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Multi Period Balance Sheet Scenario Optimisation

Information Systems Science

Master's thesis

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Abstract of master's thesis

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Abstract

The objective of this study is to formulate and model a multi period scenario optimisation problem and to obtain recommendations for optimal balance sheet item portfolio. To do this the study gathers data from the public financial statements of the case company Aktia PLC, explains the relevant framework and model specifications, describes the actual optimisation model and finally presents the analysis of the results. The process combines many aspects of finance, decision making and optimisation. The portfolio optimisation problem is done over multiple periods with simulated future scenarios by formulating a tree representing the possible future outcomes.

The academic background combines aspects from information management and finance. The frameworks are selected from several different journals and fields in business studies and economics. The study explains the theoretical framework around the main topics of financial portfolio management, decision making and scenario optimisation. For the methodology the thesis aims to form qualitative research on the theories behind the model and the case company data in the form of literature review. The more quantitative research is done on the actual optimisation model and the analysis of these results. The case company was selected to allow for quantitative research and to show that the model works in real life.

The results from the model conclude with an optimal solution and a suggestion for changes made in one year for the balance sheet item portfolio. The results were heavily affected by the inputs and thus different solutions could be obtained easily. The base line model performed very well in comparison to other options. When comparing to the actual realisation for the most recently published data, the model showed a slight reduction in profits with the strong positive effect on balance sheet growth. In this solution the profits were 2 384 000€ lower in the model than those observed in year 2013 but the balance sheet total was 1 959 281 000€ larger.

Keywords Portfolio optimisation, multi period, scenario, optimisation, optimization, scenario tree, decision making, scenario optimisation, balance sheet optimisation, simulation

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I wish to thank the now retired Professor Kallio for his instruction on how to formulate my thoughts into a mathematical optimisation model that can be presented and solved in a thesis. I want to thank my thesis instructor Professor Kuula who was very efficient in pushing me in the right direction in my work with the thesis. Since Professor Kallio has instructed Professor Kuula's master's thesis many years ago, I too would like inspire or help new people in the future with their thesis work on the field of decision making and optimisation.

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1. INTRODUCTION

Ever since the financial crisis 2007-2008, bank decision making has been under the microscope (Shiller, 2008). Although optimal bank capital and balance sheet optimisation have been a popular topic in the 20th century (Modigliani & Miller, 1958), the developing investment options and financial innovations have brought more variables to the equation (Schwartz, 2008). Thus solving optimal bank balance sheet is not easy, with difficulties in uncertain future and forecasting asset returns.

From a wider perspective, having a stable economy, where bank risk taking is not excessive, is preferred by the entire economy. The problem is that when banks aim for better profitability and growth, problems with volatility tend to arise (Hnatkovska & Loayza, 2003). From the bank decision maker's point of view, the aim is to maximise profits while avoiding bankruptcy. To counter this conflict of interests, bank regulation has been introduced to prevent banks from taking too much risk (Dothan & Williams, 1980).

In this setup the aim of this study is to find optimal bank balance sheet structure for the case company and analyse the results. The model used for this optimisation problem generates scenarios to simulate the future outcomes of the uncertain returns on assets. The model solves the optimisation problem through several time periods. The problem can be referred to as multi period balance sheet scenario optimisation.

1.1. Motivation

The reason for taking on such a challenging problem is that it offered the author a chance to combine knowledge from studies with experience from working in Santander Consumer Finance Oy. This thesis is a continuation of previous work on the author's Bachelor's thesis, where a simple one period optimisation model was formulated. The thesis combines studies in optimisation, finance and statistical tools.

Relevant master's degree courses include advanced studies in optimisation, investment science, time series analysis, advanced statistical methods and simulation. These courses are offered in the information management program under information and service management master's degree in Aalto University. Thus the thesis is a continuation of earlier work put together with more advanced studies and real life work experience.

1.2. Background of the research and target group

Previous studies on balance sheet optimisation are often portfolio optimisation and risk focused. For example Markowitz (Markowitz, 1959) portfolio theory and various mathematical models have been used to estimate and analyse optimal bank portfolios (Crane, 1971) and (Kalman & Hammer, 1967). Since the banking industry and assets have become more complex over time, more complex models have been introduced to estimate asset risks and profitability (Glantz & Kissell, 2013).

These previous studies have optimised bank balance sheet items or individual investments by maximising profits or profits with respect to risk. However, this thesis deviates from standard portfolio optimisation and uses random scenarios to solve the maximisation problem. Previous scenario optimisation research and research with large scale optimisation solver software include studies for example in the forest industry in Finland (Rämö, 2013) or South America (Kallio, et al., 2012) and real investment options (Hilli, et al., 2007). Scenario optimisation is often used in stochastic problems with uncertainty, including economics, and can be applied to portfolio optimisation problems (Mausser & Rosen, 2001).

The target group for this thesis are students and people interested in scenario optimisation. Another target group is financial and other organisations interested in balance sheet or product mix optimisation. The results of this study can be applied to budgeting problems, and with some modifications to product management. The thesis assumes some understanding on mathematical optimisation and finance.

1.3. Research problem and aims of the study

Although bank balance sheet optimisation is complex, it offers interesting study options. This thesis aims to solve a multi period scenario optimisation problem. To do this it presents a mathematical optimisation model that is built to produce realistic results that provide real life implications. These results are presented and analysed in chapter 6. In short the research goals can be stated as:

1. Obtaining, editing and analysing the data from the case company
2. Explaining relevant framework and theory
3. Selecting and formulating a mathematical optimisation model
4. Using the model to solve optimal solution for selected data and time horizon
5. Analysing the results

The expected results should be somewhat close to the real life values of the case company. Moreover, the results should yield some implications and provide useful information about the case company.

Portfolio scenario optimisation and the methods used are important as a study subject because they make complicated portfolio optimisation easy to solve with a model that can be easily modified. Adding more information and complexity in the model is fairly easy and provides more precise results. Thus the model can respond to user needs or changing problem settings. Moreover, the method of scenario optimisation deviates from the standard portfolio optimisation, making it a great tool to cross check initial optimal values obtained with other methods.

1.4. Limitations

The study is limited by the fact that the focus is on information management systems and not finance or traditional portfolio optimisation. Compared to traditional budgeting problems where balance sheet items are selected to yield optimal portfolio, the model selected for this thesis is fairly simple and uses scenarios. Although the selected method of scenario optimisation model allows adding more information to the model, the model presented in this thesis does not include many properties. Some problems and limitations may occur due to the following factors:

- Balance sheet items are cut down to five: cash, loans, other assets, savings and debt
- Time horizon is one year with 4 quartiles, in reality decision making is more continuous
- Risk is taken into account in the form of exponential utility function and decreasing marginal returns
- Future returns are forecast with a simple model based on historical averages
- Emphasis on information management and decision making rather than finance
- The data analysis is done with available case company specific data
- Quarterly profits are maximised and expected to be paid out, rather than contributing to balance sheet growth

For better end results, the model could be improved by adding more financial information in the model for example in the form of better time series analysis or more precise inputs. Moreover, if the model was used in practise, the company could provide far better estimates of asset returns, forecast returns or possible future scenarios. For these improvements further studies are encouraged.

The results of this study can be used to get indication on what are the optimal values for the selected balance sheet items with the selected model and specifications. The results obtained this way represent the average yearly changes with the generated scenarios. In reality the final outcome and the realisation is one of the endpoints of the scenario tree, rather than the average.

1.5. Structure of the thesis

The structure of this thesis is as follows. The first two chapters are introductory, including necessary information about the case company and the collected data. Research problem, questions and main findings are also included in this chapter.

The third chapter goes through the necessary framework for this thesis and the fourth presents the model specifications. These two chapters form the literature review for this thesis. They include various topics in stochastic optimisation, financial portfolio optimisation, modelling and calculus. The fifth chapter explains the mathematical model used to solve the optimisation problem. This chapter advances from introducing the scenario tree structure further on to variables and constraints.

The results chapter reports the output of the optimisation model with two different inputs. It then compares the outcomes in terms of profits and changes in the balance sheet total items. These results are compared with historical changes and the actual changes from the year 2013 financial statement. The sensitivity analysis, which uses different input variables to obtain alternative output and results, is reported at the end of this chapter.

The final chapter concludes that the model performs well and is a good option compared to the actual changes for the year 2013. The model suggests on average a higher total for the balance sheet items with some reductions in profitability. This leads to higher market share and profits in the long run, arguing that the model produces realistic and efficient results. The real life implications are thus significant and the model could be applied to the decision making process.

2. CASE COMPANY AKTIA

The case company Aktia Oyj is a fairly small Finnish bank with a market share of 3,8% on Finnish loans and 3,0% on savings (Finanssialan Keskusliitto, 2012). It was founded 1826 in Helsinki and merged later in 1991 with other local banks (Aktia, 2014).

The main reason why this case company was chosen over several other options is that it has relatively simple balance sheet. The most important drivers of Aktia's business include giving out loans, managing different assets and holding on to savings. They do not specialise in complex investment options and have for example only derivatives that aim to counter economic fluctuations (Aktia, 2013).

However, there are some complications with the data related to the case company. For one, the data used for the purpose of this study only spans from 2006 to 2012. The historic data has only 7 data points from yearly data. For better precision one should aim to use more precise and current data. Furthermore, the company could have more company specific information about the future than the public data used in this thesis can predict.

One issue regarding the historic data is that it includes the economic crisis that hit especially the banking industry, preceding the euro crisis that started later in 2009 (Hall, 2012). Even if Finland did manage to maintain high credit rating, the economic recession greatly affected the interest rates in that period (Santis, 2012). Since the data is from this high risk period with low interest rates and high fluctuations, it might be seen as a disturbance when trying to forecast future data.

2.1. Case company data

The data used in this thesis was gathered from the public financial statements of the case company Aktia from the years 2006-2012 (Aktia, 2014). It was edited to fit the purpose of this research. The values of the balance sheet items were fairly easy and straightforward to obtain from the documents, but the corresponding profits needed some calculations and assumptions. Appendix 1 and 2 show the tables for the return calculations and Table 1 below explains the key aspects.

Calculations were made for the balance sheet items in order to obtain the desired 5 classes. The company used somewhat differing notations over the years and the income for the balance sheet items often consisted of several different income variables, so some assumptions needed to be made. Moreover, since the other assets didn't really have reported income, the income was calculated with a formula:

$$\begin{aligned} Other\ Assets\ income \\ &= All\ interest\ income - Cash\ income - Income\ from\ Loans \\ &+ direct\ Other\ Asset\ income \\ &+ direct\ Other\ Asset\ \ costs\ (negative) income \end{aligned}$$

Thus the income on other assets is a sum of other interest incomes and various other income costs and profits (Appendix 2). For the purpose of the study it was important to include this variable in the model, in order to avoid having loans and cash as only assets. The returns displayed below in Table 1 show the development of asset returns in the past six years.

There are significant changes in balance sheet item values and their returns. Some of these differences can be explained by the financial crisis and some variation by the random walk in the balance sheet item returns. These balance sheet item changes and returns are used to model the problem.

Table 1: Balance sheet items and returns (1000 €)

Balance sheet	2006	2007	2008	2009	2010	2011	2012
Assets							
Cash (C_i)	307 907	235 273	506 311	340 960	273 364	475 042	587 613
Loans (L_i)	3 797 018	4 757 011	5 526 194	6 141 562	6 637 551	7 152 124	7 360 225
Other Assets (A_i)	1 385 455	2 960 529	3 507 568	4 073 317	4 108 238	3 428 897	3 292 352
Total	5 490 380	7 952 813	9 540 073	10 555 839	11 019 153	11 056 063	11 240 190
Liabilities							
Savings (Y_i)	3 340 385	3 729 991	5 015 277	4 753 586	4 356 327	4 757 179	4 689 040
Debts (D_i)	1 728 973	2 740 892	3 130 482	4 045 926	4 827 366	4 464 037	4 584 724
Equity (E_i)	249 880	339 009	316 775	466 157	497 290	523 756	657 409
Other Liabilities (O_i)	171 142	1 142 921	1 077 539	1 290 170	1 338 170	1 311 091	1 309 017
Total	5 490 380	7 952 813	9 540 073	10 555 839	11 019 153	11 056 063	11 240 190
Returns %							
Cash (C_i)	1.72 %	3.43 %	1.84 %	0.92 %	0.91 %	0.69 %	0.21 %
Loans (L_i)	3.71 %	4.39 %	5.07 %	3.10 %	2.29 %	2.60 %	2.36 %
Other Assets (A_i)	2.99 %	2.99 %	3.99 %	3.02 %	3.28 %	3.18 %	3.27 %
Savings (Y_i)	-1.85 %	-2.87 %	-3.41 %	-1.66 %	-1.25 %	-1.33 %	-1.22 %
Debts (D_i)	-2.18 %	-2.67 %	-3.36 %	-2.03 %	-1.69 %	-2.40 %	-2.30 %
Equity (E_i)	-16.80 %	-17.90 %	-1.80 %	-8.70 %	-12.00 %	-7.10 %	-8.50 %
Other Liabilities (O_i)	1.47 %	-0.28 %	-0.81 %	2.16 %	3.37 %	2.77 %	3.65 %

One important observation from this data is that the cost of borrowing is in the observation period always lower than the interest incomes on the assets. In order to be profitable, the bank needs to make money by getting higher interest for the investment than the interest rate of the debtors (Freixas & Rochet, 2008). Moreover, the bank should aim to keep the relation between the costs and income reasonable. Since the return on assets is higher than the cost of liabilities, on average it would seem profitable to increase the balance sheet items excessively.

If the bank would like increase savings or debt, the costs for borrowing would start to rise. This is due to the increasing cost of debt as debt increases by increased default risk or increased interest rates (Angbazo, 1997). Similarly, if the bank would like to invest in loans or other assets, after a while available investment options would start to yield smaller interest profits. In order to consider this, the model includes marginal effects on returns. Decreasing marginal effects mean that as the volume of a balance sheet item is increased, the return for that variable decreases.

For each increase in loans, cash or other assets, the rate of return decreases. Similarly, for each increase in debt and savings, the cost of additional increase goes up. Adding these marginal effects to the model improves the interpretation of the data and the model better predicts the real life situations. This idea of decreasing marginal effects is supported by the data presented.

Further analysis on the returns and marginal effects is done in the model specifications chapter, where the return variables are formulated and analysed. The historical prices presented in this chapter are used to formulate the model.

3. LITERATURE REVIEW AND FRAMEWORK

The field of bank and balance sheet optimisation has long been a topic of interest in the academic field, with studies from 1960s (Kalman & Hammer, 1967), (Koehn & Santomero, 1980), (Diamond & Rajan, 2000). Because of the financial crisis in 2007-2009, the topic of optimal bank capital has generated numerous new articles, journals and discussions. The theory of modern bank balance sheet optimisation dates back to Markowitz portfolio theory (Markowitz, 1952). Relevant to the topic of portfolio optimisation is the randomness and stochasticity of the future outcomes (Aoki, 1989).

In addition to portfolio optimisation theory, this thesis includes scenario optimisation (Dembo, 1991). As financial assets become more complex, it might be easier to simulate reality rather than solve absolute values. Both optimisation topics have been thoroughly covered in financial literature and this thesis aims to combine these two main topics. In addition to these two main topics, this chapter introduces topics on time series modelling, large scale optimisation software and other theoretical framework behind the mathematical model presented in chapter 3.

3.1. Stochasticity

Stochasticity refers to a situation where a system or a process is non-deterministic and must be analysed using probability theory (Aoki, 1989). In other words there is a random element or an unpredictable parameter in the system. The randomness must be controlled as this thesis does with the use of mathematical programming (Luhandjula, 2004). In this thesis the unknown parameters are simulated from the selected distribution, thus taking into account the stochasticity.

In this study the randomness comes in the form of unknown future returns. This can be seen as a result of the banking industry being prone to fluctuations. Crises and changing economic conditions greatly affect the industry and investment returns (Eickmeier, et al., 2006). In order to take this randomness into account, this study generates random scenarios of different economic conditions.

Historical data is used to predict possible future returns and scenarios are simulated in order to give possible future outcomes for returns. The different future scenarios are then used to solve the optimal solution. The stochasticity and uncertainty can in this way be controlled, modelled and taken into account. In this thesis the stochasticity is countered with scenario optimisation and simulation.

Another option to solve problems with uncertainty is by using solving a theoretical absolute value. For example, in portfolio optimisation expected profits are often maximised with respect to risk (Markowitz, 1952). In scenario optimisation future forecasts can be simulated using historic data and the complicated problem can be solved fairly easily. This way the actual optimisation model can be more complex and additional changes can be made easily.

3.1.1. Modelling financial time series

There are several options for modelling financial time series. As the main focus of this thesis is optimisation and not time series analysis, a simple model was selected. However, there are more sophisticated options available for analysing financial time series.

For example different autoregressive models (Akaike, 1969) assume that the future values of the dependent variable depend linearly on its previous values. More complex models include for example AutoRegressive Conditional Heteroskedasticity ARCH models (Engle, 1982) and others derived with the same principles. Moving average models are also widely used, and the model used in this thesis is somewhat related to the idea of average returns (Pindyck & Rubinfeld, 1998). These models often show a clear trend in the data.

When modelling and forecasting financial time series, it is important to choose an appropriate model. However, no matter how well the model performs on historical data, it might fail to predict changes or shocks to the economy (Grabel, 2003). Financial models often include some degree of random walk (Spitzer, 1964). Most often a model is formed to give an estimate for future outcomes and the random walk occurs around this estimate.

A non-complex solution for forecasting future values for prices or returns R is by a simple model using only the mean μ and an error term ε:

$$R = \mu + \varepsilon$$

This simple model does not take directly into account the momentums or the movement of the time series data, but rather uses the mean to estimate future values. For better results, one might consider formulating a more complex model. Moreover, this model does not include any other variables but the mean. For example using a separate variable for general economic conditions might help in forecasting.

The graph below illustrates the observed historical data and provides simulated future forecasts that are generated randomly using the model selected for this thesis. This comparison is done to illustrate that the model produces reasonable returns by simulation. The model is presented later on in chapter 3.

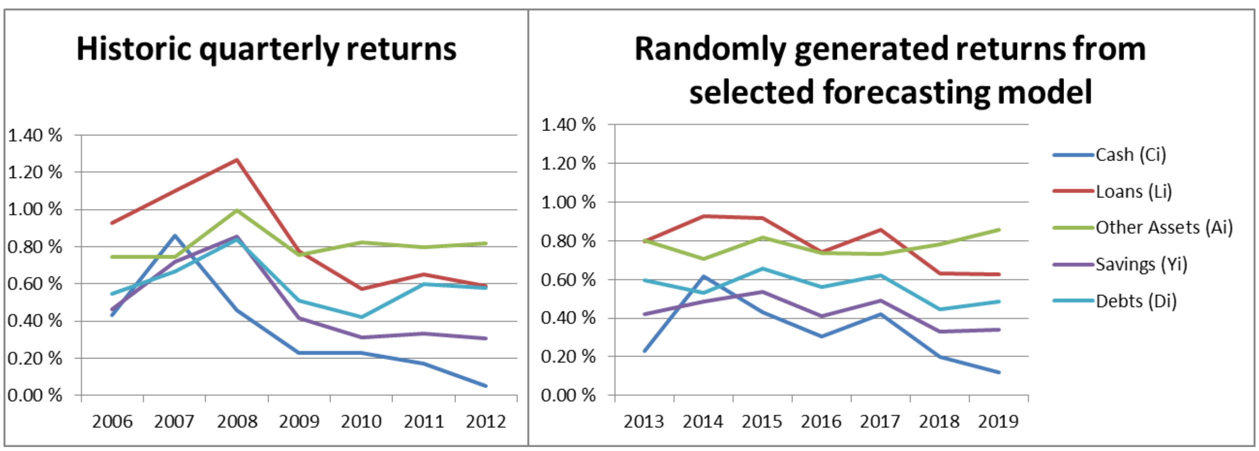

Figure 1: Historic quarterly returns and forecast made with the thesis model

This figure captures the element of randomness and uncertainty in financial forecasting and provides an estimate for future values. The output of the model does not efficiently predict or forecast similar economic recessions as can be seen in the historic data 2006-2008, but rather relies more on the average values and random walk. Relying on the average values is the downside of the model used in this thesis.

