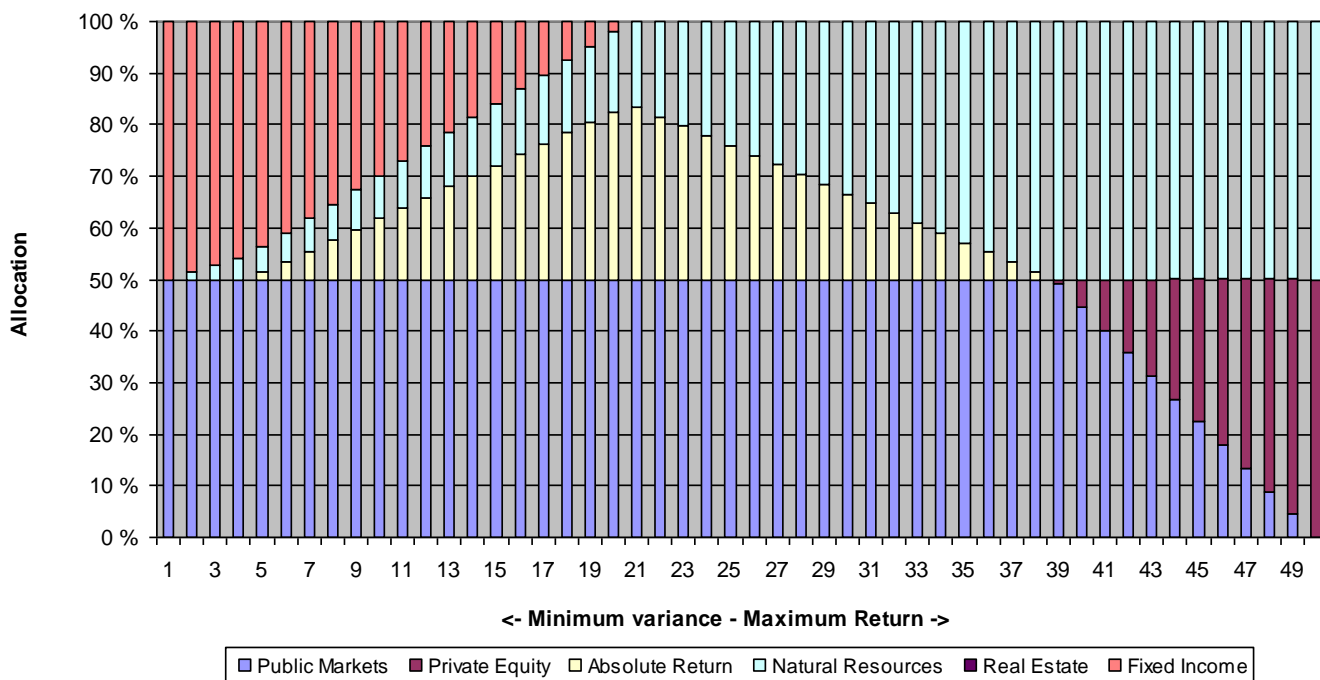


Figure 13: Efficient Portfolios with Adjusted Returns, 50% Equity Oriented Portfolio (case 3b)

With 50% allocation to equity assets, PE gets allocated to the riskier portfolios on the frontier even with purely historical data and with adjusted returns the allocations are even more diverse in the higher risk portfolios.

5.3 Analysis by Using Correlation Estimates from Literature

As stated earlier, my biggest drawback of this analysis is the short time series I have at my disposal and because of that, the correlations between the assets might not be accurate. Bekkers, Doerswijk & Lam (2009) studied the optimal mix in 10 asset portfolio which also contained PE. They used PE indices to measure the covariance and risk associated with PE between different assets classes. The debate whether or not PE indices are a good measure of correlations and risk is beyond the scope of this thesis, but similar approach was also taken by Ibbotson (2007).

Since the assets used by Bekkers, Doerswijk & Lam (2009) do not exactly match with the assets in the SE, I have to use some of the correlations as proxy. According to SMC, their NR asset class consists mostly of commodities, so I will use the standard deviations and correlations of

commodities in the study to proxy the NR asset class in SE. The same goes for fixed assets which are proxied by bonds, and absolute return is proxied by hedge funds. The correlations and standard deviations are shown in table 19 in appendix A. With correlations and standard deviations we can form the covariance matrix.

As previously I start the analysis by simulating the efficient frontier portfolios with historical returns, but this time by using the risk component (covariance) taken from the 2009 study by Bekkers, Doerswijk & Lam. In this setting the policy portfolio of SE becomes even worse than in chapter 5.2 and asset dispersion inside the return/risk universe even more monotonous.