However, for the purpose of this study, this model predicts future values with sufficient precision. Moreover, for scenario optimization it can be seen as a desirable quality to have the development of the returns close to the mean and to have generally less extreme outcomes. In other words the model generates numerous random scenarios. Some of these scenarios will have extreme values such as those observed in times of economic turbulence, but on average it provides forecasts close to the average values. When there is no time trend, the scenario tree branches do not often wander off to far extremes.

3.2. Scenario optimisation

Scenario optimisation is an important tool when dealing with uncertainty. In scenario optimisation, when faced with uncertainty, possible outcomes and their corresponding probabilities are estimated (Dembo, 1991). Two important topics discussed in this thesis under scenario optimisation are large scale optimisation problems and optimisation software.

The model used in this thesis estimates 5 random scenarios for each time period. The probabilities for each outcome are the same and thus the decision tree follows a random walk with the number of decision nodes increasing exponentially. Thus for the scenario optimisation setup the scenarios are random, given equal probabilities and increasing exponentially in numbers each time period.

3.2.1. Large scale optimisation problems

Since scenario optimisation is used over multiple periods, the number of decision nodes is increasing exponentially. For a one period model with four quartiles and 5 scenarios on each period, the number of individual final outcomes is 5^4 = 625. If 5 scenarios was used in a monthly data for modelling a yearly decision making process, the last stage alone would have 5^12 = 244 140 625 decision nodes. Since this is a model with multiple decision periods, decisions need to be made on every period.

There can be thousands of decisions to be made in a multi period scenario optimisation model. Moreover, the decisions made earlier often affect the decisions later on in the scenario tree. For each node, there are previous decisions, possible future scenarios and current scenario situation that affect the decision making process. Thus the problems become quickly very complex and complicated to solve with traditional optimisation models.

In order to solve large scale scenario optimisation problems, one needs to use software in order to avoid infinite solver calculation process.

3.2.2. AMPL optimisation software

AMPL provides optimisation software for large scale optimisation problems for linear and nonlinear problems (Fourer, et al., 2002). The program includes numerous different solvers that work in AMPL to find optimal solutions. These different solvers can be used to solve linear, nonlinear, quadratic, mixed integer and other types of optimisation problems (AMPL Optimization LLC, 2014). The distribution of different solver software and different solvers can be seen in the following graph (NEOS Server, 2013):

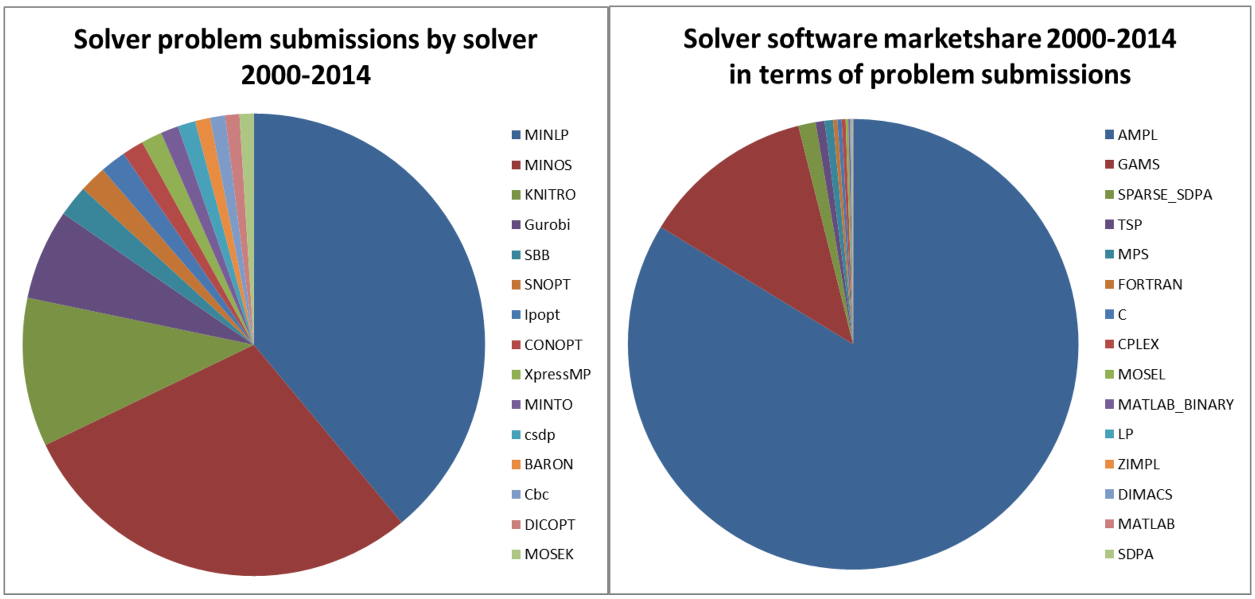

Figure 2: Market share of solver software and solvers 2000-2014 (NEOS Server 2013)

Since it is possible for anyone to create a solver using AMPL, there are plenty of options to choose from. However, there are a few software and solvers that dominate the market in terms of market share. In software AMPL and GAMS are most used, when in solvers MINLP and MINOS are most preferred. For the purpose of this study, the solver MOSEK was selected, as it offers a free and efficient solver for nonlinear optimisation.

The model needs to be written in AMPL code language in order to solve the large scale optimisation problem. Different solver software and solvers have different requirements for how the code should be represented. The model is presented in the way it is inserted into AMPL solver software using MOSEK solve (Appendix 6, 7 and 8)

The problem consists of three separate files: the model (Appendix 6), the data (Appendix 7) and the run (Appendix 8) file. The run file is used to operate the other two files and print the output. For each of the files, the code is written on the left hand side and comments indicated with # are written on the right hand side. They have no operational function but are only included to explain the code. In addition to the AMPL software, in order to operate, one needs to obtain the MOSEK solver.

The run file orders the commands to operate the model and the data files. Moreover, it prints the output of the optimisation solution, presented in Appendix 5. The output includes returns from the data file, all the decision variables and expected profits, final stage outcome node expected profits and at the end the expected average yearly changes in each balance sheet item.

3.3. Portfolio optimisation

Portfolio optimisation refers to a set of assets which are chosen to find an optimal solution, most often by maximising profits with respect to risk. As mentioned before, this theory dates back to Markowitz theory explained in his book (Markowitz, 1959). As the financial markets have developed throughout the years, the portfolio optimisation problem settings and models have become more complex, offering for example solutions for large scale financial problems (Perold, 1984).

The focus in the field of bank balance sheet optimisation is often in portfolio management and investment decisions, where risk with respect to maximum profits is being optimised (Brodt, 1978). Previous portfolio optimisation research include studies with generally two main themes:

theoretical and mathematical emphasis often related to operations research (Oguzsoy & Guven, 1997) or more practical background with focus on analytics and financial studies (Selhausen, 1977). In addition to portfolio optimisation, scenario optimisation is used with simulation in economics (Pflug, 2001). However, studies using scenario optimization with simulation in multi period model for bank balance sheet portfolio optimisation were not discovered.

Scenario tree optimization might not always be the best alternative. For example, the use of more advanced computer algorithms may be able to forecast with the given data more precisely, leading to better results (Beraldi, et al., 2013). The more advanced forecasting methods and tools may be superior in financial forecasting. On the other hand, often in finance the key to success is personal experience with the markets and the ability to forecast economic conditions and financial situations (Selhausen, 1977). To counter the problem of relying too much on data or expert knowledge, models that combine expert opinions with mathematical optimisation modelling have been produced (Lutgens & Schotman, 2010).

To answer the question of which optimisation method to use there is no single answer. Different options have different pros and cons. Simple models can usually be easily expanded where as more complex methods usually provide greater precision. To select one model over the others one needs to justify the reasons for selecting the model.

One of the most used portfolio optimisation models is the Markowitz model, that optimises portfolio returns with respect to risk (Markowitz, 1959). Figure 3 illustrates the Markowitz optimisation of profits with respect to risk. However, as Markowitz model is nonlinear, there have been several suggestions to a linear programming portfolio optimisation model. Two main linear programming models include the mean absolute deviation model (MAD) (Konno & Yamazaki, 1991) and the conditional value at risk model (CVaR) (Rockafellar & Uryasev, 2000).

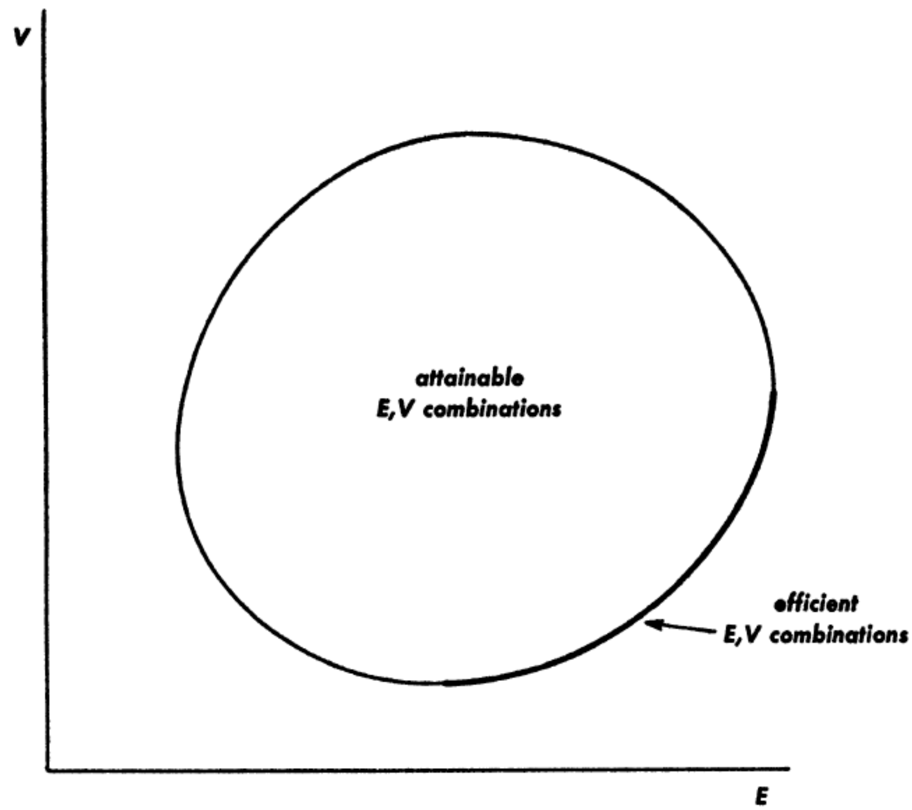

Figure 3: Markowitz portfolio optimisation and the efficient frontier (Markowitz, 1952)

As seen from the figure, balance sheet optimisation can be seen as a nonlinear trade-off between profits and risk. In the picture, the expected profits E is on the horizontal axis and volatility V is on the vertical axis. The circle represents the possible portfolios, with corresponding expected profits and volatility. For the decision maker, the best options lie on the frontier that has the highest expected profits or lowest volatility. The efficient frontier is marked on bottom right with a thicker line, where no point is dominated by lower volatility and higher profits. This curve represents feasible solutions for an optimisation problem. The decision maker must choose the optimal point according to his profits per volatility trade off.

The different optimisation models often combine at least one of the two: profits and risk. However, this thesis uses a nonlinear model which includes the cost of risk within the model in the parameter returns. Thus it aims to maximise the profits, where the risk is included in the model in the form of decreasing returns and exponential utility function. These topics are discussed further in this chapter.

3.4. Bank regulation and capital requirements[1]

The idea of bank regulation aims to prevent banks from taking too much risk. It should be clear that with the financial crisis in 2007 the banks were taking too much risk and operating in a way that was not optimal for the whole economy or the banks in the long run (Miles, et al., 2013). As a result of the crisis it can be seen that the bank regulatory framework which was based on the Basel II recommendations was unsuccessful in preventing the crisis (Danielsson, et al., 2001).

When talking about optimal solution for banks' balance sheet it is important to see that the optimal level for bank capital is different from the banks' point of view when compared to the whole society's interests. The society would prefer a stable banking sector with no crises, but the banks need to take risk in order to stay competitive. The banks with better loan and service prices attract more customers and thus banks are encouraged to take more risk. To solve this problem of increasing risks, bank regulation aims to prevent banks from taking too much risk. Stricter bank regulation results in a more stable banking sector, but there are also costs related to it.

Although the mathematical model in this thesis does not include bank capital, it is important to understand how it affects decision making and how bank regulation works.

3.4.1. Bank equity and leverage

Bank decision making is about managing different balance sheet items or the bank portfolio in changing market situations. A decision maker chooses to change the different bank balance sheet items by taking into account the future prospects of the returns on these items. Looking at it this way the optimisation problem for the banks' decision maker is to maximise the expected utility of

[1] This master's thesis is a direct continuation to my earlier work on bachelor's thesis and I wanted this chapter to be published with some modifications in this framework section.

the profits by changing certain decision variables on the balance sheet. (Fletcher, 1995) and (Sinkey, 2002)

Capital requirements and bank regulation work as constraints in the optimisation problem for the decision maker. For example Berger, Herring and Szegö discuss the role of capital in financial institutions in their article (Berger, et al., 1995). Bank capital most often refers to the bank capital stock which consists of shareholders' equity and disclosed reserves (Basel Committee on Banking Supervision, 2006). Another form of bank regulation is minimum requirements for cash, which can be directly and easily imposed on banks.

Capital requirements mean that banks have to hold a certain amount of equity with respect to their total assets. The capital ratio is defined as the share of equity in total assets $\frac{Equity}{Total\ Assets}$. Setting a minimum capital requirement means a bank has to acquire equity and cannot expand unlimitedly by borrowing money and increasing debt.

Because this method of measuring capital ratio does not take into account the risks related to different assets, most of the capital requirement models use weights to valuate risk. To estimate the risk weighted assets, the banks categorise their assets and have each category multiplied with the corresponding risk weight. The Basel accords set the minimum requirement capital according to $\frac{Equity}{Risk\ weighted\ assets}$ and they have different categories for equity – Tier 1 capital, Tier 2 capital etc. (Balin, 2008).

Leverage is closely related to the idea of capital requirements and is defined as $\frac{Total\ Assets}{Equity} = \left(\frac{Equity}{Total\ Assets}\right)^{-1}$. Thus in a highly leveraged company the equity is small compared to the total assets. This means that imposing a capital requirement would mean setting a maximum value for leverage. To understand the effects of capital requirements one needs to understand how leverage affects the banks.

Whether high leverage is a good or a bad thing depends on the market situation (Chew, 1996). This means that in a good economic situation the return on equity increases with leverage since the profits of a profitable company, with high leverage, are shared with a small amount of shareholders. Similarly a high leverage in poor economic situation causes the return on equity to decrease and even turn negative. The article by the Committee on the Global Financial System (Committee on the Global Financial System, 2009) discusses how leverage is closely linked with risk as high leverage increases risk but also the potential profits and amplifies the effects of the economic situation.

Limiting leverage is important for the stability of the banking sector, which has a huge impact on the welfare of the country (Miles, et al., 2013). When bank regulation and capital requirements are increased, the decision makers will have to increase the capital stock beyond the original optimal value for the bank (Koehn & Santomero, 1980). The total effect from increased bank regulation is a more stable banking sector, but decreased returns for the individual banks.

3.4.2. Bank regulation development

The reason why individual banks would not operate on the optimal bank capital level for the whole economy comes from the fact that they need to take risk in order to stay competitive on the markets (Boyd & De Nicoló, 2005). Bank regulation was introduced to solve this issue. By forcing banks with laws to have enough equity and cash in the bank the government can ensure a safe banking sector.

In general, this means a bank would be required to have enough cash and other liquid assets to cover for the customers' need for cash. If the amount of cash and other liquid assets drops or remains low for a long period of time, the bank is likely to have liquidity problems and may experience a bank run (Diamond & Dybvig, 1983). The cash constraint introduced in the model specifications is an important influencer in the optimal solution of the model presented in this thesis.

In addition to the bank run, another disaster scenario is when the bank does not have enough capital or equity. When a bank runs out of equity to cover for the losses of falling asset prices it goes bankrupt. A bank that goes bankrupt has a significant negative effect on the whole economy. (Gruber & Warner, 1977). This is what happened in the financial crisis in 2007 when the losses from risky assets grew large and the banks ran out of equity to cover for the losses. Equity is dropped out of the optimisation problem due to its complexity and the bad fit with the model specifications and other balance sheet items.

Comparing the two effects of better stability of the banking sector and poorer profitability explains the main issue of bank regulation. Banks need to take risks in order to stay competitive, but the risks related to highly leveraged banks can cause bankruptcy or bank runs that have major negative impacts to the whole society. One of the most used models for estimating the cost of capital was presented in an article by Modigliani and Miller (Modigliani & Miller, 1958).

In addition to the problem of optimal bank capital, there are significant challenges in the implementation of bank regulation. The problem with capital requirements is that as banks have an incentive to increase leverages beyond the values of regulative limits to maximize profits, they will find ways to increase risks and leverages in other ways in order to avoid the higher capital requirements (Tett, 2010). For example before the financial crisis, banks increased risks and leverages with new financial innovations as explained in the article discussing the causes of the financial crisis (Baily, et al., 2008). The increase in risks was mainly possible because of financial innovations that enabled banks to increase risks and leverages (Culp & Neves, 1998).

When the capital requirements fail to effectively stop banks from increasing leverages above the optimal solutions for the whole society, regulation is needed. There are several suggestions on how to improve the regulation on the banking sector to efficiently stop the banks from taking too much risk.

The two major proposals on how to improve the regulation include the Basel Accords and the Independent Banking Commission’s report - also known as Vickers report. They all aim to solve the problem with the banking sector by combining higher capital requirements with stricter regulation. The Basel I and Basel II agreements had been widely implemented already before the financial crisis (Drumond, 2009), but as they did not prevent the financial crisis they have received criticism (Balin, 2008). Moreover, even before the financial crisis, some of the weak points of Basel II were already pointed out by researchers (Danielsson, et al., 2001).

Basel II already included a capital requirement of 8% out of risk weighted assets. This meant that the assets were given weights according to their risk. As explained before the use of financial innovations allowed the banks to work around such capital requirements and thus the Basel II was insufficient to prevent the financial crisis alone. The Basel III agreement aimed to update the Basel accords in order to successfully prevent future crises.

Vickers report was released September 2011 and the implementation of the recommendations was started in the UK immediately. The recommendations were aimed for the UK's government and were even more complex than those of Basel recommendations (Independent Commission on Banking, 2011) .

Both Basel III and Independent Banking Commission's recommendations included higher capital requirements combined with other regulatory changes to prevent the banks from taking too much risk and increasing leverages beyond the values optimal for the whole economy. The Basel I and II failed to prevent the financial crisis mainly because they lacked regulation and the banks were able to go around the capital requirements and increase risk beyond the point where they can still bear the losses.

The Basel III and Vickers recommendations aim to fix this problem and introduce a sufficient regulatory reform for the financial sector to prevent future crises spreading from the banking sector to the real economy. In addition to these capital requirements and stricter bank regulation, both Basel III and the Vickers report included liquidity requirements.

The following table shows the capital requirements for different capital ratios in the Basel III framework.

Table 2: Basel III requirements for bank capital

	Basel III requirement	Additional countercyclical buffer
Tier 1 Capital/Risk Weighted Assets	8 %	
Tier 1 + 2 Capital / RWA	6 %	+2,5%
Common equity / RWA	4,5 %	+2,5%

As Aktia's $\frac{Tier\ 1\ Capital}{Risk\ Weigted\ Assets} = 11{,}8\%$, it clearly has a good financial standing and any capital constraints on equity would not be constraining (Aktia, 2013). Most Nordic banks have very high levels of capital. Moreover, since equity is not included in the model because it fits poorly the model setting, trying to force any equity constraints would be illogical. Thus, bank regulation should be included in the model in the form of cash constraints.

4. MODEL SPECIFICATIONS AND METHODS

This thesis formulates a mathematical optimisation model to solve a multi period balance sheet optimisation problem. Since the model is stochastic, scenario optimisation is used. The framework necessary for modelling is introduced in chapter 2 and the model is presented in the following chapter. In order to solve the problem, this thesis uses the AMPL optimisation software. In other words the methodology of this thesis is quite straight forward: the necessary framework is explained, then the model is presented and finally the obtained results are analysed.

The model selected for thesis uses several theories in order to solve the maximisation problem. The framework provided in the following subchapters is aimed to explain some concepts that are required to fully understand the model. In addition to explaining the theories, some justification for the selection of these theories is provided.

4.1.1. Lognormal distribution and lognormal returns

The lognormal distribution refers to a variable whose logarithm is normally distributed (Balakrishnan & Chen, 1999). Lognormal distribution **X** values can be created from normal distribution **Y**. It is possible to express the lognormal distribution **Y** as the logarithm of the normal distribution **X** or the other way around:

$$\boldsymbol{Y} = \log(\boldsymbol{X}) \quad \text{and thus also} \quad \boldsymbol{X} = exp(\boldsymbol{Y})$$

The values of the lognormal distribution **X** are always positive with a mean and variance that can be defined. It is important that the return distribution follows a lognormal distribution with strictly positive values and long tail representing unlikely crisis situations. Since the returns need to follow a lognormal distribution, the return vector **R** for all assets i can be generated using normal distribution, with the observed mean μ and the variances from the covariance matrix V.

$$\log(\boldsymbol{R}) \sim N(\mu, V)$$

The lognormal distribution can be drawn as a graph. Here is a simple illustration of the lognormal distribution with a standard normal distribution of mean 0 and variance of 1.

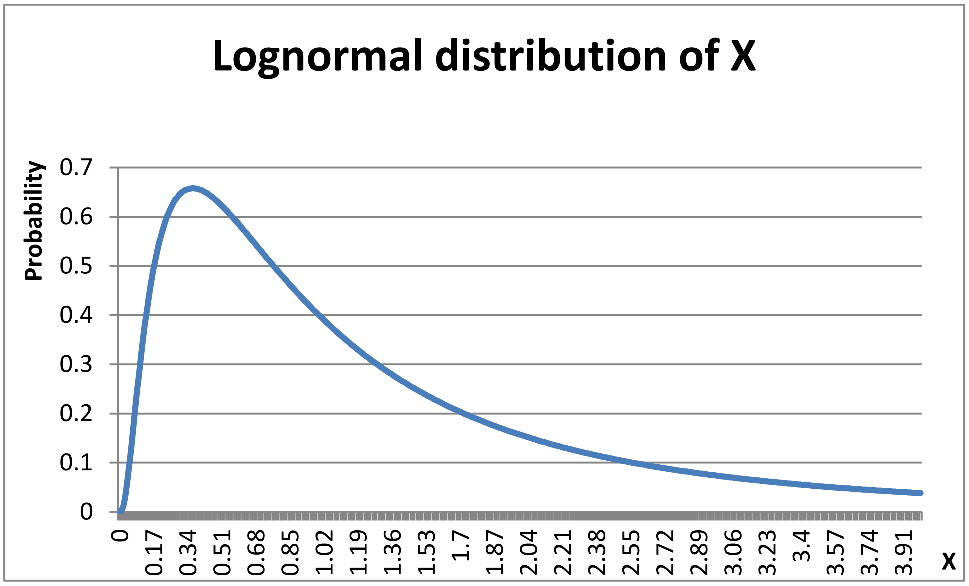

Figure 4: Lognormal distribution with standard normal distribution N(0,1)

The reason why lognormal distribution was selected for this study is that it produces returns that are always positive and they simulate returns realistically. This means that returns are generated with the same average and corresponding variances, with some extreme values in high returns or returns close to 0. Scenarios with very high returns are possible but not very likely, similarly scenarios with returns that are close to 0 are possible. This would be the case for example in a financial crisis, where assets returns might have strong fluctuations.

4.1.2. Exponential utility function

Utility theory is often used in economics and has a background on behavioural sciences. It aims to provide a measurement of utility for monetary gain (von Neumann & Morgenstern, 1953). Important theories include Von Neumann–Morgenstern utility theorem with four key axioms for utility theorem and Bernoulli's formulation, originally published in Latin 1738 and later translated in 1954 (Bernoulli, 1954).

In a stochastic system that includes uncertainty, using exponential utility function and maximising expected utility is often used in portfolio optimisation (Pratt, 1964). Using this method, each possible outcome is denoted with a utility according to the selected utility function. The expected utility is then calculated multiplying the utility of the outcome by the probability of that event. The total sum of different expected utilities is the expected utility of the decision.

Exponential utility function can be expressed in terms of portfolio profits p in a single time period and the Arrow-Pratt coefficient of absolute risk aversion γ (Arrow, 1965) (Pratt, 1964).

$$u(p) = -e^{-\gamma p}$$

The exponential utility function resembles a risk averse decision maker preference, where the utility function is exponentially concave (Rabin, 2000). In this function very small values are given more weight, having a large impact on the decision making where as high returns yield much less additional utility. The decreasing marginal utilities represent the risk averseness of the decision maker.

A concave exponential utility function resembles the decision maker's relation to risk. For a larger risk factor γ, the curve is sharper, resulting in more risk averse behaviour. Similarly, if γ is very small, the decision maker is more risk neutral. For a convex utility function the decision maker can be classified as a risk taker. To estimate the risk averseness, the Arrow-Pratt measure of absolute risk-aversion (ARA) can be calculated (Pratt, 1964) and (Arrow, 1965):

$$A(P) = -\frac{u''(p)}{u'(p)} = \gamma$$

Since the absolute risk-aversion coefficient is a constant γ and not a function of profits p, it can be concluded that changes in profits P do not affect preferences of risk (Pratt & Zeckhauser, 1987). The fact that profits do not affect risk preferences can be argued to be unrealistic, but for the profits obtained in this thesis, the problem is quite insignificant.

Decision makers in the banking sector are often trying to minimise volatility and maximise profits according to portfolio theory (Prigent, 2007). To include this risk averseness in the optimisation model, this thesis uses exponential utility function. However, exponential utility function has received some criticism for the reason that the absolute risk aversion is assumed constant (French & Buccola, 1978).

The gamma factor γ is estimated so that the utilities from the profits form a reasonable concave curve. This can be challenging when the true gamma factor of the company is unknown. When the selected –γ is multiplied by profits (p) and included inside an exponential function, the results from the utilities form a curve that resembles the risk tolerance of the bank (Pratt, 1964).

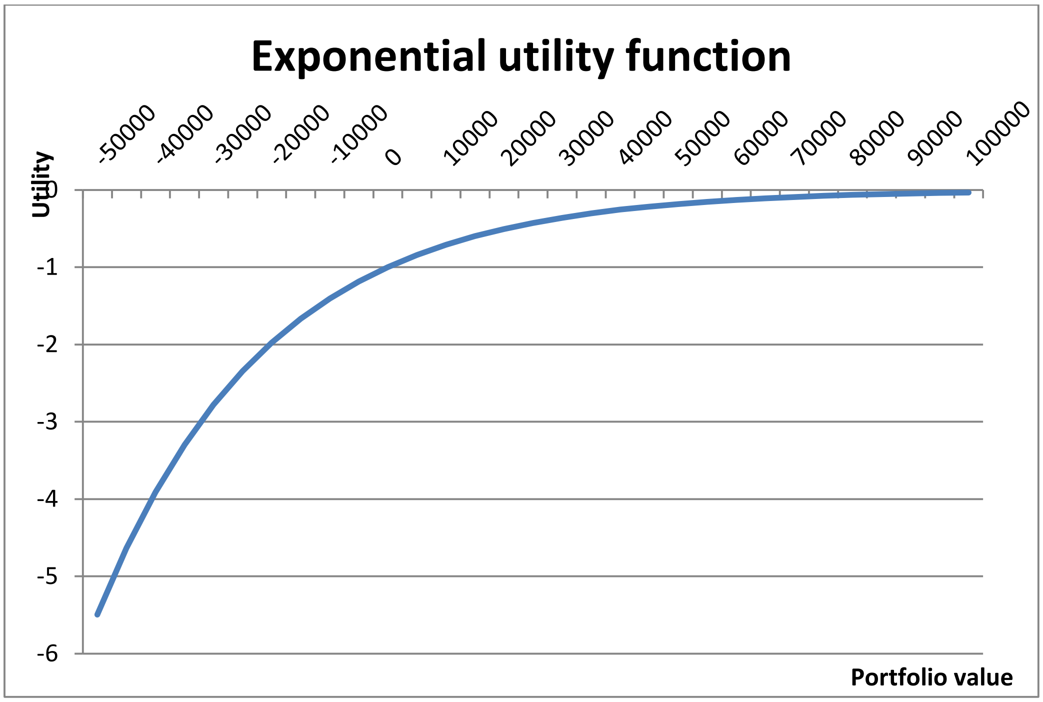

Figure 5: Exponential utility function randomised example with γ = 3,4E-05

In order to form a reasonable utility curve, the gamma factor γ is selected so that the exponent –γ*p is close to negative one. The reason why the utility function is scaled to this number is that since the utility is changing exponentially, e raised to the power of one result in reasonable changes in the utility for values below and above the profits p. If some other number was selected, the steepest part of the curve would move from the middle towards one of the ends.

The selected scaling factor proposes an estimation of the decision maker preferences, but for better results, one should select a utility curve that matches the company's unknown true gamma factor. The value for the gamma factor can be estimated with the profits from historical data. The profits per quartile are one fourth of the average yearly returns. Using the average yearly returns (calculated from Appendix 1), the gamma factor is estimated to be:

$$\gamma = \frac{1}{p} \quad => \quad \gamma = \frac{1}{\frac{average\ yearly\ returns}{4}} = \frac{4}{117345} \approx 0{,}000034$$

Using this gamma yields a utility function that gives the portfolio values reasonable concave functions. Other gamma factors can be used and the risk tolerance should be estimated and selected by the decision maker. However, since there is no knowledge of Aktia's true gamma factor or risk tolerance, this rough estimate is used for the purpose of this study.

4.1.3. **Cholesky factorisation**

To use the Cholesky decomposition one needs a positive definite covariance matrix from the data (Watkins, 2004). This matrix can be obtained by calculating the covariance between error terms. As mentioned before, the model uses lognormal returns, mean and error term to model future returns. The returns can be expressed in vector form **R** = (R_i) for a single time period. The model for the returns can thus be written as:

$$\log(\boldsymbol{R}) = \mu + \varepsilon \qquad where \qquad \log(\boldsymbol{R}) \equiv \{\log(R_i)\}$$

The observed error terms of this model form m-by-n matrix denoted the error term matrix **E**. In the data used, this matrix was 5-by-7 and included the data from 2006-2012 and 5 different balance sheet items. The error terms are calculated from the mean.

$$\boldsymbol{E} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \cdots \\ \varepsilon_{21} & \cdots & \cdots \\ \cdots & \cdots & \varepsilon_{mn} \end{bmatrix} where$$

$$\epsilon_m = \{\varepsilon_{m1}, \varepsilon_{m2}, \dots, \varepsilon_{mn}, \}$$

The error term covariance matrix V can be calculated from the error term matrix **E** by solving the covariance between the rows of the balance sheet item error terms ϵ_m. In other terms the covariance is calculated between two balance sheet item rows at a time from the data. This error term covariance matrix is denoted V and can be expressed as:

$$V = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots \\ \sigma_{21} & \cdots & \cdots \\ \cdots & \cdots & \sigma_{mm} \end{bmatrix} where$$

$$\sigma_{kl} = cov(\epsilon_k, \epsilon_l)$$

$$\sigma_{kl} = \sigma_{lk}$$

$$\sigma_k^2 = \sigma_{kk} = var(\epsilon_k)$$

To use the Cholesky decomposition, one needs to solve the Cholesky factor C from the covariance matrix V = C^TC. Here the covariance matrix is decomposed into upper and lower triangular Cholesky matrixes. The lower matrix, named Cholesky factor C, is the matrix that need to be obtained. In the following calculations c_{kk} denotes the elements in the Cholesky factor C, with arbitrary c and k denoting specific row and column element. The solution can be found by using matrix calculations that are explained further in the literature. (Watkins, 2004).

$$V = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} & \sigma_{25} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} & \sigma_{35} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} & \sigma_{45} \\ \sigma_{51} & \sigma_{52} & \sigma_{53} & \sigma_{54} & \sigma_{55} \end{bmatrix} = CC^T$$

$$= \begin{bmatrix} c_{11} & 0 & 0 & 0 & 0 \\ c_{21} & c_{22} & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 \\ c_{41} & c_{42} & c_{43} & c_{44} & 0 \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} \end{bmatrix} * \begin{bmatrix} c_{11} & c_{21} & c_{31} & c_{41} & c_{51} \\ 0 & c_{22} & c_{32} & c_{42} & c_{52} \\ 0 & 0 & c_{33} & c_{43} & c_{53} \\ 0 & 0 & 0 & c_{44} & c_{54} \\ 0 & 0 & 0 & 0 & c_{55} \end{bmatrix}$$

Now for the diagonal elements:

$c_{kk} = \sqrt{\sigma_{kk} - \sum_{n=1}^{k-1} l_{kn}^2}$ for example $c_{33} = \sqrt{\sigma_{33} - (c_{31}^2 + c_{32}^2)}$

For the elements below the diagonal line, where l > k in c_{lk} then:

$$c_{lk} = \frac{1}{c_{kk}}\sqrt{\sigma_{lk} - \sum_{n=1}^{k-1} c_{ln}c_{kn}} \qquad \text{for example} \qquad c_{32} = \frac{1}{c_{22}}\sqrt{\sigma_{32} - c_{31}c_{21}}$$

Thus solution for the Cholesky factorisation $C = \begin{bmatrix} c_{11} & 0 & 0 & 0 & 0 \\ c_{21} & c_{22} & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 \\ c_{41} & c_{42} & c_{43} & c_{44} & 0 \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} \end{bmatrix}$ can be obtained.

The Cholesky factor C can be used to solve linear equations. Moreover, multiplying the Cholesky factor C with a random uncorrelated vector U of standard normal variables, the output vector CU has the same covariance properties as the original covariance vector V (Rubinstein & Kroese, 2011). As mentioned before, this random vector **R** follows a multivariate lognormal distribution, since the logarithm of returns is used and the error terms are lognormally distributed.

The simulation of new returns can be performed as a function of C, U, the identity matrix I and the mean vector μ of historical logarithmic returns:

$$\log(\boldsymbol{R}) = \mu + CU \qquad where \qquad U \sim (0, I)$$

Here the Cholesky decomposition is used to correlate randomly generated lognormally distributed error terms of the returns. The error terms are used to simulate returns that follow a correlated lognormal distribution. The returns are thus similar in terms of mean values and correlations to those of the historical values.

5. MULTI PERIOD SCENARIO OPTIMISATION MODEL

To model a financial stochastic model, this thesis uses multi period scenario optimisation model. It aims to optimise case company Aktia's balance sheet by creating scenarios based on historical data and finding the optimal solution over a time period of 4 quartiles.

The returns for scenarios are randomly created using error terms that are lognormally distributed. Lognormal distribution is used so that the returns would follow a realistic nonnegative and long tailed distribution. Return for each scenario is created separately and different scenarios do not follow any pre-set pattern. However, based on historical data, the model does use methods that affect the randomness of the returns described later in chapter 5.2. These methods are used in order to take into account several real life factors, such as the drift in he observed data and the decreasing marginal returns.

This chapter explains the formulation of the optimisation model. It starts with the structure of the scenario tree and the decision variables. It then moves on to the return simulations, which use the theories presented in the previous chapter to simulate future returns. The marginal effects and the drift factor is shown in here since they are a part of the return simulation. Next the objective function is explained and the constraints are listed. The chapter finishes with the AMPL files used to solve this problem.

5.1. Scenario tree structure

The time horizon for this problem is one year, including four quartiles. Different time periods are denoted with the STAGE variable and for example STAGE = 0 represents the start of the year and the first quartile. The final time period, STAGE = 4, does not include any decisions but is used to assess the outcomes of the final decision period, denoted STAGE = 3. Each of the quartiles represents one stage, with the first stage only including decisions and the last stage only including results. The stage can be represented as a set with STAGE = {0, 1, 2, 3, 4}

For each stage there are five possible future scenarios. Scenarios represent different outcomes in the returns for the balance sheet items. There are 5 balance sheet items: Loans (L), Other Assets

(O), Savings (S), Debt (D) and Cash (Cs). For each stage the decision maker must choose any changes to be made for any of the five balance sheet items. Nodes represent different scenarios and decisions that need to be made. Thus the scenario tree can be drawn (with the nodes after stage 1 overlapping):

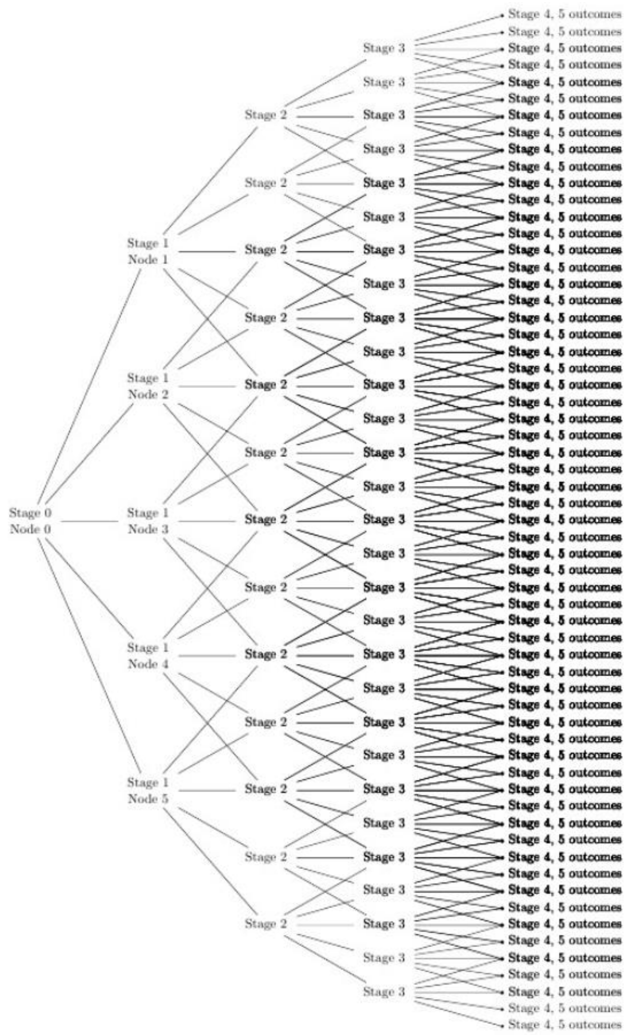

Figure 6: Decision tree structure with individual nodes overlapping after stage 1

Each node in each stage has an identification number. The first stage has one node 0 and the following stage has nodes 1, 2, 3, 4 and 5. Each decision node has 5 scenarios in the next stage. For example next nodes for node 1 are 7, 8, 9, 10 and 11 on stage 2.

The total number of nodes is growing exponentially. Looking from the current moment at stage 0 and forecasting future returns for four periods leads to hundreds of different possible outcomes. For each time period a decision should be made with these forecast future scenarios. The total number of nodes per stage can be expressed as:

Table 3: Number of nodes in each stage

	Nodes
Stage 0	1
Stage 1	5
Stage 2	25
Stage 3	125
Stage 4	625

For example in stage 3 there are 125 separate decision nodes, with 5 outcomes each. The decision maker has to make a decision based on the possible future scenarios in each node. After the decision, there are 5 outcomes for each decision node in stage 3, resulting in a total of 625 final outcomes for the problem in stage 4. In this model, the decisions are made in stages 0, 1, 2 and 3 whereas the outcomes of each decision can be calculated for the next stages, namely 1, 2, 3 and 4.