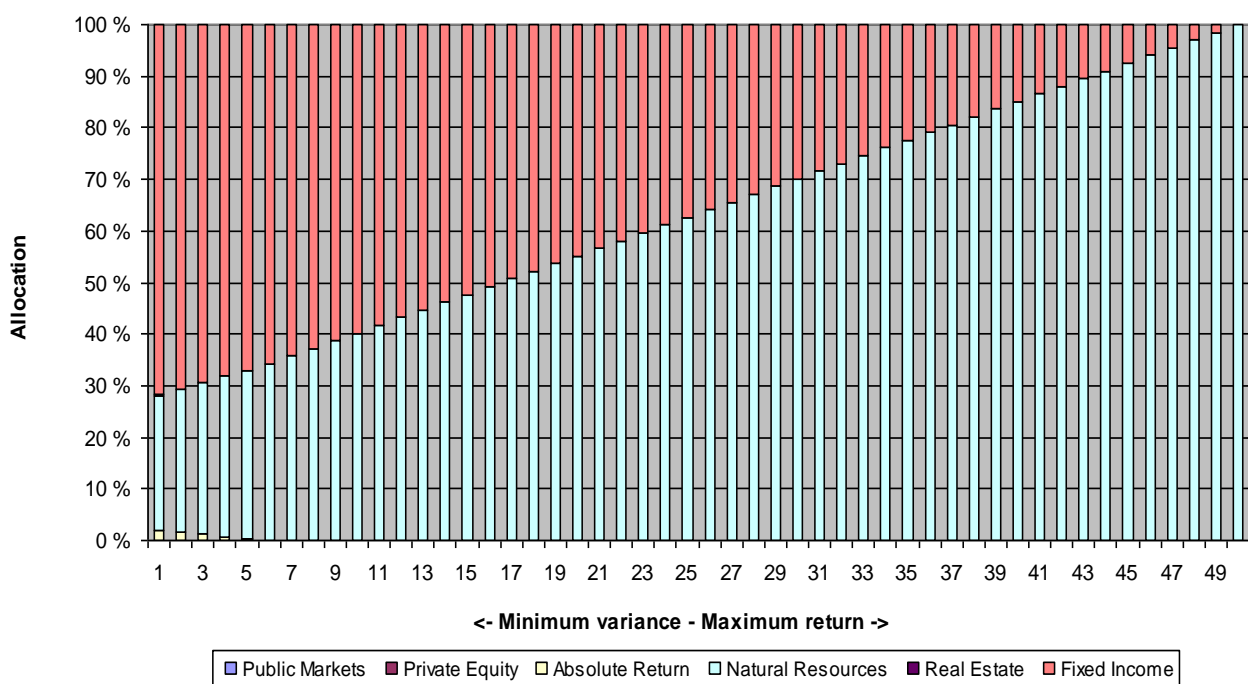


Figure 14: Efficient Portfolios with Historical Return and Risk Component from Literature (case 2a)

The situation do not change dramatically even with adjusted returns (see figure 22, appendix A). With 50% equity allocation constraint PE once again gets allocated to higher risk portfolios (figure 15). This situation does not again change much if we use adjusted returns, except that the allocation to PE start emerging on less riskier portfolios (figure 23, appendix A).

So even with adjusted risk component, PE does not get any more lucrative as an asset class except with those high risk equity portfolios. The question now arises, which of the correlation matrices used better represent the true correlations? Since there really is no way to determining this in this setting and because the allocation did not change dramatically by using adjusted risk parameters, I will use the historical correlations in my analysis in section 5.4.

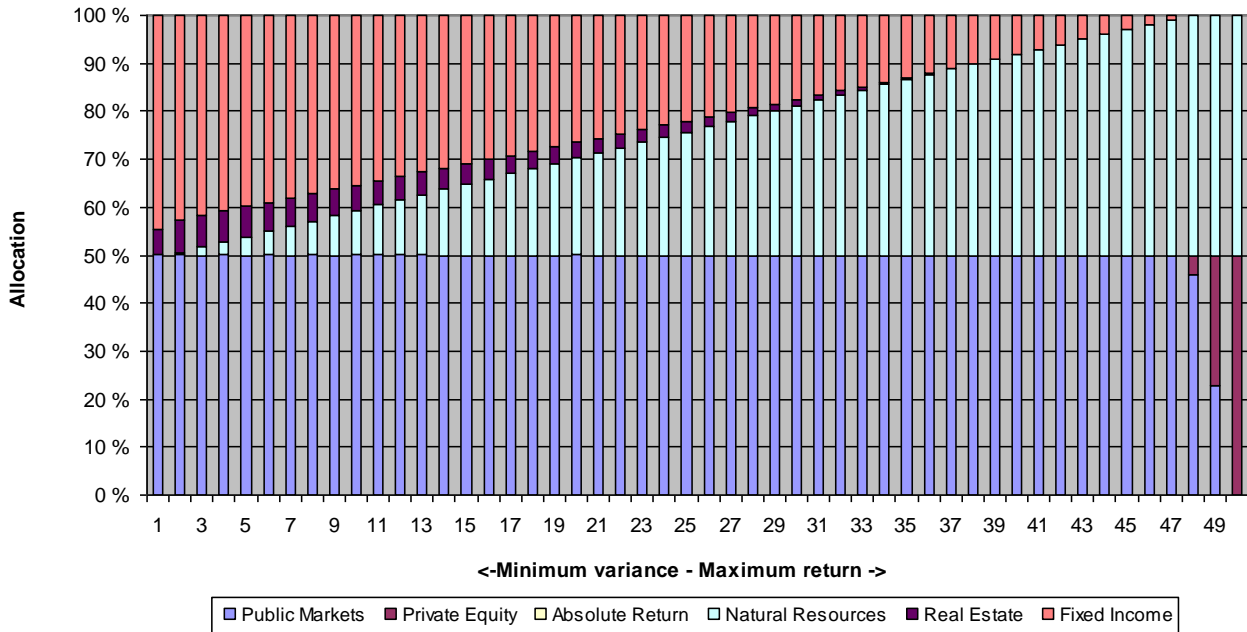


Figure 15: Efficient Portfolios with Historical Returns, Risk Component from Literature and 50% equity constraint (case 4a)

5.4 Optimization with Asset Constraints

In the previous sections I have studied the allocation to efficient frontier portfolios by using only grouped equity constraints. This however usually leads to badly diversified portfolios and the old saying "all eggs should not be in the same basket" is not realized. In this chapter I analyse the PE allocation when no single asset can have more than 20% allocation in the total portfolio and all the assets should have at least 5% allocated to them. I will only use historical return data in this analysis and since the use of adjusted correlations did not change the analysis much either, I will use the covariance from the historical data.

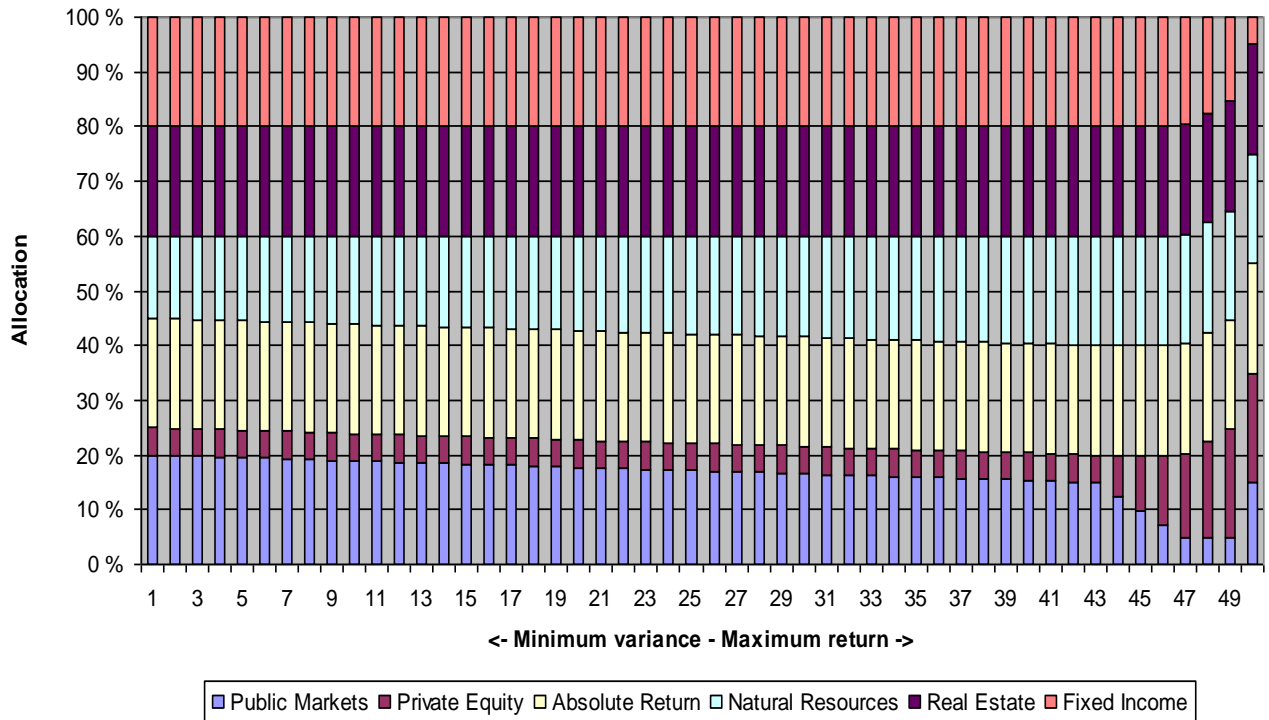


Figure 16: Efficient Portfolios with Purely Historical Data, Asset Bounds 5-20% (case 5)

When we constrain the asset limits we of course get more diversified portfolio. However in this setting we see the same thing as we did in sections 5.2 and 5.3 when we grouped the equity assets; again PE gets increasing allocations with higher risk portfolios.

In the next case, equity assets are once again bound 50% of total portfolio, but both PE and public equity inside the group can change between 5-95%. The rest of the assets in the second group have the same 5-20% allocation limit.