Throughout the thesis, node[t] refers to the set of nodes in period t. Similarly pre[k] stands for the previous node of node k and next[k] refers to the set of nodes that are the succeeding scenarios of node k. For example the next nodes of node 0 are 1, 2, 3, 4 and 5, while the previous node of 4 is 0.

5.1.1. Decision variables and budget constraints

The decision variables, denoted X_{ik}, are chosen for each balance sheet item (i) for each decision node (k) in stages 0, 1, 2 and 3. Each decision variable represents the quarterly change for a balance sheet item i in thousand euros (1000 €). They are chosen in order to maximise the utility of the expected future profits as explained in the objective function. Each decision variable X_{ik} is multiplied by the corresponding return affected by the decreasing marginal effects. In addition to these returns from the decision variables, the total profits include returns from the entire portfolio, denoted Y_{ik} and explained later.

The decision variables X_{ik} represent the changes for Loans (L), Other Assets (O), Savings (S) and Debt (D) for each stage. Since Cash (Cs) depends on the other assets, this constraint can be written as a function of the decision variables X_{ik}, where i denotes different balance sheet items and k different decision nodes:

$$X_{1k} = -X_{2k} - X_{3k} + X_{4k} + X_{5k}$$

In other words, cash can be increased by taking in savings or debt and decreased by issuing loans or buying other assets. These five assets are the decision variables to be chosen in each decision node to maximise expected future profits.

One important aspect of a multi period portfolio optimisation problem is that for each decision, the increase in portfolio affects the returns for future periods. In this model the decision variable Y_{ik} represents the total portfolio value for balance sheet item i in node k. The value of the portfolio is always the value in the previous period plus the selected optimal increase $X_{ik.}$ Notice that since the objective is to maximise the utility of profits and these profits are expected to be paid out to investors, the only increases in the portfolio come from X_{ik}, rather than increases from the returns of the portfolio.

In the optimisation software AMPL, this is done by adding a separate decision variable Y_{ik} for the total portfolio value, with k denoting the node and pre[k] denoting the previous node of k and i denoting balance sheet item. This constraint can be expressed as a function of the portfolio value variable Y_{ik}, where pre[k] stands for the previous node preceding node k:

$$Y_{i0} = portolio\ value\ at\ 2012 \qquad and\ for\ k > 0\text{:}\ Y_{ik} = Y_{ipre[k]} + X_{ipre[k]}$$

Although this portfolio value variable Y_{ik} is directly dependent on the decision variables X_{ik}, they need to be separately defined for optimisation purposes. This way the model takes into account the direct effects of increases or decreases in balance sheet items with the variable X_{ik} and captures the effect of portfolio development with the variable Y_{ik}.

5.1.2. Cash constraint

Constraints are used in the model in order to impose certain rules to the maximisation problem. Here the most important rules deal with the minimum amount of cash and how cash depends on increases in liabilities and assets. The budget constraint, discussed in under decision variables is also considered as a constraint and a decision variable. The cash constraint is another important part of the optimisation problem, but it is not considered as a decision variable.

The bank is required to have certain amount of its assets in cash or other liquid assets so that it can allow its customers to withdraw money or make other transactions. These requirements are often directly imposed on the banks by legislation (Fama, 1980). For the model the bank regulatory minimum cash requirement is included as a simple constraint for the Cash asset.

In this thesis, since the real cash per assets requirement is unknown, the cash per assets ratio is estimated from most recent historical data. In other words it is assumed that the future cash per assets ratio must be higher than the future cash per assets ratios. This constraint can be calculated from the year 2012 historical data (Appendix 1) and be expressed with cash (Cs), all assets (a) and total yearly changes in the balance sheet (Δ). The changes in cash (ΔCs) and total balance sheet assets (Δa) can be easily calculated from the data. In the AMPL model, the easiest way to solve the change in total balance sheet is from the liabilities, Savings and Debt.

$$\frac{Cs}{a} \leq \frac{Cs + \Delta Cs}{a + \Delta a} \qquad => \qquad \Delta Cs \geq \frac{Cs}{a} * \Delta a$$

Since the total amount of assets and cash are always positive in a balance sheet, it is possible to calculate the value of cash per assets and formulate the constraint. From the data of 2012 the value of cash per assets is approximately 5.23%. If the sum of the decision variables for all the assets is multiplied by the cash per asset value, then it should always be greater or equal to the decision variable for cash. Thus the constraint for cash per assets can be written as:

$$\Delta Cs \geq \frac{Cs}{a} * \Delta a = 0.052 * \Delta a$$

The change in cash assets should always be greater or equal to the change in the sum of all assets multiplied by a constant of 0.052. Since bank regulation gives requirements for liquid assets such as cash, these assets can and should be implemented in the model in the form of constraints. The decision maker and the bank has usually more precise information on the cash, but for this thesis a simple cash requirement is used. For more realistic results, more regulative rules on cash or any other asset can be implemented in the model in the form of constraint.

5.2. Simulating returns

The method for simulating future returns is selected to model the observed data. This thesis uses the covariance table of the error terms to simulate new error terms by using Cholesky factorisation (Rubinstein & Kroese, 2011). This leads to the error terms of the simulated returns to have same covariance properties as the original data. Moreover, returns are expressed as logarithms to simulate realistic, non-negative values. Thus the simulated returns have the same mean and correlation with the historical data.

The returns are treated as positive quarterly net returns for all balance sheet items, ranging from 1.5% to 0.005% (Appendix 8). For the liabilities, the return values are changed into negative for the final return values. This is due to the fact that the lognormal distribution used for the return simulation (Figure 4) creates positive returns with the specified mean and variance. The first three balance sheet items result in positive returns and the remaining two result in negative costs, since the final simulated returns always yield negative values.

The returns are net returns, suggesting that the total quarterly return for any balance sheet item is obtained by multiplying the volume of that balance sheet item multiplied by the return. Further in the model specifications we learn that the marginal effects affect the return for the specific quarterly change in the balance sheet. Thus the total return is affected by the simulated return, the amount of quarterly change and the marginal effect affecting the return of this increase in balance sheet item.

5.2.1. **Simulating error terms and generating returns for assets**

First the logarithms of the returns from historical data (RH_{ij}) are calculated. The notation used refers to different assets with the subscript i and for different historical time periods with j. Next the error terms (ε_{ij}) of each observation from the mean (μ_i) are calculated. Thus the returns can be expressed as a function of the mean and the error term:

$$\log \mathrm{RH}_{ij} = \mu_i + \varepsilon_{ij} \qquad \text{with} \qquad \varepsilon_{ij} \sim N(0, \sigma_i^2)$$

With this simple model the error terms can be calculated. The logarithm of returns, mean of these logarithms and the calculated error term matrix **E** is reported in Appendix 3. The error terms can be seen to be approximately normally distributed. The normal distribution is later used to generate new error terms and it was selected since it is the best distribution with the following historic distribution of error terms:

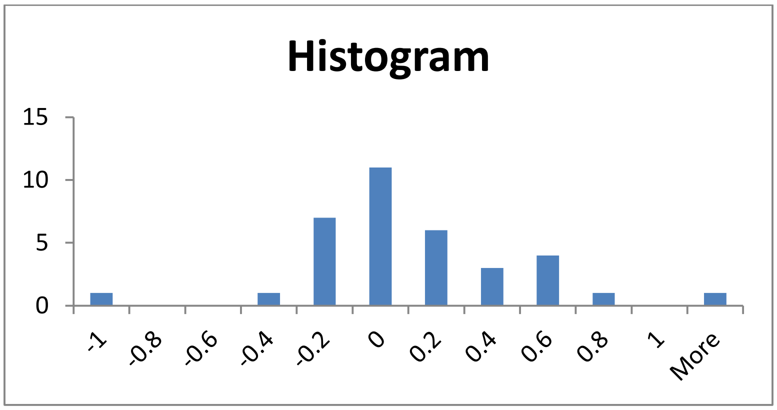

Figure 7: Evidence of normal error term frequency distribution

Next, a covariance matrix V is calculated with these error terms. As shown before, the covariance matrix V required to solve the Cholesky factorisation can be obtained from the error term matrix **E**. Each covariance presented in the covariance matrix is the covariance between two balance sheet item error term row vectors.

$$V = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots \\ \sigma_{21} & \cdots & \cdots \\ \cdots & \cdots & \sigma_{mm} \end{bmatrix}$$

Using this covariance matrix V it is possible to calculate the Cholesky decomposition $V = CC^T$ (Watkins, 2004). The matrix C has the property that when multiplied with a random vector of standard normal variables, the resulting vector has the same variances and covariance as the original covariance matrix V. This means that by calculating the Cholesky factor C, one can multiply it with vector U to generate randomly simulated error terms that have the same covariance as the historic data. The vector U variables are distributed normally, with I denoting the identity matrix.

$$U = \begin{bmatrix} u_1 \\ \cdots \\ u_n \end{bmatrix} \sim \mathrm{N}(0, \mathrm{I}) \quad \textit{thus when simulating new values } \log \mathrm{R}_{ik} = \mu_i + (CU)_k$$

By first calculating the covariance matrix V, then solving the Cholesky factor C and multiplying it with U, one can simulate new error term values. For each decision node k, a new random vector U should be created. Since the error terms were historically approximately normally distributed, variables following the normal distribution should be selected to generate the new values. The random vector U is then normally distributed with a mean of 0 and variance equal to that of the identity matrix I, and the returns are lognormally distributed.

Simulating correlated returns is important since the data supports the idea of the individual balance sheet items having significant correlations. For example in good and bad economic situations the returns on assets tend to move together. The covariance matrix V and the Cholesky factor C are reported in Appendix 4. The following figure shows the importance of correlation, as the historic data seems to move together, having significant correlation.

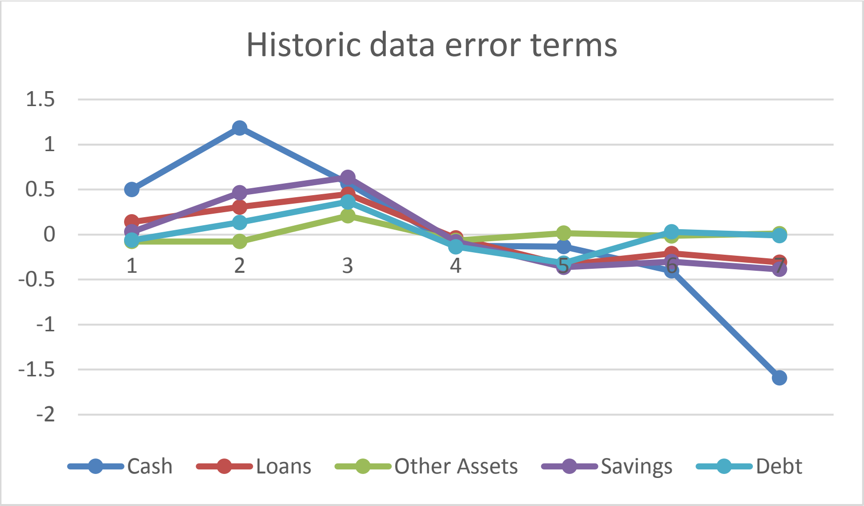

Figure 8: Error term distribution, signs of time trend

This effect of the asset returns moving together can essentially be captured by using the Cholesky decomposition. The historic error terms displayed in Figure 8 support the idea of a time trend in the returns. This trend can be included in the model with the drift factor, explained in chapter 5.2.3.

Further deriving the formula for the new simulated returns yields the following formula, with k denoting nodes and i balance sheet items. The final outcome for returns follows a lognormal distribution.

$$\mathrm{R}_{ik} = e^{\mu_i + (CU)_k}$$

By using the formula above, the Cholesky factor C and a random vector U with normally distributed values it is possible to simulate new returns that are lognormally distributed, have the same mean and same correlation as the original historical data. Here R_{ik} represents the return R for balance sheet item i in node k.

The returns for all the scenarios are generated using random numbers. For scenario optimisation, it is possible to generate pre-selected scenarios, for example good, average and bad. It is also possible to select different scenarios directly. These two methods can easily be added to the model, but they require the decision maker to make the choices different scenarios. For the purpose of this thesis, random scenarios are used. In order to create selected scenarios, more knowledge from the industry is required.

5.2.2. Decreasing marginal effect on returns

The idea of decreasing marginal effects comes from real life situations. In order to get more loan applications from customers or more depositors, the bank must offer better rates or invest in marketing. Similarly if the bank wants to increase its debt, give out more loans or buy more various assets, the quality of these options become worse as volume increases. These lead to increased costs, increased default risk or decreased returns. For all the balance sheet items, as volume is increased the returns are decreasing. (Knight, 1944).

The marginal effects make it impossible to exploit such situations where the return on loans is higher than cost of debt – as the historical data suggests – by increasing loan amounts infinitely. It offers better real life implications in the multi period model as it makes large scale changes less likely and more costly. Increasing any asset yields profits only to the point where it can be financed with the balance sheet liabilities. Increasing it further will decrease profits.

Since this thesis uses public data from Aktia PLC, the marginal effects need to be estimated from the available data (Aktia, 2014). The methods used in this thesis to estimate the marginal effects is limited to the small amount of data available. For better results, one should use company specific data obtained from the managers.

In the simulation all the returns and marginal effects are considered positive. The change is done after the returns are simulated for the liabilities, so that the returns are negative and can be considered as costs. However, in and throughout the simulation the marginal effects are positive in all observations. The returns with marginal effects r_{ik} for the decision variables X_{ik}, which represent the selected quarterly changes in the balance sheet, can be expressed with the return without marginal effects R_{ik}, where i denotes the balance sheet item, k denotes the node and pre[k] denotes the previous node.

$$r_{ik} = R_{ik} - X_{ipre[k]} * m_i$$

The marginal effects m_i for each asset can be estimated using historic data. In reality, marginal effects can depend on many factors and can be difficult to measure. Here total historical balance sheet item is denoted as a_{ij} with the subscript i for each balance sheet item and j for different historical time periods. The change in a balance sheet item i for a specific year j can be expressed as Δa_{ij} and the historical returns RH_{ij}. Moreover, while RH_{ij} is the year specific return, rh_{ij} represents the return with marginal effect m_i. The mean for the returns of all the balance sheet items for a specific year is μ_j.

$$rh_{ij} = RH_{ij} - \Delta a_{ij} * m_i$$

Notice that since the return simulation uses positive numbers for both assets and liabilities until the final phase, all returns, mean returns and marginal effects are considered positive in this phase. Moreover it makes the marginal effect estimation simpler when one does not need to consider assets and liabilities separately. Since only yearly data is available, the yearly returns are transformed into quarterly returns by dividing the yearly returns by four.

To estimate the marginal effects, the model assumes that RH_{ij} represents the return without marginal effects and can be calculated from the data. In reality the return calculated by dividing total returns is usually slightly less because the marginal effects decrease the return for the newly increased balance sheet item Δa_{ij}. However, since the marginal effects and Δa_{ij} is relatively small to the entire portfolio, this estimate is used.

$$\mathrm{RH}_{ij} = historical\ returns = \ total\ income\ for\ asset\ i\ in\ year\ j/a_{ij}$$

Next the impact of the marginal effect is estimated. To achieve this one should consider the following graphical presentation of the return. The marginal effect represents the decrease in return that depends on the change in the balance sheet item Δa_{ij} and change in the return:

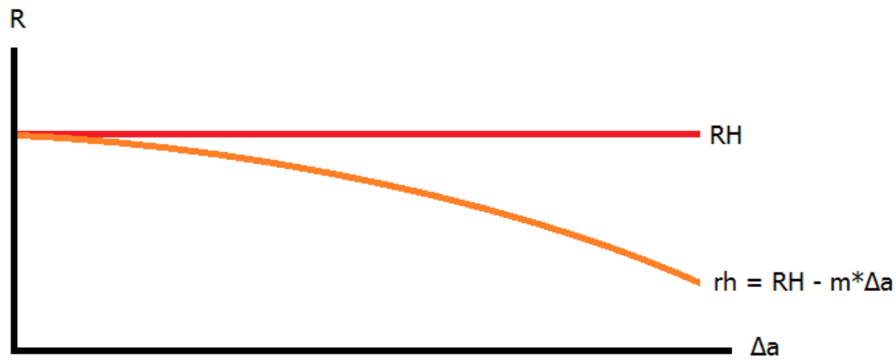

Figure 9: the effect of marginal effect on balance sheet item return

The marginal effect can be obtained by estimating the relation between the changes in return and change in balance sheet item increase. The change in balance sheet item can be directly obtained from the data, but the change in the return is more difficult to estimate.

For the purpose of this study and considering the optimisation model used, it is estimated that the marginal effects tend to fix the balance sheet item returns towards the mean. This means that the decision maker would increase the balance sheet items to the point where the asset returns are close to the cost of liabilities. This rough estimate cannot be applied to all situations or claimed to be accurate, but on average with data from several years it yields results that can be used for the purpose of this study.

The marginal effect for one year can be written by using the change in the asset, historical return without marginal effects and the historical mean return. The change in balance sheet item return is here expressed as the deviation from the average return. Thus the marginal effect can be estimated as the change in the difference between the returns divided by the change in balance sheet item.

$$m_{ij} = \frac{\mathrm{RH}_{ij} - \mu_j}{\Delta a_{ij}}$$

When calculated for each year one can see that the marginal effects vary from year to year. For the purpose of this study one needs to obtain one single marginal effect for each balance sheet item. To get this estimate for the marginal effect, the average from all the years T in the historic dataset is calculated:

$$m_i = \frac{1}{T}\sum_{j=1}^{T} m_{ij}$$

Thus the marginal effect m_i for a specific balance sheet item can be estimated by calculating the average of all the marginal effects for a specific balance sheet items through all the historical time periods. The resulting marginal effects for each balance sheet item is reported in the following table:

Table 4: Marginal effects for each year and average marginal effects m_i

	2006	2007	2008	2009	2010	2011	2012	Average
Cash	8.72E-08	8.09E-08	9.40E-09	3.84E-09	7.64E-09	1.83E-09	5.43E-09	2.80E-08
Loans	1.33E-08	8.55E-09	1.35E-08	9.77E-09	7.93E-09	9.88E-09	2.26E-08	1.22E-08
Other Assets	7.14E-09	4.74E-09	2.00E-08	6.36E-09	8.14E-09	1.79E-08	5.74E-08	1.74E-08
Savings	6.55E-09	2.49E-08	8.13E-09	2.21E-08	1.22E-08	1.16E-08	6.12E-08	2.10E-08
Debt	9.87E-09	9.08E-09	2.65E-08	7.31E-09	7.62E-09	2.01E-08	5.68E-08	1.96E-08

5.2.3. Drift factor

The drift factor φ improves the model by generating returns that are affected by the previous returns, instead of generating random returns. Thus for example shocks or extreme values tend to affect the economy longer than just one period. For a factor of 0 the error terms are generated with given covariance from a random lognormal distribution. However, for a factor of 0.5 the return is the average of the randomly simulated return and the previous return.

The data suggests there is some drift, where new returns are affected by the returns for the previous year. Adding this drift factor to the model modifies it to be very close to the Autoregressive model (Madsen, 2007). This is a simple model used to simulate simple drift, where the new return is a certain percentage of the old return and the rest is the newly simulated return. In this formulation i stands for different assets, k for different decision nodes and pre[k] for the previous node of node k.

$$\log \mathrm{R}_{ik} = (1-\varphi)(\mu_i + \varepsilon_{ik}) + \varphi\left(\log \mathrm{R}_{ipre[k]}\right)$$

Whereas the Autoregressive model would state:

$$\log \mathrm{R}_{ik} = \mu_i + \varphi \log \mathrm{R}_{ipre[k]} + \varepsilon_{ik}$$

To avoid increasing the returns over time, the mean and error term are multiplied by (1 – φ) ensuring that the mean remains close to the historic mean μ_i. A reasonable drift factor of $\varphi = 0.25$ is selected for the purpose of this study in order to get more realistic results. This means that 75% of the returns are random, and the remaining 25% is carried from previous returns.