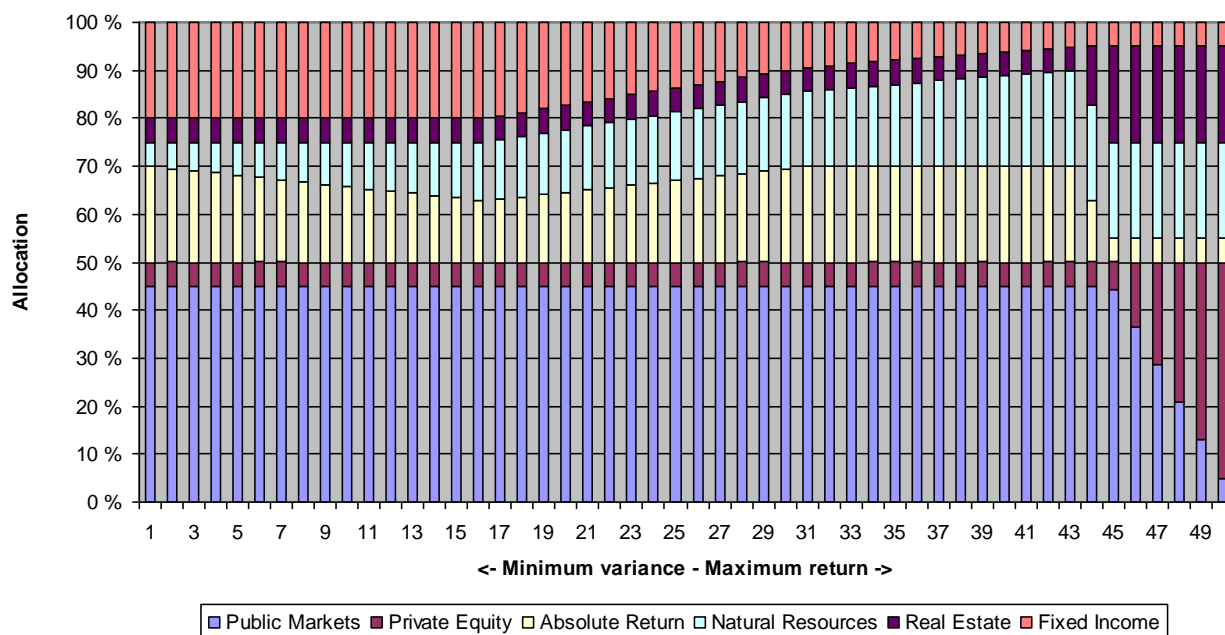


Figure 17: Efficient Portfolios with Purely Historical Data, Bounded Asset Allocations, 50% Equity Constraint (case 6)

A pattern is emerging from sections 5.2-5.4. It seems that in an unconstrained optimization PE does not get any allocation at all, unless the expected return is high enough for it to replace natural resources as the high yield asset. Whatever the case, it is clear that PE should only be allocated to riskier, high return portfolios, preferably with high equity emphasis.

5.5 Speculating the Results

My goal in the analysis part was to answer the question if the 12% allocation to PE in SE could be justified in the light of current literature on the subject and also regarding the historical data.

In light of current literature like Ibbotson (2007) and Ennis & Sebastian (2005) we could conclude that the 12% allocation is justified, since SE matches the characteristics of those entities that could consider an allocation of 10% or more to PE. However the existing literature on this topic is extremely scarce and much more research would be needed to answer this question with high confidence.

The empirical analysis gives mixed results. On one hand, we can argue that with purely historical data the whole asset mix has been wrong like we saw in section 5.2. With individual asset

constraints in section 5.4 there exists portfolios pretty similar to that of SE policy portfolio (see figure 17, portfolio #46) but this analysis heavily relies on the assumptions made in the simulation regarding different asset constraints. What we can conclude from the analysis is that when PE gets emphasis on the allocation, it is always on high risk portfolios, regardless of the expectation of the future returns (which is understandable given the high risk involved), especially in the equity constrained portfolios. This finding is consistent with that of Ibbotson (2007) and Ennis & Sebastian (2005), which suggested that only equity oriented high risk portfolios, gets allocations to PE.

If we think about SE, first of all it is an endowment fund. Endowment funds get cash inflow from endowments made by philanthropist, alumni, corporations, government and even from tuition fees. Their cash inflow is not essentially depended on the performance of the fund. Even if the year is bad, they would still have incoming cash flows (although things like state of economy could affect the amount). Endowments also have perpetual investment horizon and the only liquidity they need is to keep the operations of the endowment funds owner in operation. All these, consistent cash flows, perpetual investment horizon and relatively small liquidity constraints would imply that endowment funds like SE are capable of bearing much greater risk than other institutional investors.

There have also been some accusations that endowment funds have shifted their focus to more risky, high return investments and have abandoned their traditional less risky investment schemes, they have become greedier (Tellus Institute, 2010). When it comes to liquidity, we can see that SE has had a large allocation to stocks in the 10 year period which are the most liquid assets in their portfolio (with equity assets combined around 50% all time).

So if we think that SE can indeed sustain large risk in its policy portfolio and that they indeed emphasize the equity proportion in their endowment, then in the light of section 5.4 the 12 % allocation could indeed be justifiable. However in order to answer the question with certainty I would need much better dataset with quarterly returns and more accurate information on the investment policies and goals of SMC. However because the only one who can answer to this question is SMC, I will conclude that 12% PE allocation in SE has been justifiable if we think that the endowment can carry high risk and emphasized equities in their portfolio.

6 Conclusion

PE as an asset class is obscure and much skill and knowledge is required from the investor in order to commit to this asset. The high capital commitments also limit this asset class to the larger institutional investors or very wealthy individuals. Most investments in PE are done via limited partner – general partner contract in which the limited partner provides the general partner (the investment firm) the capital to invest in his/hers behalf to different PE assets.

The study of PE in academic literature is very limited and scarce. The main reason for this is that there is very little available, non-biased data. Some investment groups publish their own studies about PE, but those studies can be regarded as heavily biased, since this material can also be used in “road trips” to convince possible investors to invest in the company funds. There also exists strong academic debate over the characteristics of PE, most notable its returns. The actual study of PE in the framework of strategic asset allocation is very minimal and only very few studies have been conducted. Those studies show that PE should only be allocated by a moderate size investor with high risk bearing capabilities and strong emphasis on equity assets.

The academic debate over the returns of PE is wide. Some studies show excess returns over S&P 500, other show similar or underperformance compared to S&P 500. In current literature PE is considered a risky asset. The mixed views on the returns and the known high risk of the asset have left people wondering why anyone should invest in the asset. Mispricing, learning, side benefits and conspiracy theories between the limited partner and general partner contracts have all been offered as explanation to this phenomenon.

My empirical study shows that the allocation of 12% could be justified in the light of the current literature and historical data only if we can consider Stanford endowment as high risk bearer with strong emphasis on equity assets.

Much more study is required in PE as an asset class and also in its role in the strategic asset allocation. The empirical study of this thesis could be broadened to include more complex, multi-period models, but no matter how PE is studied the problem of data scarcity remains. Further study of PE in general is highly dependent on the data available and because no regulatory changes to the industry are on sight, the future research is still heavily dependent on unique datasets that might contain appraisal or other type of bias.

References

- Bekkers, N., Doeswijk, R. Q., & Lam, T. W. (2009). Strategic Asset Allocation: Determining the Optimal Portfolio with Ten Asset Classes. *Working Paper*
- Brison, G. P., Hood, R. L., & Beebower, G. L. (1995). Determinants of Portfolio Performance. *Financial Analysts Journal* , 133.
- Brown, K. C., Garlappi, L., & Tiu, C. (2010). Asset Allocation and Portfolio Performance: Evidence from University Endowment Funds. *Journal of Financial Markets Vol 13. (2)* , 268-294.
- Cheffins, B. R., & Armour, J. (2007). The Eclipse of Private Equity. *ECGI - Law Working Paper No. 082/2007* .
- Cochrane, J. H. (2005). The Risk and Return of Venture Capital. *Journal of Financial Economics vol 75, no. 1* , 3-52.
- Company, H. M. (2009-2011). *Endowment Report*. Retrieved from www.hmc.harvard.edu
- Ennis, R. M., & Sebastian, M. J. (2005). Asset Allocation with Private Equity. *The Journal of Private Equity* , 81-87.
- EVCA. European Private Equity and Venture Capital Association. www.EVCA.eu
- Gompers, P. A., & Lerner, J. (1997). Risk and Reward in Private Equity Investments: The Challenge of Performance Assessment. *Journal of Private Equity vol 1 issue 2* , 5-12.
- Harris, R. S., Jenkinson, T., & Kaplan, S. N. (2012). Private Equity Performance: What Do We Know? *Fama-Miller Working Paper; Chicago Booth Research Paper No. 11-44; Darden Business School Working Paper No. 1932316* .
- Higson, C., & Stucke, R. (2012). The Performance of Private Equity. *Working Paper*
- HMC. (2009-2011). *Endowment Report*. Retrieved from Harvard Management Company: www.hmc.harvard.edu
- Ibbotson. (2007). Private Equity and Strategic Asset Allocation. *Research Paper* .
- Ibbotson, R. G., & Kaplan, P. D. (2000). Does Asset Allocation Policy Explain 40, 90, 100 Percent of Performance. *Financial Analysts Journal vol. 56 no.1* .

JP Morgan (2008). Private Equity for Institutional Investors: Current Environment and Trends. Retrieved from www.jpmorgan.com

Kaplan, S. N., & Schoar, A. (2005). Private Equity Performance: Returns, Persistence and Capital Flows. *Journal of Finance* , 1791-1823.