Notice that when using the drift factor φ, the previous returns affect the simulation of the new returns. Thus for the first simulated returns, the drift factor does not affect the simulation. For better results, another option would be to use the latest returns, obtained from the historical data. However, this does not have significant impact on the end results and thus the first period simulated returns do not include the drift factor.

5.3. Objective function

The objective function defines the maximisation problem, where the decision maker tries to maximise the utility of the portfolio profits P in the multi period problem with respect to set constraints. The utility of the profits are calculated in each time period and each node p_k. Thus the utility of the entire optimisation problem is referred to as $u(P)$ and the utility of profits in individual nodes k is referred to as $u(p_k)$. Each utility of individual profits is multiplied by the probability of the node occurring, giving the expected utility of the entire system $u(P)$.

The profits are maximised and they are not invested back into the system. In this model, it is expected that the obtained profits are paid out to the shareholders. The reason why profits are the focus of this thesis and not equity or portfolio growth is that the thesis is from the department of Information and Service Management. Thus the optimisation objective is closer to operations research and profit optimisation rather than financial portfolio optimisation. For further studies the model could take into account the growth in the balance sheet items, so that the balance sheet items are increasing with respect to the corresponding returns and maximising equity would be the objective.

The model maximises the expected utility of the portfolio on each stage. Since the profits are calculated in periods 1, 2, 3 and 4, with 5 scenarios the probabilities are always 5^t depending on the stage t. By using ke[t] for the last node of stage t and thus kf[t] = ke[t-1]+1 for the first node on stage t, the total expected utilities can be written as:

$$u[P] = \sum_{t=1}^{T} \sum_{\mathrm{k=kf[t]}}^{ke[t]} \left[\frac{1}{5^t} * u(p_k)\right]$$

The profits depend on the returns and the value of the balance sheet items. As discussed before in chapter 3.2 the returns are generated with the specified model for each asset (i) and the marginal effects (m_i) affect this simulation. The total profits in node k can be written as a function of the decision variables X_{ij}, the returns with marginal effects r_{ik}, returns without marginal effects R_{ik} and the current balance sheet item values in the portfolio Y_{ik}, where pre[k] stands for the previous node:

$$p_k = \sum_{i=1}^{5} \left[X_{ipre[k]} * r_{ik} + Y_{ipre[k]} * R_{ik}\right]$$

The notation here includes the increases in the asset in the previous year $X_{ipre[k]}$, the marginal effects m_i, the portfolio value for each balance sheet item in the previous time period $Y_{ipre[k]}$ and the simulated returns R_{ik}.

The notation should be interpreted such that the amount of selected change in the previous time period $X_{ipre[k]}$ is multiplied by the return, which is obtained by reducing the total marginal effect from the simulated return. The total marginal effect is calculated by multiplying the marginal effect with the volume of change in the previous time period, $X_{ipre[k]}$. In addition to these profits, the total portfolio profits in the previous time period $Y_{ipre[k]}$ are calculated by multiplying the portfolio value with the simulated return.

This way it is possible to calculate the profits in each node. For the optimisation problem, the utility function or $u(p) = -e^{-\gamma * p}$ presented in the next chapter is needed. One important aspect of the optimisation problem noticeable in the total profits function above, is its concave nature. Modifying the formula it is possible to see the negative quadratic form using vectors:

$$p_k = \sum_{i=1}^{5}\left[-X_{ipre[k]}^2 * m_i + X_{ipre[k]} * R_{ik} + Y_{ipre[k]} * R_{ik}\right]$$

The objective function is a sum of concave functions. In other words increasing any balance sheet item will at some point start to decrease profits. Notice that even when the portfolio values include previous decision variables $Y_{ipre[k]}$, it can be assumed concave and thus the profits are decreasing if any decision variable is increased excessively. The second derivative of the function above is always negative and the function is concave. Objective function concavity is important when determining optimal solution. For a concave objective function, the found maximum solution is known to be global. When compared to Markowitz portfolio theory, this model has one optimal solution instead of an efficient frontier. (Dantzig, 1955).

The complete objective function can be written by combining the exponential utility function, the utility of the portfolio in node k and the probabilities of each node through all the stages. In this formulation ke[t] is used to denote the last node of stage t, kf[t] for the first node on stage t and the function utility function $u(p) = -e^{-\gamma P}$ denoting the utility of all portfolio profits in all stages.

$$u(P) = \sum_{t=1}^{T} \frac{1}{5^t} \sum_{\mathrm{k=kf[t]}}^{ke[t]} -e^{-\gamma * \left(\sum_{i=1}^{5}\left[X_{ipre[k]} * r_{ik} + Y_{ipre[k]} * R_{ik}\right]\right)}$$

The objective of the optimisation problem is to maximise the expected utility of the portfolio profits over the selected time period. This can be achieved by maximising $u(P)$ presented above.

5.3.1. Optimisation problem mathematical presentation

The entire optimisation problem and the constraints can be expressed mathematically in the following formulation.

$$\max \sum_{t=1}^{T} \frac{1}{5^t} \sum_{\mathrm{k=kf[t]}}^{ke[t]} -e^{-\gamma * \left(\sum_{i=1}^{5}\left[X_{ipre[k]} * r_{ik} + Y_{ipre[k]} * R_{ik}\right]\right)}$$

$Subject\ To$:

$$X_{1k} + X_{2k} + X_{3k} = X_{4k} + X_{5k}$$

$$X_{1k} \geq 0{,}052 * (X_{4k} + X_{5k})$$

$$Y_{ik} = Y_{ipre[k]} + X_{ipre[k]} \qquad for\ 0 < k$$

$$Y_{i0} = a_{i2012}$$

$where$:

$k\ denotes\ different\ nodes$

$i\ denotes\ different\ balance\ sheet\ items$

$t\ stands\ for\ different\ time\ periods, quartiles$

$kf[t]\ stands\ for\ first\ node\ of\ time\ period\ t$

$ke[t]\ stands\ for\ last\ node\ of\ time\ period\ t$

$\gamma\ denotes\ gamma\ factor$

$r_{ik}\ denotes\ return\ on\ the\ change\ on\ balannce\ sheet\ asset\ with\ marginal\ effects$

$R_{ik}\ denotes\ return\ on\ the\ balance\ sheet\ item\ in\ previous\ time\ period\ without\ marginal\ effects$

6. MODEL RESULTS AND ANALYSIS

The model explained in the previous chapter is used to simulate balance sheet item returns for multiple periods and for 5 scenarios branching off from each node. This means that the problem follows a decision tree with 4 stages of decision nodes. As discussed before, this method leads to having 156 decision nodes and 780 nodes with resulting profits.

This chapter compares the results from two models with different outputs. The results are analysed and compared in order to estimate model efficiency. To provide better comparison, these obtained results are compared with historical data and most recent data from 2013. The profits and balance sheet totals are compared. Finally this chapter presents the sensitivity analysis, which aims to study the effect of changing the input variables on the final output.

6.1. Comparison of two models with different inputs

Here the model is solved with two different sets of inputs. Since these inputs are based on estimates and should in reality be chosen by the decision maker, the outputs offer alternative views. Two solutions are given in order to perform comparison and analysis on the differences.

The first solution presents a simple model with no drift factor $\varphi = 0$ and the marginal effects are equal to the averages of the marginal effects estimated from the data, as explained in the previous section. The second model offers an alternative view with more precise inputs. It solves the optimal solution using a drift factor of $\varphi = 0.25$ (which means 25% of the return on a balance sheet item is carried to the next period) together with the estimated marginal effects from year 2009.

6.1.1. Results with no drift factor $\varphi = 0$ and average decreasing marginal effects calculated from historical data

To look at the results of this model with no drift factor and selected marginal effects, one needs to start from the balance sheet item return simulations. The marginal effects calculated from the data are presented earlier in chapter 5.2.2.

The results of the simulation of asset returns are shown in graph 10. The graph shows the distribution of the simulated prices and how they fluctuate in different scenarios in all nodes. The simulation produces some extreme values, which would resemble crises or boom periods in real life.

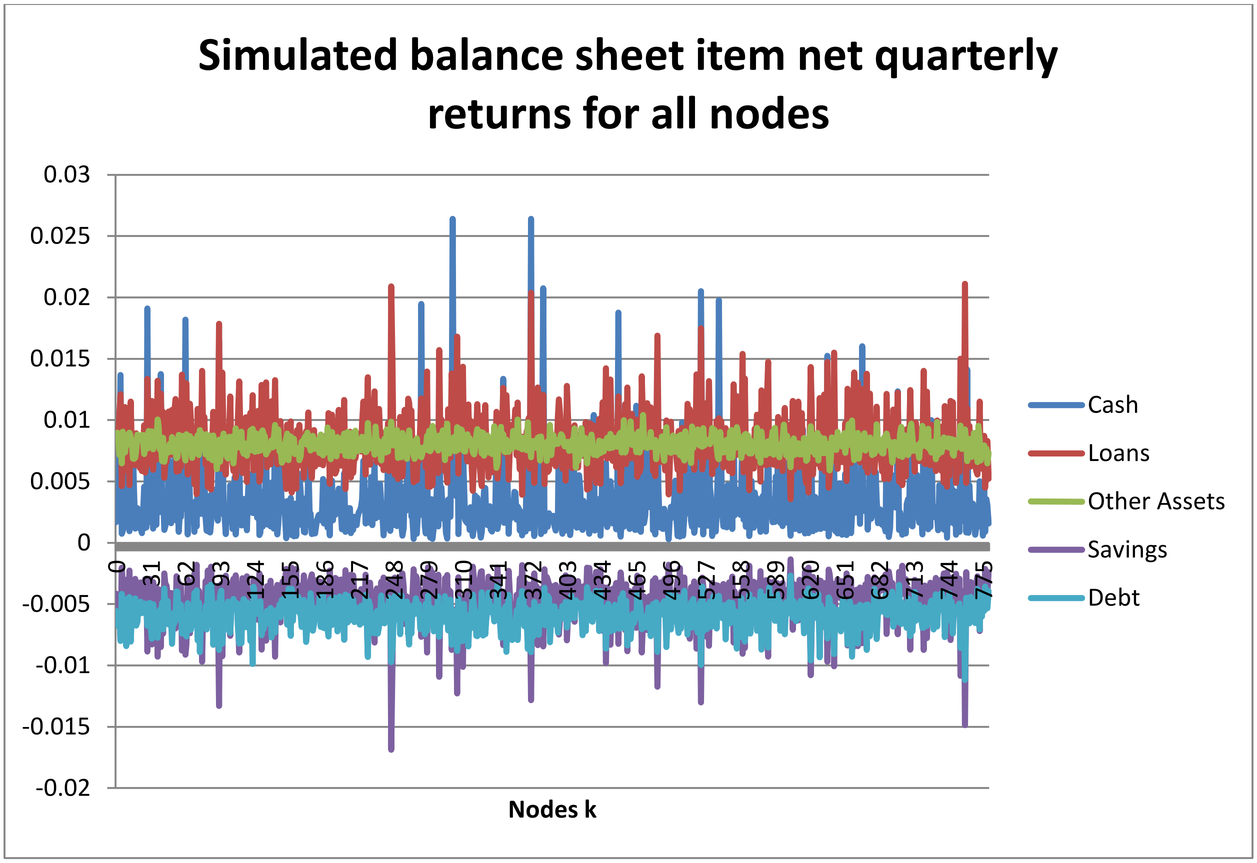

Figure 10: Model 1 simulated balance sheet item returns for all nodes

This graph can be used to analyse the nature of the optimisation problem. Giving out loans yields on average the best returns, but other assets have smaller volatility. Cash is the least profitable, with return smaller than the cost of debt or savings. However, bank regulation dictates that in order for bank to operate and meet the minimum liquidity requirements, it needs to hold certain percentage amount of its assets in cash.

On the liabilities side of the balance sheet it is possible to see that savings cost less on average, but have slightly higher volatility compared to the cost of debt. Debt on the other hand is linked to the bank's leverage and might cause trouble if increased excessively (Baxter, 1967). Similarly increasing savings might have drawbacks as the bank is required to hold higher level of cash reserves and it might be costly to try to increase savings in a competitive market. The average returns and variances for each asset can be seen in the following table:

Table 5: Balance sheet item historical mean net quarterly returns and variance with corresponding simulated values

	Historical mean	Simulated mean	Historical variance	Simulated variance
Cash	3.44E-03	3.48E-03	6.89E-06	1.10E-05
Loans	8.29E-03	8.28E-03	6.80E-06	6.20E-06
Other Assets	8.01E-03	8.02E-03	7.33E-07	5.58E-07
Savings	-4.82E-03	-4.78E-03	4.53E-06	3.73E-06
Debt	-5.89E-03	-5.87E-03	1.69E-06	1.44E-06

The optimisation software AMPL makes decisions with the returns simulated in the previous Figure 10. It takes into account all the future values of the successor scenarios and chooses balance sheet items X_{ik}, where i stands for balance sheet items and k for nodes. This optimal solution is listed in Appendix 5. The expected yearly increase can be calculated by multiplying the decision variable with the probability of occurrence and compared with the historical changes:

Table 6: Optimal solution expected yearly changes and historical yearly changes, 1000 €

	Expected total asset change in a year:	Historical average yearly changes 2006-2012:
Cash	56 021	43 421
Loans	326 719	583 867
Other Assets	306 327	327 957
Savings	493 561	340 068
Debt	195 505	517 561

From expected yearly changes in the balance sheet items (Table 6), it can be seen that the model output is somewhat close to the historical averages. However, the historical values seem to be putting more emphasis on giving out loans and debt while savings are increasing less.

This difference can be explained with a few different answers. It is highly likely that the cost for attracting more savings is higher than estimated in the model. This means that it is more costly to attract new savings customers than the model estimates. Thus the true value of the marginal effect on savings is higher and would result in an optimal solution with fewer savings. Since savings are decreasing due to the increased negative marginal effect, more debt needs to be accumulated in order for the cash to remain positive. Suggesting that the bank aims not to increase debt may be more profitable, but not necessarily realistic because of the cash constraint.

The high historical increases in loans can be explained by strategic decisions: the bank tries to capture higher market share in loans even if they are less profitable for optimal solution. This kind of strategic decision is not taken into account in the mathematical model. Another reason could be that the profits for other assets were not clearly reported in the financial statements and there is some problem estimating the returns for other assets.

It can be seen from the balance sheet average returns that the positive return on issued loans is on average higher than the cost of debt (0.83% versus 0.59%). This is one reason why the bank might choose to increase loans and debt and it is taken into account in the model in the form of decreasing marginal effects (the m_i factor). In other words, there may be some estimation error in the marginal effects that contribute the solved values to have lower loans and debt than the historical values.

To understand the objective function and the solution to the problem one needs to look at the profits. As explained before, the model aims to maximise expected utility with exponential utility function. By selecting the decision variables X_{ik}, the profits for each node can be calculated. The following graph presents the profits on each node for the simulated prices with marginal effects and solved balance sheet item changes X_{ik}. The objective is to select decision variables to maximise utility.

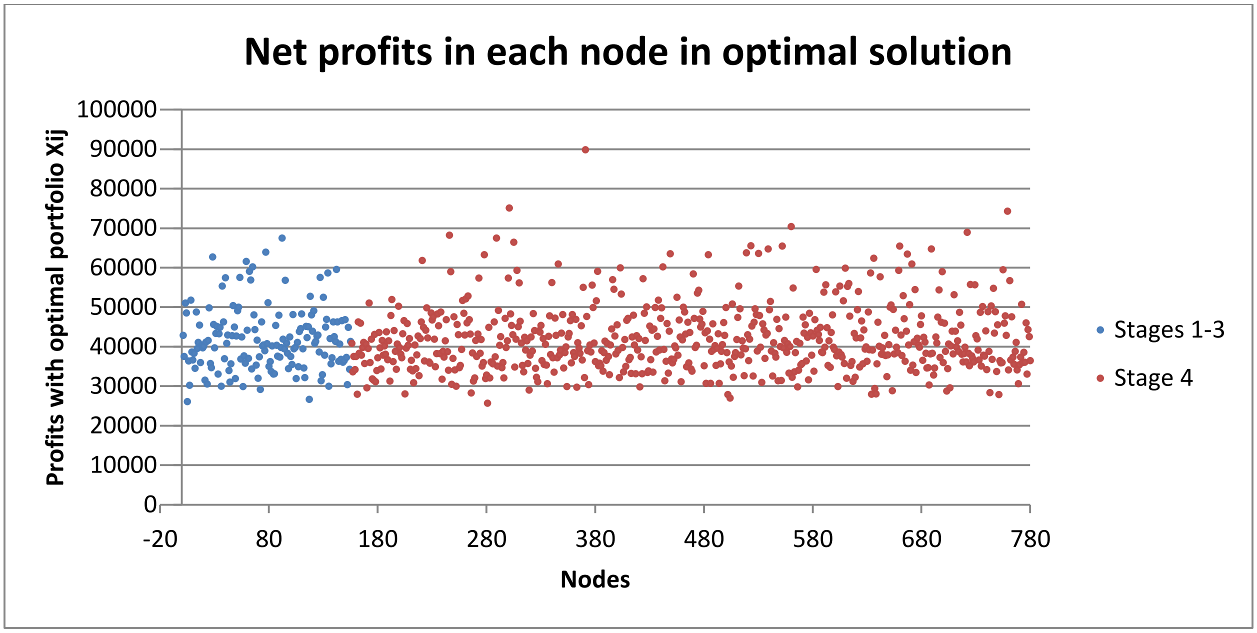

Figure 11: Profits in each node using a solved optimal portfolio X_{ik}

The graph presents the profits from the decision variables X_{ik} and illustrates the direct profits from the decisions made. Notice that here, the portfolio profits are excluded from this illustration. By maximising the expectation of the utilities from the profits on all the nodes, the system finds the optimal solution with maximal utility. As explained before, the expected utility of the whole system can be calculated by multiplying utility of the profits in each node with the probability of its occurrence. In the solution presented here, the solver gives the following output for the maximisation problem:

Table 7: MOSEK solver optimal solution for φ=0 and average marginal effects

Primal objective :	-1.0108
Dual objective :	-1.01021

This output does not really tell much about the actual problem, but it can be compared later with other models to see which performs better with the chosen utility function. For more information about the optimal solution, one should look at balance sheet item returns and average yearly changes presented before in graph 10 and table 6.

6.1.2. Results with drift factor of φ = 0.25 and marginal effects from year 2009

In this simulation, the returns are simulated with a drift factor. This means that a return has 25% impact on the next return and thus also 6.25% impact on the return after the next return and so forth. There tends to exist less extreme values as seen from the graph, but the time trend is clearer. Consecutive returns tend to be close to one another and a shock persists longer, affecting returns on several years.