Kaplan, S. N., & Strömberg, P. (2008). Leveraged Buyouts And Private Equity. *Working Paper*

Lerner, J., Schoar, A., & Wong, W. (2007). Smart Institutions, Foolish Choices: The Limited Partner Performance Puzzle. *The Journal of Finance Vol 72. No.2* , 731-764.

Ljungqvist, A., & Richardson, M. P. (2003). The Cash Flow, Return and Risk Characteristics of Private Equity. *Working Paper*

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance Vol.7, No.1* , 77-91.

Metrick, A., & Yasuda, A. (2009). The Economics of Private Equity Funds. *Review of Financial Studies 23 (6)* , 2303-2341.

Metrick, A., & Yasuda, A. (2010). Venture Capital and Other Private Equity: A Survey. *Working Paper*

NACUBO. *Commonfund Study of Endowment Returns*. Association of College and University Business Officers: www.nacubo.org

Partners Group (2012). Understanding Private Equity's Outperformance in Difficult Times. Retrieved from www.partnersgroup.com

Phalippou, L. (2007). Investing in Private Equity Funds: A Survey. *The research Foundation of CFA institute* .

Phalippou, L., & Gottschal, O. (2009). The Performance of Private Equity Funds. *Review of Financial Studies 22 (4)* , 1747-1776.

Phillips, Hager & North. (2008). Private Equity: An Overview. *Research Paper* .

SMC. (2001-2011). *Endowment Reports*. Retrieved from Stanford Management Company: www.stanfordmanage.org

SMC. (2011). *Stanford Management Company*. Retrieved from Annual Reports: www.stanfordmanage.org

Stucke, R. (2011). Updating History. *University of Oxford - Working Paper* .

Tellus Institute (2010). Educational Endowments and the Financial Crisis: Social Costs and Systemic Risks in the Shadow Banking System. Retrieved from www.tellus.org

Yale. (2009-2011). *Endowment Updates*. Retrieved from Yale Investment Office: www.yio.yale.edu

Appendix A

	Public Equity	Private Equity	Absolute Return	Natural Resources	Real Estate	Fixed Income
2001	-26 %	-43 %	14 %	3 %	13 %	15 %
2002	-8 %	-31 %	10 %	17 %	7 %	5 %
2003	2 %	-7 %	14 %	9 %	20 %	12 %
2004	24 %	12 %	18 %	20 %	19 %	1 %
2005	13 %	48 %	17 %	71 %	29 %	8 %
2006	23 %	20 %	13 %	61 %	19 %	1 %
2007	24 %	37 %	13 %	29 %	27 %	12 %
2008	0 %	7 %	2 %	44 %	2 %	13 %
2009	4 %	12 %	9 %	23 %	9 %	9 %
2010	17 %	17 %	23 %	6 %	-8 %	14 %
2011	24 %	36 %	15 %	27 %	20 %	6 %
StDev	0,16	0,28	0,05	0,22	0,11	0,05
Mean	0,09	0,10	0,13	0,28	0,14	0,09

Figure 18: Historical returns, standard deviations and mean returns

Estimated Correlations and standard deviations (Source Bekkers, Doeswijk, Lam 2009)

	Stocks	Private Equity	Real Estate	Hedge Funds	Commodities	High Yield	Credits	Bonds	Inflation linked bonds	stdev
Stocks	1									0,2
Private Equity	0,8	1								0,3
Real Estate	0,6	0,6	1							0,16
Hedge Funds	0,6	0,7	0,4	1						0,12
Commodities	0	0,1	0	0,4	1					0,26
High Yield	0,6	0,7	0,7	0,6	0	1				0,11
Credits	0,4	0,4	0,4	0,3	0	0,5	1			0,09
Bonds	0,2	0,2	0,1	-0,1	0	0,2	0,8	1		0,07
Inflation linked bonds	0	0,1	0,3	0,1	0,3	0,3	0,8	0,6	1	0,07

Figure 19: Estimated Correlations and standard deviations (Bekkers, Doeswijk, Lam 2009)

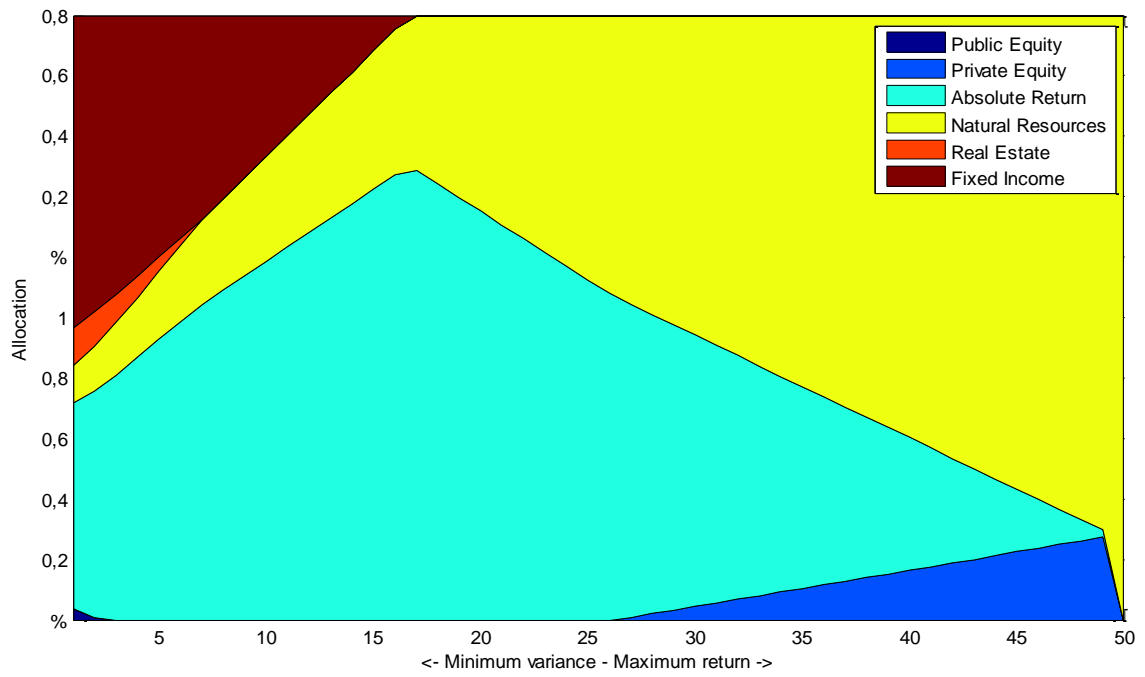


Figure 20: Efficient frontier allocations with purely historical data, PE expected return 27%

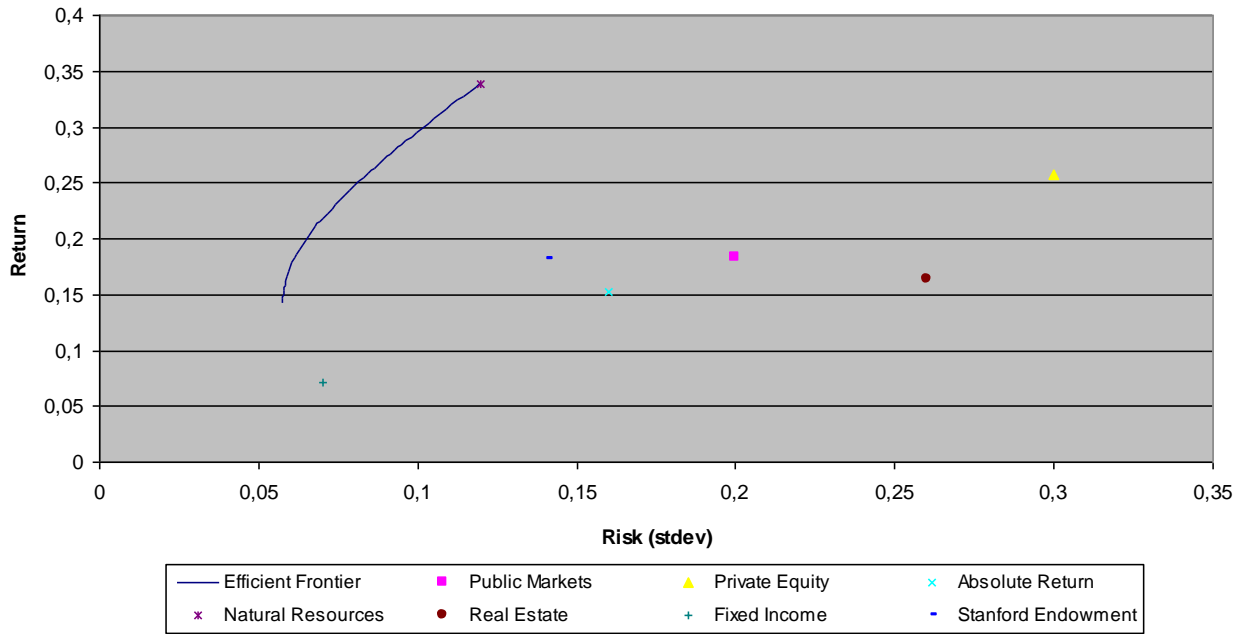


Figure 21: Efficient frontier with Different Assets with Purely Historical Returns, Risk Component from Literature