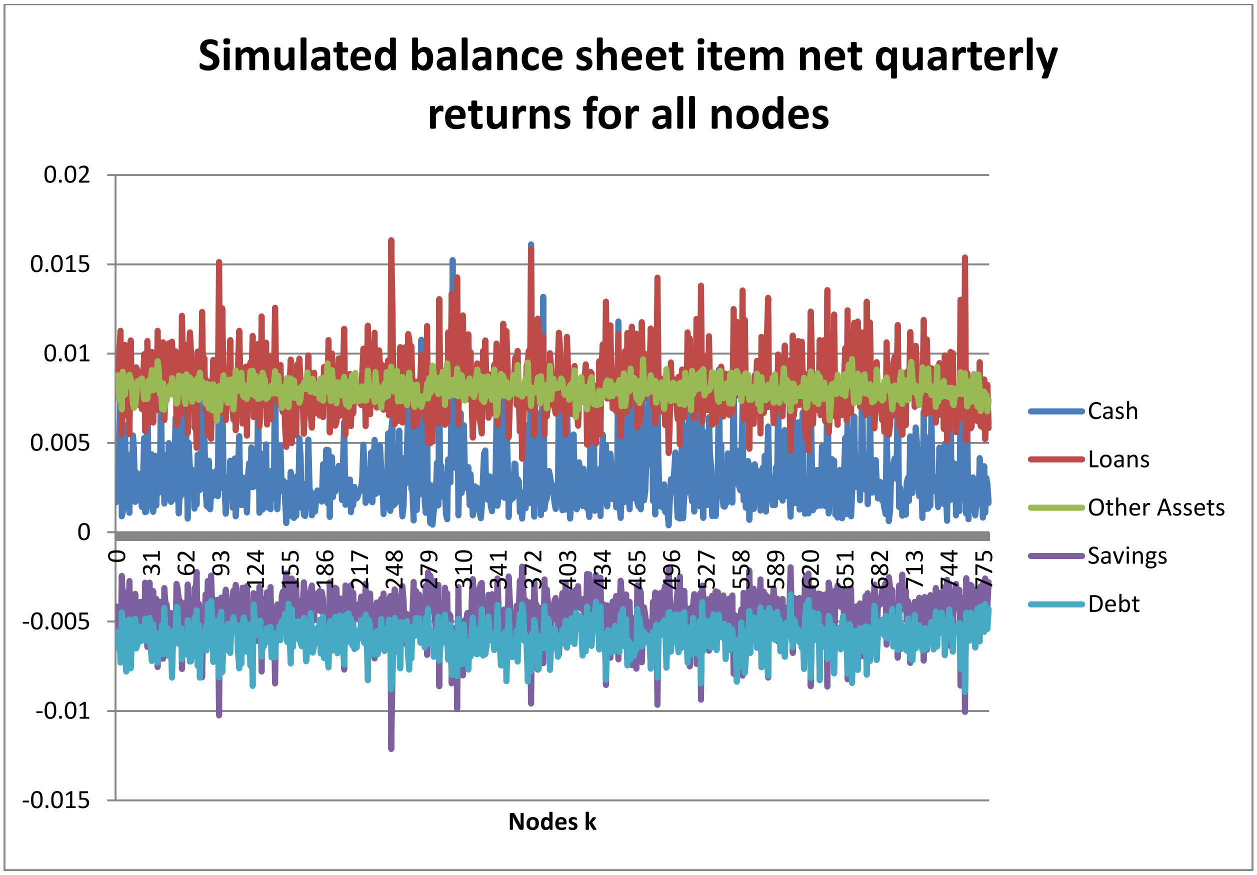

Figure 12: Simulated balance sheet item returns for all nodes with a drift factor of 25%

Because of the time trend the mean net quarterly returns and the variances are slightly affected. The distributions are clearly different to those without a drift factor. To fully understand the distributions one should look at the mean and variance of each of these simulated returns. These can be seen in the following table:

Table 8: Historical and simulated mean net quarterly returns and variances with drift

	Historical mean	Simulated mean	Historical variance	Simulated variance
Cash	3.44E-03	3.06E-03	6.89E-06	4.37E-06
Loans	8.29E-03	8.18E-03	6.80E-06	3.48E-06
Other Assets	8.01E-03	8.00E-03	7.33E-07	3.39E-07
Savings	-4.82E-03	-4.65E-03	4.53E-06	2.00E-06
Debt	-5.89E-03	-5.83E-03	1.69E-06	8.53E-07

One clear difference in the simulated returns with these inputs is that with the drift factor of φ=0.25 the variances are lower than those of the original data. This is due to the fact that part of the simulated returns is carried from previous time period return. Thus on average the returns are closer to the mean, leading to the decrease in volatility. Using the drift factor in return simulation affects the mean, variance and randomness. This might be seen to have a negative effect on the model results as they yield less extreme values and might lead into problems in forecasting.

By using the simulated returns, it is possible to solve the model. The expected yearly increase can be calculated and compared with the historical changes. The changes here are due to the selected inputs, namely drift factor and marginal effects.

Table 9: Optimal solution expected yearly changes and averages of historical changes (1000 €)

	Expected total asset change in a year:	Historical average yearly changes 2006-2012:
Cash	74 606	43 421
Loans	375 484	583 867
Other Assets	889 350	327 957
Savings	519 340	340 068
Debt	820 099	517 561

As seen here, the optimal solution puts a lot of emphasis on other assets, while increasing savings and debt. Moreover, the general tolerance towards risks seems to be higher as balance sheet items grow more on a yearly level than before. These outcomes can be explained by the selected inputs. Since the variances are lower with the drift factor, the volatility of the returns is smaller, leading to more risk taking. This can be seen in the following graph. There seem to be a few extreme values, and values are closer together.

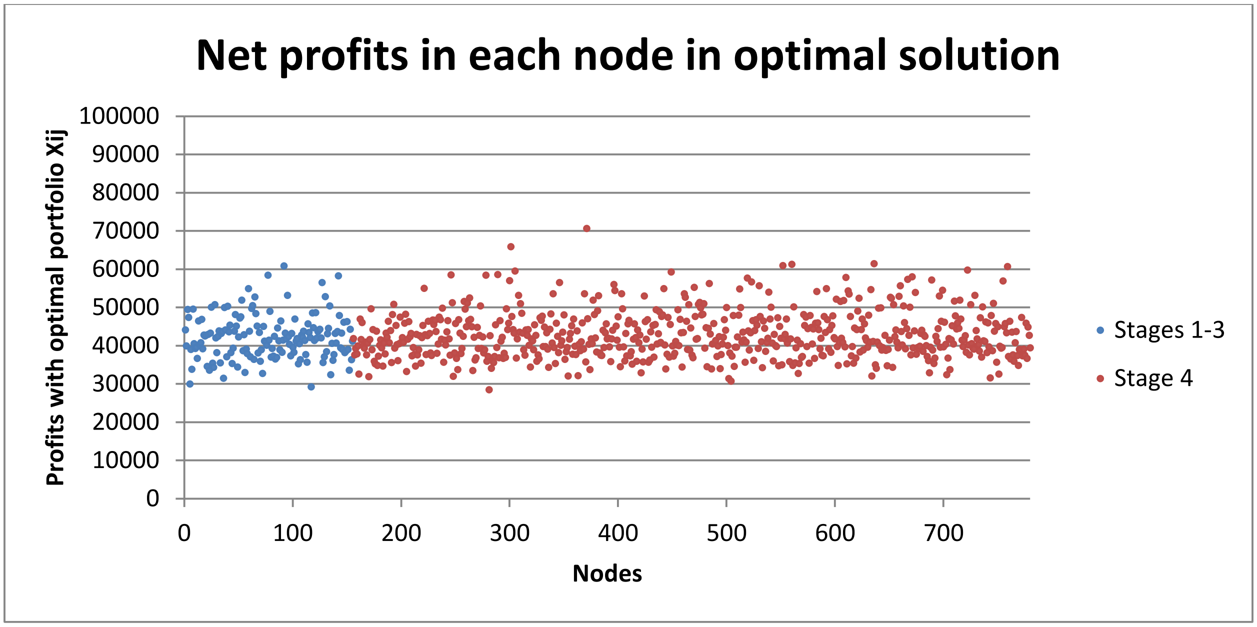

Figure 13: Profits in each node using a solved optimal portfolio Xik

The graph presents the profits from the chosen optimal balance sheet item changes X_{ik} and the returns with marginal effects m_i. With the profits closer together and with less extreme values, the decision makers are now more willing to take risk. Including the drift factor in the model is a bit problematic because it decreases the volatility leading to more risk taking. However, this idea is supported by he observed data. Thus further research on the time trend is encouraged.

It is possible to solve the optimisation problem with these profits. The model maximises the expectation of the utilities on all the nodes and finds the optimal solution with maximal utility. The output from the solver with the final objective function value is expressed in the following table.

Table 10: MOSEK solver optimal solution for φ=0.25 and marginal effects from 2009

Primal objective :	-0.97178
Dual objective :	-0.97114

This objective function value does not explain much of the problem alone, but can be compared with the solution from the first model. In this model the drift factor causes the variance in balance sheet items to be lower, leading to more risk taking. This risk taking can be seen in the form of higher proposed balance sheet average changes. Because of these higher changes, the utility in this problem solution is higher. Thus model 2 gives out a solution with more changes and higher expected profits. This is due to the inputs used in the model. In order to understand the difference between the two models, more comparisons need to be made.

6.1.3. Most recent data released by the case company after the model was formulated

The data used in formulating the model was obtained from Aktia's financial statements from the years 2006-2012 and the model used in this thesis was created with this data. During the final stages of the thesis work, new data was released from 2013. This new data provides an opportunity to compare the different optimal solutions and the actual realisation of Aktia's balance sheet. By comparing these alternatives it is also possible to look at and analyse the reasons why the model fails or succeeds in predicting certain changes in the balance sheet.

Table 11: Balance sheet items 2013 and percentage changes from 2012 (1000 €)

Balance sheet	2010	2011	2012	2013	Change	% change
Assets						
Cash (C_i)	273 364	475 042	587 613	414 328	-173 285	-29.49 %
Loans (L_i)	6 637 551	7 152 124	7 360 225	6 897 349	-462 876	-6.29 %
Other Assets (A_i)	4 108 238	3 428 897	3 292 352	3 622 129	329 777	10.02 %
Sum	11 019 153	11 056 063	11 240 190	10 933 806	-306 384	-2.73 %
Liabilities						
Savings (Y_i)	4 356 327	4 757 179	4 689 040	4 892 982	203 942	4.35 %
Debts (D_i)	4 827 366	4 464 037	4 584 724	4 106 018	-478 706	-10.44 %
Equity (E_i)	497 290	523 756	657 409	641 709	-15 700	-2.39 %
Other Liabilities (O_i)	1 338 170	1 311 091	1 309 017	1 293 097	-15 920	-1.22 %
Sum	11 019 153	11 056 063	11 240 190	10 933 806	-306 384	-2.73 %

From the table above it is possible to see the actual realisations for the yearly changes in Aktia's balance sheet items from year 2012 to year 2013. The percentage changes are also listed, to give an idea about the scale of the change. One important observation is that in general the trend in the balance sheet items and the totals is upwards up until the year 2013.

There is a 30% drop in cash assets, while only the other assets were increasing. On the liabilities side debt was cut down by 10% while more savings were accumulated. The balance sheet shrank a little which contributes to some loss of market share. To understand why the balance sheet would be decreased by the decision maker, the profits of each balance sheet item need to be considered.

Table 12: Balance sheet item profits and returns 2013 and comparison to year 2012 (1000 €)

Income	2010	2011	2012	2013	Change	% Change
Cash (C_i)	2 485	3 290	1 237	775	-462	-37.35 %
Loans (L_i)	152 164	186 132	173 496	126 325	-47 171	-27.19 %
Other Assets (A_i)	134 868	109 172	107 504	126 458	18 954	17.63 %
Sum	240 326	262 563	232 296	172 952	-59 344	-25.55 %
Savings (Y_i)	-54 411	-63 252	-57 149	-45 970	11 179	-19.56 %
Debts (D_i)	-81 692	-107 039	-105 647	-94 832	10 815	-10.24 %
Equity (E_i)	-59 675	-37 187	-55 880	-51 978	3 901	-6.98 %
Other Liabilities (O_i)	45 084	36 343	47 779	80 493	32 714	68.47 %
Sum	-91 019	-133 948	-115 017	-60 309	54 708	-47.57 %

From the table it is obvious that the returns and costs on almost all of the balance sheet items have gone down. As the returns go down, it is rational for the bank to decrease the total balance sheet. If the profits would go up, increasing would be a good solution. The returns show a significant increase in profitability of other assets, which can explain the increase in other assets. For the costs there have been some improvements in profitability. By comparison the costs have gone down more than the returns, which would argue for better profitability, combined with smaller balance sheet.

The positive changes in other assets and the negative changes in the rest of the balance sheet emphasises the importance of the market conditions. It would seem that in a bad market situation the returns go down, but since the volatility of the return on other assets is small, it becomes more desirable. Thus in a bad market situation it is rational to decrease balance sheet totals and put more emphasis on other assets. These other assets include for example buildings and other real investments.

It can be argued that some of these changes may be stochastic random walk, which can be taken into account in the simulation and some are not random, meaning that strategic planning or decision making is taking place. For random differences, one could say that for example the realisation of the balance sheet item returns were very negative. After observing and forecasting more negative future returns, the bank decision makers adjust and decrease balance sheet item values. The other explanation is that the changes were not based on a random factor, but rather a strategic decision.

The model cannot take into account strategic choices. This is because by default it maximises expected utility of the profits. However, if one would apply game theory to the process, it would no longer be possible or reasonable to solve the problem using multi period scenario optimisation (von Neumann & Morgenstern, 2007). For decision making purposes and actual budgeting issues, this model and the solutions could provide significant and important information. In the current situation however, it is very likely that the decisions are made with other methods, where strategic positioning is given more emphasis than pure maximisation of expected profits. This would argue that the problem setting follows more closely game theory than profit maximisation problem.

In the long run increasing and decreasing balance sheet yearly probably leads to high costs related to acquiring the assets and financing them with liabilities. Moreover, the benefit of steadily increasing the balance sheet and market share most likely leads to higher profits. In the model and assumptions presented in this thesis, there exists no such optimal solution in multi period optimisation where balance sheet items are increased and decreased in consecutive years.

6.1.4. Comparison of different solutions and expected profits

To compare all the results from the two models presented and the actual observed outcome, one needs to understand the big picture. For the models, the solutions depend heavily on the inputs selected. Thus one can obtain many results by using the model with different inputs. Even the same solution that can be observed in real life may be obtained by selecting the correct inputs.

When considering the observed outcome of 2013, it is the same as looking back and seeing only final node of the entire scenario tree of possible outcomes. Thus comparison of the 2013 outcome and model optimal solutions can be done, but the differences can be big because one is looking backwards where the others are looking forward. The following table summarises the changes in balance sheet items through different time periods and model solutions.

Table 13: Comparison of balance sheet between different solutions and observations (1000 €)

	Historical averages	2012 changes	2013 changes	1st model	2nd model
Cash (C_i)	43 421	112 571	-173 285	56 021	74 606
Loans (L_i)	583 867	208 101	-462 876	326 719	375 484
Other Assets (A_i)	327 957	-136 545	329 777	306 327	889 350
Total	955 246	184 127	-306 384	689 067	1 339 440
Liabilities			.		
Savings (Y_i)	340 068	-68 139	203 942	493 561	519 340
Debts (D_i)	517 561	120 687	-478 706	195 505	820 099
Total	857 628	52 548	-274 764	689 066	1 339 439

It can be seen from table above how the two model solutions are fairly close to the historical values, providing more similar optimal solutions. The selected model uses historical averages in the return simulation, which contributes to the solution values to follow the historical values to some degree. The model solutions are fairly close to the observed values in 2012 as well. However, for the year 2013 the model was unable to predict the radical change in the asset returns that caused the decrease in balance sheet items.

It is important to understand why the model did not predict the 2013 values. The 2013 balance sheet item values presented in the table represent the end values and the outcome. When making decisions with unknown outcomes, it is much harder to predict radical changes. Year 2013 data represents such radical changes where all returns decrease significantly. Even if some simulations did predict such extreme values, on average the returns were higher and the optimal solution has increasing balance sheet items. These changes can be taken into account in the model by decreasing the average returns.

The model did succeed in predicting high increases in other assets compared to the rest and the first model the increase in savings compared to debt. However, such situations where in year 2012 the savings are decreasing and debt is increasing when in year 2013 savings are increasing and debt is decreasing are very hard to predict. For any model without a separate economic variable it would be hard to predict changes that are exact opposites in consecutive years. It seems like there is some strategic agenda, or the decision maker may have done some new analysis on future economic conditions.

If for example the decision maker was expecting a growth period in 2012 and thus wanted to increase debt, loans and cash, but after some change in 2013 changed the views of the future to a more negative forecast. Thus the year 2012 and 2013 differ greatly in terms of balance sheet item yearly changes. It is also harder to solve optimal bank balance sheet item yearly changes in such situations.

For better precision that may possibly take into account the radical changes in 2013 the decision maker may adjust the expected average returns and marginal effects in the model. Furthermore, the gamma factor represents the decision makers risk tolerance and can be used to counter risky scenarios whereas the cash constraint can be used to meet regulatory requirements on cash assets. Another option would be to include an economic indicator that forecasts future economic situation. Such indicator could be for example the skirt length indicator. (van Baardwijk & Franses, 2010)

6.2. Analysis

To analyse the performance of the different solutions, one must look at the profits of the different options. For comparison, one should use the actual realisation of balance sheet item returns from year 2013. These returns can be used to estimate what would have been the impact of selecting a model and using the averages as yearly budget. It is not possible to determine the actual profits for a specific model solution, since the data is not publicly available. The profits can be calculated with the marginal effects, the returns from the yearly changes can be calculated with a simple formula, where X_i represents the selected optimal solution and R_i the return on balance sheet item i for year 2013:

$$Profits\ from\ yearly\ changes = \sum_{i=1}^{5} X_i * (R_{ik} - X_i * m_i)$$

It is important to understand that in scenario optimisation there are several time periods, and the yearly averages expressed as optimal solutions for models 1 and 2 are only averages. If in the scenario optimisation the decision maker would realize they are in a difficult node, namely in a bad economic situation, he would adjust the future possible scenarios and the optimal balance sheet items. Looking at the final changes for year 2013 or comparing profits to these values thus gives unfair disadvantage for any model that reports yearly averages. However, this way some comparison on profits can be made across different solutions or decision maker options.

Table 14: Profits from yearly changes in different solutions, with marginal effects (1000 €)

Profits from the yearly changes using returns from 2013 with marginal effects					
	Historical	2012 changes	2013 changes	1st model	2nd model
Cash (Ci)	28.37	-144.63	-1165.78	16.82	-16.46
Loans (Li)	6523.69	3281.66	-11098.30	4678.17	5152.45
Other Assets (Ai)	9579.67	-5091.33	9622.39	9063.06	17296.65
Savings (Yi)	-5618.68	542.87	-2787.75	-9742.48	-10531.92
Debts (Di)	-17203.87	-3072.86	6564.49	-5264.53	-32123.44
Total	-6690.81	-4484.29	1135.05	-1248.96	-20222.72

It can be seen from the table 14 how in the market condition of 2013 and with the corresponding returns, the bank decision making was efficient. Using the averages from model 1 or model 2 would have not been as efficient in terms of pure profit. Another significant aspect is the big difference between the two models. Model 1 outperforms the historical average values, 2012 values and model 2. Model 2 on the other hand performed worst of all the selected solutions in a bad economic situation.

The reasons behind these results lie in the economic situation. It would seem from looking at the returns that there is an economic downturn or a bad period in 2013. Model 2 has the biggest proposed total balance sheet increase, whereas year 2012 has the smallest. The model 2 solution takes most risk and 2012 solution takes least amount of risk in terms of total balance sheet changes.

This can be seen as the reason why model 2 performs worst and the 2012 solution offer better profitability with a more risk averse solution.

The most important observation is the good performance of model 1. The solution suggests increasing the total balance sheet, which can be seen beneficial in the long run as the company would lose less market share and customers. Even when it performs worse in the bad economic condition, it might be argued that it gives better setting for the future. Increasing and decreasing balance sheet totals every second year might yield better short term gains, but in the long run the multi period model aims for increasing the balance sheet totals, leading to bigger market share and profits.

6.2.1. Sensitivity analysis

A sensitivity analysis helps to understand the relationship between the input and output variables. It gives important information on how the output is affected by the input and thus the results can be better understood. Moreover, a sensitivity analysis improves the reliability of the model. (Saltelli, et al., 2000)

Because the model is not linear, a simplified analysis is performed. In this analysis, the inputs are changed to yield different results which are still in a reasonable range. The changes are done one input at a time and the amount of change is selected to show reasonable results. The base case used here is the model 1 setup without drift factor and the marginal effects. The marginal effects equal to the mean marginal effects from the data, as explained in the model specification section. The inputs that are changed in this sensitivity analysis are marginal effects m_i, cash requirements and gamma factor γ.