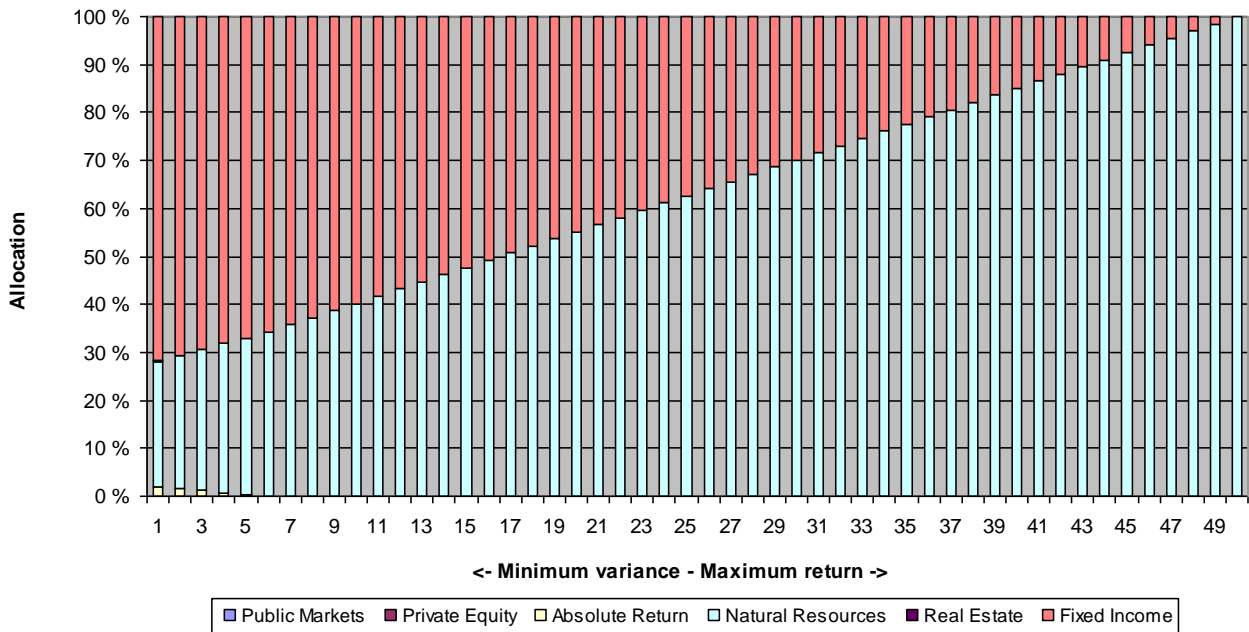


Figure 22: Efficient Portfolios with Adjusted Return and Risk Component from Literature (case 2b)

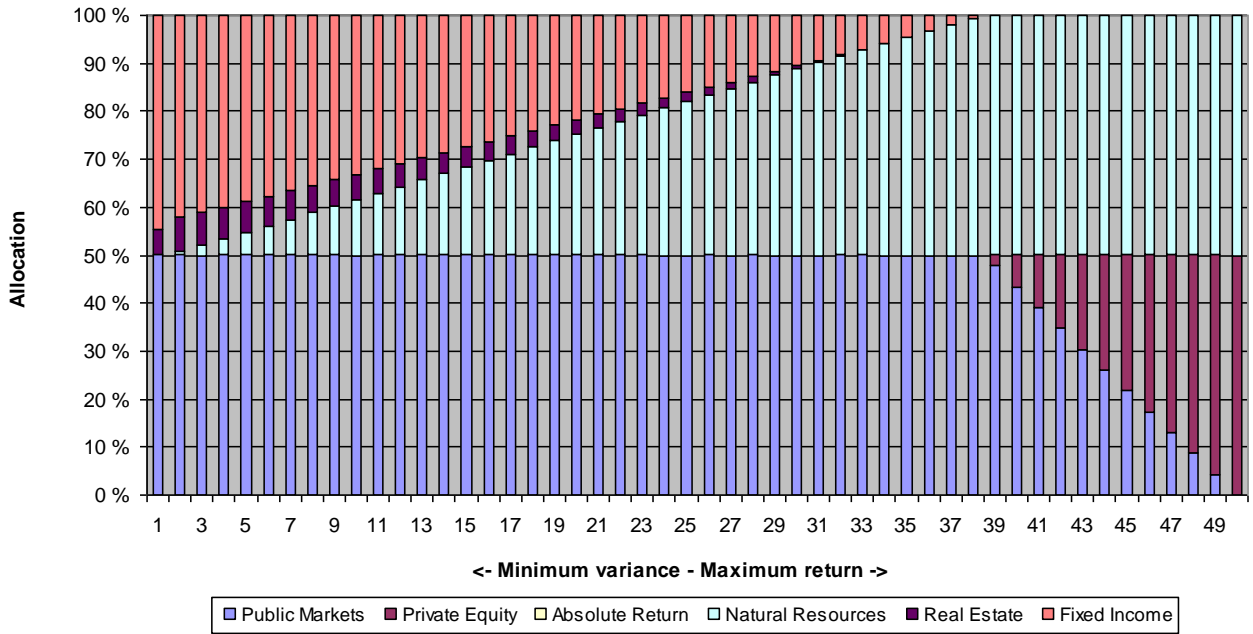


Figure 23: Efficient Portfolios with Adjusted Returns, Risk Component from Literature and 50% Equity Constraint (case 4b)