For the first analysis, the marginal effects are changed. If one considers the output of the 1st model and compares it with other results, it would seem that other assets are over emphasised, cost for acquiring more savings might be higher compared to the marginal effect of debt. In other words, acquiring more other assets might provide less profits as the investment options become fever, when on the other side it seems like it would be more costly to increase savings compared to debt. Using these ideas it is possible to give new values for the marginal effects.

Table 15: Sensitivity analysis with marginal effects increased for other assets and savings, decreased for debt

Marginal effects sensitivity analysis		
	Original m_i	New m_i
Cash	2.80E-08	2.80E-08
Loans	1.22E-08	1.22E-08
Other Assets	1.74E-08	3.74E-08
Savings	2.10E-08	4.10E-08
Debt	1.96E-08	9.60E-09

Using these new marginal effects it is possible to solve the new optimal solution. Note that here the marginal effects for other assets and savings is increased and to control these increased costs, the marginal effect for debt are decreased. This means that the relation between the liabilities savings and debt is changed so that the cost of increasing savings is higher and the cost of increasing borrowing is lower. By not changing the marginal effects too much in one direction the total balance sheet item sum should be close to the original. In this situation with the changed marginal effects, the optimal solution with average yearly changes would be:

Table 16: Average yearly changes with modified marginal effects (1000 €)

	Original m_i	New m_i	Change
Cash	56 021	63 508	7 487
Loans	326 719	442 049	115 330
Other Assets	306 327	168 967	-137 360
Savings	493 561	246 864	-246 697
Debt	195 505	427 660	232 155
Total	1 378 133	1 349 048	-29 085

As can be seen from table 17, the marginal effects directly affect the relationship and size of the proposed changes on balance sheet items. This can be argued to be one of the reasons why there are significant changes in the balance sheet throughout the years. By changing and estimating the marginal effects more precisely, one can obtain better solutions. If the relation between two balance sheet items should be changed, it is possible to increase and decrease the marginal effects which was done in this example where the marginal effect of savings was increased and marginal effect of debt was decreased

For the cash requirement the cause and effect relationship should be quite straightforward. Increasing cash requirements would cause banks to hold more of its assets in cash, which would limit investing in something more profitable. On the other hand it brings more stability as the bank is less likely to meet liquidity problems due to changing economic conditions. Bank regulation, capital and cash requirements were discussed earlier in this study.

For the purposes of this sensitivity analysis, the cash requirement constraint is increased from 0.05228 to 0.15228. This is a significant 191 % increase, but still the cash assets remain the smallest and least profitable unit. To further analyse the costs of cash constraints and bank regulation, one should run separate models and study the costs for the entire economy. The results of this sensitivity analysis can be seen below:

Table 17: Average yearly changes with model 1 and increased cash requirements (1000 €)

	Basic model	New cash requirements	Change
Cash	56 021	102 092	46 071
Loans	326 719	320 009	-6 710
Other Assets	306 327	129 223	-177 104
Savings	493 561	229 082	-264 479
Debt	195 505	322 241	126 736
Total	1 378 133	1 102 647	-275 486

It is possible to see a significant 20% drop in total sum of these balance sheet items when cash constraints are increased. This has a direct negative effect on profitability, but on the other hand the bank is less likely to meet problems with liquidity. The results could be used to further estimate the costs of increasing cash constraints. However, as this is not the emphasis of this research, it will be merely pointed out, but further research could be done with this model.

The final input factor to consider is the gamma factor γ determining the slope of the exponential utility function. For a higher gamma factor, the utility curve is steeper and it results in the decision maker to be more risk averse. Any deviation for lower profits would result in large negative utility. This would lead to the decision maker choosing safer and less volatile portfolios. These would include putting more emphasis on cash assets, other assets and savings. The results for increasing the gamma factor γ from 3.41E-05 to 7.41E-05 can be seen below:

Table 18: The effect of changing the gamma factor on average yearly changes in balance sheet

	Basic model	Increased gamma	Change
Cash	56 021	128 170	72 149
Loans	326 719	189 014	-137 705
Other Assets	306 327	438 074	131 747
Savings	493 561	590 528	96 967
Debt	195 505	164 730	-30 775
Total	1 378 133	1 510 516	132 383

As gamma factor is increased in table 19, more emphasis is put on the safer assets and liabilities. Moreover, the table argues how changing the gamma factor has complex effects on the output. Even when the decision maker is more risk averse, the balance sheet totals are increased when gamma is increased. As the gamma factor is changed, the objective function values are no longer comparable, since the values are on a different scale. Thus the decision maker should aim to choose the correct or most appropriate gamma factor in an early phase of the optimisation process.

All in all the effects of changing the inputs are significant. Changing the marginal effects and cash constraint had a more direct and predictable results, whereas changing the gamma factor had more complex effect on the resulting average balance sheet changes. By changing these inputs, the decision maker can influence the output. By selecting the correct inputs, the output can be very close to the true optimal values in real life.

The benefit of scenario optimization is seen in these results and analysis. One can make adjustments to the model fairly easily and solve the optimal solution. The closer to the real values the inputs are, the closer the output is to a theoretical situation where maximum profits are obtained. Scenario simulation proves to be a fairly efficient and fast way of finding optimal solutions for portfolio optimization problems with the methods shown in this thesis.

7. CONCLUSIONS

The aim of this thesis was to find optimal bank balance sheet structure in a multi period scenario optimisation model. In order to reach these goals, this thesis analysed and reported the public case company data that was presented in chapter 2. The next chapters summarised the most important methods, frameworks and model specifications. The model itself was presented in chapter 5. Chapter 6 reported the results from the model and the analysis on these results, including the sensitivity analysis.

To determine the success of this study the research problems need to be revisited. The research goals were set as:

1. Obtaining, editing and analysing the data from the case company
2. Explaining relevant framework and theory
3. Selecting and formulating a mathematical optimisation model
4. Using the model to solve optimal solution for selected data and time horizon
5. Analysing the results

All of the above mentioned goals were reached within the limits of this study. For better results one should consider using the company's own private data, improving the time series analysis on the data and improve the analysis on the results. These are excellent areas for further research and could each be made into a study of its own. For the purpose of this study, the goals were met successfully.

The data gathering and analysing step was quite straightforward. The data was successfully obtained from the financial statements and proved to be very useful in later parts of the study. Although the historical data was limited to only six years and only 5 balance sheet items were analysed in the end, the quality of the data was sufficient to produce accurate and significant results. Moreover it was possible to predict future returns and balance sheet changes efficiently even with the little amount of data available.

The important findings of this thesis are related to the relevant frameworks, the formulated model, obtained results and company specific analysis. The literature review revealed portfolio optimisation studies in both finance and operations management, but few combined the two or made significant connections. The third chapter summarised the main elements in stochastic optimisation, financial portfolio optimisation and scenario optimisation. Even in such challenging environment as bank portfolio optimisation, the model performed fairly well and the results obtained fell in line with the previous studies and observed outcomes.

Another important finding from the literature review was the role of bank regulation. The analysis suggested that Aktia had sufficient capital levels, but a cash constraint should be included in the model. The model itself was presented in the fifth chapter where it was successfully explained in simple terms. The main output of this chapter was the model in AMPL, presented at the end of the chapter.

In the results the importance of the inputs and the data was highlighted. For example bank regulation or changes in bank risk aversion coefficient would have direct effects on the output. The uncertainty and high importance of the input variables created challenges for the optimisation model, but the results were realistic because of the large number of simulated scenarios. Moreover, with scenario optimisation the inputs could easily be modified and changed to meet the needs of the decision maker or the market situation.

Combining portfolio optimisation with scenario optimisation in the model presented in this thesis proved to be very efficient when compared to other historical values. It can be argued that the 1st model performed very well when comparing to other alternative options. Here are the average yearly changes of the original model and profits with the later observed 2013 returns data:

Table 19: Proposed or observed yearly changes and profits related to these changes (1000 €)

	Balance sheet item yearly proposed or observed changes						Profits from yearly changes using returns from 2013 with marginal effects				
	Historical	2012 changes	2013 changes	1st model	2nd model		Historical	2012 changes	2013 changes	1st model	2nd model
C	43 421	112 571	-173 285	56 021	74 606		28.37	-144.63	-1165.78	16.82	-16.46
L	583 867	208 101	-462 876	326 719	375 484		6523.69	3281.66	-11098.30	4678.17	5152.45
O	327 957	-136 545	329 777	306 327	889 350		9579.67	-5091.33	9622.39	9063.06	17296.65
S	340 068	-68 139	203 942	493 561	519 340		-5618.68	542.87	-2787.75	-9742.48	-10531.92
D	517 561	120 687	-478 706	195 505	820 099		-17203.87	-3072.86	6564.49	-5264.53	-32123.44
Totals	1 812 874	236 675	-581 148	1 378 133	2 678 879		-6690.81	-4484.29	1135.05	-1248.96	-20222.72

The 1st model solution proposes a change where there is a significant increase in the balance sheet assets and relatively small negative effect on the total profits of the company related to these changes. On the other hand the changes are not free, as the return on these increased assets is negative. The balance sheet for the 1st model was significantly larger with assets totalling 9.1% larger than the observed 2013 data. Moreover, since the balance sheet is growing, future profits and prospects are improved. Thus one could even argue that the model solution presented in this thesis is strategically better than the one observed in the in the financial statement of 2013.

One reason for decreasing the balance sheet size might be the maximisation of short term profits in year 2013. Maximising short term profits might be more tempting for the decision maker and avoiding any kind of costs or losses related to these new changes is desirable. The model 1 solution proposed changes that would have cost the company 1.2 million whereas the 2013 changes resulted in 1.1 million profits.

This difference of approximately 2.4 million which occurred between the baseline case and the observed outcome of 2013 can be further analysed. In terms of absolute profits the 2013 was fairly efficient. However, on average over several years, the 9.1% increase in the total balance sheet would bring more profits in the future. It can be argued that this increase in the balance sheet items is an investment. Furthermore, it can be concluded that the increased balance sheet would quickly pay itself back because of the improved future profit prospects.

As explained before, since the returns on assets were higher than the cost of liabilities, increasing the balance sheet steadily should yield higher profits in the long run. The difference between short term and long term profits is vital in decision making. It might be more beneficial for the decision maker to aim for short term profits.

The model presented in this thesis is a multi-period model and optimisation is done over several periods. It does not take into account strategic short term win maximisation but provides a more general and long term solution. Moreover, looking back after observing the outcome of 2013 is the same as looking back from the final node of the decision tree and comparing it to the average solution which includes the entire decision tree.

For the results the general conclusion is clear. The model outperformed many other alternatives and even when the profits were smaller compared to the observed results of 2013 the total balance sheet was bigger. The inputs had a significant effect on the output and the decision maker could possibly reach far better results when using the correct inputs together with more precise data. With more work on the model and better data the model would perform even better.

7.1. Implications

This thesis successfully builds a multi period scenario optimisation model that is efficient and provides realistic results. The model can be implemented directly or with easy modifications to both academic research and company decision making. The model used here can be improved by using more precise inputs as the sensitivity analysis shows, in order to gain more precise results for business purposes. The model can be modified to fit the needs of either different academic interests or different business optimisation problem.

In the academic field, similar techniques have already been applied to for example the forest industry (Kallio, et al., 2012). Other research that can benefit from this study are stochastic optimisation research that use large scale optimisation software to find optimal solutions. The main contributions from this thesis for other academic research are the scenario creation process, optimisation over multiple periods and the use of AMPL optimisation software.

In business life the model could be used in different fields to solve optimal production or budgeting problems. With some modifications, the methods and model specifications could be used for example in product portfolio optimisation or financial portfolio management. The main contribution to decision makers in the field of business is the use of scenario optimisation. With the methods presented here, scenario optimisation could be applied to numerous problems. The further the field deviates from a simple portfolio optimisation problem, the more customisation and modifications need to be made in the model.

The most important thing in scenario optimisation is that the model should generate realistic scenarios that represent possible events from real life. If the scenario creation process is successful, then finding the optimal solution is fairly straightforward. Thus creating random scenarios can be very efficient when dealing with complex large scale problems. As this thesis shows, it is possible to apply scenario optimisation to a bank balance sheet portfolio optimisation problem.

7.2. Improvements in the model for future research

After the optimal solution was found there were findings as to how the model could have been improved or made better. Because of the time limitations as a master's thesis, these improvements are merely pointed out for further research and they work as a way of reporting important learning points that were discovered during the research. The thesis has a clear focus on optimisation and simulation from information management point of view. To improve he financial impact one should consider the improvements discussed in this section.

The most significant improvement in the model would be including equity in the model, such that each balance sheet item would have the returns reinvested in that balance sheet item. This way the balance sheet items and the portfolio would grow according to the simulated returns. This method would better explain the frequent changes in the balance sheet in the historical data. Including equity in the model would mean that the maximisation problem would aim to maximise the expected utility of the increase in equity (or decrease).

Equity would be equal to the increase in balance sheet asset side minus the increase in the balance sheet liabilities side. By introducing equity in the model, it would be possible to further study the relationships between bank regulation, bank equity and decision making. Moreover, this way the model would be more realistic, where instead of always paying out the profits they would be reinvested in the company, and dividends would be paid out to the investors on a yearly basis.

Another important point for future research would be to improve the time series analysis of the model. In the thesis this was done with an elementary drift factor, but further research could implement a more complex model for return simulation based on time series analysis. One option for improving the time series analysis would be to include an economic indicator to the model, such as the hemline index discussed earlier in this paper (van Baardwijk & Franses, 2010). Related to the time series analysis, one point for further study would be to improve the data analysis phase, by either obtaining more specific data or by performing deeper analytics on the data.

Other improvement options include analysing the data for better inputs as discussed in the sensitivity analysis, analysing the impact of bank regulation on the entire economy as discussed under bank regulation development or the optimal risk for the economy with the model presented in this thesis. In addition to these topics, it would be great if studies in other fields related to large scale scenario optimisation would benefit from the methods and model presented in this thesis.

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APPENDICES

Appendix 1: Balance sheet items and returns over historic data period from 2006-2012 (1000€)

Balance sheet	2006	2007	2008	2009	2010	2011	2012	
Assets								
Cash (C_i)	307 907	235 273	506 311	340 960	273 364	475 042	587 613	Inluding: Käteiset varat
Loans (L_i)	3 797 018	4 757 011	5 526 194	6 141 562	6 637 551	7 152 124	7 360 225	Inluding: Saamiset luottolaitoksilta yleisöltä ja julkisyhteisöiltä
Other Assets (A_i)	1 385 455	2 960 529	3 507 568	4 073 317	4 108 238	3 428 897	3 292 352	
Sum	5 490 380	7 952 813	9 540 073	10 555 839	11 019 153	11 056 063	11 240 190	
Liabilities								Inluding: liikkeelle lasketut velkakirjat,
Savings (Y_i)	3 340 385	3 729 991	5 015 277	4 753 586	4 356 327	4 757 179	4 689 040	muut velat luottolaitoksilta, yleisölle ja julkisyhteisöille
Debts (D_i)	1 728 973	2 740 892	3 130 482	4 045 926	4 827 366	4 464 037	4 584 724	Including: Muut rahoitusvelat
Equity (E_i)	249 880	339 009	316 775	466 157	497 290	523 756	657 409	Including: Oma pääoma
Other Liabilities (O_i)	171 142	1 142 921	1 077 539	1 290 170	1 338 170	1 311 091	1 309 017	
Sum	5 490 380	7 952 813	9 540 073	10 555 839	11 019 153	11 056 063	11 240 190	
Income								
Cash (C_i)	5 291	8 065	9 320	3 126	2 485	3 290	1 237	Income from Cash
Loans (L_i)	140 774	208 798	280 123	190 364	152 164	186 132	173 496	Income from Loans
Other Assets (A_i)	41 380	88 421	139 865	123 030	134 868	109 172	107 504	Income from Other assets
Sum	181 306	272 404	386 129	285 576	240 326	262 563	232 296	
Savings (Y_i)	-61 920	-107 152	-171 208	-78 997	-54 411	-63 252	-57 149	Interest paid on Savings
Debts (D_i)	-37 764	-73 128	-105 199	-82 170	-81 692	-107 039	-105 647	Interest paid on Debt
Equity (E_i)	-41 980	-60 683	-5 702	-40 556	-59 675	-37 187	-55 880	Cost of equity (return on equity)
Other Liabilities (O_i)	2 512	-3 246	-8 769	27 839	45 084	36 343	47 779	Other costs
Sum	-97 172	-183 526	-285 176	-133 328	-91 019	-133 948	-115 017	
Returns %								
Cash (C_i)	1.72 %	3.43 %	1.84 %	0.92 %	0.91 %	0.69 %	0.21 %	Calculated by income/balance sheet item value
Loans (L_i)	3.71 %	4.39 %	5.07 %	3.10 %	2.29 %	2.60 %	2.36 %	
Other Assets (A_i)	2.99 %	2.99 %	3.99 %	3.02 %	3.28 %	3.18 %	3.27 %	
Savings (Y_i)	-1.85 %	-2.87 %	-3.41 %	-1.66 %	-1.25 %	-1.33 %	-1.22 %	
Debts (D_i)	-2.18 %	-2.67 %	-3.36 %	-2.03 %	-1.69 %	-2.40 %	-2.30 %	
Equity (E_i)	-16.80 %	-17.90 %	-1.80 %	-8.70 %	-12.00 %	-7.10 %	-8.50 %	

Appendix 2: Other Asset return calculations (1000€)

Other Assets income and costs	2006	2007	2008	2009	2010	2011	2012
Muut korkotuotot	35 241	55 541	96 686	92 086	85 677	73 141	57 563
Osinkotuotot	1 222	1 541	1 395	607	1 105	173	53
Palkkiotuotot netto	40 061	47 346	41 034	46 346	57 030	60 565	65 319
Henkivakuutus netto	0	138 078	91 037	13 991	16 477	22 732	27 304
Rahoitusvarojen ja velkojen nettotuotot	736	2 257	-3 359	826	-5 585	-14 815	2 940
Kiinteistöt	4 880	480	6 010	406	518	266	338
Liiketoiminnan muut tuotot	5 433	3 067	14 958	3 565	7 916	4 361	4 682
Sum of income	87 573	248 310	247 761	157 827	163 138	146 423	158 199
Muut korkokulut	2 512	-3 246	-8 769	27 839	45 084	36 343	47 779
Vakuutuskorvauskulut ja vastuuvelan nettomuutos	0	-113857	-75664	0	0	0	0
Henkilöstö	-41691	-57325	-60605	-79219	-82842	-73 203	-75 352
IT/muut kulut	-26869	-35501	-38419	-44783	-50247	-26 380	-31 419
Negatiivisin liikearvon tuloutus	0	12084	0	140	0	0	0
Poistot	-3634	-5121	-5682	-6884	-7237	-5 914	-7 158
Liiketoiminnan muut kulut	-11752	-12464	-16186	-23413	-18705	-41 238	-40 291
Arvonalentumistappiot	0	0	743	-563	0	0	-1 817
Sum costs	-81434	-215 430	-204 582	-126 883	-113947	-110392	-108 258
Other Assets income calculation:							
All income from assets +	181 306	272 404	386 129	285 576	240 326	262 563	232 296
Cash (Ci) income -	5 291	8 065	9 320	3 126	2 485	3 290	1 237
Loans (Li) income -	140 774	208 798	280 123	190 364	152 164	186 132	173 496
Sum of Other Assets income +	87 573	248 310	247 761	157 827	163 138	146 423	158 199
Sum of Other Assets costs +	-81434	-215430	-204582	-126883	-113947	-110392	-108258
Other Assets Income	41 380	88 421	139 865	123 030	134 868	109 172	107 504

Appendix 3: Historic quarterly returns, logarithms of returns, means and error term matrix **E**

	Quarterly returns (1+yearly return)^(1/4)-1							
	2006	2007	2008	2009	2010	2011	2012	Mean
Cash	0.00427	0.00846	0.00457	0.00228	0.00226	0.00173	0.00053	0.00344
Loans	0.00914	0.01080	0.01244	0.00766	0.00568	0.00644	0.00584	0.00829
Other Assets	0.00738	0.00738	0.00982	0.00747	0.00811	0.00787	0.00807	0.00801
Savings	0.00460	0.00711	0.00843	0.00413	0.00311	0.00331	0.00303	0.00482
Debt	0.00542	0.00660	0.00830	0.00504	0.00420	0.00594	0.00571	0.00589
	Logarithm of quarterly returns							
	2006	2007	2008	2009	2010	2011	2012	
Cash	-5.456	-4.772	-5.388	-6.082	-6.090	-6.361	-7.550	-5.957
Loans	-4.695	-4.528	-4.387	-4.872	-5.170	-5.045	-5.143	-4.834
Other Assets	-4.908	-4.908	-4.623	-4.897	-4.815	-4.845	-4.820	-4.831
Savings	-5.381	-4.947	-4.776	-5.490	-5.774	-5.712	-5.798	-5.411
Debt	-5.218	-5.020	-4.792	-5.291	-5.472	-5.126	-5.165	-5.155
	Error terms							
	2006	2007	2008	2009	2010	2011	2012	Mean
Cash	0.501	1.185	0.569	-0.124	-0.133	-0.404	-1.593	0.007
Loans	0.139	0.306	0.447	-0.037	-0.336	-0.210	-0.309	0.026
Other Assets	-0.077	-0.077	0.208	-0.066	0.016	-0.014	0.011	0.032
Savings	0.030	0.464	0.635	-0.079	-0.363	-0.300	-0.387	0.014
Debt	-0.064	0.135	0.363	-0.136	-0.317	0.029	-0.010	0.021

Appendix 4: Covariance matrix V and Cholesky factor C

	Error Term Covariance Matrix				
	Cash	Loans	Other Assets	Savings	Debt
Cash	0.673	0.188	-0.002	0.246	0.057
Loans	0.188	0.081	0.008	0.105	0.043
Other Asset	-0.002	0.008	0.009	0.013	0.010
Savings	0.246	0.105	0.013	0.142	0.059
Debt	0.057	0.043	0.010	0.059	0.039
	Cholesky factor C				
	0.824	0	0	0	0
	0.231	0.172	0	0	0
	-0.003	0.052	0.078	0	0
	0.302	0.219	0.042	0.061	0
	0.070	0.164	0.032	0.026	0.080

Appendix 5: Optimal solution X_{ik}, 1 out of 2 pages in 1000€

Node	Asset 1	Asset 2	Asset 3	Asset 4	Asset 5
0	12271	103092	118481	176045	57799
1	11679	88520	117016	155003	62212
2	13469	142094	94609	170066	80105
3	10443	100718	82170	138353	54978
4	12983	147099	79446	158270	81259
5	12131	103990	109566	166791	58896
6	10924	107434	46345	113944	50760
7	9878	88981	45043	101506	42396
8	10695	61772	85598	107811	50254
9	9232	25026	108221	110236	32243
10	10202	63058	81706	110341	44624
11	10332	74233	69667	105618	48613
12	9269	48551	74261	95000	37080
13	9405	67818	64764	103213	38774
14	9602	79187	55572	101674	42687
15	9835	82312	48059	94325	45881
16	10714	78899	75291	113495	51410
17	11030	87744	71258	110483	59550
18	11403	109859	49665	111980	58947
19	9836	100146	38867	107585	41264
20	11739	117099	51851	114661	66028
21	10798	65983	84739	109404	52116
22	11276	92804	52408	103752	52736
23	10013	91262	46041	100419	46896
24	10565	87137	55391	94539	58554
25	8826	56440	66709	100065	31909
26	9241	47230	83207	101378	38300
27	9403	79577	53060	103546	38494
28	8806	70622	55099	100678	33849
29	8592	43183	74944	98192	28527
30	9308	70216	61120	104262	36381
31	22855	39302	24383	55044	31496
32	20232	29291	31928	54820	26631
33	17495	32711	26445	58124	18528
34	21793	26301	37148	57156	28086
35	17043	23748	30675	58191	13275
36	27225	33418	23824	56198	28269
37	19814	37175	18084	53790	21283
38	23589	57324	8052	55174	33791
39	22782	43798	19630	56176	30035
40	30345	58470	5815	50491	44139
41	17010	18235	37370	56122	16493
42	22779	40497	24144	58017	29404
43	19577	7793	53312	60211	20472
44	21096	17630	39303	56813	21217
45	19767	42751	15920	54555	23883
46	20056	9461	48544	59146	18914
47	17932	-2728	63509	60645	18068
48	19245	32833	31809	60557	23330
49	18996	19937	40609	56601	22941
50	20964	26353	29735	54795	22257
51	26193	41390	18017	52103	33497
52	18266	22913	34582	55106	20655
53	16859	11699	49302	61912	15948
54	18012	9013	49877	60666	16237
55	25440	36497	20862	50776	32023
56	26337	44964	18778	49824	40254
57	19655	29113	33294	55344	26718
58	19511	36850	15747	55335	16773
59	21403	36950	21357	53867	25843
60	18848	28282	35608	55364	27374
61	17089	11566	44391	58844	14202
62	27153	52582	9356	49988	39103
63	20656	22561	35578	61169	17625
64	29743	72496	-25537	44231	32471
65	22015	38613	27771	54712	33687
66	18722	37986	19008	52572	23144
67	15667	12167	36772	56513	8093
68	17433	47321	8924	56591	17087
69	23649	46717	14530	51172	33723
70	19154	46196	18415	54417	29347
71	17954	14512	38253	56472	14247
72	19646	35669	21671	52387	24599
73	30055	58846	2847	46685	45064
74	17801	35630	19471	53452	19450
75	16730	906	56242	59764	14115
76	27521	37873	24230	51737	37887
77	22251	46093	16475	54724	30097
78	17543	7158	54706	61519	17888
79	24525	42671	18908	55847	30256

Appendix 5: Optimal solution X_{ik}, 2 out of 2 pages

Node	Asset 1	Asset 2	Asset 3	Asset 4	Asset 5
80	25436	32553	29701	57989	29701
81	17389	39366	14459	48589	22625
82	16343	2136	54112	57557	15033
83	20462	34209	21207	52413	23466
84	29368	40129	22624	57907	34214
85	17576	10355	45400	59658	13672
86	18729	27482	28293	54589	19915
87	19326	19012	41155	55301	24192
88	23836	52283	8872	51881	33110
89	24232	47338	14212	52794	32988
90	19897	23186	37970	59136	21916
91	24062	34363	29759	54801	33383
92	22410	35698	27236	58246	27099
93	28692	34329	26211	57964	31268
94	18058	19986	38076	55025	21095
95	26437	47786	11262	54466	31019
96	18179	37950	12191	57490	10831
97	18653	10747	49439	59871	18969
98	22628	-1693	57629	58794	19770
99	22358	34416	23628	58717	21686
100	18445	7253	50147	59793	16052
101	16986	32854	20857	53525	17173
102	20309	33562	26371	55126	25117
103	17513	17677	38829	57054	16965
104	20040	38500	18374	54620	22295
105	19039	17735	39013	54336	21451
106	19235	5593	52934	58158	19604
107	19152	21973	30491	58360	13256
108	17921	16762	41270	57993	17961
109	22069	21556	41918	56956	28586
110	24913	52254	4803	48122	33848
111	33049	56834	-183	45249	44450
112	33360	36838	21788	55330	36655
113	21122	50503	1747	51580	21792
114	23135	43537	15946	55471	27147
115	19295	5931	52884	56284	21826
116	24118	35280	22469	55867	25999
117	24870	41963	20889	52887	34836
118	28867	43363	14510	51582	35158
119	16748	3526	55949	60524	15698

Node	Asset 1	Asset 2	Asset 3	Asset 4	Asset 5
120	21291	47848	14784	52189	31733
121	21633	44787	10096	55979	20538
122	18672	27012	33187	52390	26482
123	17096	8569	43660	53660	15665
124	16357	48128	6715	50073	21127
125	25884	37492	27944	56066	35254
126	17977	9510	48865	57224	19128
127	29602	54319	8549	48514	43956
128	31194	31338	25986	54559	33960
129	22544	61580	-763	49021	34340
130	19065	17301	39126	59474	16018
131	17619	21895	32091	54083	17522
132	22660	41509	16623	50788	30004
133	23635	24363	33391	54248	27141
134	22384	24428	37017	56103	27726
135	19932	28263	31686	55189	24692
136	20243	22105	39181	58982	22548
137	26920	72152	-13817	44122	41133
138	20395	30056	30822	55383	25890
139	16256	22168	36151	61463	13113
140	34819	62908	-5443	46060	46224
141	22967	28045	28314	54474	24852
142	20023	22689	35554	55743	22523
143	20117	4478	56053	58007	22640
144	21130	36262	17683	49884	25191
145	16751	38159	17164	58622	13451
146	22240	26332	32223	55786	25010
147	24889	33306	24985	50691	32489
148	18733	20059	38378	53636	23533
149	20106	36926	18469	56231	19270
150	31654	34266	21153	51757	35316
151	19124	30228	29476	58470	20359
152	17111	12537	43419	59321	13746
153	17441	37024	16154	57910	12709
154	22040	45705	13596	54117	27223
155	25324	41664	13467	48989	31466

Appendix 6: MOSEK solver optimisation model, gradu.mod file

```
#input variables
param gamma = 3.409E-05;                        # risk aversion coefficient
param factor = 0;                               # drift factor, how many % of previous price is carried to next one, default 0
param cashperasset:= 0.05228;                   # cash per assets 2012, forms the cash constrain

#formulating the tree properties
set ASSET := 1..5;                              # decision variables, asset increases
param T := 4;                                   # time horizon
param SC := 5;                                  # scenarios
param s;                                        # stage

#tree structure
set NODES := 0..((SC^(T+1)-1)/(SC-1))-1;        # nodes
set STAGE := 0..T;                              # stages
set node{t in STAGE};                           # nodes in stage t
set next{k in NODES};                           # successor nodes of k
param pre {k in NODES: k >= 0};                 # previous nodes of k
param ke {t in STAGE};                          # ke[t] t = 1..5 = last node of stage t

#returns, budget and covariance matrix
param cov {a in ASSET, b in ASSET};             # original covariance matrix
param C{a in ASSET, b in ASSET};                # cholesky factor
param R{a in ASSET, k in NODES};                # final asset returns


#variables for return calculations
param h := 7;                                   # historic data period length
param origasset{ASSET};                         # initial budget/asset values at 2012
param meanreturn{ASSET};                        # mean asset returns
param histp {a in ASSET, k in 1..h};            # historical prices
param error {a in ASSET, m in 1..h};            # error terms from historical data
param mareff{a in ASSET};                       # decreasing marginal effects of each asset, based on estimate from data

#arbitrary parameters for error terms, simulated distribution values
param solerr; param e;
param ff; param f1; param f2; param f3;
param r1; param r2; param r3; param r4; param r5;

#option randseed 0;                                             # a command to use random values in simulation

var X {a in ASSET, k in NODES: k <= ke[T-1]};                   # the decision variables, default value 30k
var Y {a in ASSET, k in NODES};                                 # the portfolio values at the end of previous time period
                                                                # portfolio is always positive in this problem setting
#utility function
maximize expectedutility:
        sum{t in STAGE: 0 < t < T+1}                            # for all stages with outcomes
                sum{k in node[t]}       (1/(SC^t))*             # for all nodes in that stage, times probability (expected value)
                (-exp(-gamma * (sum{a in ASSET}                 # calculate utility
                (X[a,pre[k]]*(R[a,k]-X[a,pre[k]]*mareff[a])     # from total return of increased assets
                +Y[a,pre[k]]*R[a,k]))));                        # and rest of the portfolio, including previous increases

subject to cashconstraint {k in NODES:k <= ke[T-1]}:            # the cash asset1 is determined as the sum of other assets
        X[1,k] = -X[2,k]-X[3,k]+X[4,k]+X[5,k];

subject to cashperassets {k in NODES:k <= ke[T-1]}:             # the bank needs to hold certain amount of cash
        X[1,k] >= (X[4,k]+X[5,k])*cashperasset;

subject to initialbudget {a in ASSET}:                          # initial balance sheet values
        Y[a,0] = origasset[a];

subject to budget {a in ASSET, k in NODES: 0 < k <= ke[T-1]}:   # previous decisions X[a,k] affect the future
        Y[a,k] = Y[a,pre[k]]+X[a,pre[k]];
```

Appendix 7: MOSEK solver gradu.dat data file

```
#original balance sheet values in 2012
param origasset :=      1 587613        2 7360225       3 3292352       4 4689040       5 4584724;

#historical quarterly returns show in appendix
param histp :   1               2               3               4               5               6               7:=
1 0.004268532   0.008461821     0.004570485     0.002284219     0.002264905     0.001726947     0.000525867
2 0.009142575   0.010797046     0.012438505     0.00766053      0.005682559     0.006443631     0.00584164
3 0.007384658   0.007384457     0.009823111     0.007466922     0.008108024     0.007866395     0.008065068
4 0.004602327   0.007105691     0.008427198     0.004128957     0.003108008     0.003307582     0.003033118
5 0.005416303   0.006604376     0.008297341     0.005039113     0.004204086     0.005941358     0.005711694;

param mareff:=
1 2.8029E-08
2 1.2232E-08
3 1.7388E-08
4 2.0958E-08
5 1.9600E-08
;

let ke[0] := 0;                                         # calculating last nodes of each stage
for {t in 1..T} {let ke[t] := ke[t-1] + SC^t};

let node[0] := 0..0;                                    # calculating nodes in stage t
for {t in 1..T}{let node[t] := ke[t-1]+1..ke[t]};

let ff := 0;                                            # calculating next nodes of node k
for {k in NODES: k <= ke[T-1]} {
  let next[k] := ff+1..ff+SC;
  let ff := ff+SC;
};

for {k in NODES: k <= ke[T-1]} {                        # calculating previous nodes of node k
  let{kk in next[k]} pre[kk] := k;
};

for {a in ASSET} {                                      # calcultaing logarithm of price
 for {k in 1..h} {
  let e := histp[a,k];
  let histp[a,k] := log(e);};};

for {a in ASSET} {                                      # calculating mean of the logarithmic returns
  let meanreturn[a] := sum{k in 1..h} histp[a,k]/h;};

for {a in ASSET} {                                      # calculating error terms in model
 for {k in 1..h} {
  let error[a,k] := histp[a,k]-meanreturn[a];};};

for {a in ASSET} {                                      # calculating covariances on error terms
 for {b in ASSET} {                                     # to be used as error term covariance matrix for cholesky composition
  let cov[a,b] := sum{k in 1..h} (error[a,k]*error[b,k])/h;};};


# compute Cholesky factorization of C (positive definite correlation matrix)
# initially C = covariance matrix
 for {l in ASSET}{
  let f2 := cov[l,l] - sum {ll in ASSET: ll < l} C[l,ll]*C[l,ll];
  let ff := (if f2 > 1e-10 then sqrt(f2) else 1.0e+50);
  let C[l,l] := ff;
  for {lll in ASSET: lll > l}{
     let f2 := sum{ll in ASSET: ll < l} C[lll,ll]*C[l,ll];
     let C[lll,l] := (cov[lll,l]-f2)/ff;
  };
};                                                      # at the end C is a lower triangular matrix and
  let{l in ASSET, ll in ASSET: ll>l} C[l,ll] := 0;      # CC^T = Cholesky factorization of the correlation matrix

   let r1:=Normal(0,1);                                 # generating scenarios, first for the node 0, then for the rest (with the drift factor):
   let r2:=Normal(0,1);                                 # notice that the drift is not used for the first time period
   let r3:=Normal(0,1);                                 # another option would be to use the 2012 returns histp[a,7]
   let r4:=Normal(0,1);
   let r5:=Normal(0,1);

      let R[1,0] := exp(meanreturn[1]+r1*C[1,1]);
      let R[2,0] := exp(meanreturn[2]+r1*C[2,1]+r2*C[2,2]);
      let R[3,0] := exp(meanreturn[3]+r1*C[3,1]+r2*C[3,2]+r3*C[3,3]);
      let R[4,0] := -exp(meanreturn[4]+r1*C[4,1]+r2*C[4,2]+r3*C[4,3]+r4*C[4,4]);
      let R[5,0] := -exp(meanreturn[5]+r1*C[5,1]+r2*C[5,2]+r3*C[5,3]+r4*C[5,4]+r5*C[5,5]);

for{t in STAGE: t<T}{                                   # generating scenarios for the rest (with the drift factor):
 for{k in node[t]}{
  for{kk in next[k]}{

   let r1:=Normal(0,1);
   let r2:=Normal(0,1);
   let r3:=Normal(0,1);
   let r4:=Normal(0,1);
   let r5:=Normal(0,1);

      let R[1,kk] := exp((1-factor)*(meanreturn[1]+r1*C[1,1])+factor*log(R[1,pre[kk]]));
      let R[2,kk] := exp((1-factor)*(meanreturn[2]+r1*C[2,1]+r2*C[2,2])+factor*log(R[2,pre[kk]]));
      let R[3,kk] := exp((1-factor)*(meanreturn[3]+r1*C[3,1]+r2*C[3,2]+r3*C[3,3])+factor*log(R[3,pre[kk]]));
      let R[4,kk] := -exp((1-factor)*(meanreturn[4]+r1*C[4,1]+r2*C[4,2]+r3*C[4,3]+r4*C[4,4])+factor*log(-R[4,pre[kk]]));
      let R[5,kk] := -exp((1-factor)*(meanreturn[5]+r1*C[5,1]+r2*C[5,2]+r3*C[5,3]+r4*C[5,4]+r5*C[5,5])+factor*log(-R[5,pre[kk]]));
  };
 };
};

display R;                                              # used to print out the simulated returns in the output
```

Appendix 8: MOSEK solver gradu.run executive file

```
reset;
model gradu.mod;
data  gradudata.dat;
option solver mosek;        # MOSEK #
param err1 := 0;            # MOSEK #
param err2 := 150;          # MOSEK #
param err3 :=  99;          # MOSEK #
solve > out;

let solerr := (if (solve_result_num=err1 or solve_result_num=err2 or solve_result_num=err3) then 0 else 1);

# The following commands merely print out the decision variables,
# profits in result nodes, and the total average changes in the balance sheet (reported as the results)

#print the decision variables in different nodes and corresponding profits
for{t in 0..T}{
 for{k in node[t]}{
if k = 0 then

# for the first stage there are no profits
{printf"Node %1.0f Stage %1.0f\n Asset1 = %8.0f\n Asset2 = %8.0f\n Asset3 = %8.0f\n Asset4 = %8.0f\n Asset5 =
%8.0f\n\n",k,t,X[1,k],X[2,k],X[3,k],X[4,k],X[5,k]}

else if t = T then
# for the last node there are no decisions to be made
{printf"Node %1.0f Stage %1.0f Profits = %8.0f\n\n",k,t,
(sum{a in ASSET} (X[a,pre[k]]*(R[a,k]-X[a,pre[k]]*mareff[a])+con[a,pre[k]]*R[a,k])); }

else
{printf"Node %1.0f Stage %1.0f\n Asset1 = %8.0f\n Asset2 = %8.0f\n Asset3 = %8.0f\n Asset4 = %8.0f\n Asset5 =
 %8.0f\nProfits = %8.0f\n\n",k,t,X[1,k],X[2,k],X[3,k],X[4,k],X[5,k],

(sum{a in ASSET} (X[a,pre[k]]*(R[a,k]-X[a,pre[k]]*mareff[a])+con[a,pre[k]]*R[a,k])); }
};
};

#print total asset changes per asset
print"Mean total asset change in a year (if T=4):";
for{a in ASSET}
{printf"Asset%8.0f = %8.0f\n",a,sum{t in STAGE} (1/(SC^t))*sum{k in node[t]: k <= ke[T-1]} X[a,k];};
```